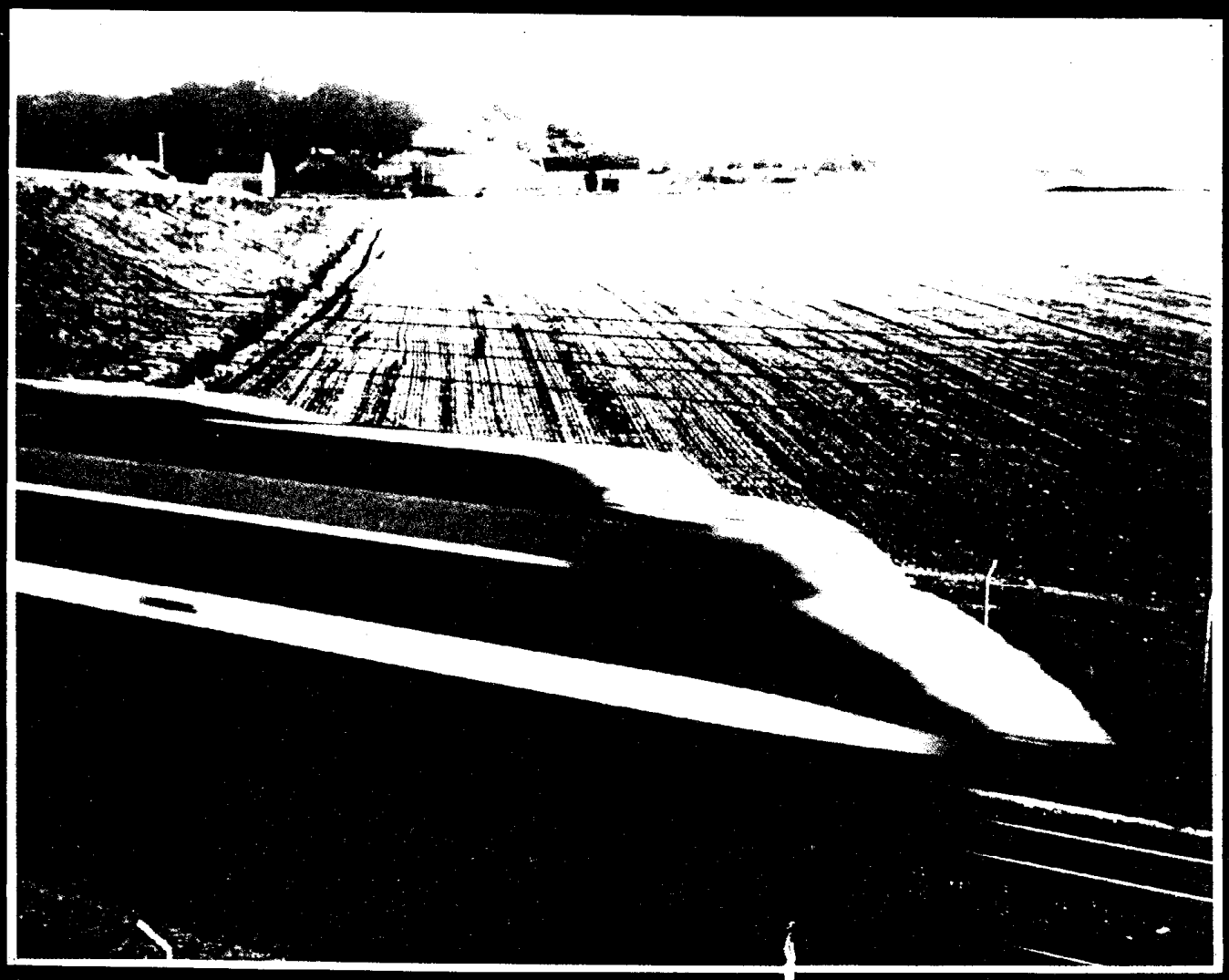
Instructor's Manual to Accompany

Vector Mechanics for Engineers

Fifth Edition

Dynamics

Ferdinand P. Beer
E. Russell Johnston, Jr.



Books
CORTESIA
Marketing Universitário
(0xx11) 3845-3105
(0xx11) 3842-6723
fax (0xx11) 3845-6095

Instructor's Manual to Accompany

Vector Mechanics for Engineers | Dynamics

Fifth Edition

FERDINAND P. BEER
Lehigh University

E. RUSSELL JOHNSTON, Jr.
University of Connecticut

McGraw-Hill Book Company
New York St. Louis San Francisco Auckland Bogotá Caracas
Colorado Springs Hamburg Lisbon London Madrid Mexico
Milan Montreal New Delhi Oklahoma City Panama Paris
San Juan São Paulo Singapore Sydney Tokyo Toronto

TO THE INSTRUCTOR:

As indicated in its preface, VECTOR MECHANICS FOR ENGINEERS: DYNAMICS is designed for a first course in dynamics. New concepts have therefore been presented in simple terms and every step explained in detail. However, because of the large number of optional sections which have been included and the maturity of approach which has been achieved, this text may also be used to teach a course which will challenge the more advanced student.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections, one or several Sample Problems, and a large number of problems to be assigned. To assist the instructor in making up a schedule of assignments that will best fit his classes, the various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Both a minimum and a maximum number of periods have been suggested and the topics which form the standard basic course in Dynamics have been separated from those which are optional. The total number of periods required to teach the standard course varies from 27 to 48, while covering the entire text would require 40 to 67 periods. In most instances, of course, the instructor will want to include some, but not all of the additional material presented in the text. If allowance is made for the time spent for review and quizzes, it is seen that this text is equally suitable for teaching the standard basic Dynamics course in 40 to 45 periods and for teaching a more complete Dynamics course to advanced students. In addition, it should be noted that STATICS and DYNAMICS may be used together to teach a combined 4- or 5-credit-hour course covering all the essential topics in Dynamics, as well as those sections of Statics which are prerequisites to the study of Dynamics (see Tables VI, VII, and VIII.

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. The instructor's attention is called to the fact that, in most cases, problems have been arranged in groups of 4 or more, all problems of the same group being closely related. This means that the instructor will easily find additional problems to amplify a particular point which he may have brought up in discussing a problem assigned for homework. In this new edition of VECTOR MECHANICS FOR ENGINEERS, a group of four problems designed to be solved with a computer has been added to the review problems at the end of each chapter. The solutions of these problems may be found in this manual; they consist of the required programs in both BASIC and FORTRAN versions and of the tabulated numerical results.

To assist in the preparation of homework assignments Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that the answers to all even-numbered problems are given at the end of the text. Because of the large number of problems available in both systems of units, the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be most desirable for a given class. To illustrate this point, a sample lesson schedule is shown in Tables III, IV, and V, together with various alternative lists of assigned homework problems. Half of the problems in each of the four lists suggested in Table III are stated in SI units and half in U.S. customary units. On the other hand, the

four lists suggested in Table IV offer a larger number of problems stated in SI units (75% if all problems are used, and 67% if only the first three or the last three problems of each assignment are used). Instructors who prefer to assign problems in SI units only may use lists A and B in Table V, while those who wish to assign problems in U.S. customary units only should use lists C and D in the same table.

It should be noted that transparencies for overhead projectors for the solutions of selected problems are available to the instructor upon request.

Since the approach used in this text differs in the number of respects from the approach used in other books, the instructor will be well advised to read the preface to VECTOR MECHANICS FOR ENGINEERS, in which the authors have outlined their general philosophy. In addition, the instructor will find in the following pages a description, chapter by chapter, of the more significant features of this text. It is hoped that this material will help the instructor in organizing his course to best fit the needs of his students.

The authors wish to acknowledge the many helpful comments and suggestions offered by the users of the previous editions of VECTOR MECHANICS FOR ENGINEERS and of MECHANICS FOR ENGINEERS. The authors also wish to thank Professor Allen C. Chao of the Department of Civil Engineering of North Carolina State University for checking their solutions and Bruce P. Johnston for assistance in programming those problems designed to be solved with a computer.

Ferdinand P. Beer
E. Russell Johnston, Jr.

DESCRIPTION OF THE MATERIAL CONTAINED IN
"VECTOR MECHANICS FOR ENGINEERS: DYNAMICS", Fifth Edition

11. Kinematics of Particles

In this chapter, the motion of bodies is studied without regard to their size; all bodies are assumed to reduce to single particles. The analysis of the effect of the size of a body and the study of the relative motion of the various particles forming a given body are postponed until Chapter 15. In order to present the simpler topics first, Chapter 11 has been divided into two parts: rectilinear motion of particles, and curvilinear motion of particles.

In Section 11.2, position, velocity and acceleration are defined for a particle in rectilinear motion. They are defined as quantities which may be either positive or negative and students should be warned not to confuse position coordinate and distance traveled, or velocity and speed. The significance of positive and negative acceleration should be stressed. Negative acceleration may indicate a loss in speed in the positive direction or a gain in speed in the negative direction.

As they begin the study of dynamics, many students are under the belief that the motion of a particle must be either uniform or uniformly accelerated. To destroy this misconception, the motion of a particle is first described under very general conditions, assuming a variable acceleration which may depend upon the time, the position, or the velocity of the particle (Sec. 11.3). To facilitate the handling of the initial conditions, definite integrals, rather than indefinite integrals, are used in the integration of the equations of motion.

The special equations relating to uniform and uniformly accelerated motion are derived in Sections 11.4 and 11.5. The students should be warned to check carefully, before using these equations, that the motion under consideration is actually a uniform or a uniformly accelerated motion.

Two important concepts are introduced in Section 11.6: (1) the concept of relative motion, which will be developed further in Sections 11.12 and 15.5, (2) the concept of dependent motions and of degrees of freedom.

The first part of Chapter 11 ends with the presentation of several graphical methods of solution of rectilinear-motion problems (Secs. 11.7 and 11.8). This material is optional and may be omitted. Several problems in which the data are given in graphical form have been included (e.g., Probs. 11.72 - 11.75).

The second part of the chapter begins with the introduction of the vectors defining the position, velocity and acceleration of a particle in curvilinear motion. The derivative of a vector function is defined and introduced at this point (Sec. 11.10). The motion of a particle is first studied in terms of rectangular components (Sec. 11.11); it is shown that in many cases (for example, projectiles) the study of curvilinear motion may be reduced to that of two independent rectilinear motions. The concept of fixed and moving frames of reference is introduced in Section 11.12 and immediately used to treat the relative motion of particles.

Sections 11.13 and 11.14 introduce respectively the use of tangential and normal components, and of radial and transverse components. Both systems of coordinates are first introduced in two dimensions and then extended to include three-dimensional space.

12. Kinetics of Particles: Newton's Second Law

As indicated earlier, this chapter and the following two are concerned only with the kinetics of particles and systems of particles. They neglect the effect of the size of the bodies considered and ignore the rotation of the bodies about their mass center. The effect of size will be taken into account in Chapters 16 through 18, which deal with the kinetics of rigid bodies.

Section 12.2 presents Newton's second law of motion and introduces the concept of a newtonian frame of reference. In Section 12.3 the concept of linear momentum of a particle is introduced, and Newton's second law is expressed in its alternate form, which states that the resultant of the forces acting on a particle is equal to the rate of change of the linear momentum of the particle. Section 12.4 reviews the two systems of units used in this text and previously discussed in Sec. 1.3, namely the SI metric units and the U.S. customary units. This section also emphasizes the difference between an absolute and a gravitational system of units.

A number of problems with two degrees of freedom have been included (Problems 12.26 through 12.33), some of which require a careful analysis of the accelerations involved (see Sample Problem 12.4).

Section 12.5 applies Newton's second law to the study of the motion of a particle in terms of rectangular components and tangential and normal components. In Section 12.6, dynamic equilibrium is presented as an alternate way of expressing Newton's second law of motion, although it will not be used in any of the Sample Problems in this text. Note that the term inertia vector is used in preference to inertia force or reversed effective force to avoid any possible confusion with actual forces.

In Section 12.7 the concept of angular momentum of a particle is introduced, and Newton's second law is used to show that the sum of the moments about a point O of the forces acting on a particle is equal to the rate of change of the angular momentum of the particle about O. Section 12.8 analyzes the motion of a particle in terms of radial and transverse components, and Section 12.9 considers the particular case of the motion of a particle under a central force. The early introduction of the concept of angular momentum greatly facilitates the discussion of this motion. Section 12.10 presents Newton's law of gravitation and its application to the study of the motion of earth satellites.

Sections 12.11 through 12.13 are optional. Section 12.11 derives the differential equation of the trajectory of a particle under a central force, while Section 12.12 discusses the trajectories of satellites and other space vehicles under the gravitational attraction of the earth. While the general equation of orbital motion is derived (Eq. 12.39), its application is restricted to launchings in which the velocity at burnout is parallel to the surface of the earth. (Oblique launchings are considered in Section 13.9.) The periodic time is found directly from the fundamental definition of areal velocity rather than by formulas requiring a previous knowledge of the properties of conic sections. The instructor will note that he may omit Sections 12.11 through 12.13 and yet assign a number of interesting space mechanics problems to his students after they have reached Section 13.9.

13. Kinetics of Particles: Energy and Momentum Methods

After a brief introduction designed to give to the students some motivation for the study of this chapter, the concept of work of a force is introduced in Section 13.2. Attention is called to the fact that the term work is always used in connection with a well-defined force. Three examples considered are the work of a weight (i.e., the work of the force exerted by the earth on a given body), the work of the force exerted by a spring on a given body, and the work of a gravitational force. Confusing statements such as the work done on a body, or the work done on a spring are avoided.

The concept of kinetic energy is introduced in Section 13.3 and the principle of work and energy is derived by integration of Newton's equation of motion. In applying the principle of work and energy, the students should be encouraged to draw separate sketches representing the initial and final positions of the body (Sec. 13.4). Section 13.5 introduces the concepts of power and efficiency.

Sections 13.6 through 13.8 are devoted to the concepts of conservative forces and potential energy, and to the principle of conservation of energy. Potential energy should always be associated with a given conservative force acting on a body. By avoiding statements such as "the energy contained in a spring" a clearer presentation of the subject is obtained, which will not conflict with the more advanced concepts that the students may encounter in later courses. In applying the principle of conservation of energy, the students should again be encouraged to draw separate sketches representing the initial and final positions of the body considered.

In Section 13.9, the principles of conservation of energy and of conservation of angular momentum are applied jointly to the solution of problems involving conservative central forces. A large number of problems of this type, dealing with the motion of satellites and other space vehicles, are available for homework assignment. As noted earlier, these problems (except the last two, indicated by asterisks) may be solved even if Sections 12.11 through 12.13 have been omitted.

The second part of Chapter 13 is devoted to the principle of impulse and momentum and to its application to the study of the motion of a particle. Section 13.10 introduces the concept of linear impulse and derives the principle of impulse and momentum from Newton's second law. The instructor should emphasize the fact that impulses and momenta are vector quantities. He should encourage his students to draw three separate sketches when applying the principle of impulse and momentum and show clearly the vectors representing respectively the initial momentum, the impulses, and the final momentum. It is only after the concept of impulsive force has been presented that the students will begin to appreciate the effectiveness of the method of impulse and momentum (Section 13.11).

Direct central impact and oblique central impact are studied in Sections 13.12 through 13.14. Note that the coefficient of restitution is defined as the ratio of the impulses during the period of restitution and the period of deformation. This more basic approach will make it possible in Section 17.12 to extend to the case of eccentric impact the results obtained here for central impact. Emphasis should be placed on the fact that, except for perfectly elastic impact, energy is not conserved. Note that the discussion of oblique central impact in Section 13.14 has been expanded to cover the case when one or both of the colliding bodies is constrained in its motion.

Section 13.15 shows how to select from the three fundamental methods studied in Chapters 12 and 13 the one best suited for the solution of a given problem. It also shows how several methods may be combined to solve a given problem. Note that problems have been included (Problems 13.162 through 13.175), which require the use of at least two of the three fundamental methods of solution.

14. Systems of Particles

Chapter 14 is devoted to the study of the motion of systems of particles. Sections 14.2 and 14.3 derive the fundamental equations (14.10) and (14.11) relating, respectively, the resultant and the moment resultant of the external forces to the rate of change of the linear and angular momentum of a system of particles. Sections 14.4 and 14.5 are devoted respectively to the motion of the mass center of a system and to the motion of the system about its mass center. Section 14.6 discusses the conditions under which the linear momentum and the angular momentum of a system of particles are conserved. Sections 14.7 and 14.8 deal with the application of the work-energy principle to a system of particles and Section 14.9 with the application of the impulse-momentum principle.

A number of challenging problems have been provided to illustrate the application of the principles discussed in Sections 14.2 through 14.9. The first group of problems (Probs. 14.1 through 14.25) deal chiefly with the conservation of the linear momentum of a system of particles and with the motion of the mass center of the system, while the second group of problems (Probs. 14.26 through 14.49) involve the combined use of the principles of conservation of energy, linear momentum, and angular momentum. However, the main purpose of these sections is to lay the proper foundation for the later study of the kinetics of rigid bodies (Chaps. 16 through 18). Depending upon the preparation and interest of the students, a greater or lesser emphasis may be placed on this part of the course. It is essential, however, that the significance of Equations (14.16) and (14.23) be pointed out to the students, in view of the role played by these equations in the study of the motion of rigid bodies.

The instructor should note the distinction made in Section 14.2 between <u>equivalent</u> systems of forces (i.e., systems of forces which have the same effect) and <u>equipollent</u> systems of forces (i.e., systems of forces which have the same resultant and the same moment resultant). The equivalence of two systems of forces has been indicated in diagrams by brown equal signs, and their equipollence by gray equal signs.

Sections 14.10 through 14.12 are optional. They are devoted to the study of variable systems of particles, with applications to the determination of the forces exerted by deflected streams and the thrust of propellers, jet engines, and rockets. Since Newton's second law $\underline{F} = m\underline{a}$ was stated for a particle with a constant mass and does not apply, in general, to a system with a variable mass (see footnote, page 682), the derivations given in Section 14.11 for a steady stream of particles and in Section 14.12 for a system gaining or losing mass are based on the consideration of an auxiliary system consisting of unchanging particles. This approach should give the students a basic understanding of the subject and lead them unconfused to more advanced courses in mechanics of fluids.

15. Kinematics of Rigid Bodies

With this chapter we start the study of the dynamics of rigid bodies. After an introduction in which the fundamental types of plane motion are defined (Section 15.1), the relations defining the velocity and the acceleration of any given particle of a rigid body are established for two particular cases: translation (Sec. 15.2) and rotation about a fixed axis (Secs. 15.3 and 15.4).

In Section 15.5 it is shown that the most general plane motion may always be considered as the sum of a translation and a rotation. This property is established by considering the relative motion of two particles of the rigid body and is immediately applied in Section 15.6 to the determination of velocities.

Section 15.7 introduces the concept of the instantaneous center of rotation. The instructor should stress the fact that, while the instantaneous-center method simplifies the solution of many problems involving velocities, it cannot be used to determine accelerations.

In Section 15.8 the concept of relative motion is used again, this time to determine accelerations in plane motion. Students should be warned against any unwarranted assumptions concerning the direction of unknown accelerations. Section 15.9 is optional. It presents an analytical method for the determination of velocities and accelerations, based on the use of a parameter.

Section 15.10 discusses the rate of change of a vector with respect to a rotating frame, and Section 15.11 applies the results obtained to the determination of Coriolis acceleration in plane motion. To make the concept of Coriolis acceleration as intuitive as possible, an example involving the motion of a collar on a rotating rod is given on page 746. It should be kept in mind, however, that Coriolis acceleration does not depend upon the existence of an actual slab or rod on which the particle moves. It will appear whenever <u>rotating axes</u> are used. There lies the fundamental difference between the approach of Section 15.11 and that of Sections 11.12 and 15.8 where moving axes of <u>fixed direction</u> were used.

The remaining sections of Chapter 15 are devoted to the kinematics of rigid bodies in three dimensions and are optional. However, they should be included in a course covering the kinetics of rigid bodies in three dimensions (Chap. 18) and then be taught either at this point or immediately before Chapter 18.

In Section 15.12 the motion of a rigid body about a fixed point is presented. The students should note that, while the finite rotations of a rigid body have magnitude and direction, they are not vectors (see also Sec. 2.3 and Fig. 2.3); on the other hand, both angular velocity and angular acceleration are vector quantities. In Section 15.13, the general motion of a rigid body is analyzed. It should be emphasized that in this section the moving frame of reference is of fixed orientation and does not rotate.

Sections 15.14 and 15.15 consider the motion of particles and rigid bodies with respect to rotating frames of reference and extend the concept of Coriolis acceleration to three-dimensional motion.

16. Plane Motion of Rigid Bodies: Forces and Accelerations

This chapter is devoted to the plane motion of rigid bodies which consist of plane slabs or which are symmetrical with respect to the reference plane. Cases involving the plane motion of nonsymmetrical bodies and, more generally, the motion of rigid bodies in three dimensions are considered in Chapter 18. If the determination of mass moments of inertia has not been covered in the previous statics course, the instructor should include material from Sections 9.11 through 9.15 of Appendix B (or from the second part of Chap. 9) at this point.

In Section 16.2 the fundamental relations derived in Chapter 14 for a system of particles are used to show that the external forces acting on a rigid body are equipollent to the vector $m\bar{a}$ attached at the mass center G of the body and the couple of moment $\bar{H}_G$. This result, which is illustrated in Fig. 16.3, is valid in the most general case of motion of a rigid body (three-dimensional as well as plane motion).

It is shown in Section 16.3 that, in the case of the plane motion of a slab or symmetrical body, the angular momentum H_G reduces to $\bar{I}\omega$ and its rate of change to $\bar{I}\alpha$. Section 16.4 is devoted to D'Alembert's principle. It is shown that the external forces acting on a rigid body are actually equivalent to the effective forces represented by the vector $m\bar{a}$ and the couple $\bar{I}\alpha$. As noted in Section 16.5, this result is obtained independently of the principle of transmissibility (Sec. 3.18) and may be used to derive this principle from the other axioms of mechanics.

At this point the students will have reached the climax of the study of rigid-body motion in two dimensions. Indeed they may solve any problem by drawing two sketches - one showing the external forces, the other the vector $m\bar{a}$ and the couple $\bar{I}\alpha$ - and then expressing that the two systems of vectors shown are equivalent. To avoid drawing two separate sketches, the method of dynamic equilibrium may be used, with a single sketch showing the external forces, the inertia vector - $m\bar{a}$ and the inertia couple - $\bar{I}\alpha$ (Sec. 16.6). However, to facilitate the transition to the study of three-dimensional motion (Chap. 18), the two-sketch method showing separately the external forces and the effective forces will be used in all sample problems.

The various types of plane-motion problems have been grouped according to their kinematic characteristics. The translation, the centroidal rotation, and the plane motion consisting of a translation and of an unrelated centroidal rotation are considered first, since they are the simplest ones to analyze. They are followed by plane motions with various kinematic constraints: non-centroidal rotation, rolling motion, and other types of plane motion. Problems involving systems of rigid bodies have been included at the end of this chapter, with either one degree of freedom (Problems 16.144 through 16.147) or two degrees of freedom (Problems 16.148 through 16.155). The instructor should stress the fact that, in spite of the different kinematic characteristics of these various motions, the approach to the kinetics of the motion is consistently the same: all problems are solved by drawing two sketches - one showing the external forces, the other the vector $m\bar{a}$ and the couple $\bar{I}\alpha$ - and then expressing that the two systems of vectors shown are equivalent.

Since the approach used in this text differs from others by the emphasis placed on the direct application of D'Alembert's principle, rather than on specialized formulas, it might be appropriate at this point to summarize the advantages derived from this approach.

(1) A single method is used, which applies to all cases of plane motion, regardless of their kinematic characteristics, and may be used safely under any conditions. This is in contrast with the equation $\Sigma M = I\alpha$ which is limited in its applications, as pointed out in Problem 16.105.

(2) By stressing the use of the free-body diagram, this method provides a better understanding of the kinetics of the motion. There will be little danger, for example, in the solution of a problem of non-centroidal rotation, that the students forget the effect of the acceleration of the mass center on the reaction at the fixed point, a mistake which occurs frequently when the specialized formula $\Sigma M_O = I_O\alpha$ is used.

(3) The method used divides the solution of a problem into two main parts, one in which the kinematic and kinetic characteristics of the problem are considered (separately if necessary), and the other in which the methods of statics are used. In this way the techniques of each separate field may be used most efficiently. For example, moment equations may be written to eliminate unwanted reactions, just as it was done in statics; this may be done independently of the kinetic characteristics of the problem.

(4) By resolving every plane motion (even a non-centroidal rotation) into a translation and a centroidal rotation, a unified approach is obtained, which will also be used in Chapter 17 with the method of work and energy and with the method of impulse and momentum, and which will be extended in Chapter 18 to the study of the three-dimensional motion of a rigid body. This approach is a basic one, which may be applied effectively throughout the study of mechanics, in advanced courses as well as in elementary ones.

17. Plane Motion of Rigid Bodies: Energy and Momentum Methods

The first portion of the chapter extends the method of work and energy, already used in Chapter 13, to the study of the plane motion of rigid bodies. The expressions for the work of a couple and for the kinetic energy of a rigid body are derived in Secs. 17.3 and 17.4. Using the results obtained in Sec. 14.7, the kinetic energy of a rigid body is separated into a translational part and a rotational part (about the mass center). The authors believe that, while it may lead to slightly longer solutions, this method is more fundamental and should be used in preference to special formulas. Indeed, it follows the basic idea of resolving every plane motion into a translation and a centroidal rotation.

It is shown in Section 17.5 that the method of work and energy is especially effective in the case of systems of rigid bodies connected by pins, inextensible cords, and meshed gears. In Section 17.6 the principle of conservation of energy is used to analyze the plane motion of rigid bodies.

In the second part of Chapter 17, the method of impulse and momentum is extended to the study of the motion of rigid bodies. The approach used is different from that of most elementary textbooks. Ready-to-use formulas are avoided; instead, the students are taught to express the general principle of impulse and momentum by means of free-body diagrams and to write the equations most appropriate to the solution of the problem considered.

The results obtained in Section 14.9 for a system of particles are directly applicable to the system of particles forming a rigid body and may be used to analyze the plane motion of rigid bodies. It is shown in Section 17.8 that the momenta of the various particles forming a rigid body reduce to a vector $m\bar{v}$ and a couple $\bar{I}\omega$ in the most general case of plane motion.

While the principle of conservation of angular momentum is discussed in Section 17.10 because of its physical and historical significance, it is not actually used in the solution of problems. To solve any problem, regardless of the type of motion, and whether it involves constant forces of finite magnitude applied for a finite time, or impulsive forces applied for a very short time interval, the students are told to draw three separate sketches showing respectively the initial momenta, the impulses of the external forces, and the final momenta. The momenta of a rigid body are represented in the most general case by a momentum vector $m\bar{v}$ attached at the mass center and a momentum couple $\bar{I}\omega$. If the students consider then the components of the vectors involved, they obtain relations between linear impulses and linear momenta. If they consider the moments of the same vectors, they obtain angular impulses and angular momenta. If, by equating moments about a point such as a pivot, they obtain an equation which does not involve any of the external forces, they will have automatically established conservation of angular momentum about that point.

The advantages derived from this approach may be summarized as follows:

(1) The students learn only one method of solution, a method based directly on a fundamental principle, a method which may be used safely under any conditions. This is in contrast with the equation $\Sigma Mt = I(\omega_2-\omega_1)$, which is limited in its applications (see Problem 17.66).

(2) The method stresses the use of free-body diagrams and thus provides a better understanding of the kinetics of the motion. It is unlikely, for example, that the students will forget an impulsive reaction at a fixed support.

(3) The students use the basic tools they learned in Statics: reduction of a system of vectors to a vector and a couple, equations relating the components or the moments of these vectors.

(4) Again, the same unified approach is used: every plane motion is resolved into a translation and a centroidal rotation. And in Chapter 18 this approach will be extended to the solution of problems involving the three-dimensional motion of rigid bodies.

Some teachers may fear that the inclusion of momentum vectors and momentum couples in the same diagrams may lead to confusion. This will not be the case,

(d) Free, damped vibrations of a particle (Sec. 19.8), and

(e) Forced, damped vibrations of a particle (Sec. 19.9). These two sections take into account the effect of friction and thus provide a more rigorous analysis of the vibrations of a particle. They are not recommended, however, to students who do not possess a basic knowledge of elementary differential equations.

In Section 19.10 the electrical analogue for a vibrating mechanical system is discussed; this section is optional and should not be assigned unless Sections 19.8 and 19.9 have also been assigned.

TABLE I. LIST OF TOPICS COVERED IN VECTOR MECHANICS FOR ENGINEERS: DYNAMICS

Sections	Topics	Suggested Number of Periods: Standard Course	Suggested Number of Periods: Additional Topics
11. KINEMATICS OF PARTICLES			
11.1-3	Rectilinear Motion	1	
11.4-6	Uniformly Accelerated Motion	1	
*11.7-8	Graphical Methods		1
11.9-12	Curvilinear Motion (Rect. Comp.)	1 - 2	
11.13	Tangential & Normal Components	0.5 - 1	
11.14	Radial & Transverse Components	0.5 - 1	
12. KINETICS OF PARTICLES: NEWTON'S SECOND LAW			
12.1-6	Equations of Motion	1 - 2	
12.7-10	Angular Momentum, Central Forces	1 - 2	
*12.11-13	Space Mechanics		1 - 2
13. KINETICS OF PARTICLES: ENERGY AND MOMENTUM METHODS			
13.1-5	Work and Kinetic Energy, Power	1 - 2	
13.6-9	Conservation of Energy, Central Forces	1 - 2	
13.10-11	Impulse and Momentum, Impulsive Motion	1 - 2	
13.12-15	Impact, Mixed Problems	1 - 2	
14. SYSTEMS OF PARTICLES			
14.1-6	Motion of Systems of Particles	0.5 - 1	
14.7-9	Energy and Momentum Methods for Systems of Particles	0.5 - 1	
*14.10-12	Variable Systems of Particles		1 - 2
15. KINEMATICS OF RIGID BODIES			
15.1-4	Translation, Rotation About Fixed Axis	1 - 1.5	
15.5-6	Velocities in Plane Motion	1 - 1.5	
15.7	Instantaneous Center	1	
15.8-*9	Accelerations in Plane Motion	1 - 2	0.5
15.10-11	Coriolis Acceleration in Plane Motion	1 - 2	
*15.12-13	Motion About Fixed Point, General Motion		1 - 2
*15.14-15	Coriolis Acceleration in Three Dimensions		1 - 2
16. PLANE MOTION OF RIGID BODIES: FORCES & ACCELERATIONS			
16.1-7	Plane Motion of Rigid Bodies	2 - 3	
16.8	Constrained Plane Motion	2 - 4	
17. PLANE MOTION OF RIGID BODIES: ENERGY & MOMENTUM METHODS			
17.1-7	Work and Energy	1 - 2	
17.8-10	Impulse and Momentum, Conservation of Angular Momentum	1 - 2	
17.11-12	Impulsive Motion, Eccentric Impact	1 - 2	
18. KINETICS OF RIGID BODIES IN THREE DIMENSIONS			
*18.1-4	Momentum and Energy in Three Dimensions		2
*18.5-8	Equations of Motion in Three Dimensions		1 - 2
*18.9-11	Gyroscopes, Motion Under No Force		2
19. MECHANICAL VIBRATIONS			
19.1-3	Vibrations of Particles	1 - 2	
*19.4	Exact Solution of Simple Pendulum		0.5
19.5	Vibrations of Rigid Bodies	1 - 2	
19.6	Energy Methods	1 - 1.5	
19.7	Forced Vibrations	1 - 1.5	
*19.8-9	Damped Vibrations		1 - 2
*19.10	Electrical Analogues		1
	Total Number of Periods	27 - 48	13 - 19

however, if the students are instructed to write separate equations involving either components or moments, as they did in Statics. The first equations will contain linear impulses and linear momenta expressed in N·s or lb·s, and the latter angular impulses and angular momenta expressed in N·m·s or lb·ft·s.

Section 17.12 is devoted to the eccentric impact of two rigid bodies, a topic seldom included in an elementary text. No special difficulty will be encountered, however, if separate sketches are used as indicated above.

18. Kinetics of Rigid Bodies in Three Dimensions

In this chapter the restrictions imposed in preceding chapters (e.g., plane motion, symmetrical bodies) are lifted and the student proceeds to the analysis of more general (and more difficult) problems, such as the rotation of nonsymmetrical bodies about fixed axes and the motion of gyroscopes.

In Section 18.1, the general result obtained in Section 16.2 is recalled, namely, that the external forces acting on a rigid body are equipollent to the vector $m\bar{a}$ attached at the mass center G of the body and the couple of moment $\dot{H}_G$. It is also pointed out that the main feature of the impulse-momentum method, namely, the reduction of the momenta of the particles of a rigid body to a linear momentum vector $m\bar{v}$ attached at G and an angular momentum H_G, remains valid, and that the work-energy principle and the principle of conservation of energy still apply in the case of the motion of a rigid body in three dimensions. The difficulties encountered in the study of the three-dimensional motion of a rigid body are related to the determination of the angular momentum H_G, of its rate of change $\dot{H}_G$, and of the kinetic energy of the body.

The determination of the angular momentum H_G of a rigid body from its angular velocity ω is discussed in Section 18.2. Since this requires the use of mass products of inertia, as well as of mass moments of inertia, the instructor should cover Sections 9.16 and 9.17 from Appendix B (or from the second part of Chapter 9) if this material has not been included in the previous statics course.

Section 18.3 is devoted to the application of the impulse-momentum principle to the three-dimensional motion of a rigid body, and Section 18.4 to the determination of its kinetic energy.

In Sections 18.5 and 18.6, the rate of change of the angular momentum H_G is computed and the equations of motion for a rigid body in three dimensions are derived. D'Alembert's principle is extended to the case of three-dimensional motion by showing that the external forces are actually equivalent to the effective forces represented by the vector $m\bar{a}$ and the couple $\dot{H}_G$. Sections 18.7 and 18.8 are devoted, respectively, to the particular cases of the motion of a rigid body about a fixed point and of the rotation of a rigid body about a fixed axis, with applications to the balancing of rotating shafts.

While Euler's equations of motion have been derived on page 907, it should be noted that the more fundamental vector relations represented by Equations (18.22), (18.23), and (18.28) are used in the actual solution of problems.

The remaining portion of this Chapter (Secs. 18.9 through 18.11) is designed for advanced students and, in general, should be omitted for ordinary classes. In Sections 18.9 and 18.10 the motion of a gyroscope is considered. At this point Eulerian angles are introduced. It should be carefully noted that the rotating system of axes Oxyz is attached to the inner gimbal; these axes are principal axes of inertia and they follow the precession and nutation of the gyroscope; they do not however spin with the gyroscope. The special case of steady precession is considered in Section 18.10. Several problems dealing with the steady precession of a top are included and, in one of them (Prob. 18.84), it is shown that the formula usually given in introductory texts is only approximate. Two problems dealing with the general motion of a top have also been included (Probs. 18.108 and 18.109).

The motion of an axisymmetrical body under no force (Sec. 18.11) introduces the student to one of the most interesting aspects of classical dynamics - an aspect which has gained widespread attention in recent years due to the interest in space vehicles and artificial satellites. In this connection, it should be pointed out that the instructor may cover the Poinsot theory of the motion of a nonsymmetrical body under no force by assigning three problems (Probs. 18.114 through 18.116). An additional problem (Prob. 18.117) relates to the stability of the rotation of a nonsymmetrical body about a principal axis.

19. Mechanical Vibrations

This chapter provides an introduction to the study of mechanical vibrations. While only one-degree-of-freedom systems are included, all the basic principles are presented. The various topics covered are as follows:

(a) Free, undamped vibrations of a particle (Sec. 19.2). The differential equation characterizing simple harmonic motion is derived and all basic terms, such as period, frequency, and amplitude, are defined. Both the analytical and the geometrical methods of solution are described. It is shown in Section 19.3 that the motion of a simple pendulum may be approximated by a simple harmonic motion. Section 19.4, which is optional, shows how an exact solution may be obtained for the period of oscillations of a simple pendulum.

(b) Free, undamped vibrations of a rigid body. The principle of equivalence of the systems of applied and effective forces is first used to determine the frequency and the period of oscillations of a rigid body (Sec. 19.5). Note that the same positive sense is assumed for the angular acceleration and displacement; this results in an apparently unrealistic assumption for the sense of the vector $m\bar{a}$ and the couple $\bar{I}\alpha$. The principle of conservation of energy is then used to solve the same type of problems (Sec. 19.6).

(c) Forced, undamped vibrations of a particle (Sec. 19.7). This section introduces the students to the concepts of natural and forced frequencies, transient and steady-state vibrations, and resonance. While all students will be able to understand this section, those with a knowledge of elementary differential equations will derive a greater benefit from it since it provides a direct application of the solution of linear nonhomogeneous equations with constant coefficients.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS

Problem Number* SI Units	U.S. Units	Problem Description
		CHAPTER 11: KINEMATICS OF PARTICLES
		RECTILINEAR MOTION OF PARTICLES
		Analyze motion of a particle, given
11.1,2	11.3,4	position = f(t)
11.5		
11.7,6		also find total distance traveled
11.9,10	11.11,8	acceleration = f(t)
11.13,12	11.15,14	acceleration = f(x)
11.17,16		
11.21	11.19,18	acceleration = f(v)
11.23,22	11.20	
11.24	11.25	velocity = f(x)
11.26	11.27	acceleration $\propto 1/r^2$
11.29	11.28	Simple harmonic motion
		Uniformly accelerated motion
11.31,30	11.33,32	motion of one particle
11.35,34		
11.39,36	11.37,38	motion of two particles
		Relative motion of particles
		systems with one degree of freedom and
11.45,40	11.42	constant velocities
11.41,44	11.43	constant accelerations
11.47,46	11.49,48	systems with two degrees of freedom
11.51,*50		
		Graphical methods
11.55,54	11.53.52	analyze motion using a given motion curve and initial conditions
11.57,56	11.59,58	motion of one particle
11.61,60		
11.62	11.64,63	motion of two particles
11.67,66	11.65	
11.68	11.69	problems involving average velocity
11.71	11.70	and rate of change of acceleration
11.73,74	11.72	problems solved by numerical integration
	11.75	problem solved by use of v-x curve
11.77,78	11.79,76	problems solved by moment-area method
		CURVILINEAR MOTION OF PARTICLES
11.83,82	11.81,80	Analyze motion of a particle given
11.84	11.85	x and y as functions of t
11.87	11.86	position vector in two dimensions
		position vector in three dimensions
		Projectiles
11.91,88	11.89.90	with horizontal initial velocity
11.95,94	11.93	with initial velocity of given magnitude and direction
11.97,96	11.99,92	given direction, find magnitude of initial velocity
	11.98	
11.103,*102	*11.101,100	given magnitude, find direction of initial velocity (solution of
11.105,104		quadratic equation required)
		Relative motion of particles with
11.107,108	11.109,106	constant velocities
	11.111,110	velocities and accelerations
11.113,112		given relative acceleration
	11.115,114	constant accelerations
11.117,116	11.119,118	two given observations of relative motion

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		Normal and tangential components
11.121	11.123,120 11.122	zero tangential acceleration
11.125,124 11.129,128	11.127,126	constant tangential acceleration find relative $\underline{v}$ and $\underline{a}$ of two particles
11.131,132	11.133,130	find radii of curvature of trajectory
*11.135	*11.134	find radius curvature of a curve in three dimensions
11.139,138	11.137,136	problems involving earth satellite in circular orbit
		Radial and transverse components
11.141,142	11.143,140	given r and θ as functions of time
11.145	11.144	
11.147,146	11.149,148	find relations among v, a, $\dot{\theta}$, and $\ddot{\theta}$
11.150	11.151	find approximate $\underline{v}$ from two observations of r and θ
		motion of particle along a spiral
11.153,152		find velocity
	11.155,156	find acceleration
11.156	11.157	motion of particle along a parabola or a circle
11.159,158	11.161,160	motion of particle in three dimensions, cylindrical coordinates
11.162	11.163	
*11.164	*11.165	find direction of binormal
11.167,166 11.171,170	11.169,168	Review problems
11.173,172 11.177,176	11.175,174	
11.C1,C2	11.C3,C4	Computer problems
		CHAPTER 12: KINETICS OF PARTICLES: NEWTON'S SECOND LAW
12.2 12.3,4	12.1	Definition of mass and weight
		Rectilinear motion
12.5,6	12.7,8	single particle
12.11,12	12.9,10	several connected bodies
12.13,14 12.15	12.17,16	
12.21,20	12.19,18	two bodies with impending relative motion
12.23,24	12.25,22	problems with variable acceleration
12.29,28	12.27,26	systems with two degrees of freedom
12.31,32	12.33,30	
		Curvilinear motion (normal and tangential components)
12.35,36	12.37,34	circular path contained in horizontal plane
12.39,38		motion defined by two criteria
*12.41	*12.40	
12.45,44	12.43,42	path contained in vertical plane
12.48	12.47,46	
12.51,50	12.49,52	cars traveling on banked curves
12.53,54		impending relative motion of particle with respect to moving support
12.55,56	12.57,58	
12.59,*60		
12.63	12.61,62	cathode-ray tubes
		Curvilinear motion (radial and transverse components)
12.66	12.65,64	find accelerations of and/or forces acting on a particle
12.69,68	12.67	for a given motion
*12.71,*70		
12.75,74	12.73,72	motion under a central force (theoretical problems)
12.79,78	12.77,76	motion with Newton's Law of Gravitation ($F = GMm/r^2$)
		Conservation of angular momentum
12.81,82	12.83,80	orbital motion of spacecraft
*12.87,86	12.85,84	mechanical devices

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		Trajectory of a particle under a central force
12.91,90	12.89,88	determination of central force for a given trajectory
12.92	12.93	analysis of spacecraft orbits
12.95,94		
12.96	12.97,98	analysis of more advanced orbital problems
12.101,100	12.99	
12.103,102	12.105,104	determination of travel time for an orbit
12.107,106		
12.109,110	12.111,108	determination of point of impact of a descending spacecraft
12.115,114	12.113,112	derivations of classical formulas
12.117,116	12.119,118	Review problems
12.121,120	12.123,124	
12.127,122	12.125,126	
12.C2,C4	12.C1,C3	Computer problems
		CHAPTER 13: KINETICS OF PARTICLES: ENERGY AND MOMENTUM METHODS
13.2	13.1	Determine kinetic energy
		Method of work and energy applied to problems involving
13.3,4		single body
13.5	13.7,6	
13.11,8	13.9,10	two connected bodies
13.15,16	13.13,12	
13.17,18	13.14	
13.20	13.19	
13.23,22	13.21,26	a body which drops out or is removed from system
13.25,24		
13.29,28	13.27,30	work of springs
13.33,34	13.31,32	work of nonlinear forces
13.35,36		
13.37,38	13.39,40	work of a gravitational force
13.41,42	13.43,44	use of F = ma
13.45,46		
13.47,48	13.49,50	Introductory problems involving power and efficiency
13.51,52	13.53,54	Power in uniform and uniformly accelerated motion
13.55,56		Power of variable force
		Method of conservation of energy applied to problems involving
13.59,58	13.57,60	springs acting in horizontal plane (no gravity)
13.63,62	13.61	
13.67,64	13.65,66	springs and effect of gravity
13.68	13.69	
13.70		
13.71,74	13.73,72	use of F = ma
13.75	*13.76	
*13.77,*78	*13.79,*80	Theory problems on conservative forces
13.81	13.82	Space mechanics problems involving gravitational potential energy
13.83,84	13.85	
	13.86	Problem involving cathode-ray tube
13.87,88		Conservation of energy and conservation of angular momentum
13.89,90	13.91,92	
		Applications to space mechanics
13.95,96	13.93,94	velocity perpendicular to radius vector
13.97	13.98	
13.99,100	13.101,102	velocity oblique to radius vector
13.103,104		
13.105,106	13.107,108	derivations and theoretical problems
13.109,*110	*13.111	

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		Method of impulse and momentum applied to problems involving
13.113,114	13.115,112	single body
13.116		
13.117,120	13.119,118	two or more connected bodies
13.121,122		
13.123,124	13.125,126	force defined by a graph
13.127,130	13.129,128	impulsive motion of a single body
13.131,132	13.135,136	impulsive motion of two bodies
13.133,134		
13.137	13.138	impulsive motion with different supports
13.141,142	13.139,140	Direct central impact
13.143,144		
13.147,148	13.145,146	Oblique central impact
13.151,152	13.149,150	Impact against fixed surfaces
13.155,156	13.153,154	
13.159,160	13.157,158	Oblique central impact with one body constrained to move in a given direction
13.161		
13.165,164	13.163,162	Problems involving conservation of energy, conservation of momentum and impact
13.167,166		
13.171,170	13.169,168	
13.172	13.173	and one body constrained to move in a given direction
*13.175	13.174	Special problems
13.177,176	13.179,180	Review problems
13.181,178		
13.183,184	13.187,182	
13.185,186		
13.C1,C3	13.C2,C4	Computer problems
		CHAPTER 14: SYSTEMS OF PARTICLES
14.3,4	14.1,2	Conservation of linear momentum (one dimension)
14.5,6		
14.9,10	14.7,8	Computation of angular momentum
14.11,14	14.13,12	Motion of mass center
14.15		
14.19,18	14.17,16	Conservation of linear momentum (2 or 3 dimensions)
14.21,20		
14.25,24	14.23,22	Derivations and proofs
14.27	14.26	Conservation of linear momentum and computations of change in kinetic energy
14.29,28	14.31,30	
14.33,34	14.37,32	Conservation of linear momentum and conservation of energy
14.35,36	14.39,38	
14.41,42	14.43,40	Conservation of linear and angular momentum
14.45,44	14.49,48	Conservation of linear momentum, angular momentum, and energy
14.47,46		

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
14.51,50	14.52	Thrust caused by diverted flow
14.53,54	14.55,56	
14.59,58	14.57,60	Reactions at supports of vane or conveyor
14.61,62		
14.63,66	14.65,64	Thrust developed by a fan, propeller, or jet engine.
14.69,68	14.67,70	
14.71,72	14.73,74	Power and efficiency
*14.75,*76	14.77	Theory problems from fluid mechanics
	14.78	Motion of sprinkler
14.79,80	14.81,82	Motion of chains or of cars of variable mass
14.85,86	14.83,84	
14.89,90	14.87,88	Thrust and acceleration of rockets
14.91,92	14.93,94	Velocity of rocket for given fuel usage
14.97,98	14.95,96	Distance traveled by rocket
14.99	14.100	Derivation of efficiency of jet engine or rocket
14.107,106	14.101,102	Review problems
14.109,108	14.103,104	
14.111,110	14.105,112	
14.C1,C4	14.C2,C3	Computer problems
		CHAPTER 15: KINEMATICS OF RIGID BODIES
		Rotation about a fixed axis
15.1,2	15.3,4	uniformly accelerated rotation
15.5		
15.7,6	15.11,8	rotation about skew axis
15.9,10		
	15.13,12	motion of the earth
15.15,14	15.19,18	motion in two dimensions
15.17,16		
15.23,20	15.21,22	rolling contact with no slipping
15.25,24		linear and angular motion
15.27,26	15.29,28	disks brought into contact, slipping occurs
	*15.30	special problem
		General plane motion - velocities
15.31,34	15.33,32	motion of single rigid body
15.35,38	15.37,36	application of vector algebra to motion of a plate.
15.39,40	15.41,42	planetary gear systems
15.43,44		
		motion of a rod connected to:
15.45,48	15.47,46	a crank and a sliding block
15.49,50	15.51,52	two cranks
15.53,54		a rolling wheel and a horizontal surface
	*15.55	special problem
		Instantaneous center of rotation
15.59,58	15.57,56	problems involving parallel velocities
15.61,60	15.63,62	single rod: angle between controlling velocities = 90°
15.65,64		
15.67,66		single rod: angle between controlling velocities $\neq$ 90°
	15.69,68	single plate guided by pins fitted into slots
15.70	15.71	problems involving two instantaneous centers
15.73,72		
15.75	15.74	space and body centrodes
15.79,76	15.77,78	previous problems solved using instantaneous center

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Problem Number* SI Units	U.S. Units	Problem Description
		General plane motion - accelerations
15.81,80		find relations among accelerations ($\omega = 0$)
15.83,82		
15.85,84	15.89,88	rolling motion
15.87,86		
		motion of a body controlled by
15.93,90	15.91,92	one crank with constant angular velocity
15.95,94	15.97,96	two cranks
15.99,98	15.101,100	a crank and a sliding block
	15.102	special problem
*15.103,*104	*15.105,*106	Analysis of plane motion in terms of a parameter
*15.107,*108	*15.109,*110	
*15.111,*112	*15.113,*114	
		Plane motion of a particle relative to a rotating frame
15.117,116	15.115,118	velocity of a particle with respect to a rotating frame
15.119,120		
		Coriolis acceleration
15.121,122	15.123,124	acceleration of particle moving at a constant speed with respect to a frame rotating with a constant angular velocity
15.125,126		
15.127,128	15.129,130	
15.131,132	15.133,134	acceleration of a particle moving with respect to a frame rotating with an angular acceleration
15.137,138	15.135,136	
*15.139,*140	*15.141,*142	special problems
		Motion about a fixed point. General motion
15.145,146	15.143,144	relation between linear and angular velocity for a rigid body with a fixed point
15.147,148	15.149,150	angular acceleration of body rotating simultaneously about two axes
15.151,152		velocity and/or acceleration of a point on a body rotating about a fixed point
15.153,154	15.155,156	angular velocities and rate of change of angular velocities given
15.159	15.157,158	
15.161,160	15.163,162	velocities of points on body specified
15.164	15.165	cone rolling on fixed surface
15.169,168	15.167,166	rod attached to moving collars: find velocities of collars (angular velocity is indeterminate)
15.171,170		
15.173,172		universal joints
15.175	15.174	rod with clevis attached at one end
*15.179,*178	*15.177,*176	rod attached to moving collars: find accelerations of collars (angular velocity and acceleration are indeterminate)
		Three dimensional motion of a particle relative to a rotating frame. Coriolis acceleration.
15.183,182	15.181,180	$\dot{\Omega} = 0$ and $\dot{u} = 0$
15.185	15.184	$\dot{\Omega} \neq 0$ and $\dot{u} \neq 0$
15.187,186	15.189,188	previous problems to be solved using rotating frames
15.193,192	15.191,190	axis of Ω in plane of mechanism
15.195,194		
15.197,198	15.199,196	frame of reference in general motion
15,203,202	15.201,200	
15.207,208	15,205,204	Review problems
15.213,212	15.209,206	
15.215,214	15.211,210	
15.C1,C4	15.C3,C2	Computer problems

*Problems which do not involve any specific system of units have been indicated by underlining their number.

TION OF PROBLEMS (CONTINUED)

ription

D BODIES:

slides

= 0)
))

diagrams for accelerated rod

educe to $m\bar{a}$ and $\bar{I}\alpha$

fective forces
oading

applied forces and gravity

d suddenly released
sliding on horizontal surface

percussion
s for a suspended body

ting with a constant angular

n for body free to rotate about

ty

$\Sigma M_C = I_C\alpha$
under gravity
ng and sliding

ation and reactions
zero

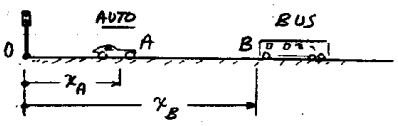
not zero
tion

eedom
reedom

iagrams for accelerated rod

have been indicated by underlining their number.

11.36



AUTO: $(x_A)_0 = 0$, $(v_A)_0 = 0$, $a_A = 0.75\ m/s^2$

$x_A = \frac{1}{2} a_A t^2 = \frac{1}{2}(0.75)t^2 = 0.375t^2$ (1)

BUS: $a_B = 0$, $v_B = -6\ m/s$, $(x_B)_{t=20s} = 0$

$x_B = (x_B)_0 + v_B t$

FOR $t = 20s$, $x_B = 0$: $0 = (x_B)_0 - (6)(20)$ $\quad (x_B)_0 = 120\ m$

THUS FOR BUS AT TIME t, WE HAVE

$x_B = 120 - 6t$ (2)

THE VEHICLES PASS WHEN $x_A = x_B$, USING EQS (1) AND (2):

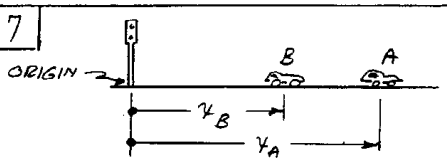
$x_A = x_B$: $\quad 0.375t^2 = 120 - 6t$

$0.375t^2 + 6t - 120 = 0$

$t = 11.6596\ s$ $\quad t = 11.60\ s$ ◀

FROM EQ(1): $x_A = 0.375t^2 = 0.375(11.598)^2$ $\quad x = 50.4\ m$ ◀

11.37



$t = 0$ WHEN AUTO A PASSES ORIGIN

AUTO A: UNIFORM MOTION. $v_A = 15\ mi/h = 22\ ft/s$

$s_A = v_A t = 22t$

AUTO B: STARTS AT $t = 2s$, WITH UNIFORM $a_B = 3\ ft/s^2$

$s_B = \frac{1}{2} a_B (t-2)^2 = \frac{3}{2}(t-2)^2$

(a) B OVERTAKES A WHEN $s_A = s_B$: $\quad 22t = 1.5(t-2)^2$

$1.5t^2 - 28t + 6 = 0$ $\quad t = 18.45\ s$ ◀

$s_A = 22(t) = 22(18.45)$ $\quad s_A = s_B = 406\ ft$ ◀

(b) SPEED OF B WHEN $t = 18.45s$

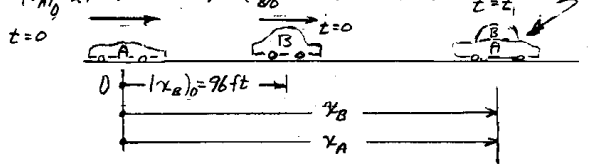
$v_B = (v_B)_0 + a_B(t-2) = 0 + (3\ ft/s^2)(18.45s - 2s)$

$v_B = 49.35\ ft/s$ $\quad v_B = 33.6\ mi/h$ ◀

11.38

$a_A = +1.8\ ft/s^2$ $\quad a_B = -1.2\ ft/s^2$ $\quad$ A OVERTAKES B

$(v_A)_0 = 24\ mi/h = 35.2\ ft/s$ $\quad (v_B)_0 = 36\ mi/h = 52.8\ ft/s$



MOTION OF AUTO A:

$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t$ (1)

$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2$ (2)

MOTION OF AUTO B:

$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t$ (3)

$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2$ (4)

(a) A OVERTAKES B AT $t = t_1$

$x_A = x_B$: $\quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$

$1.5t_1^2 - 17.6t_1 - 75 = 0$

$t_1 = -3.32s$ AND $t_1 = 15.0546$ $\quad t_1 = 15.05s$ ◀

EQ(2): $x_A = 35.2(15.05) + 0.9(15.05)^2$ $\quad x_A = 734\ ft$ ◀

11.38 CONTINUED

(b) VELOCITIES WHEN $t_1 = 15.05s$

EQ(1): $v_A = 35.2 + 1.8(15.05)$

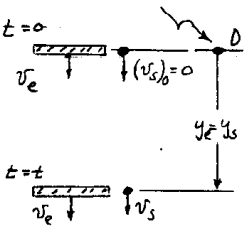
$v_A = 62.29\ ft/s$ $\quad v_A = 42.5\ mi/h \rightarrow$ ◀

EQ(3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74\ ft/s$ $\quad v_B = 23.7\ mi/h \rightarrow$ ◀

11.39

WE CHOOSE y AXIS POSITIVE DOWNWARD WITH ORIGIN AT ORIGINAL POSITION OF THE STONE.



MOTION OF ELEVATOR

UNIFORM MOTION

$a_e = 0$ $\quad v_e$ = CONSTANT

$y_e = v_e t$ (1)

MOTION OF STONE

UNIFORMLY ACCELERATED MOTION

$a_s = g = 9.81\ m/s^2$, $(v_s)_0 = 0$

$v_s = (v_s)_0 + at = 0 + gt$

$y_s = \frac{1}{2} g t^2$

(a) STONE HITS ELEVATOR

$y_e = y_s$; $\quad v_e t = \frac{1}{2} g t^2$ $\quad t = \frac{2v_e}{g}$ (2)

RELATIVE VELOCITY $= v_s - v_e$

$= gt - v_e = g\left(\frac{2v_e}{g}\right) - v_e = v_e$ ◀

(b) FOR $v_e = 7.5\ m/s$

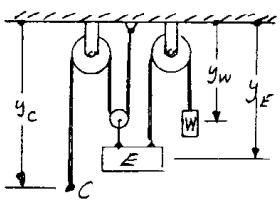
EQ(2): $t = \frac{2v_e}{g} = \frac{2(7.5\ m/s)}{9.81\ m/s^2} = 1.529$ $\quad t = 1.529s$ ◀

WHEN $t = 1.529s$

EQ(1): $y_e = v_e t = (7.5\ m/s)(1.529s)$ $\quad y = 11.47\ m$ ◀

11.40

(a) WE CHOOSE POSITIVE DIRECTION DOWNWARD



$y_C + 2y_E$ = CONSTANT

$v_C + 2v_E = 0$

BUT, $v_E = +5\ m/s$

THUS, $v_C + (2)(5\ m/s) = 0$

$v_C = -10\ m/s$

$v_C = 10\ m/s \uparrow$ ◀

(b) $y_W + y_E$ = CONSTANT

$v_W + v_E = 0$

THUS, $v_W = -v_E$

SINCE $v_E = +5\ m/s$, $v_W = -v_E = -5\ m/s$

$v_W = 5\ m/s \uparrow$ ◀

(c) RELATIVE VELOCITY OF C WITH RESPECT TO E.

$v_{C/E} = v_C - v_E = -10\ m/s - 5\ m/s = -15\ m/s$

$v_{C/E} = 15\ m/s \uparrow$ ◀

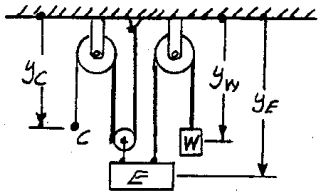
(d) RELATIVE VELOCITY OF W WITH RESPECT TO E.

$v_{W/E} = v_W - v_E = -5\ m/s - 5\ m/s = -10\ m/s$

$v_{W/E} = 10\ m/s \uparrow$ ◀

11.41

WE CHOOSE POSITIVE DIRECTION DOWNWARD.



MOTION OF COUNTERWEIGHT

$y_W = \frac{1}{2} a_W t^2$

AT $t = 5s$, $y_W = 10m$,

$10m = \frac{1}{2} a_W (5s)^2$

$a_W = 0.8 m/s^2$ $\quad a_W = 0.8 m/s^2 \downarrow$

(a) SINCE $y_W + y_E = \text{CONSTANT}$, $v_W + v_E = 0$, AND $a_W + a_E = 0$

THUS: $a_E = -a_W = -(0.8 m/s^2)$, $\quad a_E = 0.8 m/s^2 \uparrow$ ◀

ALSO, $y_C + 2y_E = \text{CONSTANT}$, $v_C + 2v_E = 0$, AND $a_C + 2a_E = 0$

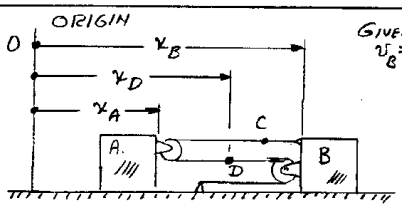
THUS: $a_C = -2a_E = -2(-0.8 m/s^2) = +1.6 m/s^2$, $\quad a_C = 1.6 m/s^2 \downarrow$ ◀

(b) VELOCITY OF ELEVATOR AFTER 5s.

$v_E = (v_E)_0 + a_E t = 0 + (-0.8 m/s^2)(5s) = -4 m/s$

$(v_E)_5 = 4 m/s \uparrow$ ◀

11.42



GIVEN: $v_B = 18 in/s \rightarrow$

(a) CABLE IS OF CONSTANT LENGTH

$3x_B - 2x_A = \text{CONSTANT}$

$3v_B - 2v_A = 0$

$3(+18 in/s) - 2v_A = 0$ $\quad v_A = +27 in/s = 27 in/s \rightarrow$ ◀

(b) CABLE BCAD IS OF CONSTANT LENGTH

$x_B + x_D - 2x_A = \text{CONSTANT}$

$v_B + v_D - 2v_A = 0$

$(+18 in/s) + v_D - 2(27 in/s) = 0$ $\quad v_D = +36 in/s = 36 in/s \rightarrow$ ◀

(c) VELOCITY OF A RELATIVE TO B:

$v_{A/B} = v_A - v_B = +27 in/s - 18 in/s = +9 in/s = 9 in/s \rightarrow$ ◀

(d) VELOCITY OF C IS SAME AS VELOCITY OF B: $v_C = v_B = 18 in/s \rightarrow$

$v_{C/D} = v_C - v_D = (18 in/s) - (36 in/s) = -18 in/s$ $\quad v_{C/D} = 18 in/s \leftarrow$ ◀

11.43

SEE FIGURE OF SOLUTION OF PROB 11.42.

SINCE A MOVES TOWARD LEFT, B ALSO MOVES LEFT

BLOCK B: $(v_B)_0 = 0$ $\quad a_B = \text{CONSTANT}$

GIVEN: $v_B = -12 in/s$ WHEN $x_B - (x_B)_0 = -24 in.$

(a) THUS: $v_B^2 = (v_B)_0^2 + 2a_B[x_B - (x_B)_0]$

$(-12 in/s)^2 = 0 + 2a_B(-24 in.)$

$a_B = -3 in/s^2$ $\quad a_B = 3 in/s^2 \leftarrow$ ◀

CABLE IS OF CONSTANT LENGTH.

$3x_B - 2x_A = \text{CONSTANT}$

$3v_B - 2v_A = 0$

$3a_B - 2a_A = 0$

$3(-3 in/s^2) - 2a_A = 0$ $\quad a_A = -4.5 in/s^2 = 4.5 in/s^2 \leftarrow$ ◀

(b) SINCE A STARTS FROM REST, $(v_A)_0 = 0$, ALSO $(x_A)_0 = 0$

AT $t = 5s$: $v_A = (v_A)_0 + a_A t = 0 + (-4.5 in/s^2)(5s)$

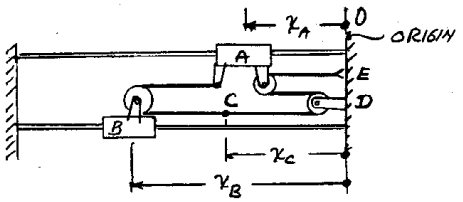
$v_A = -22.5 in/s$ $\quad v_A = 22.5 in/s \leftarrow$ ◀

$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 0 + \frac{1}{2}(-4.5 in/s^2)(5s)^2$

$x_A = -56.25 in.$ $\quad x_A = 56.3 in. \leftarrow$ ◀

11.44

WE CHOOSE THE POSITIVE DIRECTION TO THE LEFT



LENGTH OF CABLE: $2x_A + x_B + (x_B - x_A) = \text{CONSTANT}$

$x_A + 2x_B = \text{CONSTANT}$

$v_A + 2v_B = 0$ $\quad v_B = -\frac{1}{2} v_A$

$a_A + 2a_B = 0$; $\quad a_B = -\frac{1}{2} a_A$

RELATIVE MOTION: FOR v_A TO THE LEFT, $v_B = -\frac{1}{2} v_A = \frac{1}{2} v_A \rightarrow$

$v_{B/A} = v_B - v_A = -\frac{1}{2} v_A - v_A = -\frac{3}{2} v_A = \frac{3}{2} v_A \rightarrow$

WE NOTE THAT $v_{B/A}$ MUST BE TO RIGHT, i.e., $v_{B/A} = -450 mm/s$

THUS WHEN $t = 6s$: $\quad -450 mm/s = -\frac{3}{2} v_A$

$v_A = +300 mm/s$

$v_B = -\frac{1}{2} v_A = -150 mm/s$

(a) SINCE ACCELERATIONS ARE CONSTANT,

$v_A = a_A t$: $\quad +300 mm/s = a_A (6s)$

$a_A = 50 mm/s^2$ $\quad a_A = 50 mm/s^2 \leftarrow$ ◀

$v_B = a_B t$: $\quad -150 mm/s = a_B (6s)$

$a_B = -25 mm/s^2$ $\quad a_B = 25 mm/s^2 \rightarrow$ ◀

(b) WHEN $t = 8s$:

$v_B = a_B t = (-25 mm/s^2)(8s) = -200 mm/s$ $\quad v_B = 200 mm/s \rightarrow$ ◀

$x_B = \frac{1}{2} a_B t^2 = \frac{1}{2}(-25 mm/s^2)(8s)^2 = -800 mm$ $\quad x_B = 800 mm \rightarrow$ ◀

11.45

SEE FIGURE OF SOLUTION OF PROB. 11.44.

NOTE: POSITIVE DIRECTION IS TO LEFT.

(a) LENGTH OF CABLE.

$2x_A + x_B + (x_B - x_A) = \text{CONSTANT}$

$x_A + 2x_B = \text{CONSTANT}$

$v_A + 2v_B = 0$ $\quad (1)$

(a) GIVEN: $v_B = 100 mm/s \leftarrow = +100 mm/s$

EQ(1): $v_A + 2v_B = 0$ $\quad v_A + 2(100 mm/s) = 0$

$v_A = -200 mm/s$ $\quad v_A = 200 mm/s \rightarrow$ ◀

(b) LENGTH OF PORTION OF CABLE CDAE.

$x_C + 2x_A = \text{CONSTANT}$

$v_C + 2v_A = 0$

FOR $v_A = -200 mm/s$

$v_C + 2(-200 mm/s) = 0$

$v_C = 400 mm/s$ $\quad v_C = 400 mm/s \leftarrow$ ◀

(c) RELATIVE VELOCITY OF C WITH RESPECT TO B.

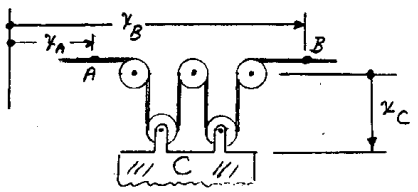
$v_C = v_B + v_{C/B}$ $\quad v_{C/B} = v_C - v_B$

WE KNOW: $v_B = +100 mm/s$ AND $v_C = 400 mm/s$:

$v_{C/B} = 400 mm/s - 100 mm/s = +300 mm/s$

$v_{C/B} = 300 mm/s \leftarrow$ ◀

11.46



LENGTH OF TAPE BETWEEN A AND B IS CONSTANT

$$x_B - x_A + 4x_C = \text{CONSTANT} \quad (1)$$

$$v_B - v_A + 4v_C = 0 \qquad v_C = \tfrac{1}{4}(v_A - v_B) \quad (2)$$

$$a_B - a_A + 4a_C = 0 \qquad a_C = \tfrac{1}{4}(a_A - a_B) \quad (3)$$

PORTION A OF TAPE: $(v_A)_0 = 600 \text{ mm/s} \rightarrow = +600 \text{ mm/s}$

WHEN $t = 6$ s, $v_A = +720$ mm/s

$v_A = (v_A)_0 + a_A t$; $720 = 600 + a_A(6)$ $\quad a_A = +20 \text{ mm/s}^2$

SINCE $v_B = 720$ mm/s = CONSTANT, $\quad a_B = 0$

MOTION OF COMPENSATOR C:

(a) EQ(3): $a_C = \frac{1}{4}(a_A - a_B) = \frac{1}{4}(20 + 0)$

$a_C = +5 \text{ mm/s}^2$ $\quad a_C = 5 \text{ mm/s}^2 \downarrow$ ◀

FOR $t = 4$ s; $v_A = (v_A)_0 + a_A t$

$v_A = 600 \text{ mm/s} + (20 \text{ mm/s}^2)(4\text{s}) = +680 \text{ mm/s}$

EQ(2): $v_C = \frac{1}{4}(v_A - v_B) = \frac{1}{4}(680 - 720)$

$= -10$ mm/s $\quad v_C = 10$ mm/s ↑ ◀

(b) FOR $t = 0$: $(v_C)_0 = \frac{1}{4}[(v_A)_0 - (v_B)_0] = \frac{1}{4}[600 - 720]$

$(v_C)_0 = -30$ mm/s

FOR $t = 6$ s: $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$

$= 0 + (-30 \text{ mm/s})(6\text{s}) + \frac{1}{2}(5 \text{ mm/s}^2)(6\text{s})^2$

$x_C = -90$ mm $\quad x_C = 90$ mm ↑ ◀

11.47

SEE SOLUTION OF PROB. 11.46 FOR SKETCH AND EQS. (1), (2), AND (3)

PORTION A OF TAPE FOR $t = 18$ s, $v_A = 720$ mm/s

$v_A = (v_A)_0 + a_A t$; $720 \text{ mm/s} = 0 + a_A(18\text{s})$ $\quad a_A = +40 \text{ mm/s}^2$

For $t = 18$ s:

$(x_A)_{18} = \frac{1}{2} a_A t^2 = \frac{1}{2}(40)(18)^2 = +6480$ mm

PORTION B OF TAPE For $t = 16$ s, $v_B = 720$ mm/s

$v_B = (v_B)_0 + a_B t$: $720 \text{ mm/s} = 0 + a_B(16\text{s})$ $\quad a_B = +45 \text{ mm/s}^2$

FOR $t = 16$ s: $(x_B)_{16} = \frac{1}{2} a_B t^2 = \frac{1}{2}(45)(16)^2 = 5760$ mm

FROM $t = 16$ s TO $t = 18$ s, $a_B = 0$, $v_B = 720$ mm/s

FOR $t = 18$ s: $(x_B)_{18} = (x_B)_{16} + v_B(2\text{s}) = 5760 + 720(2) = +7200$ mm

MOTION OF COMPENSATOR

(a) EQ(3): $a_C = \frac{1}{4}(a_A - a_B) = \frac{1}{4}(40 \text{ mm/s}^2 - 45 \text{ mm/s}^2)$

$a_C = -1.25 \text{ mm/s}^2$ $\quad a_C = 1.25 \text{ mm/s}^2 \uparrow$ ◀

FROM $t = 0$ TO $t = 10$ s, BOTH PORTIONS OF THE TAPE HAVE A CONSTANT ACCELERATION, THUS a_C IS ALSO CONSTANT

$(v_C)_{10} = (v_C)_0 + a_C(t) = 0 + (-1.25 \text{ mm/s}^2)(10\text{s})$

$(v_C)_{10} = -12.5$ mm/s $\quad (v_C)_{10} = 12.5 \text{ mm/s} \uparrow$ ◀

(b) NOTE a_C IS NOT CONSTANT FOR INTERVAL $t = 0$ TO $t = 18$ s. THEREFORE WE SHALL USE EQ(1).

(CONTINUED)

11.47 CONTINUED

EQ(1) $\quad x_B - x_A + 4x_C = \text{CONSTANT}$

CONSIDER $t = 0$ AND $t = 18$ s:

$$(x_B)_0 - (x_A)_0 + 4(x_C)_0 = (x_B)_{18} - (x_A)_{18} + 4(x_C)_{18}$$

OR: $\quad (x_C)_{18} - (x_C)_0 = \frac{1}{4}\left\{[(x_A)_{18} - (x_A)_0] - [(x_B)_{18} - (x_B)_0]\right\}$

SET INITIAL VALUES TO ZERO AND RECALL THAT

$(x_A)_{18} = +6480$ mm AND $(x_B)_{18} = +7200$ mm

$(x_C)_{18} = \frac{1}{4}[(x_A)_{18} - (x_B)_{18}] = \frac{1}{4}(+6480 \text{ mm} - 7200 \text{ mm})$

$(x_C)_{18} = -180$ mm $\quad (x_C)_{18} = 180$ mm ↑ ◀

ALTERNATE SOLUTION TO PART (b)

$0 < t < 16$ s: $\quad a_C = -1.25 \text{ mm/s}^2$ $\quad (v_C)_0 = 0$ $\quad (x_C)_0 = 0$

$(v_C)_{16} = (-1.25 \text{ mm/s}^2)(16\text{s}) = -20 \text{ mm/s}$

$(x_C)_{16} = \frac{1}{2} a_C t^2 = \frac{1}{2}(-1.25 \text{ mm/s}^2)(16\text{s})^2 = -160$ mm

$16 \text{ s} < t < 18$ s: $\quad a_A = +40 \text{ mm/s}^2$, $\quad a_B = 0$

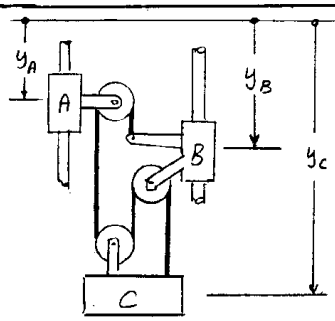
EQ(3) $\quad a_C = \frac{1}{4}(a_A - a_B) = \frac{1}{4}(40 - 0) = +10 \text{ mm/s}^2$

$(x_C)_{18} = (x_C)_{16} + (v_C)_{16}(2\text{s}) + \frac{1}{2} a_C (2\text{s})^2$

$= -160 \text{ mm} + (-20 \text{ mm/s})(2\text{s}) + \frac{1}{2}(+10 \text{ mm/s}^2)(2\text{s})^2$

$(x_C)_{18} = -160 \text{ mm} - 40 \text{ mm} + 20 \text{ mm} = -180 \text{ mm} = 180 \text{ mm} \uparrow$ ◀

11.48



LENGTH OF CORD = CONSTANT

$$L = (y_C - y_A) + (y_B - y_A) + 2(y_C - y_B) = \text{CONSTANT}$$

$$-2y_A - y_B + 3y_C = 0 \quad (1)$$

$$-2v_A - v_B + 3v_C = 0 \quad (2)$$

$$-2a_A - a_B + 3a_C = 0 \quad (3)$$

COLLAR A: $\quad a_A = -3.6 \text{ in/s}^2$

$v_A = -3.6t$ $\quad y_A = -\frac{1}{2}(3.6)t^2$

COLLAR B: $\quad a_B = 0$, $\quad v_B = 18 \text{ in/s}$, $\quad y_B = 18t$

(a)

TIME FOR $v_C = 0$:

EQ.(2): $\quad -2(-3.6t) - (18 \text{ in/s}) + 3(0) = 0$

$t = 2.5$ s ◀

(b) CHANGE IN POSITION DURING INTERVAL $t = 0$ TO $t = 2.5$ s

$y_A = -\frac{1}{2}(3.6)(2.5)^2 = -11.25$ in

$y_B = 18(2.5) = 45$ in.

FROM EQ(1) FOR CHANGE IN POSITION

$y_C = \frac{1}{3}(2y_A + y_B) = \frac{1}{3}(2(-11.25) + 45) = +7.5$ in

$y_C = 7.5$ in. ↓ ◀

11.49

SEE SKETCH AND SOLUTION OF PROB 11.48

EQ.(3) $-2a_A - a_B + 3a_C = 0$ (POSITIVE IS ↓)

COLLAR A: $(v_A)_0 = 0$ $\quad a_A = -2.5t$

COLLAR B: $(v_B)_0 = 0$ $\quad a_B = +15$ in./s

EQ(3): $a_C = \frac{1}{3}(2a_A + a_B) = \frac{1}{3}\left[2(-2.5t) + 15\right] = -\frac{5}{3}t + 5$

$(v_C)_0 = 0$: $\quad v_C = \int a_C\,dt = \int_0^t \left(-\frac{5}{3}t + 5\right)dt = -\frac{5}{6}t^2 + 5t$

FOR $v_C = 0$: $\quad 0 = -\frac{5}{6}t^2 + 5t$; $\quad t = 0$ AND $t = 6$s ◀

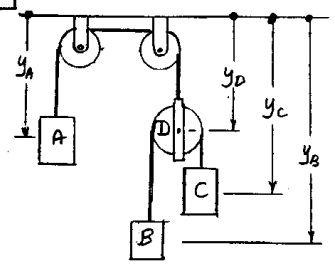
CHANGE IN POSITION OF C: $\quad y_C = \int_0^t v_C\,dt$

$y_C = \int_0^t \left(-\frac{5}{6}t^2 + 5t\right)dt = -\frac{5}{18}t^3 + \frac{5}{2}t^2$

WHEN $t = 6$s: $\quad y_C = -\frac{5}{18}(6)^3 + \frac{5}{2}(6)^2 = +30$ in.

$y_C = 30$ in ↓ ◀

*11.50



WE CHOOSE POSITIVE DIRECTION DOWNWARD.

WIRE AD

$y_A + y_D$ = CONSTANT

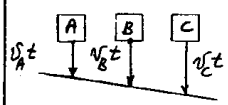
$v_A + v_D = 0$

$v_D = -v_A$

WIRE BDC $\quad (y_B - y_D) + (y_C - y_D)$ = constant ✓

$v_B + v_C - 2v_D = 0$

USE, $v_D = -v_A$: $\quad v_B + v_C - 2(-v_A) = 0$; $\quad 2v_A + v_B + v_C = 0$ (1)



FOR STRAIGHT LINE ALIGNMENT,

$v_B t = \frac{1}{2}(v_A t + v_C t)$

OR $\quad v_A - 2v_B + v_C = 0$ (2)

GIVEN: $v_{C/B} = v_C - v_B = +200$ mm/s (3)

SOLVING (1), (2), AND (3) SIMULTANEOUSLY, WE FIND

$v_A = -150$ mm/s $\quad v_A = 150$ mm/s ↑ ◀

$v_B = +50$ mm/s $\quad v_B = 50$ mm/s ↓ ◀

$v_C = +250$ mm/s $\quad v_C = 250$ mm/s ↓ ◀

11.51

SEE SOLUTION AND SKETCH OF PROB 11.50

EQ.(1): $2v_A + v_B + v_C = 0$ OR $v_A = -\frac{1}{2}(v_B + v_C)$ (1)

GIVEN: $v_{A/C} = 120$ mm/s ↑ $= -120$ mm/s

$v_{B/A} = 40$ mm/s ↑ $= -40$ mm/s

$v_A = v_C + v_{A/C}$: $\quad v_A = v_C - 120$ mm/s; $\quad v_C = v_A + 120$ (2)

$v_B = v_A + v_{B/A}$: $\quad v_B = v_A - 40$ mm/s; $\quad v_B = v_A - 40$ (3)

SUBSTITUTE FOR v_B AND v_C INTO EQ(1)

$v_A = -\frac{1}{2}(v_B + v_C) = -\frac{1}{2}\left[(v_A - 40) + (v_A + 120)\right]$

$2v_A = -2v_A - 80$

$4v_A = -80$

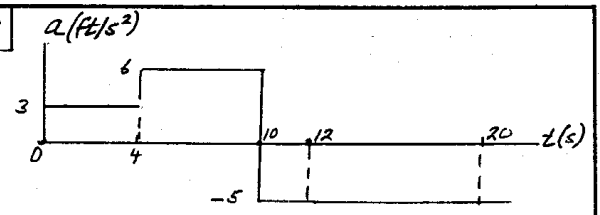
$v_A = -20$ mm/s $\quad v_A = 20$ mm/s ↑ ◀

EQ(3): $\quad v_B = +20 - 40 = -60$ mm/s $\quad v_B = 60$ mm/s ↑ ◀

EQ(2): $\quad v_C = -20 + 120 = +100$ mm/s $\quad v_C = 100$ mm/s ↓ ◀

11.52

(a)



INITIAL CONDITIONS: $t = 0$, $v_0 = -18$ ft/s, $x_0 = 0$

CHANGE IN v EQUALS AREA UNDER a-t CURVE:

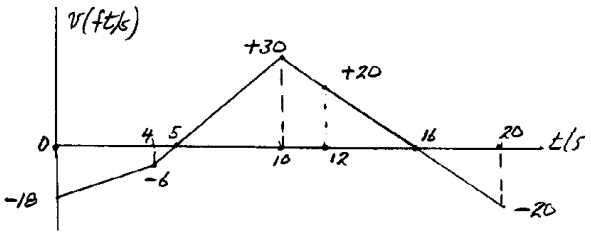
$v_0 = -18$ ft/s

$0 < t < 4$s: $\quad v_4 - v_0 = (3\text{ ft/s}^2)(4\text{s}) = +12$ ft/s $\quad v_4 = -6$ ft/s

$4\text{s} < t < 10$s: $\quad v_{10} - v_4 = (6\text{ ft/s}^2)(6\text{s}) = +36$ ft/s $\quad v_{10} = +30$ ft/s

$10\text{s} < t < 12$s: $\quad v_{12} - v_{10} = (-5\text{ ft/s}^2)(2\text{s}) = -10$ ft/s $\quad v_{12} = +20$ ft/s ◀

$12\text{s} < t < 20$s: $\quad v_{20} - v_{12} = (-5\text{ ft/s}^2)(8\text{s}) = -40$ ft/s $\quad v_{20} = -20$ ft/s



CHANGE IN x EQUALS AREA UNDER v-t CURVE

$x_0 = 0$

$0 < t < 4$s: $\quad x_4 - x_0 = \frac{1}{2}(-18 - 6)(4) = -48$ ft $\quad x_4 = -48$ ft

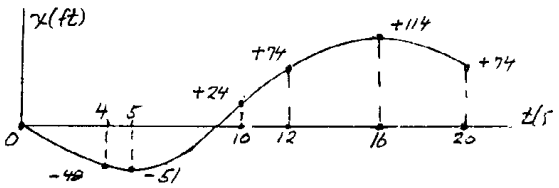
$4\text{s} < t < 5$s: $\quad x_5 - x_4 = \frac{1}{2}(-6)(1) = -3$ ft $\quad x_5 = -51$ ft

$5\text{s} < t < 10$s: $\quad x_{10} - x_5 = \frac{1}{2}(+30)(5) = +75$ ft $\quad x_{10} = +24$ ft

$12\text{s} < t < 10$s: $\quad x_{12} - x_{10} = \frac{1}{2}(+30 + 20)(2) = +50$ ft $\quad x_{12} = +74$ ft

$16\text{s} < t < 12$s: $\quad x_{16} - x_{12} = \frac{1}{2}(+20)(4) = +40$ ft $\quad x_{16} = +114$ ft

$20\text{s} < t < 16$s: $\quad x_{20} - x_{16} = \frac{1}{2}(-20)(4) = -40$ ft $\quad x_{20} = +74$ ft



(b) FROM ABOVE CURVES, WE READ

FOR $t = 12$s: $\quad v_{12} = +20$ ft/s, $\quad x_{12} = +74$ ft ◀

DISTANCE TRAVELED $t = 0$ TO $t = 12$s

FROM $t = 0$ TO $t = 6$s: DISTANCE TRAVELED = 51 ft

FROM $t = 5$s TO $t = 12$s: DISTANCE TRAVELED = $(51 + 74) = 125$ ft

TOTAL DISTANCE TRAVELED = $51 + 125 = 176$ ft ◀

11.53

SEE SOLUTION OF PROB 11.52

FROM v-t AND x-t CURVES, WE READ

(a) AT $t = 10$s: $\quad v_{MAX} = +30$ ft/s ◀

(b) AT $t = 16$s: $\quad x_{MAX} = +114$ ft ◀

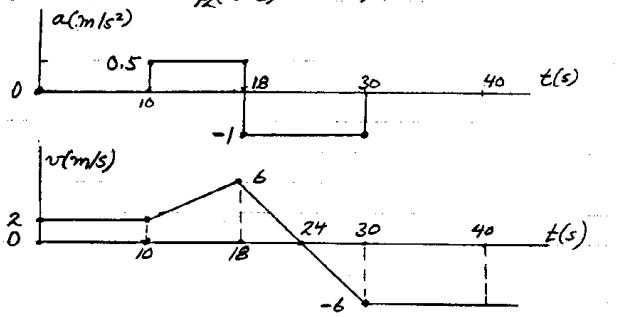
11.54

a-t CURVE : VALUE OF a = SLOPE OF v-t CURVE

$t=0$ TO $t=10s$ AND $t>30s$: $a=0$

$t=10s$ TO $t=18s$: $a=\frac{1}{8}(6-2)=+0.5\ m/s^2$

$t=18s$ TO $t=30s$: $a=\frac{1}{12}(-6-6)=-1\ m/s^2$



x-t CURVE: CHANGE IN x = AREA UNDER v-t CURVE. $x_0=-16\ m$

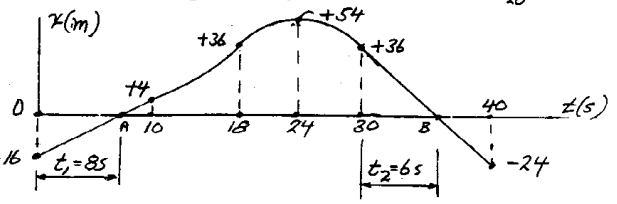
$t=0$ TO $t=10s$: $x_{10}-x_0=(2\ m/s)(10s)=20\ m$ $x_5=+4\ m$

$t=10s$ TO $t=18s$: $x_{18}-x_{10}=\frac{1}{2}(2+6\ m/s)(8s)=32\ m$ $x_9=+36\ m$

$t=18s$ TO $t=24s$: $x_{24}-x_{18}=\frac{1}{2}(6\ m/s)(6s)=18\ m$ $x_{12}=+54\ m$

$t=24s$ TO $t=30s$: $x_{30}-x_{24}=\frac{1}{2}(-6\ m/s)(6s)=-18\ m$ $x_{15}=+36\ m$

$t=30s$ TO $t=40s$: $x_{40}-x_{30}=(-6\ m/s)(10s)=-60\ m$ $x_{20}=-24\ m$



(a) WHEN $t=24s$, MAX. VALUE OF POSITION COORD. $=+54\ m$

(b) POSITION COORDINATE $=36\ m$ WHEN $t=18s$ AND $t=30s$

11.55

(a) SEE PROB 11.54 FOR a-t, v-t, AND x-t CURVES.

$t=0$ TO $t=24s$: DISTANCE TRAVELED $=16+54=70\ m$

$t=24s$ TO $t=30s$: DISTANCE TRAVELED $=54-36=18\ m$

$t=0$ TO $t=30s$: TOTAL DISTANCE TRAVELED $=88\ m$

(b) DETERMINATION OF t_1: FROM SIMILAR TRIANGLES

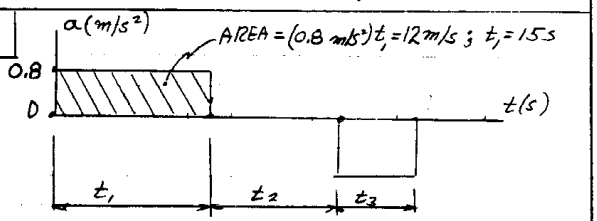
$\frac{t_1}{16}=\frac{10-t_1}{4}=\frac{10}{20}$ (ADD NUM's & DENOM's) $t_1=8s$

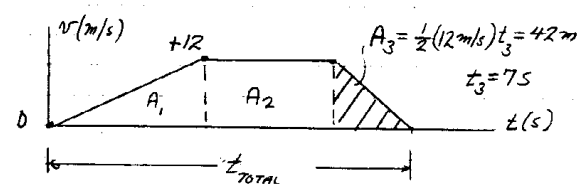
DETERMINATION OF t_2: FROM SIMILAR TRIANGLES

$\frac{t_2}{36}=\frac{10-t_2}{24}=\frac{10}{60}$, $t_2=6\ s$

FOR $x=0$, $t=30+6=36s$

11.56





TOTAL DISTANCE $=300\ m=A_1+A_2+A_3$

$300\ m=\frac{1}{2}(12\ m/s)(15s)+(12\ m/s)(t_2)+42\ m$ $t_2=14s$

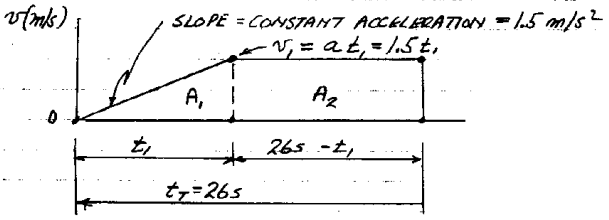
TOTAL TIME $=t_1+t_2+t_3=15s+14s+7s$ $t_{TOTAL}=36s$

11.57

SPEED: $45\ km/h=12.5\ m/s$

SINCE AN AUTO TRAVELING 12.5 m/s WILL ARRIVE AT SIGNAL B JUST AS IT TURNS GREEN, THE TIME UNTIL SIGNAL B TURNS GREEN IS

$t_T=\frac{\text{DISTANCE AB}}{12.5\ m/s}=\frac{325\ m}{12.5\ m/s}=26s$



DISTANCE TRAVELED: $A_1+A_2=325\ m$

$\frac{1}{2}(1.5t_1)t_1+(1.5t_1)(26-t_1)=325$

$-0.75t_1^2+39t_1-325=0$

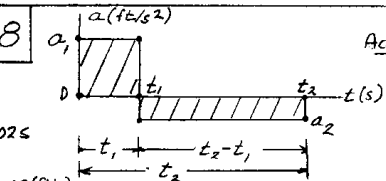
$(t_1=41.58s)$ OR $t_1=10.42s$

MAX SPEED: $v_1=1.5t_1=1.5(10.42)=15.63\ m/s$

$v_1=56.3\ km/h$

MOTORIST SHOULD ACCELERATE FOR 10.42s AT ⅔ THEN CONTINUE AT SPEED OF 56.3 km/h

11.58



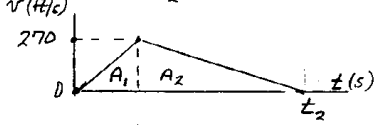
ACCELERATION PHASE

$a_1=\frac{270\ ft/s}{0.02s}$

$a_1=13{,}500\ in/s^2$

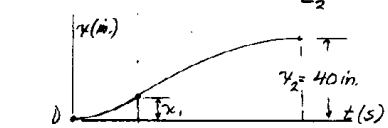
$a_1=1125\ ft/s^2$

$t_1=0.02s$



$x_1=A_1=\frac{1}{2}(270\ in/s)(0.02s)$

$x_1=2.70\ in.$



$x_2=A_1+A_2$

$40\ in.=\frac{1}{2}(270\ in/s)t_2$

$t_2=0.296s$

DECELERATION PHASE $-a_2=\frac{270}{t_2-t_1}=\frac{270}{0.296-0.02}=\frac{270}{0.276}=978.3\ in/s^2$

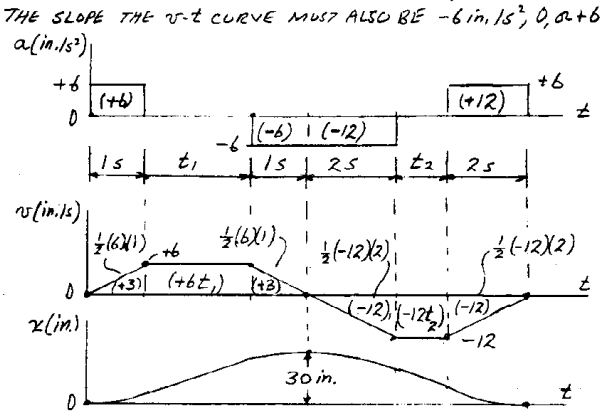
$a_2=-81.5\ ft/s^2$

11.59

WE CHOOSE POSITIVE TO THE RIGHT, THUS THE RANGE OF PERMISSIBLE VELOCITIES IS $-12\ in/s<v<6\ in/s$

SINCE ACCELERATION IS $-6\ in/s^2$, 0, OR $+6\ in/s^2$,

THE SLOPE THE v-t CURVE MUST ALSO BE $-6\ in/s^2$, 0, OR $+6\ in/s^2$.



PLANER MOVES 30 in. TO RIGHT: $+30\ in=3+6t_1+3$ $t_1=4.5$

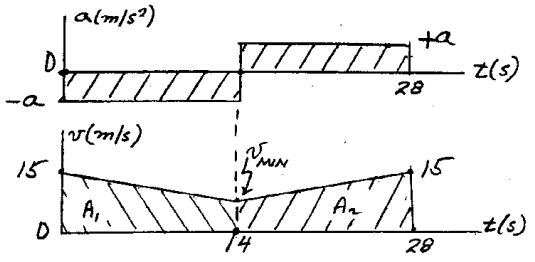
PLANER MOVES 30 in. TO LEFT: $-30\ in.=-12-12t_2+12$ $t_2=0.5s$

TOTAL TIME $=1s+4.5+1s+2.5+0.5s+2.5s=10.5s$ $t_{TOTAL}=10.5s$

11.60

DRAW v-t CURVE WITH $v_0 = v_{22} = 54\ km/h = 15\ m/s$

AREA UNDER v-t CURVE = 280 m. FOR SMALLEST ACCELERATION AND DECELERATION CHOOSE CONSTANT ACCELERATION AND CONSTANT DECELERATION OF SAME MAGNITUDE.



$A_1 = A_2$; $A_1 + A_2 = 2A_1 = 280\ m$: $A_1 = 140\ m$

$A_1 = \frac{1}{2}\left[15\ m/s + v_{MIN}\right](14\ s) = 140\ m$

$v_{min} = 5\ m/s$ $\quad v_{min} = 18\ km/h$ ◀

a = SLOPE OF v-t CURVE:

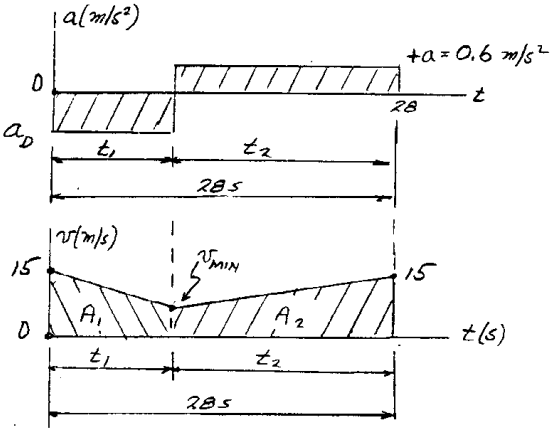
$a = \frac{15\ m/s - 5\ m/s}{14\ s}$ $\quad a = \pm 0.714\ m/s^2$ ◀

11.61

DRAW v-t CURVE WITH $v_0 = v_{22} = 54\ km/h = 15\ m/s$

AREA UNDER v-t CURVE = 280 m. WE CHOOSE CONSTANT DECELERATION AND CONSTANT ACCELERATION.

NOTE: DECELERATION ≠ ACCELERATION OF $0.6\ m/s^2$.



$A_1 + A_2 = 280\ m$: $\frac{1}{2}(15\ m/s + v_{MIN})(28\ s) = 280\ m$

$v_{MIN} = 5\ m/s$ $\quad v_m = 18\ km/h$ ◀

FOR ACCELERATION PHASE: $a = 0.6\ m/s^2$ ◀

$a = +0.6\ m/s^2$ = SLOPE OF v-t CURVE

$0.6\ m/s^2 = \frac{15\ m/s - 5\ m/s}{t_2}$

$t_2 = 16.667\ s$

$t_1 + t_2 = 28\ s$: $t_1 + 16.667\ s = 28\ s$

$t_1 = 11.333\ s$

FOR DECELERATION PHASE:

a_D = SLOPE OF v-t CURVE

$a_D = \frac{v_{MIN} - 15\ m/s}{t_1} = \frac{5\ m/s - 15\ m/s}{11.333\ s}$

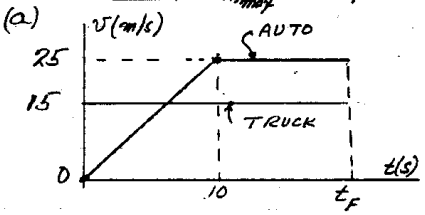
$a_D = -0.882\ m/s^2$ ◀

11.62

TRUCK: $v_B = 54\ km/h = 15\ m/s$, $a_T = 0$

AUTO: $(v_A)_{max} = 90\ km/h = 25\ m/s$

(a)



FOR TRUCK $x_B = 15\,t$

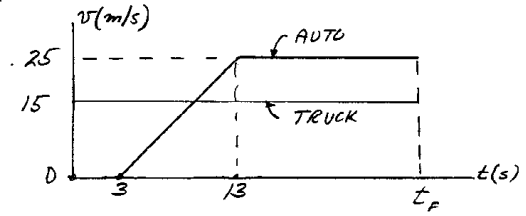
FOR AUTO: $(t > 10s)$ $x_A = \frac{1}{2}(25)(10) + 25\,(t - 10)$

WHEN $t = t_F$, $x_A = x_B$: $15\,t_F = 125 + 25(t_F - 10)$

$t_F = 12.5\ s$ ◀

$x_F = x_B = (15\ m/s)(12.5\ s) = 187.5\ m$ $\quad x_F = 187.5\ m$ ◀

(b) AUTO STARTS 3s AFTER TRUCK PASSES AUTO



FOR BUS: $x_B = 15\,t$

FOR AUTO: $(t > 13s)$ $x_A = \frac{1}{2}(25)(13 - 3) + 25\,(t - 13)$

WHEN $t = t_F$, $x_A = x_B$: $15\,t_F = 125 + 25(t_F - 13)$

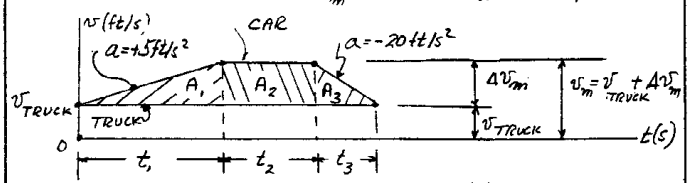
$t_F = 20\ s$ ◀

$x_F = x_B = (15\ m/s)(20\ s)$ $\quad x_F = 300\ m$ ◀

11.63

RELATIVE TO TRUCK, CAR MUST MOVE A DISTANCE: $\Delta x = 16 + 40 + 50 + 40 = 146\ ft$

ALLOWABLE INCREASE IN SPEED: $\Delta v_m = 50 - 35 = 15\ mi/h = 22\ ft/s$



ACCELERATION PHASE: $t_1 = 22/5 = 4.4\ s$ $\quad A_1 = \frac{1}{2}(22)(4.4) = 48.4\ ft$

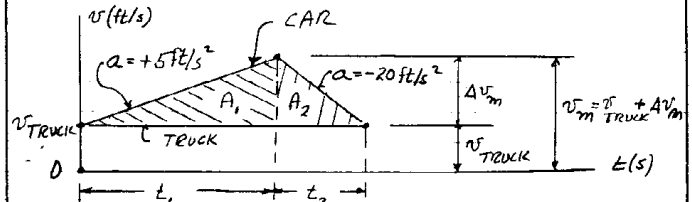
DECELERATION PHASE: $t_3 = 22/20 = 1.1\ s$ $\quad A_3 = \frac{1}{2}(22)(1.1) = 12.1\ ft$

BUT: $\Delta x = A_1 + A_2 + A_3$: $146\ ft = 48.4 + (22)t_2 + 12.1$ $\quad t_2 = 3.895\ s$

$t_{TOTAL} = t_1 + t_2 + t_3 = 4.4\ s + 3.895\ s + 1.1\ s = 9.395\ s$ $\quad t_F = 9.39\ s$ ◀

11.64

RELATIVE TO TRUCK, CAR MUST MOVE A DISTANCE: $\Delta x = 16 + 40 + 50 + 40 = 146\ ft$



$\Delta v_m = 5\,t_1 = 20\,t_2$; $t_2 = \frac{1}{4}\,t_1$

$\Delta x = A_1 + A_2$: $146\ ft = \frac{1}{2}(\Delta v_m)(t_1 + t_2)$

$146\ ft = \frac{1}{2}(5\,t_1)(t_1 + \frac{1}{4}\,t_1)$

$t_1^2 = 46.72$ $\quad t_1 = 6.835\ s$ $\quad t_2 = \frac{1}{4}\,t_1 = 1.709$

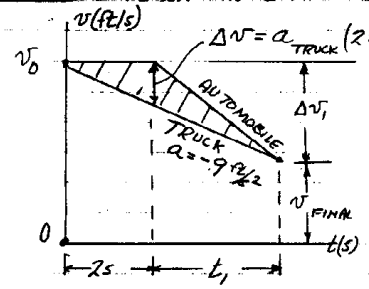
$t_{TOTAL} = t_1 + t_2 = 6.835 + 1.709$ $\quad t_F = 8.54\ s$ ◀

$\Delta v_m = 5\,t_1 = 5(6.835) = 34.18\ ft/s = 23.3$ ~~mi/h~~

SINCE $v_{TRUCK} = 35\ mi/h$, $\quad v_m = 35\ mi/h + 23.3\ mi/h$ $\quad v_m = 58.3\ mi/h$ ◀

11.65

WE DRAW v-t CURVE (ASSUME v_{FINAL} is POSITIVE)



NOTE: REAR-END COLLISION IS JUST AVOIDED. THUS WHEN DISTANCE BETWEEN AUTO AND TRUCK IS ZERO, THE TRUCK AND AUTO MUST HAVE SAME VELOCITY v_F.

SHADED AREA $= \Delta x = 30\ ft$

$\frac{1}{2}(18\ ft/s)(2s) + \frac{1}{2}(18\ ft/s)\,t_1 = 30\ ft \qquad t_1 = 1.333\ s$

$\Delta v_1 = (a_{TRUCK})(2 + t_1) = (-9\ ft/s^2)(2s + 1.333s) = 30\ ft/s$

BUT, ALSO: $\Delta v_1 = a_{AUTO}(t_1)$

$-30\ ft = a_{AUTO}(1.333s) \qquad a_{AUTO} = -22.5\ ft/s^2$

(a) $v_0 = 60\ mi/h = 88\ ft/s$

$v_{FINAL} = v_0 + \Delta v = 88\ ft/s - 30\ ft/s = 58\ ft/s$

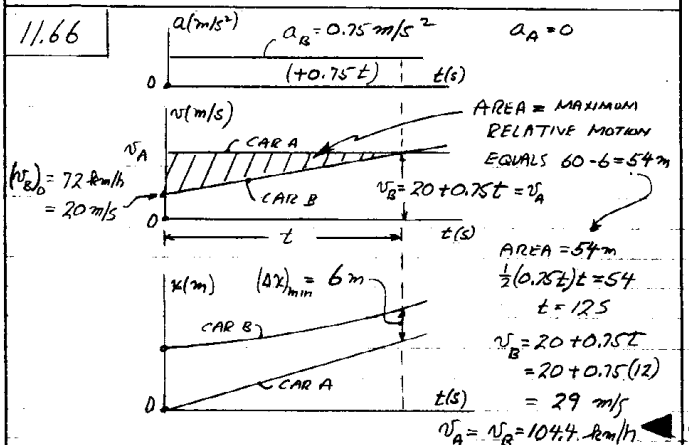
(b) $v_0 = 45\ mi/h = 66\ ft/s$

$v_{FINAL} = v_0 + \Delta v = 66\ ft/s - 30\ ft/s = 36\ ft/s$

FOR BOTH (a) AND (b) THE FINAL VELOCITY IS POSITIVE AS WE HAVE ASSUMED. THUS FOR BOTH (a) AND (b):

DECELERATION $= 22.5\ ft/s^2$ ◀

11.66



AREA $= 54\ m$

$\frac{1}{2}(0.75t)t = 54$

$t = 12\ s$

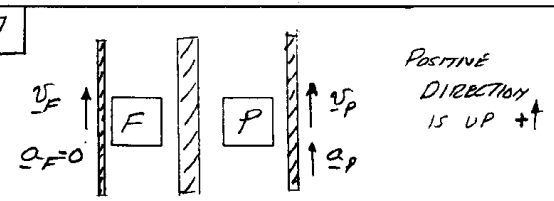
$v_B = 20 + 0.75t$

$= 20 + 0.75(12)$

$= 29\ m/s$

$v_A = v_B = 104.4\ km/h$ ◀

11.67



POSITIVE DIRECTION IS UP +↑

FREIGHT ELEVATOR MOVES UPWARD AT CONSTANT VELOCITY

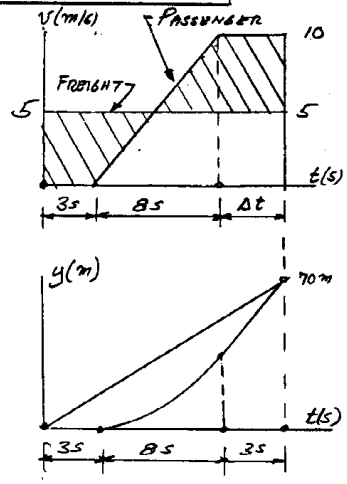
$v_F = 5\ m/s \qquad a_F = 0$

PASSENGER ELEVATOR STARTS FROM REST AND STARTS UPWARD WITH

$a_p = 1.25\ m/s^2$

THREE SECONDS AFTER IT WAS PASSED BY FREIGHT ELEVATOR. $(v_p)_{max} = 10\ m/s$. WE NOW DRAW THE v-t AND y-t CURVES (CONTINUED)

11.67 CONTINUED



PASSENGER ELEVATOR OVERTAKES FREIGHT ELEVATOR WHEN AREAS UNDER THEIR v-t CURVE ARE EQUAL. THAT IS, WHEN SHADED AREA (WHICH REPRESENTS DIFFERENCE IN AREAS) IS ZERO.

THUS $\Delta t = 3s$, AND TIME REQUIRED IS

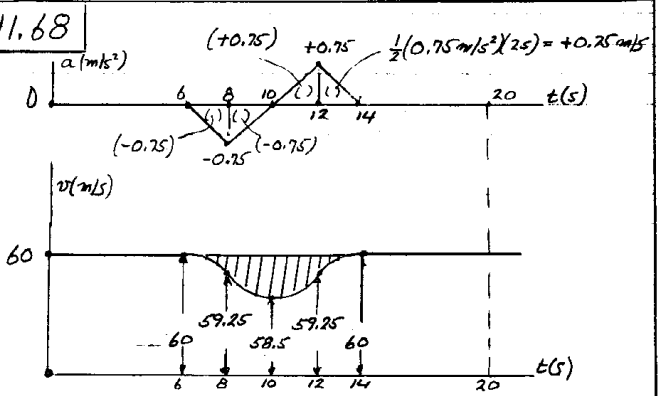
$t = 3 + 8 + 3 = 14s$ ◀

DISTANCE TRAVELED IS

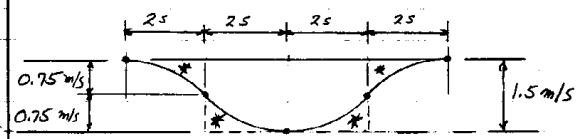
$d = v_p t = (5\ m/s)(14s)$

$d = 70\ m$ ◀

11.68



GEOMETRY OF "BELL-SHAPED" PORTION OF v-t CURVE



THE PARABOLIC SPANDRELS MARKED BY * ARE OF EQUAL AREA. THUS, TOTAL AREA OF SHADED PORTION OF v-t DIAGRAM IS:

$= \Delta x = 6\ m$

(a) WHEN $t = 20\ s$: $\qquad v_{20} = 50\ m/s$ ◀

$v_{20} = (60\ m/s)(20s) - (\text{SHADED AREA})$

$= 1200\ m - 6\ m \qquad v_{20} = 1194\ m$ ◀

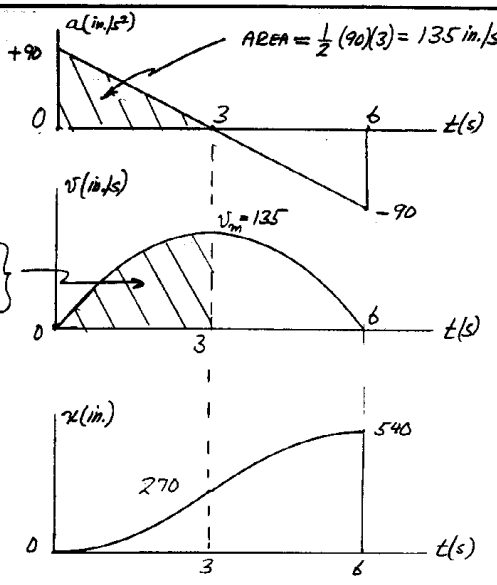
(b) FROM $t = 6\ s$ TO $t = 14s$: $\Delta t = 8s$

$\Delta x = (60\ m/s)(14\ s - 6s) - (\text{SHADED AREA})$

$= (60\ m/s)(8s) - 6m = 480\ m - 6\ m = 474\ m$

$v_{AVERAGE} = \frac{\Delta x}{\Delta t} = \frac{474\ m}{8s} \qquad v_{AVERAGE} = 59.25\ m/s$ ◀

11.69



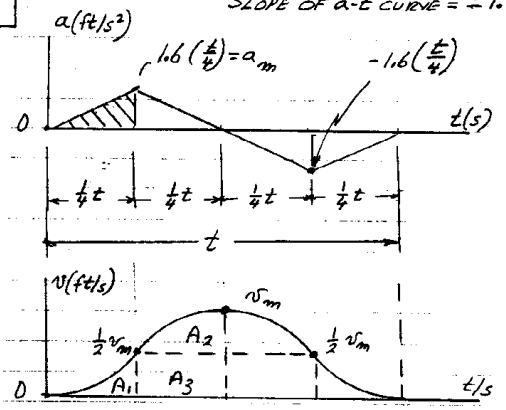
(a) MAXIMUM VELOCITY:
WHEN $t = 3s$, $v_{max} = 135$ in./s ◀

(b) POSITION WHEN $t = 6s$: $x_6 = 540$ in. ◀

(c) AVERAGE VELOCITY OVER INTERVAL $0 < t < 6s$.

$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{540 \text{ in}}{6 s}$ $\quad v_{avg} = 90$ in./s ◀

11.70



DISTANCE TRAVELED = AREA UNDER v-t CURVE

$50 \text{ ft} = 2(A_1 + A_2 + A_3)$

$50 \text{ ft} = 2\left[\frac{1}{3}\frac{v_m}{2}\frac{t}{4} + \frac{2}{3}\frac{v_m}{2}\frac{t}{4} + \frac{v_m}{2}\frac{t}{4}\right]$

$50 \text{ ft} = 2\left(\frac{v_m t}{4}\right)$; $\quad 100 = v_m t$ (1)

SHADED AREA OF a-t DIAGRAM:

$\frac{1}{2}\left(1.6\frac{t}{4}\right)\frac{t}{4} = \frac{1}{2}v_m$ $\quad v_m = \frac{1.6}{16}t^2$

SUBSTITUTE INTO EQ (1): $100 = \left(\frac{1.6}{16}t^2\right)t$

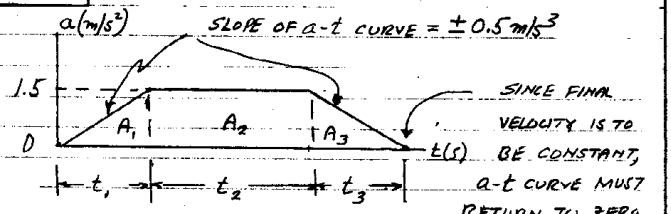
$t^3 = 1000$ $\quad t = 10s$ ◀

AVERAGE VELOCITY $= \frac{\Delta x}{t} = \frac{50 \text{ ft}}{10s} = 5$ ft/s ◀

NOTE: $v_m = \frac{1.6}{16}(10)^2 = 10$ ft/s², $\quad a_m = 1.6\left(\frac{10}{4}\right) = 4$ ft/s²

11.71

$a_m = 1.5$ m/s²



$1.5 \text{ m/s}^2 = (0.5 \text{ m/s}^3)t_1$ $\quad t_1 = 3s$

$A_1 = \frac{1}{2}(1.5 \text{ m/s}^2)(3s) = 2.25$ m/s

$-1.5 \text{ m/s}^2 = -(0.5 \text{ m/s}^3)t_3$ $\quad t_3 = 3s$

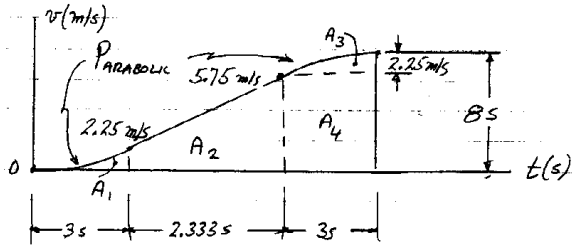
$A_3 = \frac{1}{2}(1.5 \text{ m/s}^2)(3s) = 2.25$ m/s

SINCE $v_{FINAL} = 8$ m/s,

$A_1 + A_2 + A_3 = v_{FINAL}$

$2.25 \text{ m/s} + (1.5 \text{ m/s}^2)t_2 + 2.25 \text{ m/s} = 8 \text{ m/s}$ $\quad t_2 = 2.333$ s

(a) TOTAL TIME $= t_1 + t_2 + t_3 = 3s + 2.333s + 3s$. $\quad t = 8.33s$ ◀



(b) DISTANCE TRAVELED $= A_1 + A_2 + A_3 + A_4$

$x = \frac{1}{3}(2.25 \text{ m/s})(3s) + \frac{1}{2}(2.25 \text{ m/s} + 5.75 \text{ m/s})(2.333s) + \frac{2}{3}(2.25 \text{ m/s})(3s) + (5.75 \text{ m/s})(3s)$

$x = 2.25 \text{ m} + 9.333 \text{ m} + 4.5 \text{ m} + 17.25 \text{ m}$ $\quad x = 33.3$ m ◀

(c) AVERAGE VELOCITY: $v_{AVG} = \frac{33.3 \text{ m}}{8.33s}$ $\quad v_m = 4$ m/s ◀

11.72

PROCEDURE

1. WE APPROXIMATE THE a-t CURVE BY RECTANGLES AS SHOWN ON NEXT PAGE. FOR EACH INTERVAL OF TIME $\Delta t = 1.0$ s, WE HAVE

CHANGE IN v = AREA UNDER a-t CURVE

$\Delta v = (a_{AVERAGE})\Delta t$ $\quad$ SEE COLUMN (1)

2. NOTING THAT $v_0 = 15$ mi/h $= 22$ ft/s, WE CONSTRUCT THE v-t CURVE BY SUCCESSIVELY ADDING THE CHANGES IN VELOCITY Δv. SEE COLUMN (2)

3. WE NOW APPROXIMATE THE v-t CURVE BY RECTANGLES AS SHOWN ON NEXT PAGE.

CHANGE IN x = AREA UNDER v-t CURVE

$\Delta x = (v_{AVERAGE})\Delta t$

SEE COLUMN (3)

4. NOTING THAT $x_0 = 0$, WE CONSTRUCT THE x-t CURVE BY SUCCESSIVELY ADDING THE CHANGES IN POSITION Δx. SEE COLUMN (4)

(CONTINUED)

11.72 CONTINUED

$\Delta t = 1$ s

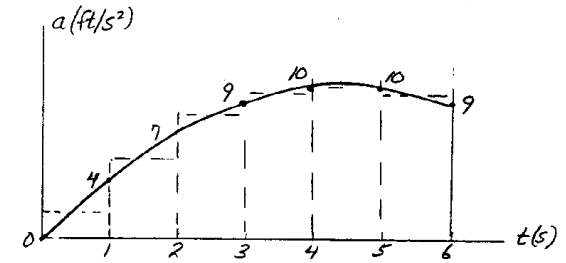
			①	②		③	④
t (s)	a (ft/s²)	$a_{AVERAGE}$	$\Delta v = a_{AVG}(\Delta t)$	v (ft/s)	$v_{AVERAGE}$	$\Delta x = v_{AVG}(\Delta t)$	x (ft)
0	0			22			0
		2	2		23	23	
1	4			24			23
		5.5	5.5		26.75	26.75	
2	7			29.5			49.75
		8	8		33.5	33.5	
3	9			37.5			83.25
		9.5	9.5		42.25	42.25	
4	10			47			125.5
		10	10		52	52	
5	10			57			177.5
		9.5	9.5		61.75	61.75	
6	9			66.5			239.25

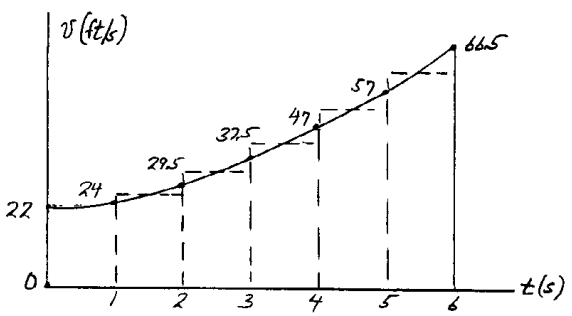
(a) VELOCITY WHEN $t = 6$ s:

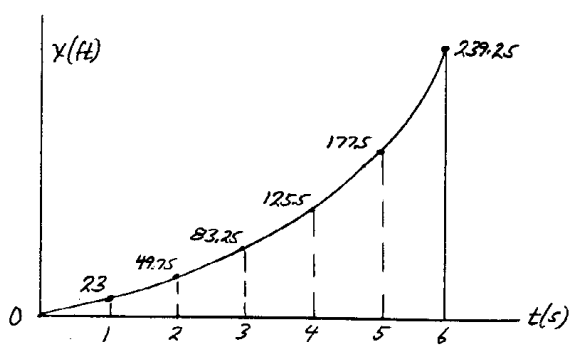
$v = 66$ ft/s $= 45$ mi/h ◀

(b) DISTANCE TRAVELED WHEN $t = 6$ s:

$x = 239$ ft ◀







11.73

PROCEDURE:

1. WE APPROXIMATE THE a-t CURVE BY THE RECTANGLES SHOWN BELOW. FOR EACH INTERVAL OF TIME $\Delta t = 0.5$ s.

CHANGE IN v = AREA UNDER a-t CURVE

$\Delta v = (a_{AVERAGE})\Delta t$ SEE COLUMN ①

2. SINCE $v = 0$ WHEN $t = 3$ s, THE INITIAL VELOCITY IS

v_0 = TOTAL AREA OF THE a-t CURVE

$v_0 = -\Sigma \Delta v = -(-38.5 \text{ m/s})$ SEE SUM OF COL. ①

3. WE CONSTRUCT THE v-t CURVE BY SUCCESSIVELY ADDING CHANGES IN VELOCITY (WHICH ARE NEGATIVE) TO THE INITIAL VELOCITY. SEE COLUMN ②

4. WE NOW APPROXIMATE THE v-t CURVE BY THE RECTANGLES SHOWN BELOW. $\Delta t = 0.5$ s.

CHANGE IN x = AREA UNDER v-t CURVE

$\Delta x = (v_{AVERAGE})\Delta t$ SEE COLUMN ③

5. SINCE $x = 0$ WHEN $t = 0$,

$x_{FINAL} = \Sigma \Delta x$ SEE SUM OF COL. ③

6. THE x-t CURVE IS THEN CONSTRUCTED BY SUCCESSIVELY ADDING THE CHANGES IN POSITION. SEE COLUMN ④

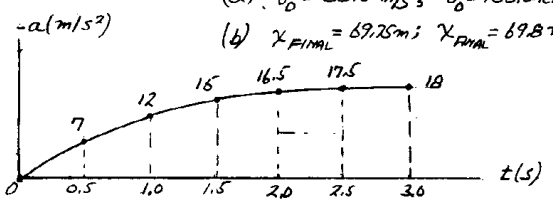
$\Delta t = 0.5$ s

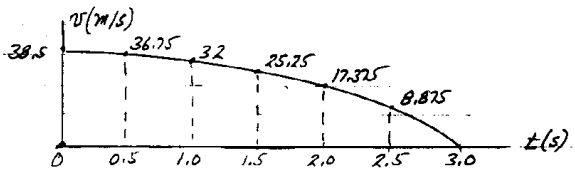
			①	②		③	④
t (s)	a (m/s²)	a_{AVG} (m/s²)	$\Delta v = (a_{AVG})\Delta t$	v (m/s)	$v_{AVERAGE}$ (m/s)	Δx (m)	x (m)
0	0			38.5			0
		-3.5	-1.75		37.625	18.813	
0.5	-7			36.75			18.813
		-9.5	-4.75		34.375	17.187	
1.0	-12			32			36.0
		-13.5	-6.75		28.625	14.313	
1.5	-15			25.25			50.313
		-15.75	-7.875		21.312	10.656	
2.0	-16.5			17.375			60.969
		-17	-8.5		13.125	6.562	
2.5	-17.5			8.875			67.531
		-17.75	-8.875		4.438	2.219	
3.0	-18			0			69.75

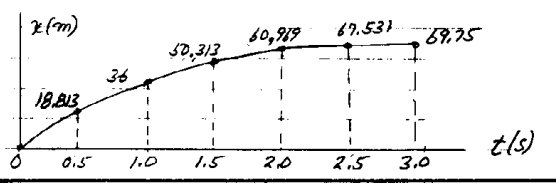
$-v_0 = \Sigma \Delta v = -38.5$ m/s

(a) $v_0 = 38.5$ m/s; $v_0 = 138.6$ km/h ◀

(b) $x_{FINAL} = 69.75$ m; $x_{FINAL} = 69.8$ m ◀







11.74

WE APPROXIMATE THE a-v CURVE BY THE SERIES OF HORIZONTAL DASHED LINES SHOWN ON THE FIGURE. FOR EACH INTERVAL THE ACCELERATION IS ASSUMED CONSTANT

$$\Delta v = a\,\Delta t \quad \text{OR} \quad \Delta t = \Delta v/a$$
$$\Delta x = v\,\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

$v_0 = 90\ \text{km/h} = 25\ \text{m/s}$

FROM $v = 25$ m/s TO $v = 20$ m/s:

$a = -2.1\ \text{m/s}^2$ $\quad \Delta v = 20 - 25 = -5\ \text{m/s}$

$$\Delta t = \frac{\Delta v}{a} = \frac{-5\ \text{m/s}}{-2.1\ \text{m/s}^2} = 2.38\ \text{s}$$

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2 = (25\ \text{m/s})(2.38\ \text{s}) + \tfrac{1}{2}(-2.1\ \text{m/s}^2)(2.38\ \text{s})^2$$
$$= +59.52\ \text{m} - 5.95\ \text{m} \qquad \Delta x = 53.57\ \text{m}$$

FROM $v = 20$ m/s TO $v = 10$ m/s:

$a = -2.5\ \text{m/s}^2$ $\quad \Delta v = 10 - 20 = -10\ \text{m/s}$

$$\Delta t = \frac{\Delta v}{a} = \frac{-10\ \text{m/s}}{-2.5\ \text{m/s}^2} = 4\ \text{s}$$

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2 = (20\ \text{m/s})(4\ \text{s}) + \tfrac{1}{2}(-2.5\ \text{m/s}^2)(4\ \text{s})^2$$
$$= 80\ \text{m} - 20\ \text{m} \qquad \Delta x = 60\ \text{m}$$

FROM $v = 10$ m/s TO $v = 0$:

$a = -3\ \text{m/s}^2$ $\quad \Delta v = 0 - 10 = -10\ \text{m/s}$

$$\Delta t = \frac{\Delta v}{a} = \frac{-10\ \text{m/s}}{-3\ \text{m/s}^2} = 3.333\ \text{s}$$

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2 = (10\ \text{m/s})(3.333\ \text{s}) + \tfrac{1}{2}(-3\ \text{m/s}^2)(3.333\ \text{s})^2$$
$$= 33.33\ \text{m} - 16.67\ \text{m} \qquad \Delta x = 16.66\ \text{m}$$

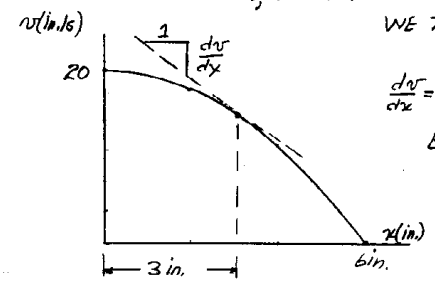
FOR ENTIRE MOTION $v = 25$ m/s TO $v = 0$

(a) $t = \Sigma\Delta t = 2.38\ \text{s} + 4\ \text{s} + 3.33\ \text{s}$ $\quad t = 9.71\ \text{s}$ ◀

(b) $x = \Sigma\Delta x = 53.57\ \text{m} + 60\ \text{m} + 16.66\ \text{m}$ $\quad x = 130.2\ \text{m}$ ◀

11.75

(a) AT THE POINT CORRESPONDING TO $x = 3$ in., WE DRAW TANGENT TO v-x CURVE



WE THEN MEASURE:

$v = 15.8$ in./s

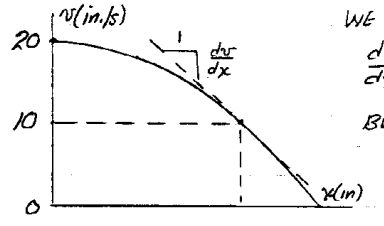
$$\frac{dv}{dx} = -3\ \frac{\text{in./s}}{\text{in.}}$$

BUT: $a = v\dfrac{dv}{dx}$

$= (15.8)(3)$

$a = -47.4\ \text{in./s}^2$ ◀

(b) AT THE POINT CORRESPONDING TO $v = 10$ in./s, WE DRAW THE TANGENT TO THE v-t CURVE.



WE THEN MEASURE:

$$\frac{dv}{dx} = -5.5\ \frac{\text{in./s}}{\text{in.}}$$

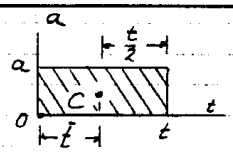
BUT $a = v\dfrac{dv}{dx}$

$= (10)(5.5)$

$a = -55\ \text{in./s}^2$ ◀

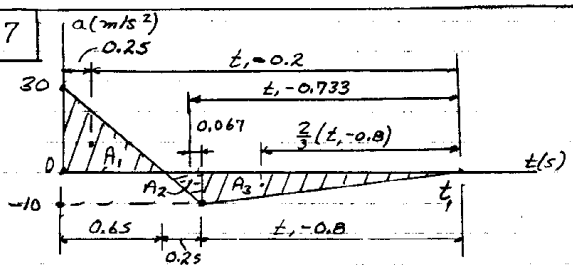
NOTE: IN ORDER TO USE THE METHOD SHOWN IN FIG 11.13, THE SAME SCALE MUST BE USED ON THE v AND x AXES. BY USING THE BASIC RELATIONSHIP $a = v\,dv/dx$ WE MAY MEASURE v AND dv/dx REGARDLESS OF THE SCALES USED

11.76



$$x = x_0 + v_0 t + A(t - \bar{t})$$
$$= x_0 + v_0 t + t(at)\left(\tfrac{t}{2}\right)$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$ ◀

11.77



(a) SINCE $v = 0$ WHEN $t = 0$ AND WHEN $t = t_1$, THE CHANGE IN v BETWEEN $t = 0$ AND $t = t_1$ IS ZERO. THUS, AREA UNDER a-t CURVE IS ZERO

$$A_1 + A_2 + A_3 = 0$$
$$\tfrac{1}{2}(30)(0.6) + \tfrac{1}{2}(-10)(0.2) + \tfrac{1}{2}(-10)(t_1 - 0.8) = 0$$
$$9 - 1 - 5t_1 + 4 = 0 \qquad t_1 = 2.4\ \text{s}$$ ◀

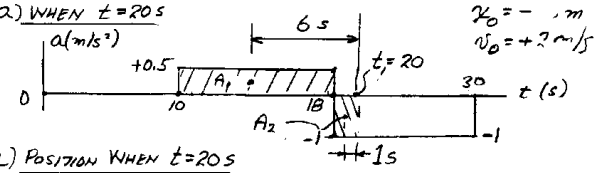
(b) POSITION WHEN $t = t_1 = 2.4$ s

$$x = x_0 + v_0 t_1 + A_1(t_1 - 0.2) + A_2(t_1 - 0.733) + A_3\left(\tfrac{2}{3}\right)(t_1 - 0.8)$$
$$= 0 + 0 + (9)(2.4 - 0.2) + (-1)(2.4 - 0.733) + \left[\tfrac{1}{2}(-10)(2.4 - 0.8)\right]\tfrac{2}{3}(2.4 - 0.8)$$
$$= 19.8\ \text{m} - 1.667\ \text{m} - 8.533\ \text{m} \qquad x = 9.60\ \text{m}$$ ◀

11.78

SEE SOLUTION OF PROB 11.54 FOR v-t CURVE

(a) WHEN $t = 20$ s



$x_0 = -\ \text{m}$

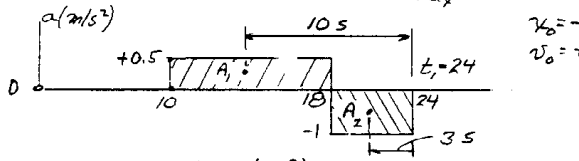
$v_0 = +2$ m/s

(a) POSITION WHEN $t = 20$ s

$$x = x_0 + v_0 t + \Sigma A(t - \bar{t})$$
$$= -16\ \text{m} + (2\ \text{m/s})(20\ \text{s}) + \left[(0.5\ \text{m/s}^2)(\ \ )\right](6\ \text{s}) + \left[(-1\ \text{m/s}^2)(2\ \text{s})\right](1\ \text{s})$$
$$= -16\ \text{m} + 40\ \text{m} + 24\ \text{m} - 2\ \text{m} \qquad x_{10} = 46\ \text{m}$$ ◀

(b) MAXIMUM POSITION COORDINATE OCCURS WHEN $v = 0$

SEE v-t CURVE AND NOTE THAT v_{max} OCCURS WHEN $t = 24$ s

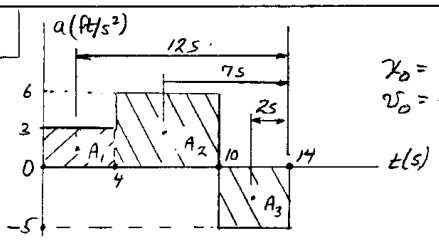


$x_0 = -16$ m

$v_0 = +2$ m/s

$$x_{MAX} = x_{t_2} = x_0 + v_0 t + \Sigma A(t_1 - \bar{t})$$
$$x_{MAX} = -16\ \text{m} + (2\ \text{m/s})(24\ \text{s}) + \left[(0.5\ \text{m/s}^2)(18\ \text{s})\right](10\ \text{s}) + \left[(-1\ \text{m/s}^2)(6\ \text{s})\right](3\ \text{s})$$
$$= -16\ \text{m} + 48\ \text{m} + 40\ \text{m} - 18\ \text{m} \qquad x_{MAX} = 54\ \text{m}$$ ◀

11.79



$v_0 = 0$

$v_0 = -18$ ft/s

WHEN $t = 14$ s:

$$x = x_0 + v_0 t + \Sigma A(t_1 - \bar{t})$$
$$= 0 - (18\ \text{ft/s})(14\ \text{s}) + \left[(3\ \text{ft/s}^2)(4\ \text{s})\right](12\ \text{s}) + \left[(6\ \text{ft/s}^2)(6\ \text{s})\right](7\ \text{s})$$
$$+ \left[(-5\ \text{ft/s})(4\ \text{s})\right](2\ \text{s})$$
$$x_{14} = -252\ \text{m} + 144\ \text{m} + 252\ \text{m} - 40\ \text{m}$$
$$x_{14} = +104\ \text{ft}$$ ◀

11.80

$x = 2t^3 - 8t^2$ $\quad y = 3t^2 - 12t$

$v_x = dx/dt = 6t^2 - 16t$ $\quad v_y = dy/dt = 6t - 12$

$a_x = dv_x/dt = 12t - 16$ $\quad a_y = dv_y/dt = 6$

(a) WHEN $t = 1$ s:

$v_x = -10$ $\quad v_y = -6$ $\quad \underline{v} = 11.66$ in./s $\angle 31.0°$

$a_x = -4$ $\quad a_y = +6$ $\quad \underline{a} = 7.21$ in./s^2 $\angle 56.3°$

(b) WHEN $t = 2$ s:

$v_x = -8$ $\quad v_y = 0$ $\quad \underline{v} = 8$ in./s $\leftarrow$

$a_x = +8$ $\quad a_y = +6$ $\quad \underline{a} = 10$ in./s^2 $\angle 36.9°$

(c) WHEN $t = 3.5$ s:

$v_x = +6$ $\quad v_y = +6$ $\quad \underline{v} = 8.49$ in./s $\angle 45.0°$

$a_x = +20$ $\quad a_y = +6$ $\quad \underline{a} = 20.9$ in./s^2 $\angle 17.7°$

11.81

WE FORM THE PRODUCT xy.

$xy = (t+1)^2[4(t+1)^{-2}] = 4$

$xy = 4$ $\quad \therefore$ PATH IS A HYPERBOLA

$x = (t+1)^2$ $\quad y = 4(t+1)^{-2}$

$v_x = 2(t+1)$ $\quad v_y = -8(t+1)^{-3}$

$a_x = 2$ $\quad a_y = +24(t+1)^{-4}$

(a) WHEN $t = 0$:

$v_x = +2$ $\quad v_y = -8$ $\quad \underline{v} = 8.25$ ft/s $\angle 76.0°$

$a_x = +2$ $\quad a_y = +24$ $\quad \underline{a} = 24.1$ ft/s^2 $\angle 85.2°$

(b) WHEN $t = 0.5$ s:

$v_x = +3$ $\quad v_y = -2.37$ $\quad \underline{v} = 3.82$ ft/s $\angle 38.3$

$a_x = +2$ $\quad a_y = +4.74$ $\quad \underline{a} = 5.14$ ft/s^2 $\angle 67.1°$

11.82

$x = t^2 - 8t + 7$ $\quad y = 0.5t^2 + 2t - 4$

$v_x = dx/dt = 2t - 8$ $\quad v_y = t + 2$

$a_x = dv_x/dt = 2$ $\quad a_y = dv_y/dt = 1$

$v^2 = v_x^2 + v_y^2 = (2t-8)^2 + (t+2)^2$

WHEN v IS MINIMUM, v^2 IS ALSO MINIMUM; THUS, WE WRITE

$\frac{d(v^2)}{dt} = 2(2t-8)(2) + 2(t+2) = 2(4t - 16 + t + 2) = 0$

$= 2(5t - 14) = 0$ $\quad t = 2.80$ s

WHEN $t = 2.8$ s:

$x = (2.8)^2 - 8(2.8) + 7$ $\quad x = -7.56$ m

$y = 0.5(2.8)^2 + 2(2.8) - 4$ $\quad y = 5.52$ m

$v_x = 2(2.8) - 8$ $\quad v_x = -2.40$ m/s

$v_y = 2.8 + 2$ $\quad v_y = +4.80$ m/s $\quad \underline{v} = 5.37$ m/s $\angle 63.4°$

11.83

$x = 48 \sin \frac{1}{2}\pi t$ $\quad y = 15t^2$

$v_x = dx/dt = 24\pi \cos \frac{1}{2}\pi t$ $\quad v_y = dy/dt = 30t$

$a_x = dv_x/dt = -12\pi^2 \sin \frac{1}{2}\pi t$ $\quad a_y = dv_y/dt = 30$

(a) WHEN $t = 1$ s:

$v_x = 24\pi \cos \frac{\pi}{2} = 0$ $\quad \underline{v} = 30$ mm/s $\uparrow$

$v_y = +30$

$a_x = -12\pi^2 \sin \frac{\pi}{2} = -12\pi^2$ $\quad \underline{a} = 122.2$ mm/s^2 $\angle 14.2°$

$a_y = +30$

(b) WHEN $t = 2$ s:

$v_x = 24\pi \cos \pi = -24\pi$ $\quad \underline{v} = 96.4$ mm/s $\angle 38.5°$

$v_y = 30(2) = 60$

$a_x = -12\pi^2 \sin \pi = 0$ $\quad \underline{a} = 30$ mm/s^2 $\uparrow$

$a_y = 30$

11.84

$x = A \cos pt$ $\quad y = B \sin pt$ $\quad$ (1)

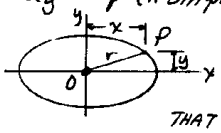
$v_x = dx/dt = -Ap \sin pt$ $\quad v_y = dy/dt = Bp \cos pt$

(a) $a_x = dv_x/dt = -Ap^2 \cos pt$ $\quad a_y = dv_y/dt = -Bp^2 \sin pt$

USING (1), WE MAY WRITE

$a_x = -p^2(A \cos pt) = -p^2 x$

$a_y = -p^2(A \sin pt) = -p^2 y$ $\quad$ THUS: $\frac{a_x}{a_y} = \frac{x}{y}$



THIS SHOWS THAT $\underline{a}$ IS DIRECTED ALONG OP. HOWEVER, EXPRESSIONS FOR a_x AND a_y SHOW THAT $\underline{a}$ IS OPPOSITE TO OP. THUS $\underline{a}$ IS DIRECTED TOWARD O.

(b) $a^2 = a_x^2 + a_y^2 = p^4(A^2 \cos^2 pt + B^2 \sin^2 pt)$ (2)

WE NOTE THAT $r^2 = x^2 + y^2 = (A^2 \cos^2 pt + B^2 \sin^2 pt)$ (3)

DIVIDING (2) BY (3), MEMBER BY MEMBER

$\frac{a^2}{r^2} = p^4$ $\quad a = p^2 r$ $\quad$ THUS $a \sim r$

11.85

$x = 16(1 - e^{-\pi t})$ $\quad y = 3 \sin 4\pi t$

$v_x = dx/dt = +16\pi e^{-\pi t}$ $\quad v_y = 12\pi \cos 4\pi t$

$a_x = dv_x/dt = -16\pi^2 e^{-\pi t}$ $\quad a_y = dv_y/dt = -48\pi^2 \sin 4\pi t$

(a) WHEN $t = 0$:

$v_x = +16\pi$ $\quad v_y = +12\pi$ $\quad \underline{v} = 62.8$ in./s $\angle 36.9°$

$a_x = -16\pi^2$ $\quad a_y = 0$ $\quad \underline{a} = 157.9$ in./s $\leftarrow$

(b) WHEN $t = 0.125$ s $= \frac{1}{8}$ s:

$v_x = +16\pi e^{-\pi/8} = 33.94$ in./s

$v_y = +12\pi \cos(4\pi/8) = 0$ $\quad \underline{v} = 33.9$ in./s $\rightarrow$

$a_x = -16\pi^2 e^{-\pi/8} = -106.63$ in./s^2

$a_y = -48\pi^2 \sin(4\pi/8) = -473.7$ in./s^2 $\quad \underline{a} = 486$ in./s^2 $\angle 77.3°$

11.86

$\underline{r} = (R \sin pt)\underline{i} + ct\underline{j} + (R \cos pt)\underline{k}$

$\underline{v} = (Rp \cos pt)\underline{i} + c\underline{j} - (Rp \sin pt)\underline{k}$

$\underline{a} = -(Rp^2 \sin pt)\underline{i} - (Rp^2 \cos pt)\underline{k}$

MAGNITUDE OF VELOCITY

$v^2 = v_x^2 + v_y^2 + v_z^2 = (Rp \cos pt)^2 + c^2 + (-Rp \sin pt)^2$

$v^2 = c^2 + R^2p^2$ $\quad v = \sqrt{c^2 + R^2p^2}$

MAGNITUDE OF ACCELERATION

$a^2 = a_x^2 + a_y^2 + a_z^2 = (-Rp^2 \sin pt)^2 + (-Rp^2 \cos pt)^2 = R^2p^4$; $\quad a = \sqrt{Rp^2}$

11.87

$\underline{r} = At\underline{i} + ABt^3\underline{j} + Bt^2\underline{k}$

OR $x = At$, $y = ABt^3$, $z = Bt^2$

WE CONFIRM THAT: $yz = (At)(Bt^2) = AB^3t^3$

FOR $A = 1$, $B = 2$:

$\underline{r} = t\underline{i} + 2t^3\underline{j} + 2t^2\underline{k}$

$\underline{v} = \underline{i} + 6t^2\underline{j} + 4t\underline{k}$

$\underline{a} = \quad + 12t\underline{j} + 4\underline{k}$

$v = \sqrt{(1)^2 + (6t^2)^2 + (4t)^2}$ $\quad v = \sqrt{1 + 36t^4 + 16t^2}$

$a = \sqrt{(12t)^2 + (4)^2}$ $\quad a = \sqrt{144t^2 + 16}$

(a) WHEN $t = 0$: $\quad v = 1$ m/s $\quad a = 4$ m/s^2

(b) WHEN $t = 2$ s: $\quad v = \sqrt{1 + 36(2)^4 + 16(2)^2} = \sqrt{641}$; $\quad v = 25.3$ m/s

$a = \sqrt{144(2)^2 + 16} = \sqrt{592}$; $\quad a = 24.3$ m/s

FOR USE IN SOLUTION OF PROB 11.135, WE COMPUTE:

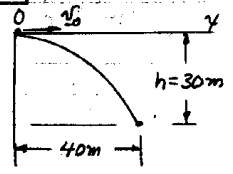
$\frac{dv}{dt} = \frac{d}{dt}\sqrt{1 + 36t^4 + 16t^2} = \frac{1}{2} \frac{144t^3 + 32t}{\sqrt{1 + 36t^4 + 16t^2}}$

WHEN $t = 0$: $\quad dv/dt = 0$ $\triangleleft$

WHEN $t = 2$ s:

$\frac{dv}{dt} = \frac{1}{2} \frac{144(2)^3 + 32(2)}{\sqrt{1 + 36(2)^4 + 16(2)^2}} = \frac{1}{2} \frac{1216}{\sqrt{641}} = 24.01$ m/s^2 $\triangleleft$

11.88



(a) VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y_0 = 0, \ (v_y)_0 = 0$

$y = \frac{1}{2} g t^2$

$30\text{m} = \frac{1}{2}(9.81\text{m/s}^2) t^2$

$t = 2.473\text{ s}$

HORIZONTAL MOTION (UNIFORM)

$x = (v_x)_0 t$

$40\text{m} = (v_x)_0 (2.473\text{ s})$

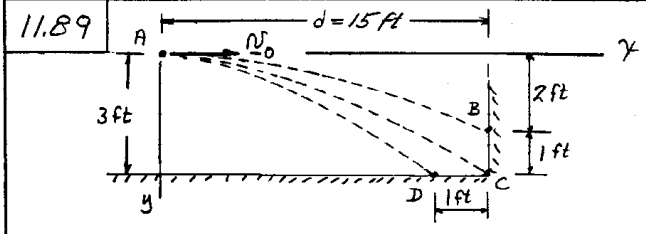
$(v_x)_0 = 16.17\text{ m/s}$ ◀

(b) FOR $h = 20\text{m}$ AND $(v_x)_0 = 16.17\text{ m/s}$, WE HAVE

$y = \frac{1}{2} g t^2 \quad 20\text{m} = \frac{1}{2}(9.81\text{m/s}^2) t^2 \quad t = 2.02\text{ s}$

$x = (v_0)_x t = (16.17\text{m/s})(2.02\text{ s}) \quad x = 32.7\text{ m}$ ◀

11.89



(a) BALL HITS C: $x_C = 15\text{ ft}, \ y_C = 3\text{ ft}$

$y_C = \frac{1}{2} g t^2 \quad 3\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.4317\text{ s}$

$x_C = v_0 t \quad 15\text{ ft} = v_0 (0.4317\text{ s}) \quad v_0 = 34.7\text{ ft/s}$ ◀

(b) BALL HITS B: $x_B = 15\text{ ft}, \ y_B = 2\text{ ft}, \ (v_0 = \text{MAXIMUM})$

$y_B = \frac{1}{2} g t^2 \quad 2\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.3525\text{ s}$

$x_B = v_0 t \quad 15\text{ ft} = v_0 (0.3525) \quad v_0 = 42.6\text{ ft/s}$ ◁

BALL HITS D: $x_D = 15 - 1 = 14\text{ ft}, \ y_D = 3\text{ ft}, \ (v_0 = \text{MINIMUM})$

$y_D = \frac{1}{2} g t^2 \quad 3\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.4317\text{ s}$

$x_D = v_0 t \quad 14\text{ ft} = v_0 (0.4317\text{ s}) \quad v_0 = 32.4\text{ ft/s}$ ◁

FOR BALL TO STRIKE CORNER BCD: $32.4\text{ ft/s} \le v_0 \le 42.6\text{ ft/s}$ ◀

11.90

SEE SKETCH OF SOLUTION OF PROB 11.89

WE NOW HAVE $d = 25\text{ ft}$

(a) BALL HITS C: $x_C = 25\text{ ft}, \ y_C = 3\text{ ft}$

$y_C = \frac{1}{2} g t^2 \quad 3\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.4317\text{ s}$

$x_C = v_0 t \quad 25\text{ ft} = v_0 (0.4317\text{ s}) \quad v_0 = 57.9\text{ ft/s}$ ◀

(b) BALL HITS B: $x_B = 25\text{ ft} \quad y_B = 2\text{ ft}$

$y_B = \frac{1}{2} g t^2 \quad 2\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.3525\text{ s}$

$x_B = v_0 t \quad 25\text{ ft} = v_0 (0.3525\text{ s}) \quad v_0 = 70.9\text{ ft/s}$ ◁

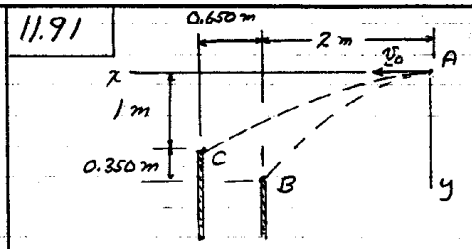
(c) BALL HITS D: $x_D = 25 - 1 = 24 \quad y_D = 3\text{ ft}$

$y_C = \frac{1}{2} g t^2 \quad 3\text{ ft} = \frac{1}{2}(32.2\text{ ft/s}^2) t^2 \quad t = 0.4317\text{ s}$

$x_C = v_0 t \quad 24\text{ ft} = v_0 (0.4317\text{ s}) \quad v_0 = 55.6\text{ ft/s}$ ◁

FOR BALL TO STRIKE CORNER BCD: $55.6\text{ ft/s} \le v_0 \le 70.9\text{ ft/s}$ ◀

11.91



SAND HITS C: LARGEST VALUE OF v_0

$x_C = 2.650\text{ m} \quad y_C = 1\text{ m}$

$y_C = \frac{1}{2} g t^2 \quad 1\text{ m} = \frac{1}{2}(9.81\text{m/s}^2) t^2 \quad t = 0.4515\text{ s}$

$x_C = v_0 t \quad 2.65\text{ m} = v_0 (0.4515\text{ s}) \quad v_0 = 5.87\text{ m/s}$ ◁

SAND HITS B: SMALLEST VALUE OF v_0

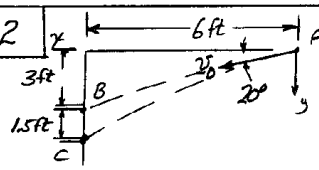
$x_B = 2\text{ m} \quad y_B = 1.350\text{ m}$

$y_B = \frac{1}{2} g t^2 \quad 1.35\text{ m} = \frac{1}{2}(9.81\text{m/s}^2) t^2 \quad t = 0.5246\text{ s}$

$x_B = v_0 t \quad 2\text{ m} = v_0 (0.5246\text{ s}) \quad v_0 = 3.81\text{ m/s}$ ◁

FOR SAND TO ENTER CHUTE: $3.81\text{ m/s} \le v_0 \le 5.87\text{ m/s}$ ◀

11.92



$(v_0)_x = v_0 \cos 20° = 0.9397 v_0$

$(v_0)_y = v_0 \sin 20° = 0.3420 v_0$

ENTERS PIPE AT B: LARGEST VALUE OF v_0 $\begin{cases} x_B = 6\text{ ft} \\ y_B = 3\text{ ft} \end{cases}$

$6\text{ ft} = 0.9397 v_0 t; \quad t = 6.385/v_0$

$3\text{ ft} = 0.3420 v_0 t + \frac{1}{2}(32.2) t^2$

SUBSTITUTE FOR t: $3 = (0.3420)(6.385) + 16.1\left(\frac{6.385}{v_0}\right)^2$

$3 = 2.1837 + 656.37/v_0^2 \quad v_0 = 28.4\text{ ft/s}$ ◁

ENTERS PIPE AT C: SMALLEST VALUE OF v_0 $\begin{cases} x_B = 6\text{ ft} \\ y_B = 4.5\text{ ft} \end{cases}$

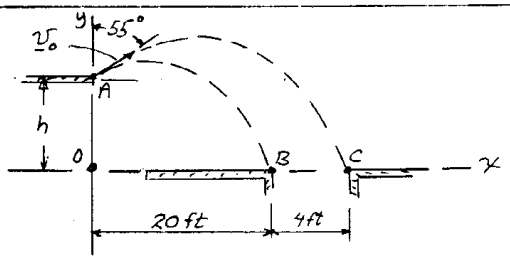
$6\text{ ft} = 0.9397 v_0 t; \quad t = 6.385/v_0$

$4.5\text{ ft} = 0.3420 v_0 t + \frac{1}{2}(32.2) t^2$

$4.5\text{ ft} = (0.3420)(6.385) + 16.1\left(\frac{6.385}{v_0}\right)^2; \quad v_0 = 16.83\text{ ft/s}$ ◁

FOR SPHERE TO ENTER PIPE: $16.83\text{ ft/s} \le v_0 \le 28.4\text{ ft/s}$ ◀

11.93



$v_0 = 25\text{ ft/s} \ \angle 35° \quad (v_x)_0 = 20.479\text{ ft/s} \quad (v_y)_0 = 14.339\text{ ft/s}$

WATER STRIKES B: SMALLEST HEIGHT h

$x_B = 20\text{ ft} \quad y_0 = h$

$x_B = (v_0)_x t \quad 20\text{ ft} = (20.479\text{ ft/s}) t \quad t = 0.9766\text{ s}$

$y_B = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

$0 = h + (14.339\text{ ft/s})(0.9766\text{ s}) - \frac{1}{2}(32.2\text{ ft/s}^2)(0.9766\text{ s})^2$

$0 = h + 14.0035 - 15.3553 \quad h = 1.359\text{ ft}$ ◁

WATER STRIKES C: LARGEST HEIGHT h

$x_C = 24\text{ ft} \quad y_0 = h$

$x_C = (v_0)_x t \quad 24\text{ ft} = (20.479\text{ ft/s}) t \quad t = 1.1719\text{ s}$

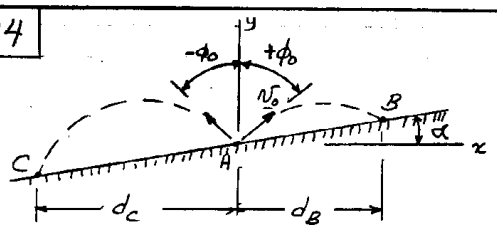
$y_C = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

$0 = h + (14.339\text{ ft/s})(1.1719\text{ s}) - \frac{1}{2}(32.2\text{ ft/s}^2)(1.1719\text{ s})^2$

$0 = h + 16.804 - 22.112 \quad h = 5.31\text{ ft}$ ◁

FOR WATER TO ENTER BC: $1.359\text{ ft} \le h \le 5.31\text{ ft}$ ◀

11.94



(a) FOR UP SLOPE WATERING ϕ_0 IS POSITIVE

$(v_x)_0 = v_0 \sin\phi_0 \qquad (v_y)_0 = v_0 \cos\phi_0$

HORIZONTAL MOTION

$a_x = 0 \qquad v_x = (v_x)_0 = v_0 \sin\phi_0$

$x = (v_x)_0 t = (v_0 \sin\phi_0)t \qquad (1)$

VERTICAL MOTION

$a_y = -g \qquad v_y = (v_y)_0 - gt = (v_0 \sin\phi_0) - gt$

$y = (v_y)_0 t - \frac{1}{2}gt^2 = (v_0 \cos\phi_0)t - \frac{1}{2}gt^2 \qquad (2)$

AT POINT B: $y = x \tan\alpha \qquad (3)$

SUBSTITUTE FROM (1) AND (2) INTO (3):

$(v_0 \cos\phi_0)t - \frac{1}{2}gt^2 = (v_0 \sin\phi_0)t(\tan\alpha)$

$t\left[v_0 \cos\phi_0 - v_0 \sin\phi_0 \tan\alpha\right] = \frac{1}{2}gt^2$

$t = 0$ AND $\quad t = \frac{2v_0}{g}(\cos\phi_0 - \sin\phi_0 \tan\alpha)$

EQ(1): $x = d_B \qquad x = d_B = (v_0 \sin\phi_0)t$

$x = d_B = \frac{2v_0^2}{g}(\sin\phi_0 \cos\phi_0 - \sin^2\phi_0 \tan\alpha) \qquad (4)$

DATA: $v_0 = 8$ m/s, $\phi_0 = 40°$, $\alpha = 10°$, $g = 9.81$ m/s²

$d_B = \frac{2(8\text{ m/s})^2}{9.81\text{ m/s}^2}\left[\sin 40° \cos 40° - \sin^2 40° \tan 10°\right]$

$d_B = (13.048\text{ m})(0.49240 - 0.07285) \qquad d_B = 5.47\text{ m}$ ◀

(b) FOR DOWN SLOPE WATERING: WE NOW HAVE $\phi_0 = -40°$

SUBSTITUTE DATA INTO EQ (4),

$x = -d_C = \frac{2(8\text{ m/s})^2}{9.81\text{ m/s}^2}\left[\sin(-40°)\cos(-40°) - \sin^2(-40°)\tan 10°\right]$

$x = -d_C = (13.048\text{ m})(-0.49240 - 0.07285) \qquad d_C = 7.38\text{ m}$ ◀

COMMENT: TO DETERMINE THE LARGEST VALUES OF d_B AND d_C WE SHALL WRITE EQ (4) IN TERMS OF ϕ

$x = \frac{2v_0^2}{g}(\sin\phi \cos\phi - \sin^2\phi \tan\alpha)$

OR:

$x = \frac{2v_0^2}{g}\left[\frac{1}{2}\sin 2\phi - \frac{1}{2}(1 - \cos 2\phi)\tan\alpha\right]$

FOR x_{max}: $\frac{dx}{d\phi} = \frac{2v_0^2}{g}\left[\cos 2\phi - \sin 2\phi \tan\alpha\right] = 0$

$1 - \frac{\sin 2\phi_m}{\cos 2\phi_m}\tan\alpha = 0 \qquad \tan 2\phi_m = \cot\alpha$

FOR $\alpha = 10°$: $\tan 2\phi_m = \cot 10° = 5.6713$

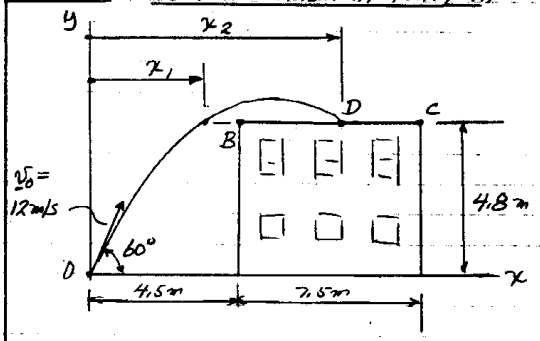
$2\phi_m = 80° \quad$ AND $\quad 2\phi_m = -100°$

$\phi_{max} = 40° \qquad \phi_{max} = -50°$

WE OBSERVE THAT A MAXIMUM DOES NOT OCCUR IN THE GIVEN RANGE $-40° < \phi_0 < 40°$. THUS, THE COMPUTED VALUES OF d_B AND d_D DO DELINEATE THE AREA OF THE SLOPE WHICH IS WATERED.

11.95

WE PLACE ORIGIN AT POINT A.



VERTICAL MOTION $\quad (v_0)_y = 12 \cos 30° = 10.39$ m/s

$y = (v_0)_y t - \frac{1}{2}gt^2$

AT D: $y = 4.8$ m $\qquad 4.8 = 10.39t - \frac{1}{2}(9.81)t^2$

$t^2 - 2.1183t + 0.9786 = 0$

$t_1 = 0.681$ s $\qquad t_2 = 1.437$ s

HORIZONTAL MOTION

$(v_0)_x = 12 \sin 30°, \qquad x = (v_0)_x t = (12 \sin 30°)t = 6t$

WHEN $t_1 = 0.681$ s $\qquad x_1 = 6(0.681) = 4.086$ m

$t_2 = 1.437$ s $\qquad x_2 = 6(1.437) = 8.622$ m

SINCE $x_1 = 4.086$ m IS LESS THAN 4.5 m, THE STREAM CLEARS THE ROOF AT B.

FROM SKETCH WE FIND:

$BD = x_2 - 4.5\text{ m} = 8.622 - 4.5 = 4.122$ m

STREAM STRIKES ROOF 4.12 m FROM B ◀

11.96

USE FIGURE OF SOLUTION OF PROB 11.95. v_0 = UNKNOWN

$(v_0)_x = v_0 \cos 60° = 0.5v_0 \qquad (v_0)_y = v_0 \sin 60° = 0.866v_0$

VERTICAL MOTION: $y = (v_0)_y t - \frac{1}{2}gt^2 = 0.866v_0 t - \frac{1}{2}(9.81)t^2 \qquad (1)$

HORIZONTAL MOTION: $x = (v_0)_x t = 0.5v_0 t \qquad (2)$

FOR STREAM TO HIT C: $\quad x_C = 12$ m, $y_C = 4.8$ m

EQ(2): $\quad 12\text{ m} = 0.5v_0 t \qquad t = 24/v_0$

EQ(1): $\quad 4.8\text{ m} = 0.866v_0 t - 4.905t^2$

$4.8 = 0.866(24) - 4.905(24/v_0)^2$

$15.98 = 4.905(24/v_0)^2 \qquad v_0 = 13.29$ m/s ◁

FOR STREAM TO HIT B: $\quad x_B = 4.5$ m $\quad y_B = 4.8$ m

EQ(2): $\quad 4.5\text{ m} = 0.5v_0 t \qquad t = 9/v_0$

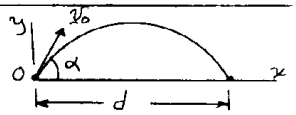
EQ(1): $\quad 4.8\text{ m} = 0.866v_0 t - 4.905t^2$

$4.8 = 0.866(9) - 4.905(9/v_0)^2$

$2.994 = 4.905(9/v_0)^2 \qquad v_0 = 11.52$ m/s ◁

FOR STREAM TO HIT ROOF: $\quad 11.52\text{ m/s} \le v_0 \le 13.29\text{ m/s}$ ◀

11.97



$(v_x)_0 = v_0 \cos\alpha$

$(v_y)_0 = v_0 \sin\alpha$

VERTICAL MOTION

$y = (v_0)_y t - \frac{1}{2}gt^2 = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$

FOR $y = 0$: $\quad 0 = t\left[v_0 \sin\alpha - \frac{1}{2}gt\right]$

$t = 0$ (AT GUN), AND $t = \frac{2v_0}{g}\sin\alpha$ (AT TARGET)

HORIZONTAL MOTION

$x = (v_x)_0 t$

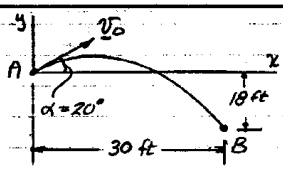
$d = (v_0 \cos\alpha)\left(\frac{2v_0}{g}\sin\alpha\right) = \frac{2v_0^2}{g}\sin\alpha\cos\alpha = \frac{v_0^2}{g}\sin 2\alpha$

$v_0^2 = gd/\sin 2\alpha$

(a) POINT B, $\alpha = 25°$, $d = 2.5$ km: $\quad v_0^2 = (9.81)(2500)/\sin 50°$; $\quad v_0 = 178.9$ m/s ◀

(b) POINT C, $\alpha = 25°$, $d = 3$ km: $\quad v_0^2 = (9.81)(3000)/\sin 50°$; $\quad v_0 = 196.0$ m/s ◀

11.98



$(v_0)_x = v_0 \cos 20° = 0.9397\, v_0$

$(v_0)_y = v_0 \sin 20° = 0.3420\, v_0$

HORIZONTAL MOTION: $x = (v_0)_x t = (0.9397\, v_0)t$

AT B: $30\text{ ft} = (0.9397\, v_0)t$ $\quad t = 31.925/v_0$ (1)

VERTICAL MOTION: $y = (v_0)_y t - \frac{1}{2} g t^2$

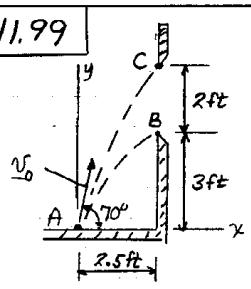
AT B: $-18\text{ ft} = v_0(0.3420)t - \frac{1}{2}(32.2)t^2$

USE EQ.(1): $-18 = (0.3420)(31.925) - (16.1)\left(\frac{31.925}{v_0}\right)^2$

$-28.918 = -(16.1)\left(\frac{31.925}{v_0}\right)^2$

$v_0 = 23.82\text{ ft/s}$ $\quad v_0 = 23.8\text{ ft/s}$ ◀

11.99



$(v_x)_0 = v_0 \cos 70° = 0.3420\, v_0$

$(v_y)_0 = v_0 \sin 70° = 0.9397\, v_0$

HORIZONTAL MOTION: $x = (v_x)_0 t = (0.3420\, v_0)t$

AT B OR C: $x = 2.5\text{ ft}$

$2.5\text{ ft} = (0.3420)v_0 t$

$t = 7.31/v_0$ (1)

VERTICAL MOTION: $y = (v_y)_0 t - \frac{1}{2} g t^2$

$y = (0.9397)v_0 t - \frac{1}{2}(32.2)t^2$

USE EQ(1): $y = (0.9397)(7.31) - 16.1\left(\frac{7.31}{v_0}\right)^2$ (2)

AT POINT B: $y_B = 3\text{ ft}$ $\quad (v_0 = \text{MINIMUM})$

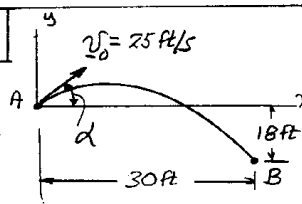
EQ(2): $3 = 6.869 - 16.1\left(\frac{7.31}{v_0}\right)^2$ $\quad v_0 = 14.91\text{ ft/s}$ ◁

AT POINT C: $y_B = 5\text{ ft}$ $\quad (v_0 = \text{MAXIMUM})$

EQ(2): $5 = 6.869 - 16.1\left(\frac{7.31}{v_0}\right)^2$ $\quad v_0 = 21.5\text{ ft/s}$ ◁

BALL ENTERS BC FOR: $14.91\text{ ft/s} \le v_0 \le 21.5\text{ ft/s}$ ◀

11.100



$(v_0)_x = v_0 \cos\alpha = 25\cos\alpha$

$(v_0)_y = v_0 \sin\alpha = 25\sin\alpha$

HORIZONTAL MOTION $\quad x = (v_x)_0 t$

AT B: $x_B = 30\text{ ft}$ $\quad 30 = (25\cos\alpha)t$

$t = \frac{30}{25\cos\alpha}$ (1)

VERTICAL MOTION $\quad y = (v_y)_0 t - \frac{1}{2} g t^2$

AT B: $y_B = -18\text{ ft}$ $\quad -18 = (25\sin\alpha)t - \frac{1}{2}(32.2)t^2$

USE EQ(1): $-18 = (25\sin\alpha)\left(\frac{30}{25\cos\alpha}\right) - 16.1\left(\frac{30}{25}\right)^2 \frac{1}{\cos^2\alpha}$

SUBSTITUTE $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

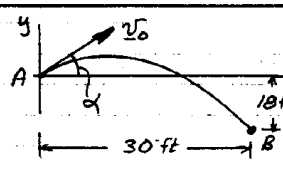
$-18 = 30\tan\alpha - 23.184(1 + \tan^2\alpha)$

$\tan^2\alpha - 1.294\tan\alpha + 0.2236 = 0$

$\tan\alpha = 0.2054$ OR $\tan\alpha = 1.0886$

$\alpha = 11.6°$ OR $\alpha = 47.4°$ ◀

*11.101



$(v_x)_0 = v_0 \cos\alpha$

$(v_y)_0 = v_0 \sin\alpha$

HORIZONTAL MOTION: $x = (v_x)_0 t = (v_0 \cos\alpha)t$

AT B: $30\text{ ft} = v_0 t \cos\alpha$ $\quad t = \frac{30}{v_0 \cos\alpha}$ (1)

VERTICAL MOTION: $y = (v_y)_0 t - \frac{1}{2} g t^2$

AT B: $-18 = (v_0 \sin\alpha)t - \frac{1}{2}(32.2)t^2$

USE EQ(1) $\quad -18 = 30\frac{\sin\alpha}{\cos\alpha} - 16.1\left(\frac{30}{v_0}\right)^2 \frac{1}{\cos^2\alpha}$

SUBSTITUTE $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

$-18 = 30\tan\alpha - 16.1\left(\frac{30}{v_0}\right)^2 (1 + \tan^2\alpha)$

$\tan^2\alpha - \left[\frac{v_0^2}{(16.1)(30)}\right]\tan\alpha + \left[1 - \frac{18 v_0^2}{(16.1)(30)^2}\right] = 0$ (2)

(a) WE SEEK THE SMALLEST VALUE OF v_0 FOR WHICH A REAL SOLUTION EXISTS FOR EQ(2). WE NOTE THAT EQ(2) IS OF THE FORM

$A\tan^2\alpha + B\tan\alpha + C = 0$

THUS, $B^2 - 4AC = 0$: $\left[\frac{v_0^2}{(16.1)(30)}\right]^2 - 4\left[1 - \frac{18 v_0^2}{(16.1)(30)^2}\right] = 0$

$(v_0^2)^2 + (4)(18)(16.1)v_0^2 - 4(16.1)^2(30)^2 = 0$

$(v_0^2)^2 + 1159\, v_0^2 - 933{,}156 = 0$

$v_0^2 = \frac{1}{2}\left[-1159 \pm \sqrt{(1159)^2 + 4(933{,}156)}\right] = \frac{1}{2}(-1159 \pm 2253)$

$v_0^2 = -1706$ (IMPOSSIBLE); $\quad v_0^2 = 547$ $\quad v_0 = 23.4\text{ ft/s}$ ◀

(b) SUBSTITUTE $v_0^2 = 547$ AND SOLVE FOR $\tan\alpha$:

$\tan^2\alpha - \left[\frac{(547)}{(16.1)(30)}\right]\tan\alpha - 4\left[1 - \frac{18(547)}{(16.1)(30)^2}\right] = 0$

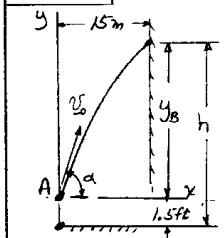
SINCE $\sqrt{B^2 - 4AC} = 0$ FOR $v_0^2 = 547$, WE HAVE

$\tan\alpha = \frac{1}{2}\left[-B \pm 0\right] = \frac{1}{2}\left[\frac{547}{(16.1)(30)}\right] = 0.5662$

$\alpha = 29.5°$ ◀

*11.102

$v_0 = 18\text{ m/s}$ $\quad (v_x)_0 = 18\cos\alpha$ $\quad (v_y)_0 = 18\sin\alpha$



HORIZONTAL MOTION $\quad x = (v_x)_0 t = (18\cos\alpha)t$

AT B: $x_B = 15\text{ m} = (18\cos\alpha)t$ $\quad t = \frac{5}{6\cos\alpha}$ (1)

VERTICAL MOTION $\quad y = (v_y)_0 t - \frac{1}{2} g t^2$

AT B: $y_B = (18\sin\alpha)t - \frac{1}{2}(9.81)t^2$

USE EQ(1): $y_B = (18\sin\alpha)\frac{5}{6\cos\alpha} - 4.905\left(\frac{5}{6\cos\alpha}\right)^2$

$y_B = 15\tan\alpha - 4.905\left(\frac{5}{6}\right)^2 \frac{1}{\cos^2\alpha}$

SUBSTITUTE $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

$y_B = 15\tan\alpha - 3.406(1 + \tan^2\alpha)$ (2)

FOR MAXIMUM h, WE SEEK MAXIMUM y_B. NOTE: $y_B = f(\tan\alpha)$

$\frac{d(y_B)}{d(\tan\alpha)} = 15 - 6.812\tan\alpha = 0$

$\tan\alpha = \frac{15}{6.812} = 2.202$ $\quad \alpha = 65.6°$ ◀

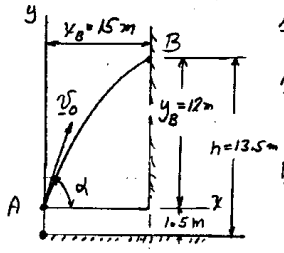
SUBSTITUTING $\tan\alpha = 2.202$ INTO EQ(2), WE HAVE

$(y_B)_{max} = 15(2.202) - 3.406\left[1 + (2.202)^2\right] = 13.11\text{ m}$

MAXIMUM h:

$h_{max} = (y_B)_{max} + 1.5\text{ m} = 13.11\text{ m} + 1.5\text{ m}$; $\quad h_{MAX} = 14.61\text{ m}$ ◀

11.103



$\underline{v}_0 = 18$ m/s $\quad (v_x)_0 = 18\cos\alpha \quad (v_y)_0 = 18\sin\alpha$

HORIZONTAL MOTION $\quad x = (v_x)_0 t = (18\cos\alpha)t$

AT B: $\quad x_B = 15\text{ m} = (18\cos\alpha)t$

$$t = \frac{0.8333}{\cos\alpha} \qquad (1)$$

VERTICAL MOTION $\quad y = (v_y)_0 t - \frac{1}{2}gt^2$

$$y = (18\sin\alpha)t - \frac{1}{2}(9.81)t^2$$

AT B: $\quad y_B = 12\text{ m} = (18\sin\alpha)\left(\frac{0.8333}{\cos\alpha}\right) - 4.905\left(\frac{0.8333}{\cos\alpha}\right)^2$

SUBSTITUTE $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

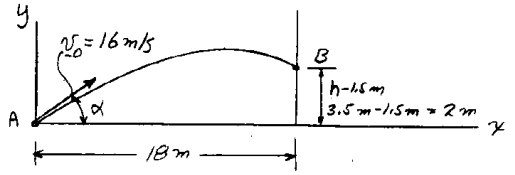
$$12 = 15\tan\alpha - 3.406(1 + \tan^2\alpha)$$

$$\tan^2\alpha - 4.404\tan\alpha + 4.523 = 0$$

$\tan\alpha = 2.773 \quad \alpha = 70.2°$ $\qquad$ $\tan\alpha = 1.631 \quad \alpha = 58.5°$

STREAM HITS B FOR $\alpha = 58.5°$ OR $\alpha = 70.2°$ ◀

11.104



$v_0 = 16$ m/s $\quad (v_x)_0 = 16\cos\alpha \quad (v_y)_0 = 16\sin\alpha$

HORIZONTAL MOTION $\quad x = (v_x)_0 t = (16\cos\alpha)t$

AT B: $\quad x_B = 18\text{ m} = (16\cos\alpha)t \qquad t = \frac{1.125}{\cos\alpha} \qquad (1)$

VERTICAL MOTION: $\quad y = (v_y)_0 t - \frac{1}{2}gt^2$

AT B: $\quad y_B = 2\text{ m} = (16\sin\alpha)t - \frac{1}{2}(9.81)t^2$

USE EQ(1): $\quad 2 = (16\sin\alpha)\left(\frac{1.125}{\cos\alpha}\right) - 4.905\left(\frac{1.125}{\cos\alpha}\right)^2$

SUBSTITUTE $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

$$2 = 18\tan\alpha - 6.2079(1 + \tan^2\alpha)$$

$$\tan^2\alpha - 2.8995\tan\alpha + 1.3222 = 0$$

$\tan\alpha = 2.3328 \quad \alpha = 66.80°$ $\qquad$ $\tan\alpha = 0.56679 \quad \alpha = 29.54°$

CHECK WHETHER BALL HITS CEILING

FOR $\alpha = 66.80°$ $\quad (v_y)_0 = 16\sin 66.80° = 14.706$ ft/s

$$v_y = (v_y)_0 - gt = 14.706 - 9.81t$$

FOR MAX HEIGHT $\quad v_y = 0 = 14.706 - 9.81t \qquad t_m = 1.499$ s

$$y = (v_0)_y t - \frac{1}{2}gt^2$$

$$y = (14.706)t - \frac{1}{2}(9.81)t^2$$

WHEN $t = t_m = 1.365$ s: $\quad y_{max} = (14.706)(1.499) - 4.905(1.499)^2$

$y_{max} = 11.027$ m

SINCE CEILING IS AT $y = 4.5$ m ABOVE A,

FOR $\alpha = 62.93°$ BALL HITS CEILING.

FOR $\alpha = 29.54°$ $\quad (v_y)_0 = 16\sin 29.54° = 7.8885$ m

$$v_y = (v_y)_0 - gt = 7.8885 - 9.81t$$

FOR MAX HEIGHT $\quad v_y = 0 = 7.8885 - 9.81t \qquad t_m = 0.8041$ s

$$y = (v_y)_0 t - \frac{1}{2}gt^2 = 7.8885(t) - \frac{1}{2}(9.81)t^2$$

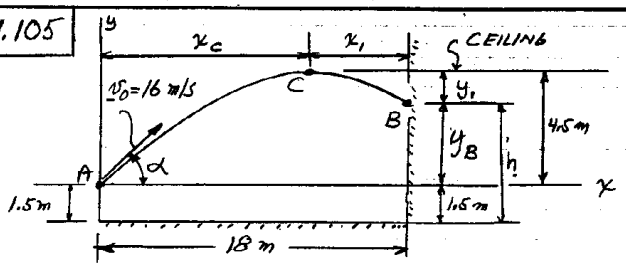
WHEN $t = t_m = 0.8125$ s: $\quad y_{max} = 7.8885(0.8041) - \frac{1}{2}(9.81)(0.8041)^2$

$y_{max} = 3.172$ m

SINCE CEILING IS AT $y = 4.5$ m ABOVE A,

BALL WILL HIT POINT B FOR $\alpha = 29.5°$ ◀

11.105



FOR MAXIMUM y_B, BALL REACHES CEILING WITH A HORIZONTAL VELOCITY.

VERTICAL MOTION (UNIFORMLY ACCELERATED)

$$v_y^2 = (v_y)_0^2 + 2a y_C$$

AT C: $v_y = 0$, $y_C = 4.5$ m: $\quad 0 = (v_y)_0^2 - 2(9.81\text{ m/s}^2)(4.5\text{ m})$

$(v_y)_0 = 9.396$ m/s

FROM A TO C: $\quad t_{CA} = \frac{(v_y)_0}{g} = \frac{9.396}{9.81} = 0.9578$ s

AT A: $\quad v_0^2 = (v_x)_0^2 + (v_y)_0^2$

$(16\text{ m/s})^2 = (v_x)_0^2 + (9.396\text{ m/s})^2 \qquad (v_x)_0 = 12.95$ m/s

HORIZONTAL MOTION:

$$x_C = (v_x)_0 t_{CA} = (12.95\text{ m/s})(0.9578\text{ s}) = 12.40\text{ m}$$

$$x_1 = 18\text{ m} - x_C = 18\text{ m} - 12.40\text{ m} = 5.60\text{ m}$$

FROM C TO B:

$$t_{CB} = \frac{x_1}{(v_x)_0} = \frac{5.60\text{ m}}{12.95\text{ m/s}} = 0.4321\text{ s}$$

VERTICAL MOTION:

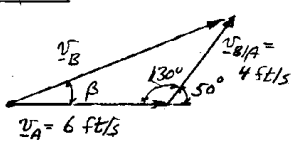
$$y_1 = \frac{1}{2}g t_{CB}^2 = \frac{1}{2}(9.81\text{ m/s}^2)(0.4321\text{ s})^2 = 0.916\text{ m}$$

FROM SKETCH WE NOTE THAT $y_B = 4.5\text{m} - 0.916\text{ m} = 3.584$ m

$h = y_B + 1.5 = 3.584\text{ m} + 1.5\text{ m} \qquad h = 5.08$ m ◀

11.106

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$$v_B^2 = 4^2 + 6^2 - 2(4)(6)\cos 130°$$

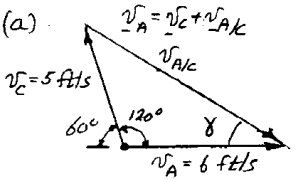
$v_B = 9.102$ ft/s

$$\frac{\sin\beta}{4\text{ ft/s}} = \frac{\sin 130°}{9.102\text{ ft/s}}$$

$\beta = 19.7°$

$\underline{v}_B = 9.10$ ft/s ∠ 19.7°

(a)



$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}$

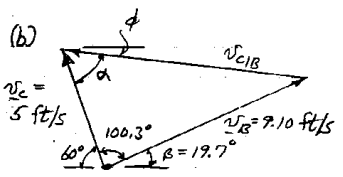
$$v_{A/C}^2 = 5^2 + 6^2 - 2(5)(6)\cos 120°$$

$v_{A/C} = 9.539$ ft/s

$$\frac{\sin\gamma}{5\text{ ft/s}} = \frac{\sin 120°}{9.539\text{ ft/s}}$$

$\gamma = 27.0°$ $\qquad$ $\underline{v}_{A/C} = 9.54$ ft/s ∠ 27.0° ◀

(b)



$$v_{C/B}^2 = 5^2 + (9.10)^2 - 2(5)(9.10)\cos 100.3°$$

$v_{C/B} = 11.14$ ft/s

$$\frac{\sin\alpha}{9.10\text{ ft/s}} = \frac{\sin 100.3°}{11.14\text{ ft/s}}$$

$\alpha = 53.5°$

$\alpha + \phi = 60°$; $\quad 53.5° + \phi = 60°$ $\quad \phi = 6.5°$ $\quad \underline{v}_{C/B} = 11.14$ ft/s ∠ 6.5° ◀

(c) IN 10-S INTERVAL: $\quad \underline{r}_{B/C} = \underline{v}_{B/C}(10\text{s}) = 111.4\text{ ft} \angle 6.5°$ ◀

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

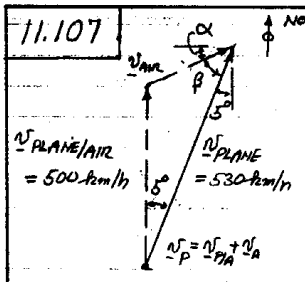
$\underline{v}_A = \underline{v}_C + \underline{v}_{C/A}$

ADD EQUATIONS

$$\underline{v}_A + \underline{v}_B + \underline{v}_C = \underline{v}_A + \underline{v}_B + \underline{v}_C + \underline{v}_{B/A} + \underline{v}_{C/B} + \underline{v}_{C/A}$$

THUS: $\quad \underline{v}_{B/A} + \underline{v}_{C/B} + \underline{v}_{C/A} = 0$ ◀

11.107



LAW OF COSINES

$v_{AIR}^2 = (500)^2 + (530)^2 - 2(500)(530)\cos 5°$

$v_{AIR} = 54.0$ km/h

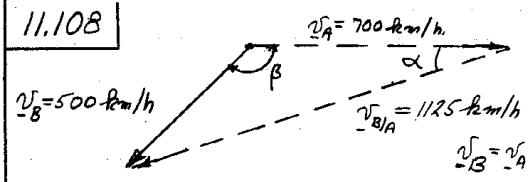
LAW OF SINES

$\frac{\sin\beta}{500} = \frac{\sin 5°}{54.0}$

$\beta = 53.8°$

$\alpha = 90° - 5° - 53.8° = 31.2°$ $\quad \underline{v}_{AIR} = 54.0$ km/h $\angle 31.2°$ ◀

11.108



LAW OF COSINES

$1125^2 = 500^2 + 700^2 - 2(500)(700)\cos\beta$

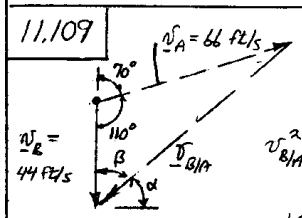
$\cos\beta = -0.7509$

$\beta = 138.7°$

WE NOTE THAT 138.7° = 48.7°

DIRECTION OF AIRPLANE B IS 48.7° WEST OF SOUTH ◀

11.109



v_A = 45 mi/h = 66 ft/s

v_B = 30 mi/h = 44 ft/s

LAW OF COSINES

$v_{B/A}^2 = 66^2 + 44^2 - 2(66)(44)\cos 110°$

$v_{B/A} = 90.99$ ft/s

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

LAW OF SINES

$\frac{\sin\beta}{66} = \frac{\sin 110°}{90.99}$ $\quad \beta = 42.97°$

$\alpha = 90° - \beta = 90° - 42.97° = 47.03°$

(a) RELATIVE VELOCITY: $\quad \underline{v}_{B/A} = 91.0$ ft/s $\angle 47.0°$ ◀

(b) CHANGE IN POSITION FOR $\Delta t = 4$ s.

$\Delta \underline{r}_{B/A} = \underline{v}_{B/A}\Delta t = (91.0 \text{ ft/s})(4\text{s})$ $\quad \underline{r}_{B/A} = 364$ ft $\angle 47.0°$ ◀

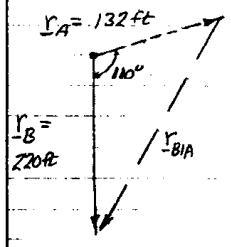
(c) DISTANCE BETWEEN AUTOS 2 seconds AFTER AUTO A HAS PASSED INTERSECTION

AUTO A TRAVELS FOR 2 s.

$\underline{v}_A = 66$ ft/s $\angle 20°$

$r_A = v_A t = (66 \text{ ft/s})(2\text{s}) = 132$ ft

$\underline{r}_A = 132$ ft $\angle 20°$



AUTO B $\quad v_B = 44$ ft/s ↓

$r_B = v_B t = (44 \text{ ft/s})(5\text{s}) = 220$ ft ↓

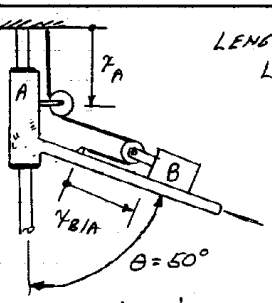
LAW OF COSINES

$r_{B/A}^2 = (132)^2 + (220)^2 - 2(132)(220)\cos 110°$

$r_{B/A} = 292.7$ ft

$\underline{r}_B = \underline{r}_A + \underline{r}_{B/A}$ $\quad$ DISTANCE BETWEEN AUTOS = 293 ft ◀

11.110



LENGTH OF CABLE = CONSTANT

$L = \gamma_A + 2\gamma_{B/A} = \text{CONSTANT}$

$v_A + 2v_{B/A} = 0$ (1)

$a_A + 2a_{B/A} = 0$ (2)

DATA: $\quad \underline{a}_A = 15$ in./s² ↓ $\quad \underline{v}_A = 9$ in./s ↓

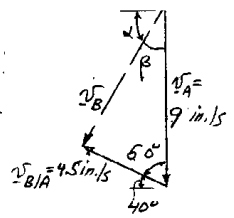
EQS (1) AND (2) $\quad a_A = -2a_{B/A}$ $\quad v_A = -2v_{B/A}$

$15 = -2a_{B/A}$ $\quad 9 = -2v_{B/A}$

$a_{B/A} = -7.5$ in./s² $\quad v_{B/A} = -4.5$ in./s

$\underline{a}_{B/A} = 7.5$ in./s² $\angle 40°$ $\quad \underline{v}_{B/A} = -4.5$ in./s $\angle 40°$

(a) VELOCITY OF B $\quad \underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



LAW OF COSINES

$v_B^2 = (9)^2 + (4.5)^2 - 2(9)(4.5)\cos 50°$

$v_B = 7.013$ in./s

LAW OF SINES

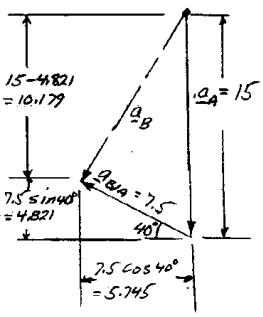
$\frac{\sin\beta}{4.5} = \frac{\sin 50°}{7.013}$ $\quad \beta = 29.44°$

$\alpha = 90° - \beta = 90° - 29.44° = 60.56°$

$\underline{v}_B = 7.01$ in./s $\angle 60.6°$ ◀

(b) ACCELERATION OF B: $\underline{a}_B$ MAY BE FOUND BY USING ANALYSIS SIMILAR TO THAT USED ABOVE FOR $\underline{v}_B$. AN ALTERNATE METHOD IS

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

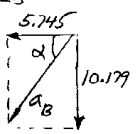


$\underline{a}_B = 15$ in./s² ↓ $+ 7.5$ in./s² $\angle 40°$

$= -15\,\underline{j} - (7.5\cos 40°)\underline{i} + (7.5\sin 40°)\underline{j}$

$= -15\,\underline{j} - 5.745\,\underline{i} + 4.821\,\underline{j}$

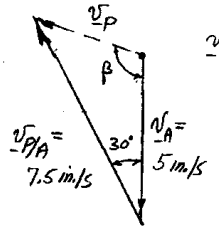
$\underline{a}_B = -5.745\,\underline{i} - 10.179\,\underline{j}$



$\underline{a}_B = 11.69$ in./s² $\angle 60.6°$ ◀

11.111

(a) FOR $\theta = 30°$, $\underline{v}_{P/A} = 7.5$ in/s ∢ 30°



$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

LAW OF COSINES

$v_P^2 = (7.5)^2 + (5)^2 - 2(5)(7.5)\cos 30°$

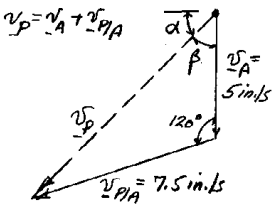
$v_P = 4.037$ in/s

LAW OF SINES

$\frac{\sin\beta}{7.5} = \frac{\sin 30°}{4.037}$ $\quad \beta = 111.7°$

$\underline{v}_P = 4.04$ in/s ∢ 21.7° ◀

(b) FOR $\theta = 120°$, $\underline{v}_{P/A} = 7.5$ in/s ∢ 30°



LAW OF COSINES

$v_P^2 = (7.5)^2 + (5)^2 - 2(5)(7.5)\cos 120°$

$v_P = 10.897$ in/s

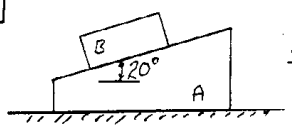
LAW OF SINES

$\frac{\sin\beta}{7.5} = \frac{\sin 120°}{10.897}$ $\quad \beta = 36.6°$

$\alpha = 90 - \beta = 90° - 36.6 = 53.4°$

$\underline{v}_P = 10.90$ in/s ∢ 53.4° ◀

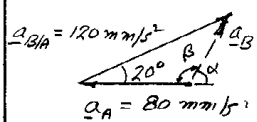
11.112



$\underline{a}_A = 80$ mm/s² ←

$\underline{a}_{B/A} = 120$ mm/s² ∢ 20°

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$



LAW OF COSINES

$a_B^2 = (80)^2 + (120)^2 - 2(80)(120)\cos 20°$

$a_B = 52.52$ mm/s²

LAW OF SINES

$\frac{\sin\beta}{120 \text{ mm/s}^2} = \frac{\sin 20°}{52.52 \text{ mm/s}^2}$ $\quad \beta = 128.6°$

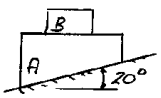
$\alpha = 180 - 128.6 = 51.4°$

(a) ACCELERATION OF B: $\underline{a}_B = 52.5$ mm/s² ∢ 51.4° ◀

(b) VELOCITY OF B: SINCE a_B = CONSTANT AND $v_B = 0$ WHEN $t = 0$, WHEN $t = 3$s: $v_B = a_B(3s) = (52.52 \text{ mm/s}^2)(3s) = 157.56$ mm/s

$\underline{v}_B = 157.6$ mm/s ∢ 51.4° ◀

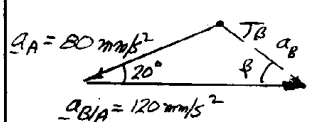
11.113



$\underline{a}_A = 80$ mm/s² ∢ 20°

$\underline{a}_{B/A} = 120$ mm/s² →

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$



LAW OF COSINES

$a_B^2 = (80)^2 + (120)^2 - 2(80)(120)\cos 20°$

$a_B = 52.52$ mm/s²

LAW OF SINES

$\frac{\sin\beta}{80 \text{ mm/s}^2} = \frac{\sin 20°}{52.52 \text{ mm/s}^2}$ $\quad \beta = 31.4°$

(a) ACCELERATION OF B: $\underline{a}_B = 52.5$ mm/s² ∢ 31.4° ◀

(b) VELOCITY OF B: SINCE a_B = CONSTANT AND $v_B = 0$ WHEN $t = 0$, WHEN $t = 3$s: $v_B = a_B(3s) = (52.52 \text{ mm/s}^2)(3s)$

$v_B = 157.56$ mm/s

$\underline{v}_B = 157.6$ mm/s ∢ 31.4° ◀

11.114

AS SAND HITS THE BELT:

$(v_S)_x = (v_{SAND})_x$ = CONSTANT = 6 ft/s ←

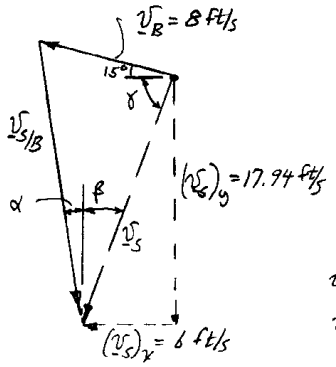
SINCE SAND FALLS 5 ft, (UNIF. ACCEL. MOTION)

$(v_S)_y = (v_{SAND})_y = \sqrt{2gh} = \sqrt{2(32.2 \text{ ft/s}^2)(5\text{ft})}$

$= 17.94$ ft/s ↓

RELATIVE VELOCITY $\quad \underline{v}_B = \underline{v}_{BELT} = 8$ ft/s ∢ 15°

$\underline{v}_S = \underline{v}_B + \underline{v}_{S/B}$



$\tan\beta = \frac{6 \text{ ft/s}}{17.94 \text{ ft/s}} = 0.3344$

$\beta = 18.49°$

$\gamma = 90° - \beta = 90° - 18.49$

$\gamma = 71.51°$

$v_S = \sqrt{(6)^2 + (17.94)^2}$

$v_S = 18.92$ ft/s

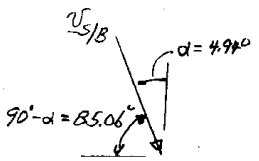
LAW OF COSINES

$v_{S/B}^2 = (18.91)^2 + (8)^2 - 2(18.91)(8)\cos(15° + 71.51°)$

$v_{S/B} = 20.08$ ft/s

LAW OF SINES

$\frac{\sin(\alpha+\beta)}{8 \text{ ft/s}} = \frac{\sin(15°+\gamma)}{20.08 \text{ ft/s}}$; $\frac{\sin(\alpha+18.49)^\circ}{8 \text{ ft/s}} = \frac{\sin(15°+71.51°)}{20.08 \text{ ft/s}}$



$(\alpha + 18.49°) = 23.43°$

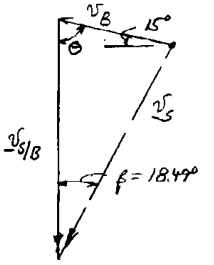
$\alpha = 4.94°$

$\underline{v}_{S/B} = 20.1$ ft/s ∢ 85.1° ◀

11.115

FROM PROB 11.114 SAND HITS BELT WITH $v_S = 18.92$ ft/s ∢ 71.5

(a) FOR $\underline{v}_{S/B}$ TO BE VERTICAL



$\underline{v}_S = \underline{v}_B + \underline{v}_{S/B}$

$\theta = 90° - 15° = 75°$

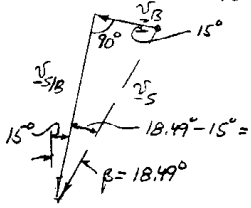
LAW OF SINES

$\frac{v_S}{\sin\theta} = \frac{v_B}{\sin 18.49°}$

$\frac{18.92}{\sin 75°} = \frac{v_B}{\sin 18.49°}$ $\quad v_B = 6.212$ ft/s

$\underline{v}_B = 6.21$ ft/s ∢ 15° ◀

(b) FOR MINIMUM $v_{S/B}$, WE MAKE $\underline{v}_{S/B} \perp \underline{v}_B$.



$v_B = v_S \sin 3.49°$

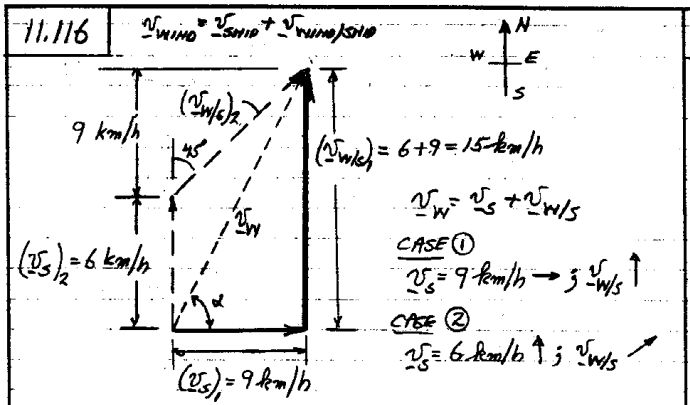
$= (18.92 \text{ ft/s}) \sin 3.49°$

$= 1.152$ ft/s

$\underline{v}_B = 1.152$ ft/s ∢ 15° ◀

11.116

$\underline{v}_{WIND} = \underline{v}_{SHIP} + \underline{v}_{WIND/SHIP}$



$\underline{v}_W = \underline{v}_S + \underline{v}_{W/S}$

CASE ① $\underline{v}_S = 9$ km/h → ; $\underline{v}_{W/S}$ ↑

CASE ② $\underline{v}_S = 6$ km/h ↑ ; $\underline{v}_{W/S}$ ↗

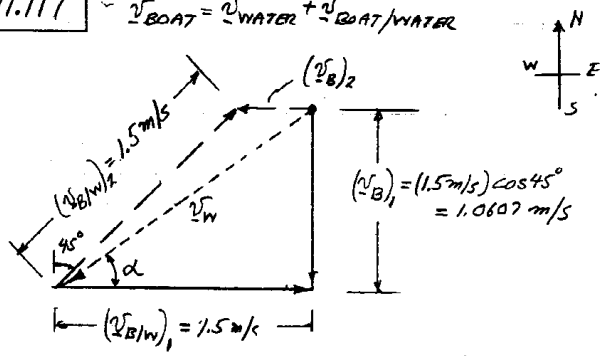
$\tan\alpha = \frac{15}{9} = 1.6667 \qquad \alpha = 59.0°$

$v_W = \sqrt{9^2 + 15^2} = 17.49$ km/h

$\underline{v}_W = 17.49$ km/h ∠ 59.0° ◀

11.117

$\underline{v}_{BOAT} = \underline{v}_{WATER} + \underline{v}_{BOAT/WATER}$



CASE ① : $\underline{v}_{B/W} = 1.5$ m/s → ; $\underline{v}_B$ = ↕

CASE ② : $\underline{v}_{B/W} = 1.5$ m/s ↗ ; $\underline{v}_B$ = ↔

FROM VECTOR DIAGRAM: $\tan\alpha = \frac{1.0607}{1.5} = 0.707 \quad \alpha = 35.3°$

$v_W = \sqrt{(1.5)^2 + (1.0607)^2} = 1.837$ m/s

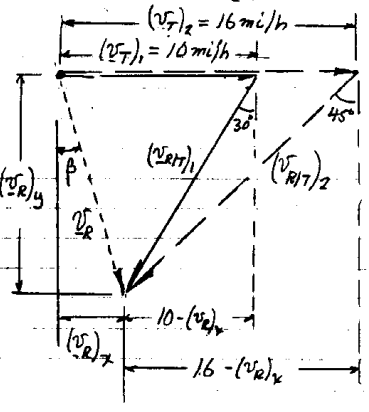
$\underline{v}_W = 1.837$ m/s ∠ 35.3° ◀

11.118

$\underline{v}_{RAIN} = \underline{v}_{TRAIN} + \underline{v}_{RAIN/TRAIN}$

CASE ① : $\underline{v}_T = 10$ mi/h → ; $\underline{v}_{R/T}$ ∠ 30°

CASE ② : $\underline{v}_T = 16$ mi/h → ; $\underline{v}_{R/T}$ ∠ 45°



CASE ① : $(v_R)_y \tan 30° = 10 - (v_R)_x$ (1)

CASE ② : $(v_R)_y \tan 45° = 16 - (v_R)_x$ (2)

SUBTRACT (1) FROM (2)

$(\underline{v}_R)_y (\tan 45° - \tan 30°) = 6$

$(\underline{v}_R)_y = 14.196$ mi/h

EQ(2): $(14.196)\tan 45° = 16 - (\underline{v}_R)_x$

$(\underline{v}_R)_x = 1.804$ mi/h

$\tan\beta = \frac{1.804}{14.196}$

$\beta = 7.24°$

$v_R = \frac{14.196}{\cos 7.24} = 14.31$ mi/h $= 20.98$ ft/s

$\underline{v}_R = 21.0$ ft/s ∠ 82.8° ◀

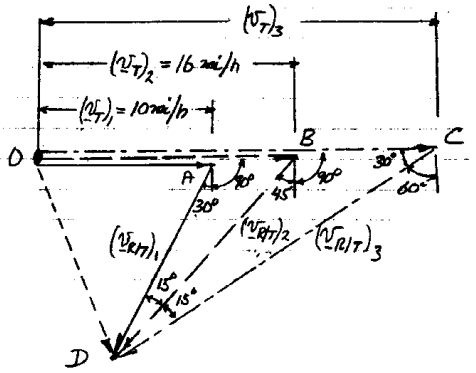
11.119

$\underline{v}_{RAIN} = \underline{v}_{TRAIN} + \underline{v}_{RAIN/TRAIN}$

CASE ① : $v_T = 10$ mi/h → ; $\underline{v}_{R/T}$ ∠ 30°

CASE ② : $v_T = 16$ mi/h → ; $\underline{v}_{R/T}$ ∠ 45°

CASE ③ : v_T = ? ; $\underline{v}_{R/T}$ ∠ 60°



FROM VECTOR DIAGRAM, WE WRITE

IN △ ABD: $AB = 16 - 10 = 6$ mi/h

LAW OF SINES:

$\frac{(v_{R/T})_2}{\sin 120°} = \frac{6 \text{ mi/h}}{\sin 15°} \qquad (v_{R/T})_2 = (6 \text{ mi/h})\frac{\sin 120°}{\sin 15°}$

IN △ BCD:

$\frac{BC}{\sin 15°} = \frac{(v_{R/T})_2}{\sin 30°} \qquad BC = (v_{R/T})_2 \frac{\sin 15°}{\sin 30°}$

$BC = \left[(6 \text{ mi/h})\frac{\sin 120°}{\sin 15°}\right]\frac{\sin 15°}{\sin 30°} = 10.39$ mi/h

FROM VECTOR DIAGRAM:

$(\underline{v}_T)_3 = (16 \text{ mi/h}) + BC = 16 \text{ mi/h} + 10.39 \text{ mi/h} = 26.39$ mi/h

$(\underline{v}_T)_3 = 26.4$ mi/h → ◀

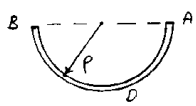
11.120

$a_n = \frac{v^2}{\rho} \qquad a_n = 2.4$ ft/s²

$v = 45$ mi/h $= 66$ ft/s

$2.4 \text{ ft/s}^2 = \frac{(66 \text{ ft/s})^2}{\rho} \qquad \rho = 1815$ ft ◀

11.121



ARC ADC $= 0.8$ km $= 800$ m

$\therefore 800 \text{ m} = \rho\,\pi$

$\rho = \frac{800}{\pi} = 254.65$ m

GIVEN: $a_n = 0.1g = 0.1(9.81 \text{ m/s}^2) = 0.981 \text{ m/s}^2$

$a_n = \frac{v^2}{\rho}$; $0.981 \text{ m/s}^2 = \frac{v^2}{254.65 \text{ m}}$

$v^2 = 249.8 \qquad v = 15.8054$ m/s $\qquad v = 56.9$ km/h ◀

11.122

$a_n = 10g = 10(32.2 \text{ ft/s}^2) = 322 \text{ ft/s}^2 \qquad \rho = 25$ ft

$a_n = \frac{v^2}{\rho}$; $322 \text{ ft/s}^2 = \frac{v^2}{25 \text{ ft}}$

$v^2 = 8050 \qquad v = 89.72$ ft/s $\qquad v = 89.7$ ft/s ◀

11.123

(a) DRUM B $\quad r = 0.9$ in. $= 0.075$ ft $\quad a_n = 480$ ft/s²

$a_n = \frac{v^2}{r}$; $480 \text{ ft/s}^2 = \frac{v^2}{0.075 \text{ ft}} \qquad v^2 = 36$

$v = 6$ ft/s ◀

(b) DRUM A $\quad r = 1.5$ in. $= 0.125$ ft $\quad v = 6$ ft/s

$a_n = \frac{v^2}{r} = \frac{(6 \text{ ft/s})^2}{0.125 \text{ ft}} = 288 \text{ ft/s}^2 \qquad a_n = 288$ ft/s² ◀

11.124

$v_0 = 72\ km/h = 20\ m/s$, $a_t = -1.25\ m/s^2$ $\rho = 350\ m$

(a) $t = 0$: $a_n = \frac{v^2}{\rho} = \frac{(20\ m/s)^2}{350\ m} = 1.1429\ m/s^2$

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(1.1429)^2 + (1.25)^2}$ $a = 1.694\ m/s^2$ ◀

(b) $t = 4\ s$: TANGENTIAL MOTION IS UNIFORMLY ACCELERATED

$v = v_0 + a_t(t)$ $v = 20\ m/s + (-1.25\ m/s^2)(4\ s) = 15\ m/s$

$a_n = \frac{v^2}{\rho} = \frac{(15\ m/s)^2}{350\ m} = 0.64286\ m/s^2$

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(0.64286)^2 + (1.25)^2}$ $a = 1.406\ m/s^2$ ◀

11.125

$\rho = 300\ m$ $a_t = 0.75\ m/s^2$

FOR $a = 0.9\ m/s^2$: $a^2 = a_n^2 + a_t^2$

$(0.9\ m/s^2)^2 = a_n^2 + (0.75\ m/s)^2$ $a_n = 0.4975\ m/s^2$

$a_n = \frac{v^2}{\rho}$; $0.4975\ m/s^2 = \frac{v^2}{300\ m}$

$v^2 = 149.25$ $v = 12.217\ m/s$

BUS STARTS FROM REST $v_0 = 0$

$v = v_0 + a_t t$ $12.217\ m/s = 0 + (0.75t)\ m/s$

$t = 16.289\ s$ $t = 16.29\ s$ ◀

$v^2 = v_0^2 + 2a(x - x_0)$: $149.25 = 0 + 2(0.75)x$

$x = 99.5\ m$ $x = 99.5\ m$ ◀

11.126

TANGENTIAL MOTION (UNIFORMLY ACCELERATED)

$v_0 = 60\ mi/h = 88\ ft/s$ $x_0 = 0$

$v_1 = 75\ mi/h = 110\ ft/s$ $x_1 = 600\ ft$

$v_1^2 = v_0^2 + 2a_t(x_1 - x_0)$

$110^2 = 88^2 + 2a_t(600)$ $a_t = \frac{110^2 - 88^2}{2(600)} = 3.63\ ft/s^2$

AT $x_2 = 400\ ft$

$v_2^2 = v_0^2 + 2a_t(x_2 - x_0) = 88^2 + 2(3.63)(400)$

$v_2^2 = 10,648$ $v_2 = 103.19\ ft/s = 70.36\ mi/h$

TOTAL ACCELERATION AT $x_2 = 400\ ft$ $\rho = 800\ ft$

$a_n = \frac{v_2^2}{\rho} = \frac{10,648}{800} = 13.31\ ft/s^2$ $a_t = 3.63\ ft/s$

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(13.31)^2 + (3.63)^2} = 13.796\ ft/s^2$ $a = 13.80\ ft/s^2$ ◀

11.127

WHEN $v = 45\ mi/h = 66\ ft/s$,

$\rho = 1200\ ft$ $a_n = \frac{v^2}{\rho} = \frac{(66\ ft/s)^2}{1200\ ft} = 3.63\ ft/s^2$

WE NOTE THAT MAXIMUM TOTAL ACCELERATION IS $4\ ft/s^2$

$a^2 = a_n^2 + a_t^2$

$(4\ ft/s^2)^2 = (3.63\ ft/s^2)^2 + a_t^2$ $a_t = 1.680\ ft/s^2$ ◀

a_t IS CONSTANT FOR ENTIRE MOTION.

AS TRAIN STARTS: $v_0 = 0$, $x_0 = 0$

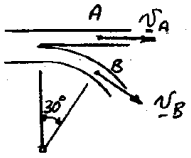
x_1 = DISTANCE TRAVELED BEFORE $v_1 = 66\ ft/s$

$v_1^2 = v_0^2 + 2a_t(x_1 - x_0)$

$(66)^2 = 0 + 2(1.680)(x_1 - 0)$

$x_1 = 1296.4\ ft$ $x_1 = 1296\ ft$ ◀

11.128



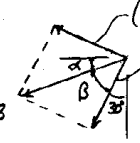
RADIUS OF PATH B = 150 m

AUTO A: $v_A = 75\ km/h \rightarrow$ $a_A = 1.5\ m/s^2 \rightarrow$

AUTO B: $v_B = 40\ km/h \searrow = 11.111\ m/s \measuredangle 30°$

$(a_B)_n = \frac{v_B^2}{\rho} = \frac{11.111^2}{150} = 0.823\ m/s^2 \swarrow 60°$

$(a_B)_t = 0.9\ m/s^2 \searrow 30°$

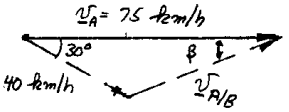


$a_B = \sqrt{(0.9)^2 + (0.823)^2} = 1.2196\ m/s^2$

$\beta = 47.6°$ $\alpha = 90° - 30° - 47.6° = 12.4°$

$a_B = 1.2196\ m/s^2 \measuredangle 12.4°$

(a) VELOCITIES $v_A = v_B + v_{A/B}$ (WE USE km/h)



LAW OF COSINES

$v_{A/B}^2 = 75^2 + 40^2 - 2(40)(75)\cos 30°$

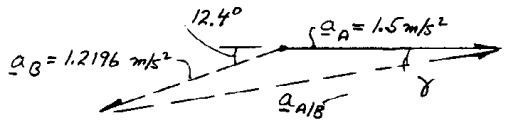
$v_{A/B} = 45.04\ km/h$

LAW OF SINES

$\frac{\sin\beta}{40} = \frac{\sin 30°}{45.04}$ $\beta = 26.36°$

$v_{A/B} = 45.0\ km/h \measuredangle 26.4°$ ◀

(b) ACCELERATIONS $a_A = a_B + a_{A/B}$ (WE USE m/s^2)



LAW OF COSINES

$a_{A/B}^2 = (1.2196)^2 + (1.5)^2 - 2(1.2196)(1.5)\cos(180° - 12.4°)$

$a_{A/B} = 2.704\ m/s^2$

LAW OF SINES

$\frac{\sin\gamma}{1.2196} = \frac{\sin(180° - 12.4°)}{2.704}$ $\gamma = 5.56°$

$a_{A/B} = 2.70\ m/s^2 \measuredangle 5.6°$ ◀

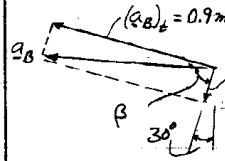
11.129

SEE SKETCH AND SOLUTION OF PROB 11.128

AUTO A: $v_A = 75\ km/h \rightarrow$ $a_A = 1.5\ m/s^2 \rightarrow$

AUTO B $v_B = 20\ km/h = 5.556\ m/s \measuredangle 30°$

$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(5.556)^2}{150} = 0.2058\ m/s^2 \swarrow 60°$ $(a_B)_t = 0.9\ m/s^2 \searrow 30°$



$a_B = \sqrt{(0.9)^2 + (0.2058)^2}$

$a_B = 0.9232\ m/s^2$

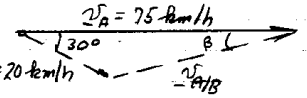
$\beta = 77.1°$

γ = ANGLE WITH HORIZONTAL

$\gamma = (77.1° + 30°) - 90° = 17.1°$

$a_B = 0.9232\ m/s^2 \measuredangle 17.1°$

(a) VELOCITIES $v_A = v_B + v_{A/B}$ (WE USE km/h)



LAW OF COSINES

$v_{A/B}^2 = 20^2 + 75^2 - 2(20)(75)\cos 30°$

$v_{A/B} = 58.53\ km/h$

LAW OF SINES

$\frac{\sin\beta}{20} = \frac{\sin 30°}{58.53}$ $\beta = 9.84°$

$v_{A/B} = 58.5\ km/h \measuredangle 9.8°$ ◀

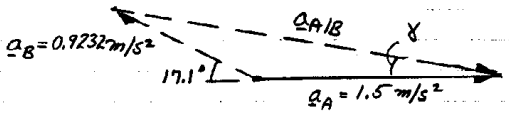
(CONTINUED)

11.129 CONTINUED (b) ACCELERATIONS

WE HAVE: $\underline{a}_A = 1.5\ m/s^2 \rightarrow$

$\underline{a}_B = 0.9232\ m/s^2 \ \angle 17.1°$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$



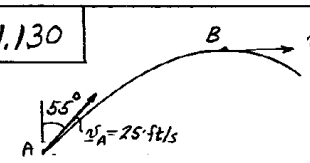
LAW OF COSINES $a_{A/B}^2 = (0.9232)^2 + (1.5)^2 - 2(0.9232)(1.5)\cos(180° - 17.1°)$

$a_{A/B} = 2.398\ m/s^2$

LAW OF SINES $\dfrac{\sin\gamma}{0.9232} = \dfrac{\sin(180° - 17.1°)}{2.398}$ $\gamma = 6.5°$

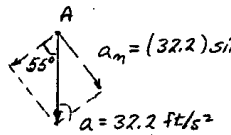
$\underline{a}_{A/B} = 2.40\ m/s^2 \ \angle 6.5°$ ◀

11.130



$\underline{v}_B = (\underline{v}_A)_\chi = (25\ ft/s)\sin 55° = 20.48\ ft/s \rightarrow$

AT A: SINCE $a = g = 32.2\ ft/s^2 \downarrow$



$a_n = \dfrac{v_A^2}{\rho_A}$

$26.377\ ft/s^2 = \dfrac{(25\ ft/s)^2}{\rho_A}$

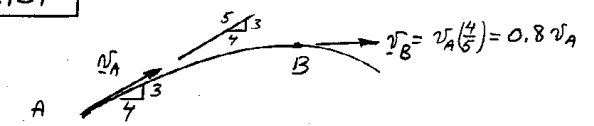
$\rho_A = 23.7\ ft$ ◀

AT B: $a = g = \dfrac{v_B^2}{\rho_B}$

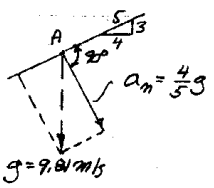
$32.2\ ft/s^2 = \dfrac{(20.48\ ft/s)^2}{\rho_B}$

$\rho_B = 13.03\ ft$ ◀

11.131



(a) AT A: $\rho_A = 35\ m$ $g = 9.81\ m/s^2 \downarrow$



$a_n = \dfrac{v_A^2}{\rho_A}$

$0.8(9.81\ m/s^2) = \dfrac{v_A^2}{35\ m}$

$v_A^2 = 274.6$

$v_A = 16.573\ m/s$ $\underline{v}_A = 16.57\ m/s \nearrow$ ◀

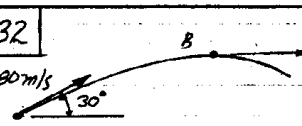
(b) AT B: $v_B = 0.8\,v_A = 0.8(16.573\ m/s) = 13.259\ m/s \rightarrow$

$a_n = g = 9.81\ m/s^2 \downarrow$ $a_n = \dfrac{v_B^2}{\rho_B}$

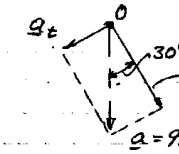
$9.81\ m/s^2 = \dfrac{(13.259\ m/s)^2}{\rho_B}$

$\rho_B = 17.92\ m$ ◀

11.132



AT O: (WHERE PROJECTILE IS FIRED)



$a_n = \dfrac{v_0^2}{\rho_0}$; $8.496\ m/s^2 = \dfrac{(180\ m/s)^2}{\rho_0}$ $\rho_0 = 3810\ m$ ◀

AT B: (MAXIMUM ELEVATION) $a_n = g = 9.81\ m/s^2 \downarrow$

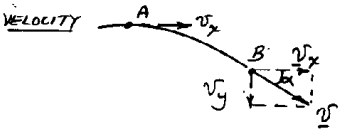
$a_B = g = \dfrac{v_B^2}{\rho_B}$; $9.81\ m/s^2 = \dfrac{(155.88\ m/s)^2}{\rho_B}$ $\rho_B = 2480\ m$ ◀

11.133

(a) SEE SAMPLE PROB 11.11.

(b) HORIZONTAL MOTION IS UNIFORM: $v_\chi = $ CONSTANT

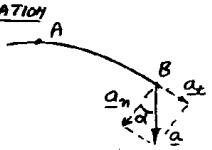
VELOCITY



$v_\chi = v\cos\alpha$

$v = \dfrac{v_\chi}{\cos\alpha}$

ACCELERATION



ACCELERATION = CONSTANT: $a = g$

$a_n = g\cos\alpha$

$a_n = \dfrac{v^2}{\rho}$; $\rho = \dfrac{v^2}{a_n}$

$\rho = \dfrac{(v_\chi/\cos\alpha)^2}{g\cos\alpha} = \dfrac{v_\chi^2}{g}\dfrac{1}{\cos^3\alpha}$

AT A: $\alpha = 0$, $\cos\alpha = 1$ $\rho_{min} = \dfrac{v_\chi^2}{g}$

$\rho = \rho_{min}\left(\dfrac{1}{\cos^3\alpha}\right)$ ◀

*11.134

EQ. 11.39: $\underline{a} = \dfrac{dv}{dt}\underline{e}_t + \dfrac{v^2}{\rho}\underline{e}_n$

SINCE $\underline{e}_t \perp \underline{e}_n$: $a^2 = \left(\dfrac{dv}{dt}\right)^2 + \left(\dfrac{v^2}{\rho}\right)^2$ (1)

FROM PROB 11.85: $a = Rp^2$, $v = \sqrt{r^2 + R^2p^2}$, ALSO $\dfrac{dv}{dt} = 0$

EQ(1): $(Rp^2)^2 = 0 + \left(\dfrac{r^2 + R^2p^2}{\rho}\right)^2$

$\rho = \dfrac{r^2 + R^2p^2}{Rp^2}$ $\rho = R + \dfrac{r^2}{Rp^2}$ ◀

*11.135

EQ. 11.39: $\underline{a} = \dfrac{dv}{dt}\underline{e}_t + \dfrac{v^2}{\rho}\underline{e}_n$

SINCE $\underline{e}_t \perp \underline{e}_n$: $a^2 = \left(\dfrac{dv}{dt}\right)^2 + \left(\dfrac{v^2}{\rho}\right)^2$ (1)

SEE SOLUTION OF PROB 11.87 FOR VALUES OF a, v, AND dv/dt

(a) WHEN $t = 0$: $a = 4\ m/s^2$, $v = 1\ m/s$, $dv/dt = 0$

EQ(1): $(4\ m/s)^2 = 0 + \left[\dfrac{(1\ m/s)^2}{\rho}\right]^2$

$4 = \dfrac{1}{\rho}$ $\rho = 0.25\ m$ ◀

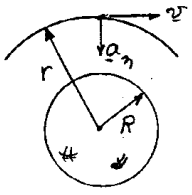
(b) WHEN $t = 2s$: $a = \sqrt{592}\ m/s^2$, $v = \sqrt{641}\ m/s$, $\dfrac{dv}{dt} = 24.01\ m/s^2$

EQ(1): $\left(\sqrt{592}\right)^2 = (24.01)^2 + \left[\dfrac{(\sqrt{641})^2}{\rho}\right]^2$

$\dfrac{641}{\rho} = \sqrt{592 - 576.48} = \sqrt{15.52} = 3.94$ $\rho = 162.7\ m$ ◀

11.136

$v = 16{,}000 \text{ mi/h} = 23{,}467 \text{ ft/s}$



$$a_n = g\left(\frac{R}{r}\right)^2 = 32.2\,\frac{(3960\times5280)^2}{r^2}$$

BUT $\quad a_n = \dfrac{v^2}{r} = \dfrac{(23{,}467)^2}{r}$

$$\therefore \quad \frac{(23{,}467)^2}{r} = 32.2\,\frac{(3960\times5280)^2}{r^2}$$

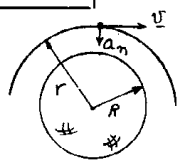
$$r = 32.2\,\frac{(3960\times5280)^2}{(23{,}467)^2} = 25.562\times10^6 \text{ ft}$$

$$r = 4841.3 \text{ mi}$$

HEIGHT ABOVE SURFACE $= 4841.3 - 3960 = 881.3$ mi

$h = 881$ mi. ◀

11.137



$$a_n = \frac{v^2}{\rho} = \frac{v^2}{r}$$

BUT: $a_n = g\left(\frac{R}{r}\right)^2$

$$\frac{v^2}{r} = g\left(\frac{R}{r}\right)^2 \qquad v^2 = \frac{gR^2}{r}$$

DATA: $r = (3960 \text{ mi} + 140 \text{ mi}) = 4100 \text{ mi} = 21.648\times10^6$ ft

$R = 3960 \text{ mi} = 20.909\times10^6$ ft

$g = 32.2 \text{ ft/s}^2$

$$v^2 = \frac{gR^2}{r} = \frac{(32.2)(20.909\times10^6)^2}{21.648\times10^6} = 650.3\times10^6$$

$v = 25{,}501 \text{ ft/s} = 17{,}387 \text{ mi/h} \qquad v = 17{,}390 \text{ mi/h}$ ◀

11.138

(a) WE KNOW: $a_n = \dfrac{v^2}{r}$ AND $a_n = g\left(\dfrac{R}{r}\right)^2$

SETTING $a_n = a_n$: $\dfrac{v^2}{r} = g\left(\dfrac{R}{r}\right)^2 \qquad v^2 = g\dfrac{R^2}{r}$ (1)

THUS v IS PROPORTIONAL TO $\dfrac{1}{\sqrt{r}}$ Q.E.D.

(b) LENGTH OF ONE ORBIT: $s = 2\pi r$

PERIOD (OF TIME) FOR ONE ORBIT: $t = \dfrac{s}{v} = \dfrac{2\pi r}{v}$ (2)

EQ(1): $v^2 = g\dfrac{R^2}{r} \qquad v = R\sqrt{\dfrac{g}{r}}$

EQ(2) $\quad t = \dfrac{2\pi r}{v} = \dfrac{2\pi r}{R\sqrt{g/r}} = \dfrac{2\pi}{R}\,\dfrac{r^{3/2}}{g^{1/2}}$

NEGLECTING ATMOSPHERE, WE CHOOSE $r_{min} = R$

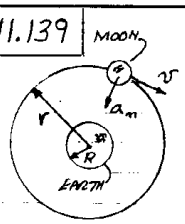
$$t_{minimum} = \frac{2\pi}{R}\,\frac{R^{3/2}}{g^{1/2}} = 2\pi\sqrt{R/g}$$

$R = 6370 \text{ km} = 6.37\times10^6$ m

$$t_{minimum} = 2\pi\sqrt{(6.37\times10^6)/9.81} = 5063 \text{ s}$$

$t_{minimum} = 84.4$ minutes ◀

11.139



$r = 384\times10^3 \text{ km} = 384\times10^6$ m

$R = 6370 \text{ km} = 6.37\times10^6$ m

SEE SOLUTION OF PROB 11.137 FOR DERIVATION OF EQ(1)

$$v^2 = g\frac{R^2}{r} = (9.81 \text{ m/s}^2)\,\frac{(6.37\times10^6 \text{ m})^2}{384\times10^6 \text{ m}}$$

$v^2 = 1.0366\times10^6 \qquad v = 1018$ m/s

$v = 3665$ km/h $\qquad v = 3670$ km/h ◀

NOTE SINCE DIAMETER OF MOON ≈ 3480 km, THE MOON MOVES RELATIVE TO THE EARTH A DIAMETER IN ABOUT ONE HOUR

11.140

$r = 60t^2 - 20t^3 \qquad \theta = 2t^2$

$\dot r = 120t - 60t^2 \qquad \dot\theta = 4t$

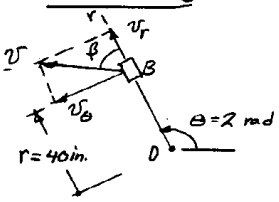
$\ddot r = 120 - 120t \qquad \ddot\theta = 4$

WHEN $t = 1$ s: $\quad r = 60 - 20 = 40$ in. $\qquad \theta = 2$ rad

$\dot r = 120 - 60 = 60$ in./s $\qquad \dot\theta = 4$ rad/s

$\ddot r = 120 - 120 = 0 \qquad \ddot\theta = 4$ rad/s^2

(a) VELOCITY OF B:



$v_r = \dot r = 60$ in/s

$v_\theta = r\dot\theta = (40\text{in.})(4\text{rad/s}) = 160$ in./s

$$\tan\beta = \frac{v_\theta}{v_r} = \frac{160 \text{ in/s}}{60 \text{ in/s}} = 2.667$$

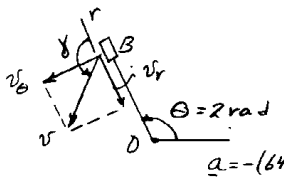
$\beta = 69.4^\circ \qquad v = 170.9$ in./s

$\underline{v} = (60 \text{ in./s})\underline{e}_r + (160 \text{ in./s})\underline{e}_\theta$ ◀

(b) ACCELERATION OF B:

$a_r = \ddot r - r\dot\theta^2 = 0 - (40\text{in.})(4 \text{ rad/s})^2 = -640 \text{ in./s}^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = (40\text{in.})(+4\text{rad/s}^2) + 2(60\text{in./s})(4\text{rad/s}) = +640 \text{ in./s}^2$



$a = 640\sqrt{2} = 905.1 \text{ in/s}^2$

$\gamma = 90^\circ + 45^\circ = 135^\circ$

$\underline{a} = -(640 \text{ in./s}^2)\underline{e}_r + (640 \text{ in./s}^2)\underline{e}_\theta$ ◀

(c) ACCELERATION OF B RELATIVE TO ROD

$a_{B/OA} = \ddot r$, SINCE $\ddot r = 0 \qquad a_{B/OA} = 0$ ◀

11.141

$r = 1.25t^2 - 0.9t^3 \qquad \theta = \frac{1}{2}\pi(4t - 3t^2)$

$\dot r = 2.5t - 2.7t^2 \qquad \dot\theta = \frac{1}{2}\pi(4 - 6t)$

$\ddot r = 2.5 - 5.4t \qquad \ddot\theta = \frac{1}{2}\pi(-6)$

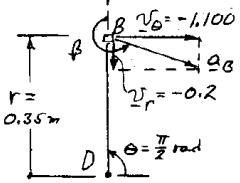
WHEN $t = 1$ s: $\quad r = 1.25 - 0.9 = 0.35$ m $\qquad \theta = \frac{1}{2}\pi$ rad

$\dot r = 2.5 - 2.7 = -0.2$ m/s $\qquad \dot\theta = -\pi$ rad/s

$\ddot r = 2.5 - 5.4 = -2.9$ m/s^2 $\qquad \ddot\theta = -3\pi$ rad/s^2

(a) VELOCITY OF B:

$v_r = \dot r = -0.2$ m/s $\qquad v_\theta = r\dot\theta = (0.35\text{m})(-\pi \text{ rad/s}) = -1100$ m/s



$\beta = 180^\circ + \tan^{-1}\dfrac{1.1}{0.2} = 259.7^\circ$

$v_B = \sqrt{(-0.2)^2 + (-1.100)^2} = 1.118$ m/s

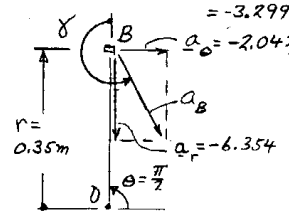
$v_B = 1.118$ m/s, $\beta = 259.7^\circ$ ◀

(b) ACCELERATION OF B:

$a_r = \ddot r - r\dot\theta^2 = (-2.9 \text{ m/s}^2) - (0.35\text{m})(-\pi \text{ rad/s})^2 = -6.354$ m/s^2

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = (0.35\text{m})(-3\pi \text{ rad/s}^2) + 2(-0.2 \text{ m/s})(-\pi \text{ rad/s})$

$= -3.299 + 1.257 = -2.042$ m/s^2



$\gamma = 180^\circ + \tan^{-1}\dfrac{2.042}{6.354} = 197.8^\circ$

$a_B = \sqrt{(-6.354)^2 + (-2.042)^2} = 6.674$ m/s^2

$a_B = 6.67$ m/s^2, $\gamma = 197.8^\circ$ ◀

(c) ACCELERATION OF B RELATIVE TO THE ROD

$a_{B/OA} = \ddot r = -2.90$ m/s^2 $\qquad a_{B/OA} = 2.90$ m/s^2, $\gamma = 180^\circ$ ◀

11.142

$r = 2b\cos\omega t$ $\quad \theta = \omega t$

$\dot r = -2b\omega\sin\omega t$ $\quad \dot\theta = \omega$

$\ddot r = -2b\omega^2\cos\omega t$ $\quad \ddot\theta = 0$

(a) VELOCITY

$v_r = \dot r = -2b\omega\sin\omega t$ $\quad v_\theta = r\dot\theta = 2b\omega\cos\omega t$

$v^2 = v_r^2 + v_\theta^2 = (2b\omega)^2\left[(-\sin\omega t)^2 + (\cos\omega t)^2\right] = (2b\omega)^2$

$v = 2b\omega$ ◀

ACCELERATION

$a_r = \ddot r - r\dot\theta^2 = -2b\omega^2\cos\omega t - (2b\cos\omega t)(\omega)^2$

$= -4b\omega^2\cos\omega t$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = (2b\cos\omega t)(0) + 2(-2b\omega\sin\omega t)(\omega)$

$= -4b\omega^2\sin\omega t$

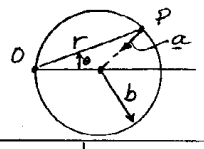
$a^2 = a_r^2 + a_\theta^2 = (-4b\omega^2)^2(\cos^2\omega t + \sin^2\omega t) = (4b\omega^2)^2$

$a = 4b\omega^2$ ◀

(b) SINCE $v = 2b\omega$ = CONSTANT, $a_t = 0$

THUS: $a_n = a = 4b\omega^2$

$a_n = \frac{v^2}{\rho}$; $\quad \rho = \frac{v^2}{a_n} = \frac{(2b\omega)^2}{4b\omega^2} = b$ $\quad \rho = b$



FOR THE PATH ρ = CONSTANT

THUS, PATH IS A CIRCLE ◀

11.143

ARCHIMEDEAN SPIRAL

$r = 10t$ $\quad \theta = 2\pi t$

$\dot r = 10$ $\quad \dot\theta = 2\pi$

$\ddot r = 0$ $\quad \ddot\theta = 0$

VELOCITY $v_r = \dot r = 10$ in./s $\quad v_\theta = r\dot\theta = (10t)(2\pi) = 20\pi t$

ACCELERATION $a_r = \ddot r - r\dot\theta^2 = 0 - (10t)(2\pi)^2 = -40\pi^2 t$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(10)(2\pi) = 40\pi$

(a) WHEN $t = 0$: $\quad v_r = 10$ in./s, $\quad v_\theta = 0$; $\quad \underline{v} = (10\text{ in./s})\underline{e}_r$ ◀

$a_r = 0$, $\quad a_\theta = 40\pi = 125.7$ in./s^2; $\quad \underline{a} = (125.7\text{ in./s}^2)\underline{e}_\theta$ ◀

(b) WHEN $t = \frac{1}{4}$ s $\quad$ FOR β AND γ SEE FIGURE BELOW.

$v_r = 10$ in./s, $\quad v_\theta = r\dot\theta = \left[10\left(\frac{1}{4}\right)\right]2\pi = 5\pi = 15.71$ in./s

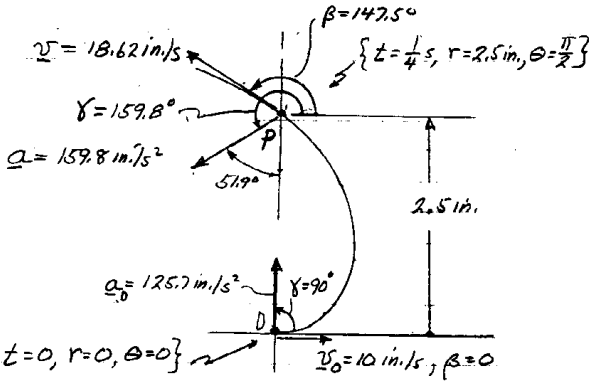
$\beta = 90° + \tan^{-1}\frac{15.71}{10} = 147.5°$; $\quad v = \sqrt{v_r^2 + v_\theta^2} = 18.62$ in./s

$\underline{v} = (10\text{ in./s})\underline{e}_r + (15.71\text{ in./s})\underline{e}_\theta$ ◀

$a_r = -40\pi^2\left(\frac{1}{4}\right) = -10\pi^2 = -98.7$ in./s^2 $\quad a_\theta = 40\pi = 125.7$ in./s^2

$\gamma = 270° - \tan^{-1}\frac{125.7}{98.7} = 218.1°$ $\quad a = \sqrt{a_r^2 + a_\theta^2} = 159.8$ in./s^2

$\underline{a} = -(98.7\text{ in./s}^2)\underline{e}_r + (125.7\text{ in./s}^2)\underline{e}_\theta$ ◀



11.144

$r = 2b(1 + \cos\frac{1}{2}\pi t)$ $\quad \theta = \frac{1}{2}\pi t$

$\dot r = -\pi b\sin\frac{1}{2}\pi t$ $\quad \dot\theta = \frac{1}{2}\pi$

$\ddot r = -\frac{1}{2}\pi^2 b\cos\frac{1}{2}\pi t$ $\quad \ddot\theta = 0$

VELOCITY $v_r = \dot r = -\pi b\sin\frac{1}{2}\pi t$ △

$v_\theta = r\dot\theta = \pi b(1 + \cos\frac{1}{2}\pi t)$ △

ACCELERATION $a_r = \ddot r - r\dot\theta^2$

$= -\frac{1}{2}\pi^2 b\cos\frac{1}{2}\pi t - \frac{1}{2}\pi^2 b(1 + \cos\frac{1}{2}\pi t)$

$a_r = -\frac{1}{2}\pi^2 b(1 + 2\cos\frac{1}{2}\pi t)$ △

$a_\theta = r\ddot\theta + 2\dot r\dot\theta$

$= 0 + 2(-\pi b\sin\frac{1}{2}\pi t)(\frac{1}{2}\pi)$

$a_\theta = -\pi^2 b\sin\frac{1}{2}\pi t$ △

(a) WHEN $t = 0$: $\quad r = 2b(1+1) = 4b$ $\quad \theta = 0$

VELOCITY $v_r = 0$ $\quad v_\theta = \pi b(1+1) = 2\pi b$

$\underline{v} = 2\pi b\,\underline{e}_r$ ◀

ACCELERATION $a_r = -\frac{1}{2}\pi^2 b(1+2) = -\frac{3}{2}\pi^2 b$ $\quad a_\theta = 0$

$\underline{a} = -\frac{3}{2}\pi^2 b\,\underline{e}_\theta$ ◀

(b) WHEN $t = 1$ s: $\quad r = 2b(1 + \cos\frac{\pi}{2}) = 2b$ $\quad \theta = \frac{\pi}{2} = 90°$

VELOCITY $v_r = -\pi b\sin\frac{\pi}{2} = -\pi b$

$v_\theta = \pi b(1 + \cos\frac{\pi}{2}) = \pi b$

$\underline{v} = -\pi b\,\underline{e}_r + \pi b\,\underline{e}_\theta$ ◀

ACCELERATION $a_r = -\frac{1}{2}\pi^2 b(1 + 2\cos\frac{\pi}{2}) = -\frac{1}{2}\pi^2 b$

$a_\theta = -\pi^2 b\sin\frac{\pi}{2} = -\pi^2 b$

$\underline{a} = -\frac{1}{2}\pi^2 b\,\underline{e}_r - \pi^2 b\,\underline{e}_\theta$ ◀

11.145

SEE SOLUTION OF PROB 11.145 FOR $r, v_r, v_\theta, a_r, a_\theta$

(a) WHEN $t = \frac{1}{2}$ s: $\quad r = 2b(1 + \cos\frac{\pi}{4}) = 3.414b$, $\quad \theta = 45°$

$v_r = -\pi b\sin\frac{\pi}{4} = -0.707\pi b$ $\quad v_\theta = \pi b(1 + \cos\frac{\pi}{4}) = 1.707\pi b$

$\underline{v} = -0.707\pi b\,\underline{e}_r + 1.707\pi b\,\underline{e}_\theta$ ◀

$a_r = -\frac{1}{2}\pi^2 b(1 + 2\cos\frac{\pi}{4}) = -1.207\pi^2 b$ $\quad a_\theta = -\pi^2 b\sin\frac{\pi}{4} = -0.707\pi^2 b$

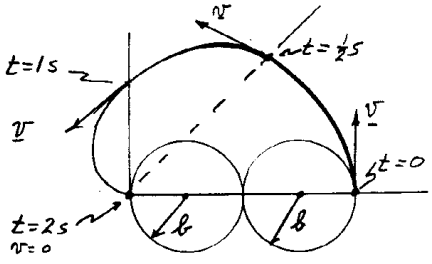
$\underline{a} = -1.207\pi^2 b\,\underline{e}_r - 0.707\pi^2 b\,\underline{e}_\theta$ ◀

(b) WHEN $t = 2$ s $\quad r = 0$ $\quad \theta = \pi = 180°$

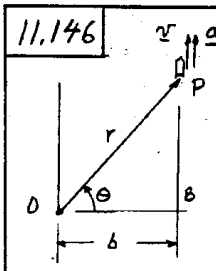
$v_r = -\pi b\sin\pi = 0$ $\quad v_\theta = \pi b(1 + \cos\pi) = 0$ $\quad v = 0$ ◀

$a_r = -\frac{1}{2}\pi^2 b(1 + 2\cos\pi) = \frac{1}{2}\pi^2 b$ $\quad a_\theta = -\pi^2 b\sin\pi = 0$

$\underline{a} = \frac{1}{2}\pi^2 b\,\underline{e}_\theta$ ◀



11.146



$$r = \frac{b}{\cos\theta} = b\sec\theta$$

$$v_r = \dot r = b\frac{\sin\theta}{\cos^2\theta}\dot\theta = \frac{b}{\cos\theta}\tan\theta\,\dot\theta$$

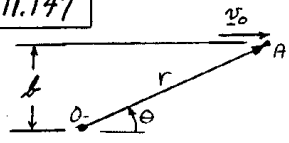
$$v_\theta = r\dot\theta = \frac{b}{\cos\theta}\dot\theta$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \frac{b}{\cos\theta}\sqrt{1+\tan^2\theta}\;\dot\theta$$

BUT $\sqrt{1+\tan^2\theta} = \frac{1}{\cos\theta}$ $\qquad v = \frac{b\dot\theta}{\cos\theta}\left(\frac{1}{\cos\theta}\right) = \frac{b}{\cos^2\theta}\dot\theta$

OR: $\qquad v = b\sec^2\theta\,\dot\theta$ ◀

11.147



$$r = \frac{b}{\sin\theta}$$

$$v_r = \dot r = -b\frac{\cos\theta}{\sin^2\theta}\dot\theta$$

$$v_\theta = r\dot\theta = \frac{b}{\sin\theta}\dot\theta$$

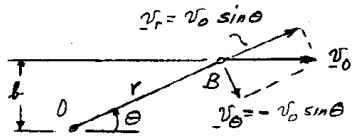
$$v_B^2 = v_0^2 = v_r^2 + v_\theta^2 = \left[\frac{\cos^2\theta}{\sin^4\theta} + \frac{1}{\sin^2\theta}\right]b^2\dot\theta^2$$

$$v_0^2 = \left[\frac{\cos^2\theta + \sin^2\theta}{\sin^4\theta}\right]b^2\dot\theta^2 = \left[\frac{1}{\sin^4\theta}\right]b^2\dot\theta^2$$

$$v_0 = \pm\frac{b\dot\theta}{\sin^2\theta}$$ SINCE $v_0 \rightarrow$ CORRESPONDS TO $\theta\downarrow$, CHOOSE MINUS SIGN

$$v_0 = -\frac{b\dot\theta}{\sin^2\theta} \qquad \dot\theta = -\left(\frac{v_0}{b}\right)\sin^2\theta$$ ◀

ALTERNATE SOLUTION



WE RESOLVE v_0 INTO v_r AND v_θ COMPONENTS AS SHOWN AND NOTE THAT $b = r\sin\theta$ OR $r = \frac{b}{\sin\theta}$

$$v_\theta = r\dot\theta = -v_0\sin\theta$$

$$\left(\frac{b}{\sin\theta}\right)\dot\theta = -v_0\sin\theta \qquad \dot\theta = -\left(\frac{v_0}{b}\right)\sin^2\theta$$ ◀

11.148

SINCE THE ROCKET MOVES IN A VERTICAL STRAIGHT LINE, ITS ACCELERATION IS CONSTANT IN DIRECTION. FOR THIS RECTILINEAR MOTION, WE HAVE:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[b\sec^2\theta\,\dot\theta\right]$$

$$= 2b\sec\theta(\sec\theta\tan\theta)\dot\theta^2 + b\sec^2\theta\,\ddot\theta$$

$$a = b\sec^2\theta\left[\ddot\theta + 2\dot\theta^2\tan\theta\right]$$ ◀

ALTERNATE SOLUTION

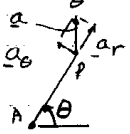
$$a_r = \ddot r - r\dot\theta^2 = \frac{d}{dt}\left[b\sec\theta\tan\theta\,\dot\theta\right] - b\sec\theta\,\dot\theta^2$$

$$= b\sec\theta\tan^2\theta\,\dot\theta^2 + b\sec^3\theta\,\dot\theta^2 + b\sec\theta\tan\theta\,\ddot\theta - b\sec\theta\,\dot\theta^2$$

$$a_r = b\sec\theta\tan\theta\left[\ddot\theta + 2\tan\theta\,\dot\theta^2\right]$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta = b\sec\theta\,\ddot\theta + 2b\sec\theta\tan\theta\,\dot\theta^2$$

$$a_\theta = b\sec\theta\left[\ddot\theta + 2\tan\theta\,\dot\theta^2\right]$$



$$a = \sqrt{a_r^2 + a_\theta^2} = b\sec\theta\left[\ddot\theta + 2\tan\theta\,\dot\theta^2\right]\sqrt{1+\tan^2\theta}$$

BUT: $\sec\theta = \sqrt{1+\tan^2\theta}$

$$a = b\sec^2\theta\left[\ddot\theta + 2\dot\theta^2\tan\theta\right]$$ ◀

11.149

SEE SOLUTION AND FIGURE OF SOLUTION OF PROB. 11.147.

WE NOTE THAT AIRPLANE MOVES AT CONSTANT SPEED IN A STRAIGHT LINE, THUS $a_A = 0$ AND $a_\theta = 0$

EQ 11.46: $\quad a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 \qquad \ddot\theta = -\frac{2\dot r\dot\theta}{r}$ (1)

FROM SOLUTION OF PROB 11.147, WE RECALL

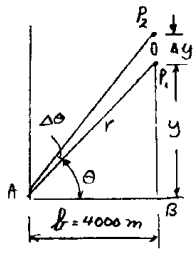
$$r = \frac{b}{\sin\theta} \qquad \dot r = -b\frac{\cos\theta}{\sin^2\theta}\dot\theta \qquad \dot\theta = -\left(\frac{v_0}{b}\right)\sin^2\theta$$

SUBSTITUTING IN (1) THE EXPRESSIONS FOR r AND $\dot r$:

$$\ddot\theta = -2\left[-b\frac{\cos\theta}{\sin^2\theta}\dot\theta\right]\dot\theta\left[\frac{1}{b/\sin\theta}\right] = 2\frac{\cos\theta}{\sin\theta}\dot\theta^2$$

$$\ddot\theta = 2\frac{\cos\theta}{\sin\theta}\left[\left(\frac{v_0}{b}\right)\sin^2\theta\right]^2 \qquad \ddot\theta = 2\frac{v_0^2}{b^2}\sin^3\theta\cos\theta$$ ◀

11.150



$t = 0$: $\theta = 47.0°$

$t = 0.5s$: $\theta = 48.0°$

$\Delta t = 0.5s$; $\Delta\theta = 48.0° - 47.0° = 1.0°$

$$\dot\theta = \frac{\Delta\theta}{\Delta t} = \frac{1.0°}{0.5s} = 2.0 \text{ degrees/s}$$

$$\dot\theta = 0.03491 \text{ rad/s}$$

DURING INTERVAL Δt:

$$\theta_{AVG} = \tfrac{1}{2}(47.0° + 48.0°) = 47.5° = 0.829 \text{ rad}$$

FROM PROB 11.146, WE HAVE

$$v = b\sec^2\theta\,\dot\theta = (4000 \text{ m})(\sec 47.5)^2(0.03491 \text{ rad/s})$$

$$v = 305.9 \text{ m/s} \qquad \underline{v} = 306 \text{ m/s} \uparrow$$ ◀

ALTERNATE SOLUTION WE COMPUTE Δy DURING THE INTERVAL $\Delta t = 0.5s$

$t = 0$, $\theta = 47.0°$: $\quad y = b\tan\theta = 4000\tan 47° = 4289.47 \text{ m}$

$t = 0.5s$, $\theta = 48.0°$: $\quad y + \Delta y = 4000\tan 48.0° = 4442.45$

$$\Delta y = 4442.45 - 4289.47 = 152.98 \text{ m}$$

$$v = \frac{\Delta y}{\Delta t} = \frac{152.98 \text{ m}}{0.5s} \qquad \underline{v} = 306 \text{ m/s} \uparrow$$ ◀

11.151

FOR THE INTERVAL OF TIME $\Delta t = 2s$, WE WRITE: $\Delta r = 13{,}600 \text{ ft} - 12{,}600 \text{ ft} = 1000 \text{ ft}$

$$\Delta\theta = 28.3° - 31.2° = -2.9° = -0.05061 \text{ rad}$$

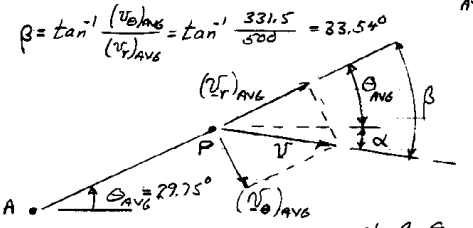
$$(\underline{v}_r)_{AVG} = \frac{\Delta\underline{r}}{\Delta t} = \frac{1000 \text{ ft}}{2s} = 500 \text{ ft/s} \nearrow$$

$$(v_\theta)_{AVG} = (r_{AVG})\frac{\Delta\theta}{\Delta t} = \left[\frac{12{,}600 + 13{,}600}{2}\right]\left(\frac{-0.05061 \text{ rad}}{2s}\right) = 331.5 \text{ ft/s} \searrow$$

$$v_{AVG} = \sqrt{(v_r)_{AVG}^2 + (v_\theta)_{AVG}^2} = \sqrt{(500 \text{ ft/s})^2 + (331.5 \text{ ft/s})^2} = 599.9 \text{ ft/s}$$

$$v_{AVG} = 409.0 \text{ mi/h}$$

$$\beta = \tan^{-1}\frac{(v_\theta)_{AVG}}{(v_r)_{AVG}} = \tan^{-1}\frac{331.5}{500} = 33.54°$$



$$\theta_{AVG} = \tfrac{1}{2}(31.2° + 28.3°) = 29.75°$$

$$\alpha = \beta - \theta_{AVG} = 33.54° - 29.75°$$

$$\alpha = 3.79°$$

$$\underline{v} = 409 \text{ mi/h} \measuredangle 3.8°$$ ◀

11.152 HYPERBOLIC SPIRAL $r = \frac{b}{\theta}$

$$\dot{r} = \frac{dr}{dt} = -\frac{b}{\theta^2}\frac{d\theta}{dt} = -\frac{b}{\theta^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{b}{\theta^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{b}{\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = b\dot{\theta}\sqrt{\left(-\frac{1}{\theta^2}\right)^2 + \left(\frac{1}{\theta}\right)^2} = \frac{b\dot{\theta}}{\theta^2}\sqrt{1+\theta^2}$$

$$v = \frac{b}{\theta^2}\sqrt{1+\theta^2}\,\dot{\theta} \quad \blacktriangleleft$$

11.153 LOGARITHMIC SPIRAL $r = e^{b\theta}$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\frac{d\theta}{dt} = be^{b\theta}\dot{\theta}$$

$$v_r = \dot{r} = be^{b\theta}\dot{\theta} \qquad v_\theta = r\dot{\theta} = e^{b\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = e^{b\theta}\dot{\theta}\sqrt{b^2+1} \qquad v = e^{b\theta}\sqrt{1+b^2}\,\dot{\theta} \quad \blacktriangleleft$$

11.154 HYPERBOLIC SPIRAL $r = \frac{b}{\theta}$

FROM PROB 11.151: $\dot{r} = -\frac{b}{\theta^2}\dot{\theta}$

$$\ddot{r} = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2 - \frac{b}{\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b}{\theta}\ddot{\theta} + 2\left(-\frac{b}{\theta^2}\dot{\theta}\right)\dot{\theta} = \frac{b}{\theta}\ddot{\theta} - 2\frac{b}{\theta^2}\dot{\theta}^2$$

SINCE $\dot{\theta} = \omega = \text{CONSTANT}$, $\ddot{\theta} = 0$, AND WE WRITE:

$$a_r = +\frac{2b}{\theta^3}\omega^2 - \frac{b}{\theta}\omega^2 = \frac{b\omega^2}{\theta^3}(2-\theta^2)$$

$$a_\theta = -2\frac{b}{\theta^2}\omega^2 = -\frac{b\omega^2}{\theta^3}(2\theta)$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{b\omega^2}{\theta^3}\sqrt{(2-\theta^2)^2 + (2\theta)^2} = \frac{b\omega^2}{\theta^3}\sqrt{4 - 4\theta^2 + \theta^4 + 4\theta^2}$$

$$a = \frac{b\omega^2}{\theta^3}\sqrt{4+\theta^4} \quad \blacktriangleleft$$

11.155 LOGARITHMIC SPIRAL $r = e^{b\theta}$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\dot{\theta}$$

$$\ddot{r} = be^{b\theta}\ddot{\theta} + b^2e^{b\theta}\dot{\theta}^2 = be^{b\theta}\left(\ddot{\theta} + b\dot{\theta}^2\right)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = be^{b\theta}\left(\ddot{\theta} + b\dot{\theta}^2\right) - e^{b\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{b\theta}\ddot{\theta} + 2\left(be^{b\theta}\dot{\theta}\right)\dot{\theta}$$

SINCE $\dot{\theta} = \omega = \text{CONSTANT}$, $\ddot{\theta} = 0$, AND WE WRITE

$$a_r = be^{b\theta}(b\omega^2) - e^{b\theta}\omega^2 = e^{b\theta}(b^2-1)\omega^2$$

$$a_\theta = 2be^{b\theta}\omega^2$$

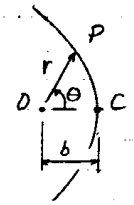
$$a = \sqrt{a_r^2 + a_\theta^2} = e^{b\theta}\omega^2\sqrt{(b^2-1)^2 + (2b)^2}$$

$$= e^{b\theta}\omega^2\sqrt{b^4 - 2b^2 + 1 + 4b^2} = e^{b\theta}\omega^2\sqrt{b^4 + 2b^2 + 1}$$

$$= e^{b\theta}\omega^2\sqrt{(b^2+1)^2} = e^{b\theta}\omega^2(b^2+1)$$

$$a = (1+b^2)\omega^2 e^{b\theta} \quad \blacktriangleleft$$

11.156 $r = \frac{2b}{1+\cos kt}$ $\qquad \theta = kt$



$$\dot{r} = \frac{2bk\sin kt}{(1+\cos kt)^2} \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

$$\ddot{r} = \frac{2bk}{(1+\cos kt)^4}\left[(1+\cos kt)^2 k\cos kt + (\sin kt)2(1+\cos kt)(k\sin kt)\right]$$

(a) WHEN $\theta = kt = 0$:

$r = b \quad \dot{r} = 0 \quad \ddot{r} = \frac{2bk}{(2)^4}\left[(2)^2 k(1) + 0\right] = \frac{1}{2}bk^2$

$\theta = 0 \quad \dot{\theta} = k \quad \ddot{\theta} = 0$

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = bk \qquad \underline{v} = bk\,\underline{e}_\theta \quad \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{1}{2}bk^2 - bk^2 = -\frac{1}{2}bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = b(0) + 2(0) = 0$$

$$\underline{a} = -\frac{1}{2}bk^2\,\underline{e}_r \quad \blacktriangleleft$$

(b) WHEN $\theta = kt = 90°$:

$r = 2b \quad \dot{r} = 2bk \quad \ddot{r} = \frac{2bk}{1^4}\left[0 + 2k\right] = 4bk^2$

$\theta = 90° \quad \dot{\theta} = k \quad \ddot{\theta} = 0$

$$v_r = \dot{r} = 2bk \qquad v_\theta = r\dot{\theta} = 2bk$$

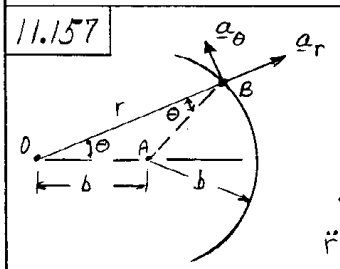
$$\underline{v} = 2bk\,\underline{e}_r + 2bk\,\underline{e}_\theta \quad \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4bk^2 - 2bk^2 = 2bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2b(0) + 2(2bk)k = 4bk^2$$

$$\underline{a} = 2bk^2\,\underline{e}_r + 4bk^2\,\underline{e}_\theta \quad \blacktriangleleft$$

11.157



FROM SKETCH:

$$r = 2b\cos\theta$$

$$\dot{r} = -2b\sin\theta\,\dot{\theta}$$

SINCE $\dot{\theta} = \text{CONSTANT}$, $\ddot{\theta} = 0$

$$\ddot{r} = -2b\cos\theta\,\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\cos\theta\,\dot{\theta}^2 - (2b\cos\theta)\dot{\theta}^2$$

$$a_r = -4b\cos\theta\,\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)(0) + 2(-2b\sin\theta)\dot{\theta}^2$$

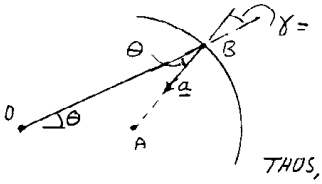
$$a_\theta = -4b\sin\theta\,\dot{\theta}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4b\dot{\theta}^2\sqrt{(-\cos\theta)^2 + (-\sin\theta)^2}$$

$$a = 4b\dot{\theta}^2$$

SINCE BOTH b AND $\dot{\theta}$ ARE CONSTANT, WE FIND THAT

$a = \text{CONSTANT}$ ◀



$$\gamma = \tan^{-1}\frac{a_\theta}{a_r} = \tan^{-1}\left(\frac{-4b\sin\theta\,\dot{\theta}^2}{-4b\cos\theta\,\dot{\theta}^2}\right)$$

$$\gamma = \tan^{-1}(\tan\theta)$$

$$\gamma = \theta$$

THUS, $\underline{a}$ IS DIRECTED TOWARD A ◀

11.158

$R = A \quad \theta = 2\pi t \quad z = B\sin 2\pi nt$
$\dot{R} = 0 \quad \dot{\theta} = 2\pi \quad \dot{z} = 2\pi nB\cos 2\pi nt$
$\ddot{R} = 0 \quad \ddot{\theta} = 0 \quad \ddot{z} = -4\pi^2 n^2 B\sin 2\pi nt$

VELOCITY (EQ 11.49)

$\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$

$\underline{v} = +A(2\pi)\underline{e}_\theta + 2\pi nB\cos 2\pi nt\,\underline{k}$

$v = 2\pi\sqrt{A^2 + n^2B^2\cos^2 2\pi nt}$ ◀

ACCELERATION (EQ 11.50)

$\underline{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$

$\underline{a} = -4\pi^2 A\underline{e}_R - 4\pi^2 n^2 B\sin 2\pi nt\,\underline{k}$

$a = 4\pi^2\sqrt{A^2 + n^4B^2\sin^2 2\pi nt}$ ◀

11.159

FROM PROB 11.158 FOR $n = 1$ AND $\theta = 2\pi t$

$\underline{v} = 2\pi A\underline{e}_\theta + 2\pi B\cos\theta\,\underline{k} = 2\pi[A\underline{e}_\theta + B\cos\theta\,\underline{k}]$

$\underline{a} = -4\pi^2 A\underline{e}_R - 4\pi^2 B\sin\theta\,\underline{k} = 4\pi^2[-A\underline{e}_R - B\sin\theta\,\underline{k}]$

DIRECTION OF BINORMAL IS PARALLEL TO $\underline{v}\times\underline{a}$

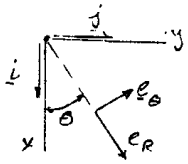
FROM FIG 11.26, WE OBSERVE THAT

$\underline{e}_R \times \underline{e}_\theta = \underline{k} \quad \underline{e}_\theta \times \underline{k} = \underline{e}_R \quad \underline{k}\times\underline{e}_R = \underline{e}_\theta$

$\underline{v}\times\underline{a} = 8\pi^3[A\underline{e}_\theta + B\cos\theta\,\underline{k}]\times[-A\underline{e}_R - B\sin\theta\,\underline{k}]$

$= 8\pi^3[A^2\underline{k} - AB\cos\theta\,\underline{e}_\theta - AB\sin\theta\,\underline{e}_R] \quad (1)$

WE SHALL EXPRESS $\underline{v}\times\underline{a}$ USING THE x, y, z COORDINATES



$\underline{e}_R = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$

$\underline{e}_\theta = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j}$

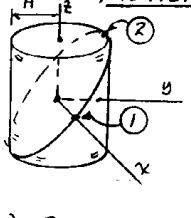
SUBSTITUTE INTO EQ(1):

$\underline{v}\times\underline{a} = 8\pi^3[A^2\underline{k} - AB(-\cos\theta\sin\theta\,\underline{i} + \cos^2\theta\,\underline{j}) - AB(\sin\theta\cos\theta\,\underline{i} + \sin^2\theta\,\underline{j})]$

$= 8\pi^3[A^2\underline{k} - AB(\cos^2\theta + \sin^2\theta)\underline{j}]$

$\underline{v}\times\underline{a} = 8\pi^3[A^2\underline{k} - AB\,\underline{j}]$

(a) WE NOTE THAT SINCE A AND B ARE CONSTANTS, THE DIRECTION OF THE BINORMAL (PARALLEL TO $\underline{v}\times\underline{a}$) IS CONSTANT AND THUS THE MOTION IS CONTAINED IN A PLANE ◀



SINCE THE INTERSECTION OF A PLANE AND A CYLINDER IS AN ELLIPSE,

PATH IS AN ELLIPSE

(b) RADII OF CURVATURE FROM PROB 11.158 FOR $n = 1$

$v = 2\pi\sqrt{A^2 + B^2\cos^2 2\pi t} \qquad a = 4\pi^2\sqrt{A^2 + B^2\sin^2 2\pi t}$

$\frac{dv}{dt} = 2\pi\left(\frac{1}{2}\right)\frac{2B^2\cos 2\pi t(-\sin 2\pi t)2\pi}{\sqrt{A^2 + B^2\cos^2 2\pi t}} = \frac{-4\pi^2B^2\sin 2\pi t\cos 2\pi t}{\sqrt{A^2 + B^2\cos^2 2\pi t}}$

(CONTINUED)

11.159 CONTINUED

SINCE $2\pi t = \theta$, WE MAY WRITE

$v = 2\pi\sqrt{A^2 + B^2\cos^2\theta} \qquad a = 4\pi^2\sqrt{A^2 + B^2\sin^2\theta}$

$\frac{dv}{dt} = \frac{-4\pi^2B^2\sin\theta\cos\theta}{\sqrt{A^2 + B^2\cos^2\theta}}$

MAXIMUM RADIUS OF CURVATURE OCCURS AT ①

SINCE AT ①, $\theta = 0$

$v^2 = 4\pi^2(A^2 + B^2) \qquad a = 4\pi^2 A \qquad \frac{dv}{dt} = 0$

THUS:

$a = a_n = \frac{v^2}{\rho_{MAX}}; \quad \rho_{MAX} = \frac{v^2}{a} = \frac{4\pi^2(A^2+B^2)}{4\pi^2 A} \qquad \rho_{MAX} = \frac{A^2+B^2}{A}$ ◀

MINIMUM RADIUS OF CURVATURE OCCURS AT ②

SINCE AT ②, $\theta = \frac{1}{2}\pi$

$v^2 = 4\pi^2A^2, \qquad a = 4\pi^2\sqrt{A^2+B^2} \qquad \frac{dv}{dt} = 0$

THUS:

$a = a_n = \frac{v^2}{\rho_{MIN}}; \quad \rho_{MIN} = \frac{v^2}{a} = \frac{4\pi^2A^2}{4\pi^2\sqrt{A^2+B^2}} \qquad \rho_{MIN} = \frac{A^2}{\sqrt{A^2+B^2}}$ ◀

11.160

$R = ht\tan\beta \quad \theta = 2\pi t \quad z = ht$
$\dot{R} = h\tan\beta \quad \dot{\theta} = 2\pi \quad \dot{z} = h$
$\ddot{R} = 0 \quad \ddot{\theta} = 0 \quad \ddot{z} = 0$

$\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k} = h\tan\beta\,\underline{e}_R + ht(\tan\beta)2\pi\underline{e}_\theta + h\underline{k}$

$v = h\sqrt{\tan^2\beta + 4\pi^2t^2\tan^2\beta + 1}$

$= h\sqrt{\tan^2\beta(1 + 4\pi^2t^2) + \tan^2\beta\cot^2\beta}$

$v = h\tan\beta\sqrt{(1 + 4\pi^2t^2) + \cot^2\beta}$ ◀

$\underline{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$

$= (-ht\tan\beta)(4\pi^2)\underline{e}_R + (2h\tan\beta)2\pi\underline{e}_\theta$

$a^2 = (4\pi^2 ht\tan\beta)^2 + (4\pi h\tan\beta)^2$

$a = 4\pi h\tan\beta\sqrt{1 + \pi^2t^2}$ ◀

11.161

$R = A(1 - e^{-t}) \quad \theta = 2\pi t \quad z = B(1 - e^{-t})$
$\dot{R} = Ae^{-t} \quad \dot{\theta} = 2\pi \quad \dot{z} = Be^{-t}$
$\ddot{R} = -Ae^{-t} \quad \ddot{\theta} = 0 \quad \ddot{z} = -Be^{-t}$

VELOCITY (EQ 11.49)

$\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$

$\underline{v} = Ae^{-t}\underline{e}_R + 2\pi A(1 - e^{-t})\underline{e}_\theta + Be^{-t}\underline{k}$

(a) WHEN $t = 0$: $e^{-t} = e^0 = 1$; $\underline{v} = A\underline{e}_R + B\underline{k}$ $\quad v = \sqrt{A^2 + B^2}$ ◀

(b) WHEN $t = \infty$: $e^{-t} = e^{-\infty} = 0$ $\quad \underline{v} = 2\pi A\underline{e}_\theta$ $\quad v = 2\pi A$ ◀

ACCELERATION (EQ 11.50)

$\underline{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$

$= [-Ae^{-t} - A(1 - e^{-t})4\pi^2]\underline{e}_R + [0 + 2Ae^{-t}(2\pi)]\underline{e}_\theta - Be^{-t}\underline{k}$

(a) WHEN $t = 0$: $e^{-t} = e^0 = 1$

$\underline{a} = -A\underline{e}_R + 4\pi A\underline{e}_\theta - B\underline{k}$

$a = \sqrt{A^2 + (4\pi A)^2 + B^2} \qquad a = \sqrt{(1 + 16\pi^2)A^2 + B^2}$ ◀

(b) WHEN $t = \infty$: $e^{-t} = e^{-\infty} = 0$

$\underline{a} = -4\pi^2 A\underline{e}_R \qquad a = 4\pi^2 A$ ◀

11.162

$R = A \quad \theta = 2\pi t \quad z = A\sin^2 \pi t$

$\dot{R} = 0 \quad \dot{\theta} = 2\pi \quad \dot{z} = 2\pi A \sin \pi t \cos \pi t = \pi A \sin 2\pi t$

$\ddot{R} = 0 \quad \ddot{\theta} = 0 \quad \ddot{z} = 2\pi^2 A \cos 2\pi t$

VELOCITY (EQ 11.49)

$\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k} = \pi A \underline{e}_\theta + \pi A \sin 2\pi t \, \underline{k}$

$v = \pi A\sqrt{4 + \sin^2 2\pi t}$ ◀

ACCELERATION (EQ 11.50)

$\underline{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$

$= -4\pi^2 A \underline{e}_R + 2\pi^2 A \cos 2\pi t \, \underline{k}$

$a = 2\pi^2 A\sqrt{4 + \cos^2 2\pi t}$ ◀

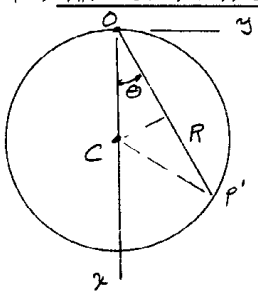
11.163

$R = 2k\cos t \quad \theta = t \quad z = pt$

$\dot{R} = -2k\sin t \quad \dot{\theta} = 1 \quad \dot{z} = p$

$\ddot{R} = -2k\cos t \quad \ddot{\theta} = 0 \quad \ddot{z} = 0$

(a) PATH OF PARTICLE $OP = r \quad \theta = t$



CONSIDER PROJECTION ON THE xy PLANE.

$x = R\cos\theta = 2k\cos^2\theta$

$x = 2k\left(\tfrac{1}{2}\right)(1 + \cos 2\theta)$

$(x - k) = k\cos 2\theta$

$y = R\sin\theta = 2k\cos\theta\sin\theta$

$y = 2k\left(\tfrac{1}{2}\right)\sin 2\theta$

$y = k\sin 2\theta$

WE NOTE THAT $(x-k)^2 + y^2 = k^2(\cos^2 2\theta + \sin^2 2\theta) = k^2$

THIS IS EQUATION OF A CIRCLE OF RADIUS k WITH CENTER AT $x = k$, $y = 0$. SINCE THE VELOCITY COMPONENT $\dot{z} = p$ IS CONSTANT, THE PATH IS A:

HELIX, VERTICAL AXIS AT $\theta = 0$, $R = k$ ◀

(b) VELOCITY AND ACCELERATION

EQ 11.49: $\underline{v} = \dot{R}\underline{e}_R + R\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$

$\underline{v} = (-2k\sin t)\underline{e}_R + (2k\cos t)\underline{e}_\theta + p\underline{k}$

SINCE $\underline{e}_R, \underline{e}_\theta, \underline{k}$ FORM AN ORTHOGONAL TRIAD

$v = \sqrt{v_R^2 + v_\theta^2 + v_z^2} = \sqrt{(-2k\sin t)^2 + (2k\cos t)^2 + p^2}$

$= \sqrt{4k^2(\sin^2 t + \cos^2 t) + p^2} \qquad v = \sqrt{4k^2 + p^2}$ ◀

WE NOTE THAT v = CONSTANT, THUS $dv/dt = 0$

EQ 11.50: $\underline{a} = (\ddot{R} - R\dot{\theta}^2)\underline{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$

$= (-2k\cos t - 2k\cos t)\underline{e}_R + (0 - 4k\sin t)\underline{e}_\theta + 0$

$= -(4k\cos t)\underline{e}_R - (4k\sin t)\underline{e}_\theta$

$a = \sqrt{a_R^2 + a_\theta^2 + a_z^2} = 4k\sqrt{\cos^2 t + \sin^2 t} \qquad a = 4k$ ◀

(c) RADIUS OF CURVATURE WE RECALL THAT $dv/dt = 0$,

EQ 11.39: $\underline{a} = \left(\frac{dv}{dt}\right)\underline{e}_t + \frac{v^2}{\rho}\underline{e}_n = 0 + \frac{v^2}{\rho}\underline{e}_n$

$a = \frac{v^2}{\rho} \qquad \rho = \frac{v^2}{a} = \frac{4k^2 + p^2}{4k}$

$\rho = k + \frac{p^2}{4k}$ ◀

*11.164

FROM SOLUTION OF PROB 11.86

$\underline{v} = (Rp\cos pt)\underline{i} + c\underline{j} - (Rp\sin pt)\underline{k}$

$\underline{a} = -(Rp^2\sin pt)\underline{i} \quad -(Rp^2\cos pt)\underline{k}$

SINCE BOTH $\underline{v}$ AND $\underline{a}$ LIE IN THE OSCULATING PLANE, WE NOTE THAT THE BINORMAL IS PARALLEL TO $\underline{v}\times\underline{a}$.

$$\underline{v}\times\underline{a} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ (Rp\cos pt) & c & -(Rp\sin pt) \\ -(Rp^2\sin pt) & 0 & -(Rp^2\cos pt) \end{vmatrix}$$

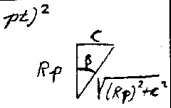
$= -cRp^2\cos pt\,\underline{i} + cRp^2\sin pt\,\underline{k} + R^2p^3(\sin^2 pt + \cos^2 pt)\underline{j}$

$\underline{v}\times\underline{a} = Rp^2\left[-c\cos pt\,\underline{i} + Rp\,\underline{j} + c\sin pt\,\underline{k}\right]$

LET β = ANGLE BETWEEN $\underline{e}_b$ AND THE y AXIS:

$$\cos\beta = \frac{(\underline{v}\times\underline{a})\cdot\underline{j}}{|\underline{v}\times\underline{a}|(1)} = \frac{Rp^2[Rp]}{Rp^2\sqrt{(c\cos pt)^2 + (Rp)^2 + (c\sin pt)^2}}$$

$$\cos\beta = \frac{Rp}{\sqrt{(Rp)^2 + c^2}} \qquad \text{OR:} \quad \tan\beta = \frac{c}{Rp}$$



SINCE BINORMAL ⊥ OSCULATING PLANE, THE ANGLE BETWEEN OSCULATING PLANE AND THE y AXIS IS $\gamma = 90° - \beta$.

$\tan\gamma = \tan(90° - \beta) = \cot\beta = \frac{Rp}{c} \qquad \gamma = \tan^{-1}\frac{Rp}{c}$ ◀

*11.165

FROM SOLUTION OF PROB 11.87:

$\underline{v} = \underline{i} + 6t^2\underline{j} + 4t\underline{k}$

$\underline{a} = 12t\underline{j} + 4\underline{k}$

SINCE BOTH $\underline{v}$ AND $\underline{a}$ LIE IN THE OSCULATING PLANE, WE NOTE THAT BINORMAL IS PARALLEL TO $\underline{v}\times\underline{a}$.

$$\underline{v}\times\underline{a} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 6t^2 & 4t \\ 0 & 12t & 4 \end{vmatrix} = -24t^2\underline{i} - 4\underline{j} + 12t\underline{k}$$

(a) WHEN $t = 0$: $\underline{v}\times\underline{a} = -4\underline{j}$

BINORMAL IS UNIT VECTOR: $\underline{e}_b = -\underline{j}$ $\quad \theta_x = \theta_z = 90°, \ \theta_y = 180°$ ◀

(b) WHEN $t = 2s$: $\underline{v}\times\underline{a} = -96\underline{i} - 4\underline{j} + 24\underline{k}$

$|\underline{v}\times\underline{a}| = \sqrt{(-96)^2 + (-4)^2 + (24)^2} = 99.035$

BINORMAL IS UNIT VECTOR $\underline{e}_b$:

$$\underline{e}_b = \frac{\underline{v}\times\underline{a}}{|\underline{v}\times\underline{a}|} = \frac{1}{99.035}\left[-96\underline{i} - 4\underline{j} + 24\underline{k}\right]$$

DIRECTION ANGLES OF BINORMAL:

$\cos\theta_x = \frac{-96}{99.035} = -0.96935 \qquad \theta_x = 165.8°$ ◀

$\cos\theta_y = \frac{-4}{99.035} = -0.04039 \qquad \theta_y = 92.3°$ ◀

$\cos\theta_z = \frac{+24}{99.035} = 0.24234 \qquad \theta_z = 76.0°$ ◀

11.166

TANGENTIAL MOTION (UNIFORM ACCELERATION)

AT $s_0 = 0$, $v_0 = 54\ km/h = 15\ m/s$

AT $s_1 = 150\ m$, $v_1 = 90\ km/h = 25\ m/s$

$$v_1^2 = v_0^2 + 2a_t(s_1 - s_0)$$

$$(25)^2 = (15)^2 + 2a_t(150 - 0) \qquad a_t = 1.333\ ft/s^2$$

AT $s = 100\ m$: $\quad v^2 = v_0^2 + 2a_t(s - s_0)$

$$v^2 = (15)^2 + 2(1.333)(100 - 0) = 491.67$$

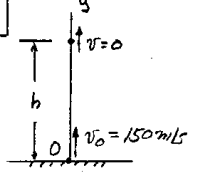
$$v = 22.17\ m/s$$

$$a_n = \frac{v^2}{\rho} = \frac{22.17^2}{240} = 2.049\ m/s^2$$

TOTAL ACCELERATION $\quad a = \sqrt{a_t^2 + a_n^2}$

$$a = \sqrt{(1.333)^2 + (2.049)^2} = 2.444\ m/s^2 \qquad a = 2.44\ m/s^2$$ ◀

11.167



LET $k = +0.0005$

THEN $a = -(g + kv^2)$

$$v\frac{dv}{dy} = -(g + kv^2)$$

$$\int_{v_0}^{0} -\frac{v\,dv}{(g + kv^2)} = \int_0^h dy$$

$$\left| -\frac{1}{2k} \ln(g + kv^2) \right|_{v_0}^{0} = h$$

$$h = \left| \frac{1}{2k} \ln(g + kv^2) \right|_0^{v_0} = \frac{1}{2k} \ln \frac{g + kv_0^2}{g} = \frac{1}{2k} \ln\left(1 + \frac{kv_0^2}{g}\right)$$

DATA: $k = 0.0005$, $v_0 = 150\ m/s$, $g = 9.81\ m/s^2$

$$\frac{kv_0^2}{g} = \frac{(0.0005)(150)^2}{9.81} = 1.1468$$

$$h = \frac{1}{2k} \ln\left(1 + \frac{kv_0^2}{g}\right) = \frac{1}{2(0.0005)} \ln(1 + 1.1468)$$

$$h = (1000)(0.7640) \qquad h = 764\ m$$ ◀

11.168

$$\underline{r} = (8\sin\pi t)\underline{i} - (2\cos 2\pi t)\underline{j}$$

$$\underline{v} = (8\pi\cos\pi t)\underline{i} + (4\pi\sin 2\pi t)\underline{j}$$

$$\underline{a} = -(8\pi^2\sin\pi t)\underline{i} + (8\pi^2\cos 2\pi t)\underline{j}$$

(a) WHEN $t = 1\ s$:

$$\underline{v} = (8\pi\cos\pi)\underline{i} + (4\pi\sin 2\pi)\underline{j}$$

$$\underline{v} = -8\pi\underline{i} + 0 \qquad \underline{v} = -(8\pi\ m/s)\underline{i}$$ ◀

$$\underline{a} = -(8\pi^2\sin\pi)\underline{i} + (8\pi^2\cos 2\pi)\underline{j}$$

$$\underline{a} = 0 + 8\pi^2\underline{j} \qquad \underline{a} = (8\pi^2\ m/s^2)\underline{j}$$ ◀

(b) PATH OF PARTICLE

SINCE $\underline{r} = x\underline{i} + y\underline{j}$; $\quad x = 8\sin\pi t \qquad y = -2\cos 2\pi t$

RECALL THAT $\cos 2\theta = 1 - 2\sin^2\theta$ AND WRITE

$$y = -2\cos 2\pi t = -(1 - 2\sin^2\pi t) \qquad (1)$$

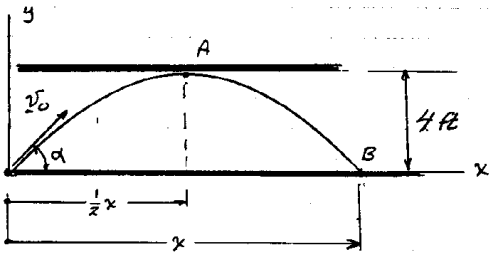
BUT, SINCE $x = 8\sin\pi t$,

OR $\sin\pi t = x/8$, EQ(1) YIELDS

$$y = -\left[1 - 2\left(\frac{x}{8}\right)^2\right] \qquad y = \frac{x^2}{32} - 1$$

PARABOLA Q.E.D. ◀

11.169



VERTICAL MOTION (UNIFORMLY ACCELERATED)

WE NOTE THAT MAXIMUM HEIGHT = 4 ft, THUS

AT $y = 4\ ft$, $v_y = 0$:

$$v_y^2 = (v_0)_y^2 + 2a(y - y_0)$$

$$0 = (v_0)_y^2 + 2(-32.2\ ft/s^2)(4\ ft)$$

$$(v_0)_y^2 = 257.6\ ft^2/s^2 \qquad (v_0)_y = 16.05\ ft/s$$

SINCE $v_0 = 40\ ft/s$, WE WRITE

$$v_0^2 = (v_0)_x^2 + (v_0)_y^2$$

$$(40\ ft/s)^2 = (v_0)_x^2 + 257.6\ ft^2/s^2 \qquad (v_0)_x = 36.64\ ft/s$$

LET t_A = TIME FOR WATER TO REACH POINT A

AT A: $v_y = 0 \qquad v_y = (v_y)_0 - g\,t_A$

$$0 = 16.05 - 32.2\,t_A$$

$$t_A = 0.4984\ s$$

FOR WATER TO REACH B: $t_B = 2t_A = 0.9969\ s$

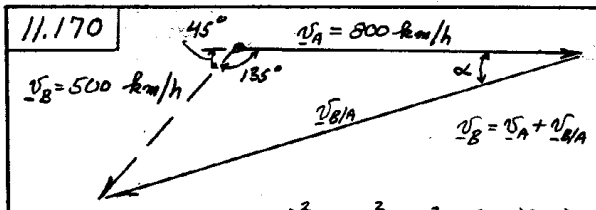
HORIZONTAL MOTION (UNIFORM MOTION)

$$x = (v_0)_x t_B = (36.64\ ft/s)(0.9969\ s) = 36.53\ ft$$

$$x = 36.5\ ft$$ ◀

MAKRON Books
CORTESIA
Marketing Universitario
(0xx11) 3645-3105
(0xx11) 3842-6729
fax: (0xx11) 3845-6995

11.170



LAW OF COSINES $v_{B/A}^2 = 800^2 + 500^2 - 2(800)(500)\cos 135°$

$v_{B/A} = 1206.5$ km/h

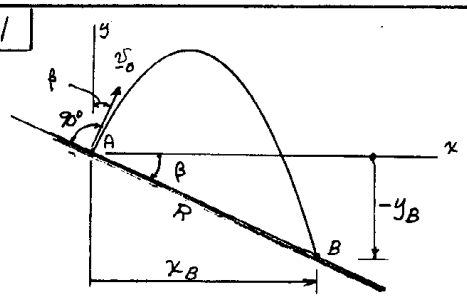
LAW OF SINES $\frac{\sin\alpha}{500} = \frac{\sin 135°}{1206.5}$ $\alpha = 17.04°$

CHANGE IN POSITION IN 3 MINUTES $= \frac{3}{60}$ hr $= 0.05$ hr

$\Delta r_{B/A} = v_{B/A}\Delta t = (1206.5 \text{ km/h})(0.05 \text{ hr}) = 60.325$ km

$\Delta r_{B/A} = 60.3$ km $\angle 17.0°$ ◀

11.171



AT A: SINCE $v_0 \perp$ SURFACE OF INCLINE,

$(v_x)_0 = v_0 \sin\beta$ $(v_y)_0 = v_0\cos\beta$

HORIZONTAL MOTION (UNIFORM MOTION)

$x = (v_0)_x t = (v_0 \sin\beta)t$

VERTICAL MOTION (UNIFORMLY ACCELERATED)

$y = (v_0)_y t - \frac{1}{2}gt^2$

$y = (v_0\cos\beta)t - \frac{1}{2}gt^2$

AT B: FOR BALL TO STRIKE B AT $t = t_1$:

$-y_B = x_B \tan\beta$

OR $y_B = -x_B\frac{\sin\beta}{\cos\beta}$

$(v_0\cos\beta)t_1 - \frac{1}{2}gt_1^2 = -\left[(v_0\sin\beta)t_1\right]\frac{\sin\beta}{\cos\beta}$

$(v_0\cos^2\beta)t_1 - \frac{1}{2}gt_1^2(\cos\beta) = -(v_0\sin^2\beta)t_1$

$t_1 = 0$ AND $t_1 = \frac{2v_0}{g}\frac{(\cos^2\beta + \sin^2\beta)}{\cos\beta} = \frac{2v_0}{g}\frac{1}{\cos\beta}$

$x_B = (v_0\sin\beta)t_1 = (v_0\sin\beta)\frac{2v_0}{g}\frac{1}{\cos\beta} = \frac{2v_0^2}{g}\tan\beta$

BUT, R = DISTANCE AB ALONG INCLINE,

$R = \frac{x_B}{\cos\beta}$ $R = \frac{2v_0^2}{g}\frac{\tan\beta}{\cos\beta}$ ◀

11.172

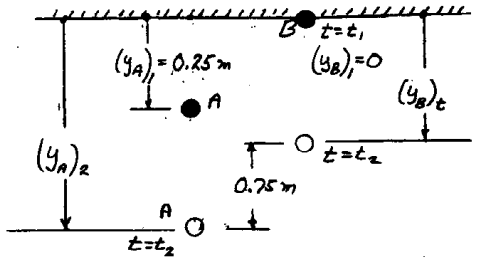
SEE SOLUTION OF PROB 11.171

GIVEN: $v_0 = 10$ m/s, $\beta = 30°$

$R = \frac{2v_0^2}{g}\frac{\tan\beta}{\cos\beta} = \frac{2(10)^2}{9.81}\frac{\tan 30°}{\cos 30°}$ $R = 13.59$ m ◀

11.173

UNIFORMLY ACCELERATED MOTIONS ORIGIN AT FAUCET



DROP A: BEGINS TO FALL WHEN $t = 0$.

t_1 = TIME WHEN $(y_A)_1 = 0.25$ m

$(y_A)_1 = \frac{1}{2}gt_1^2$; $0.25 \text{ m} = \frac{1}{2}(9.81 \text{ m/s}^2)t_1^2$

$t_1^2 = \frac{0.5}{g}$ $t_1 = 0.2258$ s

DROP B: BEGINS TO FALL WHEN $t = t_1$

$y_B = \frac{1}{2}g(t - t_1)^2$

DROPS B AND A ARE 0.75 m APART WHEN $t = t_2$

$(y_A)_2 - (y_B)_2 = 0.75$ m

$\frac{1}{2}gt_2^2 - \frac{1}{2}g(t_2 - t_1)^2 = 0.75$ m

$\cancel{\frac{1}{2}gt_2^2} - \cancel{\frac{1}{2}gt_2^2} + gt_2t_1 - \frac{1}{2}gt_1^2 = 0.75$ m

SOLVE FOR t_2: $t_2 = \frac{1}{gt_1}\left(0.75 \text{ m} + \frac{1}{2}gt_1^2\right)$

$(y_A)_2 = \frac{1}{2}gt_2^2 = \frac{1}{2}\frac{g}{g^2t_1^2}\left(0.75 \text{ m} + \frac{1}{2}gt_1^2\right)^2$

SUBSTITUTE $t_1^2 = \frac{0.5}{g}$ $(y_A)_2 = \frac{1}{2}\left(\frac{1}{0.5}\right)\left(0.75 + \frac{1}{2}(0.5)\right) = 1$ m

DROP A WILL HAVE FALLEN 1 m ◀

11.174

$r = 6\cos 2\pi t$ $\theta = \pi t$

$\dot r = -12\pi\sin 2\pi t$ $\dot\theta = \pi$

$\ddot r = -24\pi^2\cos 2\pi t$ $\ddot\theta = 0$

VELOCITY $v_r = \dot r = -12\pi\sin 2\pi t$ $v_\theta = r\dot\theta = 6\pi\cos 2\pi t$

ACCELERATION $a_r = \ddot r - r\dot\theta^2 = -24\pi^2\cos 2\pi t - 6\pi^2\cos 2\pi t$

$a_r = -30\pi^2\cos 2\pi t$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(-12\pi\sin 2\pi t)\pi$

$a_\theta = -24\pi^2\sin 2\pi t$

(a) WHEN $t = 0$: $v_r = 0$ $v_\theta = 6\pi$ $v = 6\pi$

$\underline v = (18.85 \text{ in./s})\underline e_\theta$ ◀

$a_r = -30\pi^2$ $a_\theta = 0$ $a = -30\pi^2$

$\underline a = -(296 \text{ in./s}^2)\underline e_r$ ◀

(b) WHEN $t = \frac{1}{4}$ s: $r = 6\cos\frac{\pi}{4} = 0$

$v_r = -12\pi\sin\pi/2 = -12\pi$

$v_\theta = 6\pi\cos\pi/2 = 0$

$v = -12\pi$

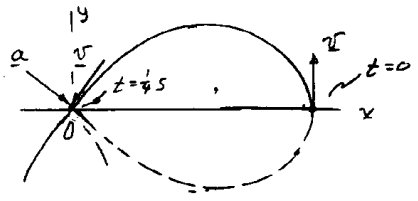
$\underline v = -(37.7 \text{ in./s})\underline e_r$ ◀

$a_r = -30\pi^2\cos\frac{\pi}{2} = 0$

$a_\theta = -24\pi^2\sin\pi/2 = -24\pi^2$

$a = -24\pi^2$

$\underline a = -(237 \text{ in./s}^2)\underline e_\theta$ ◀



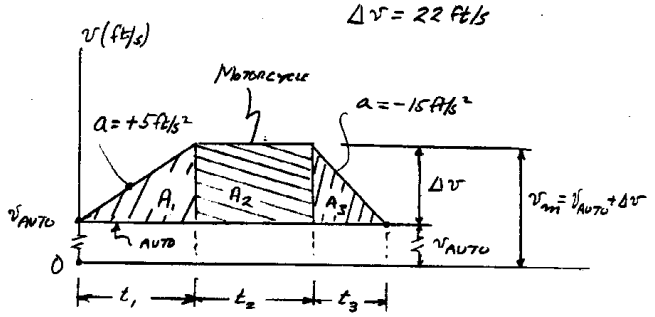
11.175

RELATIVE TO AUTOMOBILE THE MOTORCYCLE MUST MOVE A TOTAL DISTANCE:

$$\Delta x = 8\text{ ft} + 60\text{ ft} + 15\text{ ft} + 60\text{ ft} = 143\text{ ft}$$

ALLOWABLE INCREASE IN SPEED $= 55\text{ mi/h} - 40\text{ mi/h} = 15\text{ mi/h}$

$$\Delta v = 22\text{ ft/s}$$



ACCELERATION PHASE: $t_1 = \frac{\Delta v}{a} = \frac{22\text{ ft/s}}{5\text{ ft/s}^2} = 4.4\text{ s}$

$$A_1 = \tfrac{1}{2}(22\text{ ft/s})(4.4\text{ s}) = 48.4\text{ ft}$$

DECELERATION PHASE: $t_3 = \frac{\Delta v}{a} = \frac{22\text{ ft/s}}{15\text{ ft/s}^2} = 1.467\text{ s}$

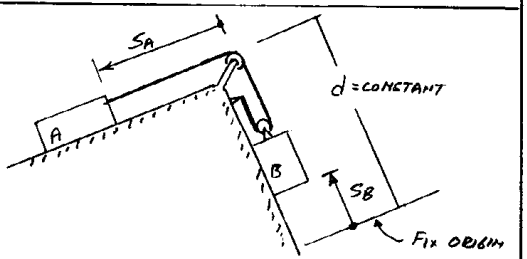
$$A_3 = \tfrac{1}{2}(22\text{ ft/s})(1.467\text{ s}) = 16.14\text{ ft}$$

BUT: $\Delta x = A_1 + A_2 + A_3$

$$143\text{ ft} = 48.4\text{ ft} + (22\text{ ft/s})t_2 + 16.14\text{ ft} \qquad t_2 = 3.567\text{ s}$$

$$t_{TOTAL} = 4.4\text{ s} + 3.567\text{ s} + 1.467\text{ s} \qquad t_{TOTAL} = 9.43\text{ s}$$ ◀

11.176



LENGTH OF CABLE

$$s_A + 2(d - s_B) + \text{constant}$$

$\frac{d}{dt}$: $\quad v_A - 2v_B = 0$ OR $v_B = \frac{1}{2}v_A$

FOR USE IN PROB 11.177 WE AGAIN DIFFERENTIATE WITH RESPECT TO t AND OBTAIN:

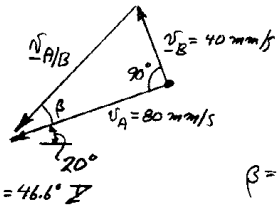
$$a_A - 2a_B = 0 \quad \text{OR} \quad a_B = \tfrac{1}{2}a_A$$

(a) FOR $v_A = 80\text{ mm/s}$ ↙:

$$v_B = \tfrac{1}{2}v_A = \tfrac{1}{2}(80\text{ mm/s}) = 40\text{ mm/s}$$

$$\underline{v}_B = 40\text{ mm/s} \measuredangle 70°$$ ◀

(b)



$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

$$v_{A/B} = \sqrt{40^2 + 80^2}$$

$$v_{A/B} = 89.44\text{ mm/s}$$

$$\beta = \tan^{-1}\frac{40}{80} = 26.6°$$

$$20° + \beta = 46.6° \measuredangle$$

FOR $\Delta t = 4\text{ s}$: $r_{A/B} = v_{A/B}\Delta t = (89.44\text{ mm/s})(4\text{ s}) = 357.8\text{ mm}$

$$\underline{r}_{A/B} = 358\text{ mm} \measuredangle 46.6°$$ ◀

11.177

SEE SOLUTION OF PROB 11.176 FOR SKETCH AND FOLLOWING RELATIONS:

$$v_B = \tfrac{1}{2}v_A$$

$$a_B = \tfrac{1}{2}a_B$$

(a) GIVEN $a_A = 48\text{ mm/s}^2$ ↙

$$a_B = \tfrac{1}{2}a_A = \tfrac{1}{2}(48\text{ mm/s}^2) = 24\text{ mm/s}^2$$

$$\underline{a}_B = 24\text{ mm/s}^2 \measuredangle 70°$$ ◀

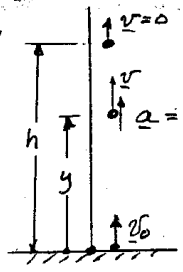
(b) VELOCITY OF B WHEN $t = 5\text{ s}$.

$$v_B = a_B t = (24\text{ mm/s}^2)(5\text{ s}) = 120\text{ mm/s}$$

$$\underline{v}_B = 120\text{ mm/s} \measuredangle 70°$$ ◀

PROBLEM 11.C1

UPWARD MOTION
(POSITIVE SENSE ↑)



$$a = v\frac{dv}{dy} = -(g+kv^2)$$

$$\int_{v_0}^{0} -\frac{v\,dv}{(g+kv^2)} = \int_0^h dy$$

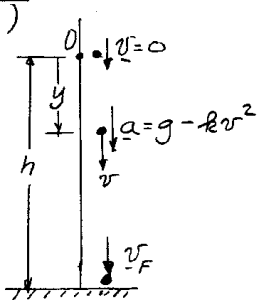
$$\left|\frac{1}{2k}\ln(g+kv^2)\right|_{v_0}^{0} = h$$

$$h = \frac{1}{2k}\ln\left(\frac{g+kv_0^2}{g}\right)$$

$$h = \frac{1}{2k}\ln\left(1+\frac{kv_0^2}{g}\right) \quad \triangleleft$$

AT $y=0$, $v=v_0$
AT $y=h$, $v=0$

DOWNWARD MOTION
(POSITIVE SENSE ↓)



$$a = v\frac{dv}{dy} = g-kv^2$$

$$\int_0^{v_F} \frac{v\,dv}{(g-kv^2)} = \int_0^h dy$$

$$\left|-\frac{1}{2k}\ln(g-kv^2)\right|_0^{v_F} = h$$

$$h = -\frac{1}{2k}\ln\left(\frac{g-kv_F^2}{g}\right)$$

$$\left(1-\frac{kv_F^2}{g}\right) = e^{-2hk}$$

$$v_F = \sqrt{\frac{g}{k}\left(1-e^{-2hk}\right)} \quad \triangleleft$$

AT $y=0$, $v=0$
AT $y=h$, $v=v_F$

```
10 '*****************************************************************
20 '*'                      PROBLEM 11.C1                            *
30 '*                       BASIC VERSION                           *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '            VO = initial velocity of rocket (m/s)
90 '             H = maximum height reached by rocket  (m)
100 '            A = acceleration of rocket  (m/^2)
110 '
120 '          Data
130             K = .0005
140             G = 9.810001
150 '
160       PRINT "    VO              H              VF    "
170       PRINT "  (m/s)            (m)            (m)    ":PRINT
180 '
190       FOR VO = 0 TO  400   STEP 25
200 '
210 '           Maximum height reached by rocket
220         H = (.5/K)*LOG(1+K*(VO^2)/G)
230 '
240 '           Velocity of rocket as it hits ground
250         VF = SQR(G/K*(1 - EXP(-2*H*K)))
260 '
270         PRINT USING "   ###     ";VO;:
280         PRINT USING "    ####.##    ";H,VF
290       NEXT
```

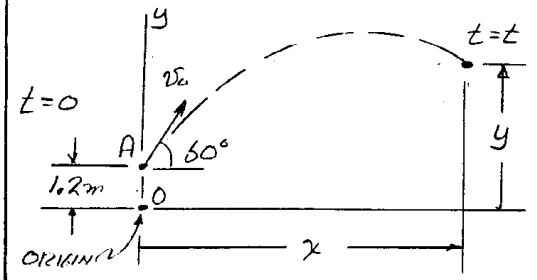
VO (m/s)	H (m)	VF (m)
0	0.00	0.00
25	31.36	24.61
50	119.93	47.09
75	252.08	66.12
100	411.90	81.39
125	585.77	93.26
150	763.97	102.38
175	940.36	109.36
200	1111.44	114.73
225	1275.44	118.91
250	1431.63	122.20
275	1579.90	124.81
300	1720.47	126.92
325	1853.72	128.63
350	1980.12	130.04
375	2100.15	131.22
400	2214.29	132.20

```
C*****************************************************************
C*'                      PROBLEM 11.C1                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C                    Definition of Symbols
C
C                 VO = initial velocity of rocket  (m/s)
C                  H = maximum height reached by rocket  (m)
C                  A = acceleration of rocket  (m**2)
C
      REAL K, VO
C                    Data
                     K = 0.0005
                     G = 9.81
C
      WRITE (*,'(/,2X,A\)') '  V0                H                VF'
      WRITE (*,'(/,2X,A\)') ' (m/s)              (m)              (m)'
      WRITE (*,'(/,2X,A\)')
C
      DO 10 I = 0,400,25
                     VO = I
C
C                    Maximum height reached by rocket
           H = (0.5/K)*LOG(1.+K*(VO**2)/G)
C
C                    Velocity of rocket as it hits ground
          VF = SQRT(G/K*(1. - EXP(-2.*H*K)))
C
          WRITE (*,20) VO,H,VF
   20     FORMAT (3X,F5.1,12X,F7.2,12X,F6.2)
   10 CONTINUE
      END
```

V0 (m/s)	H (m)	VF (m)
.0	.00	.00
25.0	31.36	24.61
50.0	119.93	47.09
75.0	252.08	66.12
100.0	411.90	81.39
125.0	585.77	93.26
150.0	763.97	102.38
175.0	940.36	109.36
200.0	1111.44	114.73
225.0	1275.44	118.91
250.0	1431.63	122.20
275.0	1579.90	124.81
300.0	1720.47	126.92
325.0	1853.72	128.63
350.0	1980.12	130.04
375.0	2100.15	131.22
400.0	2214.29	132.20

PROBLEM 11.C2

PATH OF STREAM OF WATER



HORIZONTAL MOTION (UNIFORM)

$v_x = v_0 \cos 60°$ $\quad x = v_0 t = (v_0 \cos\theta)t = 0.5 v_0 t$ (1)

VERTICAL MOTION (UNIFORMLY ACCELERATED)

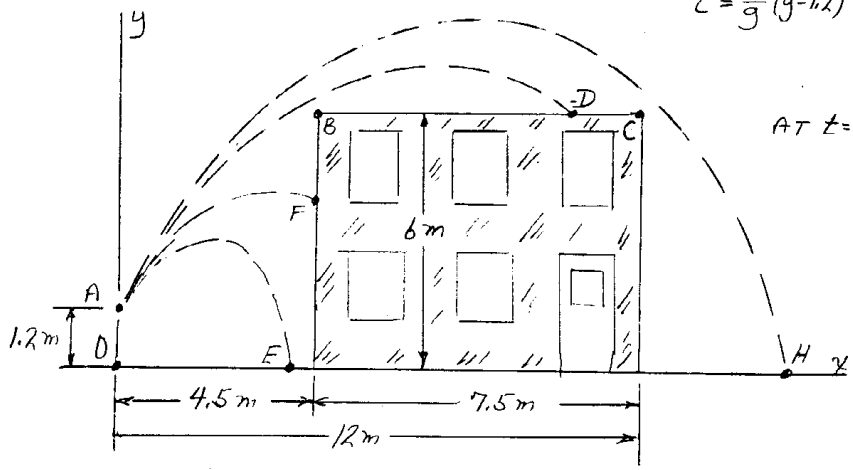
$v_y = v_0 \sin 60° - g$

$y = (1.2\,m) + (v_0 \sin 60°)t - \frac{1}{2} g t^2$ (2)

REARRANGE: $t^2 - \left(\frac{2v_0}{g}\sin 60°\right)t + \frac{2}{g}(y - 1.2) = 0$

$t^2 + Bt + C = 0$: $B = -\frac{2v_0}{g}\sin 60° = \frac{2v_0}{g}\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}\frac{v_0}{g}$

$C = \frac{2}{g}(y - 1.2)$ $\qquad t = 0.5\left(-B + \sqrt{B^2 - 4C}\right)$ (3)



AT $t=0$, PARTICLE OF WATER LEAVES A WITH VELOCITY v_0 AND STRIKES THE GROUND OR BUILDING AT TIME t.

AS MAGNITUDE OF v_0 INCREASES WE HAVE:

FOR $y = 0$ AND $x < 4.5\,m$, WATER STRIKES GROUND AT F; ELSE →

FOR $x = 4.5\,m$ AND $y < 6\,m$, WATER STRIKES WALL AT F; ELSE →

FOR $y = 6\,m$ AND $x < 12\,m$, WATER STRIKES ROOF AT D; ELSE →

FOR $y = 0$, WATER STRIKES GROUND AT H.

$B = -\sqrt{3}\,v_0/g$; $C = 2(-1.2)/g = -2.4/g$
USE (3) TO FIND t, THEN USE (1) TO FIND x.

FROM (1): $4.5 = 0.5 v_0 t$, FIND $t = 9/v_0$
USE (2) TO OBTAIN y.

$B = -\sqrt{3}\,v_0/g$; $C = 2(6-1.2)/g = 9.6/g$
USE (3) TO FIND t, THEN USE (1) TO FIND x; THEN $BD = 12 - x$

$B = -\sqrt{3}\,v_0/g$; $C = 2(-1.2)/g = -2.4/g$
USE (3) TO FIND t, THEN USE (1) TO FIND x

```
10 '*************************************************************
20 '*                      PROBLEM 11.C2                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '          X,Y = coordinates: origin at the feet of the firefighter
70 '           VO = initial velocity of the water
80 '            T = time for water to move from the nozzle to
90 '                where it strikes the ground or the building
100 '         B,C = coefficicents in quadratic formula
110 ' -----------------------------------------------------------
120 '   Note: Initial velocity forms angle of 60 degrees with
130 '         horizontal; therefore
140 '           VO sub x = VO*cos 60 = VO*0.5
150 '           VO sub y = VO*sin 60 = VO*0.86603 = V)*(SQR(3))/2
160 ' -----------------------------------------------------------
170       G = 9.810001:  I = 0:  K = 0:  J = 0
180 '
190       PRINT "Water strikes ground at a distance x from firefighter"
200 '           Note: If x > 4.5 meters, water will strike building.
210 '
```

CONTINUED

```
C*************************************************************
C*'                     PROBLEM 11.C2                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C                Definition of Symbols
C
C                  X,Y = coordinates: origin at the feet of the fireman
C                   VO = initial velocity of water
C                    T = time for water to move from the nozzle to
C                        where it strikes the ground or the building
C                  B,C = coefficient in quadratic formula
      REAL VO
      G = 9.81
      I = 0
      J = 0
      K = 0
C
      WRITE (*,'(/,2X,2A\)')  'Water strikes ground at',
     +                        ' a distance x  from firefighter'
      WRITE (*,'(/,A\)')
C           Note : If x > 4.5 meters, water will strike building.
C
```

CONTINUED

PROBLEM 11.C2 CONTINUED

PROGRAM BASIC VERSION CONTINUED

```
220       FOR VO = 4 TO 16
230          B = -SQR(3)*VO/G:    C = -2.4/G
240          T = .5*(-B+SQR(B^2 -4*C)):   X = T*VO/2
250          WHILE X <4.5
260             PRINT USING "   Vo = ## m/s   ";VO;
270             PRINT USING "    x = ##.### m";X
280             GOTO 610
290          WEND
300          I = I + 1: IF I > 1 GOTO 340
310          PRINT
320          PRINT "Water strikes the wall at a distance y above ground"
330 '             Note: x = 4.5 m  and  y < 6 meters
340          T = 9/VO
350          Y = 1.2 + .86603*VO*T -.5*G*(T)^2
360          WHILE Y < 6
370             PRINT USING "   Vo = ## m/s   ";VO;
380             PRINT USING "    y = ##.### m";Y
390             GOTO 610
400          WEND
410          J = J + 1: IF J > 1 GOTO 450
420          PRINT
430          PRINT "Water strikes the roof at a distance BD from B"
440 '             Note: Height of roof is y = 6 m; x < 12 meters.
450          B =-SQR(3)*VO/G:   C = 9.600001/G
460          T = .5*(-B+SQR(B^2 -4*C)):   X = .5*VO*T
470          WHILE X < 12
480             BD = X - 4.5
490             PRINT USING "   Vo = ## m/s   ";VO;
500             PRINT USING "   BD = ##.### m";BD
510             GOTO 610
520          WEND
530          K = K + 1: IF K >1 GOTO 570
540          PRINT
550          PRINT
             "Water strikes ground at a distance x from firefighter"
560 '             Note: Water goes completely over building
570          B = -SQR(3)*VO/G:    C = -2.4/G
580          T = .5*(-B+SQR(B^2 -4*C)):   X = T*VO/2
590          PRINT USING "   Vo = ## m/s   ";VO;
600          PRINT USING "    y = ##.### m";X
610       NEXT
```

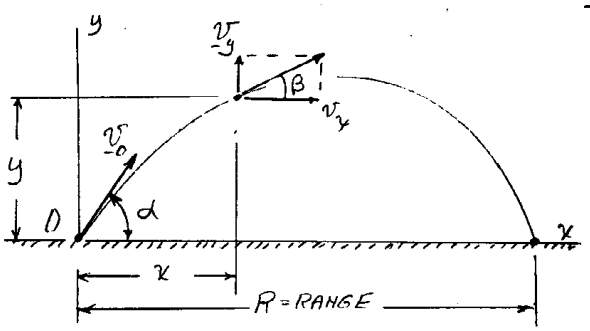
```
RUN
Water strikes ground at a distance x from firefighter
   Vo =  4 m/s        x =  1.922 m
   Vo =  5 m/s        x =  2.761 m
   Vo =  6 m/s        x =  3.763 m

Water strikes the wall at a distance y above ground
   Vo =  7 m/s        y =  0.886 m
   Vo =  8 m/s        y =  2.786 m
   Vo =  9 m/s        y =  4.089 m
   Vo = 10 m/s        y =  5.021 m
   Vo = 11 m/s        y =  5.711 m

Water strikes the roof at a distance BD from B
   Vo = 12 m/s       BD =  4.130 m
   Vo = 13 m/s       BD =  6.741 m

Water strikes ground at a distance x from firefighter
   Vo = 14 m/s        y = 17.970 m
   Vo = 15 m/s        y = 20.533 m
   Vo = 16 m/s        y = 23.272 m
```

PROGRAM FORTRAN VERSION CONTINUED

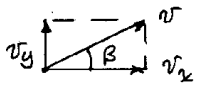
```
      DO 610 M = 4,16
            VO = M
             B = -SQRT(3.)*VO/G
             C = -2.4/G
             T = 0.5*(-B+SQRT(B**2 - 4.*C))
             X = T*VO/2.
         IF (X.LT.4.5) THEN
             WRITE (*,20) VO,X
   20        FORMAT (9X,'Vo  = ',F3.0,' m/s',9X,' x  = ',F6.3,' m ')
             GOTO 610
           ELSE
             GOTO 300
         ENDIF
C
  300    I = I + 1
         IF (I.GT.1) GOTO 340
         WRITE (*,'(//,2X,2A\)') 'Water strikes the wall at',
     +                           ' distance y above ground'
         WRITE (*,'(/,A\)')
C                        Note : x = 4.5 m. and y = 6 m.
  340    T = 9./VO
         Y = 1.2 + 0.86603*VO*T - 0.5*G*(T**2)
C
         IF (Y.LT.6.) THEN
             WRITE (*,30) VO,Y
   30        FORMAT (9X,'Vo  = ',F3.0,' m/s',9X,' x  = ',F6.3,' m ')
             GOTO 610
           ELSE
             GOTO 410
         ENDIF
C
  410    J = J + 1
         IF (J.GT.1) GOTO 450
         WRITE (*,'(//,2X,2A\)') 'Water strikes the roof at',
     +                           ' a distance BD from B'
         WRITE (*,'(/,A\)')
C                        Note : Height of roof is y = 6 m. ; x < 12 m
  450    B = -SQRT(3.)*VO/G
         C = 9.6/G
         T = 0.5*(-B+SQRT(B**2 - 4.*C))
         X = 0.5 * VO*T

         IF (X.LT.12.) THEN
             BD = X - 4.5
             WRITE (*,40) VO,BD
   40        FORMAT (9X,'Vo  = ',F3.0,' m/s',9X,' BD  = ',F6.3,' m ')
             GOTO 610
           ELSE
             GOTO 530
         ENDIF
C
  530    K = K + 1
         IF (K.GT.1) GOTO 570
         WRITE (*,'(//,2X,2A\)') 'Water strikes ground at',
     +                           ' a distance x from firefighter'
         WRITE (*,'(/,A\)')
C                        Note : Water goes completely over building
  570    B = -SQRT(3.)*VO/G
         C = -2.4/G
         T = 0.5*(-B+SQRT(B**2 - 4.*C))
         X = T*VO/2.
         WRITE (*,50) VO,X
   50    FORMAT (9X,'Vo  = ',F3.0,' m/s',9X,' x  = ',F6.3,' m ')
  610 CONTINUE
      END
```

```
Water strikes ground at a distance x from firefighter

        Vo  =  4. m/s          x  =  1.922 m
        Vo  =  5. m/s          x  =  2.761 m
        Vo  =  6. m/s          x  =  3.763 m


Water strikes the wall at distance y above ground

        Vo  =  7. m/s          x  =   .886 m
        Vo  =  8. m/s          x  =  2.786 m
        Vo  =  9. m/s          x  =  4.089 m
        Vo  = 10. m/s          x  =  5.021 m
        Vo  = 11. m/s          x  =  5.711 m


Water strikes the roof at a distance BD from B

        Vo  = 12. m/s          BD  =  4.130 m
        Vo  = 13. m/s          BD  =  6.741 m


Water strikes ground at a distance x from firefighter

        Vo  = 14. m/s          x  = 17.970 m
        Vo  = 15. m/s          x  = 20.533 m
        Vo  = 16. m/s          x  = 23.272 m
```

PROBLEM 11.C3

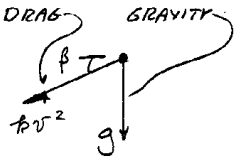


VELOCITY:



$$v_x = v\cos\beta \qquad v_y = v\sin\beta$$ ◁

ACCELERATION:

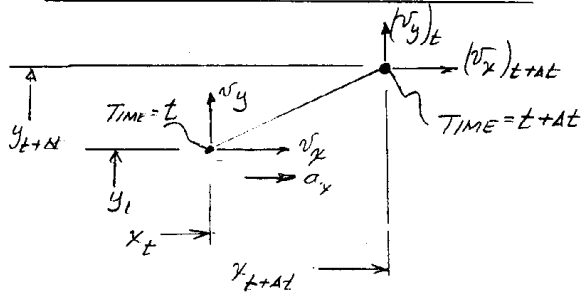


FOR THE CHOSEN POSITIVE SENSES:

$$a_x = -kv^2\cos\beta$$ ◁

$$a_y = -g - kv^2\sin\beta$$ ◁

MOTION DURING TIME INTERVAL Δt



ASSUME a_x AND a_y ARE CONSTANT DURING EACH TIME INTERVAL Δt.

$$(v_x)_{t+\Delta t} = (v_x)_t + a_x\Delta t$$ ◁

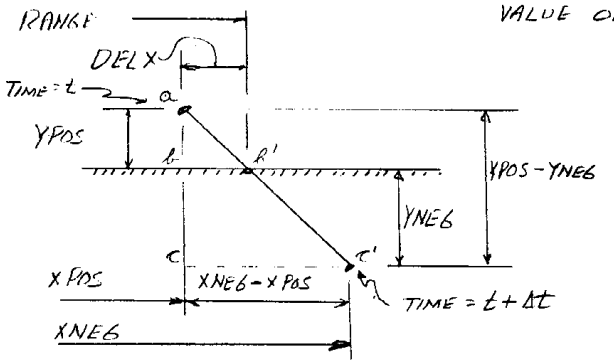
$$(v_y)_{t+\Delta t} = (v_y)_t + a_y\Delta t$$ ◁

$$(x)_{t+\Delta t} = (x)_t + (v_x)_t\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$ ◁

$$(x)_{t+\Delta t} = (x)_t + \left[(v_x)_t + \tfrac{1}{2}a_x\Delta t\right]\Delta t$$ ◁

$$(y)_{t+\Delta t} = (y)_t + \left[(v_y)_t + \tfrac{1}{2}a_y\Delta t\right]\Delta t$$ ◁

INTERPOLATION TO FIND RANGE: ASSUME PATH IS A STRAIGHT LINE DURING INTERVAL Δt WHEN VALUE OF y CHANGES FROM POSITIVE TO NEGATIVE.



SIMILAR TRIANGLES a-b-b' AND a-c-c':

$$\frac{DELX}{YPOS} = \frac{XNEG - XPOS}{YPOS - YNEG}$$

$$DELX = (YPOS)\,\frac{XNEG - XPOS}{YPOS - YNEG}$$ ◁

RANGE: $R = XPOS + DELX$ ◁

WE MAY FIND THE TIME OF IMPACT BY WRITING

$$DELT = \Delta t\,\frac{YPOS}{YPOS - YNEG}$$

TIME OF IMPACT = $t + DELT$

CONTINUED

PROBLEM 11.C3 CONTINUED

```
10 '*****************************************************************
20 '*'                        PROBLEM 11.C3                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '                Definition of Symbols
70 '
80 '       X,Y = coordinates of projectile
90 '         V = velocity of projectile
100 '   VX,VY = components of the velocity of the projectile
110 '       A = acceleration of projectile
120 '   AX,AY = components of acceleration of the projectile
121 '
130 '       K = constant related to drag of projectile
140 '   ALPHA = initial angle between path of projectile and horizontal
150 '      V0 = initial velocity of projectile
160 '    BETA = angle between path of projectile and horizontal
170 '       T = elapsed time after firing of projectile
180 '
190       PI = 4*ATN(1)
200       V0 = 1200:    ALPHA = 40:     G = 32.2:    DT = .5
210       PRINT "Projectile with drag"
220       INPUT "Constant related to drag: K = ";K:   PRINT
230      PRINT "  T       VX            VY            X           Y"
240      PRINT "  s       m/s           m/s           m           m"
250      PRINT
260 '
270 ' Initial components of velocity
280       VX = V0*COS(ALPHA*PI/180)
290       VY = V0*SIN(ALPHA*PI/180)
300 '
310 ' Print initial values
320       GOTO 440
330 '
340 ' Compute new values after increment of time" DT = 0.5 s
350       V = SQR(VX^2 + VY^2):     BETA = ATN(VY/VX)
360       AX = -K*V^2*COS(BETA):     AY = - G - K*V^2*SIN(BETA)
370       T = T + DT
380       X = X +(VX + .5*AX*DT)*DT
390       Y = Y +(VY + .5*AY*DT)*DT
400       VX = VX + AX*DT:     VY = VY + AY*DT
410 '
420 ' Print values at 5-second intervals
430       I = I + 1: TEST = I/10:     IF TEST < 1 GOTO 490
440       PRINT USING " ##    ";T;:PRINT USING "####.##       ";VX,VY;:
450       PRINT USING "#####       ";X,Y
460       I = 0
470 '
480 ' Check whether shell has hit ground
490       IF Y <0  GOTO 550 ELSE GOTO 350
500 ' - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
510 ' We now interpolate to find range. NEG refers to the first time
520 ' at which the y coordinate is negative. POS refers to that time
530 ' immediately before the elevation was first negative.
540 '
550       XNEG = X:     YNEG = Y
560       X = X -(VX + .5*AX*DT)*DT
570       Y = Y -(VY + .5*AY*DT)*DT
580       VX = VX - AX*DT:     VY = VY - AY*DT
590       XPOS = X: YPOS = Y
600       PRINT " "
610       PRINT USING "T = ##.## s     ";T,
620       PRINT USING "YNEG = ####.###      ";YNEG,
630       PRINT USING "YPOS = ####.###      ";YPOS
640 '
650 ' Straight-line interpolation to find range and time at impact
660       DELT = DT*(YPOS)/(YPOS - YNEG)
670       DELX = YPOS*(XNEG - XPOS)/(YPOS - YNEG)
680       PRINT " "
690       RANGE = XPOS + DELX
700       PRINT USING"Range = ##### meters";RANGE
710       PRINT " "
720       FINALT = (T - DT) + DELT
730       PRINT USING "Time at impact = ##.## s";FINALT
740       END
```

SEE NEXT PAGE FOR PROGRAM RUNS

CONTINUED

```
C*****************************************************************
C*'                        PROBLEM 11.C3                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C               Definition of Symbols
C
C       X,Y = coordinates of projectile
C         V = velocity of projectile
C     VX,VY = components of velocity of the projectile
C         A = acceleration of projectile
C     AX,AY = components of acceleration of the projectile
C
C         K = constant related to drag of projectile
C     ALPHA = initial angle between path of projectile and horizontal
C        VO = initial velocity of projectile
C      BETA = angle between path of projectile and horizontal
C         T = elapsed time after firing of projectile
C
      REAL TEST, K
      PI = 4.*ATAN(1.)
      V0 = 1200.
      ALPHA = 40.
      G = 32.2
      DT = 0.5
      WRITE (*,'(/,2X,A\)') 'Projectile with drag'
      WRITE (*,'(/,2X,A\)') 'Constant related to drag : K  = '
      READ (*,*) K
      WRITE (*,'(/,2X,A\)') '  T       VX          VY          X          Y'
      WRITE (*,'(/,2X,A\)') '  s       m/s         m/s         m          m'
      WRITE (*,'(/,A\)')
C
      I = 0
      T = 0.
C Initial components of velocity
      VX = V0*COS(ALPHA*PI/180.)
      VY = V0*SIN(ALPHA*PI/180.)
C
C Print initial values
      GOTO 310
C
C Compute new values after increment of time  DT = 0.5 s
  220 V = SQRT(VX**2 + VY**2)
      BETA = ATAN(VY/VX)
      AX = (-K*V**2)*COS(BETA)
      AY = -G - K*(V**2)*SIN(BETA)
      T = T + DT
      X = X + (VX + 0.5*AX*DT)*DT
      Y = Y + (VY + 0.5*AY*DT)*DT
      VX = VX + AX*DT
      VY = VY + AY*DT
C
C Print values at 5-second intervals
      I = I + 1
      TEST = I /10.
      IF (TEST.LT.1.) GOTO 360
  310 WRITE (*,20) T,VX,VY,X,Y
   20 FORMAT (3X,F3.0,4X,F6.2,4X,F7.2,5X,F6.0,4X,F5.0)
      I = 0
C
C Check whether shell has hit ground
  360 IF (Y.LT.0.) THEN
         GOTO 420
      ELSE
         GOTO 220
      ENDIF
C-----------------------------------------------------------------------
C       We now interpolate to find range. NEG refers to the first time
C       at which the y coordinate is negative. POS refers to that time
C       immediately before the elevation was first negative.
C
  420 XNEG = X
      YNEG = Y
      X = X - (VX + 0.5*AX*DT)*DT
      Y = Y - (VY + 0.5*AY*DT)*DT
      VX = VX - AX*DT
      VY = VY - AY*DT
      XPOS = X
      YPOS = Y
      WRITE (*,30) T,YNEG,YPOS
   30 FORMAT (/,5X,'T  = ',F5.2,' s.',10X,'YNEG  = ',F9.3,10X,'YPOS  =
     +',F7.3)
C
C Straight-line interpolation to find range and time at impact
      DELT = DT*(YPOS)/(YPOS - YNEG)
      DELX = YPOS*(XNEG - XPOS)/(YPOS - YNEG)
      RANGE = XPOS + DELX
      WRITE (*,40) RANGE
   40 FORMAT (/,5X,'Range  = ',F6.0,' m.')
      FINALT = (T - DT) + DELT
      WRITE (*,50) FINALT
   50 FORMAT (/,5X,'Time at impact  = ',F5.2,' s.')
      END
```

CONTINUED

Problem 11.C 3 continued

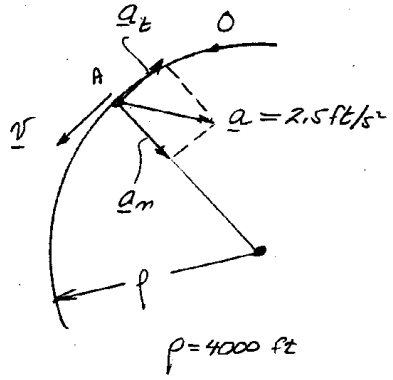
Program Run: Basic version

```
Projectile with drag
Constant related to drag: K = ? 0

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0     919.25      771.35          0          0
   5     919.25      610.35       4596       3454
  10     919.25      449.35       9193       6103
  15     919.25      288.35      13789       7948
  20     919.25      127.35      18385       8987
  25     919.25      -33.65      22981       9221
  30     919.25     -194.65      27578       8650
  35     919.25     -355.65      32174       7275
  40     919.25     -516.65      36770       5094
  45     919.25     -677.65      41366       2108

T = 48.00 s     YNEG =  -69.821     YPOS =  321.331

Range = 44042 meters

Time at impact = 47.91 s

Projectile with drag
Constant related to drag: K = ? 2E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0     919.25      771.35          0          0
   5     908.74      602.34       4570       3433
  10     899.27      435.81       9089       6028
  15     890.61      271.30      13564       7795
  20     882.54      108.49      17996       8743
  25     874.80      -52.82      22390       8882
  30     867.12     -212.72      26744       8218
  35     859.26     -371.13      31060       6757
  40     851.01     -527.86      35336       4509
  45     842.22     -682.65      39570       1482

T = 47.50 s     YNEG = -320.532     YPOS =   62.893

Range = 41320 meters

Time at impact = 47.08 s

Projectile with drag
Constant related to drag: K = ? 10E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0     919.25      771.35          0          0
   5     868.82      571.94       4466       3351
  10     827.16      386.93       8703       5743
  15     791.94      212.51      12748       7238
  20     761.17       46.05      16630       7882
  25     733.09     -113.98      20365       7709
  30     706.22     -268.12      23963       6752
  35     679.43     -416.15      27427       5039
  40     651.99     -557.35      30756       2602

T = 44.50 s     YNEG = -179.898     YPOS =  162.287

Range = 33468 meters

Time at impact = 44.24 s


Projectile with drag
Constant related to drag: K = ? 20E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0     919.25      771.35          0          0
   5     823.24      537.22       4344       3255
  10     751.56      335.65       8273       5426
  15     695.69      155.02      11886       6646
  20     649.77      -11.46      15246       7000
  25     609.46     -167.23      18393       6549
  30     571.66     -313.29      21345       5344
  35     534.44     -449.05      24110       3434
  40     496.95     -573.33      26689        873

T = 41.50 s     YNEG =  -13.669     YPOS =  293.303

Range = 27415 meters

Time at impact = 41.48 s
```

Program Run: Fortran version

```
Projectile with drag
Constant related to drag : K  = 0

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0.    919.25      771.35          0.         0.
   5.    919.25      610.35       4596.      3454.
  10.    919.25      449.35       9193.      6103.
  15.    919.25      288.35      13789.      7948.
  20.    919.25      127.35      18385.      8987.
  25.    919.25      -33.65      22981.      9221.
  30.    919.25     -194.65      27578.      8650.
  35.    919.25     -355.65      32174.      7275.
  40.    919.25     -516.65      36770.      5094.
  45.    919.25     -677.65      41366.      2108.

   T  = 48.00 s.          YNEG  =   -69.822          YPOS  =  321.331

   Range  = 44042. m.

   Time at impact  = 47.91 s.

Projectile with drag
Constant related to drag : K  =  2E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0.    919.25      771.35          0.         0.
   5.    908.74      602.34       4570.      3433.
  10.    899.27      435.81       9089.      6028.
  15.    890.61      271.30      13564.      7795.
  20.    882.54      108.49      17996.      8743.
  25.    874.80      -52.82      22390.      8882.
  30.    867.12     -212.72      26744.      8218.
  35.    859.26     -371.13      31060.      6757.
  40.    851.01     -527.86      35336.      4509.
  45.    842.23     -682.65      39570.      1482.

   T  = 47.50 s.          YNEG  =  -320.528          YPOS  =   62.897

   Range  = 41320. m.

   Time at impact  = 47.08 s.

Projectile with drag
Constant related to drag : K  =  10E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0.    919.25      771.35          0.         0.
   5.    868.82      571.94       4466.      3351.
  10.    827.16      386.93       8703.      5743.
  15.    791.94      212.51      12748.      7238.
  20.    761.17       46.05      16630.      7882.
  25.    733.09     -113.98      20365.      7709.
  30.    706.22     -268.12      23963.      6752.
  35.    679.43     -416.15      27427.      5039.
  40.    651.99     -557.35      30756.      2602.

   T  = 44.50 s.          YNEG  =  -179.897          YPOS  =  162.289

   Range  = 33468. m.

   Time at impact  = 44.24 s.


Projectile with drag
Constant related to drag : K  =  20E-6

   T       VX          VY          X          Y
   s       m/s         m/s         m          m

   0.    919.25      771.35          0.         0.
   5.    823.24      537.22       4344.      3255.
  10.    751.56      335.65       8273.      5426.
  15.    695.69      155.01      11886.      6646.
  20.    649.77      -11.46      15246.      7000.
  25.    609.46     -167.23      18393.      6549.
  30.    571.66     -313.29      21345.      5344.
  35.    534.44     -449.05      24110.      3434.
  40.    496.95     -573.33      26689.       873.

   T  = 41.50 s.          YNEG  =   -13.671          YPOS  =  293.301

   Range  = 27415. m.

   Time at impact  = 41.48 s.
```

PROBLEM 11.C4



$$a^2 = a_t^2 + a_n^2 \qquad a_n = \frac{v^2}{\rho} = \frac{v^2}{4000\,ft}$$

$$(2.5)^2 = a_t^2 + a_n^2$$

$$a_t = -\sqrt{(2.5)^2 - a_n^2}$$

$$\Delta v = a_t \Delta t$$

FOR CONSTANT a_t DURING INTERVAL Δt:

$$\Delta x = v\Delta t + \frac{1}{2} a_t \Delta t^2 = v\Delta t + \frac{1}{2}(a_t \Delta t)\Delta t$$

$$= v\Delta t + \frac{1}{2}(\Delta v)\Delta t$$

$$\Delta x = \left(v + \frac{1}{2}\Delta v\right)\Delta t$$

$$x_{t+\Delta t} = x_t + \Delta x$$

```
10 '*****************************************************************
20 '*'                        PROBLEM 11.C4                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '             Definition of Symbols
70 '
80 '          V = velocity  (ft/s)
90 '          X = distance traveled by train  (ft)
100 '    AN,AT = components of acceleration  (ft/s^2)
110 '          T = time since brakes were first applied   (seconds)
120 '      DELT = time interval at which brakes are reset (1 second)
130 '        DT = final time interval until V becomes zero  (seconds)
140 '
150       V = 88:    'Initial velocity: 60 mi/h = 88 ft.s'
160       DELT = 1:  'Brake setting interval in seconds'
170 '
180           'Calculate V and X at one-second intervals until
190           '    the velocity becomes negative.
200       WHILE V > 0
210           AN = V^2/4000
220           AT = -SQR(2.5^2 - AN^2)
230           DV = (AT)*DELT
240           DX = (V + DV/2)*DELT
250           V = V + DV
260           X = X + DX
270           T = T + DELT
280       WEND
290 '
300           'Assume uniformly accelerated motion during one-second
310           'interval when velocity becomes zero.
320       T = T - DELT:      X = X - DX:      V = V - DV
330       DT = V/ABS(AT)
340       DX = (V/2)*DT
350       T = T + DT:      X = X + DX
360 '
370       PRINT "Total time required to stop train:"
380       PRINT USING "                    T = ##.## seconds";T:PRINT
390       PRINT "Total distance traveled by the train:"
400       PRINT USING "                    X = ####.## feet";X


RUN
Total time required to stop train:
                    T = 38.39 seconds

Total distance traveled by the train:
                    X = 1788.55 feet
```

```
C*****************************************************************
C*'                        PROBLEM 11.C4                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C                    Definition of Symbols
C
C            V = velocity  (ft/s)
C            X = distance traveled by train  (ft)
C        AN,AT = components of acceleration  (ft/s**2)
C            T = time since brakes were first applied  (seconds)
C         DELT = time interval at which brakes are reset (1 second)
C           DT = final time interval until V becomes zero  (seconds)
C
C     Initial velocity : 60 mi/h = 88 ft.s.
      V = 88.
C     Brake setting interval
      DELT = 1.
C
   10 IF (V.GT.0.) THEN
         AN = V**2/4000.
         AT = -SQRT(2.5**2 - AN**2)
         DV = (AT)*DELT
         DX = (V + DV/2.)*DELT
         V = V + DV
         X = X + DX
         T = T + DELT
      ELSE
         GOTO 20
      ENDIF
      GOTO 10
C
C           Assume uniformly accelerated motion during the one-second
C           interval when velocity become zero.
   20 T = T - DELT
      X = X - DX
      V = V - DV
      DT = V/(ABS(AT))
      DX = (V/2.)*DT
      T = T + DT
      X = X + DX
C
      WRITE (*,30) T
   30 FORMAT (/,5X,'Total time required to stop train :',/,20X,'T  = ',F
     +5.2,' seconds')
      WRITE (*,40) X
   40 FORMAT (/,5X,'Total distance traveled by the train :',/,20X,'X  =
     +',F7.2,' feet')
      END



 Total time required to stop train :
                T  = 38.39 seconds

 Total distance traveled by the train :
                X  = 1788.55 feet
```

12.1 $g = 32.09(1 + 0.0053 \sin^2\phi)$ ft/s²

$\phi = 0°$: $g = 32.09$ ft/s²
$\phi = 45°$: $g = 32.175$ ft/s²
$\phi = 90°$: $g = 32.26$ ft/s²

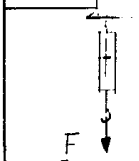
Mass: At all latitudes: $m = 5.000$ lb ◀
or $m = \frac{5.00 \text{ lb}}{32.175 \text{ ft/s}^2}$ $m = 0.1554$ lb·s²/ft ◀

Weight: $W = mg$
$\phi = 0°$: $W = (0.1554 \text{ lb·s}^2/\text{ft})(32.09 \text{ ft/s}^2) = 4.987$ lb ◀
$\phi = 45°$: $W = (0.1554 \text{ lb·s}^2/\text{ft})(32.175 \text{ ft/s}^2) = 5.000$ lb ◀
$\phi = 90°$: $W = (0.1554 \text{ lb·s}^2/\text{ft})(32.26 \text{ ft/s}^2) = 5.013$ lb ◀

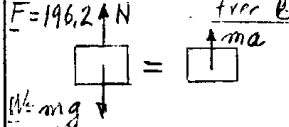
12.2 (a) Weight: $W = mg = (2 \text{ kg})(1.62 \text{ m/s}^2)$
$W = 3.24$ N ◀

(b) Mass: Same as on earth: $m = 2$ kg ◀

12.3 (a) Spring Scale

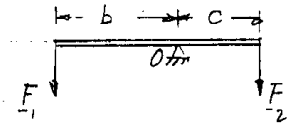


When the spring scale indicates a mass of 20 kg, the force $\underline{F}$ exerted on the spring must be equal to the weight of a 20-kg mass:
$F = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2$ N



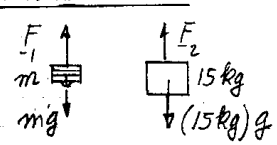
Free Body: Box
$+\uparrow \Sigma F_y = ma$: $196.2 \text{ N} - mg = ma$
$(15 \text{ kg})a = 196.2 \text{ N} - (15 \text{ kg})(9.81 \text{ m/s}^2)$
$a = 13.08 \text{ m/s}^2 - 9.81 \text{ m/s}^2$
$a = 3.27 \text{ m/s}^2 \uparrow$ ◀

(b) Lever Scale
Assume unequal lever arms



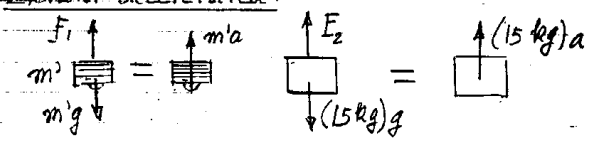
$+\circlearrowleft \Sigma M_O = 0$: $bF_1 - cF_2 = 0$
$F_1 = \frac{c}{b} F_2$ (1)

Elevator at rest:



$F_1 = mg$ $F_2 = (15 \text{ kg})g$
Substitute into (1):
$mg = \frac{c}{b}(15 \text{ kg})g$
$m = \frac{c}{b}(15 \text{ kg})$ (2)

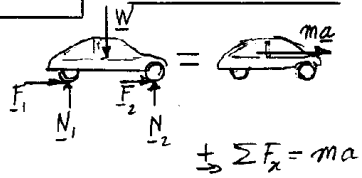
Elevator accelerated:



$F_1 - m'g = m'a$
$F_1 = m'(g + a)$
$F_2 - (15 \text{ kg})g = (15 \text{ kg})a$
$F_2 = (15 \text{ kg})(g + a)$
Substitute into (1):
$m'(g + a) = \frac{c}{b}(15 \text{ kg})(g + a)$ $m' = \frac{c}{b}(15 \text{ kg})$ (3)
Comparing (2) and (3), we have $m' = m$. Thus the same mass is used to balance box. Mass indicated is still 15 kg ◀

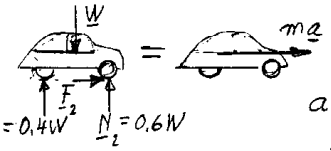
12.4 Mass of satellite is independent of gravity:
$m = 500$ kg
$v = 26.1 \times 10^3$ km/h
$= (26.1 \times 10^6 \text{ m/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 7.25 \times 10^3$ m/s
$L = mv = (500 \text{ kg})(7.25 \times 10^3 \text{ m/s})$
$L = 3.63 \times 10^6$ kg·m/s ◀

12.5 (a) Four-Wheel Drive



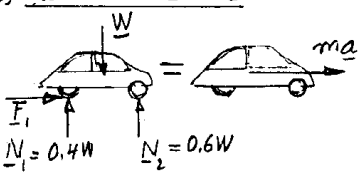
$+\uparrow \Sigma F_y = 0$: $N_1 + N_2 - W = 0$
$F_1 + F_2 = \mu N_1 + \mu N_2$
$= \mu(N_1 + N_2) = \mu W$
$\pm \Sigma F_x = ma$ $F_1 + F_2 = ma$
$\mu W = ma$
$a = \frac{\mu W}{m} = \mu \frac{mg}{m} = \mu g = 0.80(9.81)$
$a = 7.848 \text{ m/s}^2$
$v^2 = 2ax = 2(7.848 \text{ m/s}^2)(60 \text{ m}) = 941.76 \text{ m}^2/\text{s}^2$
$v = 30.69$ m/s $= 110.5$ km/h ◀

(b) Front-Wheel Drive



$F_2 = ma$
$\mu(0.6W) = ma$
$a = \frac{0.6\mu W}{m} = \frac{0.6\mu mg}{m}$
$= 0.6\mu g = 0.6(0.80)(9.81)$
$a = 4.709 \text{ m/s}^2$
$v^2 = 2ax = 2(4.709 \text{ m/s}^2)(60 \text{ m}) = 565.1 \text{ m}^2/\text{s}^2$
$v = 23.77$ m/s $= 85.6$ km/h ◀

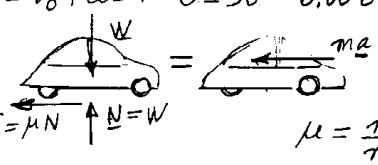
(b) Rear-Wheel Drive



$F_1 = ma$
$\mu(0.4W) = ma$
$a = \frac{0.4\mu W}{m} = \frac{0.4\mu mg}{m}$
$= 0.4\mu g = 0.4(0.80)(9.81)$
$a = 3.139 \text{ m/s}^2$
$v^2 = 2ax = 2(3.139 \text{ m/s}^2)(60 \text{ m}) = 376.7 \text{ m}^2/\text{s}^2$
$v = 19.41$ m/s $= 69.9$ km/h ◀

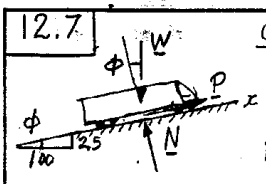
12.6 (a) Uniformly accelerated motion:
$v_0 = 108$ km/h $= 30$ m/s, $v = 0$
$x = 75$ m
$v^2 = v_0^2 + 2ax$: $0 = (30)^2 + 2a(75)$
$a = -6.00 \text{ m/s}^2$
$\underline{a} = 6.00 \text{ m/s}^2 \leftarrow$
$v = v_0 + at$: $0 = 30 - 6.00t$ $t = 5.00$ s ◀

(b)



$F = ma$
$\mu W = ma$
$\mu = \frac{ma}{mg} = \frac{6.00 \text{ m/s}^2}{9.81 \text{ m/s}^2}$
$\mu = 0.612$ ◀

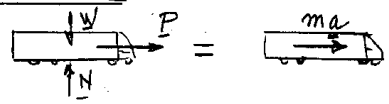
12.7 Going up: Truck in equilibrium since speed is constant.



$+\nearrow \Sigma F_x = 0$: $P - W\sin\phi = 0$

$P = W\sin\phi \approx W\tan\phi = 0.025W$ (1)

On level section:



$+\rightarrow \Sigma F_x = ma$: $P = ma$

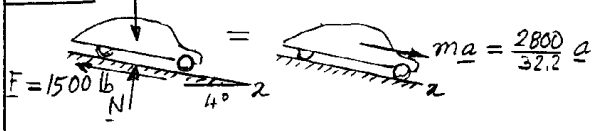
Substitute for P from (1):

$0.025W = ma$

$$a = \frac{0.025W}{m} = 0.025g = 0.025(32.2\ \text{ft/s}^2)$$

$$a = 0.805\ \text{ft/s}^2$$ ◀

12.8 $W = 2800$ lb



$+\searrow \Sigma F_x = ma$: $W\sin\theta - F = ma$

$(2800\ \text{lb})\sin 4° - 1500\ \text{lb} = \frac{2800}{32.2}a$

$195.3 - 1500 = 86.96a$

$$a = -15.00\ \text{ft/s}^2$$

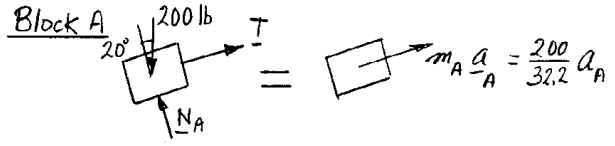
$v_0 = 55\ \text{mi/h} = 80.67\ \text{ft/s}$ $\quad v = 0$

$v^2 = v_0^2 + 2ax$: $0 = (80.67)^2 + 2(-15.00)x$

$$x = 217\ \text{ft}$$ ◀

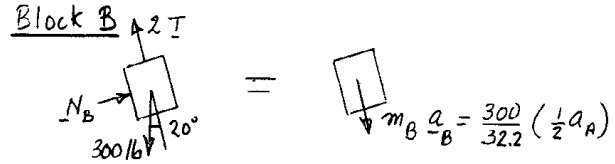
12.9 We note that $a_B = \frac{1}{2}a_A$.

Block A



$+\nearrow \Sigma F_x = m_A a_A$: $T - (200\ \text{lb})\sin 20° = \frac{200}{32.2}a_A$ (1)

Block B



$+\nwarrow \Sigma F_y = m_B a_B$: $(300\ \text{lb})\cos 20° - 2T = \frac{300}{32.2}\left(\frac{1}{2}a_A\right)$ (2)

Multiply (1) by 2 and add (2):

$-2(200\ \text{lb})\sin 20° + (300\ \text{lb})\cos 20° = 2\frac{200}{32.2}a_A + \frac{300}{32.2}\left(\frac{1}{2}a_A\right)$

$145.10 = \frac{550}{32.2}a_A$, $a_A = 8.495$, $\underline{a}_A = 8.49\ \text{ft/s}^2 \nearrow$ ◀

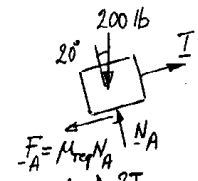
$a_B = \frac{1}{2}a_A = \frac{1}{2}(8.495\ \text{ft/s}^2)$, $\underline{a}_B = 4.25\ \text{ft/s}^2 \searrow$ ◀

(b) From (1): $T = (200\ \text{lb})\sin 20° + \frac{200}{32.2}(8.495)$

$$T = 121.2\ \text{lb}$$ ◀

12.10 We first find the value of μ_s required to maintain equilibrium:

Block A:



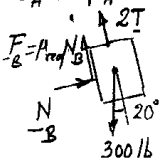
$+\nwarrow \Sigma F_y = 0$: $N_A - 200\cos 20° = 0$

$N_A = 187.94$ lb

$+\rightarrow \Sigma F_x = 0$: $T - \mu_{req}(187.94) - 200\sin 20° = 0$

$T - \mu_{req}(187.94) - 68.40 = 0$ (1)

Block B:



$+\nearrow \Sigma F_x = 0$: $N_B - 300\sin 20° = 0$

$N_B = 102.61$ lb

$+\nwarrow \Sigma F_y = 0$: $2T + \mu_{req}(102.61) - 300\cos 20° = 0$

$2T + \mu_{req}(102.61) - 281.91 = 0$ (2)

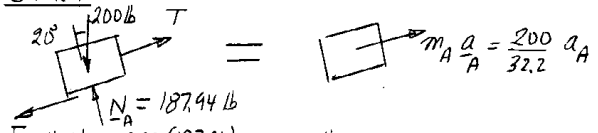
Multiply (1) by 2 and subtract from (2):

$\mu_{req}(478.49) - 145.11 = 0$ $\quad \mu_{req} = 0.303$

Since $\mu_{req} > \mu_s$, blocks will move (A up, B down)

We note that $a_B = \frac{1}{2}a_A$

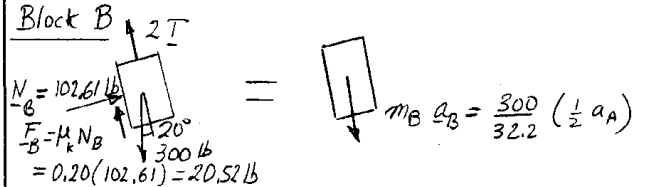
Block A



$F_A = \mu_k N_A = 0.20(187.94) = 37.59$ lb

$+\rightarrow \Sigma F_x = m_A a_A$: $T - 200\sin 20° - 37.59 = \frac{200}{32.2}a_A$ (3)

Block B



$= 0.20(102.61) = 20.52$ lb

$+\searrow \Sigma F_y = m_B a_B$: $300\cos 20° - 2T - 20.52 = \frac{300}{32.2}\left(\frac{1}{2}a_A\right)$ (4)

Multiply (3) by 2 and add (4):

$-2(200)\sin 20° - 2(37.59) + 300\cos 20° - 20.52 = 2\left(\frac{200}{32.2}a_A\right) + \frac{150}{32.2}a_A$

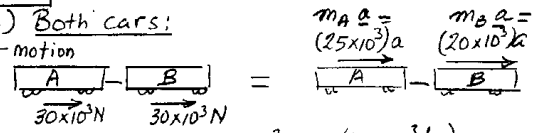
$49.40 = \frac{550}{32.2}a_A$, $a_A = 2.892\ \text{ft/s}^2$, $\underline{a}_A = 2.89\ \text{ft/s}^2 \nearrow$ ◀

$a_B = \frac{1}{2}a_A = \frac{1}{2}(2.892)$, $\underline{a}_B = 1.446\ \text{ft/s}^2 \searrow$ ◀

(b) From (3): $T = 200\sin 20° + 37.59 + \frac{200}{32.2}(2.892)$, $T = 124.0$ lb ◀

12.11 $v_0 = 90\ \text{km/h} = 90/3.6 = 25\ \text{m/s}$

(a) Both cars:



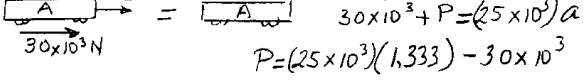
$+\rightarrow \Sigma F_x = \Sigma ma$: $60\times10^3\ \text{N} = (45\times10^3\ \text{kg})a$

$$\underline{a} = 1.333\ \text{m/s}^2 \rightarrow$$

$v^2 = v_0^2 + 2ax$: $0 = (25)^2 + 2(-1.333)x$

$$x = 234\ \text{m}$$ ◀

(b) Car A:



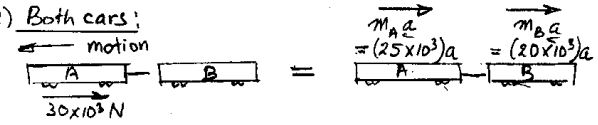
$+\rightarrow \Sigma F_x = ma$: $30\times10^3 + P = (25\times10^3)a$

$P = (25\times10^3)(1.333) - 30\times10^3$

$P = +3332$ N $\quad P = 3.33$ kN T ◀

12.12 $v_0 = 90\ km/h = 90/3.6 = 25\ m/s$

(a) Both cars:



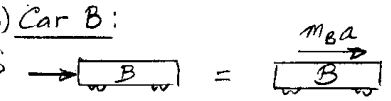
$\pm \Sigma F_x = \Sigma ma$: $30\times10^3\ N = (45\times10^3\ kg)a$

$a = 0.6667\ m/s^2 \rightarrow$

$v^2 = v_0^2 + 2ax$: $0 = (25)^2 + 2(-0.6667)x$

$x = 469\ m$ ◀

(b) Car B:



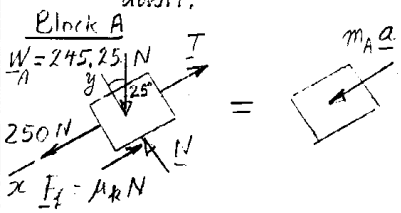
$\pm \Sigma F_x = m_B a$: $P = m_B a$

$= (20\times10^3\ kg)(0.6667\ m/s^2)$

$P = +13\,333\ N$, $\underline{P} = 13.33\ kN\ C$ ◀

12.13 Assume motion, with block A moving down.

Block A



$W_A = m_A g$

$W_A = (25\ kg)(9.81\ m/s^2) = 245.25\ N$

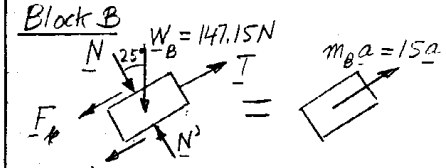
$+\uparrow \Sigma F_y = 0$: $N - (245.25\ N)\cos 25° = 0$, $N = 222.3\ N$

$F_k = \mu_k N = 0.15(222.3\ N) = 33.34\ N$

$+\leftarrow \Sigma F_x = ma$: $250 + (245.25)\sin 25° - 33.34 - T = 25a$

$320.31 - T = 25a$ (1)

Block B



$W_B = m_B g$

$= (15\ kg)(9.81\ m/s^2) = 147.15\ N$

$+\uparrow \Sigma F_y = 0$: $N' - N - (147.15)\cos 25° = 0$

$N' = 222.3 + 133.4 = 355.7\ N$

$F'_k = \mu_k N' = 0.15\ (355.7) = 53.35\ N$

$\pm \Sigma F_x = ma$: $T - F_k - F'_k - W_B \sin 25° = m_B a$

$T - 33.34 - 53.35 - (147.15)\sin 25° = 15a$

$T - 148.88 = 15a$ (2)

(a) Add Eqs. (1) and (2):

$171.43 = 40a$

$a = +4.286\ m/s^2$ $\underline{a_A} = 4.29\ m/s^2 \swarrow$ ◀

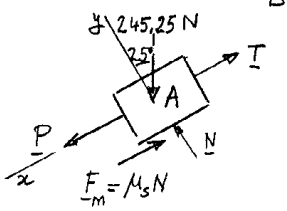
(b) Substitute for a in Eq. (1):

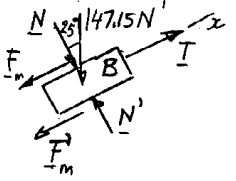
$320.31 - T = 25(4.286)$ $T = 213\ N$ ◀

Check. We should verify that blocks actually move by determining the value of the force $\underline{P}$ for which motion is impending. (CONTINUED)

12.13 CONTINUED Find $\underline{P}$ for impending motion:

Both blocks are in equilibrium





From $\Sigma F_y = 0$, we find again

$N = 222.3\ N$ $N' = 355.7\ N$

and thus

$F_m = \mu_s N = 0.20(222.3\ N) = 44.46\ N$

$F'_m = \mu_s N' = 0.20(355.7\ N) = 71.14\ N$

Free Body: Block A

$+\uparrow \Sigma F_y = 0$: $P + 245.25\sin 25° - 44.46 - T = 0$ (3)

Free Body: Block B

$+\nearrow \Sigma F_x = 0$: $T - 147.15\sin 25° - 44.46 - 71.14 = 0$ (4)

Add Eqs. (3) and (4):

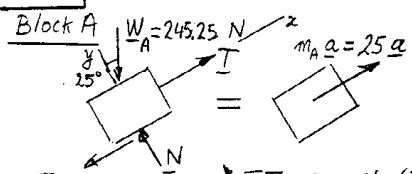
$P + (245.25 - 147.15)\sin 25° - 2(44.46) - 71.14 = 0$

$P = 118.6\ N$

Since actual value of P (250 N) is larger than value for impending motion (118.6 N), motion takes place as assumed.

12.14 Assume motion, with block A moving up.

Block A



See solution of Prob. 12.13 for computation of W_A and W_B

$+\uparrow \Sigma F_y = 0$: $N - (245.25)\cos 25° = 0$

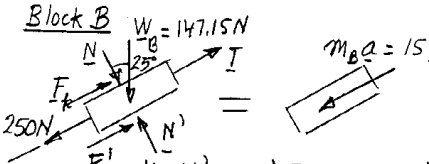
$N = 222.3\ N$

$F_k = \mu_k N = 0.15(222.3\ N) = 33.34\ N$

$+\nearrow \Sigma F_x = ma$: $T - (245.25)\sin 25° - 33.34 = 25a$

$T - 136.99 = 25a$ (1)

Block B



$+\uparrow \Sigma F_y = 0$: $N' - N - (147.15)\cos 25° = 0$

$N' = 222.3 + 133.4 = 355.7\ N$

$F'_k = \mu_k N' = 0.15(355.7\ N) = 53.35\ N$

$+\nearrow \Sigma F_x = ma$: $250 - F_k - F'_k + W_B \sin 25° - T = 15a$

$250 - 33.34 - 53.35 + (147.15)\sin 25° - T = 15a$

$225.50 - T = 15a$ (2)

Add Eqs. (1) and (2): $225.50 - 136.99 = 40a$

$a = +2.213\ m/s^2$ $\underline{a_A} = 2.21\ m/s^2 \nearrow$ ◀

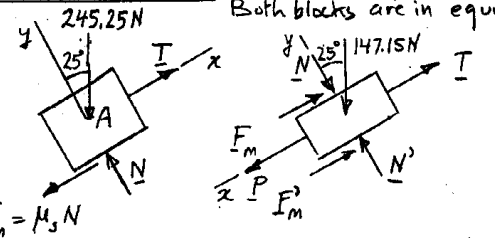
(b) Substitute for a in Eq. (1):

$T - 136.99 = 25(2.213)$ $T = 192.3\ N$ ◀

Check. We should verify that blocks actually move as assumed by determining the value of $\underline{P}$ for which motion of A is impending upward. (CONTINUED)

12.14 CONTINUED

Find P for impending motion: Both blocks are in equilibrium



$F_m = \mu_s N$

From $\Sigma F_y = 0$, we find again $N = 222.3$ N, $N' = 355.7$ N

and thus $F_m = \mu_s N = 0.20(222.3\text{ N}) = 44.46$ N

$F'_m = \mu_s N' = 0.20(355.7\text{ N}) = 71.14$ N

Free Body: Block A

$\pm\rightarrow \Sigma F_x = 0$: $T - 245.25\sin 25° - 44.46 = 0$ (3)

Free Body: Block B

$+\uparrow \Sigma F_y = 0$: $P + 147.15\sin 25° - T - 44.46 - 71.14 = 0$ (4)

Add Eqs. (3) and (4):

$P - (245.25 - 147.15)\sin 25° - 2(44.46) - 71.14 = 0$

$P = 201.5$ N

Since actual value of P (250 N) is larger than value for impending motion (201.5 N), motion takes place as assumed.

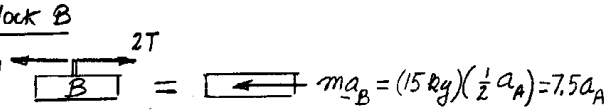
12.15

Referring to solution of Prob. 11.44 we note that

$$a_B = -\tfrac{1}{2} a_A \quad (1)$$

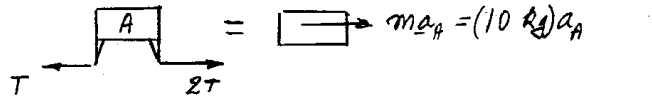
where minus sign indicates that a_A and a_B have opposite sense.

Block B



$\pm\rightarrow \Sigma F = ma$: $25 - 2T = 7.5 a_A$ (2)

Block A



$\pm\rightarrow \Sigma F = ma$: $2T - T = 10 a_A$

$T = 10 a_A$ (3)

(a) Substituting for T from (3) into (2):

$25 - 2(10 a_A) = 7.5 a_A$

$25 = 27.5 a_A$ $\quad a_A = 0.909\text{ m/s}^2 \rightarrow$

$v_A = (v_A)_0 + a_A t = 0 + 0.909(1.2)$, $\quad v_A = 1.091$ m/s $\rightarrow$ ◀

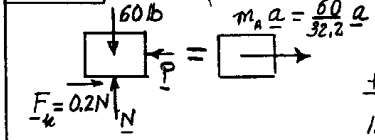
(b) Substituting $a = 0.909$ into Eq. (1): $a_B = 0.455\text{ m/s}^2 \leftarrow$

$v_B = (v_B)_0 + a_B t = 0 + 0.455(1.2)$

$v_B = 0.545$ m/s $\leftarrow$ ◀

12.16

Package A



$+\uparrow \Sigma F_y = 0$: $N - 60\text{ lb} = 0$

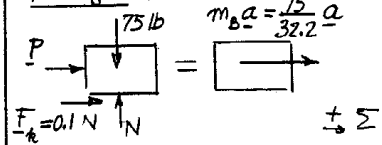
$N = 60$ lb

$F_k = 0.2(60\text{ lb}) = 12$ lb

$\pm\rightarrow \Sigma F_x = ma$:

$12 - P = \frac{60}{32.2} a$ (1)

Package B



$+\uparrow \Sigma F_y = 0$: $N - 75\text{ lb} = 0$

$N = 75$ lb

$F_k = 0.1(75\text{ lb}) = 7.5$ lb

$\pm\rightarrow \Sigma F_x = ma$:

$P + 7.5 = \frac{75}{32.2} a$ (2)

(a) Add (1) and (2): $12 + 7.5 = \frac{60+75}{32.2} a$

$a = 4.651$ $\quad a = 4.65\text{ ft/s}^2 \rightarrow$ ◀

(b) Substitute into (2):

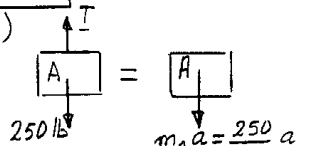
$P + 7.5 = \frac{75}{32.2}(4.651)$ $\quad P = +3.33$ lb

$P = 3.33$ lb $\rightarrow$ ◀

Note: Since P is positive, the two packages are actually pushing on each other.

12.17

(1)



Block A: $+\downarrow \Sigma F_y = ma$: $250 - T = \frac{250}{32.2} a$ (1)

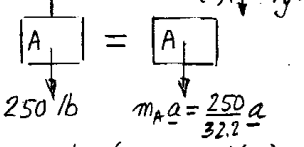
Block B: $+\uparrow \Sigma F_y = ma$: $T - 150 = \frac{150}{32.2} a$ (2)

(a) Add (1) and (2): $100 = \frac{400}{32.2} a$, $\quad a = 8.05\text{ ft/s}^2 \downarrow$ ◀

(b) $v = at = (8.05\text{ ft/s}^2)(2\text{ s})$ $\quad v = 16.10$ ft/s $\downarrow$ ◀

(c) $v^2 = 2ax = 2(8.05)(8) = 128.8$ $\quad v = 11.35$ ft/s $\downarrow$ ◀

(2)



(a) $+\downarrow \Sigma F_y = ma$: $250 - 150 = \frac{250}{32.2} a$

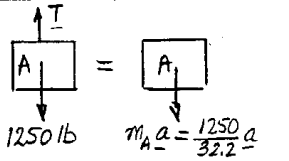
$100 = \frac{250}{32.2} a$

$a = 12.88\text{ ft/s}^2 \downarrow$ ◀

(b) $v = at = (12.88\text{ ft/s}^2)(2\text{ s})$ $\quad v = 25.8$ ft/s $\downarrow$ ◀

(c) $v^2 = 2ax = 2(12.88)(8) = 206.08$ $\quad v = 14.36$ ft/s $\downarrow$ ◀

(3)



Block A: $+\downarrow \Sigma F_y = ma$: $1250 - T = \frac{1250}{32.2} a$ (3)

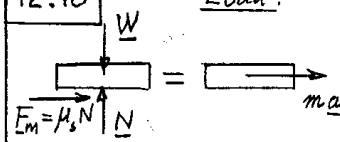
Block B: $+\uparrow \Sigma F_y = ma$: $T - 1150 = \frac{1150}{32.2} a$ (4)

(a) Add (3) and (4): $100 = \frac{2400}{32.2} a$, $\quad a = 1.342\text{ ft/s}^2 \downarrow$ ◀

(b) $v = at = (1.342\text{ ft/s}^2)(2\text{ s})$ $\quad v = 2.68$ ft/s $\downarrow$ ◀

(c) $v^2 = 2ax = 2(1.342)(8) = 21.472$ $\quad v = 4.63$ ft/s $\downarrow$ ◀

12.18 *Load:* We assume that sliding of load relative to trailer is impending:



$F = F_m = \mu_s N$

Deceleration of load is same as deceleration of trailer, which is the maximum allowable deceleration a_{max}

$+\uparrow \Sigma F_y = 0$: $N - W = 0$ $\quad N = W$

$F_m = \mu_s N = 0.40\,W$

$\pm\rightarrow \Sigma F_x = ma$: $\quad F_m = ma_{max}$

$0.40\,W = \frac{W}{g} a_{max}$ $\quad a_{max} = 12.88\ ft/s^2$

$\underline{a}_{max} = 12.88\ ft/s^2 \rightarrow$

Uniformly accelerated motion:

$v^2 = v_0^2 + 2ax$ with $v = 0$ $\quad v_0 = 45\ mi/h = 66\ ft/s$

$a = -a_{max} = -12.88\ ft/s^2$

$0 = (66)^2 + 2(-12.88)x$ $\quad x = 169.1\ ft$ ◀

12.19 See solution of Prob. 12.18 for determination of maximum allowable deceleration of trailer if load is not to shift:

$\underline{a}_{max} = 12.88\ ft/s^2 \rightarrow$

Uniformly accelerated motion of trailer:

$v = v_0 + at$ with $v = 0$ $\quad v_0 = 60\ mi/h = 88\ ft/s$

$a = a_T$ $\quad t = 4\ s$

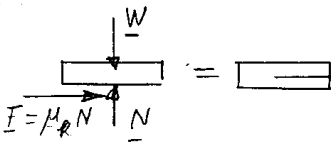
$0 = 88 + a_T(4)$ $\quad a_T = -22.0\ ft/s^2$

$\underline{a}_T = 22.0\ ft/s^2 \rightarrow$

(a) Since $a_T > a_{max}$, load will shift ◀

(b) *Load:* Since load slides relative to trailer,

$F = F_k = \mu_k N$



where $\underline{a}_L$ = absolute acceler. of load

$+\uparrow \Sigma F_y = 0$: $N - W = 0$ $\quad N = W$

$F = \mu_k N = 0.30\,W$

$\pm\rightarrow \Sigma F_x = ma$: $\quad F = ma_L$

$0.30W = \frac{W}{g} a_L$

$a_L = 0.30g = 9.66\ ft/s^2$

$\underline{a}_L = 9.66\ ft/s^2 \rightarrow$

Motion of load relative to trailer

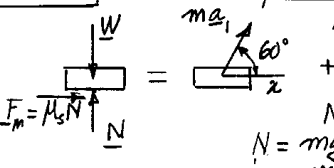
$\underline{a}_{L/T} = \underline{a}_L - \underline{a}_T = 9.66 \rightarrow - (-22.0) \leftarrow = 12.34\ ft/s^2$

Uniformly accelerated motion:

$v_{L/T}^2 = (v_{L/T})_0^2 + 2a_{L/T}x_{L/T} = 0 + 2(12.34)(10)$

$= 246.80$ $\quad v_{L/T} = 15.71\ ft/s \leftarrow$ ◀

12.20 *Accelerated phase of motion:* Assume sliding impending



$+\uparrow \Sigma F_y = ma_y$:

$N - W = ma_1 \sin 60°$

$N = mg + ma_1 \sin 60°$

$= m(g + a_1 \sin 60°)$

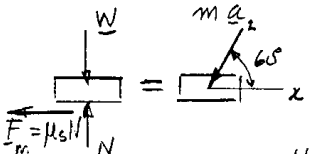
$F_m = \mu_s N = 0.30\,N = 0.30m(g + a_1 \sin 60°)$

$\pm\rightarrow \Sigma F_x = ma_x$: $\quad 0.30m(g + a_1 \sin 60°) = ma_1 \cos 60°$

$a_1 = \frac{0.30(9.81\ m/s^2)}{\cos 60° - 0.30 \sin 60°}$ (1)

$\underline{a}_1 = 12.25\ m/s^2 \nearrow$ ◀

Decelerated phase of motion:



Assume again that sliding is impending

$+\uparrow \Sigma F_y = ma_y$:

$N - W = -ma_2 \sin 60°$

$N = mg - ma_2 \sin 60°$

$= m(g - a_2 \sin 60°)$

$F_m = \mu_s N = 0.30\,N = 0.30m(g - a_2 \sin 60°)$

$\pm\leftarrow \Sigma F_x = ma_x$: $\quad 0.30m(g - a_2 \sin 60°) = ma_2 \cos 60°$

$a_2 = \frac{0.30(9.81\ m/s^2)}{\cos 60° + 0.30 \sin 60°}$ (2)

$\underline{a}_2 = 3.87\ m/s^2 \swarrow$ ◀

12.21 See solution of Prob. 12.20 for derivation of Eqs. (1) and (2) defining largest allowable acceleration and deceleration

(a) *Maximum velocity reached*

Accelerated phase:

$v^2 = v_0^2 + 2ax$

$v_{max}^2 = 0 + 2a_1 x_1$, $\quad v_{max}^2 = \frac{0.6(9.81)x_1}{\cos 60° - 0.30 \sin 60°}$ (3)

Decelerated phase:

$v^2 = v_0^2 + 2ax$

$0 = v_{max}^2 - 2a_2 x_2$, $\quad v_{max}^2 = \frac{0.6(9.81)x_2}{\cos 60° + 0.30 \sin 60°}$ (4)

Adding numerators and denominators in (3) and (4), and noting that $x_1 + x_2 = (3m)/\sin 60°$:

$v_{max}^2 = \frac{0.6(9.81)(x_1 + x_2)}{2\cos 60°} = \frac{0.6(9.81)(3)}{2\cos 60° \sin 60°} = 20.39$

$v_{max} = 4.516\ m/s$, $\quad v_{max} = 4.52\ m/s$ ◀

(b) *Required time*

Accelerated phase: $x_1 = \frac{1}{2} v_{max} t_1$, $\quad t_1 = \frac{2x_1}{v_{max}}$

Decelerated phase: $x_2 = \frac{1}{2} v_{max} t_2$, $\quad t_2 = \frac{2x_2}{v_{max}}$

Adding:

$t = t_1 + t_2 = \frac{2(x_1 + x_2)}{v_{max}} = \frac{2(3\ m)/\sin 60°}{4.516\ m/s}$

$t = 1.534\ s$ ◀

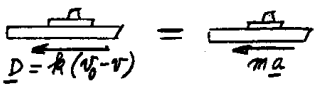
12.22 Drag $= D = k(v_0 - v)$, where v = vel. of ship

Before chain breaks: Ship is in equilibrium, with $v = a$

$\pm \Sigma F_x = 0$: $T_0 - kv_0 = 0$

$k = T_0/v_0$ (1)

After chain breaks:



$\pm \Sigma F_x = ma$: $k(v_0 - v) = ma$

Recalling (1): $(T_0/v_0)(v_0 - v) = ma$

$$\frac{dv}{dt} = a = \frac{T_0}{mv_0}(v_0 - v)$$

$$\frac{dv}{v_0 - v} = \frac{T_0}{mv_0}\,dt$$

Integrating from $t = 0$, $v = 0$ to $t = t$, $v = \frac{1}{2}v_0$:

$$\int_0^{\frac{1}{2}v_0} \frac{dv}{v_0 - v} = \frac{T_0}{mv_0}\int_0^t dt$$

$$\left[-\ln(v_0 - v)\right]_0^{\frac{1}{2}v_0} = \frac{T_0}{mv_0}\,t$$

$$t = \frac{mv_0}{T_0}\ln\frac{v_0}{v_0 - \frac{1}{2}v_0} = \frac{mv_0}{T_0}\ln\frac{v_0}{\frac{1}{2}v_0} \qquad t = \frac{mv_0}{T_0}\ln 2$$ ◀

12.23 $F = ma$:

$$40\times10^3\ \text{N} - 2.25v^2 = (25\times10^3\ \text{kg})a$$

Substituting $a = v\frac{dv}{dx}$:

$$40\times10^3 - 2.25v^2 = (25\times10^3)v\frac{dv}{dx}$$

$$\int_0^{x_1} dx = \int_0^{v_1} \frac{(25\times10^3)v\,dv}{40\times10^3 - 2.25v^2}$$

$$x_1 = -\frac{25\times10^3}{2(2.25)}\left[\ln(40\times10^3 - 2.25v^2)\right]_0^{v_1}$$

$$= \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25v_1^2}$$

For $v_1 = 240$ km/h $= 66.67$ m/s

$$x_1 = \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25(66.67)^2} = 5.556\ln 1.333$$

$$= 1.5982\times10^3\ \text{m} \qquad x_1 = 1.598\ \text{km}$$ ◀

12.24 $F = ma$:

$$40\times10^3\ \text{N} - 2.25v^2 = (25\times10^3\ \text{kg})a$$

Substituting $a = \frac{dv}{dt}$:

$$40\times10^3 - 2.25v^2 = (25\times10^3)\frac{dv}{dt}$$

$$\int_0^{t_1} dt = \int_0^{v_1} \frac{(25\times10^3)\,dv}{40\times10^3 - 2.25v^2} = (11.111\times10^3)\int_0^{v_1} \frac{dv}{17.778\times10^3 - v^2}$$

$$t_1 = \frac{11.111\times10^3}{2\sqrt{17.778\times10^3}}\left[\ln\frac{\sqrt{17.778\times10^3} + v}{\sqrt{17.778\times10^3} - v}\right]_0^{v_1}$$

$$= 41.667\ln\frac{133.33 + v_1}{133.33 - v_1}$$

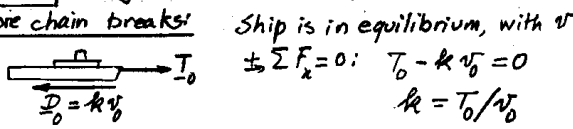
For $v_1 = 240$ km/h $= 66.67$ m/s

$$t_1 = 41.667\ln\frac{133.33 + 66.67}{133.33 - 66.67} = 41.667\ln 3$$

$$= 41.667(1.0986)$$

$$t_1 = 45.8\ \text{s}$$ ◀

12.25



$\pm \Sigma F = ma$:

$$P - kv = ma$$

$$a = \frac{1}{m}(P - kv) \qquad (1)$$

Initial conditions: $t = 0$, $v = 0$.

$$\frac{dv}{dt} = a = \frac{1}{m}(P - kv)$$

$$\int_0^v \frac{dv}{P - kv} = \frac{1}{m}\int_0^t dt$$

$$-\frac{1}{k}\left[\ln(P - kv)\right]_0^v = \frac{t}{m}$$

$$\ln\frac{P - kv}{P} = -\frac{kt}{m}$$

$$P - kv = Pe^{-\frac{kt}{m}} \qquad (2)$$

Substituting for $P - kv$ from (2) into (1):

$$a = \frac{P}{m}e^{-\frac{kt}{m}}$$

Solving (2) for v:

$$v = \frac{P}{k}\left(1 - e^{-\frac{kt}{m}}\right) \qquad (3)$$

Making $v = dx/dt$ in (3):

$$\frac{dx}{dt} = \frac{P}{k}\left(1 - e^{-\frac{kt}{m}}\right)$$

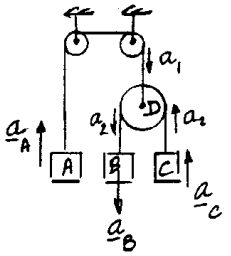
$$x = \int_0^t \frac{P}{k}\left(1 - e^{-\frac{kt}{m}}\right)dt = \frac{P}{k}t + \frac{Pm}{k^2}\left[e^{-\frac{kt}{m}}\right]_0^t$$

$$x = \frac{P}{k}t - \frac{Pm}{k^2}\left(1 - e^{-\frac{kt}{m}}\right)$$ ◀

12.26 Relations between accelerations

Let a_1 be accel. of pulley D (+ downward)

Let a_2 be accel. of B relative to D (+ downward)



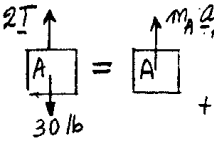
Then:

$$\underline{a}_A = a_1\uparrow$$

$$\underline{a}_B = (a_2 - a_1)\downarrow \qquad (1)$$

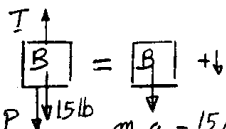
$$\underline{a}_C = (a_2 - a_1)\uparrow$$

Let T be tension in cord BC. Then tension in cord AD is $2T$ since D is of negligible weight.



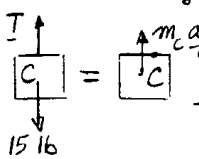
Block A:

$+\uparrow \Sigma F = ma$: $2T - 30 = \frac{30}{g}a_1$ (2)



Block B:

$+\downarrow \Sigma F = ma$: $P + 15 - T = \frac{15}{g}(a_2 + a_1)$ (3)



Block C:

$+\uparrow \Sigma F = ma$: $T - 15 = \frac{15}{g}(a_2 - a_1)$ (4)

Add Eqs. (2) and (3), and subtract (4):

$$2T - 30 + P + 15 - T - T + 15 = \frac{30}{g}a_1 + \frac{15}{g}(a_2 + a_1) - \frac{15}{g}(a_2 - a_1)$$

(CONTINUED)

12.26 CONTINUED The last equation yields

$$P = \frac{30}{g}a_1 + \frac{15}{g}a_2 + \frac{15}{g}a_1 - \frac{15}{g}a_2 + \frac{15}{g}a_1$$

$$P = \frac{60}{g}a_1 \qquad a_1 = \frac{P}{60}g \qquad (5)$$

Add now Eqs. (3) and (4):

$$P + 15 - P + P - 15 = \frac{15}{g}(a_2 + a_1 + a_2 - a_1)$$

$$P = \frac{30}{g}a_2 \qquad a_2 = \frac{P}{30}g \qquad (6)$$

Substitute for a_1 and a_2 from (5) and (6) into the relations (1):

$$a_A = a_1 = \frac{P}{60}g \qquad \underline{a}_A = \frac{P}{60}g\uparrow \qquad (7)$$

$$a_B = a_2 + a_1 = \frac{P}{30}g + \frac{P}{60}g \qquad \underline{a}_B = \frac{P}{20}g\downarrow \qquad (8)$$

$$a_C = a_2 - a_1 = \frac{P}{30}g - \frac{P}{60}g \qquad \underline{a}_C = \frac{P}{60}g\uparrow \qquad (9)$$

(a) Motion of B relative to A

From Eqs. (7) and (8):

$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = \frac{P}{20}g\downarrow - \frac{P}{60}g\uparrow = \frac{P}{20}g\downarrow + \frac{P}{60}g\downarrow$$

$$\underline{a}_{B/A} = \frac{P}{15}g\downarrow = \frac{3}{15}(32.2) = 6.44\ \text{ft/s}^2\downarrow$$

$$v_{B/A} = (v_{B/A})_0 + a_{B/A}t = 0 + (6.44\ \text{ft/s}^2)(4\text{s}) = 25.76\ \text{ft/s}$$

$$\underline{v}_{B/A} = 25.8\ \text{ft/s}\downarrow$$ ◀

(b) Motion of C relative to A

From Eqs. (7) and (9):

$$\underline{a}_{C/A} = \underline{a}_C - \underline{a}_A = \frac{P}{60}g\uparrow - \frac{P}{60}g\uparrow = 0$$

$$v_{C/A} = (v_{C/A})_0 + a_{C/A}t = 0 + 0(4) = 0, \quad \underline{v}_{C/A} = 0$$ ◀

12.27 See solution of Prob. 12.26 for the derivation of Eqs. (1) through (6) and (8).

Motion of B (uniformly accelerated)

$$+\downarrow y_B = (v_B)_0 t + \frac{1}{2}a_B t^2$$

Make $y_B = 12$ ft, $(v_B)_0 = 0$, $t = 2$ s

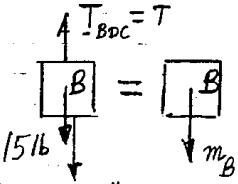
$$12\ \text{ft} = \frac{1}{2}a_B(2\text{s})^2 \qquad a_B = 6\ \text{ft/s}^2$$

(a) Substitute $a_B = 6$ into Eq. (8):

$$6 = \frac{P}{20}g, \qquad P = \frac{120}{32.2} = 3.727\ \text{lb}$$

$$P = 3.73\ \text{lb}$$ ◀

(b)



$$m_B\underline{a}_B = \frac{15}{g}(6) = \frac{90}{32.2} = 2.795\ \text{lb}$$

$P = 3.727$ lb

$$+\downarrow \Sigma F = ma: \quad 15 + 3.727 - T = 2.795$$

$$T = 15.932\ \text{lb}$$

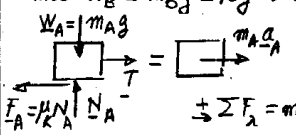
Since weight of pulley D is negligible,

$$T_{AD} = 2T = 2(15.932\ \text{lb}) \qquad T_{AD} = 31.9\ \text{lb}$$ ◀

12.28 We first check that static equil. is not maintained:

$$(F_A)_m + (F_C)_m = \mu_s(m_A + m_C)g = 0.24(5+10)g = 3.6g$$

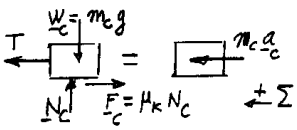
Since $W_B = m_B g = 10g > 3.6g$, equil. is not maintained.



Block A: ΣF_y: $N_A = m_A g$

$F_A = \mu_K N_A = 0.2 m_A g$

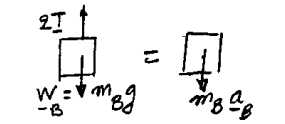
$\pm\Sigma F_x = m_A a_A$: $T - 0.2 m_A g = m_A a_A$ (1)



Block C: ΣF_y: $N_C = m_C g$

$F_C = \mu_K N_C = 0.2 m_C g$

$\pm\Sigma F_x = m_C a_C$: $T - 0.2 m_C g = m_C a_C$ (2)



Block B: $+\downarrow\Sigma F_y = m_B a_B$

$m_B g - 2T = m_B a_B$ (3)

From kinematics: $\underline{a}_B = \frac{1}{2}(\underline{a}_A + \underline{a}_C)$ (4)

(a) Tension in cord. Given Data: $m_A = 5$ kg, $m_B = m_C = 10$ kg

Eq. (1): $T - 0.2(5)g = 5a_A$ $\quad a_A = 0.2T - 0.2g$ (5)

Eq. (2): $T - 0.2(10)g = 10a_C$ $\quad a_C = 0.1T - 0.2g$ (6)

Eq. (3): $10g - 2T = 10a_B$ $\quad a_B = g - 0.2T$ (7)

Substitute into (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g + 0.1T - 0.2g)$$

$$1.2g = 0.35T \qquad T = \frac{24}{7}g = \frac{24}{7}(9.81\ \text{m/s}^2) \qquad T = 33.6\ \text{N}$$ ◀

(b) Substitute for T into (5), (7), and (6):

$a_A = 0.2(\frac{24}{7}g) - 0.2g = 0.4857(9.81\ \text{m/s}^2)$ $\quad \underline{a}_A = 4.76\ \text{m/s}^2\rightarrow$ ◀

$a_B = g - 0.2(\frac{24}{7}g) = 0.3143(9.81\ \text{m/s}^2)$ $\quad \underline{a}_B = 3.08\ \text{m/s}^2\downarrow$ ◀

$a_C = 0.1(\frac{24}{7}g) - 0.2g = 0.14286(9.81\ \text{m/s}^2)$ $\quad \underline{a}_C = 1.401\ \text{m/s}^2\leftarrow$ ◀

12.29 We first check that static equil. is not maintained.

$$(F_A)_m + (F_C)_m = \mu_s(m_A + m_C)g = 0.24(5+20)g = 6g$$

Since $W_B = m_B g = 10g > 6g$, equil. is not maintained.

We shall assume that all 3 blocks move and use Eqs. (1), (2), (3), (4) derived in solution of Prob. 12.28.

(a) Tension in cord.

Given Data: $m_A = 5$ kg, $m_B = 10$ kg, $m_C = 20$ kg

Eq. (1): $T - 0.2(5)g = 5a_A$ $\quad a_A = 0.2T - 0.2g$ (5')

Eq. (2): $T - 0.2(20)g = 20a_C$ $\quad a_C = 0.05T - 0.2g$ (6')

Eq. (3): $10g - 2T = 10a_B$ $\quad a_B = g - 0.2T$ (7')

Substitute into (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g + 0.05T - 0.2g)$$

$$1.2g = 0.325T \qquad T = \frac{48}{13}g = \frac{48}{13}(9.81\ \text{m/s}^2) \qquad T = 36.2\ \text{N}$$ ◁

Substitute into (6'): $a_C = 0.05(\frac{48}{13}g) - 0.2g = -0.0154g$ (impossible)

This means that our assumption that block C moves was wrong.

Assuming now that C does not move, we have $a_C = 0$. ◁

Substituting $a_C = 0$ and from (5') and (6') into Eq. (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g)$$

$$1.1g = 0.3T \qquad T = \frac{11}{3}g = \frac{11}{3}(9.81\ \text{m/s}^2) = 35.97\ \text{N}$$

$$T = 36.0\ \text{N}$$ ◁

Substituting for T into Eqs. (5') and (7'):

$a_A = 0.2(\frac{11}{3}g) - 0.2g = 0.5333(9.81\ \text{m/s}^2)$ $\quad \underline{a}_A = 5.23\ \text{m/s}^2\rightarrow$ ◀

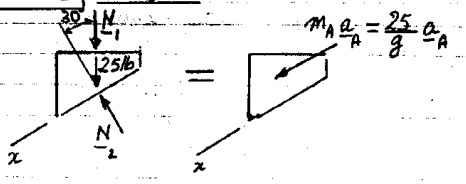
$a_B = g - 0.2(\frac{11}{3}g) = 0.2667(9.81\ \text{m/s}^2)$ $\quad \underline{a}_B = 2.62\ \text{m/s}^2\downarrow$ ◀

We also check that $T = \frac{11}{3}g$ is less than $(F_C)_m = 0.24(20g) = 4.8g$.

The value assumed for a_C is thus correct: $\underline{a}_C = 0$ ◀

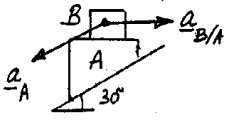
as well as that obtained for T: $T = 36.0$ N ◀

12.31 Wedge A



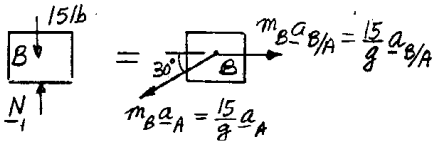
$+\swarrow \Sigma F_x = ma$: $(N_1 + 25)\sin 30° = \frac{25}{g} a_A$ (1)

Block B. The acceleration of B may be resolved into two components as shown



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

It follows that the vector $m_B \underline{a}_B$ may be represented as shown:



$+\downarrow \Sigma F_y = ma_y$: $15 - N_1 = \frac{15}{g} a_A \sin 30°$ (2)

$\pm\rightarrow \Sigma F_x = ma_x$: $0 = \frac{15}{g} a_{B/A} - \frac{15}{g} a_A \cos 30°$

$a_{B/A} = a_A \cos 30°$ (3)

(a) Multiply (2) by $\sin 30° = \frac{1}{2}$ and add to (1):

$12.5 + 7.5 = \frac{25 + 3.75}{g} a_A$

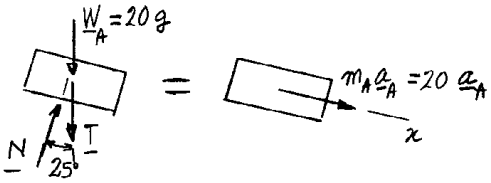
$a_A = \frac{20}{28.75}(32.2 \text{ ft/s}^2) = 22.40 \text{ ft/s}^2$

$\underline{a}_A = 22.4 \text{ ft/s}^2 \measuredangle 30°$ ◀

(b) From (3): $a_{B/A} = 22.40 \cos 30° = 19.40 \text{ ft/s}^2$

$\underline{a}_{B/A} = 19.40 \text{ ft/s}^2 \rightarrow$ ◀

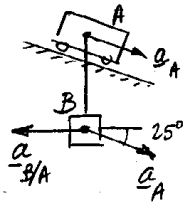
12.32 Cart A



$\pm\searrow \Sigma F_x = ma$: $(T + W_A)\sin 25° = m_A a_A$

$(T + 20g)\sin 25° = 20 a_A$ (1)

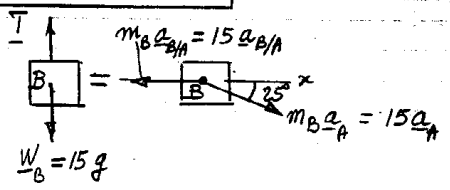
Block B. The acceleration of B is the sum of the acceleration of A and of the acceleration of B relative to A:



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

(CONTINUED)

12.32 CONTINUED



$+\downarrow \Sigma F_y = ma_y$: $15g - T = 15 a_A \sin 25°$ (2)

$\pm\rightarrow \Sigma F_x = ma_x$: $0 = -15 a_{B/A} + 15 a_A \cos 25°$

$a_{B/A} = a_A \cos 25°$ (3)

Multiply (2) by $\sin 25°$ and add to (1):

$20g \sin 25° + 15g \sin 25° = (20 + 15 \sin^2 25°) a_A$

$a_A = \frac{35 \sin 25°}{20 + 15 \sin^2 25°}(9.81) = 6.398 \text{ m/s}^2$

$\underline{a}_A = 6.40 \text{ m/s}^2 \measuredangle 25°$ ◀

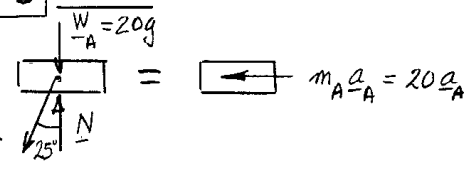
From (2): $T = 15(9.81) - 15(6.398)\sin 25° = 106.59 \text{ N}$

$T = 106.6 \text{ N}$ ◀

Note: From (3), we have

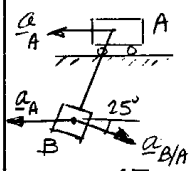
$a_{B/A} = 6.398 \cos 25° = 5.80 \text{ m/s}^2$ $\quad \underline{a}_{B/A} = 5.80 \text{ m/s}^2 \leftarrow$

12.33 Cart A

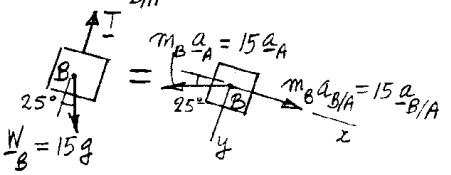


$\pm\leftarrow \Sigma F_x = ma$: $T \sin 25° = 20 a_A$ (1)

Block B. The acceleration of B is the sum of the acceleration of A and of the acceleration of B relative to A:



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$



$+\swarrow \Sigma F_y = ma_y$: $15g \cos 25° - T = 15 a_A \sin 25°$ (2)

$\pm\rightarrow \Sigma F_x = ma_x$: $15g \sin 25° = 15 a_{B/A} - 15 a_A \cos 25°$

$a_{B/A} = g \sin 25° + a_A \cos 25°$ (3)

Multiply (2) by $\sin 25°$ and add to (1):

$15g \sin 25° \cos 25° = (20 + 15 \sin^2 25°) a_A$

$a_A = \frac{15 \sin 25° \cos 25°}{20 + 15 \sin^2 25°}(9.81) = 2.4852 \text{ m/s}^2$

$\underline{a}_A = 2.49 \text{ m/s}^2 \leftarrow$ ◀

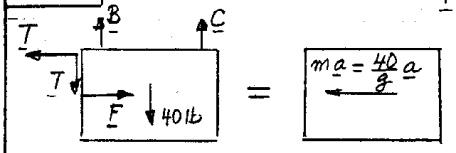
From (1): $T = \frac{20(2.4852)}{\sin 25°} = 117.6 \text{ N}$

$T = 117.6 \text{ N}$ ◀

Note: From (3), we have

$a_{B/A} = 9.81 \sin 25° + 2.4852 \cos 25°$ $\quad \underline{a}_{B/A} = 6.40 \text{ m/s}^2 \measuredangle 25°$

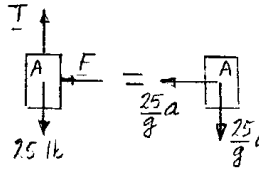
12.33 (a) **Panel:** $\underline{F}$ = Force exerted by counterweight



$\pm \Sigma F_x = ma$: $\quad T - F = \frac{40}{g}a \qquad (1)$

Counterweight A: Its acceleration has two components:

$$\underline{a}_A = \underline{a}_P + \underline{a}_{A/P} = \underline{a} + a\downarrow$$



$\pm \Sigma F_x = ma_x$: $\quad F = \frac{25}{g}a \qquad (2)$

$+\downarrow \Sigma F_y = ma_y$: $\quad 25 - T = \frac{25}{g}a \qquad (3)$

Add (1), (2), and (3):

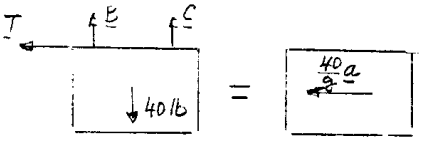
$$T - F + F + 25 - T = \frac{40 + 25 + 25}{g}a$$

$$a = \frac{25}{90}g = \frac{25}{90}(32.2) \qquad \underline{a} = 8.94\ \text{ft/s}^2 \leftarrow$$

Substitute for a into (3):

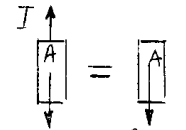
$$25 - T = \frac{25}{g}\left(\frac{25}{90}g\right) \qquad T = 25 - \frac{625}{90} \qquad T = 18.06\ \text{lb}$$

(b) **Panel:**



$\pm \Sigma F_x = ma$: $\quad T = \frac{40}{g}a \qquad (1)$

Counterweight A:



$+\downarrow \Sigma F_y = ma$: $\quad 25 - T = \frac{25}{g}a \qquad (2)$

Add (1) and (2):

$$\cancel{T} + 25 - \cancel{T} = \frac{40 + 25}{g}a$$

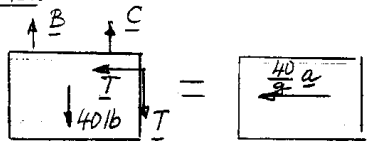
$$a = \frac{25}{65}g \qquad \underline{a} = 12.38\ \text{ft/s}^2 \leftarrow$$

Substitute for a into (1):

$$T = \frac{40}{g}\left(\frac{25}{65}g\right) = \frac{1000}{65} \qquad T = 15.38\ \text{lb}$$

(c) Since panel is accelerated to the left, there is no force exerted by panel on counterweight and vice versa.

Panel:



$\pm \Sigma F_x = ma$: $\quad T = \frac{40}{g}a \qquad (1)$

Counterweight A: Same free body as in part (b):

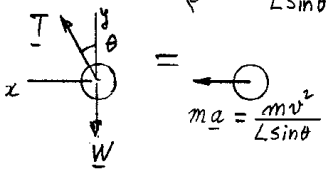
$+\downarrow \Sigma F_y = ma$: $\quad 25 - T = \frac{25}{g}a \qquad (2)$

Since Eqs. (1) and (2) are the same as in (b), we get same answers:

$$\underline{a} = 12.38\ \text{ft/s}^2 \leftarrow\ ; \quad T = 15.38\ \text{lb}$$

12.34 Since ball describes horizontal circle of radius $\rho = L\sin\theta$ at a constant speed, its acceleration is horizontal and equal to

$$a = a_n = \frac{v^2}{\rho} = \frac{v^2}{L\sin\theta} \qquad \underline{a} = \frac{v^2}{L\sin\theta} \leftarrow$$



$\pm \Sigma F_x = ma$: $\quad T\sin\theta = \frac{mv^2}{L\sin\theta}$

$$T\sin^2\theta = \frac{Wv^2}{gL} \qquad (1)$$

$+\uparrow \Sigma F_y = 0$: $\quad T\cos\theta - W = 0 \qquad (2)$

Given: $L = 3$ ft, $T = 10$ lb, $W = 4$ lb

Eq. (2): $\quad 10\cos\theta - 4 = 0$

$$\cos\theta = 0.4 \qquad \theta = 66.42° \qquad \theta = 66.4°$$

Solving Eq. (1) for v^2:

$$v^2 = \frac{gL}{W}(T\sin^2\theta) = \frac{32.2(3)}{4}(10\sin^2 66.42°) = 202.86$$

$$v = 14.24\ \text{ft/s}$$

12.35 See solution of Prob. 12.34 for derivation of Eqs. (1) and (2). Solving (2) for T and substituting into (1):

$$\frac{W\sin^2\theta}{\cos\theta} = \frac{Wv^2}{gL}$$

$$\sin^2\theta = \frac{v^2}{gL}\cos\theta$$

Setting $\sin^2\theta = 1 - \cos^2\theta$ and rearranging the terms:

$$\cos^2\theta + \frac{v^2}{gL}\cos\theta - 1 = 0 \qquad (3)$$

Substituting the given data, we have

$$\frac{v^2}{gL} = \frac{(1.5\ \text{m/s})^2}{(9.81\ \text{m/s}^2)(0.6\ \text{m})} = 0.38226$$

and Eq. (3) becomes

$$\cos^2\theta + 0.38226\cos\theta - 1 = 0$$

Solving for $\cos\theta$:

$$\cos\theta = \frac{-0.38226 \pm \sqrt{(0.38226)^2 + 4}}{2}$$

$$\cos\theta = 0.82697, \quad \theta = 34.21° \qquad \theta = 34.2°$$

or $\cos\theta = -1.2092$ (impossible)

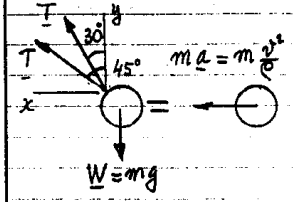
Substituting for $\cos\theta$ in Eq. (2) of Prob. 12.34, and making $W = (2\ \text{kg})(9.81\ \text{m/s}^2) = 19.62$ N:

$$T\cos\theta = W$$

$$T(0.82697) = 19.62\ \text{N}$$

$$T = 23.7\ \text{N}$$

12.36



$+\leftarrow \Sigma F_x = ma$:

$T(\sin 30° + \sin 45°) = \frac{mv^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$:

$T(\cos 30° + \cos 45°) - mg = 0$

$T(\cos 30° + \cos 45°) = mg$ (2)

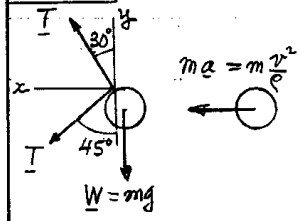
Divide (1) by (2):

$$\frac{\sin 30° + \sin 45°}{\cos 30° + \cos 45°} = \frac{v^2}{\rho g}$$

$v^2 = 0.76733\,\rho g = 0.76733(1.6\text{ m})(9.81\text{ m/s}^2) = 12.044\text{ m}^2/\text{s}^2$

$v = 3.47\text{ m/s}$ ◀

12.37



$\pm \Sigma F_x = ma$:

$T(\sin 30° + \sin 45°) = \frac{mv^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$:

$T(\cos 30° - \cos 45°) - mg = 0$

$T(\cos 30° - \cos 45°) = mg$ (2)

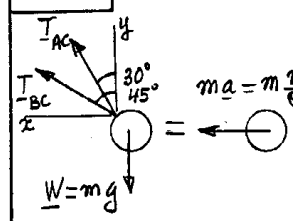
Divide (1) by (2):

$$\frac{\sin 30° + \sin 45°}{\cos 30° - \cos 45°} = \frac{v^2}{\rho g}$$

$v^2 = 7.5958\,\rho g = 7.5958(5\text{ ft})(32.2\text{ ft/s}^2) = 1222.9\text{ ft}^2/\text{s}^2$

$v = 35.0\text{ ft/s}$ ◀

12.38



$\pm \Sigma F_x = ma$:

$T_{AC}\sin 30° + T_{BC}\sin 45° = \frac{mv^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$:

$T_{AC}\cos 30° + T_{BC}\cos 45° - mg = 0$

$T_{AC}\cos 30° + T_{BC}\cos 45° = mg$ (2)

Divide (1) by (2):

$$\frac{T_{AC}\sin 30° + T_{BC}\sin 45°}{T_{AC}\cos 30° + T_{BC}\cos 45°} = \frac{v^2}{\rho g} \quad (3)$$

When AC is slack, $T_{AC} = 0$. Eq. (3) yields

$v_1^2 = \rho g \tan 45° = (1.6\text{ m})(9.81\text{ m/s}^2)\tan 45° = 15.696\text{ m}^2/\text{s}^2$

$v_1 = 3.96\text{ m/s}$

Wire AC will remain taut if $v \le v_1$, that is, if

$v \le 3.96\text{ m/s}$ ◁

When BC is slack, $T_{BC} = 0$. Eq. (3) yields

$v_2^2 = \rho g \tan 30° = (1.6\text{ m})(9.81\text{ m/s}^2)\tan 30° = 9.0621\text{ m}^2/\text{s}^2$

$v_2 = 3.01\text{ m/s}$

Wire BC will remain taut if $v \ge v_2$, that is, if

$v \ge 3.01\text{ m/s}$ ◁

Combining the results obtained, we conclude that both wires remain taut for

$3.01\text{ m/s} \le v \le 3.96\text{ m/s}$ ◀

12.39 See solution of Prob. 12.38 for derivation of Eqs. (1) and (2):

$T_{AC}\sin 30° + T_{BC}\sin 45° = \frac{mv^2}{\rho}$ (1)

$T_{AC}\cos 30° + T_{BC}\cos 45° = mg$ (2)

Subtract (1) from (2). Since $\sin 45° = \cos 45°$, we obtain

$T_{AC}(\cos 30° - \sin 30°) = mg - \frac{mv^2}{\rho}$ (4)

Making $T_{AC} = 40$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s², we find the value v_1 of v for which $T_{AC} = 40$ N:

$40(\cos 30° - \sin 30°) = 5(9.81) - \frac{5v_1^2}{1.6}$

$14.641 = 49.05 - \frac{v_1^2}{0.32}$ $\quad v_1^2 = 11.011$ $\quad v_1 = 3.32\text{ m/s}$

We have $T_{AC} \le 40$ N for $v \ge v_1$, that is, for $v \ge 3.32$ m/s ◁

Multiply (1) by $\cos 30°$, (2) by $\sin 30°$, and subtract:

$T_{BC}(\sin 45°\cos 30° - \cos 45°\sin 30°) = \frac{mv^2}{\rho}\cos 30° - mg\sin 30°$

$T_{BC}\sin 15° = \frac{mv^2}{\rho}\cos 30° - mg\sin 30°$ (5)

Making $T_{BC} = 40$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s², we find the value v_2 of v for which $T_{BC} = 40$ N:

$40\sin 15° = \frac{5v_2^2}{1.6}\cos 30° - 5(9.81)\sin 30°$

$10.353 = 2.7063v_2^2 - 24.525$, $\quad v_2^2 = 12.888$ $\quad v_2 = 3.59$ m/s

We have $T_{BC} \le 40$ N for $v \le v_2$, that is, for $v \le 3.59$ m/s ◁

Combining the results obtained, we conclude that the range of allowable values is

$3.32\text{ m/s} \le v \le 3.59\text{ m/s}$ ◀

***12.40** See next page.

***12.41** From the solution of Prob. 12.38 we find that both wires remain taut for $3.01\text{ m/s} \le v \le 3.96\text{ m/s}$ ◁

To determine the values of v for which the tension in either wire will not exceed 60 N, we recall Eqs. (4) and (5) in the solution of Prob. 12.39.

Making $T_{AC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s² in Eq. (4), we find the value v_1 of v for which $T_{AC} = 60$ N:

$60(\cos 30° - \sin 30°) = 5(9.81) - \frac{5v_1^2}{1.6}$

$21.962 = 49.05 - \frac{v_1^2}{0.32}$ $\quad v_1^2 = 8.668$, $\quad v_1 = 2.94$ m/s

We have $T_{AC} \le 60$ N for $v \ge v_1$, that is, for $v \ge 2.94$ m/s ◁

Making $T_{BC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s² in Eq. (5), we find the value v_2 of v for which $T_{BC} = 60$ N:

$60\sin 15° = \frac{5v_2^2}{1.6}\cos 30° - 5(9.81)\sin 30°$

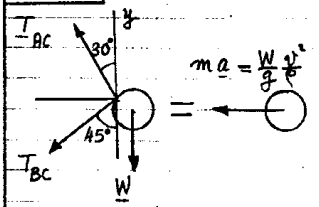
$15.529 = 2.7063v_2^2 - 24.525$ $\quad v_2^2 = 14.80$, $\quad v_2 = 3.85$ m/s

We have $T_{BC} \le 60$ N for $v \le v_2$, that is, for $v \le 3.85$ m/s ◁

Combining the results obtained, we conclude that the range of allowable values is

$3.01\text{ m/s} \le v \le 3.85\text{ m/s}$ ◀

***12.40**



$\pm \Sigma F_x = ma$:

$T_{AC}\sin 30° + T_{BC}\sin 45° = \frac{W}{g}\frac{v^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$:

$T_{AC}\cos 30° - T_{BC}\cos 45° - W = 0$

$T_{AC}\cos 30° - T_{BC}\cos 45° = W$ (2)

Add Eqs. (1) and (2). Since $\sin 45° = \cos 45°$, we obtain

$$T_{AC}(\cos 30° + \sin 30°) = W\left(1 + \frac{v^2}{g\rho}\right)$$

Substituting given data, $W = 8$ lb, $\rho = 5$ ft, $g = 32.2$ ft/s^2;

$$1.3660\, T_{AC} = 8\left(1 + \frac{v^2}{161}\right)$$

$$v^2 = 27.491\, T_{AC} - 161 \quad (3)$$

For $T_{AC} = 0$: $v_1^2 = -161$ (impossible)

For $T_{AC} = 15$ lb: $v_2^2 = 27.491(15) - 161 = 251.36$

$v_2 = 15.85$ ft/s

For $0 \le T_{AC} \le 15$ lb, we must have

$v \le 15.85$ ft/s ◁

For the conditions regarding wire BC, we multiply Eq. (1) by $\cos 30°$, Eq. (2) by $\sin 30°$ and subtract:

$$T_{BC}(\sin 45°\cos 30° + \cos 45°\sin 30°) = W\left(\frac{v^2}{g\rho}\cos 30° - \sin 30°\right)$$

$$T_{BC}\sin 75° = W\left(\frac{v^2}{g\rho}\cos 30° - \sin 30°\right)$$

With given data:

$$0.96593\, T_{BC} = 8\left(0.866\frac{v^2}{161} - 0.5\right)$$

$$v^2 = 92.956 + 22.447\, T_{BC} \quad (4)$$

For $T_{BC} = 0$: $v_3^2 = 92.956$ $\quad v_3 = 9.64$ ft/s

For $T_{BC} = 15$ lb: $v_4^2 = 92.956 + 22.447(15) = 429.66$

$v_4 = 20.73$ ft/s

For $0 \le T_{BC} \le 15$ lb, we must have

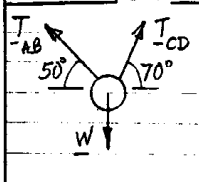
9.64 ft/s $\le v \le 20.73$ ft/s ◁

Combining the results obtained, we conclude that range of allowable values is

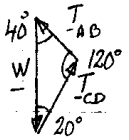
9.64 ft/s $\le v \le 15.85$ ft/s ◀

***12.41** See previous page

12.42 (a) Before AB is cut. (Equilibrium)



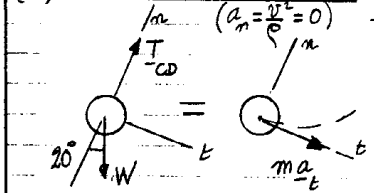
Force Triangle: Law of sines



$$\frac{T_{CD}}{\sin 40°} = \frac{W}{\sin 120°}$$

$T_{CD} = 0.742W$ ◀

(b) After AB has been cut. (Accelerated motion)



$+\nearrow \Sigma F_n = ma_n = 0$:

$T_{CD} - W\cos 20° = 0$

$T_{CD} = W\cos 20°$

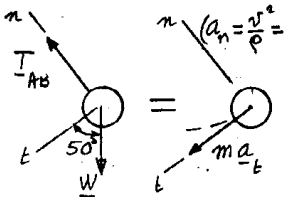
$T_{CD} = 0.940W$ ◀

12.43 (a) Before CD is cut. (Equilibrium)

See solution of Prob. 12.42 for FB-diagram and force triangle.

Law of sines: $\frac{T_{AB}}{\sin 20°} = \frac{W}{\sin 120°}$ $\quad T_{AB} = 0.395W$ ◀

(b) After CD has been cut. (Accelerated motion)



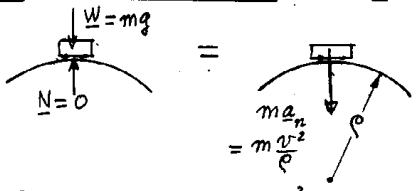
$+\nwarrow \Sigma F_n = ma_n = 0$:

$T_{AB} - W\sin 50° = 0$

$T_{AB} = W\sin 50°$

$T_{AB} = 0.766W$ ◀

12.44 (a) Train leaves track when $N = 0$:

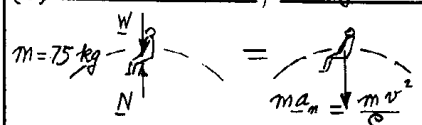


$+\downarrow \Sigma F_n = ma_n$: $\quad mg = \frac{mv^2}{\rho}$

Data: $v = 382$ km/h $= \frac{382}{3.6}$ m/s $= 106.11$ m/s

$\rho = \frac{v^2}{g} = \frac{(106.11\text{ m/s})^2}{9.81\text{ m/s}^2} = 1148$ m $\quad \rho = 1148$ m ◀

(b) Force exerted on passenger at top of hill:



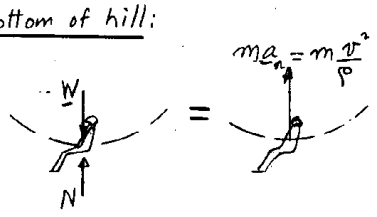
$v = 300$ km/h $= \frac{300}{3.6}$ m/s $= 83.33$ m/s

$+\downarrow \Sigma F_n = ma_n$: $\quad W - N = \frac{mv^2}{\rho}$

$$N = W - \frac{mv^2}{\rho} = (75\text{ kg})(9.81\text{ m/s}^2) - (75\text{ kg})\frac{(83.33\text{ m/s})^2}{1148\text{ m}}$$

$N = 735.75\text{ N} - 453.69\text{ N} = 282.06$ N $\quad \underline{N} = 282$ N ↑ ◀

At bottom of hill:



$+\uparrow \Sigma F_n = ma_n$: $\quad N - W = \frac{mv^2}{\rho}$

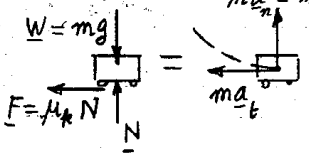
$N = W + \frac{mv^2}{\rho} = 735.75\text{ N} + 453.69\text{ N}$

$= 1189.4$

$\underline{N} = 1189$ N ↑ ◀

12.45

(a)



$+\uparrow \Sigma F_n = ma_n$:

$N - mg = m\frac{v^2}{\rho}$

$N = m\left(g + \frac{v^2}{\rho}\right)$

$F = \mu_k N = \mu_k m\left(g + \frac{v^2}{\rho}\right)$

$\pm \Sigma F_t = ma_t$: $F = ma_t$

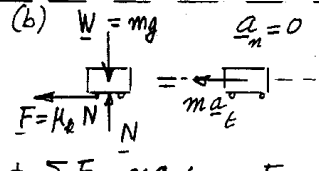
$a_t = \frac{F}{m} = \mu_k\left(g + \frac{v^2}{\rho}\right)$

Given Data: $\mu_k = 0.20$, $v = 72\ km/h = 20\ m/s$

$g = 9.81\ m/s^2$ $\rho = 30\ m$

$a_t = 0.20\left[9.81 + \frac{(20)^2}{30}\right]$ $a_t = 4.63\ m/s^2$ ◀

(b)



$W = mg$ $a_n = 0$ $+\uparrow \Sigma F_n = ma_n = 0$:

$N - mg = 0$

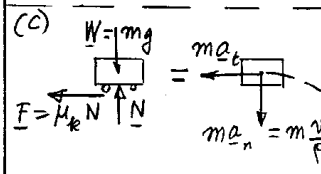
$N = mg$

$F = \mu_k N = \mu_k mg$

$\pm \Sigma F_t = ma_t$: $F = ma_t$

$a_t = \frac{F}{m} = \mu_k g = 0.20(9.81)$ $a_t = 1.962\ m/s^2$ ◀

(c)



$+\downarrow \Sigma F_n = ma_n$:

$mg - N = \frac{mv^2}{\rho}$

$N = m\left(g - \frac{v^2}{\rho}\right)$

$F = \mu_k N = \mu_k m\left(g - \frac{v^2}{\rho}\right)$

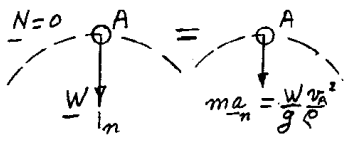
$\pm \Sigma F_t = ma_t$: $F = ma_t$

$a_t = \frac{F}{m} = \mu_k\left(g - \frac{v^2}{\rho}\right) = 0.20\left[9.81 - \frac{(20)^2}{45}\right]$

$a_t = 0.1842\ m/s^2$ ◀

12.46

Note: "Apparent weight" of pilot is measured by the force N exerted on pilot by his seat. "Weightlessness" means that $N = 0$.



At point A:

$+\downarrow \Sigma F_n = ma_n$:

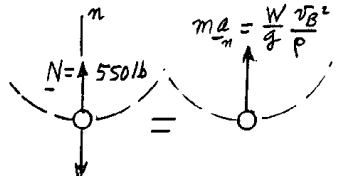
$W = \frac{W}{g}\frac{v_A^2}{\rho}$

$v_A^2 = g\rho = (32.2\ ft/s^2)(500\ ft)$

$v_A^2 = 16{,}100$ $v_A = 126.89\ ft/s$

$v_A = 86.5\ mi/h$ ◀

At point B:



$+\uparrow \Sigma F_n = ma_n$:

$N - W = \frac{W}{g}\frac{v_B^2}{\rho}$

$550 - 160 = \frac{160}{32.2}\frac{v_B^2}{500}$

$v_B^2 = (32.2)(500)\frac{390}{160}$

$= 39{,}244$

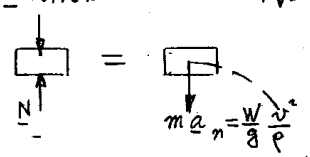
$v_B = 198.1\ ft/s$

$v_B = 135.1\ mi/h$ ◀

12.47

We note that, since speed of belt is constant, $a_t = 0$ as long as package does not slip.

(a)



$+\downarrow \Sigma F_n = ma_n$:

$W - N = ma_n$

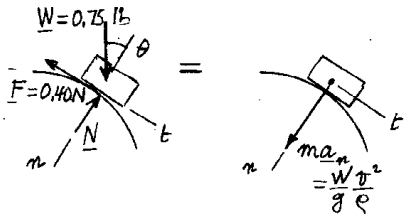
$W - N = \frac{W}{g}\frac{v^2}{\rho}$

$N = W\left(1 - \frac{v^2}{g\rho}\right)$

$N = (0.75\ lb)\left[1 - \frac{(4)^2}{32.2\left(\frac{10}{12}\right)}\right]$

$N = 0.303\ lb \uparrow$ ◀

(b) When slipping is impending, we have $F = F_m = \mu_s N = 0.40N$ and still $a_t = 0$



$+\searrow \Sigma F_t = ma_t = 0$: $W\sin\theta - F = 0$

$0.75\sin\theta - 0.4N = 0$

$N = 1.875\sin\theta$ (1)

$+\swarrow \Sigma F_n = ma_n$: $W\cos\theta - N = \frac{W}{g}\frac{v^2}{\rho}$

$0.75\cos\theta - N = \frac{0.75}{32.2}\frac{(4)^2}{(10/12)}$ (2)

Substituting for N from (1) into (2):

$0.75\cos\theta - 1.875\sin\theta = 0.44720$

$\cos\theta = 2.5\sin\theta + 0.59627$

Squaring both members, and setting $\cos^2\theta = 1 - \sin^2\theta$:

$1 - \sin^2\theta = 6.25\sin^2\theta + 2.98135\sin\theta + 0.35554$

$7.25\sin^2\theta + 2.98135\sin\theta - 0.64446 = 0$

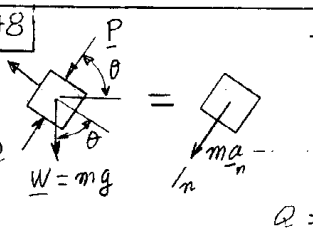
$\sin^2\theta + 0.41122\sin\theta - 0.088891 = 0$

$\sin\theta = \frac{-0.41122 \pm 0.72434}{2}$

$\sin\theta = 0.15656$ $\theta = 9.01°$ ◀

or $\sin\theta = -0.56778$ $\theta = -34.6°$ (extraneous)

12.48



$+\swarrow \Sigma F_n = ma_n$:

$P + mg\sin\theta - Q = m\frac{v^2}{\rho}$

To have contact with specified surface, we need $Q \geq 0$, or

$Q = P + mg\sin\theta - \frac{mv^2}{\rho} \geq 0$

$\sin\theta \geq \frac{1}{g}\left(\frac{v^2}{\rho} - \frac{P}{m}\right)$ (1)

Data: $m = 0.250\ kg$, $v = 3\ m/s$, $P = 1.5\ N$, $\rho = 0.9\ m$

Substitute into (1):

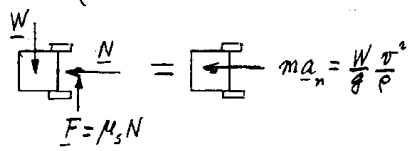
$\sin\theta \geq \frac{1}{9.81}\left[\frac{(3)^2}{0.9} - \frac{1.5}{0.25}\right]$

$\sin\theta \geq 0.40775$

$24.1° \leq \theta \leq 155.9°$ ◀

12.49 $v = 45\ \text{mi/h} = 66\ \text{ft/s}$

$\rho = 60\text{ft} - 2\text{ ft} = 58\text{ ft}$



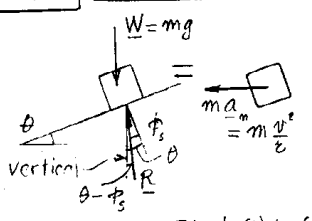
$+\uparrow \Sigma F_y = 0$: $\quad \mu_s N - W = 0 \quad \mu_s N = W \quad (1)$

$\pm \Sigma F_n = ma_n$: $\quad N = \frac{W}{g}\frac{v^2}{\rho} \quad (2)$

Divide (1) by (2):

$$\mu_s = \frac{g\rho}{v^2} = \frac{(32.2)(58)}{(66)^2} \qquad \mu_s = 0.43 \blacktriangleleft$$

12.50 Minimum safe speed:



$\pm \Sigma F_n = ma_n$:

$$R\sin(\theta - \phi_s) = \frac{mv^2}{r} \quad (1)$$

$+\uparrow \Sigma F_y = 0$:

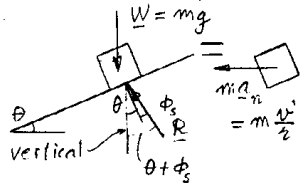
$$R\cos(\theta - \phi_s) - mg = 0$$

$$R\cos(\theta - \phi_s) = mg \quad (2)$$

Divide (1) by (2): $\tan(\theta - \phi_s) = \frac{v^2}{gr}$

$$v = \sqrt{gr\tan(\theta - \phi_s)} \blacktriangleleft$$

Maximum safe speed:



$\pm \Sigma F_n = ma_n$:

$$R\sin(\theta + \phi_s) = \frac{mv^2}{r} \quad (1)$$

$+\uparrow \Sigma F_y = 0$:

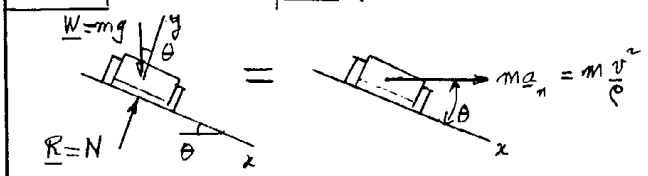
$$R\cos(\theta + \phi_s) - mg = 0$$

$$R\cos(\theta + \phi_s) = mg \quad (2)$$

Divide (1) by (2): $\tan(\theta + \phi_s) = \frac{v^2}{gr}$

$$v = \sqrt{gr\tan(\theta + \phi_s)} \blacktriangleleft$$

12.51 (a) At rated speed ($F = 0$)



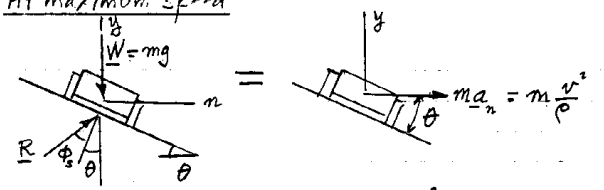
$\pm \Sigma F_x = ma_x$: $\quad mg\sin\theta = m\frac{v^2}{\rho}\cos\theta$

$$\tan\theta = \frac{v^2}{g\rho}$$

Given: $v = 180\ \text{km/h} = 50\ \text{m/s}$, $\rho = 200\text{m}$

$\tan\theta = \frac{(50)^2}{9.81(200)} = 1.2742 \quad \theta = 51.88° \quad \theta = 51.9° \blacktriangleleft$

(b) At maximum speed



$\pm \Sigma F_n = ma_n$: $\quad R\sin(\theta + \phi_s) = m\frac{v^2}{\rho}$

$+\uparrow \Sigma F_y = 0$: $\quad R\cos(\theta + \phi_s) - mg = 0$

(CONTINUED)

12.51 CONTINUED We have therefore

$$R\sin(\theta + \phi_s) = m\frac{v^2}{\rho} \quad (1)$$

$$R\cos(\theta + \phi_s) = mg \quad (2)$$

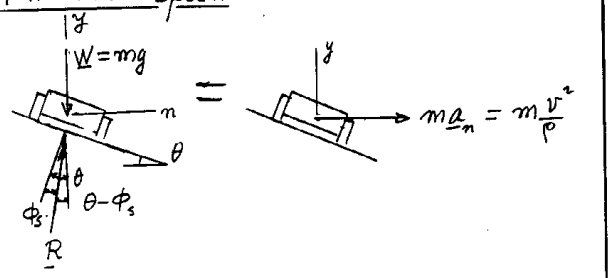
Divide (1) by (2): $\tan(\theta + \phi_s) = \frac{v^2}{g\rho}$

Given: $\rho = 200\text{ m}$, $v = 320\ \text{km/h} = 88.889\ \text{m/s}$, $\theta = 51.88°$

$\tan(51.88° + \phi_s) = \frac{(88.889)^2}{9.81(200)} = 4.0271 \quad 51.88° + \phi_s = 76.05°$

$\phi_s = 76.05° - 51.88° = 24.17° \quad \mu_s = \tan\phi_s = 0.45 \blacktriangleleft$

(c) At minimum speed



$\pm \Sigma F_n = ma_n$: $\quad R\sin(\theta - \phi_s) = m\frac{v^2}{\rho} \quad (3)$

$+\uparrow \Sigma F_y = 0$: $\quad R\cos(\theta - \phi_s) - mg = 0$

$$R\cos(\theta - \phi_s) = mg \quad (4)$$

Divide (3) by (4): $\tan(\theta - \phi_s) = \frac{v^2}{g\rho}$

$$v^2 = g\rho\tan(\theta - \phi_s)$$

Given: $\rho = 200\text{ m}$, $\theta - \phi_s = 51.88° - 24.17° = 27.71°$

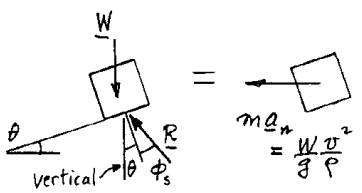
$v^2 = 9.81(200)\tan 27.71° = 1031$

$v = 32.10\ \text{m/s} \qquad v = 115.6\ \text{km/h} \blacktriangleleft$

12.52 $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.75 = 36.87°$

$\theta = 18°$

$\rho = 400\text{ ft}$



$\pm \Sigma F_n = ma_n$: $\quad R\sin(\theta + \phi_s) = \frac{W}{g}\frac{v^2}{\rho} \quad (1)$

$+\uparrow \Sigma F_y = 0$: $\quad R\cos(\theta + \phi_s) - W = 0$

$$R\cos(\theta + \phi_s) = W \quad (2)$$

Divide (1) by (2):

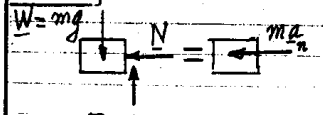
$$\tan(\theta + \phi_s) = \frac{v^2}{g\rho}$$

$v^2 = g\rho\tan(\theta + \phi_s) = 32.2(400)\tan(18° + 36.87°)$

$= 32.2(400)\tan 54.87° = 18{,}306$

$v = 135.30\ \text{ft/s}, \quad v = 92.2\ \text{mi/h} \blacktriangleleft$

12.53



$+\leftarrow \Sigma F_n = ma_n$: $\quad N = ma_n \quad (1)$

$+\uparrow \Sigma F_y = 0$: $\quad F_m - W = 0$

$\mu_s N - mg = 0 \quad (2)$

Substitute for N from (1) into (2):

$\mu_s ma_n - mg = 0$

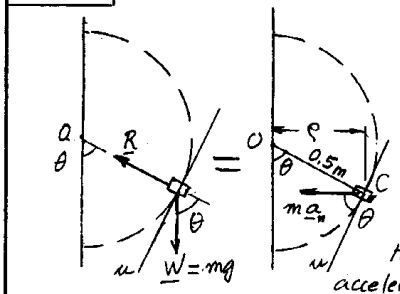
$a_n = \frac{g}{\mu_s}$

or, since $a_n = \frac{v^2}{\rho}$:

$v^2 = \rho a_n = \frac{\rho g}{\mu_s} = \frac{(0.20\ m)(9.81\ m/s^2)}{0.25} = 7.848\ m^2/s^2$

$v = 2.80\ m/s$ ◀

12.54



$\rho = (0.5m)\sin\theta$

$v = \rho(7.5\ rad/s)$

$= 3.75\sin\theta$

$ma_n = m\frac{v^2}{\rho}$

$= m\frac{(3.75\sin\theta)^2}{0.5\sin\theta}$

$= 28.125m\sin\theta$

Since collar is not to slide on rod, only acceleration is $\underline{a}_n$ as shown.

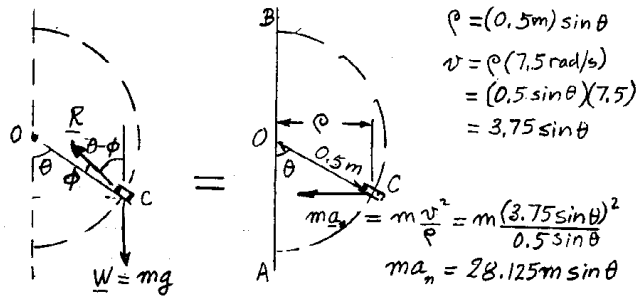
$+\nearrow \Sigma F_u = ma_u$: $\quad mg\sin\theta = ma_n\cos\theta$

$mg\sin\theta = (28.125m\sin\theta)\cos\theta$

$\sin\theta = 0$ $\quad \theta = 0°$ and $\theta = 180°$ ◀

and $\cos\theta = \frac{9.81}{28.125} = 0.3488$ $\quad \theta = 69.6°$ ◀

12.55 Assume that collar does not slide and that force $\underline{R}$ exerted by rod on collar is directed to the right of line CO as shown.



$\rho = (0.5m)\sin\theta$

$v = \rho(7.5\ rad/s)$

$= (0.5\sin\theta)(7.5)$

$= 3.75\sin\theta$

$ma_n = m\frac{v^2}{\rho} = m\frac{(3.75\sin\theta)^2}{0.5\sin\theta}$

$ma_n = 28.125m\sin\theta$

$+\leftarrow \Sigma F_n = ma_n$: $\quad R\sin(\theta-\phi) = 28.125m\sin\theta \quad (1)$

$+\uparrow \Sigma F_y = 0$: $\quad R\cos(\theta-\phi) - mg = 0$

$R\cos(\theta-\phi) = 9.81m \quad (2)$

Divide (1) by (2): $\quad \tan(\theta-\phi) = \frac{28.125}{9.81}\sin\theta \quad (3)$

For impending motion, we make $\phi = \phi_s$ in Eq. (3)

(a) $\underline{\theta = 90°}$: $\quad \tan(90° - \phi_s) = \frac{28.125}{9.81} = 2.867$

$90° - \phi_s = 70.77°$ $\quad \phi_s = 19.23°$, $\mu_s = \tan\phi_s = 0.349$ ◀

Since $\phi_s > 0$, $\underline{R}$ is directed as shown.

Motion impending downward ◀

(CONTINUED)

12.55 CONTINUED

(b) $\underline{\theta = 60°}$: $\quad \tan(60° - \phi_s) = \frac{28.125}{9.81}\sin 60° = 2.4829$

$60° - \phi_s = 68.06°$, $\quad \phi_s = -8.06°$, $\quad \mu_s = |\tan\phi_s| = 0.142$ ◀

Since $\phi_s < 0$, $\underline{R}$ is directed to the left of line CO.

Motion impending upward ◀

(c) $\underline{\theta = 45°}$: $\quad \tan(45° - \phi_s) = \frac{28.125}{9.81}\sin 45° = 2.0273$

$45° - \phi_s = 63.74°$, $\quad \phi_s = -18.74°$, $\quad \mu_s = |\tan\phi_s| = 0.339$ ◀

Since $\phi_s < 0$, $\underline{R}$ is directed to the left of line CO.

Motion impending upward ◀

12.56 See solution of Prob. 12.55 for diagram and derivation of Eq. (3).

(a) $\underline{\theta = 75°}$: Assuming no sliding, we use Eq. (3):

$\tan(75° - \phi) = \frac{28.125}{9.81}\sin 75° = 2.7693$

$75° - \phi = 70.15°$ $\quad \phi = -4.85°$

But $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.04°$.

Since $|\phi| < \phi_s$, $\underline{\text{collar does not slide}}$ ◀

From Eq. (2): $R\cos 70.15° = (9.81\ m/s^2)(0.25\ kg)$

$R = 7.223\ N$ (directed to the left of CO, since $\phi < 0$)

$F = R\sin\phi = (7.223\ N)\sin(-4.85°) = -0.611\ N$

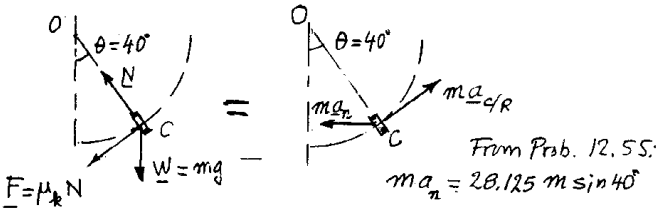
$\underline{F} = 0.611\ N \measuredangle 75°$ ◀

(b) $\underline{\theta = 40°}$: Assume again that collar does not slide and use Eq. (3) of Prob. 12.55:

$\tan(40° - \phi) = \frac{28.125}{9.81}\sin 40° = 1.8429$

$40° - \phi = 61.51°$ $\quad \phi = -21.51°$. Since $\phi < 0$ and $|\phi| > \phi_s$, $\underline{\text{collar slides up}}$ ◀

Since collar slides up the rod, it will have a relative acceleration $\underline{a}_{C/R}$ which should be taken into account:



From Prob. 12.55:

$ma_n = 28.125m\sin 40°$

Using components along CO:

$N - mg\cos 40° = ma_n\sin 40°$

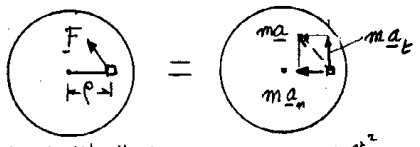
$N = mg\cos 40° + ma_n\sin 40° = mg\cos 40° + 28.125m\sin^2 40°$

$N = m(9.81\cos 40° + 28.125\sin^2 40°) = (0.25\ kg)(19.135\ m/s^2)$

$N = 4.784\ N$ $\quad F = \mu_k N = 0.20(4.784\ N)$

$\underline{F} = 0.957\ N \measuredangle 40°$ ◀

12.57



$$F = \mu_s N = \mu_s W = \mu_s mg \qquad ma_n = m\frac{v^2}{\rho} = m\frac{(a_t t)^2}{\rho}$$

$$ma = m\sqrt{a_n^2 + a_t^2} = m\sqrt{\frac{a_t^4 t^4}{\rho^2} + a_t^2}$$

$$ma = ma_t\sqrt{\frac{a_t^2 t^4}{\rho^2} + 1}$$

$F = ma$: $\mu_s mg = ma_t\sqrt{\frac{a_t^2 t^4}{\rho^2} + 1}$

$$\mu_s^2 g^2 = a_t^2\left(\frac{a_t^2 t^4}{\rho^2} + 1\right)$$

Solving for t^4:

$$t^4 = \left(\frac{\rho}{a_t}\right)^2\left[\left(\frac{\mu_s g}{a_t}\right)^2 - 1\right]$$

(a) Given data: $\rho = 1.5$ ft, $\mu_s = 0.50$, $a_t = 4.5$ ft/s², $g = 32.2$ ft/s²

$$t^4 = \left(\frac{1.5}{4.5}\right)^2\left[\left(\frac{0.50 \times 32.2}{4.5}\right)^2 - 1\right] = \frac{1}{9}(12.80 - 1) = 1.3112$$

$t = 1.0701$ s $\qquad t = 1.070$ s ◀

(b) $v = a_t t = 4.5(1.0701)$ $\qquad v = 4.82$ ft/s ◀

12.58

Referring to diagram of Prob. 12.57

$F = ma$: $\mu_s mg = m\sqrt{a_n^2 + a_t^2}$

$$(\mu_s g)^2 = a_n^2 + a_t^2 \qquad (1)$$

But $a_n = \frac{v^2}{\rho} = \frac{(5\text{ ft/s})^2}{1.5\text{ ft}} = 16.667$ ft/s²

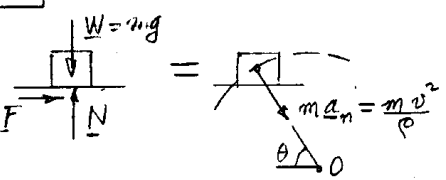
and $\mu_s g = 0.60(32.2\text{ ft/s}^2) = 19.32$ ft/s²

Substituting into Eq. (1):

$(19.32)^2 = (16.667)^2 + a_t^2$ $\qquad a_t^2 = 95.485$, $a_t = 9.772$ ft/s²

$v = a_t t$: $\quad 5\text{ ft/s} = (9.772\text{ ft/s}^2)t$ $\qquad t = 0.512$ s ◀

12.59



$\pm\rightarrow \Sigma F_x = ma_x$: $\quad F = \frac{mv^2}{\rho}\cos\theta$ $\qquad (1)$

$+\uparrow \Sigma F_y = ma_y$: $\quad N - mg = -\frac{mv^2}{\rho}\sin\theta$

$$N = mg - \frac{mv^2}{\rho}\sin\theta \qquad (2)$$

For block not to slide, we must have $F \le \mu_s N$. Substituting from (1) and (2):

$$\frac{mv^2}{\rho}\cos\theta \le \mu_s\left(mg - \frac{mv^2}{\rho}\sin\theta\right)$$

$$v^2 \le \frac{\mu_s g\rho}{\cos\theta + \mu_s\sin\theta} \qquad (3)$$

The smallest value of the right-hand member of (3) is obtained when $\cos\theta + \mu_s\sin\theta$ is maximum, i.e., when

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0$$

$$-\sin\theta + \mu_s\cos\theta = 0 \qquad \tan\theta = \mu_s \qquad (4)$$

(CONTINUED)

12.59 CONTINUED

Given data: $\mu_s = 0.40$, $\rho = 0.4$ m

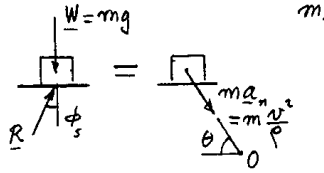
Eq. (4): $\tan\theta = 0.40$ $\qquad \theta = 21.8°$ ◀

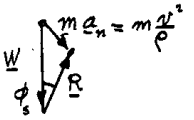
Substituting into Eq. (3):

$$v^2 \le \frac{0.40(9.81)(0.4)}{\cos 21.8° + 0.40\sin 21.8°} = 1.457 \qquad v_{max} = 1.207\text{ m/s}$$ ◀

For $\underline{F}$ directed to the left, we replace μ_s by $-\mu_s$ in Eqs. (3) and (4) and obtain same value for v_{max} and $\theta = 158.2°$ ◀

Alternate Solution: For impending sliding, the vector $m\underline{a}_n$ is the sum of $\underline{W}$ and $\underline{R}$:





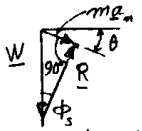
Smallest value of v for which sliding is impending is obtained by drawing $m\underline{a}_n \perp \underline{R}$. We then have $\theta = \phi_s = \tan^{-1}0.40$ $\qquad \theta = \phi_s = 21.8°$ ◀

and

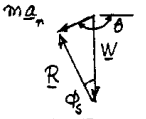
$ma_n = W\sin\phi_s$ $\qquad m\frac{v^2}{\rho} = mg\sin\phi_s$

$v^2 = 9.81(0.4)\sin 21.8° = 1.457$

$v_{max} = 1.207$ m/s ◀



For $-\underline{R}$ directed to the left, we obtain



$\theta = 180° - \phi_s = 180° - 21.8°$ $\qquad \theta = 158.2°$ ◀

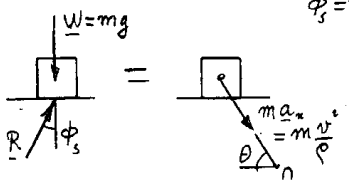
and again

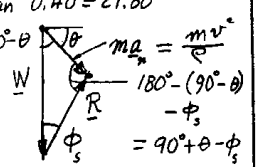
$m\frac{v^2}{\rho} = mg\sin\phi_s$ $\qquad v_{max} = 1.207$ m/s ◀

12.60

When block is just about to slide, reaction $\underline{R}$ forms angle ϕ_s with normal. We express that vector $m\underline{a}_n$ is the sum of $\underline{R}$ and $\underline{W}$:

$\phi_s = \tan^{-1}0.40 = 21.80°$



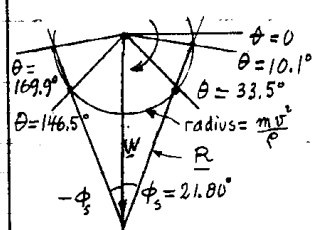


Law of sines:

$$\frac{\sin(90° + \theta - \phi_s)}{W} = \frac{\sin\phi_s}{ma_n} \quad \text{or} \quad \frac{\cos(\theta - \phi_s)}{mg} = \frac{\sin\phi_s}{mv^2/\rho}$$

$$\cos(\theta - \phi_s) = \frac{g\rho}{v^2}\sin\phi_s = \frac{9.81(0.4)}{(1.22)^2}\sin 21.80° = 0.9791$$

$\theta - 21.80° = \pm 11.73°$ $\qquad \theta = 10.1°$ or $\theta = 33.5°$



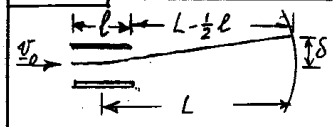
Since rod rotates ↻, value $\theta = 10.1°$ will be reached first. Block starts sliding left for $\theta = 10.1°$ and is still sliding for $\theta = 33.5°$. (Further analysis would show that it does stop before reaching next starting point.) Replacing ϕ_s by $-\phi_s$,

we find $\theta = 180° - 33.5° = 146.5°$ and $\theta = 180° - 10.1° = 169.9°$.

Since rod rotates ↻, block starts sliding right for $\theta = 146.5°$.

Thus, block starts sliding for $\theta = 10.1°$ and $\theta = 146.5°$ ◀

12.61 Horizontal motion (uniform)



Time between plates: $t_1 = \frac{\ell}{v_0}$

Time beyond plates: $t_2 = \frac{1}{v_0}\left(L - \frac{1}{2}\ell\right)$

Vertical motion

Between plates: uniformly accelerated motion

$F = \frac{eV}{d} = ma_y$: $\quad \frac{eV}{d} = ma_y \quad a_y = \frac{eV}{md}$

As electron leaves plates:

$(v_y)_1 = a_y t_1 = \frac{eV}{md}\frac{\ell}{v_0}$

$y_1 = \frac{1}{2}a_y t_1^2 = \frac{1}{2}\frac{eV}{md}\left(\frac{\ell}{v_0}\right)^2 \qquad (1)$

Beyond plates: uniform motion

$\delta = y_2 = y_1 + (v_y)_1 t_2 = \frac{1}{2}\frac{eV\ell^2}{mdv_0^2} + \frac{eV}{md}\frac{\ell}{v_0}\frac{1}{v_0}\left(L - \frac{1}{2}\ell\right)$

$= \frac{eV\ell}{mdv_0^2}\left(\frac{\ell}{2} + L - \frac{\ell}{2}\right), \quad \delta = \frac{eV\ell L}{mv_0^2 d}$ ◀

12.62 See solution of Prob. 12.61 for derivation of Eq. (1) which defines vertical distance y_1 traveled by electron as it leaves plates:

$$y_1 = \frac{1}{2}\frac{eV}{md}\left(\frac{\ell}{v_0}\right)^2$$

If electron is not to strike positive plate, we must have

$$y_1 \le \frac{d}{2}$$

$$\frac{1}{2}\frac{eV}{md}\frac{\ell^2}{v_0^2} \le \frac{d}{2}$$

$$\frac{d^2}{\ell^2} \ge \frac{eV}{mv_0^2} \qquad \frac{d}{\ell} \ge \sqrt{\frac{eV}{mv_0^2}}$$

Smallest allowable value: $\frac{d}{\ell} = \sqrt{\frac{eV}{mv_0^2}}$ ◀

12.63 From solution of Prob. 12.61:

$$\delta = \frac{eV\ell L}{mv_0^2 d}$$

Denote by ℓ' and L' the new lengths of plate and tube. Since δ and all other characteristics are to remain unchanged, we must have

$$\delta = \delta' \qquad \frac{eV\ell L}{mv_0^2 d} = \frac{eV\ell' L'}{mv_0^2 d}$$

$$\frac{\ell'}{\ell} = \frac{L}{L'}$$

Since $\frac{L'}{L} = \frac{1}{2}$, we have

$$\frac{\ell'}{\ell} = \frac{2}{1} \qquad \ell' = 2\ell$$

The length of the plates should be doubled ◀

12.64 Kinematics

$$r = 3t^2 - t^3 \qquad \theta = 2t^2$$
$$\dot{r} = 6t - 3t^2 \qquad \dot{\theta} = 4t$$
$$\ddot{r} = 6 - 6t \qquad \ddot{\theta} = 4$$

(a) When $t = 0$:

$$r = 0 \qquad \theta = 0$$
$$\dot{r} = 0 \qquad \dot{\theta} = 0$$
$$\ddot{r} = 6\ \text{ft/s}^2 \qquad \ddot{\theta} = 4\ \text{rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6\ \text{ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

(b) When $t = 1$ s:

$$r = 3(1)^2 - (1)^3 = 2\ \text{ft} \qquad \theta = 2(1)^2 = 2\ \text{rad}$$
$$\dot{r} = 6(1) - 3(1)^2 = 3\ \text{ft/s} \qquad \dot{\theta} = 4(1) = 4\ \text{rad/s}$$
$$\ddot{r} = 6 - 6(1) = 0 \qquad \ddot{\theta} = 4\ \text{rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(4)^2 = -32\ \text{ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(4) + 2(3)(4) = +32\ \text{ft/s}^2$$

Kinetics

(a) When $t = 0$:

$$F_r = ma_r = \frac{4}{32.2}(6) \qquad F_r = 0.745\ \text{lb}$$ ◀

$$F_\theta = ma_\theta = \frac{4}{32.2}(0) \qquad F_\theta = 0$$ ◀

(b) When $t = 1$ s:

$$F_r = ma_r = \frac{4}{32.2}(-32) \qquad F_r = -3.98\ \text{lb}$$ ◀

$$F_\theta = ma_\theta = \frac{4}{32.2}(+32) \qquad F_\theta = 3.98\ \text{lb}$$ ◀

12.65 Kinematics. See the expressions obtained in the solution of Prob. 12.64 for $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$

When $t = 3$ s:

$$r = 3(3)^2 - (3)^3 = 0 \qquad \theta = 2(3^2) = 18\ \text{rad}$$
$$\dot{r} = 6(3) - 3(3)^2 = -9\ \text{ft/s} \qquad \dot{\theta} = 4(3) = 12\ \text{rad/s}$$
$$\ddot{r} = 6 - 6(3) = -12\ \text{ft/s}^2 \qquad \ddot{\theta} = 4\ \text{rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -12 - (0)(12)^2 = -12\ \text{ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0(4) + 2(-9)(12) = -216\ \text{ft/s}^2$$

Kinetics

$$F_r = ma_r = \frac{4}{32.2}(-12) \qquad F_r = -1.491\ \text{lb}$$ ◀

$$F_\theta = ma_\theta = \frac{4}{32.2}(-216) \qquad F_\theta = -26.8\ \text{lb}$$ ◀

12.66 Kinematics

$r = 2(1+\cos 2\pi t)$ $\quad \theta = 2\pi t$

$\dot{r} = -4\pi \sin 2\pi t$ $\quad \dot{\theta} = 2\pi$

$\ddot{r} = -8\pi^2 \cos 2\pi t$ $\quad \ddot{\theta} = 0$

(a) When $t = 0$:

$r = 2(1+1) = 4\ m$ $\quad \theta = 0$

$\dot{r} = 0$ $\quad \dot{\theta} = 2\pi\ rad/s$

$\ddot{r} = -8\pi^2\ m/s^2$ $\quad \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = -8\pi^2 - 4(2\pi)^2 = -24\pi^2\ m/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

(b) When $t = 0.75s$:

$r = 2\left(1+\cos\frac{3\pi}{2}\right) = 2(1+0) = 2\ m$

$\theta = 2\pi(0.75) = 1.5\pi$

$\dot{r} = -4\pi \sin\frac{3\pi}{2} = +4\pi$ $\quad \dot{\theta} = 2\pi$

$\ddot{r} = -8\pi^2 \cos\frac{3\pi}{2} = 0$ $\quad \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(2\pi)^2 = -8\pi^2\ m/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(4\pi)(2\pi) = 16\pi^2\ m/s^2$

Kinetics

When $t = 0$:

$F_r = ma_r = (0.5\ kg)(-24\pi^2\ m/s^2)$, $F_r = -118.4\ N$ ◀

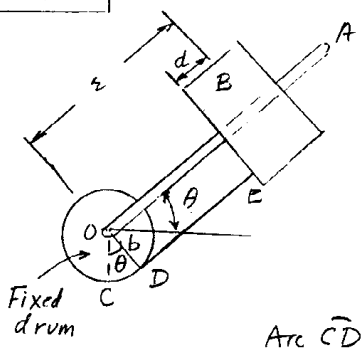
$F_\theta = ma_\theta = (0.5\ kg)(0)$ $\quad F_\theta = 0$ ◀

When $t = 0.75s$:

$F_r = ma_r = (0.5\ kg)(-8\pi^2\ m/s^2)$, $F_r = -39.5\ N$ ◀

$F_\theta = ma_\theta = (0.5\ kg)(16\pi^2\ m/s^2)$, $F_\theta = 79.0\ N$ ◀

12.67



Kinematics

Since the length of cord CDE must remain constant, we have

Arc $\overline{CD}$ + DE = constant

$b\theta + r - d = 0 + r_0 - d$

Where r_0 = length DE when $\theta = 0$.

$r = r_0 - b\theta$ $\quad (1)$

Differentiate: $\dot{r} = -b\dot{\theta} = -b\dot{\theta}_0$

$\ddot{r} = 0$ $\quad \ddot{\theta} = 0$

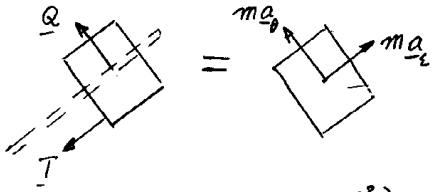
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\dot{\theta}_0^2$ $\quad a_r = -r\dot{\theta}_0^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-b\dot{\theta}_0)\dot{\theta}_0 = -2b\dot{\theta}_0^2$

(CONTINUED)

12.67 CONTINUED

Kinetics - Motion of Block



(a) $\Sigma F_r = ma_r$: $-T = ma_r = m(-r\dot{\theta}_0^2) = -mr\dot{\theta}_0^2$

$T = mr\dot{\theta}_0^2$ ◀

(b) $\Sigma F_\theta = ma_\theta$: $Q = ma_\theta = m(-2b\dot{\theta}_0^2) = -2mb\dot{\theta}_0^2$

$|Q| = 2mb\dot{\theta}_0^2$ ◀

Note: From Eq. (1), we note that B describes an Archimedian spiral.

12.68 See solution of Prob. 12.67 to obtain

$T = mr\dot{\theta}_0^2$ $\quad |Q| = 2mb\dot{\theta}_0^2$

With given data

$m = 2\ kg$ $\quad r = 0.5m$ $\quad b = 0.06\ m$

$\dot{\theta}_0 = 8\ rad/s$

We have:

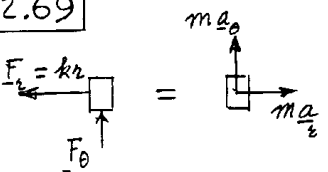
(a) $T = (2\ kg)(0.5\ m)(8\ rad/s)^2$

$T = 64.0\ N$ ◀

(b) $|Q| = 2(2\ kg)(0.06\ m)(8\ rad/s)^2$

$|Q| = 15.36\ N$ ◀

12.69



Where

$a_r = \ddot{r} - r\dot{\theta}^2$ (1)

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ (2)

(a) $+\rightarrow \Sigma F_r = ma_r$: $-kr = ma_r$

$a_r = -\frac{kr}{m} = -\frac{(36\ N/m)(0.4m)}{(0.2\ kg)} = -72\ m/s^2$ ◀

Eq (2): $a_\theta = (0.4m)(0) + 2(1.8\ m/s)(12\ rad/s) = +43.2\ m/s^2$ ◀

(b) From Eq. (1): $\ddot{r} = a_r + r\dot{\theta}^2$

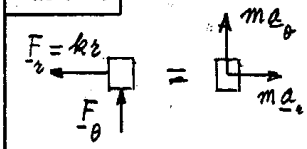
$\ddot{r} = -72\ m/s^2 + (0.4m)(12\ rad/s)^2$ $\quad \ddot{r} = -14.40\ m/s^2$ ◀

(c) $+\uparrow \Sigma F_\theta = ma_\theta$: $F_\theta = ma_\theta = (0.2\ kg)(43.2\ m/s^2)$

$F_\theta = 8.64\ N$ ◀

Note that the acceleration a_r of the slider and its relative acceleration $\ddot{r}$ with respect to arm AB are not equal.

*12.70



Where
$a_r = \ddot{r} - r\dot{\theta}^2$ (1)
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ (2)

(a) $+\rightarrow \Sigma F_r = ma_r$: $-kr = ma_r$ $\quad a_r = -\frac{k}{m}r$ (3)

Substitute this value for a_r in Eq. (1):

$-\frac{k}{m}r = \ddot{r} - r\dot{\theta}^2$ $\quad \ddot{r} = (\dot{\theta}^2 - \frac{k}{m})r$

$\ddot{r} = \left[(12\text{ rad/s})^2 - \frac{36\text{ N·m}}{0.2\text{ kg}}\right]r = -36r$

But $\ddot{r} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = v_r\frac{dv_r}{dr}$

Thus: $v_r\frac{dv_r}{dr} = -36r$

$\int_0^{v_r} v_r\,dv_r = -\int_{0.5\text{m}}^{0.3\text{m}} 36r\,dr$

$\frac{1}{2}v_r^2 = \left[18r^2\right]_{0.3}^{0.5}$

$v_r^2 = 36(0.25 - 0.09) = 5.76\text{ m}^2/\text{s}^2$

$v_r = \pm 2.40\text{ m/s}$

Since r is decreasing: $v_r = -2.40\text{ m/s}$ ◀

$v_\theta = r\dot{\theta} = (0.3\text{m})(12\text{ rad/s})$ $\quad v_\theta = +3.60\text{ m/s}$ ◀

(b) From Eq. (3):

$a_r = -\frac{k}{m}r = -\frac{36\text{ N·m}}{0.2\text{ kg}}(0.3\text{m})$ $\quad a_r = -54\text{ m/s}^2$ ◀

From Eq. (2), since $\dot{\theta} = \dot{\theta}_0 = \text{constant}$:

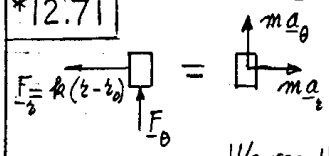
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2v_r\dot{\theta}_0 = 2(-2.40\text{ m/s})(12\text{ rad/s})$

$a_\theta = -57.6\text{ m/s}^2$ ◀

(c) $+\uparrow \Sigma F_\theta = ma_\theta$: $F_\theta = (0.2\text{ kg})(-57.6\text{ m/s}^2)$

$F_\theta = -11.52\text{ N}$ ◀

*12.71



We now have
$F_r = k(r - r_0)$ ←
with $k = 36\text{ N·m}$
and $r_0 = 0.045\text{ m}$

We recall that $a_r = \ddot{r} - r\dot{\theta}^2$ (1)
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ (2)

(a) $+\rightarrow \Sigma F_r = ma_r$: $-k(r - r_0) = ma_r$

$a_r = -\frac{k(r - r_0)}{m} = -\frac{(36\text{ N·m})}{0.2\text{ kg}}(r + 0.045\text{m})$

$a_r = -180r - 8.10$ (3)

Substitute for a_r from (3) into (1):

$-180r - 8.10 = \ddot{r} - r\dot{\theta}^2$

$= \ddot{r} - r(12\text{ rad/s})^2$

$\ddot{r} = -36r - 8.10$

But $\ddot{r} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = v_r\frac{dv_r}{dr}$

Thus: $v_r\frac{dv_r}{dr} = -36r - 8.10$

$\int_0^{v_r} v_r\,dv_r = -\int_{0.5\text{m}}^{0.3\text{m}} (36r + 8.10)\,dr$

$\frac{1}{2}v_r^2 = \left[18r^2 + 8.10r\right]_{0.3}^{0.5}$

(CONTINUED)

*12.71 CONTINUED

$v_r^2 = 36(0.25 - 0.09) + 16.2(0.5 - 0.3) = 9\text{ m}^2/\text{s}^2$

$v_r = \pm 3.00\text{ m/s}$

Since r is decreasing: $v_r = -3.00\text{ m/s}$ ◀

$v_\theta = r\dot{\theta} = (0.3\text{m})(12\text{ rad/s})$ $\quad v_\theta = +3.60\text{ m/s}$ ◀

(b) From Eq. (3): $a_r = -180(0.3) - 8.10$ $\quad a_r = -62.1\text{ m/s}^2$ ◀

From Eq. (2), since $\dot{\theta} = \dot{\theta}_0 = \text{constant}$:

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2v_r\dot{\theta}_0 = 2(-3.00\text{ m/s})(12\text{ rad/s})$

$a_\theta = -72.0\text{ m/s}^2$ ◀

(c) $+\uparrow \Sigma F_\theta = ma_\theta$: $F_\theta = (0.2\text{ kg})(-72.0\text{ m/s}^2)$

$F_\theta = -14.40\text{ N}$ ◀

12.72

Equation of path: $r = r_0\cos\theta$

Eq. (12.27): $r^2\dot{\theta} = h$ where $h = r_0^2\dot{\theta}_0 = r_0 v_0$

or $\dot{\theta} = \frac{h}{r^2} = \frac{r_0 v_0}{r_0^2\cos^2\theta} = \frac{v_0}{r_0\cos^2\theta}$ (1)

$v_r = \dot{r} = (-r_0\sin\theta)\dot{\theta} = (-r_0\sin\theta)\frac{v_0}{r_0\cos^2\theta}$

$v_r = -\frac{v_0\sin\theta}{\cos^2\theta}$

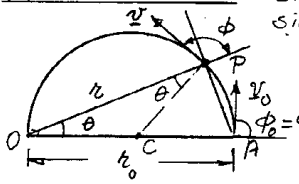
$v_\theta = r\dot{\theta} = r_0\cos\theta\frac{v_0}{r_0\cos^2\theta} = \frac{v_0\cos\theta}{\cos^2\theta}$

$v = \sqrt{v_r^2 + v_\theta^2} = \frac{v_0}{\cos^2\theta}\sqrt{\sin^2\theta + \cos^2\theta} = \frac{v_0}{\cos^2\theta}$

or: $v = \frac{v_0 r_0^2}{r_0^2\cos^2\theta}$ $\quad v = \frac{v_0 r_0^2}{r^2}$ (Q.E.D.) ◀

Alternate solution: Since $r\dot{\theta} = v_\theta = v\sin\phi$, and since $\phi_0 = 90°$, Eq. (12.18) yields

$r v\sin\phi = r_0 v_0$



Observing that $OP \perp AP$ and $v \perp CP$, we have

$\phi = 90° + \theta$, $\sin\phi = \cos\theta$

Thus: $r v\cos\theta = r_0 v_0$ (2)

But, from equation of path:

$r = r_0\cos\theta$ $\quad \cos\theta = \frac{r}{r_0}$

Substituting for $\cos\theta$ in (2):

$\frac{r^2 v}{r_0} = r_0 v_0$ $\quad v = \frac{v_0 r_0^2}{r^2}$ (Q.E.D.) ◀

12.73

Equation of path: $r = r_0/\cos 2\theta$

Eq. (12.27): $r^2\dot{\theta} = h$ where $h = r_0^2\dot{\theta}_0 = r_0 v_0$

or $\dot{\theta} = \frac{h}{r^2} = \frac{r_0 v_0}{(r_0/\cos 2\theta)^2} = \frac{v_0}{r_0}\cos^2 2\theta$

$v_r = \dot{r} = \left(\frac{2r_0\sin 2\theta}{\cos^2 2\theta}\right)\dot{\theta} = \left(\frac{2r_0\sin 2\theta}{\cos^2 2\theta}\right)\left(\frac{v_0}{r_0}\cos^2 2\theta\right)$

$v_r = 2v_0\sin 2\theta$ ◀

$v_\theta = r\dot{\theta} = \left(\frac{r_0}{\cos 2\theta}\right)\left(\frac{v_0}{r_0}\cos^2 2\theta\right)$

$v_\theta = v_0\cos 2\theta$ ◀

12.74 (a) $F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$ $\quad F_\theta = 0$

We recall from the solution of Prob. 12.72 that

$$\dot{\theta} = \frac{v_0}{r_0 \cos^2\theta} \qquad (1)$$

and that the equation of the path is

$$r = r_0 \cos\theta$$

$$\dot{r} = (-r_0 \sin\theta)\dot{\theta} = (-r_0 \sin\theta)\frac{v_0}{r_0\cos^2\theta} = -\frac{v_0 \sin\theta}{\cos^2\theta}$$

$$\ddot{r} = -v_0 \frac{\cos^3\theta + 2\cos\theta\sin^2\theta}{\cos^4\theta}\dot{\theta} = -v_0\frac{\cos^2\theta + 2\sin^2\theta}{\cos^3\theta}\left(\frac{v_0}{r_0\cos^2\theta}\right)$$

$$= -\frac{v_0^2}{r_0}\frac{\cos^2\theta + 2\sin^2\theta}{\cos^5\theta}$$

Thus:

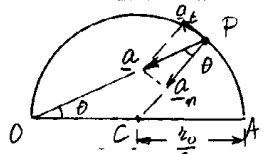
$$F_r = m\left[-\frac{v_0^2}{r_0}\frac{\cos^2\theta + 2\sin^2\theta}{\cos^5\theta} - r_0\cos\theta\left(\frac{v_0}{r_0\cos^2\theta}\right)^2\right]$$

$$= -\frac{mv_0^2}{r_0}\frac{2\cos^2\theta + 2\sin^2\theta}{\cos^5\theta} = -\frac{2mv_0^2}{r_0\cos^5\theta}$$

or, since $r = r_0\cos\theta$ and $F = -|F_r|$: $\quad F = \frac{2mv_0^2 r_0^4}{r^5}$ (Q.E.D.)

(b) For $\theta = 0$: $\quad r = r_0$, $\quad F = \frac{2mv_0^2 r_0^4}{r_0^5} = \frac{2mv_0^2}{r_0}$

Alternate solution for part (a). Acceleration of P is directed toward O and normal component toward C. Thus: $a = a_n/\cos\theta$.



But $a_n = \frac{v^2}{\rho} = \frac{v^2}{r_0/2} = \frac{2v^2}{r_0}$

Recalling expression obtained for v in Prob. 12.72:

$$a_n = \frac{2}{r_0}\left(\frac{v_0 r_0^2}{r^2}\right)^2 = \frac{2v_0^2}{r_0}\frac{r_0^4}{r^4}$$

$$a = \frac{a_n}{\cos\theta} = \frac{2v_0^2}{r_0\cos\theta}\frac{r_0^4}{r^4} = \frac{2v_0^2}{r}\frac{r_0^4}{r^4} = \frac{2v_0^2 r_0^4}{r^5}$$

$F = ma$: $\quad F = \frac{2mv_0^2 r_0^4}{r^5}$ (Q.E.D.)

12.75 (a) $F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$ $\quad F_\theta = 0$

We recall from the solution of Prob. 12.73 that

$$r = r_0/\cos 2\theta \quad \text{and} \quad \dot{\theta} = \frac{v_0}{r_0}\cos^2 2\theta$$

and that $v_r = \dot{r} = 2v_0 \sin 2\theta$

Differentiating:

$$\ddot{r} = (4v_0\cos 2\theta)\dot{\theta} = (4v_0\cos 2\theta)\frac{v_0}{r_0}\cos^2 2\theta$$

$$\ddot{r} = \frac{4v_0^2}{r_0}\cos^3 2\theta$$

Thus:

$$F_r = m\left[\frac{4v_0^2}{r_0}\cos^3 2\theta - \frac{r_0}{\cos 2\theta}\left(\frac{v_0}{r_0}\cos^2 2\theta\right)^2\right]$$

$$= \frac{3mv_0^2}{r_0}\cos^3 2\theta$$

or, since $\cos 2\theta = \frac{r_0}{r}$:

$$F_r = \frac{3mv_0^2}{r_0}\left(\frac{r_0}{r}\right)^3 \qquad F = |F_r| = \frac{3mv_0^2 r_0^2}{r^3}$$

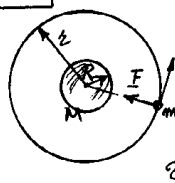
(Q.E.D.)

(b) For $\theta = 0$: $\quad r = r_0$,

$$F = \frac{3mv_0^2 r_0^2}{r_0^3} = \frac{3mv_0^2}{r_0}$$

12.76 For circular orbit: $\quad F = ma_n$

Substitute for F from Eq. (12.2B) and set $a_n = v^2/r$:



$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}} \qquad (1)$$

$$\tau = \frac{2\pi r}{v} = \frac{2\pi}{\sqrt{GM}}r^{3/2} \qquad (2)$$

τ is minimum for $r = R$:

$$\tau_{min} = \frac{2\pi}{\sqrt{GM}}R^{3/2}$$

But $M = \frac{4}{3}\pi R^3 \rho$. Thus:

$$\tau_{min} = \frac{2\pi R^{3/2}}{G^{1/2}\left(\frac{4}{3}\right)^{1/2}\pi^{1/2}R^{3/2}\rho^{1/2}} = \left(\frac{3\pi}{G\rho}\right)^{1/2} \qquad (Q.E.D.)$$

12.77 See solution of Prob. 12.76 for sketch and derivation of Eq. (2). Recalling from Eq. (12.30) that at surface of the earth $GM = gR^2$, we have

$$\tau = \frac{2\pi}{\sqrt{GM}}r^{3/2} = \frac{2\pi}{g^{1/2}R}r^{3/2}$$

$$r^3 = \frac{gR^2\tau^2}{4\pi^2} \qquad (3)$$

Given data (SI units): $g = 9.81\ m/s^2$, $R = 6.37\times10^6\ m$

$$\tau = (27.3\ days)(24\ h/day)(3600\ s/h) = 2.36\times10^6\ s$$

$$r^3 = \frac{9.81(6.37\times10^6)^2(2.36\times10^6)^2}{4\pi^2} = 56.16\times10^{24}$$

$$r = 383\times10^6\ m = 383\times10^3\ km$$

Given data (U.S. units): $g = 32.2\ ft/s^2$, $\tau = 2.36\times10^6\ s$

$$R = (3960\ mi)(5280\ ft/mi) = 20.9\times10^6\ ft$$

$$r^3 = \frac{32.2(20.9\times10^6)^2(2.36\times10^6)^2}{4\pi^2} = 1.984\times10^{27}$$

$$r = 1.257\times10^9\ ft = 238{,}000\ mi$$

12.78 (a) Altitude of satellite

See solution of Probs. 12.76 and 12.77 for derivation of Eq. (3):

$$r^3 = \frac{gR^2\tau^2}{4\pi^2}$$

Given data (SI units): $g = 9.81\ m/s^2$, $R = 6.37\times10^6\ m$

$$\tau = 23\ h\ 56\ min = (23.93\ h)(3600\ s/h) = 86.15\times10^3\ s$$

$$r^3 = \frac{9.81(6.37\times10^6)^2(86.15\times10^3)^2}{4\pi^2} = 74.83\times10^{21}$$

$$r = 42.14\times10^6\ m = 42.14\times10^3\ km$$

$$\text{Altitude} = (42.14 - 6.37)10^3 = 35.77\times10^3\ km$$

Given data (U.S. units): $g = 32.2\ ft/s^2$, $\tau = 86.15\times10^3\ s$

$$R = (3960\ mi)(5280\ ft/mi) = 20.9\times10^6\ ft$$

$$r^3 = \frac{32.2(20.9\times10^6)^2(86.15\times10^3)^2}{4\pi^2} = 2.644\times10^{24}$$

$$r = 138.3\times10^6\ ft = 26.19\times10^3\ mi$$

$$\text{Altitude} = (26.19 - 3.96)10^3\ mi = 22{,}230\ mi$$

(b) Velocity of satellite

SI units: $\quad v = \frac{2\pi r}{\tau} = \frac{2\pi(42.14\times10^6\ m)}{86.15\times10^3\ s}$

$$v = 3070\ m/s$$

U.S. units: $\quad v = \frac{2\pi r}{\tau} = \frac{2\pi(138.3\times10^6\ ft)}{86.15\times10^3\ s}$

$$v = 10{,}090\ ft/s$$

12.79 See solution of Prob. 12.76 for sketch and derivation of Eq. (2):

$$\tau = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

or $$GM = \frac{4\pi^2 r^3}{\tau^2} \quad (1)$$

(a) For Callisto: $r = 1.884 \times 10^6$ km $= 1.884 \times 10^9$ m

$\tau = 16$ days 16 h 32 min $= 1.442 \times 10^6$ s

Eq. (1): $$GM = \frac{4\pi^2 (1.884 \times 10^9 \text{ m})^3}{(1.442 \times 10^6 \text{ s})^2} = 126.96 \times 10^{15} \text{ m}^3/\text{s}^2$$

But $G = 66.73 \times 10^{-12}$ m^3/kg·s^2. Thus

$$M = \frac{126.96 \times 10^{15} \text{ m}^3/\text{s}^2}{66.73 \times 10^{-12} \text{ m}^3/\text{kg}\cdot\text{s}^2} = 1.9026 \times 10^{27} \text{ kg}$$

Mass of Jupiter $= 1.90 \times 10^{27}$ kg ◀

(This is 319 times the mass of the earth)

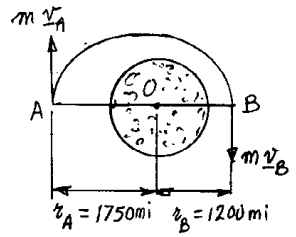
(b) For Io: $\tau = 1$ day 18 h 28 min $= 152.88 \times 10^3$ s

From Eq. (1) and recalling value obtained for GM:

$$r^3 = \frac{GM}{4\pi^2} \tau^2 = \frac{126.96 \times 10^{15} \text{ m}^3/\text{s}^2}{4\pi^2} (152.88 \times 10^3 \text{ s})^2$$

$r^3 = 75.164 \times 10^{24}$ m^3 $\quad r = 422 \times 10^3$ km ◀

12.80 (a) Elliptic path AB:



Conservation of angular momentum between A and B. Since $\underline{v}_A \perp OA$ and $\underline{v}_B \perp OB$:

$$H_0 = r_A m v_A = r_B m v_B$$

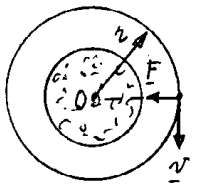
$$(v_B)_{ell} = \left(\frac{r_A}{r_B}\right)(v_A)_{ell}$$

But $(v_A)_{ell} = 2950$ mi/h $- 290$ mi/h $= 2660$ mi/h

Thus: $$(v_B)_{ell} = \frac{1750 \text{ mi}}{1200 \text{ mi}} (2660 \text{ mi/h}) = 3879.2 \text{ mi/h}$$

$(v_B)_{ell} = 3880$ mi/h ◀

(b) Circular orbits:



For any circular orbit:

$$F = ma_n$$

But from Eq. (12.19)

$$F = \frac{GMm}{r^2}$$

Also: $$a_n = \frac{v^2}{r}$$

Substituting in $F = ma_n$: $$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v_{circ} = \sqrt{\frac{GM}{r}} \quad (1)$$

For circular orbits through A and through B:

$$(v_A)_{circ} = \sqrt{\frac{GM}{r_A}} \qquad (v_B)_{circ} = \sqrt{\frac{GM}{r_B}}$$

$$\frac{(v_B)_{circ}}{(v_A)_{circ}} = \sqrt{\frac{r_A}{r_B}} \qquad (v_B)_{circ} = (v_A)_{circ}\sqrt{\frac{r_A}{r_B}}$$

$$(v_B)_{circ} = (2950 \text{ mi/h})\sqrt{\frac{1750 \text{ mi}}{1200 \text{ mi}}} = 3562.5 \text{ mi/h}$$

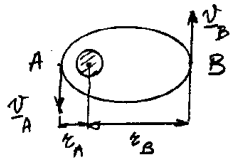
Decrease in $v = 3879.2$ mi/h $- 3562.5$ mi/h $= 317$ mi/h ◀

12.81 $r_A = 3400 + 2200 = 5600$ km $= 5.60 \times 10^6$ m

$r_B = 3400 + 100\,000 = 103\,400$ km $= 103.4 \times 10^6$ m

$r_C = 3400 + 1000 = 4400$ km $= 4.40 \times 10^6$ m

First Transfer Orbit



$v_A = 2771$ m/s $+ 1046$ m/s

$= 3817$ m/s

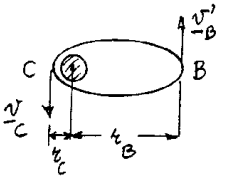
Cons. of angular momentum:

$$r_A m v_A = r_B m v_B$$

$$(5.60 \times 10^6)(3817) = (103.4 \times 10^6) v_B$$

$v_B = 206.7$ m/s

Second Transfer Orbit



$v'_B = v_B + \Delta v_B$

$= 206.7 - 22.0 = 184.7$ m/s

Cons. of angular momentum:

$$r_B m v'_B = r_C m v_C$$

$$(103.4 \times 10^6)(184.7) = (4.40 \times 10^6) v_C$$

$v_C = 4340$ m/s

After Last Maneuver

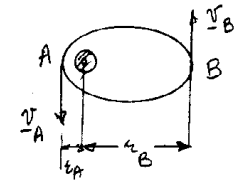
$v = v_C + \Delta v_C = 4340 + 660 = 5000$ m/s ◀

12.82 $r_A = 3400 + 2200 = 5600$ km $= 5.60 \times 10^6$ m

$r_B = 3400 + 100\,000 = 103\,400$ km $= 103.4 \times 10^6$ m

$r_C = 3400 + 300 = 3700$ km $= 3.70 \times 10^6$ m

First Transfer Orbit



$v_A = 2771$ m/s $+ 1046$ m/s

$= 3817$ m/s

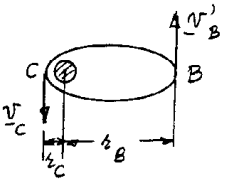
Cons. of angular momentum:

$$r_A m v_A = r_B m v_B$$

$$(5.60 \times 10^6)(3817) = (103.4 \times 10^6) v_B$$

$v_B = 206.7$ m/s

Second Transfer Orbit



$v'_B = v_B + \Delta v_B$

$= 206.7 - 37.2 = 169.5$ m/s

Cons. of angular momentum:

$$r_B m v'_B = r_C m v_C$$

$$(103.4 \times 10^6)(169.5) = (3.70 \times 10^6) v_C$$

$v_C = 4740$ m/s

After Last Maneuver

$v = v_C + \Delta v_C = 4740 + 660 = 5400$ m/s ◀

12.83 Circular orbits

See solution of Prob. 12.80 for derivation of

$$v_{circ} = \sqrt{\frac{GM}{r}} \qquad (1)$$

But from Eq. (12.30):

$$GM = gR^2 = (32.2 \text{ ft/s}^2)(3960 \times 5280 \text{ ft})^2 = 14.08 \times 10^{15}$$

Orbit through A:

$$r_A = 340 \text{ mi} + 3960 \text{ mi} = 4300 \text{ mi} = 22.70 \times 10^6 \text{ ft}$$

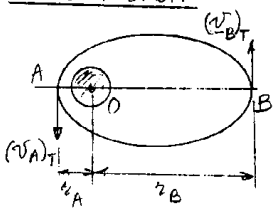
Orbit through B:

$$r_B = 22,230 \text{ mi} + 3960 \text{ mi} = 26,190 \text{ mi} = 138.28 \times 10^6 \text{ ft}$$

$$(v_A)_{circ} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.08 \times 10^{15}}{22.70 \times 10^6}} = 24,900 \text{ ft/s}$$

$$(v_B)_{circ} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.08 \times 10^{15}}{138.28 \times 10^6}} = 10,090 \text{ ft/s}$$

Transfer orbit



$$(v_A)_T = (v_A)_{circ} + \Delta v_A$$
$$= 24,900 \text{ ft/s} + 7740 \text{ ft/s}$$
$$= 32,640 \text{ ft/s}$$

Conservation of ang. momentum between A and B:

$$H_O = r_A m (v_A)_T = r_B m (v_B)_T$$

$$(v_B)_T = \frac{r_A}{r_B}(v_A)_T = \frac{22.70 \times 10^6}{138.28 \times 10^6}(32,640 \text{ ft/s}) = 5360 \text{ ft/s}$$

$$\Delta v_B = (v_B)_{circ} - (v_B)_T = 10,090 - 5360 = 4730 \text{ ft/s}$$

$$\Delta v_B = 4730 \text{ ft/s}$$ ◀

12.84 (a) Since no force acts on the ball after the cord has been cut and until the ball hits stop B,

$$F_r = F_\theta = 0 \qquad a_r = a_\theta = 0$$ ◀

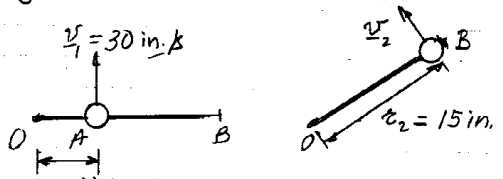
(Until it hits stop B, the ball will move in a straight line with the absolute velocity $\underline{v}_1$.)

(b) Recalling that $a_r = \ddot{r} - r\dot{\theta}^2$ we have

$$\ddot{r} = a_r + r\dot{\theta}^2 = 0 + r_1\left(\frac{v_1}{r_1}\right)^2 = \frac{v_1^2}{r_1} = \frac{(30 \text{ in./s})^2}{3 \text{ in.}}$$

$$\ddot{r} = +300 \text{ in./s}^2$$ ◀

(c) Since the force exerted by stop B on the ball is directed toward O, we have conservation of angular momentum about O:

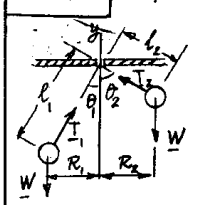


$$H_O = r_1 m v_1 = r_2 m v_2$$

$$v_2 = \frac{r_1}{r_2} v_1 = \frac{3 \text{ in.}}{15 \text{ in.}}(30 \text{ in./s})$$

$$v_2 = 6 \text{ in./s}$$ ◀

12.85



(a) Since force $\underline{T}$ exerted by cord and weight $\underline{W}$ have zero moment about y axis, we write Eq. (12.19) in terms of y comp.:

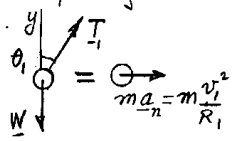
$$\dot{H}_y = \Sigma M_y = 0$$

and thus $H_y = Rmv = \text{constant}$

Therefore: $R_1 m v_1 = R_2 m v_2$

$$(\ell_1 \sin\theta_1) v_1 = (\ell_2 \sin\theta_2) v_2$$

Squaring both members: $\ell_1^2 \sin^2\theta_1 \, v_1^2 = \ell_2^2 \sin^2\theta_2 \, v_2^2$ (1)



As ball describes circle of radius R_1:

$$+\leftarrow \Sigma F_n = ma_n: \quad T_1 \sin\theta_1 = m\frac{v_1^2}{R_1}$$

$$T_1 \sin\theta_1 = m\frac{v_1^2}{\ell_1 \sin\theta_1}, \quad v_1^2 = \frac{T_1 \ell_1}{m}\sin^2\theta_1 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad T_1 \cos\theta_1 - W = 0, \quad T_1 = \frac{W}{\cos\theta_1} = \frac{mg}{\cos\theta_1}$$

Substitute for T_1 into (2): $v_1^2 = g\ell_1 \sin^2\theta_1 / \cos\theta_1$ (3)

Similarly: $v_2^2 = g\ell_2 \sin^2\theta_2 / \cos\theta_2$ (4)

Substitute for v_1^2 and v_2^2 from (3) and (4) into (1):

$$\ell_1^2 \sin^2\theta_1 \left(g\ell_1 \sin^2\theta_1 / \cos\theta_1\right) = \ell_2^2 \sin^2\theta_2 \left(g\ell_2 \sin^2\theta_2 / \cos\theta_2\right)$$

$$\ell_1^3 \sin^3\theta_1 \tan\theta_1 = \ell_2^3 \sin^3\theta_2 \tan\theta_2$$ ◀

(b) Making $\ell_1 = 30$ in., $\theta_1 = 30°$, $\ell_2 = 20$ in.:

$$(30)^3 \sin^3 30° \tan 30° = (20)^3 \sin^3\theta_2 \tan\theta_2$$

$$\sin^3\theta_2 \tan\theta_2 = 0.24357$$

Solving by trial and error: $\theta_2 = 40.9°$ ◀

12.86 (a) Transverse Component of Velocity.

Since the only horizontal force exerted on collar is the force $\underline{F}$ exerted by spring, we have $F_\theta = 0$ and the angular momentum is conserved about the shaft:

$$r_0 m (v_\theta)_0 = r m v_\theta \qquad v_\theta = \frac{r_0}{r}(v_\theta)_0$$

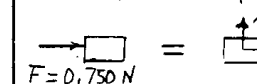
Since $(v_\theta)_0 = r_0 \dot{\theta}_0$: $v_\theta = \frac{r_0^2}{r}\dot{\theta}_0 = \frac{(0.150 \text{ m})^2}{0.600 \text{ m}}(12 \text{ rad/s})$

$$v_\theta = 0.450 \text{ m/s}$$ ◀

(b) Components of Acceleration.

Only horizontal force exerted on collar is $\underline{F}$ exerted by spring. For $r = 0.6$ m:

$$F = kx = (5 \text{ N/m})(0.75 \text{ m} - 0.6 \text{ m}) = 0.750 \text{ N}$$



$$\pm\rightarrow \Sigma F_r = ma_r: \quad 0.750 \text{ N} = (0.3 \text{ kg})a_r$$

$$a_r = +2.50 \text{ m/s}^2$$ ◀

$$+\uparrow \Sigma F_\theta = ma_\theta, \quad 0 = ma_\theta, \quad a_\theta = 0$$ ◀

(c) Acceleration Relative to Rod

We first note that, for $r = 0.6$ m,

$$v_\theta = r\dot{\theta}, \quad \dot{\theta} = \frac{v_\theta}{r} = \frac{0.450 \text{ m/s}}{0.6 \text{ m}} = 0.75 \text{ rad/s}$$

We recall that $a_r = \ddot{r} - r\dot{\theta}^2$

Accel. relative to rod $= \ddot{r} = a_r + r\dot{\theta}^2$

$$= \ddot{r} = +2.50 \text{ m/s}^2 + (0.6 \text{ m})(0.75 \text{ rad/s})^2 = +2.8375 \text{ m/s}^2$$

Accel. relative to rod $= +2.84 \text{ m/s}^2$ ◀

***12.87** (a) Radial component of velocity

For an arbitrary distance $r \le 0.75$ m from the shaft, the spring is compressed by $(0.75\text{m} - r)$ and the force exerted by the spring on the collar is

$$\pm\rightarrow \quad F_r = +(5\text{ N/m})(0.75\text{m} - r)$$

On the other hand, we have $a_r = \ddot{r} - r\dot{\theta}^2$. Thus

$$F_r = m a_r: \quad 5(0.75 - r) = (0.300\text{ kg})(\ddot{r} - r\dot{\theta}^2) \quad (1)$$

But, since $F_\theta = 0$, we have conservation of angular momentum about the shaft:

$$r^2\dot{\theta} = r_0^2\dot{\theta}_0$$

or, since $r_0 = 0.15$ m and $\dot{\theta}_0 = 12$ rad/s,

$$r^2\dot{\theta} = (0.15)^2(12) = 0.270 \qquad \dot{\theta} = \frac{0.27}{r^2}$$

Substituting for $\dot{\theta}$ into Eq. (1) and solving for $\ddot{r}$,

$$\ddot{r} = \frac{5}{0.3}(0.75 - r) + r\left(\frac{0.27}{r^2}\right)^2$$

$$\ddot{r} = 12.5 - 16.667r + \frac{0.0729}{r^3}$$

But $\ddot{r} = \dfrac{dv_r}{dt} = \dfrac{dv_r}{dr}\dfrac{dr}{dt} = v_r\dfrac{dv_r}{dr}$

Thus:

$$v_r\,dv_r = \left(12.5 - 16.667r + \frac{0.0729}{r^3}\right)dr$$

Integrating in v_r from 0 to v_r and in r from 0.15 m to 0.6 m, we have

$$\int_0^{v_r} v_r\,dv_r = \int_{0.15}^{0.6}\left(12.5 - 16.667r + \frac{0.0729}{r^3}\right)dr$$

$$\frac{1}{2}v_r^2 = \left[12.5r - 8.333r^2 - \frac{0.0729}{2r^2}\right]_{0.15}^{0.6}$$

$$\frac{1}{2}v_r^2 = 12.5(0.6 - 0.15) - 8.333(0.6)^2 + 8.333(0.15)^2 - \frac{0.0729}{2(0.6)^2} + \frac{0.0729}{2(0.15)^2}$$

$$= 5.625 - 3 + 0.1875 - 0.10125 + 1.620 = 4.33125$$

$$v_r^2 = 8.6625 \qquad v_r = 2.9432 \qquad v_r = +2.94\text{ m/s} \blacktriangleleft$$

(b) Value of $\ddot{\theta}$

Since $F_\theta = 0$:

$$F_\theta = m a_\theta: \quad 0 = m a_\theta \qquad a_\theta = 0$$

But $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$. Thus

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad (2)$$

But $r = 0.6$ m $\qquad \dot{r} = v_r = 2.9432$ m/s

$$\dot{\theta} = \frac{0.27}{r^2} = \frac{0.27}{(0.6)^2} = 0.750\text{ rad/s}$$

Substituting into Eq. (2):

$$(0.6\text{ m})\ddot{\theta} + 2(2.9432\text{ m/s})(0.75\text{ rad/s}) = 0$$

$$\ddot{\theta} = -7.358\text{ rad/s}^2 \qquad \ddot{\theta} = -7.36\text{ rad/s}^2 \blacktriangleleft$$

12.88

$$r = r_0\cos\theta \quad (1)$$

$$u = \frac{1}{r} = \frac{1}{r_0\cos\theta} \quad (2)$$

$$\frac{du}{d\theta} = \frac{\sin\theta}{r_0\cos^2\theta}$$

$$\frac{d^2u}{d\theta^2} = \frac{\cos^2\theta + 2\cos\theta\sin^2\theta}{r_0\cos^4\theta} = \frac{1 + 2\tan^2\theta}{r_0\cos\theta} \quad (3)$$

From Eq. (12.37):

$$F = mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) \quad (4)$$

Substituting from (2) and (3) into (4):

$$F = \frac{mh^2}{r_0^2\cos^2\theta}\left(\frac{1 + 2\tan^2\theta + 1}{r_0\cos\theta}\right) = \frac{2mh^2(1 + \tan^2\theta)}{r_0^3\cos^3\theta}$$

or, since $1 + \tan^2\theta = \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$$F = \frac{2mh^2}{r_0^3\cos^5\theta} = \frac{2mh^2r_0^2}{(r_0\cos\theta)^5}$$

and, recalling Eq. (1):

$$F = +\frac{2mh^2r_0^2}{r^5} \quad \text{(Q.E.D.)} \blacktriangleleft$$

Note: The + sign indicates that F is attractive

12.89

$$r = \frac{r_0}{\cos 2\theta} \quad (1)$$

$$u = \frac{1}{r} = \frac{1}{r_0}\cos 2\theta \quad (2)$$

$$\frac{du}{d\theta} = -\frac{2}{r_0}\sin 2\theta$$

$$\frac{d^2u}{d\theta^2} = -\frac{4}{r_0}\cos 2\theta \quad (3)$$

From Eq. (12.28): $\quad F = mh^2u^2\left(\dfrac{d^2u}{d\theta^2} + u\right) \quad (4)$

Substituting from (2) and (3) into (4):

$$F = \frac{mh^2}{r_0^2}\cos^2 2\theta\left(-\frac{4}{r_0}\cos 2\theta + \frac{1}{r_0}\cos 2\theta\right) = -\frac{3mh^2}{r_0^3}\cos^3 2\theta$$

or, recalling Eq. (1):

$$F = -\frac{3mh^2}{r^5} \quad \text{(Q.E.D.)} \blacktriangleleft$$

Note: The − sign indicates that F is repulsive

12.90

$$r = \frac{b}{\theta} \quad (1)$$

$$u = \frac{1}{r} = \frac{\theta}{b} \quad (2)$$

$$\frac{du}{d\theta} = \frac{1}{b}$$

$$\frac{d^2u}{d\theta^2} = 0 \quad (3)$$

From Eq. (12.37):

$$F = mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) \quad (4)$$

Substituting from (2) and (3) into (4):

$$F = mh^2\left(\frac{\theta}{b}\right)^2\left(0 + \frac{\theta}{b}\right) = mh^2\left(\frac{\theta}{b}\right)^3$$

or, recalling Eq. (1):

$$F = +\frac{mh^2}{r^3} \quad \text{(Q.E.D.)} \blacktriangleleft$$

Note: The + sign indicates that F is attractive.

12.91

$$r = r_0 e^{b\theta} \quad (1)$$

$$u = \frac{1}{r} = \frac{1}{r_0} e^{-b\theta} \quad (2)$$

$$\frac{du}{d\theta} = -\frac{b}{r_0} e^{-b\theta}$$

$$\frac{d^2u}{d\theta^2} = +\frac{b^2}{r_0} e^{-b\theta} \quad (3)$$

From Eq. (12.37):

$$F = mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) \quad (4)$$

Substituting from (2) and (3) into (4):

$$F = mh^2 \frac{1}{r_0^2} e^{-2b\theta}\left(\frac{b^2}{r_0} e^{-b\theta} + \frac{1}{r_0} e^{-b\theta}\right)$$

$$F = \frac{mh^2}{r_0^3}(b^2+1)\, e^{-3b\theta} = \frac{mh^2(1+b^2)}{r_0^3 e^{3b\theta}}$$

or, recalling Eq. (1):

$$F = +\frac{mh^2(1+b^2)}{r^3} \quad \text{(Q.E.D.)}$$

Note: The + sign indicates that F is attractive.

12.92 For a parabolic trajectory, we have from Eq. (12.43):

$$v_0 = v_{esc} = \sqrt{\frac{2GM}{r_0}} \quad (1)$$

or $$M = \frac{r_0 v_0^2}{2G} = \frac{(350\times10^6 \text{ m})(26.9\times10^3 \text{ m/s})^2}{2(66.73\times10^{-12} \text{ m}^3/\text{kg}\cdot\text{s}^2)}$$

$$M = 1.90\times10^{27} \text{ kg}$$

12.93 For Tethys, we have from Eq. (12.44):

$$v_T = v_{circ} = \sqrt{\frac{GM}{r_T}} \quad \text{or} \quad v_T^2 = \frac{GM}{r_T} \quad (1)$$

where M = mass of Saturn.

For Voyager 1, we use Eq. (12.39') with $\theta = 0$:

$$\frac{1}{r_V} = \frac{GM}{h^2}(1+\varepsilon\cos 0) = \frac{GM}{h^2}(1+\varepsilon)$$

But $h = r_V v_V$. Substituting:

$$\frac{1}{r_V} = \frac{GM}{r_V^2 v_V^2}(1+\varepsilon) \qquad v_V^2 = \frac{(1+\varepsilon)GM}{r_V} \quad (2)$$

Dividing (2) by (1), member by member:

$$\frac{v_V^2}{v_T^2} = (1+\varepsilon)\frac{r_T}{r_V} \qquad 1+\varepsilon = \frac{r_V}{r_T}\left(\frac{v_V}{v_T}\right)^2$$

Substituting the given data:

$$1+\varepsilon = \frac{115\times10^3 \text{ mi}}{183\times10^3 \text{ mi}}\left(\frac{68.8 \text{ ft/s}}{37.2 \text{ ft/s}}\right)^2 = 2.150, \quad \varepsilon = 1.150$$

12.94 Using Eq. (1) of Prob. 12.92 for each of the Voyager spacecrafts, we have

$$v_1 = \sqrt{\frac{2GM}{r_1}} \qquad v_2 = \sqrt{\frac{2GM}{r_2}}$$

where M = mass of Jupiter. We write

$$\frac{v_2}{v_1} = \sqrt{\frac{r_1}{r_2}}$$

$$v_2 = v_1\sqrt{\frac{r_1}{r_2}} = (26.9 \text{ km/s})\sqrt{\frac{350\times10^3 \text{ km}}{715\times10^3 \text{ km}}} = 18.82 \text{ km/s}$$

12.95

For Voyager 1, we still have

$$v_1 = v_{esc} = \sqrt{\frac{2GM}{r_1}} \quad (1)$$

For Voyager 2, we use Eq. (12.39') with $\theta = 0$:

$$\frac{1}{r_2} = \frac{GM}{h^2}(1+\varepsilon\cos 0) = \frac{GM}{h^2}(1+\varepsilon)$$

But $h = r_2 v_2$. Substituting and letting $\varepsilon = 1.50$,

$$\frac{1}{r_2} = \frac{GM}{r_2^2 v_2^2}(1+1.50) \qquad v_2 = \sqrt{\frac{2.50GM}{r_2}} \quad (2)$$

Dividing (2) by (1), member by member:

$$\frac{v_2}{v_1} = \sqrt{\frac{1.25 r_1}{r_2}}$$

$$v_2 = v_1\sqrt{\frac{1.25 r_1}{r_2}} = (26.9 \text{ km/s})\sqrt{\frac{(1.25)(350\times10^3 \text{ km})}{715\times10^3 \text{ km}}}$$

$$v_2 = 21.04 \text{ km/s}$$

12.96 Letting $\theta = 0$ in Eq. (12.39'):

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \quad (1)$$

Letting $\theta = 180°$ in same equation:

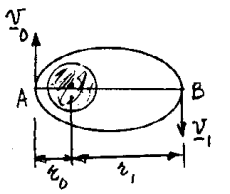
$$\frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon) \quad (2)$$

Adding (1) and (2), member to member:

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad \text{(Q.E.D.)}$$

12.97 Elliptic orbit. We use the equation derived in previous problem:

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad (1)$$



with $r_0 = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi} = 21.120\times10^6 \text{ ft}$

$r_1 = 3960 \text{ mi} + 240 \text{ mi} = 4200 \text{ mi} = 22.176\times10^6 \text{ ft}$

$$GM = gR^2 = (32.2 \text{ ft/s}^2)(3960\times5280)^2 = 14.0771\times10^{15}$$

Substitute into (1): $\frac{1}{21.120\times10^6} + \frac{1}{22.176\times10^6} = \frac{2(14.0771\times10^{15})}{h^2}$

$$\frac{28.1542\times10^{15}}{h^2} = 47.3485\times10^{-9} + 45.0938\times10^{-9} = 92.4423\times10^{-9}$$

$$h^2 = 304.560\times10^{21} \qquad h = 551.869\times10^9$$

Since $h = r_0 v_0$, $v_0 = \frac{551.869\times10^9}{21.120\times10^6} = 26.1302\times10^3 \text{ ft/s}$

Since $h = r_1 v_1$, $v_1 = \frac{551.869\times10^9}{22.176\times10^6} = 24.8859\times10^3 \text{ ft/s}$

Circular orbit. From Eq. (12.35):

$$v_{circ} = \sqrt{\frac{GM}{r_1}} = \sqrt{\frac{14.0771\times10^{15}}{22.176\times10^6}} = 25.1950\times10^3 \text{ ft/s}$$

(a) Speed v_0:

$$v_0 = 26.1302\times10^3 \text{ ft/s}, \quad v_0 = 26.1\times10^3 \text{ ft/s}$$

(b) Increase in speed at B:

$$\Delta v_B = v_{circ} - v_1 = 25.1950\times10^3 \text{ ft/s} - 24.8859\times10^3 \text{ ft/s}$$

$$= 0.3091\times10^3 \text{ ft/s}, \quad \Delta v_B = 309 \text{ ft/s}$$

12.98 (a) Original Orbit. We use equation of Prob 12.96:

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2} \qquad (1)$$

with $r_0 = r_A = 3960\text{ mi} + 1500\text{ mi} = 5460\text{ mi} = 28.829\times10^6\text{ ft}$

$r_1 = r_B = 3960\text{ mi} + 6000\text{ mi} = 9960\text{ mi} = 52.589\times10^6\text{ ft}$

$GM = gR^2 = (32.2\text{ ft/s}^2)(3960\times5280\text{ ft})^2 = 14.077\times10^{15}\text{ ft}^3/\text{s}^2$

$$\frac{1}{28.829\times10^6}+\frac{1}{52.589\times10^6}=\frac{2(14.077\times10^{15})}{h^2},\quad h = 724.06\times10^9\text{ ft}^2/\text{s}$$

But $h = r_A v_A$: $724.06\times10^9 = (28.829\times10^6)v_A$

$v_A = 25{,}116\text{ ft/s}$ $\quad v_A = 17{,}120\text{ mi/h}$ ◀

(b) Modified Orbit. After 8% increase in speed:

$v_A' = 1.08\,v_A = 1.08\,(25{,}116\text{ ft/s}) = 27{,}125\text{ ft/s}$

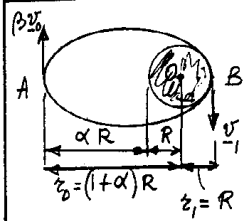
$h = r_A v_A' = (28.829\times10^6\text{ ft})(27.125\text{ ft/s}) = 781.98\times10^9\text{ ft}^2/\text{s}$

Substitute into Eq. (1) and make $r_0 = r_A = 28.829\times10^6\text{ ft}$, $r_1 = r_{B'}$:

$$\frac{1}{28.829\times10^6}+\frac{1}{r_{B'}}=\frac{2(14.077\times10^{15})}{(781.98\times10^9)^2},\quad r_{B'} = 88{,}074\times10^6\text{ ft} = 16{,}680\text{ mi}$$

Maximum Altitude $= r_{B'} - R = 16{,}680 - 3960 = 12{,}720\text{ mi}$ ◀

12.99



Consider elliptic orbit which brings probe to graze surface of planet. We use equation derived in Prob. 12.96:

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2} \qquad (1)$$

where $r_0 = (1+\alpha)R$, $r_1 = R$

$h = r_0(\beta v_0)$ $\quad h^2 = r_0^2\beta^2 v_0^2$

But v_0 is velocity on circular orbit. Therefore, from Eq. (12.44): $v_0^2 = \frac{GM}{r_0}$

$$h^2 = r_0^2\beta^2\left(\frac{GM}{r_0}\right) = \beta^2 r_0 GM = \beta^2(1+\alpha)RGM$$

Substituting for r_0, r_1, and h^2 in Eq. (1):

$$\frac{1}{(1+\alpha)R}+\frac{1}{R}=\frac{2GM}{\beta^2(1+\alpha)RGM}$$

Multiplying all terms by $(1+\alpha)R$:

$$1+1+\alpha=\frac{2}{\beta^2} \qquad \beta=\sqrt{\frac{2}{2+\alpha}}$$ ◀

12.100 Velocity of approach

Since approach trajectory is parabolic, we use Eq. (12.43):

$$v_0 = v_{esc} = \sqrt{\frac{2GM_{Venus}}{r_A}}$$

But $(GM)_{Venus} = 0.82\,(GM)_{Earth}$

and $(GM)_{Earth} = gR^2 = (9.81\text{ m/s}^2)(6.37\times10^6\text{ m})^2 = 398\times10^{12}\text{ m}^3/\text{s}^2$

Thus: $$v_0 = \sqrt{\frac{2(0.82)(398\times10^{12}\text{ m}^3/\text{s}^2)}{15\times10^6\text{ m}}} = 6596.6\text{ m/s}$$

Velocity on circular orbit of radius $r_C = 9000\text{ km}$

Using Eq. (12.35):

$$v_{circ} = \sqrt{\frac{GM_{Venus}}{r_C}} = \sqrt{\frac{0.82(398\times10^{12}\text{ m}^3/\text{s}^2)}{9\times10^6\text{ m}}}$$

$v_{circ} = 6021.8\text{ m/s}$ (CONTINUED)

12.100 CONTINUED First transfer orbit

We use equation derived in Prob. 12.96 with $r_0 = r_A$ and $r_1 = r_B$:

$$\frac{2GM}{h^2}=\frac{1}{r_A}+\frac{1}{r_B}=\frac{1}{15\times10^6\text{ m}}+\frac{1}{300\times10^6\text{ m}} = 70\times10^{-9}$$

$$h^2 = \frac{2GM}{70\times10^{-9}} = \frac{2(0.82)(398\times10^{12})}{70\times10^{-9}},\quad h = 96.564\times10^9$$

Since $h = r_A v_A$: $v_A = \frac{h}{r_A} = \frac{96.564\times10^9}{15\times10^6} = 6437.6\text{ m/s}$

Since $h = r_B v_B$: $v_B = \frac{h}{r_B} = \frac{96.564\times10^9}{300\times10^6} = 321.88\text{ m/s}$

(a) Speed reduction at A:

$\Delta v_A = v_A - v_0 = 6437.6\text{ m/s} - 6596.6\text{ m/s}$

$\Delta v_A = -159\text{ m/s}$ ◀

Second transfer orbit

We use equation of Prob. 12.96 with $r_0 = r_B$ and $r_1 = r_C$:

$$\frac{2GM}{h^2}=\frac{1}{r_B}+\frac{1}{r_C}=\frac{1}{300\times10^6\text{ m}}+\frac{1}{9\times10^6\text{ m}} = 114.44\times10^{-9}$$

$$h^2 = \frac{2GM}{114.44\times10^{-9}} = \frac{2(0.82)(398\times10^{12})}{114.44\times10^{-9}} \qquad h = 75.521\times10^9$$

Since $h = r_B v_B'$: $v_B' = \frac{h}{r_B} = \frac{75.521\times10^9}{300\times10^6} = 251.74\text{ m/s}$

Since $h = r_C v_C$: $v_C = \frac{h}{r_C} = \frac{75.521\times10^9}{9\times10^6} = 8391.2\text{ m/s}$

(b) Speed reduction at B:

$\Delta v_B = v_B' - v_B = 251.74\text{ m/s} - 321.88\text{ m/s} = -70.1\text{ m/s}$ ◀

(c) Speed reduction at C:

$\Delta v_C = v_{circ} - v_C = 6021.8\text{ m/s} - 8391.2\text{ m/s} = -2370\text{ m/s}$ ◀

12.101 First transfer orbit

We use equation derived in Prob. 12.96 with $r_0 = r_A$, $r_1 = r_B$:

$$\frac{1}{r_A}+\frac{1}{r_B}=\frac{2GM}{h^2} \qquad (1)$$

With $r_A = 15\times10^6\text{ m}$ $\quad v_A = 6500\text{ m/s}$

$h = r_A v_A = (15\times10^6)(6500) = 97.5\times10^9\text{ m}^2/\text{s}$

See solution of Prob. 12.100 for computation of GM:

$$(GM)_{Venus} = 0.82(398\times10^{12}\text{ m}^3/\text{s}^2)$$

Substituting into Eq. (1):

$$\frac{1}{15\times10^6}+\frac{1}{r_B}=\frac{2(0.82)(398\times10^{12})}{(97.5\times10^9)^2} = 68.6622\times10^{-9}$$

$$\frac{1}{r_B} = 68.6622\times10^{-9} - 66.6667\times10^{-9} = 1.9955\times10^{-9}$$

(a) $r_B = 501.1\times10^6\text{ m}$ $\quad r_B = 501\times10^3\text{ km}$ ◀

Since $h = r_B v_B$: $v_B = \frac{h}{r_B} = \frac{97.5\times10^9}{501.1\times10^6} = 194.57\text{ m/s}$

(CONTINUED)

12.101 CONTINUED Second transfer orbit

We use Eq. of Prob. 12.96 with $r_0 = r_B$ and $r_1 = r_C$:

$$\frac{2GM}{h^2} = \frac{1}{r_B} + \frac{1}{r_C} = 1.9955\times10^{-9} + \frac{1}{9\times10^6\text{ m}} = 113.11\times10^{-9}$$

$$h^2 = \frac{2GM}{113.11\times10^{-9}} = \frac{2(0.82)(398\times10^{12})}{113.11\times10^{-9}}, \quad h = 75.966\times10^9$$

Since $h = r_B v_B'$: $v_B' = \frac{h}{r_B} = \frac{75.966\times10^9}{501.1\times10^6} = 151.60\text{ m/s}$

Since $h = r_C v_C$: $v_C = \frac{h}{r_C} = \frac{75.966\times10^9}{9\times10^6} = 8440.7\text{ m/s}$

(b) Speed reduction at B:

$$\Delta v_B = v_B' - v_B = 151.60\text{ m/s} - 194.57\text{ m/s} = -43.0\text{ m/s}$$ ◀

(c) Speed reduction at C:

See solution of Prob. 12.100 for determination of velocity on circular orbit:

$$v_{circ} = 6021.8\text{ m/s}$$

$$\Delta v_C = v_{circ} - v_C = 6021.8\text{ m/s} - 8440.7\text{ m/s} = -2420\text{ m/s}$$ ◀

12.102 From solution of Prob. 12.100, for first transfer orbit:

$$h = 96.564\times10^9$$

$$a = \tfrac{1}{2}(r_A + r_B) = \tfrac{1}{2}(15\times10^6\text{ m} + 300\times10^6\text{ m}) = 157.5\times10^6\text{ m}$$

$$b = \sqrt{r_A r_B} = \sqrt{15\times300}\,(10^6\text{ m}) = 67.082\times10^6\text{ m}$$

Using Eq. (12.45):

$$\text{Time from A to B} = \tfrac{1}{2}\tau = \frac{\pi ab}{h} = \frac{\pi(157.5\times10^6)(67.082\times10^6)}{96.564\times10^9}$$

$$= 343.73\times10^3\text{ s} = 3\text{ days } 23\text{ h } 29\text{ min}$$ ◀

12.103 From solution of Prob. 12.100 for second transfer orbit:

$$h = 75.521\times10^9$$

$$a = \tfrac{1}{2}(r_B + r_C) = \tfrac{1}{2}(300\times10^6\text{ m} + 9\times10^6\text{ m}) = 154.5\times10^6\text{ m}$$

$$b = \sqrt{r_B r_C} = \sqrt{300\times9}\,(10^6\text{ m}) = 51.9615\times10^6\text{ m}$$

Using Eq. (12.45):

$$\text{Time from B to C} = \tfrac{1}{2}\tau = \frac{\pi ab}{h} = \frac{\pi(154.5\times10^6)(51.9615\times10^6)}{75.521\times10^9}$$

$$= 333.96\times10^3\text{ s} = 3\text{ days } 20\text{ h } 46\text{ min}$$ ◀

12.104 (a) From solution of Prob. 12.97 for circular orbit:

$$v_{circ} = 25.1950\times10^3\text{ ft/s}, \quad r_1 = 22.176\times10^6\text{ ft}$$

$$\tau = \frac{2\pi r_1}{v_{circ}} = \frac{2\pi(22.176\times10^6\text{ ft})}{25.1950\times10^3\text{ ft/s}} = 5530\text{ s}$$

$$\tau = 1\text{ h } 32\text{ min } 10\text{ s}$$ ◀

(b) From solution of Prob. 12.97 for elliptic orbit:

$$h = 551.869\times10^9$$

$$a = \tfrac{1}{2}(r_0 + r_1) = \tfrac{1}{2}(21.120 + 22.176)10^6\text{ ft} = 21.648\times10^6\text{ ft}$$

$$b = \sqrt{r_0 r_1} = \sqrt{21.120\times22.176}\,(10^6\text{ ft}) = 21.642\times10^6\text{ ft}$$

Using Eq. (12.45):

$$\text{Time from A to B} = \tfrac{1}{2}\tau = \frac{\pi ab}{h} = \frac{\pi(21.648\times10^6)(21.642\times10^6)}{551.869\times10^9}$$

$$= 2667\text{ s} = 44\text{ min } 27\text{ s}$$ ◀

12.105 We apply Kepler's third law to the orbits and periodic times of Earth and Halley's comet:

$$\left(\frac{\tau_H}{\tau_E}\right)^2 = \left(\frac{a_H}{a_E}\right)^3$$

Thus $a_H = a_E\left(\frac{\tau_H}{\tau_E}\right)^{2/3} = r_E\left(\frac{76\text{ years}}{1\text{ year}}\right)^{2/3} = 17.94\,r_E$

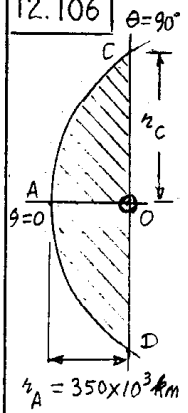
But $a_H = \tfrac{1}{2}(r_{min} + r_{max})$

$$17.94\,r_E = \tfrac{1}{2}\left(\tfrac{1}{2}r_E + r_{max}\right)$$

$$r_{max} = 2(17.94\,r_E) - \tfrac{1}{2}r_E = (35.88 - 0.5)r_E = 35.38\,r_E$$

$$r_{max} = 35.38(92.9\times10^6\text{ mi}) = 3.29\times10^9\text{ mi}$$ ◀

12.106



For parabolic trajectory ($\varepsilon = 1$), Eq. (12.39') yields

$$\frac{1}{r} = \frac{GM}{h^2}(1 + \cos\theta)$$

For $\theta = 0$: $\frac{1}{r_A} = \frac{GM}{h^2}(1+1) = \frac{2GM}{h^2}$

For $\theta = 90°$: $\frac{1}{r_C} = \frac{GM}{h^2}(1+0) = \frac{GM}{h^2}$

Thus: $r_C = 2r_A$

Shaded area $= A = \frac{2}{3}r_A(2r_C)$

$$A = \frac{2}{3}r_A(4r_A) = \frac{8}{3}r_A^2$$

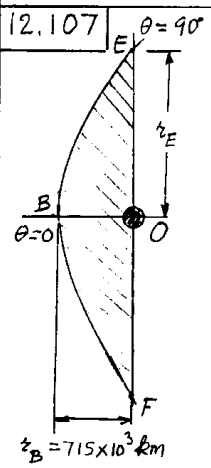
Areal velocity $= \frac{dA}{dt} = \frac{1}{2}h = \frac{1}{2}r_A v_A$

Time to travel from C to D $= t_{CD} = \frac{A}{dA/dt}$

$$t_{CD} = \frac{\frac{8}{3}r_A^2}{\frac{1}{2}r_A v_A} = \frac{16}{3}\frac{r_A}{v_A} = \frac{16}{3}\frac{350\times10^3\text{ km}}{26.9\text{ km/s}} = 69\,393\text{ s}$$

$$t_{CD} = 19\text{ h } 17\text{ min}$$ ◀

12.107



For parabolic trajectory ($\varepsilon = 1$), Eq. (12.39') yields

$$\frac{1}{r} = \frac{GM}{h^2}(1 + \cos\theta)$$

For $\theta = 0$: $\frac{1}{r_B} = \frac{GM}{h^2}(1+1) = \frac{2GM}{h^2}$

For $\theta = 90°$: $\frac{1}{r_E} = \frac{GM}{h^2}(1+0) = \frac{GM}{h^2}$

Thus: $r_E = 2r_B$

Shaded area $= A = \frac{2}{3}r_B(2r_E)$

$$A = \frac{2}{3}r_B(4r_B) = \frac{8}{3}r_B^2$$

Areal velocity $= \frac{dA}{dt} = \frac{1}{2}h = \frac{1}{2}r_B v_B$

Time to travel from E to F:

$$t_{EF} = \frac{A}{dA/dt} = \frac{\frac{8}{3}r_B^2}{\frac{1}{2}r_B v_B} = \frac{16}{3}\frac{r_B}{v_B}$$

We recall from the solution of Prob. 12.94 that

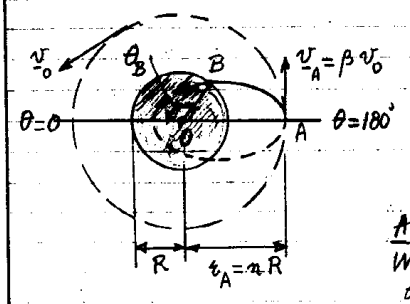
$$v_B = v_{max} = v_2 = 18.82\text{ km/s}$$

Thus:

$$t_{EF} = \frac{16}{3}\frac{715\times10^3\text{ km}}{18.82\text{ km/s}} = 202.62\times10^3\text{ s} = 2\text{ days } 8\text{ h } 17\text{ min}$$ ◀

MAKRON Books
COPIED
Marketing Universitário
(0xx11) 3645-3105
(0xx11) 3642-6729
(0xx11) 3645-6095

12.108



Circular orbit:
Eq. (12.44) with $r = nR$

$$v_0 = \sqrt{\frac{GM}{nR}} \quad (1)$$

Elliptic crash trajectory:
Eq. (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \quad (2)$$

At point A:
We have $\theta = 180°$, $r = nR$

Substituting in (2):

$$\frac{1}{nR} = \frac{GM}{h^2} - C \quad \text{or} \quad C = \frac{GM}{h^2} - \frac{1}{nR} \quad (3)$$

At point B: We have $\theta = \theta_B$ $r = R$. Substituting in (2):

$$\frac{1}{R} = \frac{GM}{h^2} + C\cos\theta_B \quad \text{or} \quad C\cos\theta_B = \frac{1}{R} - \frac{GM}{h^2} \quad (4)$$

Dividing (4) by (3), member by member:

$$\cos\theta_B = \frac{\frac{1}{R} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{nR}} \quad (5)$$

We note that

$$h = r_A v_A = (nR)(\beta v_0), \qquad h^2 = n^2R^2\beta^2 v_0^2$$

Substituting for v_0 from Eq. (1):

$$h^2 = n^2R^2\beta^2 \frac{GM}{nR} = nR\beta^2 GM$$

and

$$\frac{GM}{h^2} = \frac{GM}{nR\beta^2 GM} = \frac{1}{nR\beta^2} \quad (6)$$

Substituting from (6) into (5):

$$\cos\theta_B = \frac{\frac{1}{R} - \frac{1}{nR\beta^2}}{\frac{1}{nR\beta^2} - \frac{1}{nR}} = \frac{n\beta^2 - 1}{1 - \beta^2}$$

Since $\sphericalangle(AOB) = 180° - \theta_B$, we have $\cos(AOB) = -\cos\theta_B$

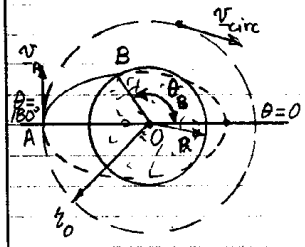
and

$$\cos(AOB) = \frac{1 - n\beta^2}{1 - \beta^2} \quad \blacktriangleleft$$

which shows that $\sphericalangle(AOB)$ depends only upon n and β.

12.109

Circular orbit of command module



We use Eq. (12.44):

$$v_{circ} = \sqrt{\frac{GM}{r_0}} \quad (1)$$

with $(GM)_{Moon} = 0.01230(GM)_{Earth}$

But $(GM)_{Earth} = gR^2$

$= (9.81\ m/s^2)(6.37\times10^6\ m)^2$

$= 398\times10^{12}\ m^3/s^2$

Thus:

$$v_{circ} = \sqrt{\frac{0.01230(398\times10^{12})}{r_0}} \quad (2)$$

But $r_0 = 1740\ km + 120\ km = 1860\ km = 1.860\times10^6\ m$

Substituting for r_0 in (2), we find

$v_{Command} = v_{circ} = 1622.3\ m/s$ (CONTINUED)

12.109 CONTINUED

Elliptic trajectory of LEM

We use Eq. (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta$$

At A (apogee), $\theta = 180°$ and $r = r_0$:

$$\frac{1}{r_0} = \frac{GM}{h^2} - C \quad \text{or} \quad C = \frac{GM}{h^2} - \frac{1}{r_0} \quad (3)$$

At B, $\theta = \theta_B$ and $r = R$:

$$\frac{1}{R} = \frac{GM}{h^2} + C\cos\theta_B \quad \text{or} \quad C\cos\theta_B = \frac{1}{R} - \frac{GM}{h^2} \quad (4)$$

Dividing (4) by (3), member by member:

$$\cos\theta_B = \frac{\frac{1}{R} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{r_0}} \quad (5)$$

But $h = r_0 v_A$ $\qquad h^2 = r_0^2 v_A^2$

and from (1): $\qquad GM = r_0 v_{circ}^2$

Thus:

$$\frac{GM}{h^2} = \frac{r_0 v_{circ}^2}{r_0^2 v_A^2} = \frac{1}{r_0}\left(\frac{v_{circ}}{v_A}\right)^2$$

Substituting into (5) and multiplying numerator and denominator by r_0, we have

$$\cos\theta_B = \frac{\frac{r_0}{R} - \left(\frac{v_{circ}}{v_A}\right)^2}{\left(\frac{v_{circ}}{v_A}\right)^2 - 1} \quad (6)$$

Given: $\sphericalangle AOB = 50°$, $\theta_B = 180° - 50° = 130°$, $\cos\theta_B = -0.64279$

$r_0 = 1860\ km$, $R = 1740\ km$, $\frac{r_0}{R} = 1.06897$

Substituting into Eq. (6):

$$-0.64279 = \frac{1.06897 - \left(\frac{v_{circ}}{v_A}\right)^2}{\left(\frac{v_{circ}}{v_A}\right)^2 - 1}$$

$$\left(\frac{v_{circ}}{v_A}\right)^2(1 - 0.64279) = 1.06897 - 0.64279 \qquad \left(\frac{v_{circ}}{v_A}\right)^2 = 1.1931$$

$$v_A = 0.91551\, v_{circ}$$

Recalling the value found for v_{circ}:

$v_{LEM} = v_A = 0.91551(1622.3\ m/s) = 1485.2\ m/s$

$v_{LEM/Command} = 1485.2 - 1622.3 = -137.1\ m/s$ $\blacktriangleleft$

Note: Minus sign means that LEM is coast toward the rear.

12.110

See solution of Prob. 12.109 for derivation of Eqs. (2) and (6).

Given: $R = 1740\ km$

$r_0 = 1740\ km + 150\ km = 1890\ km = 1.89\times10^6\ m$

$$\frac{r_0}{R} = \frac{1890}{1740} = 1.08621$$

From Eq. (2):

$$v_{circ} = \sqrt{\frac{0.01230(398\times10^{12})}{1.89\times10^6}} = 1609.4\ m/s$$

$v_A = v_{LEM} = 1609.4\ m/s - 80\ m/s = 1529.4\ m/s$

$$\left(\frac{v_{circ}}{v_A}\right)^2 = \left(\frac{1609.4\ m/s}{1529.4\ m/s}\right)^2 = 1.10735$$

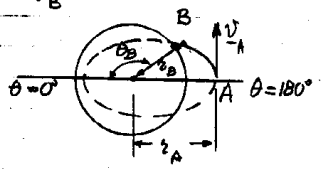
Substituting for r_0/R and $(v_{circ}/v_A)^2$ into Eq. (6):

$$\cos\theta_B = \frac{1.08621 - 1.10735}{1.10735 - 1} = -0.19693, \quad \theta_B = 101.4°$$

$\sphericalangle AOB = 180° - \theta_B = 180° - 101.4°$ $\quad \sphericalangle AOB = 78.6°$ $\blacktriangleleft$

12.111

$\theta_B = 180° - 50° = 130°$



From Eq. (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \quad (1)$$

At A: $\theta = 180°$, $r = r_A$:

$$\frac{1}{r_A} = \frac{GM}{h^2} - C$$

or $$C = \frac{GM}{h^2} - \frac{1}{r_A} \quad (2)$$

At B: $\theta = \theta_B$, $r = r_B$

$$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B \quad \text{or} \quad C\cos\theta_B = \frac{1}{r_B} - \frac{GM}{h^2} \quad (3)$$

Dividing (3) by (2), member by member:

$$\cos\theta_B = \frac{\frac{1}{r_B} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{r_A}} \quad (4)$$

We have $h = r_A v_A$ or, since $v_A = \frac{94}{100} v_{circ}$,

$$h = r_A(0.94 v_{circ}) \qquad h^2 = 0.8836\, r_A^2 v_{circ}^2$$

But, from Eq. (12.35), $v_{circ}^2 = \frac{GM}{r_A}$ Thus

$$h^2 = 0.8836\, r_A^2\left(\frac{GM}{r_A}\right) = 0.8836\, r_A GM$$

and $$\frac{GM}{h^2} = \frac{GM}{0.8836 r_A GM} = \frac{1}{0.8836 r_A}$$

Substituting into (4) and multiplying numerator and denominator by $0.8836 r_A$, and recalling that $\theta_B = 130°$:

$$\cos 130° = \frac{0.8836(r_A/r_B) - 1}{1 - 0.8836}$$

$$0.8836\frac{r_A}{r_B} = 1 + (1 - 0.8836)\cos 130° = 0.92518$$

$$r_B = \frac{0.8836}{0.92518} r_A = \frac{0.8836}{0.92518}(3960 + 200) = 3973.0 \text{ mi}$$

Altitude of B $= 3973.0$ mi $- 3960$ mi $= 13.0$ mi ◀

12.112 At apogee or perigee the normal to the orbit passes through the center of force O. Thus, at each of these points, we have

$$F_n = F = \frac{GMm}{r^2}$$

Also, $$a_n = \frac{v^2}{\rho}$$

Newton's second law:

$$F_n = m a_n: \quad \frac{GMm}{r^2} = m\frac{v^2}{\rho}$$

or $$\frac{1}{\rho} = \frac{GM}{r^2 v^2} = \frac{GM}{h^2} \quad (1)$$

We recall from Prob. 12.97 that

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad (2)$$

Comparing Eqs. (1) and (2), we conclude that at apogee or perigee, we have

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$$

(Q.E.D.) ◀

12.113 (a) We use Eq. (12.39'):

$$\frac{1}{r} = \frac{GM}{h^2}(1 + \varepsilon\cos\theta)$$

For $\theta = 0$, $r = r_0$:

$$\frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon) \quad (1)$$

For $\theta = 180°$, $r = r_1$:

$$\frac{1}{r_1} = \frac{GM}{h^2}(1 - \varepsilon) \quad (2)$$

Divide (1) by (2), member by member:

$$\frac{r_1}{r_0} = \frac{1+\varepsilon}{1-\varepsilon}$$

$$r_1 - r_1\varepsilon = r_0 + r_0\varepsilon$$

$$\varepsilon = \frac{r_1 - r_0}{r_1 + r_0} \quad (3)$$ ◀

Note: This equation may also be obtained by recalling from geometry that $\varepsilon = c/a$, where

c = semi-focal distance $= \frac{1}{2}(r_1 - r_0)$

Thus: $$\varepsilon = \frac{c}{a} = \frac{\frac{1}{2}(r_1 - r_0)}{\frac{1}{2}(r_1 + r_0)} = \frac{r_1 - r_0}{r_1 + r_0}$$

(b) From solution of Prob. 12.100 and Eq. (3):

First transfer orbit:

$r_0 = r_A = 15 \times 10^3$ km, $r_1 = r_B = 300 \times 10^3$ km

$$\varepsilon_1 = \frac{300 - 15}{300 + 15} \qquad \varepsilon_1 = 0.905$$ ◀

Second transfer orbit:

$r_0 = r_C = 9 \times 10^3$ km, $r_1 = r_B = 300 \times 10^3$ km

$$\varepsilon_2 = \frac{300 - 9}{300 + 9} \qquad \varepsilon_2 = 0.942$$ ◀

12.114 See solution of Prob. 12.96 for derivation of

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$$

from Eq. (12.39'). We may also write this equation as

$$\frac{r_0 + r_1}{r_0 r_1} = \frac{2GM}{h^2}$$

Recalling from Eq. (12.46) that $r_0 + r_1 = 2a$ and from Eq. (12.47) that $r_0 r_1 = b^2$ we have

$$\frac{2a}{b^2} = \frac{2GM}{h^2} \quad \text{or} \quad h^2 = \frac{GMb^2}{a}$$

From Eq. (12.45):

$$\tau = \frac{2\pi ab}{h}$$

$$\tau^2 = \frac{4\pi^2 a^2 b^2}{h^2} = \frac{4\pi^2 a^2 b^2}{GMb^2/a} = \frac{4\pi^2 a^3}{GM}$$

For all planets, GM has the same value, since M represents the mass of the sun and G is a universal constant. Therefore

$$\tau^2 \sim a^3 \qquad \text{(Q.E.D.)}$$ ◀

12.115 We recall Eq. (12.39'):

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

For $\theta = 0$, $r = r_0$:

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \quad \text{or} \quad r_0 = \frac{h^2}{GM}\frac{1}{1+\varepsilon} \qquad (1)$$

For $\theta = 180°$, $r = r_1$:

$$\frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon) \quad \text{or} \quad r_1 = \frac{h^2}{GM}\frac{1}{1-\varepsilon} \qquad (2)$$

Recalling the definition of "a" given in Eq. (12.46) and using (1) and (2):

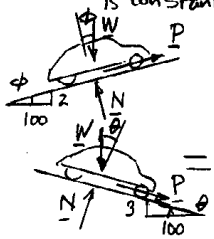
$$a = \frac{1}{2}(r_0 + r_1) = \frac{1}{2}\frac{h^2}{GM}\left(\frac{1}{1+\varepsilon} + \frac{1}{1-\varepsilon}\right) = \frac{1}{2}\frac{h^2}{GM}\frac{1-\varepsilon+1+\varepsilon}{(1+\varepsilon)(1-\varepsilon)}$$

$$a = \frac{1}{2}\frac{h^2}{GM}\frac{2}{1-\varepsilon^2} \quad \text{or} \quad a = \frac{h^2}{GM}\frac{1}{1-\varepsilon^2}$$

Solving for h:

$$h = \sqrt{GMa(1-\varepsilon^2)} \qquad \text{(Q.E.D.)}$$

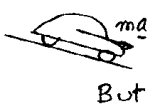
12.116 Going up: Car in static equilibrium, since speed is constant.



$+\nearrow \Sigma F_x = 0$: $P - W\sin\Phi = 0$

$P = W\sin\Phi \approx W\tan\Phi = 0.02\,W \quad (1)$

Going down:



$+\searrow \Sigma F_x = ma$

$P + W\sin\theta = ma$

But $\sin\theta \approx \tan\theta = 0.03$

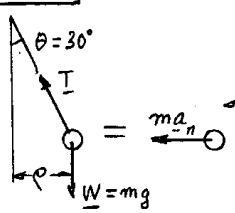
Thus: $P + 0.03\,W = ma$

Substitute for P from (1):

$0.02W + 0.03W = ma$

$$a = 0.05\frac{W}{m} = 0.05g = 0.05(9.81\ \text{m/s}^2) = 0.491\ \text{m/s}^2$$

12.117 Initial velocity of drop = velocity of bucket



$\Sigma F_y = 0$: $T\cos 30° = mg \quad (1)$

$+\swarrow \Sigma F_n = ma_n$: $T\sin 30° = ma_n \quad (2)$

Divide (2) by (1):

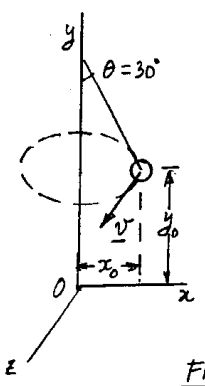
$$\tan 30° = \frac{a_n}{g} = \frac{v^2/\rho}{g}$$

Thus: $v^2 = \rho g \tan 30°$

But: $\rho = L\sin 30° = (1.2\text{m})\sin 30° = 0.6\text{m}$

Thus: $v^2 = 0.6(9.81)\tan 30° = 3.398$

$v = 1.843$ m/s



Assuming the bucket to rotate clockwise (when viewed from above), and using the axes shown, we find that the components of the initial velocity of the drop are

$(v_0)_x = 0$, $(v_0)_y = 0$, $(v_0)_z = 1.843$ m/s

Free fall of drop

$$y = y_0 + (v_0)_y t - \tfrac{1}{2}gt^2 \qquad y = y_0 - \tfrac{1}{2}gt^2$$

When drop strikes floor: $y = 0$ $\quad y_0 - \frac{1}{2}gt^2 = 0$

(CONTINUED)

12.117 CONTINUED

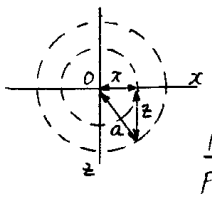
But $y_0 = 2L - L\cos 30° = 2(1.2) - 1.2\cos 30° = 1.361$ m

Thus: $1.361 - \frac{1}{2}(9.81)t^2 = 0$ $\quad t = 0.527$s

Projection on horizontal floor (uniform motion)

$x = x_0 + (v_0)_x t = L\sin 30° + 0$, $\quad x = 0.6$m

$z = z_0 + (v_0)_z t = 0 + 1.843(0.527) = 0.971$m



Radius of circle: $a = \sqrt{x^2 + z^2}$

$a = \sqrt{(0.6)^2 + (0.971)^2}$

$a = 1.141$ m

Note: The drop travels in a vertical plane parallel to the yz plane.

12.118 From solution of Prob. 11.157 we conclude that

$$a_r = -4b\cos\theta\,\dot\theta_0^2 \qquad a_\theta = -4b\sin\theta\,\dot\theta_0^2$$

(a) $F_r = ma_r = -4mb\,\dot\theta_0^2\cos\theta$

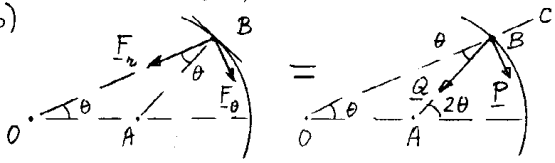
$F_\theta = ma_\theta = -4mb\,\dot\theta_0^2\sin\theta$

Given Data: $m = \frac{(4/16)}{32.2} = \frac{0.25}{32.2}$, $b = \frac{18}{12}\text{ft} = 1.5\text{ft}$, $\dot\theta_0 = 12$ rad/s

$F_r = -4\left(\frac{0.25}{32.2}\right)(1.5)(12)^2\cos\theta$, $\quad F_r = -(6.71\text{ lb})\cos\theta$

$F_\theta = -4\left(\frac{0.25}{32.2}\right)(1.5)(12)^2\sin\theta$, $\quad F_\theta = -(6.71\text{ lb})\sin\theta$

(b)



Project along BO: $F_r = Q\cos\theta$

$6.71\cos\theta = Q\cos\theta$ $\quad Q = 6.71\text{ lb} \measuredangle 2\theta$

Project along perpendicular to BO: $F_\theta = P + Q\sin\theta$

$P = F_\theta - Q\sin\theta = 6.71\sin\theta - 6.71\sin\theta$

$P = 0$

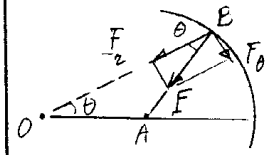
We note that $P \neq F_\theta$. This is because F_r and F_θ are rectangular components of the force F exerted on pin B, while Q and P are oblique components of F.

Remark. The same results could have been obtained by recalling from the solution of Prob. 11.157 that the acceleration a of pin B is directed toward A and has the magnitude

$$a = 4b\,\dot\theta_0^2 = 4(1.5\text{ ft})(12\text{ rad/s})^2 = 864\text{ ft/s}^2$$

Thus the force F exerted on B is also directed toward A and has the magnitude

$$F = ma = \frac{4/16}{32.2}(864) = 6.71\text{ lb}$$



It follows that

$F_r = -(6.71\text{ lb})\cos\theta$, $F_\theta = -(6.71\text{ lb})\sin\theta$

and also that

$Q = F = 6.71\text{ lb} \measuredangle 2\theta$ and $P = 0$

12.119 The smallest distance r_2 corresponds to a force F in the string equal to the 6-lb breaking strength. Thus

$$F = ma_n: \quad 6\text{ lb} = \frac{5/16}{32.2}\frac{v_2^2}{r_2}$$

$$v_2^2 = 618.24\, r_2 \qquad (1)$$

As the string is pulled through the hole at O, the only force acting in the horizontal plane is the force F exerted by the string on the ball. Thus, we have a central force and may use Eq. (12.18):

$$r_2^2\dot{\theta}_2 = r_1^2\dot{\theta}_1 \quad \text{or} \quad r_2 v_2 = r_1 v_1$$

$$v_2 = \frac{r_1 v_1}{r_2} = \frac{(2\text{ ft})(4\text{ ft/s})}{r_2} \quad \text{or} \quad v_2 = \frac{8}{r_2} \qquad (2)$$

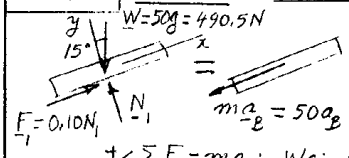
(a) Substitute for v_2 from (2) into (1):

$$\frac{64}{r_2^2} = 618.24\, r_2 \qquad r_2^3 = 0.10352$$

$$r_2 = 0.46954\text{ ft} = 5.63\text{ in.} \blacktriangleleft$$

(b) From (2): $v_2 = \frac{8}{0.46954}$ $\qquad v_2 = 17.04\text{ ft/s} \blacktriangleleft$

12.120 Part (a) Plate B: $\mu_1 = \tan\phi = 0.10$



$\phi = 5.71° < 15°$

Plate B will slide

$\Sigma F_y = 0$: $N_1 = 490.5\cos 15°$

$N_1 = 473.8\text{ N}$

$+\nearrow \Sigma F_x = ma_x$: $W\sin 15° - F_1 = ma_B$

$490.5\sin 15° - 0.10(473.8) = 50a_B$

$a_B = 1.591\text{ m/s}^2\ \nearrow \blacktriangleleft$

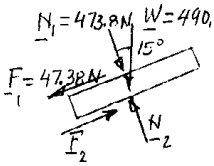


Plate A: Assume no sliding

$+\nearrow \Sigma F_x = 0$: $F_2 - F_1 - W\sin 15° = 0$

$F_2 - 47.38 - 490.5\sin 15° = 0$

$F_2 = 174.33\text{ N}$

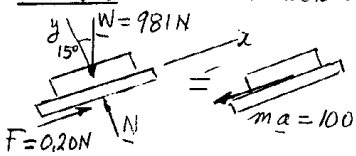
$+\nwarrow \Sigma F_y = 0$: $N_2 - N_1 - W\cos 15° = 0$

$N_2 - 473.8 - 490.5\cos 15° = 0, \quad N_2 = 947.6\text{ N}$

$(F_2)_{max} = \mu_2 N_2 = 0.20\, N_2 = 0.20(947.6\text{ N}) = 189.5\text{ N}$

Since F_2 (174.33 N) is smaller than $(F_2)_{max}$ (189.5 N), assumption of no sliding is correct and $a_A = 0$ $\blacktriangleleft$

Part (b) Plates are welded together.



$\mu = \tan\phi = 0.20$

$\phi = 11.31° < 15°$

Plates will slide

$+\nwarrow \Sigma F_y = 0$:

$N = 981\cos 15° = 947.6\text{ N}$

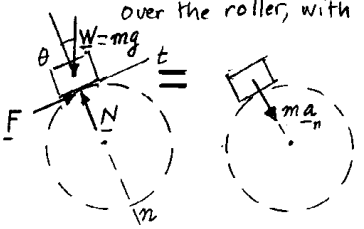
$+\nearrow \Sigma F_x = ma_x$: $W\sin 15° - F = ma$

$981\sin 15° - 0.20(947.6) = 100a$

$a = 0.6439\text{ m/s}^2$

$a_A = a_B = 0.644\text{ m/s}^2\ \nearrow \blacktriangleleft$

12.121 Consider the package in an arbitrary position over the roller, with $0 \le \theta \le 20°$.



Since v = constant, $a_t = 0$

$$a = a_n = \frac{v^2}{\rho} = \frac{v^2}{0.2\text{ m}}$$

For maximum v, slipping is impending:

$F = \mu N = 0.75N$

$+\searrow \Sigma F_t = ma_t$: $F - mg\sin\theta = 0$

$0.75N - mg\sin\theta = 0$

$N = \frac{4}{3}mg\sin\theta$

$+\searrow \Sigma F_n = ma_n$: $mg\cos\theta - N = ma_n$

$mg\cos\theta - \frac{4}{3}mg\sin\theta = m\frac{v^2}{0.2}$

$v^2 = 0.2g\left(\cos\theta - \frac{4}{3}\sin\theta\right)$

Let us determine the derivative of v^2:

$$\frac{d(v^2)}{d\theta} = -0.2g\left(\sin\theta + \frac{4}{3}\cos\theta\right)$$

We note that $d(v^2)/d\theta < 0$ over entire interval $0 \le \theta \le 20°$. Thus, v^2 decreases from $\theta = 0$ to $\theta = 20°$ and its smallest value occurs for $\theta = 20°$:

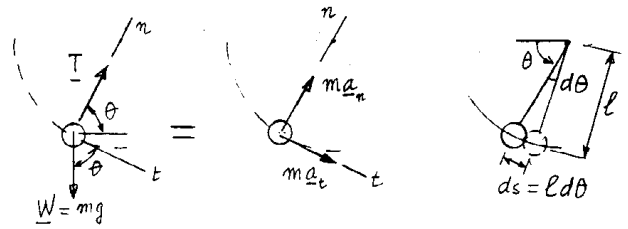
$v^2 = 0.2g\left(\cos 20° - \frac{4}{3}\sin 20°\right) = 0.9490$

$v = 0.974\text{ m/s}$

This smallest value is the value is the value which should not be exceeded. Thus, critical location is where belt starts curving over roller and

$v_{max} = 0.974\text{ m/s}$ $\blacktriangleleft$

12.122



(a) $+\searrow \Sigma F_t = ma_t$: $mg\cos\theta = ma_t$ $\qquad a_t = g\cos\theta$ $\blacktriangleleft$

$$v\frac{dv}{ds} = a_t = g\cos\theta$$

$$v\,dv = g\cos\theta\,(\ell\,d\theta)$$

$$\int_0^v v\,dv = g\ell\int_0^\theta \cos\theta\,d\theta$$

$$\frac{1}{2}v^2 = g\ell\sin\theta$$

$v^2 = 2g\ell\sin\theta$ $\qquad v = \sqrt{2g\ell\sin\theta}$ $\blacktriangleleft$

(b) $+\nwarrow \Sigma F_n = ma_n$: $T - mg\sin\theta = ma_n$

$T - mg\sin\theta = m\frac{v^2}{\ell}$

$T = mg\sin\theta + m\frac{2g\ell\sin\theta}{\ell} = 3mg\sin\theta$

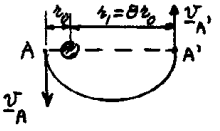
Making $T = 2W = 2mg$:

$2mg = 3mg\sin\theta, \quad \sin\theta = \frac{2}{3}, \quad \theta = 41.8°$ $\blacktriangleleft$

12.123 Station S_1: Circular orbit of radius r_0

By Eq. (12.44): $v_0 = v_{circ} = \sqrt{\frac{GM}{r_0}}$ (1)

Motion of Vehicle



See solution of Prob. 12.96 for derivation of equation

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$$

Making $r_1 = 8r_0$ and $h = r_0 v_A$:

$$\frac{1}{r_0} + \frac{1}{8r_0} = \frac{2GM}{r_0^2 v_A^2}, \quad \frac{9}{8r_0} = \frac{2GM}{r_0^3 v_A^2}, \quad v_A^2 = \frac{16GM}{9r_0}$$

Recalling Eq. (1): $v_A = \frac{4}{3}\sqrt{\frac{GM}{r_0}}$ $\quad v_A = \frac{4}{3}v_0$ (2)

(a) Launching velocity $= v_A - v_0 = \frac{4}{3}v_0 - v_0 = \frac{1}{3}v_0$ ◀

Expressing conservation of angular momentum for vehicle between A and A':

$$h = r_0 v_A = r_1 v_A' \quad \text{or} \quad r_0 v_A = 8r_0 v_A'$$

Thus: $v_A' = \frac{1}{8}v_A = \frac{1}{8}\frac{4}{3}v_0 = \frac{1}{6}v_0$

Station S_2: $v_2 = v_{circ} = \sqrt{\frac{GM}{r_1}} = \sqrt{\frac{GM}{8r_0}}$

Recalling Eq. (1): $v_2 = \frac{1}{\sqrt{8}}v_0 = 0.3536\,v_0$ (3)

Note: As the vehicle approaches station S_2, its velocity is $v_A' = \frac{1}{6}v_0$, while velocity of S_2 is $v_2 = 0.3536\,v_0$. Thus $v_A' < v_2$ and vehicle must increase its speed by $\Delta v = 0.3536\,v_0 - 0.1667v_0 = 0.1869v_0$ in order to dock.

(b) Time of vehicle travel

Using Eq. (12.45): $t = \frac{\tau}{2} = \frac{\pi ab}{h}$

where $a = \frac{1}{2}(r_0 + r_1) = \frac{1}{2}(r_0 + 8r_0) = 4.5r_0$

$b = \sqrt{r_0 r_1} = \sqrt{r_0(8r_0)} = r_0\sqrt{8}$

$h = r_0 v_A = r_0\left(\frac{4}{3}v_0\right) = \frac{4}{3}r_0 v_0$

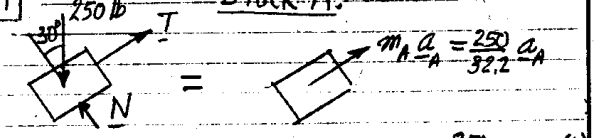
Thus: $t = \frac{\tau}{2} = \frac{\pi(4.5r_0)r_0\sqrt{8}}{\frac{4}{3}r_0 v_0} = \frac{27\sqrt{8}}{8}\frac{\pi r_0}{v_0}$ (4)

In the time t, S_2 will describe an arc of circle of length $v_2 t$. This arc subtends the angle $\theta = v_2 t / r_1$. Recalling Eqs. (3) and (4):

$$\theta = \frac{v_2 t}{r_1} = \frac{(v_0/\sqrt{8})}{8r_0}\left(\frac{27\sqrt{8}}{8}\frac{\pi r_0}{v_0}\right) = \frac{27}{64}\pi \text{ radians}$$

or $\theta = \frac{27}{64}(180°)$ $\quad \theta = 75.9°$ ◀

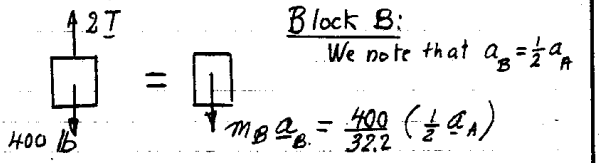
12.124 Block A:



$\rightarrow \Sigma F_x = m_A a_A$: $T - (250\text{ lb})\sin 30° = \frac{250}{32.2} a_A$ (1)

Block B:

We note that $a_B = \frac{1}{2}a_A$



$+\downarrow \Sigma F_y = m_B a_B$: $400\text{ lb} - 2T = \frac{400}{32.2}\left(\frac{1}{2}a_A\right)$ (2)

(a) Multiply (1) by 2 and add (2):

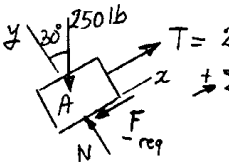
$$-2(250)\sin 30° + 400 = 2\frac{250}{32.2}a_A + \frac{400}{32.2}\left(\frac{1}{2}a_A\right)$$

$$150 = \frac{700}{32.2}a_A \qquad a_A = 6.90\text{ ft/s}^2 \nearrow$$ ◀

$$a_B = \frac{1}{2}a_A = \frac{1}{2}(6.90\text{ ft/s}^2), \quad a_B = 3.45\text{ ft/s}^2 \downarrow$$ ◀

(b) From (1): $T - 250\sin 30° = \frac{250}{32.2}(6.90)$ $\quad T = 178.6\text{ lb}$ ◀

12.125 We first determine whether the blocks move by computing the friction force required to maintain block A in equilibrium.



$T = 200\text{ lb}$ when B in equilibrium

$\rightarrow \Sigma F_x = 0$: $200 - 250\sin 30° - F_{req} = 0$

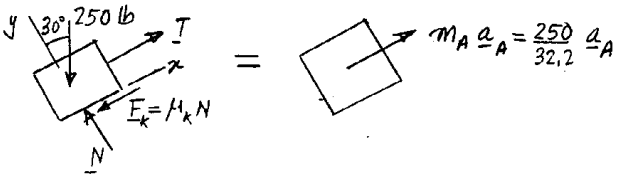
$F_{req} = 75.0\text{ lb}$

$+\uparrow \Sigma F_y = 0$: $N - 250\cos 30° = 0$ $\quad N = 216.5\text{ lb}$

$F_m = \mu_s N = 0.25(216.5\text{ lb}) = 54.13\text{ lb}$

Since $F_{req} > F_m$, blocks will move (A up and B down)

Block A



$+\nwarrow \Sigma F_y = 0$: $N - (250\text{ lb})\cos 30° = 0$ $\quad N = 216.5\text{ lb}$

$F_k = \mu_k N = 0.20(216.5\text{ lb}) = 43.30\text{ lb}$

$\rightarrow \Sigma F_x = m_A a_A$: $T - (250\text{ lb})\sin 30° - 43.30\text{ lb} = \frac{250}{32.2}a_A$ (1)

Block B: See solution of Prob. 12.124. We obtain

$$400\text{ lb} - 2T = \frac{400}{32.2}\left(\frac{1}{2}a_A\right) \quad (2)$$

(a) Multiply (1) by 2 and add (2):

$$-2(250)\sin 30° - 2(43.30) + 400 = 2\frac{250}{32.2}a_A + \frac{400}{32.2}\left(\frac{1}{2}a_A\right)$$

$63.4 = \frac{700}{32.2}a_A$, $\quad a_A = 2.916$, $\quad a_A = 2.92\text{ ft/s}^2 \nearrow$ ◀

$a_B = \frac{1}{2}a_A = \frac{1}{2}(2.916\text{ ft/s}^2)$, $\quad a_B = 1.458\text{ ft/s}^2 \downarrow$ ◀

(b) From (1): $T - 250\sin 30° - 43.3 = \frac{250}{32.2}(2.916)$, $\quad T = 190.9\text{ lb}$ ◀

12.126 At A: $r_0 = 3960\text{ mi} + 170\text{ mi} = 4130\text{ mi}$

$= (4130)(5280) = 21.8064 \times 10^6\text{ ft}$

$v_0 = 32.6 \times 10^3\text{ ft/s}$

$h = r_0 v_0 = (21.8064 \times 10^6\text{ ft})(32.6 \times 10^3\text{ ft/s}) = 710.89 \times 10^9\text{ ft}^2/\text{s}$

(a) Use equation derived in Prob. 12.96:

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \quad \text{or} \quad \frac{1}{r_0} + \frac{1}{r_1} = \frac{2gR^2}{h^2}$$

$$\frac{1}{21.8064 \times 10^6} + \frac{1}{r_1} = \frac{2(32.2)(3960 \times 5280)^2}{(710.89 \times 10^9)^2}$$

$$\frac{1}{r_1} = 55.711 \times 10^{-9} - 45.858 \times 10^{-9} = 9.853 \times 10^{-9}$$

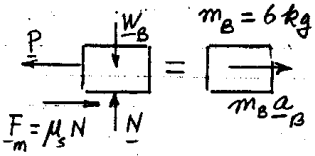
$$r_1 = 101.49 \times 10^6\text{ ft} = 19{,}222\text{ mi}$$

Max. altitude $= r_1 - R = 19{,}222 - 3960 = 15{,}260\text{ mi}$ ◀

(b) $r_1 v_1 = h$: $(101.49 \times 10^6\text{ ft})\, v_1 = 710.89 \times 10^9\text{ ft}^2/\text{s}$

$v_1 = 7.0045 \times 10^3\text{ ft/s}$ $\quad v_1 = 7000\text{ ft/s}$ ◀

12.127 Block B



For maximum P, sliding is impending and

$F_m = \mu_s N = \mu_s W_B$

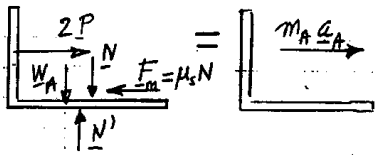
$= 0.30(6\text{ kg})(9.81\text{ m/s}^2)$

$F_m = 17.658\text{ N}$ (1)

$\pm \Sigma F_x = ma$: $\quad F_m - P = m_B a_B$

$P = F_m - m_B a_B$, (2)

Bracket A



$\pm \Sigma F_x = ma$: $\quad 2P - F_m = m_A a_A$ (3)

Substitute for P from (2) into (3):

$$2(F_m - m_B a_B) - F_m = m_A a_A$$

$$m_A a_A + 2 m_B a_B = F_m$$

Substituting for F_m from (1) and for m_A and m_B, and observing that $a_B = a_A$, since block does not slide,

$$10 a_A + 2(6) a_A = 17.658$$

$a_B = a_A = 0.80264$ $\quad \underline{a}_A = 0.803\text{ m/s}^2 \rightarrow$ ◀

From (2): $P = 17.658 - 6(0.80264)$

$= 12.84\text{ N}$ $\quad \underline{P} = 12.84\text{ N} \rightarrow$ ◀

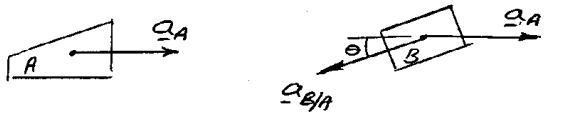
PROBLEM 12.C1

ASSUME BOTH BLOCK AND WEDGE MOVE. LET μ = COEFFICIENT OF FRICTION AT ALL SURFACE

$\theta = 30°$

KINEMATICS

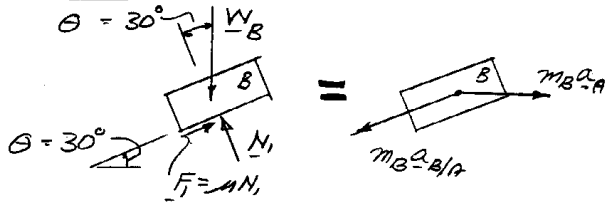
(SEE ALSO SAMPLE PROBLEM 12.4)



$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

$$\underline{a}_B = [a_A \rightarrow] + [a_{B/A} \measuredangle \theta]$$

KINETICS BLOCK B (ASSUME MOTION $\measuredangle \theta$)



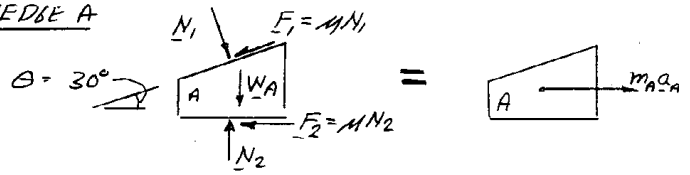
$$+\nwarrow^{30°} \Sigma F = m_B a_B: \quad N_1 - W_B \cos\theta = -m_B a_A \sin\theta$$

$$N_1 = W_B \cos\theta - \frac{W_B}{g} a_A \sin\theta \qquad (1)$$

$$+\angle_{30°} \Sigma F = m_B a_B: \quad -W_B \sin\theta + \mu N_1 = m_B a_A \cos\theta - m_B a_{B/A}$$

$$\frac{W_B}{g} a_{B/A} = +\frac{W_B}{g} a_A \cos\theta + W_B \sin\theta - \mu N_1 \qquad (2)$$

WEDGE A



$$\rightarrow \Sigma F_x = m_A a_x$$

$$N_1 \sin\theta - \mu N_1 \cos\theta - \mu N_2 = m_A a_A \qquad (3)$$

$$+\uparrow \Sigma F_y = m_A a_y$$

$$N_2 - N_1 \cos\theta - \mu N_1 \sin\theta - W_A = 0$$

$$N_2 = N_1(\cos\theta + \mu \sin\theta) + W_A = 0 \qquad (4)$$

SUBSTITUTE N_2 FROM (4) INTO (3):

$$N_1 \sin\theta - \mu N_1 \cos\theta - \mu N_1(\cos\theta + \mu \sin\theta) + W_A = m_A a_A$$

$$N_1 \left[\sin\theta - 2\mu\cos\theta - \mu^2 \sin\theta\right] - \mu W_A = \frac{W_A}{g} a_A$$

LET: BRACKET $= \left[\sin\theta - 2\mu\cos\theta - \mu^2\sin\theta\right]$ THUS WE WRITE $N_1[\text{BRACKET}] - \mu W_A = \frac{W_A}{g} a_A \qquad (5)$

SUBSTITUTE N_1 FROM (1) INTO (5): $\left(W_B \cos\theta - \frac{W_B}{g} a_A \sin\theta\right)[\text{BRACKET}] - \mu W_A = \frac{W_A}{g} a_A$

$$W_B \cos\theta\,[\text{BRACKET}] - \mu W_A = \frac{W}{g} a_A + \frac{W_B}{g} a_A \sin\theta\,[\text{BRACKET}]$$

SOLVE FOR a_A:

$$a_A = \frac{W_B \cos\theta\,[\text{BRACKET}] - \mu W_A}{W_A + W_B \sin\theta\,[\text{BRACKET}]} g \qquad \triangleleft$$

SUBSTITUTE N_1 FROM (1) INTO (2): $\frac{W_B}{g} a_{B/A} = \frac{W_B}{g} a_A \cos\theta + W_B \sin\theta - \mu\left(W_B \cos\theta - \frac{W_B}{g} a_A \sin\theta\right)$

DIVIDE BY W_B/g:

$$a_{B/A} = a_A(\cos\theta + \mu \sin\theta) + g(\sin\theta - \mu\cos\theta) \qquad (6) \quad \triangleleft$$

IF $a_A < 0$, THEN WEDGE DOES NOT MOVE AND $\triangleleft$

FROM (6) WITH $a_A = 0$, WE HAVE $a_{B/A} = g(\sin\theta - \mu\cos\theta)$ $\triangleleft$

IF $a_{B/A} < 0$, NEITHER WEDGE NOR BLOCK MOVES: NO MOTION $\triangleleft$

SEE NEXT PAGE FOR PROGRAMS AND RUNS

PROBLEM 12.C1 CONTINUED

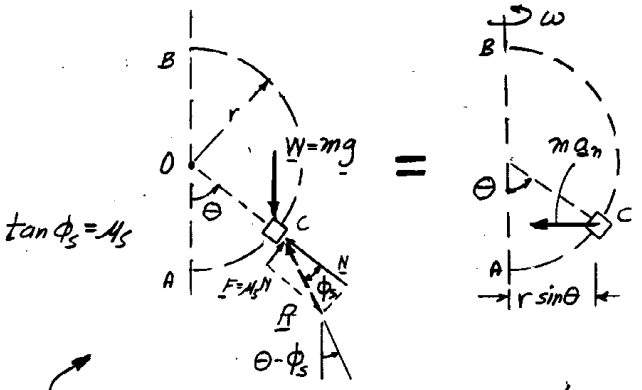
```
10 '*****************************************************************
20 '*'                        PROBLEM 12.C1                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '              Definition of Symbols
70 '
80 '         WA = weight of wedge   (pounds)
90 '         WB = weight of block   (pounds)
100 '        MU = coefficient of friction at all surfaces
110 '
120 '            (Accelerations are expressed in ft/s^2)
130 '        AA = acceleration of wedge A, positive to the right
140 '       ABA = acceleration of block with respect to wedge,
150 '             directed down along the top surface of the wedge
160 '         G = acceleration of gravity
170 '
180 '        TH = angle formed by wedge and horizontal (30 degrees)
190 '   BRACKET = function of MU shown in derivation
200 '
210    K = 4*ATN(1)/180
220 '
230    WA = 30:     WB = 12
240    G = 32.2:    TH = 30
250    PRINT "   MU          AA          ABA"
260    PRINT "             ft/s^2       ft/s^2":PRINT
270 '
280    FOR MU = 0 TO 1 STEP .01
290       BRACKET = (SIN(K*TH) -2*MU*COS(K*TH) -MU^2*SIN(K*TH))
300       AA = ((WB*COS(K*TH)*BRACKET - MU*WA)/
                                  (WA + WB*SIN(K*TH)*BRACKET))*G
310       ABA = AA*(COS(K*TH) + MU*SIN(K*TH)) + G*(SIN(K*TH)
                                                  - MU*COS(K*TH))
320       IF AA < 0 GOTO 350
330       PRINT USING "##.###      ";MU,AA,ABA
340    NEXT
350    PRINT: PRINT"     (Wedge does not move)":AA = 0
360 '
370         'Determine value of MU at smallest 0.1 interval which is
380         'greater that the last value of MU at which wedge moved.
390    MU = (INT(MU*10) + 1)/10
400    FOR MU =  MU TO 1 STEP .1
410       ABA  = G*(SIN(K*TH) - MU*COS(K*TH))
420       IF ABA < 0 GOTO 460
430       PRINT USING "##.###      ";MU,AA,ABA
440    NEXT
450 '
460    PRINT: PRINT "          (No Motion)": AA = 0: ABA = 0
470    PRINT USING "##.###      ";MU,AA,ABA
```

```
RUN
   MU          AA          ABA
               ft/s^2      ft/s^2

  0.000       5.070      20.491
  0.010       4.616      19.842
  0.020       4.158      19.185
  0.030       3.696      18.519
  0.040       3.230      17.846
  0.050       2.760      17.165
  0.060       2.286      16.475
  0.070       1.807      15.777
  0.080       1.325      15.070
  0.090       0.838      14.354
  0.100       0.347      13.629

      (Wedge does not move)
  0.200       0.000      10.523
  0.300       0.000       7.734
  0.400       0.000       4.946
  0.500       0.000       2.157

           (No Motion)
  0.600       0.000       0.000
```

```
C*****************************************************************
C*'                        PROBLEM 12.C1                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C                    Definition of Symbols
C
C          WA = weight of wedge     (lbs)
C          WB = weight of block     (lbs)
C          MU = coefficient of friction at all surfaces
C
C               (Accelerations are expressed in ft/s**2)
C           AA = acceleration of wedge A, positive to the right
C          ABA = acceleration of block with respect to wedge,
C                directed down along the top surface of the wedge
C            G = acceleration of gravity
C
C          TH = angle formed by wedge and horizontal (30 degrees)
C     BRACKET = Function of MU shown in derivation
C
      REAL MU, K
C
      WA = 30.
      WB = 12.
      G = 32.2
      TH = 30.
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,'(/,2X,A\') '   MU              AA              ABA'
      WRITE (*,'(/,2X,A\') '                ft/s**2         ft/s**2'
      WRITE (*,'(/,A\')
C
      MU = 0.
      STEP = 0.01
   10 BRACKET = (SIN(K*TH) - 2.*MU*COS(K*TH) - MU**2*SIN(K*TH))
      AA = ((WB*COS(K*TH)*BRACKET-MU*WA)/(WA+WB*SIN(K*TH)*BRACKET))*G
      ABA = AA*(COS(K*TH)+MU*SIN(K*TH) + G*(SIN(K*TH)-MU*COS(K*TH))
      IF (AA.LT.0.) GOTO 320
      WRITE (*,20) MU,AA,ABA
   20 FORMAT (3X,F6.3,10X,F6.3,10X,F6.3)
      MU = MU + STEP
      IF (MU.LT.1.) GOTO 10
C
  320 WRITE (*,'(/,14X,A\)') '(Wedge does not move)'
      WRITE (*,'(/,A\)')
      AA = 0.
C
C           Determine value of MU at smallest 0.1 interval which is
C           greater than the last value of MU for which wedge moved.
      MU = (INT(MU*10.)+1.)/10.
C
      STEP = 0.1
   30 ABA = G*(SIN(K*TH) - MU*COS(K*TH))
      IF (ABA.LT.0.) GOTO 430
      WRITE (*,40) MU,AA,ABA
   40 FORMAT (3X,F6.3,10X,F6.3,10X,F6.3)
      MU = MU + STEP
      IF (MU.LT.1.) GOTO 30
C
  430 WRITE (*,'(/,18X,A\)') '(No Motion)'
      WRITE (*,'(/,A\)')
      AA = 0.
      ABA = 0.
      WRITE (*,50) MU,AA,ABA
   50 FORMAT (3X,F6.3,10X,F6.3,10X,F6.3)
      END
```

```
      MU          AA          ABA
                ft/s**2       ft/s**2

     .000        5.070       20.491
     .010        4.616       19.842
     .020        4.158       19.185
     .030        3.696       18.519
     .040        3.230       17.846
     .050        2.760       17.165
     .060        2.286       16.475
     .070        1.807       15.777
     .080        1.325       15.070
     .090         .838       14.354
     .100         .347       13.629

           (Wedge does not move)

     .200         .000       10.523
     .300         .000        7.734
     .400         .000        4.946
     .500         .000        2.157

              (No Motion)

     .600         .000        .000
```

PROBLEM 12.C2



$\tan\phi_s = \mu_s$

LET ω = CONSTANT RATE OF ROTATION OF ROD ABOUT AB

$$v = (r\sin\theta)\,\omega$$

$$a_n = \frac{v^2}{(r\sin\theta)} = \frac{r^2\sin^2\theta\,\omega^2}{r\sin\theta} = r\omega^2\sin\theta$$

$$+\leftarrow \Sigma F_n = ma_n: \quad R\sin(\theta-\phi_s) = m\,a_n$$

$$R\sin(\theta-\phi_s) = m\,r\,\omega^2\sin\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad R\cos(\theta-\phi_s) - mg = 0$$

$$R\cos(\theta-\phi_s) = mg \qquad (2)$$

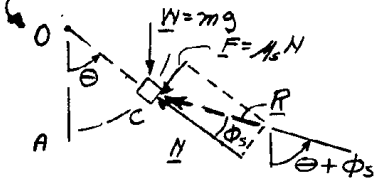
DIVIDE (1) BY (2): $\tan(\theta-\phi_s) = (r\omega^2/g)\sin\theta$

TO SOLVE FOR VALUES OF θ, WE WRITE

$$Q = \tan(\theta-\phi_s) - (r\omega^2/g)\sin\theta$$

AND SEEK VALUES OF θ FOR WHICH $Q=0$

FOR MOTION OF COLLAR IMPENDING ↗



SAME AS ABOVE EXCEPT WE REPLACE $(\theta-\phi_s)$ BY $(\theta+\phi_s)$

THUS, WE MUST ALSO SEEK VALUES OF θ FOR WHICH

$$Q = \tan(\theta+\phi_s) - (r\omega^2/g)\sin\theta = 0$$

```
10 '*************************************************************
20 '*                        PROBLEM 12.C2                      *
30 '*                        BASIC VERSION                      *
40 '*************************************************************
50 '
60 '                  Definition of Symbols
70 '
80 '          TH = angle theta  (degrees)
90 '          MU = coeffifient of static friction
100 '        PHI = angle of static friction  (degrees)
110 '     OMEGA = Angular velocity of rod
120 '         R = radius of semicircular rod  (m)
130 '
140      K = 4*ATN(1)/180
150 '
160      LET MU = .25
170      LET OMEGA = 7.5
180      LET R = .5
190      G = 9.810001
200      PHI = ATN(MU)/K
210 '  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
220 '  We first seek the zeros of the function Q using a minus sign
230 '  in the first term.
240      PRINT " Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
250      PRINT " "
260      PRINT "Theta                Q"
270      PRINT " "
280      FOR TH = Ø TO 180 STEP 10
290         Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)
300         PRINT USING "###             ####.###";TH,Q
310      NEXT
320      PRINT "Record range(s) of the value of theta for which Q = Ø
330      INPUT "In order to proceed, enter any digit.      ",P :PRINT
340      PRINT " Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
350      PRINT " "
360      INPUT "Lower limit of range  ", LL
370      INPUT "Upper limit of range  ", UL
380      DEL = 1: M = 1:   PRINT " "
390         Q = TAN((LL - PHI)*K) - ((R*OMEGA^2)/G)*SIN(LL*K)
400         IF Q > Ø THEN 410 ELSE 420
410      N = -1: GOTO 430
420      N = 1
430      FOR TH = LL TO UL STEP DEL
440            M = M + 1: IF M > 99 GOTO 990
450         Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)
460         IF Q > Ø GOTO 480
470      NEXT
480      TH = TH - DEL:    DEL = DEL/10
490      LL = TH:          UL = TH + (11)*DEL
500         M = M + 1: IF M > 99 GOTO 990
510      IF ABS(Q) < .0001 GOTO 530
520      GOTO 430
530      PRINT USING "Motion impends for theta = ###.### degrees.";TH
540      PRINT: PRINT "Do you have another range of theta to  "
550      PRINT "investigate for a zero of
560      PRINT " Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
```

CONTINUED

```
C*************************************************************
C*                        PROBLEM 12.C2                      *
C*                        FORTRAN VERSION                    *
C*************************************************************
C
C                  Definition of Symbols
C
C          TH = angle theta  (degrees)
C          MU = coefficient of static friction
C         PHI = angle of static friction   (degrees)
C       OMEGA = angular velocity of rod
C           R = radius of semicircular rod  (m)
C
      REAL MU, K, LL, N
      INTEGER T
      MU = Ø.25
      OMEGA = 7.5
      R = Ø.5
      G = 9.81
      K = 4.*ATAN(1.)/180.
      PHI = ATAN(MU)/K
C
C     We first seek the zeros of the function Q using a minus sign
C     in the first term.
      WRITE (*,*)'Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)'
      WRITE (*,*)' '
      DO 10 T = Ø,180,10
        TH = T
        Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)
        WRITE (*,11) TH,Q
   11   FORMAT (3X,F4.0,15X,F15.3)
   10 CONTINUE
      WRITE (*,*)'Record range(s) of value of theta for which Q = Ø'
      WRITE (*,'(2X,A\)')'In order to proceed, enter any digit.    '
      READ (*,*) DIGIT
      WRITE (*,*) ' '
      WRITE (*,*)'Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SI (TH*K)'
   12 WRITE (*,'(/,2X,A\)') 'Lower limit of range = '
      READ (*,*) LL
      WRITE (*,'(2X,A\)') 'Upper limit of range = '
      READ (*,*) UL
      DEL = 1.
      M = 1
C
      Q = TAN((LL - PHI)*K) -'((R*OMEGA**2)/G)*SIN(LL*K)
        IF (Q.GT.Ø.Ø) THEN
          N = -1.
        ELSE
          N = 1.
        ENDIF
   15 TH = LL
   20 Q = (TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K))*N
      M = M + 1
      IF (M.GT.99) GOTO 998
      IF (Q.GT.Ø.Ø) GOTO 30
      TH = TH + DEL
      IF(TH.LT.UL) GOTO 20
```

CONTINUED

PROBLEM 12.C2

CONTINUATION OF BASIC PROGRAM

```
570     PRINT " ":INPUT  "If YES enter 1, if NO enter 0.      ",W
580     IF W = 1 GOTO 360
590 '   ------------------------------------------------------------
600 '   We now seek the zeros of the function Q using a plus sign
610 '   in the first term.
620     PRINT " "
630     PRINT " Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
640     PRINT " "
650     PRINT "Theta                Q"
660     PRINT " "
670     FOR TH = 0 TO 180 STEP 10
680        Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)
690        PRINT USING "###            ####.###";TH,Q
700     NEXT
710     PRINT "Record range(s) of the value of theta for which Q = 0
720     INPUT "In order to proceed, enter any digit.      ";P: PRINT
730     PRINT " Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
740     PRINT " "
750     INPUT "Lower limit of range  ", LL
760     INPUT "Upper limit of range  ", UL
770     DEL = 1: M = 1:   PRINT " "
780        Q = TAN((LL + PHI)*K) - ((R*OMEGA^2)/G)*SIN(LL*K)
790        IF Q > 0 THEN 800 ELSE 810
800     N = -1: GOTO 820
810     N = 1
820     FOR TH = LL TO UL STEP DEL
830           M = M + 1: IF M > 99 GOTO 990
840        Q = (TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K))*N
850        IF Q => 0 GOTO 870
860     NEXT
870     TH = TH - DEL:    DEL = DEL/10
880     LL = TH:          UL = TH + (10)*DEL
890        M = M + 1: IF M > 99 GOTO 990
900     IF ABS(Q) < .000001 GOTO 920
910     GOTO 820
920     PRINT USING "Motion impends for theta = ###.### degrees.";TH
930     PRINT: PRINT "Do you have another range of theta to  "
940     PRINT "investigate for a zero of
950     PRINT " Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K) "
960     PRINT " ":INPUT  "If YES enter 1, if NO enter 0.      ",WW
970     IF WW = 1 GOTO 750
980     GOTO 1000
990     PRINT"Check ranges of theta that you used. Then try again.'
1000    END
```

```
Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

Theta                Q

 0              -0.250
 10             -0.568
 20             -0.876
 30             -1.147
 40             -1.356
 50             -1.471
 60             -1.449
 70             -1.214
 80             -0.581
 90              1.133
 100            11.348
 110           -12.267
 120            -5.979
 130            -4.250
 140            -3.221
 150            -2.400
 160            -1.656
 170            -0.944
 180            -0.250

Record range(s) of the value of theta for which Q = 0
In order to proceed, enter any digit.    7

 Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

Lower limit of range  80
Upper limit of range  90

Motion impends for theta =  84.731 degrees.

Do you have another range of theta to
investigate for a zero of
 Q = TAN((TH - PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

If YES enter 1, if NO enter 0.      0
```

CONTINUED

CONTINUATION OF FORTRAN PROGRAM

```
   30 TH = TH - DEL
      DEL = DEL/10
      LL = TH
      UL = TH + (11.)*DEL
      IF(ABS(Q).GE.0.0001) GOTO 15
      WRITE (*,50) TH
   50 FORMAT (/,2X,'Motion impends for theta =  ',F7.3,' degrees')
      WRITE (*,*)' '
      WRITE (*,*)'Do you have another range of theta to investigate'
      WRITE (*,*)'for a zero of'
      WRITE (*,*)'Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)'
      WRITE (*,'(/,2X,A\)') 'If YES enter 1, if NO enter 0.     '
      READ (*,*) W
      If (W.EQ.1.) GOTO 12
      WRITE (*,*) '------------------------------------------------'
C     We now seek the zeros of the function Q using a plus sign
C     in the first term.
      WRITE (*,*)'Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)'
      WRITE (*,*)' '
      DO 60 T = 0,180,10
        TH = T
        Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)
        WRITE (*,11) TH,Q
   61   FORMAT (3X,F4.0,15X,F15.3)
   60 CONTINUE
      WRITE (*,*)'Record range(s) of value of theta for which Q = 0'
      WRITE (*,'(2X,A\)') 'In order to proceed, enter any digit.     '
      READ (*,*) P
      WRITE (*,*)' '
      WRITE (*,*)'Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)'
   65 WRITE (*,'(/,2X,A\)') 'Lower limit of range = '
      READ (*,*) LL
      WRITE (*,'(2X,A\)') 'Upper limit of range = '
      READ (*,*) UL
      DEL = 1.
      M = 1
C
      Q = TAN((LL + PHI)*K) - ((R*OMEGA**2)/G)*SIN(LL*K)
        IF (Q.GT.0.0) THEN
           N = -1.
        ELSE
           N = 1.
        ENDIF
   70 TH = LL
   80 Q = (TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K))*N
      M = M + 1
      IF (M.GT.99) GOTO 998
      IF(Q.GT.0.0) GOTO 100
      TH = TH + DEL
      IF(TH.LT.UL) GOTO 80
  100 TH = TH - DEL
      DEL = DEL/10
      LL = TH
      UL = TH + (11.)*DEL
      IF(ABS(Q).GE.0.000001) GOTO 70
      WRITE (*,120) TH
  120 FORMAT (/,2X,'Motion impends for theta =  ',F7.3,' degrees')
      WRITE (*,*)
      WRITE (*,*)'Do you have another range of theta to investigate'
      WRITE (*,*)'for a zero of'
      WRITE (*,*)'Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)'
      WRITE (*,'(/,2X,A\)') 'If YES enter 1, if NO enter 0.     '
      READ (*,*) WW
      IF (WW.EQ.1.) GOTO 65
      GOTO 999
  998 WRITE (*,*)'Check range of values of theta. Then try again.'
  999 END
```

```
Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

   0.                    -.250
  10.                    -.568
  20.                    -.876
  30.                   -1.147
  40.                   -1.356
  50.                   -1.471
  60.                   -1.449
  70.                   -1.214
  80.                    -.581
  90.                    1.133
 100.                   11.348
 110.                  -12.267
 120.                   -5.979
 130.                   -4.250
 140.                   -3.221
 150.                   -2.400
 160.                   -1.656
 170.                    -.944
 180.                    -.250
Record range(s) of value of theta for which Q = 0
 In order to proceed, enter any digit.    3
```

CONTINUED

PROBLEM 12.C2

CONTINUATION OF RUN OF BASIC PROGRAM

```
Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

Theta            Q

 0              0.250
 10            -0.052
 20            -0.305
 30            -0.467
 40            -0.465
 50            -0.143
 60             1.013
 70             6.879
 80           -16.995
 90            -6.867
100            -5.066
110            -4.175
120            -3.517
130            -2.922
140            -2.330
150            -1.720
160            -1.085
170            -0.427
180             0.250


Record range(s) of the value of theta for which Q = 0
In order to proceed, enter any digit.      7

 Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

Lower limit of range  0
Upper limit of range  10

Motion impends for theta =   8.197 degrees.

Do you have another range of theta to
investigate for a zero of
 Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.      1
Lower limit of range  50
Upper limit of range  60

Motion impends for theta =  52.124 degrees.

Do you have another range of theta to
investigate for a zero of
 Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.      1
Lower limit of range  170
Upper limit of range  180

Motion impends for theta = 176.339 degrees.

Do you have another range of theta to
investigate for a zero of
 Q = TAN((TH + PHI)*K) - ((R*OMEGA^2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.      0
Ok
```

CONTINUATION OF RUN OF FORTRAN PROGRAM

```
Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 Lower limit of range = 80
 Upper limit of range = 90

 Motion impends for theta =   84.731 degrees

Do you have another range of theta to investigate
for a zero of
Q = TAN((TH - PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.    0
-------------------------------------------------
Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

  0.                          .250
 10.                         -.052
 20.                         -.305
 30.                         -.467
 40.                         -.465
 50.                         -.143
 60.                         1.013
 70.                         6.879
 80.                       -16.995
 90.                        -6.867
100.                        -5.066
110.                        -4.175
120.                        -3.517
130.                        -2.922
140.                        -2.330
150.                        -1.720
160.                        -1.085
170.                         -.427
180.                         .250
Record range(s) of value of theta for which Q = 0
 In order to proceed, enter any digit.    3

Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 Lower limit of range = 0
 Upper limit of range = 10

 Motion impends for theta =    8.197 degrees

Do you have another range of theta to investigate
for a zero of
Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.    1

 Lower limit of range = 50
 Upper limit of range = 60

 Motion impends for theta =   52.124 degrees

Do you have another range of theta to investigate
for a zero of
Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.    1

 Lower limit of range = 170
 Upper limit of range = 180

 Motion impends for theta =   176.339 degrees

Do you have another range of theta to investigate
for a zero of
Q = TAN((TH + PHI)*K) - ((R*OMEGA**2)/G)*SIN(TH*K)

 If YES enter 1, if NO enter 0.    0
```

WE HAVE FOUND THAT MOTION IMPENDS AT:

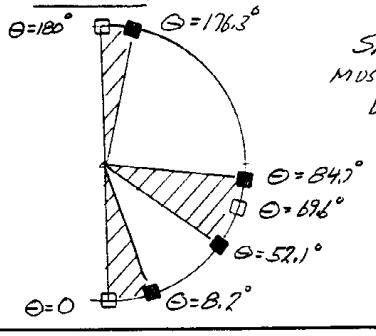
- $\Theta = 8.2^\circ$ ■
- $\Theta = 52.1^\circ$ ■
- $\Theta = 84.7^\circ$ ■
- $\Theta = 176.3^\circ$ ■

FROM ANSWER OF PROB 12.54 WE FIND THAT FOR $\mu_s = 0$, COLLAR IS IN EQUILIBRIUM AT:

$\Theta = 0$, $\Theta = 69.6^\circ$, AND $\Theta = 180^\circ$ □

(THESE VALUES CAN BE CALCULATED BY RUNNING THE ABOVE PROGRAM WITH $\mu_s = 0$)

SUMMARY



□ = EQUILIBRIUM WITH $\mu_s = 0$

SINCE THE VALUES FOR $\mu_s = 0$ MUST FALL WITHIN THE DESIRED RANGES, WE HAVE

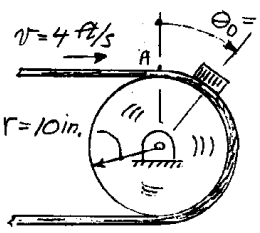
RANGES OF Θ FOR EQUILIBRIUM

$0 \le \Theta \le 8.2^\circ$ ▲

$52.1^\circ \le \Theta \le 84.7^\circ$ ▲

$176.3^\circ \le \Theta \le 180^\circ$ ▲

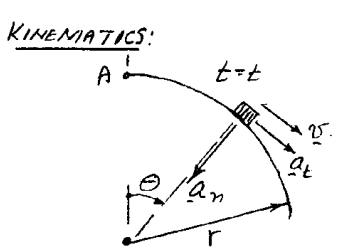
PROBLEM 12.C3

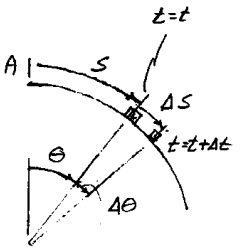


ANGLE AT WHICH BLOCK FIRST SLIPS FOR $\mu_s = 0.40$ (SEE PROB 12.48)

For $\theta > \theta_0 = 9.01°$, BLOCK SLIPS RELATIVE TO THE BELT AND COEFFICIENT OF FRICTION IS $\mu = \mu_k = 0.35$

KINEMATICS:





$\Delta v = a_t \Delta t$; $\Delta\theta = \frac{\Delta s}{r}$ ◁

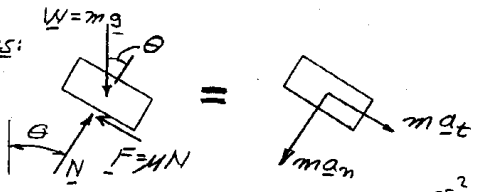
WHEN $t = t + \Delta t$: $v = v + \Delta v$

$\Delta s = v\Delta t + \frac{1}{2} a_t \Delta t^2 = (v + \frac{1}{2}\Delta v)\Delta t$ ◁

$\Delta\theta = \Delta s / r$ ◁

$\theta = \theta + \Delta\theta$ ◁

KINETICS:



$+\nearrow \Sigma F = ma_n$: $mg\cos\theta - N = ma_n = m\frac{v^2}{r}$

$N = m\left(g\cos\theta - \frac{v^2}{r}\right)$ ◁ (1)

$+\searrow \Sigma F = ma_t$: $mg\sin\theta - \mu N = ma_t$

$a_t = g\sin\theta - \mu\frac{N}{m}$; $a_t = g\sin\theta - \frac{\mu}{m}(m)(g\cos\theta - \frac{v^2}{r})$

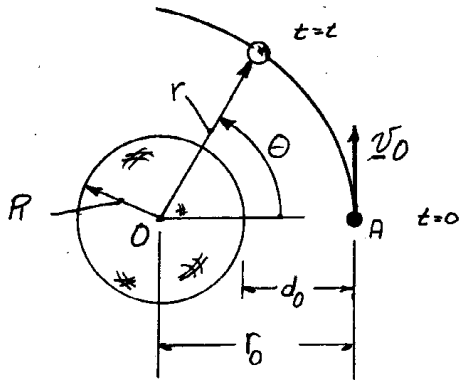
$\Delta v = a_t \Delta t = \left(g\sin\theta - g\mu\cos\theta + \mu v^2/r\right)\Delta t$ ◁

INCREMENT AT STEPS: $\Delta t = 0.001$ s, CALCULATE AT EACH STEP VALUE OF N USING (1). BLOCK WILL LEAVE BELT WHEN $N < 0$.

```
10 '*****************************************************************
20 '*'                      PROBLEM 12.C3                            *
30 '*                       BASIC VERSION                           *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '             TH = angle theta  (degrees)
90 '             MU = coefficient of kinetic friction
100 '             V = velocity of belt  (ft/s)
110 '             R = Radius of pulley  (feet)
120 '            DT = Interval of time for integration  (seconds)
130 '
140     K = 4*ATN(1)/180
150     MU = .35
160     V = 4
170     R = 10/12
180     DT = .001
190 '
200 '       Angle at which package first slips with respect to belt:
210             TH0 = 9.01      '(degrees)      See Prob. 12.48.
220 '
230     TH = TH0:     G = 32.2
240 '
250     WHILE N =>0
260         DV = (G*SIN(K*TH) - G*MU*COS(K*TH) + MU*V^2/R)*DT
270         V = V + DV
280         DS = (V + DV/2)*DT
290         DTH = DS/R     'increment of angle theta in radians
300         TH = TH + (DTH)/K
310         N = (G*COS(K*TH) - V^2/R)
320     WEND
330 '
340     PRINT "For given increment of time: DT = 0.001 seconds";PRINT
350     PRINT "Package leaves belt when:"
360     PRINT USING "                          theta = ##.## degrees";TH
370     PRINT
380     PRINT "Corresponding speed of package:"
390     PRINT USING "                                  v = ##.### ft/s";V

RUN
For given increment of time: DT = 0.001 seconds

Package leaves belt when:
                       theta = 33.84 degrees

Corresponding speed of package:
                          v =  4.724 ft/s
```

```
C*****************************************************************
C*'                      PROBLEM 12.C3                            *
C*                       FORTRAN VERSION                         *
C*****************************************************************
C
C                    Definition of Symbols
C
C                TH = angle theta   (degrees)
C                MU = coefficient of kinetic friction
C                 V = velocity of belt   (ft/s)
C                 R = radius of pulley   (feet)
C                DT = interval of time for integration   (seconds)
C
      REAL MU, K, N
      MU = 0.35
      V = 4.
      R = 10./12.
      DT = 0.001
C
C       Angle theta at which package first slips with respect to belt:
      THO = 9.01
C                (degrees)          See Prob. 12.48
C
      K = 4.*ATAN(1.)/180.
      TH = THO
      G = 32.2
C
      N = 0.
  240 IF (N.GE.0.) THEN
          DV = (G*SIN(K*TH) - G*MU*COS(K*TH) + MU*V**2/R)*DT
          V = V + DV
          DS = (V+DV/2.)*DT
          DTH = DS/R
C           Increment of angle in radians
          TH = TH + (DTH)/K
          N = (G*COS(K*TH) - V**2/R)
      ELSE
          GOTO 330
      ENDIF
      GOTO 240
C
  330 WRITE (*,'(/,5X,2A\)') 'For given increment of time :',
     +                       '  DT = 0.001 second'
      WRITE (*,'(/,5X,A\)') 'Package leaves belt when :'
      WRITE (*,40) TH
   40 FORMAT (/,20X,'Theta  = ',F5.2,' degrees')
      WRITE (*,'(/,5X,A\)') 'Corresponding speed of package :'
      WRITE (*,50) V
   50 FORMAT (/,24X,'v  = ',F5.3,' ft/s')
      END

  For given increment of time : DT = 0.001 second
  Package leaves belt when :
                   Theta  = 33.84 degrees

  Corresponding speed of package :
                        v  = 4.724 ft/s
```

PROBLEM 12.C4



FREE FLIGHT OF SPACECRAFT STARTS AT A. v_0 IS PARALLEL TO SURFACE OF EARTH. ALTITUDE AT TIME t IS DENOTED BY d. DISTANCE FROM CENTER OF EARTH IS

$$r = R + d$$

EQUATION OF PATH IS

EQ.(12.39): $$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta$$

OR, $$r = 1\Big/\left(\frac{GM}{h^2} + C\cos\theta\right)$$ ◁

WHERE [EQ.(12.41)]: $$h = r_0 v_0$$ ◁

AND [EQ.(12.42)]: $$C = \frac{1}{r_0} - \frac{GM}{h^2}$$ ◁

TIME OF FREE FLIGHT:

EQ.(12.27): $$r^2\left(\frac{d\theta}{dt}\right) = h$$

SOLVE FOR dt: $$dt = \left(\frac{r^2}{h}\right)d\theta \quad \text{or} \quad \Delta t = \frac{r^2}{h}\Delta\theta$$ ◁

WE SUM Δt TO FIND TIME ELAPSED FROM LAUNCHING

NOTE: v_0 IS ENTERED IN km/h AND CONVERTED TO m/s IN EACH OF THE FOLLOWING PROGRAMS

```
10 '*************************************************************
20 '*                      PROBLEM 12.C4                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '              Definition of Symbols
70 '
80 '      TH = angle theta  (degrees)
90 '      RE = radius of the earth  (km)
100 '      R = distance from center of earth to spacecraft  (km)
110 '      D = altitude of spacecraft   (km)
120 '     D0 = altitute of spacecraft at launch   (km)
130 '     V0 = velocity of launch  (km/h)
140 '     R0 = distance from center of earth to spacecraft
150 '          at launch   (km)
160 '
170 '     GM = (mass of earth)*(constant of gravitation)
180 '        = 398 * 10^12    (m^3/s^2)
190 '
200 '      N = counter
210 '    NBR = number of increments in a printing interval
220 '
230 '          DATA
240 '
250     INPUT "Enter launch velocity v0 expressed in km/h.  ",V0
260     INPUT "Enter largest value of theta (in degrees)  ",THMAX:PRINT
270     IF V0 <> 38750! GOTO 290
280     PRINT USING "PARABOLIC TRAJECTORY: v0 = ##### km/h";V0:GOTO 320
290     IF V0 > 38750! GOTO 310
300     PRINT USING "ELLIPTIC ORBIT: v0 = ##### km/h"; V0:GOTO 320
310     PRINT USING "HYPERBOLIC TRAJECTORY: v0 = ##### km/h"; V0
320 '
```

CONTINUED

```
C*************************************************************
C*                      PROBLEM 12.C4                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C                 Definition of Symbols
C
C         TH = angle theta  (degrees)
C         RE = radius of the earth   (km)
C          R = distance from center of earth to spacecraft  (km)
C          D = altitude of spacecraft   (km)
C         D0 = altitude of the spacecraft at launch   (km)
C         V0 = velocity of spacecraft at launch  (km/h)
C         R0 = distance from center of earth to spacecraft
C              at launching  (km)
C
C         GM = (mass of earth)*(constant of gravitation)
C            = 398 * 10**12   (m**3/s**3)
C
C          N = counter
C        NBR = number of increments in a printing interval
C
      REAL H, K, N
C         DATA
C
      WRITE (*,'(2X,A\)') 'Enter launch velocity V0 expressed in km/h '
      READ (*,*)  V0
      WRITE (*,'(2X,A\)') 'Enter largest value of theta (in degrees)  '
      READ (*,*)  THMAX
      WRITE (*,*) ' '
C
      IF (V0.NE.38750)  GOTO 20
   10 FORMAT (2X,'PARABOLIC TRAJECTORY: V0 = ',F6.0' km/h')
   20 IF (V0.GT.38750)  GOTO 40
      WRITE (*,30) V0
   30 FORMAT (2X,'ELLIPTIC ORBIT: V0 = ',F6.0' km/h')
      GOTO 60
   40 WRITE (*,50) V0
   50 FORMAT (2X,'HYPERBOLIC TRAJECTORY: V0 = ',F6.0' km/h')
```

CONTINUED

PROBLEM 12.C4

CONTINUATION OF BASIC PROGRAM

```
330       K = 4*ATN(1)/180
340       RE = 6.37 *10^6     ' radius of earth in meters
350       GM = 398*10^12      ' m^3/s^2
360 '
370       D0 = 500 *10^3      ' altitude at launch in meters
380       R0 = RE + D0        ' value of R at launch in meters
390 '
400    PRINT " "
410    PRINT " theta        altitude        time"
420    PRINT "degrees       km*1000         minutes"
430    PRINT  " "
440    V0 = V0/3.6      'express v0 in m/s
450    DTH = .1
460    NBR = 50
470    WHILE TH <= THMAX
480       H = R0 * V0
490       C = 1/R0 - GM/H^2
500       R = 1/(GM/H^2 + C*COS(TH*K))
510       D = (R - RE)/10^6
520       DT = R^2 *(DTH*K)/H
530       IF INT(N/NBR) = N/NBR THEN 540 ELSE 550
540       PRINT USING "  ###      ####.###        #####.###";TH, D ,T
550       N = N + 1
560       T = T + DT/60
570       TH = N*DTH
580    WEND
590    V0 = V0*3.6
600    IF V0 => 38750! GOTO 630
610       T = T - DT/60
620       PRINT USING "Periodic Time  = ####.# min";2*T
630    END
```

```
Enter launch velocity v0 expressed in km/h.  36900
Enter largest value of theta (in degrees)  180

ELLIPTIC ORBIT: v0 = 36900 km/h

 theta        altitude        time
degrees       km*1000         minutes

    0          0.500          0.000
    5          0.512          0.976
   10          0.547          1.958
   15          0.607          2.954
   20          0.691          3.971
   25          0.801          5.016
   30          0.939          6.098
   35          1.107          7.225
   40          1.306          8.410
   45          1.539          9.662
   50          1.811         10.997
   55          2.125         12.431
   60          2.486         13.983
   65          2.901         15.677
   70          3.377         17.541
   75          3.922         19.610
   80          4.547         21.927
   85          5.264         24.547
   90          6.089         27.536
   95          7.040         30.982
  100          8.138         34.995
  105          9.412         39.717
  110         10.892         45.336
  115         12.617         52.097
  120         14.631         60.322
  125         16.988         70.443
  130         19.745         83.029
  135         22.962         98.838
  140         26.694        118.860
  145         30.976        144.361
  150         35.796        176.897
  155         41.055        218.239
  160         46.524        270.137
  165         51.794        333.839
  170         56.287        409.368
  175         59.348        494.786
  180         60.439        585.942
Periodic Time  = 1171.9 min
```

CONTINUED

CONTINUATION OF FORTRAN PROGRAM

```
C
   60 K = 4.*ATAN(1.)/180.
      RE = 6.37*10E5
      GM = 398*10E11
C
      D0 = 5.00E5
      R0 = RE + D0
C
      WRITE (*,*) ' '
      WRITE (*,*) '  theta       altitude         time'
      WRITE (*,*) ' degrees      km*1000          minutes'
      WRITE (*,*) ' '
      V0 = V0/3.6
      DTH = 0.1
      NBR = 50
C
   80 IF (TH.LE.THMAX) THEN
        H = R0 * V0
        C = 1/R0 - GM/H**2
        R = 1/(GM/H**2 + C*COS(TH*K))
        D = (R - RE)/10**6
        DT = (R**2)*(DTH*K)/H
          IF (INT(N/NBR).NE.(N/NBR)) THEN
              GOTO 100
          ENDIF
        WRITE (*,90) TH, D, T
   90   FORMAT (2X,F4.0,6X,F8.3,8X,F8.2)
  100   N = N + 1
        T = T + DT/60
        TH = N*DTH
        GOTO 80
      ELSE
        GOTO 200
      ENDIF
  200 V0 = V0*3.6
      IF (V0.GE.38750) GOTO 300
        T = T - DT/60
        WRITE (*,210) 2*T
  210   FORMAT (5X,'Periodic Time = ',F6.1,' min')
  300 END
```

```
Enter launch velocity V0 expressed in km/h 36900
Enter largest value of theta (in degrees)  180

ELLIPTIC ORBIT: V0 = 36900. km/h

 theta        altitude        time
degrees       km*1000         minutes

    0.          .500            .00
    5.          .512            .98
   10.          .547           1.96
   15.          .607           2.95
   20.          .691           3.97
   25.          .801           5.02
   30.          .939           6.10
   35.         1.107           7.23
   40.         1.306           8.41
   45.         1.539           9.66
   50.         1.811          11.00
   55.         2.125          12.43
   60.         2.486          13.98
   65.         2.901          15.68
   70.         3.377          17.54
   75.         3.922          19.61
   80.         4.547          21.93
   85.         5.264          24.55
   90.         6.089          27.54
   95.         7.040          30.98
  100.         8.138          34.99
  105.         9.412          39.72
  110.        10.892          45.34
  115.        12.617          52.10
  120.        14.631          60.32
  125.        16.988          70.44
  130.        19.745          83.03
  135.        22.962          98.84
  140.        26.694         118.86
  145.        30.976         144.36
  150.        35.796         176.90
  155.        41.055         218.24
  160.        46.524         270.14
  165.        51.794         333.84
  170.        56.287         409.37
  175.        59.348         494.79
  180.        60.439         585.94
   Periodic Time = 1171.9 min
```

CONTINUED

PROBLEM 12.C4

CONTINUATION OF RUN OF BASIC PROGRAM

```
Enter launch velocity v0 expressed in km/h.  38750
Enter largest value of theta (in degrees)  170

PARABOLIC TRAJECTORY: v0 = 38750 km/h
```

theta degrees	altitude km*1000	time minutes
0	0.500	0.000
5	0.513	0.929
10	0.553	1.866
15	0.619	2.817
20	0.714	3.790
25	0.838	4.793
30	0.993	5.836
35	1.183	6.928
40	1.410	8.083
45	1.679	9.313
50	1.994	10.635
55	2.362	12.070
60	2.790	13.640
65	3.288	15.378
70	3.868	17.320
75	4.545	19.514
80	5.337	22.023
85	6.268	24.928
90	7.369	28.338
95	8.681	32.398
100	10.256	37.311
105	12.167	43.362
110	14.510	50.960
115	17.425	60.715
120	21.107	73.551
125	25.847	90.931
130	32.087	115.261
135	40.530	150.689
140	52.341	204.795
145	69.574	292.466
150	96.128	445.690
155	140.158	741.982
160	221.161	1401.920
165	395.912	3228.645
170	893.187	10605.960

```
Enter launch velocity v0 expressed in km/h.  47500
Enter largest value of theta (in degrees)  115

HYPERBOLIC TRAJECTORY: v0 = 47500 km/h
```

theta degrees	altitude km*1000	time minutes
0	0.500	0.000
5	0.517	0.759
10	0.570	1.525
15	0.660	2.307
20	0.788	3.113
25	0.958	3.954
30	1.174	4.840
35	1.443	5.784
40	1.771	6.803
45	2.169	7.916
50	2.650	9.150
55	3.232	10.537
60	3.939	12.122
65	4.805	13.966
70	5.877	16.157
75	7.222	18.821
80	8.943	22.151
85	11.204	26.456
90	14.275	32.258
95	18.647	40.514
100	25.303	53.173
105	36.547	74.870
110	59.332	119.806
115	128.903	260.802

CONTINUATION OF RUN OF FORTRAN PROGRAM

```
Enter launch velocity V0 expressed in km/h 38750
Enter largest value of theta (in degrees)  170

PARABOLIC TRAJECTORY: V0 = 38750. km/h
```

theta degrees	altitude km*1000	time minutes
0.	.500	.00
5.	.513	.93
10.	.553	1.87
15.	.619	2.82
20.	.714	3.79
25.	.838	4.79
30.	.993	5.84
35.	1.183	6.93
40.	1.410	8.08
45.	1.679	9.31
50.	1.994	10.64
55.	2.362	12.07
60.	2.790	13.64
65.	3.288	15.38
70.	3.868	17.32
75.	4.545	19.51
80.	5.337	22.02
85.	6.268	24.93
90.	7.369	28.34
95.	8.681	32.40
100.	10.256	37.31
105.	12.167	43.36
110.	14.510	50.96
115.	17.425	60.71
120.	21.107	73.55
125.	25.847	90.93
130.	32.087	115.26
135.	40.530	150.69
140.	52.341	204.80
145.	69.574	292.47
150.	96.128	445.69
155.	140.158	741.90
160.	221.163	1401.93
165.	395.916	3228.69
170.	893.216	10606.33

```
Enter launch velocity V0 expressed in km/h 47500
Enter largest value of theta (in degrees)  115

HYPERBOLIC TRAJECTORY: V0 = 47500. km/h
```

theta degrees	altitude km*1000	time minutes
0.	.500	.00
5.	.517	.76
10.	.570	1.52
15.	.660	2.31
20.	.788	3.11
25.	.958	3.95
30.	1.174	4.84
35.	1.443	5.78
40.	1.771	6.80
45.	2.169	7.92
50.	2.650	9.15
55.	3.232	10.54
60.	3.939	12.12
65.	4.805	13.97
70.	5.877	16.16
75.	7.222	18.82
80.	8.943	22.15
85.	11.204	26.46
90.	14.275	32.26
95.	18.647	40.51
100.	25.303	53.17
105.	36.547	74.87
110.	59.331	119.81
115.	128.903	260.80

13.1

MASS OF SATELLITE: $\frac{100\,lb}{32.2\,ft/s^2} = 3.1055\ lb\cdot s^2/ft$

VELOCITY $= v = 15{,}000\ mi/h = 22{,}000\ ft/s$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(3.1055\ lb\cdot s^2/ft)(22{,}000\ ft/s)^2$$

$$T = 751.55\times10^6\ lb\cdot ft \qquad T = 752\times10^6\ lb\cdot ft$$

NOTE: ACCELERATION OF GRAVITY HAS NO EFFECT ON THE MASS OF THE SATELLITE

13.2

(a) ON THE EARTH:

$m = 3\ kg$ $\qquad W = mg = (3\ kg)(9.81\ m/s^2)$

$v = 30\ m/s$ $\qquad W = 29.43\ N$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(3\ kg)(30\ m/s)^2 = 1350\ J$$

$T = 1350\ J$

$T = Wh$: $\quad h = \frac{T}{W} = \frac{1350\ J}{29.43\ N}$ $\qquad h = 45.9\ m$

(b) ON THE MOON:

MASS IS UNCHANGED: $m = 3\ kg$

THUS, T IS UNCHANGED: $\qquad T = 1350\ J$

BUT, WEIGHT ON THE MOON IS:

$$W = mg = (3\ kg)(1.62\ m/s^2) = 4.86\ N$$

$T = Wh$: $\quad h = \frac{T}{W} = \frac{1350\ J}{4.86\ N}$ $\qquad h = 278\ m$

13.3

(a) FRONT-WHEEL DRIVE: SINCE 60% OF WEIGHT IS DISTRIBUTED OVER THE FRONT WHEELS, THE MAXIMUM FORCE TO MOVE AUTO IS:

$$P = F = \mu N = 0.8(0.6\,W) = 0.8(0.6\,mg)$$

FOR 100 m:

$$U_{1-2} = P(100\ m) = 0.8(0.6\,mg)(100\ m) = 48\,mg$$

$T_1 = 0$

$$T_1 + U_{1-2} = T_2$$

$$0 + 48\,mg = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2(48g) = 96\,(9.81\ m/s)^2 = 941.76$$

$v_2 = 30.68\ m/s$ $\qquad v_2 = 110.5\ km/h$

(b) REAR-WHEEL DRIVE: USE SAME SOLUTION USED FOR PART a, EXCEPT 40% WEIGHT IS DISTRIBUTED OVER THE REAR WHEELS.

$$P = F = \mu N = 0.8(0.4\,W) = 0.8(0.4\,mg)$$

FOR 100 m:

$$U_{1-2} = P(100\ m) = 0.8(0.4\,mg)(100\ m) = 32\,mg$$

$T_1 = 0$:

$$T_1 + U_{1-2} = T_2$$

$$0 + 32\,mg = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2(32g) = 64(9.81\ m/s^2) = 627.84$$

$v_2 = 25.056\ m/s$ $\qquad v_2 = 90.2\ km/h$

(c) SEE NEXT COLUMN

NOTE: THE AUTOMOBILE IS TREATED AS A PARTICLE IN THIS PROBLEM. THE WEIGHT DISTRIBUTION IS ASSUMED TO BE THE SAME FOR STATIC AND DYNAMIC CONDITIONS. COMPARE WITH SAMPLE PROBLEM 16.1 WHERE THE VEHICLE IS TREATED AS A RIGID BODY.

(CONTINUED)

13.3 CONTINUED

(c) FOUR-WHEEL DRIVE

MAXIMUM FORCE REQUIRED TO MOVE AUTO IS:

$$P = F = 0.8\,W = 0.8\,mg$$

FOR 100 m: $\quad U_{1-2} = P(100\ m) = (0.8\,mg)(100\ m) = 80\,mg$

$T_1 = 0$:

$$T_1 + U_{1-2} = T_2$$

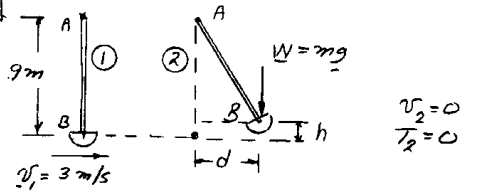
$$0 + 80\,mg = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2(80g) = 160(9.81\ m/s^2) = 1569.6$$

$v_2 = 39.62\ m/s$ $\qquad v_2 = 142.6\ km/h$

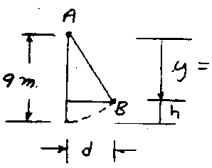
NOTE: SINCE THIS SOLUTION DOES NOT DEPEND ON THE WEIGHT DISTRIBUTION OVER THE WHEELS, THE SAME RESULT WOULD BE OBTAINED IF THE AUTO WERE CONSIDERED TO BE A RIGID BODY (cf. SAMPLE PROB 16.1)

13.4



$$T_1 + U_{1-2} = T_2: \quad \frac{1}{2}mv_1^2 - (mg)h = 0$$

$$h = \frac{v_1^2}{2g} = \frac{(3\ m/s)^2}{2(9.8\ m/s^2)} = 0.4587\ m$$



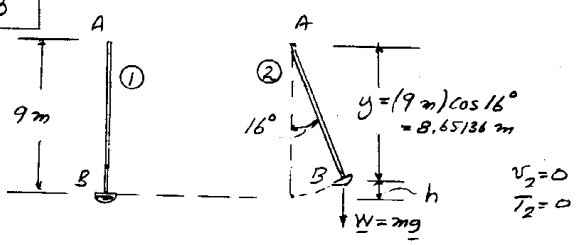
$y = 9\ m - h = 9 - 0.4587 = 8.5413\ m$

$$AB^2 = (9\ m)^2 = y^2 + d^2$$

$$9^2 = (8.5413\ m)^2 + d^2$$

$d = 2.837\ m$ $\qquad d = 2.84\ m$

13.5



$h = 9\ m - y$

$= 9\ m - 8.65136\ m$

$h = 0.34864\ m$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}mv_1^2 - (mg)(0.34864\ m) = 0$$

$$v_1^2 = 2(9.81\ m/s^2)(0.34864\ m)$$

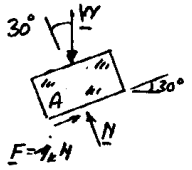
$$v_1^2 = 6.8405\ m^2/s^2$$

$v_1 = 2.6154\ m/s$ $\qquad v_1 = 2.62\ m/s$

13.6

FROM A TO B: Position 1 AT A, $v_1 = 4$ ft/s

$T_1 = \frac{1}{2}\frac{W}{g}v_1^2 = \frac{1}{2}\frac{W}{g}(4\text{ ft/s})^2 = 8\frac{W}{g}$

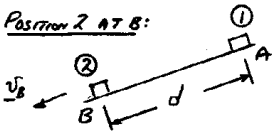


$+\uparrow \Sigma F = 0 \quad N - W\cos 30° = 0$

$N = W\cos 30°$

$F = \mu_k N = 0.25\,W\cos 30°$

Position 2 AT B:



$U_{1-2} = (W\sin 30° - F)d$

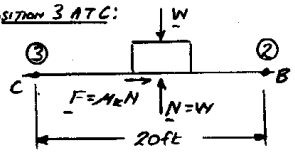
$= (W\sin 30° - 0.25W\cos 30°)d$

$T_2 = \frac{1}{2}\frac{W}{g}v_B^2$

$T_1 + U_{1-2} = T_2: \quad 8\frac{W}{g} + (W\sin 30° - 0.25W\cos 30°)d = T_2$

FROM B TO C Position 2: T_2 = SEE ABOVE

Position 3 AT C:



$U_{2-3} = -F(20\text{ ft}) = \mu_k N(20\text{ ft})$

$= -0.25\,W(20\text{ ft}) = -5W$

$v_C = v_B = 8$ ft/s

$T_3 = \frac{1}{2}\frac{W}{g}v_3^2 = \frac{1}{2}\frac{W}{g}(8)^2 = 32\frac{W}{g}$

$T_2 + U_{2-3} = T_3$

$\left[8\frac{W}{g} + (W\sin 30° - 0.25W\cos 30°)d\right] - 5W = 32\frac{W}{g}$

$\frac{8}{g} + (0.5 - 0.2165)d - 5 = \frac{32}{g}$

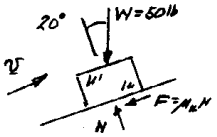
$0.2836\,d = \frac{24}{g} + 5 = 0.7453 + 5 = 5.7453$

$d = 20.3$ ft ◀

13.7

(a) AT A: Position 1. $v_1 = v_A = 40$ ft/s

$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{50\text{ lb}}{32.2\text{ ft/s}^2}(40\text{ ft/s})^2 = 1242.2$ ft·lb



$+\uparrow \Sigma F = 0 \quad N - (50\text{ lb})\cos 20° = 0$

$N = 46.985$ lb

$F = \mu_k N = (0.15)(46.985\text{ lb}) = 7.048$ lb

AT B: Position 2, $v = 0$

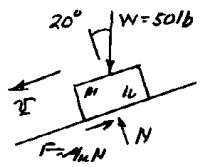
$U_{1-2} = (-50\sin 20° - 7.048)x = -24.149x$ ft·lb

$T_1 + U_{1-2} = T_2: \quad 1242.2\text{ ft·lb} - 24.149x = 0$

$x = 51.44$ ft $\quad x = 51.4$ ft ◀

(b) PACKAGE RETURNS TO A: AT B: Position 2, $v = 0$ $T_2 = 0$

BACK AT A: Position 3, $T_3 = \frac{1}{2}mv_3^2 = \frac{1}{2}\frac{50}{32.2}v_3^2 = 0.7764v_3^2$



$N = 50\cos 20° = 46.985$ lb

$F = \mu_k N = 0.15(46.985) = 7.048$ lb

$U_{2-3} = (+W\sin 20° - F)x$

$= (+50\sin 20° - 7.048)(51.44\text{ ft})$

$= (10.053\text{ lb})(51.44\text{ ft}) = 517.13$ ft·lb

$T_2 + U_{2-3} = T_3: \quad 0 + 517.13\text{ ft·lb} = 0.7764\,v_3^2$

$v_A = v_3 = 25.81$ ft/s $\quad v_A = 25.8$ ft/s ◀

(c) TOTAL ENERGY DISSIPATED DUE TO FRICTION IS EQUAL TO DIFFERENCE OF INITIAL AND FINAL KINETIC ENERGIES.

$T_1 - T_3 = \frac{1}{2}m(v_1^2 - v_3^2) = \frac{1}{2}\left(\frac{50\text{ lb}}{32.2\text{ ft/s}^2}\right)\left[(40\text{ ft/s})^2 - (25.81\text{ ft/s})^2\right]$

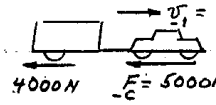
$= 725.1$ ft·lb

ENERGY DISSIPATED = 725 ft·lb ◀

13.8

(a) CAR-TRAILER SYSTEM $T_2 = 0$

$v_1 = 72$ km/h $= 20$ m/s



$m_{car} = 1400$ kg

$m_{trailer} = 1200$ kg

$T_1 = \frac{1}{2}(m_t + m_c)v_1^2$

$= \frac{1}{2}(1200\text{ kg} + 1400\text{ kg})(20\text{ m/s})^2$

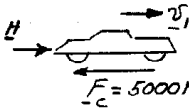
$T_1 = 520$ kJ

$T_1 + U_{1-2} = T_2: \quad 520\text{ kJ} - (4000\text{ N} + 5000\text{ N})x = 0$

$x = 57.78$ m $\quad x = 57.8$ m ◀

(b) CAR CONSIDERED SEPARATELY $T_2 = 0$

$v_1 = 72$ km/h $= 20$ m/s



$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1400\text{ kg})(20\text{ m/s})^2$

$T_1 = 280$ kJ

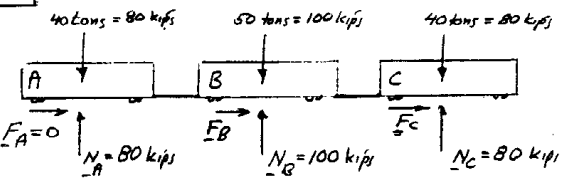
WE RECALL FROM PART a THAT $x = 57.78$ m

$T_1 + U_{1-2} = T_2: \quad 280\text{ kJ} - (5000\text{ N} - H)(57.78\text{ m}) = 0$

$5000\text{ N} - H = 4846 \quad H = +154$ N

HORIZONTAL COMPONENT OF FORCE EXERTED BY TRAILER HITCH ON CAR: $H = 154$ N → ◀

13.9



$\mu_k = 0.35 \quad F_B = (0.35)(100\text{ kips}) = 35$ kips

$F_C = (0.35)(80\text{ kips}) = 28$ kips

$v_1 = 30$ mi/h $= 44$ ft/s ← $\quad v_2 = 0 \quad T_2 = 0$

(a) ENTIRE TRAIN:

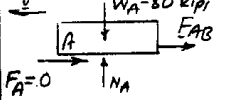
$T_1 + U_{1-2} = T_2$

$\frac{1}{2}\frac{(80\text{ kips} + 100\text{ kips} + 80\text{ kips})}{32.2\text{ ft/s}^2}(44\text{ ft/s})^2 - (28\text{ kips} + 35\text{ kips})x = 0$

$x = 124.07$ ft $\quad x = 124.1$ ft ◀

(b) FORCE IN EACH COUPLING: RECALL THAT $x = 124.07$ ft

CAR A: ASSUME F_{AB} TO BE IN TENSION



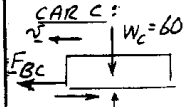
$T_1 + U_{1-2} = T_2$

$\frac{1}{2}\frac{80\text{ kips}}{32.2}(44)^2 - F_{AB}(124.07\text{ ft}) = 0$

$F_{AB} = +19.38$ kips

$F_{AB} = 19.38$ kips T ◀

CAR C:



$T_1 + U_{1-2} = T_2$

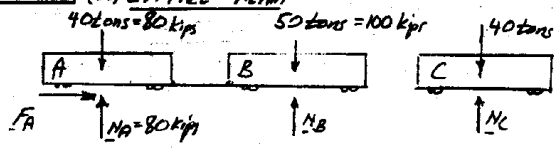
$\frac{1}{2}\frac{80\text{ kips}}{32.2}(44)^2 + (F_{BC} - 28\text{ kips})(124.07\text{ ft}) = 0$

$F_{BC} - 28\text{ kips} = -19.38$ kips

$F_{BC} = +8.62$ kips $\quad F_{BC} = 8.62$ kips T ◀

13.10

(a) ENTIRE TRAIN



$F_A = \mu N_A = (0.35)(80 \text{ kips}) = 28 \text{ kips}$

$v_1 = 30 \text{ mi/h} = 44 \text{ ft/s} \leftarrow \quad v_2 = 0 \quad T_2 = 0$

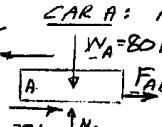
$T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{(80 \text{ kips} + 100 \text{ kips} + 80 \text{ kips})}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips})x = 0$$

$x = 279.1 \text{ ft} \qquad x = 279 \text{ ft}$ ◀

(b) FORCE IN EACH COUPLING:

CAR A: ASSUME F_{AB} TO BE IN TENSION



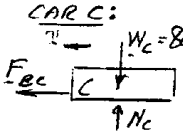
$T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips} + F_{AB})(279.1 \text{ ft}) = 0$$

$28 \text{ kips} + F_{AB} = +8.62 \text{ kips}$

$F_{AB} = -19.38 \text{ kips} \qquad F_{AB} = 19.38 \text{ kips C}$ ◀

CAR C:



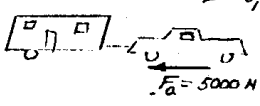
$T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 + F_{BC}(279.1 \text{ ft}) = 0$$

$F_{BC} = -8.617 \text{ kips} \qquad F_{BC} = 8.62 \text{ kip C}$ ◀

13.11

(a) CAR-TRAILER SYSTEM



$m_{car} = 1400 \text{ kg}$

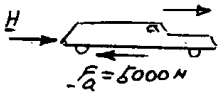
$m_{trailer} = 1200 \text{ kg}$

$$T_1 = \frac{1}{2}(m_t + m_c)v_1^2 = \frac{1}{2}(1200 \text{ kg} + 1400 \text{ kg})(20 \text{ m/s})^2 = 520 \text{ kJ}$$

$T_1 + U_{1-2} = T_2: \quad v_2 = 0 \quad T_2 = 0$

$520 \text{ kJ} - (5000 \text{ N})x = 0 \qquad x = 104.0 \text{ m}$ ◀

(b) CAR CONSIDERED SEPARATELY



$v_1 = 72 \text{ km/h} = 20 \text{ m/s}$

$v_2 = 0 \quad T_2 = 0$

$m_{car} = 1400 \text{ kg}$

$$T_1 = \frac{1}{2}(1400 \text{ kg})(20 \text{ m/s})^2 = 280 \text{ kJ}$$

$$U_{1-2} = -(5000 \text{ N})(104.0 \text{ m}) + H(104.0 \text{ m})$$
$$= -520 \text{ kJ} + H(104.0 \text{ m})$$

$T_2 = 0$

$T_1 + U_{1-2} = T_2$

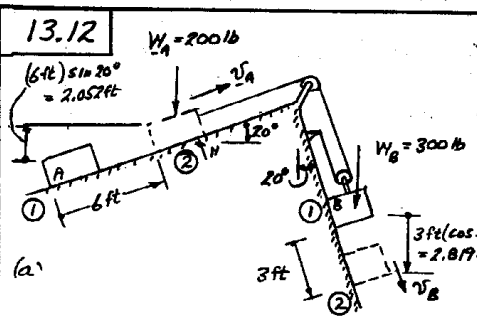
$280 \text{ kJ} - 520 \text{ kJ} + H(104.0 \text{ m}) = 0$

$H = 2.308 \text{ kN} \qquad H = 2.31 \text{ kN}$

HORIZONTAL COMPONENT OF FORCE EXERTED BY TRAILER HITCH ON CAR:

$\underline{H} = 2.31 \text{ kN} \rightarrow$ ◀

13.12



KINEMATICS $v_B = \frac{1}{2}v_A$

INITIALLY: $v_1 = 0 \quad T_1 = 0$

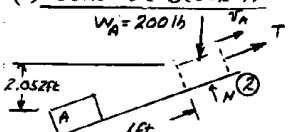
$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}\frac{200}{32.2}v_A^2 + \frac{1}{2}\frac{300}{32.2}\left(\frac{v_A}{2}\right)^2$$
$$= 3.1056 v_A^2 + 1.1646 v_A^2 = 4.2702 v_A^2$$

$U_{1-2} = -(200 \text{ lb})(2.052 \text{ ft}) + (300 \text{ lb})(2.819 \text{ ft}) = 435.3 \text{ ft·lb}$

$T_1 + U_{1-2} = T_2: \quad 0 + 435.3 \text{ ft·lb} = 4.2702 v_A^2$

$v_A = 10.096 \text{ ft/s} \qquad \underline{v_A} = 10.10 \text{ ft/s} \nearrow$ ◀

(b) CONSIDER BLOCK A:



POSITION 1: $v_1 = 0 \quad T_1 = 0$

POSITION 2: $v_2 = v_A = 10.096 \text{ ft/s}$

$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}\frac{200}{32.2}(10.096)^2 = 316.6 \text{ ft·lb}$$

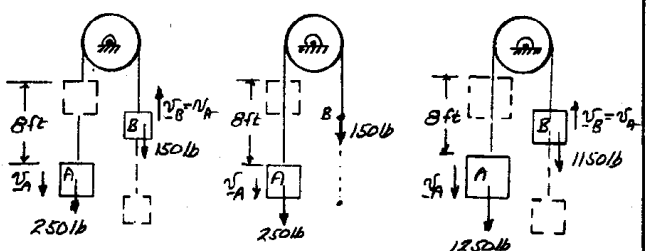
$U_{1-2} = T(6 \text{ ft}) - (200 \text{ lb})(2.052 \text{ ft})$
$= 6T - 410.4$

$T_1 + U_{1-2} = T_2: \quad 0 + 6T - 410.4 = 316.6$

$T = 121.16 \text{ lb} \qquad T = 121.2 \text{ lb}$ ◀

13.13

$v_1 = 0 \quad T_1 = 0$ KINEMATICS $v_B = v_A$



CASE (1): $T_1 + U_{1-2} = T_2$

$$0 + (250 \text{ lb})(8 \text{ ft}) - (150 \text{ lb})(8 \text{ ft}) = \frac{1}{2}\frac{250}{32.2}v_A^2 + \frac{1}{2}\frac{150}{32.2}v_A^2$$

$800 \text{ ft·lb} = \frac{1}{2}\frac{400}{32.2}v_A^2 \qquad v_A^2 = 128.8 \qquad \underline{v_A} = 11.35 \text{ ft/s} \downarrow$ ◀

CASE (2): $T_1 + U_{1-2} = T_2$

$$0 + (250 \text{ lb})(8 \text{ ft}) - (150 \text{ lb})(8 \text{ ft}) = \frac{1}{2}\frac{250}{32.2}v_A^2$$

$800 \text{ ft·lb} = \frac{1}{2}\frac{250}{32.2}v_A^2 \qquad v_A^2 = 206.1 \qquad \underline{v_A} = 14.36 \text{ ft/s} \downarrow$ ◀

CASE (3): $T_1 + U_{1-2} = T_2$

$$0 + (125 \text{ lb})(8 \text{ ft}) - (115 \text{ lb})(8 \text{ ft}) = \frac{1}{2}\frac{125}{32.2}v_A^2 + \frac{1}{2}\frac{115}{32.2}v_A^2$$

$800 \text{ ft·lb} = \frac{1}{2}\frac{240}{32.2}v_A^2 \qquad v_A^2 = 21.47$

$\underline{v_A} = 4.63 \text{ ft/s} \downarrow$ ◀

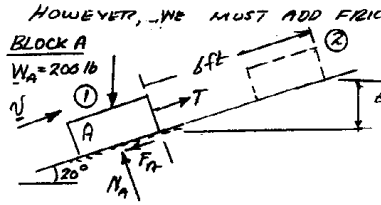
13.14

(a) SEE SOLUTION OF PROB 13.12. WE STILL HAVE THE SAME GEOMETRY AND KINEMATICS, ALSO, THE INITIAL AND FINAL KINETIC ENERGIES ARE NOT CHANGED,

$T_1 = 0 \qquad T_2 = 4.2702\, v_A^2$

HOWEVER, WE MUST ADD FRICTION FORCES TO THE BLOCKS.

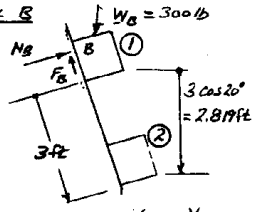
BLOCK A



$\Sigma F = 0 \quad N_A = 200 \cos 20° = 187.94$ lb

$F_A = \mu_k N_A = 0.20(187.94\text{ lb}) = 37.59$ lb

BLOCK B



$\nearrow \Sigma F = 0: \quad N_B = 300 \sin 20°$

$N_B = 102.61$ lb

$F_B = \mu_k N_B = 0.2(102.61\text{ lb}) = 20.52$ lb

WORK: $U_{1-2} = -(200\text{ lb})(2.052\text{ ft}) - (37.59\text{ lb})(6\text{ ft}) + (300\text{ lb})(2.819\text{ ft}) - (20.52\text{ lb})(3\text{ ft}) = +148.2\text{ ft·lb}$

WORK-ENERGY: $T_1 + U_{1-2} = T_2$

$0 + 148.2 = 4.2702\, v_A^2 \qquad v_A = 5.89\text{ ft/s} \angle 20°$ ◀

(b) CONSIDER ABOVE SKETCH OF BLOCK A.

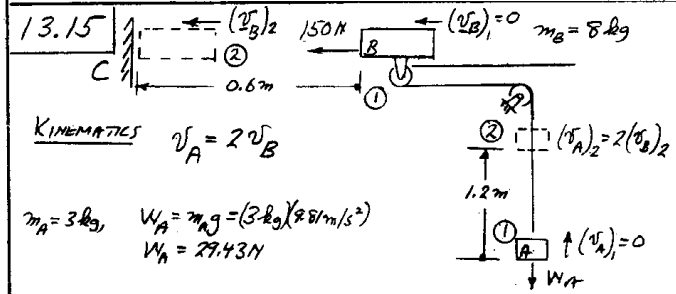
$T_1 = 0 \qquad T_2 = \frac{1}{2} m_A v_A^2 = \frac{1}{2}\left(\frac{200}{32.2}\right)(5.89)^2 = 107.7\text{ lb·ft}$

$U_{1-2} = -(200\text{ lb})(2.052\text{ ft}) - (37.59\text{ lb})(6\text{ ft}) + T(6\text{ ft})$

$= -635.94 + 6T$

$T_1 + U_{1-2} = T_2: \quad 0 - 635.94 + 6T = 107.7 \qquad T = 123.9\text{ lb}$ ◀

13.15



KINEMATICS $\quad v_A = 2 v_B$

$m_A = 3$ kg, $\quad W_A = m_A g = (3\text{ kg})(9.81\text{ m/s}^2)$

$W_A = 29.43$ N

(a) 150-N FORCE ACT THROUGH ENTIRE 0.6-m MOTION OF B

$U_{1-2} = (150\text{ N})(0.6\text{ m}) - (29.43\text{ N})(1.2\text{ m}) = 54.68\text{ J}$

$T_1 = 0 \qquad T_2 = \frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2$

$= \frac{1}{2}(3\text{ kg})\left[2(v_B)_2\right]^2 + \frac{1}{2}(8\text{ kg})(v_B)_2^2 = 10\,(v_B)_2^2$

$T_1 + U_{1-2} = T_2: \quad 0 + 54.68 = 10\,(v_B)_2^2$

$(v_B)_2 = 2.338\text{ m/s} \qquad v_B = 2.34\text{ m/s} \leftarrow$ ◀

(b) INITIAL AND FINAL VELOCITIES ARE ZERO.

$T_1 = 0 \qquad T_2 = 0$

REMOVE 150-N FORCE AFTER B MOVES DISTANCE d.

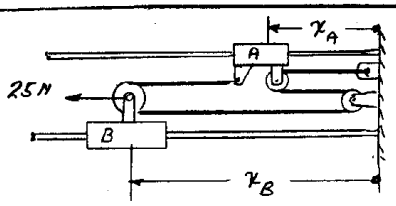
$U_{1-2} = (150\text{N})d - (29.43\text{N})(1.2\text{m})$

$= 150d - 35.316\text{ J}$

$T_1 + U_{1-2} = T_2: \quad 0 + 150d - 31.316 = 0 \qquad d = 0.2354\text{ m}$

$d = 235\text{ mm}$ ◀

13.16



LENGTH OF CABLE = CONSTANT

$2x_A + (x_B - x_A) + x_B = \text{CONSTANT}$

$x_A + 2x_B = \text{CONSTANT} \qquad \Delta x_B = -\frac{1}{2}\Delta x_A$

$v_A + 2v_B = 0 \qquad v_B = -\frac{1}{2} v_A$

INITIAL VELOCITY = 0: $\quad T_1 = 0$

(a) FOR $\quad \Delta x_A = 400\text{ mm} \rightarrow = -400\text{ mm}$

$\Delta x_B = -\frac{1}{2}(-400\text{ mm}) = +200\text{ mm} = 200\text{ mm} \leftarrow$

$U_{1-2} = (25\text{ N})(\Delta x_B) = (25\text{ N})(0.200\text{ m}) = 5\text{ J}$

$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$= \frac{1}{2}(9\text{ kg})\, v_A^2 + \frac{1}{2}(12\text{ kg})\left(-\frac{1}{2} v_A\right)^2$

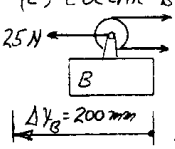
$= 4.5\, v_A^2 + 1.5\, v_A^2 = 6\, v_A^2$

$T_1 + U_{1-2} = T_2: \quad 0 + 5\text{ J} = 6\, v_A^2$

$v_A^2 = 0.8333 \qquad v_A = 0.91287 \qquad \underline{v}_A = 913\text{ mm/s} \rightarrow$ ◀

(b) $\quad v_B = -\frac{1}{2} v_A = -\frac{1}{2}(0.91287) = 0.4564 \qquad \underline{v}_B = 456\text{ mm/s} \leftarrow$ ◀

(c) COLLAR B:



$v_1 = 0 \qquad T_1 = 0$

$v_2 = 0.4564\text{ m/s}$

$T_2 = \frac{1}{2} m_B v_2^2 = \frac{1}{2}(12\text{ kg})(0.4564\text{ m/s})^2 = 1.250\text{ J}$

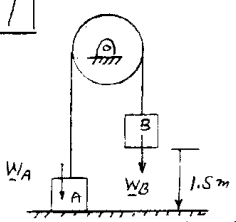
$U_{1-2} = (25\text{ N} - 2T)\Delta x_B = (25\text{ N} - 2T)(0.2\text{ m})$

$T_1 + U_{1-2} = T_2:$

$0 + (25\text{ N} - 2T)0.2\text{ m} = 1.25\text{ J}$

$5 - 0.4\,T = 1.25 \qquad T = 9.375\text{ N} \qquad \underline{T} = 9.38\text{ N}$ ◀

13.17



(a)

$W_A = m_A g = (8\text{ kg})g = 78.48\text{ N}$

$W_B = m_B g = (12\text{ kg})g = 117.72\text{ N}$

$v_1 = 0 \qquad T_1 = 0$

ENERGY DISSIPATED = 10 J

$U_{1-2} = -W_A(1.5\text{ m}) + W_B(1.5\text{ m}) - 10\text{ J}$

$= -(78.48)(1.5) + (117.72)(1.5) - 10 = 58.86\text{ J} - 10\text{ J} = 48.86\text{ J}$

$T_2 = \frac{1}{2}(m_A + m_B) v_2^2 = \frac{1}{2}(8\text{ kg} + 12\text{ kg}) v_2^2 = 10\, v_2^2$

$T_1 + U_{1-2} = T_2: \quad 0 + 48.86\text{ J} = 10 v_2^2 \qquad v_2^2 = 4.886 \qquad v_2 = 2.21\text{ m/s} \downarrow$ ◀

(b)

BLOCK A:

$T_1 = 0, \quad T_2 = \frac{1}{2} m_A v_2^2 = \frac{1}{2}(8\text{ kg})(4.886) = 19.544\text{ J}$

$U_{1-2} = (F_A - W_A)1.5\text{ m} = (F_A - 78.48\text{N})(1.5\text{m})$

$T_1 + U_{1-2} = T_2:$

$0 + (F_A - 78.48\text{ N})(1.5\text{ m}) = 19.544\text{ J}$

$F_A - 78.48\text{ N} = 13.03\text{ N} \qquad \underline{F}_A = 91.5\text{ N} \uparrow$ ◀

BLOCK B:

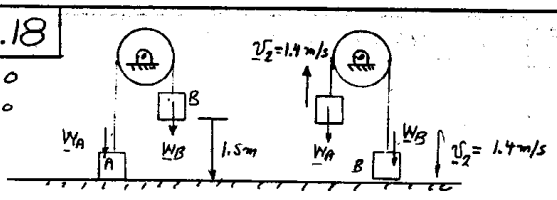
$T_1 = 0 \qquad T_2 = \frac{1}{2} m_B v_2^2 = \frac{1}{2}(12\text{ kg})(4.886) = 29.316\text{ J}$

$U_{1-2} = (W_B - F_B)(1.5\text{ m}) = (117.72\text{ N} - F_B)(1.5\text{ m})$

$T_1 + U_{1-2} = T_2: \quad 0 + (117.72\text{N} - F_B)(1.5\text{m}) = 29.316\text{ J}$

$117.72\text{N} - F_B = 19.54 \qquad \underline{F}_B = 98.2\text{ N} \uparrow$ ◀

13.18



(a)

$m_A = 12$ kg $\quad W_A = m_A g = (12)(9.81) = 117.72$ N

$m_B = 15$ kg $\quad W_B = m_B g = (15)(9.81) = 147.15$ N

$T_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(12\text{ kg} + 15\text{ kg})(1.4\text{ m/s})^2 = 26.46$ J

DENOTE BY E THE ENERGY DISSIPATED

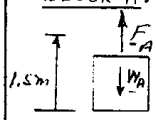
$U_{1-2} = (W_B - W_A)(1.5\text{ m}) - E$

$= (147.15\text{ N} - 117.72\text{ N})(1.5\text{ m}) - E = 44.145\text{ J} - E$

$T_1 + U_{12} = T_2$: $\quad 0 + 44.145\text{ J} - E = 26.46\text{ J}$

ENERGY DISSIPATED $= E = 17.68$ J ◀

BLOCK A:



$T_1 = 0$, $\quad T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(12\text{ kg})(1.4\text{ m/s})^2 = 11.76$ J

$U_{1-2} = (F_A - W_A)(1.5\text{ m}) = (F_A - 117.72\text{ N})(1.5\text{ m})$

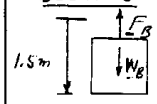
$T_1 + U_{12} = T_2$:

$0 + (F_A - 117.72\text{ N})(1.5\text{ m}) = 11.76$ J

$F_A - 117.72\text{ N} = 7.84$ N

$F_A = 125.56$ N $\quad \underline{F_A} = 125.6$ N ↑ ◀

BLOCK B:



$T_1 = 0$, $\quad T_2 = \frac{1}{2}m_B v_2^2 = \frac{1}{2}(15\text{ kg})(1.4\text{ m/s})^2 = 14.70$ J

$U_{1-2} = (W_B - F_B)(1.5\text{ m}) = (147.15\text{ N} - F_B)(1.5\text{ m})$

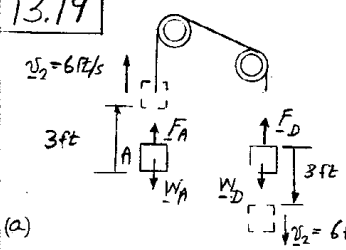
$T_1 + U_{1-2} = T_2$:

$0 + (147.15\text{ N} - F_B)(1.5\text{ m}) = 14.70$ J

$147.15\text{ N} - F_B = 9.80$ N

$F_B = 137.35$ N $\quad \underline{F_B} = 137.4$ N ↑ ◀

13.19



(a)

$W_A = 100$ lb

$W_D = 250$ lb

$v_1 = 0$, $\quad T_1 = 0$

BLOCK A:

$U_{1-2} = (F_A - W_A)(3\text{ ft}) = (F_A - 100\text{ lb})(3\text{ ft})$

$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}\frac{100\text{ lb}}{32.2}(6\text{ ft/s})^2 = 55.90$ ft·lb

$T_1 + U_{12} = T_2$: $\quad 0 + (F_A - 100\text{ lb})(3\text{ ft}) = 55.90$ ft·lb

$F_A - 100\text{ lb} = 18.634$

$F_A = 118.634$ lb $\quad \underline{F_A} = 118.6$ lb ↑ ◀

BLOCK D:

$U_{1-2} = (W_D - F_D)(3\text{ ft}) = (250\text{ lb} - F_D)(3\text{ ft})$

$T_2 = \frac{1}{2}m_B v_2^2 = \frac{1}{2}\frac{250\text{ lb}}{32.2}(6\text{ ft/s})^2 = 139.75$ ft·lb

$T_1 + U_{1-2} = T_2$: $\quad 0 + (250\text{ lb} - F_D)(3\text{ ft}) = 139.75$ ft·lb

$250\text{ lb} - F_D = 46.58$ lb $\quad F_D = 203.4$ lb

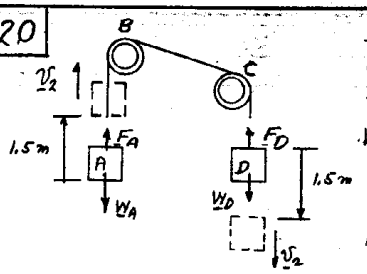
(b) EQ 8.13: $\beta = 180° = \pi$ radians $\quad \underline{F_D} = 203$ lb ↑ ◀

$\mu\beta = \ln\frac{F_D}{F_A} = \ln\frac{203.4\text{ lb}}{118.6\text{ lb}} = 0.5395$; $\quad \mu = \frac{0.5395}{\pi} = 0.172$

$\mu = 0.17$ ◀

(c) ENERGY DISSIPATED $= (F_D - F_A)(3\text{ ft}) = (203.4 - 118.6)(3) = 254$ ft·lb ◀

13.20



$m_A = 20$ kg

$W_A = m_A g = (20)g = 196.2$ N

$m_D = 55$ kg

$W_D = m_D g = (55)g = 539.6$ N

$v_1 = 0$ $\quad T_1 = 0$

ASSUME SYSTEM MOVES *

EQ 8.14: $\beta = 180° = \pi$ radians $\quad \mu_k = 0.25$

$\frac{F_D}{F_A} = e^{\mu\beta} = e^{0.25\pi} = 2.193$ $\quad F_D = 2.193\,F_A$ (1)

BLOCK A: $\quad T_1 = 0$ $\quad T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(20\text{ kg})v_2^2 = 10\,v_2^2$

$U_{1-2} = (F_A - W_A)(1.5\text{ m})$

$T_1 + U_{1-2} = T_2$: $\quad 0 + (F_A - W_A)(1.5) = 10\,v_2^2$

$v_2^2 = \frac{1.5}{10}(F_A - W_A)$ (2)

BLOCK D: $\quad T_1 = 0$ $\quad T_2 = \frac{1}{2}m_D v_2^2 = \frac{1}{2}(55\text{ kg})v_2^2 = 27.5\,v_2^2$

$U_{1-2} = (W_D - F_D)(1.5\text{ m})$

$T_1 + U_{1-2} = T_2$: $\quad 0 + (W_D - F_D)(1.5\text{ m}) = 27.5\,v_2^2$

$v_2^2 = \frac{1.5}{27.5}(W_D - F_D)$ (3)

FROM EQS (2) AND (3):

$v_2^2 = v_2^2$: $\quad \frac{1.5}{10}(F_A - W_A) = \frac{1.5}{27.5}(W_D - F_D)$

FROM EQ(1) SUBSTITUTE FOR F_D

$F_A - W_A = \frac{10}{27.5}(W_D - 2.193\,F_A)$

$F_A - 196.2\text{ N} = 0.3636(539.6\text{ N} - 2.193\,F_A)$

$F_A + 0.7974\,F_A = 196.2\text{ N} + 196.2\text{ N}$

$1.7974\,F_A = 392.4$ N $\quad F_A = 218.32$ N $\quad \underline{F_A} = 218$ N ↑ ◀

EQ(1): $\quad F_D = 2.193(218.32\text{ N}) = 478.8$ N $\quad \underline{F_D} = 479$ N ↑ ◀

EQ(2) $\quad v_2^2 = \frac{1.5}{27.5}(F_A - W_A) = \frac{1.5}{27.5}(218.32 - 196.2) = 1.2065$

$v_A = v_2 = 1.0984$ m/s $\quad \underline{v_A} = 1.098$ m/s ↑ ◀

ENERGY DISSIPATED $= (F_D - F_A)(1.5\text{ m})$

$= (478.8\text{ N} - 218.3\text{ N})(1.5\text{ m}) = 391$ J ◀

---

* WE MAY CHECK THAT SYSTEM DOES INDEED MOVE. FIND SMALLEST COEFFICIENT FOR WHICH EQUILIBRIUM IS MAINTAINED.

FOR EQUILIBRIUM $F_A = W_A = m_A g = 20\,g$

$F_D = W_D = m_D g = 55\,g$

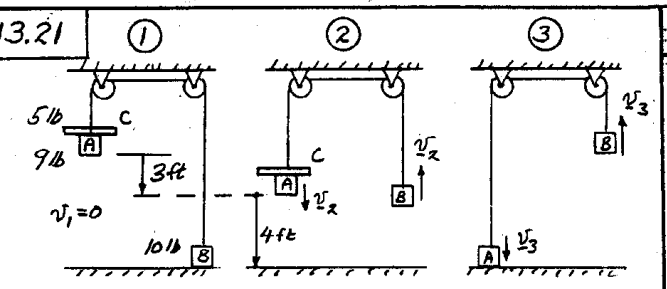
EQ 8.13: $\beta = \pi$ radians

$\mu\beta = \ln\frac{F_D}{F_A} = \ln\frac{55\,g}{20\,g} = \ln 2.75 = 1.0116$

$\mu_{\text{REQUIRED}} = \frac{1.0116}{\pi} = 0.322$

SINCE $\mu_{\text{REQ}} > \mu_s = 0.25$, SYSTEM MOVES.

13.21



FROM ① TO ②: $T_1=0$ $T_2=\frac{1}{2}\left(\frac{5lb+9lb+10lb}{g}\right)v_2^2=\frac{12}{g}v_2^2$

$U_{1-2}=(5lb+9lb)3ft-(10lb)3ft=12\ ft\cdot lb$

$T_1+U_{1-2}=T_2$: $0+12=\frac{12}{g}v_2^2$ $\quad v_2^2=g$

FROM ② TO ③: (COLLAR C HAS BEEN REMOVED)

$T_2'=\frac{1}{2}\left(\frac{9lb+10lb}{g}\right)v_2^2=9.5\frac{v_2^2}{g}=9.5\left(\frac{1}{g}\right)g=9.5\ ft\cdot lb$

$T_3=\frac{1}{2}\left(\frac{9lb+10lb}{g}\right)v_3^2=9.5\frac{v_3^2}{g}$

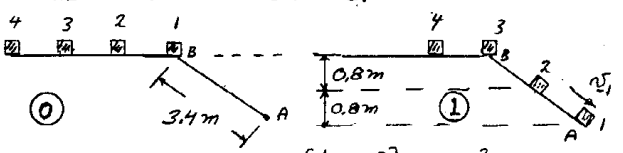
$U_{2-3}=(9lb)4ft-(10lb)4ft=-4\ ft\cdot lb$

$T_2'+U_{2-3}=T_3$: $9.5\ ft\cdot lb-4\ ft\cdot lb=9.5\frac{v_3^2}{g}$

$v_3^2=\frac{5.5}{9.5}g=10.642$ $\quad v_3=4.32\ ft/s\downarrow$ ◀

13.22

PACKAGE 1 MOVES TO A AND FALLS OFF BELT, ALL FOUR PACKAGES MOVE.



$v_0=0$, $T_0=0$; $T_1=4\left[\frac{1}{2}mv_1^2\right]=2mv_1^2$

ONLY WEIGHTS OF PACKAGES 1 AND 2 DO WORK

$U_{0-1}=mg(1.6m)+mg(0.8m)=2.4mg$

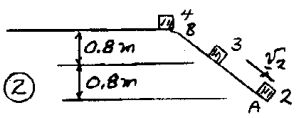
$T_0+U_{0-1}=T_1$: $0+2.4mg=2mv_1^2$

$v_1^2=1.2g=1.2(9.81)=11.772$ $\quad v_1=3.43\ m/s$ ◀

PACKAGE 2 MOVES TO A (PACKAGE 1 HAS FALLEN OFF THE BELT. ITS KINETIC ENERGY IS LOST TO THE SYSTEM.)

KINETIC ENERGY OF REMAINING 3 PACKAGES

$T_1'=3\left[\frac{1}{2}mv_1^2\right]=\frac{3}{2}m(11.772)=17.658\ m$



WEIGHTS OF PACKAGES 2 AND 3 DO WORK

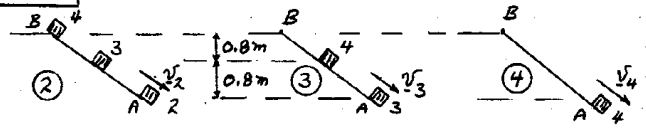
$U_{1-2}=mg(0.8m)+mg(0.8m)=1.6mg=15.696\ m$

$T_1'+U_{1-2}=T_2$: $17.658\ m+15.696\ m=3\left[\frac{1}{2}mv_2^2\right]$

$v_2^2=22.236$ $\quad v_2=4.72\ m/s$ ◀

13.23

SEE SOLUTION OF PROB 13.22



PACKAGE 3 MOVES TO A: (PACKAGES 1 AND 2 HAVE FALLEN OFF OF THE BELT). KINETIC ENERGY OF REMAINING PACKAGES 3 AND 4 WITH $v_2=4.72\ m/s$, IS

$T_2=2\left[\frac{1}{2}mv_2^2\right]=mv_2^2=m(4.72)^2=22.28m$

WEIGHTS OF PACKAGES 3 AND 4 DO WORK.

$U_{2-3}=mg(0.8m)+mg(0.8m)=1.6m(9.81)=15.696m$

$T_2+U_{2-3}=T_3$: $22.28\ m+15.696m=2\left[\frac{1}{2}mv_3^2\right]$

$v_3^2=37.976$ $\quad v_3=6.16\ m/s$ ◀

PACKAGE 4 MOVE TO A: ONLY PACKAGE 4 REMAINS TO BE CONSIDERED.

$T_3'=\frac{1}{2}mv_3^2=\frac{1}{2}m(37.976)=18.988\ m$

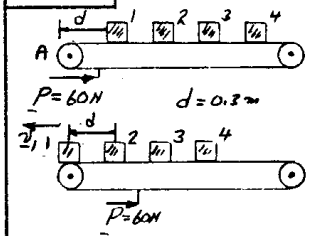
$U_{3-4}=mg(0.8m)=(0.8)m(9.81)=7.848\ m$

$T_3'+U_{3-4}=T_4$: $18.988\ m+7.848m=\frac{1}{2}mv_4^2$

$v_4^2=53.672$ $\quad v_4=7.33\ m/s$ ◀

13.24

PACKAGE 1 MOVES TO A.



POSITION "0": $v_0=0$, $T_0=0$

SYSTEM MOVES $d=0.3m$ TO

POSITION 1:

$U_{0-1}=Pd=(60N)(0.3m)=18J$

$T_1=4\left(\frac{1}{2}mv_1^2\right)$

$=4\left[\frac{1}{2}(4kg)(v_1^2)\right]=8v_1^2$

$T_0+U_{0-1}=T_1$

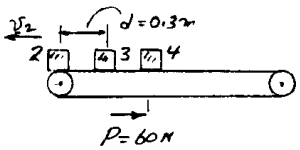
$0+18J=8v_1^2$

$v_1^2=2.25$ $\quad v_1=1.5\ m/s$ ◀

PACKAGE 2 MOVES TO A. (PACKAGE 1 HAS FALLEN OFF OF THE BELT. ITS KINETIC ENERGY IS LOST TO THE SYSTEM.)

KINETIC ENERGY OF 3 REMAINING PACKAGES IS

$T_1=3\left[\frac{1}{2}mv_1^2\right]=3\left[\frac{1}{2}(4kg)(2.25)\right]=13.5J$



POSITION 2

FORCE P HAS MOVED THROUGH ANOTHER 0.3m

$U_{1-2}=Pd=(60N)(0.3m)=18J$

$T_2=3\left[\frac{1}{2}mv_2^2\right]=3\left[\frac{1}{2}(4kg)v_2^2\right]=6v_2^2$

$T_1+U_{1-2}=T_2$

$13.5J+18J=6v_2^2$

$v_2^2=5.25$ $\quad v_2=2.29\ m/s$ ◀

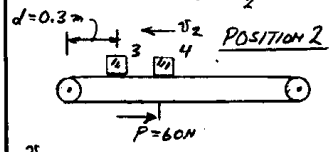
13.25

PACKAGE 3 MOVES TO A. PACKAGES 1 AND 2 HAVE FALLEN OFF OF THE BELT. THEIR KINETIC ENERGY IS LOST TO THE SYSTEM.

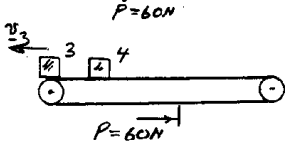
SINCE $v_2 = 2.29$ m/s THE KINETIC ENERGY OF PACKAGES 3 AND 4 IS:

$$T_2 = 2\left[\tfrac{1}{2} m v_2^2\right] = 2\left[\tfrac{1}{2}(4\text{ kg})(2.29\text{ m/s})^2\right]$$

$$T_2 = 21.0\text{ J}$$



POSITION 2



POSITION 3 FORCE P HAS MOVED THROUGH $d = 0.3$ m

$$U_{2-3} = Pd = (60\text{N})(0.3\text{m}) = 18\text{ J}$$

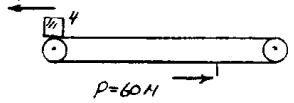
$$T_3 = 2\left[\tfrac{1}{2} m v_3^2\right] = 2\left[\tfrac{1}{2}(4)v_3^2\right] = 4v_3^2$$

$$T_2 + U_{2-3} = T_3: \quad 21\text{J} + 18\text{J} = 4v_3^2$$

$$v_3^2 = 9.75 \qquad \underline{v_3} = 3.12\text{ m/s} \blacleftarrow$$

PACKAGE 4 MOVES TO A. PACKAGE 3 HAS FALLEN OFF OF THE BELT. ITS KINETIC ENERGY IS LOST TO SYSTEM. KINETIC ENERGY OF REMAINING PACKAGE 4 IS:

$$T_3 = \tfrac{1}{2} m v_3^2 = \tfrac{1}{2}(4\text{kg})(9.75) = 19.5\text{ J}$$



POSITION 4 FORCE P HAS AGAIN MOVED THROUGH $d = 0.3$ m.

$$U_{3-4} = Pd = (60\text{N})(0.3\text{m}) = 18\text{J}$$

$$T_4 = \tfrac{1}{2} m v_4^2 = \tfrac{1}{2}(4\text{kg}) v_4^2 = 2v_4^2$$

$$T_3 + U_{3-4} = T_4: \quad 19.5\text{J} + 18\text{J} = 2v_4^2$$

$$v_4^2 = 18.75 \qquad \underline{v_4} = 4.33\text{ m/s} \blacleftarrow$$

13.26

SEE SKETCH OF SOLUTION OF PROBL 13.21 REPLACE 5 lb COLLAR BY COLLAR OF WEIGHT W_C

FROM ① TO ②: $T_1 = 0$ $\quad T_2 = \tfrac{1}{2}(W_C + 9\text{lb} + 10\text{lb})\dfrac{v_2^2}{g}$

$$U_{1-2} = (W_C + 9\text{lb})3\text{ft} - (10\text{lb})3\text{ft} = 3W_C - 3$$

$$T_1 + U_{1-2} = T_2: \quad 0 + (3W_C - 3) = \tfrac{1}{2}(W_C + 19)\frac{v_2^2}{g}$$

$$\frac{v_2^2}{g} = \frac{2(3W_C - 3)}{W_C + 19} = \frac{6W_C - 6}{W_C + 19}$$

FROM ② TO ③: (COLLAR C HAS BEEN REMOVED)

$$T_2' = \tfrac{1}{2}(9\text{lb} + 10\text{lb})\frac{v_2^2}{g} = \frac{19}{2}\left(\frac{6W_C - 6}{W_C + 19}\right)$$

WE NOW WISH TO HAVE $v_3 = 0$, THUS $T_3 = 0$

$$U_{2-3} = (9\text{lb})4\text{ft} - (10\text{lb})4\text{ft} = -4\text{ ft·lb}$$

$$T_2' + U_{2-3} = T_3: \quad \frac{19}{2}\left(\frac{6W_C - 6}{W_C + 19}\right) - 4\text{ ft·lb} = 0$$

$$19(6W_C - 6) - 8(W_C + 19) = 0$$

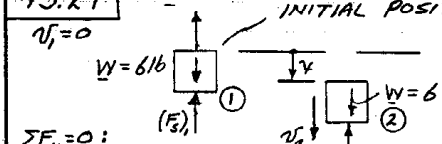
$$114W_C - 114 - 8W_C - 152 = 0$$

$$106W_C - 266 = 0$$

$$W_C = 2.509\text{ lb}$$

$$W_C = 2.51\text{ lb} \blacleftarrow$$

13.27



$k = 8\text{ lb/in} = 96\text{ lb/ft}$

$v_1 = 0$

$\Sigma F_y = 0$: $\quad (F_s)_1 = 6 - 3 = 3\text{ lb}$ C

$F_s = (F_s)_1 + kx = 3\text{ lb} + (96\text{ lb/ft})x$

$v_1 = 0$ $\quad T_1 = 0$: $\quad T_2 = \tfrac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)v_2^2 = 0.09317\,v_2^2$

FOR WEIGHT: $U_{1-2} = (6\text{ lb})x = 6x$

FOR SPRING: $U_{1-2} = -\int_0^x (3 + 96x)\,dx = -3x - 48x^2$

$$T_1 + U_{1-2} = T_2: \quad 0 + 6x - 3x - 48x^2 = 0.09317\,v_2^2$$

$$3x - 48x^2 = 0.09317\,v_2^2 \qquad (1)$$

(a) For x_m, $v_2 = 0$: $\quad 3x - 48x^2 = 0$

$$x = 0, \quad x_m = \frac{3}{48} = \frac{1}{16}\text{ ft} \qquad x_m = 0.75\text{ in.} \downarrow \blacleftarrow$$

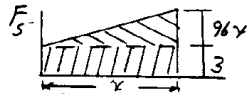
(b) For v_m ING SEEK MAXIMUM OF $U_{1-2} = 3x - 48x^2$

$$\frac{dU_{1-2}}{dx} = 3 - 96x = 0 \qquad x = \frac{3}{96}\text{ ft} = \frac{1}{32}\text{ ft}$$

EQ. (1): $\quad 3\left(\frac{1}{32}\text{ ft}\right) - 48\left(\frac{1}{32}\text{ ft}\right)^2 = 0.09317\,v_m^2$

$$v_m^2 = 0.5031 \qquad v_m = 0.7093\text{ ft/s} \qquad \underline{v_m} = 8.51\text{ in/s} \uparrow \blacleftarrow$$

NOTE U_{1-2} FOR THE SPRING MAY BE COMPUTED USING F_s-x CURVE



U_{1-2} = AREA $= 3x + \tfrac{1}{2}96x^2$

13.28

OUTER SPRING:

$$U_{1-2} = -\tfrac{1}{2}kx^2 = -\tfrac{1}{2}(3000\text{ N/m})(0.150\text{ m})^2 = -33.75\text{ J}$$

INNER SPRING: $U_{1-2} = -\tfrac{1}{2}(10\,000\text{ N/m})(0.06\text{ m})^2 = -18.00\text{ J}$

GRAVITY: $U_{1-2} = W(h + 0.150\text{ m})$

$$= (8\text{ kg} \times 9.81\text{ m/s}^2)(h + 0.150\text{ m})$$

$$= 78.48h + 11.772$$

TOTAL WORK: $U_{1-2} = 78.48h + 11.772 - 33.75 - 18.00$

$$= 78.48h - 39.978$$

SINCE $T_1 = T_2 = 0$, $\quad T_1 + U_{1-2} = T_2$ REDUCES TO

$U_{1-2} = 0$: $\quad 78.48h - 39.978 = 0 \qquad h = 0.5094\text{ m}$

$$h = 509\text{ mm} \blacleftarrow$$

13.29

x = VERTICAL DEFLECTION OF OUTER SPRING

ASSUME $x > 90\text{ mm} = 0.090\text{ m}$

OUTER SPRING: $U_{1-2} = -\tfrac{1}{2}(3000\text{ N/m})x^2 = -1500x^2$

INNER SPRING: $U_{1-2} = -\tfrac{1}{2}(10\,000\text{ N/m})(x - 0.090\text{ m})^2$

GRAVITY: FOR $h = 600\text{ mm} = 0.6\text{ m}$

$$U_{1-2} = (8\text{ kg} \times 9.81\text{ m/s}^2)(x + 0.6\text{ m})$$

$$= 78.48x + 47.09$$

TOTAL WORK: $U_{1-2} = 78.48x + 47.09 - 1500x^2 - \tfrac{1}{2}(10000)(x - 0.090)^2$

$$U_{1-2} = 78.48x + 47.09 - 1500x^2 - 5000x^2 + 900x - 40.5$$

$$= -6500x^2 + 978.48x + 6.59$$

SINCE $T_1 = T_2 = 0$, $\quad T_1 + U_{1-2} = T_2$ REDUCES TO

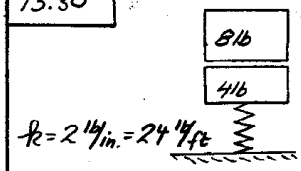
$U_{1-2} = 0$: $\quad -6500x^2 + 978.48x + 6.59 = 0$

$x = -0.0065\text{ m}$ (IMPOSSIBLE)

$x = 0.1570\text{ m}$

MAXIMUM DEFLECTION = 157.0 mm ◀

13.30

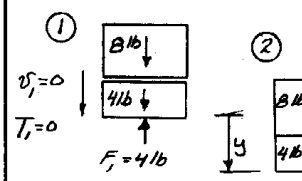


8-lb BLOCK RELEASED WITH $v_1 = 0$

INITIAL FORCE IN SPRING

$F_1 = 4$ lb

y = ADDITIONAL DEFORMATION OF SPRING



$F = F_1 + ky = 4 + (24\ \text{lb/ft})y$

$$T_2 = \frac{1}{2}\frac{(8+4)\text{lb}}{g}v_2^2 = \frac{6}{g}v_2^2$$

WORK: BLOCKS: $U_{1-2} = (8+4)y = 12y$

FORCE EXERTED BY SPRING:

$$U_{1-2} = -\int_0^y (4+24y)\,dy = -4y - 12y^2$$

$$T_1 + U_{1-2} = T_2: \quad 0 + 12y - 4y - 12y^2 = \frac{6}{g}v_2^2$$

$$8y - 12y^2 = \frac{6}{g}v_2^2 \qquad (1)$$

(a) MAXIMUM VELOCITY occurs when

$$\frac{d}{dy}(8y - 12y^2) = 0; \quad 8 - 24y = 0; \quad y = \tfrac{1}{3}\text{ ft}$$

EQ. (1): $8\left(\frac{1}{3}\right) - 12\left(\frac{1}{3}\right)^2 = \frac{6}{32.2}v_{max}^2$

$v_{max}^2 = 7.156$ $\quad v_{max} = 2.67$ ft/s ◀

(b) MAXIMUM FORCE EXERTED BY SPRING occurs when y is MAX.

FOR y = MAXIMUM, $v_2 = 0$, THUS:

$8y - 12y^2 = 0$ $\quad y = \frac{2}{3}$ ft

$F_{MAX} = 4 + 24y = 4 + 24\left(\frac{2}{3}\right)$ $\quad F_{MAX} = 20$ lb ◀

13.31

$$T_1 = \frac{1}{2}\frac{(90\text{ tons})(2000\text{ lb/ton})}{32.2\text{ ft/s}^2}\left(\frac{88}{60}\text{ ft/s}\right)^2 = 6012\text{ ft-lb}$$

$T_2 = 0$ $\quad T_1 + U_{1-2} = T_2$

$6012\text{ ft-lb} + U_{1-2} = 0$ $\quad U_{1-2} = -6012\text{ ft-lb} = -72.15$ kip-in.

FENDER A:

FROM DIAGRAM: $F = -kx$ $\quad k = \frac{60\text{ kips}}{12\text{ in.}} = 5$ kips/in.

$$U_{1-2} = \int_0^x F\,dx = -\int_0^x 5x\,dx = -\frac{5}{2}x^2 = -72.15$$

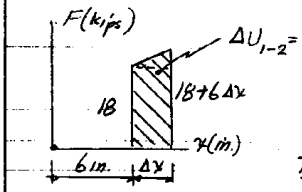
$x^2 = 28.86$ $\quad x = 5.37$ in. ◀

FENDER B: WE DIVIDE AREA UNDER CURVE B INTO TRAPEZOIDS

	PARTIAL AREA	TOTAL AREA
FROM $x=0$ TO $x=2$ in.:	$\frac{1}{2}(2\text{in.})(4\text{ kips}) = 4$ kip-in.	4 kip-in.
FROM $x=2$ in. TO $x=4$ in.:	$\frac{1}{2}(2\text{in.})(4+10) = 14$ kip-in.	18 kip-in.
FROM $x=4$ in. TO $x=6$ in.:	$\frac{1}{2}(2\text{in.})(10+18) = 28$ kip-in.	46 kip-in.

WE STILL NEED $\Delta U = 72.15 - 46 = 26.15$ kip-in.

EQUATION OF STRAIGHT LINE APPROXIMATING CURVE B FROM $x = 6$ in. TO $x = 8$ in. IS: $\frac{\Delta x}{x} = \frac{F-18}{30-18}$ $\quad F = 18 + 6\Delta x$



$\Delta U_{1-2} = 18\Delta x + \frac{1}{2}(6\Delta x)\Delta x = 26.15$ kip-in.

$(\Delta x)^2 + 6\Delta x - 8.716 = 0$

$\Delta x = 1.209$ in.

THUS: $x = 6\text{in.} + 1.209\text{in.} = 7.209$ in.

$x = 7.21$ in. ◀

13.32

(a) $v_1 = 90\text{ mi/h} = 132\text{ ft/s}$

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{15000\text{ lb}}{32.2\text{ ft/s}^2}\right)(132\text{ ft/s})^2 = 4.058\times10^6\text{ ft-lb} = 4058\text{ ft-kips}$$

$T_2 = 0$: $\quad T_1 + U_{1-2} = T_2$

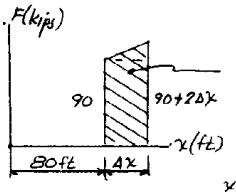
$4058\text{ ft-lb} + U_{1-2} = 0$ $\quad U_{1-2} = -4058$ ft-kips

WE SEEK DISTANCE x FOR WHICH AREA UNDER F-x CURVE IS $-U_{1-2} = 4058$ ft-kips. THE ESTIMATE OF THE AREA IS FACILITATED IF WE DIVIDE THE AREA INTO TRAPEZOIDS

	PARTIAL AREA	TOTAL AREA
FROM $x=0$ TO $x=20$ ft:	$\frac{1}{2}(20\text{ft})(18\text{ kips}) = 180$ ft-kips	180 ft-kips
FROM $x=20$ TO $x=40$ ft:	$\frac{1}{2}(20\text{ft})(18+38) = 560$ ft-kips	740 ft-kips
FROM $x=40$ TO $x=60$ ft:	$\frac{1}{2}(20\text{ft})(38+60) = 980$ ft-kips	1720 ft-kips
FROM $x=60$ TO $x=80$ ft:	$\frac{1}{2}(20\text{ft})(60+90) = 1500$ ft-kips	3220 ft-kips

WE STILL NEED $\Delta U = 4058 - 3220 = 838$ ft-kips.

EQUATION OF STRAIGHT LINE APPROXIMATING CURVE, FROM $x = 80$ ft TO $x = 100$ ft



$\frac{\Delta x}{20} = \frac{F-90}{130-90}$ $\quad F = 90 + 2\Delta x$

$\Delta U = 90\Delta x + \frac{1}{2}(2\Delta x)(\Delta x) = 838$ ft-kips

$(\Delta x)^2 + 90\Delta x - 838 = 0$

$\Delta x = 8.51$ m

$x = 80 + 8.51 = 88.51$ ft $\quad x = 88.5$ ft ◀

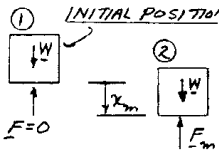
(b) MAXIMUM FORCE occurs when $x = 88.51$ ft

$|F_{MAX}| = 90 + 2\Delta x = 90 + 2(8.51) = 107.02$ kips

$|F_{MAX}| = m|a_{MAX}|$: $\quad 107,020\text{ lb} = \frac{150001\text{b}}{32.2\text{ ft/s}^2}|a_{MAX}|$

$|a_{MAX}| = 229.7$ ft/s^2 $\quad |a_{MAX}| = 230$ ft/s^2 ◀

13.33



$W = mg = (S\cdot\rho)g$ $\quad W = 49.05$ N

SINCE $T_1 = T_2 = 0$, $T_1 + U_{1-2} = T_2$ YIELDS $U_{1-2} = 0$

$$U_{1-2} = W x_m - \int_0^{x_m} F\,dx = 49.05 x_m - \int_0^{x_m} F\,dx = 0 \qquad (1)$$

(a) FOR $F = kx$, $k = (3000\text{ N/m})x$

EQ. (1): $\quad 49.05 x_m - \int_0^{x_m} 3000x\,dx = 0$

$49.05 x_m - 1500 x_m^2 = 0$ $\quad x_m = 32.7\times10^{-3}\text{ m} = 32.7$ mm ◀

$F_m = 3000 x_m = 3000(32.7\times10^{-3})$ $\quad F_m = 98.1$ N ◀

(b) FOR $F = (3000\text{ N/m})x(1 + 160x^2)$

EQ. (1): $\quad 49.05 x_m - \int_0^{x_m} 3000(x + 160x^3)\,dx = 0$

$$49.05 x_m - 3000\left(\tfrac{1}{2}x_m^2 + 40x_m^4\right) = 0 \qquad (2)$$

SOLVE BY TRIAL: $x_m = 30.44\times10^{-3}$ m $\quad x_m = 30.4$ mm ◀

$F_m = (3000)(30.44\times10^{-3})\left[1 + 160(30.44\times10^{-3})^2\right]$ $\quad F_m = 104.9$ N ↑ ◀

13.34

(a) SEE SOLUTION OF PROB 13.33(a).

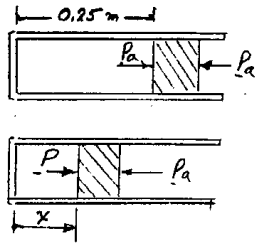
(b) FOR $F = (3000\text{ N/m})x(1 - 160x^2)$:

EQ. (2) OF PROB 13.34 BECOMES: $49.05 x_m - 3000\left(\frac{1}{2}x_m^2 - 40x_m^4\right) = 0$

SOLVE BY TRIAL: $x_m = 36.63\times10^{-3}$ m $\quad x_m = 36.6$ mm ↓ ◀

$F_m = (3000)(36.63\times10^{-3})\left[1 - 160(36.63\times10^{-3})^2\right]$ $\quad F_m = 86.3$ N ↑ ◀

13.35



$m = 4\ kg$
DIAMETER = 100 mm
$p_a = 101.3\ kN/m^2$

FOR $x = 250\ mm = 0.25\ m$
INSIDE PRESSURE = p_a

RESULTANT OF PRESSURE FORCES ON OUTSIDE FACE

$P_a = p_a A = (101.3\ kN/m^2)\frac{\pi}{4}(0.100\ m)^2 = 795.6\ N$

ON INSIDE FACE, PRESSURE VARIES INVERSELY AS x

$P = pA = \left(p_a \frac{0.25\ m}{x}\right)A = p_a A\left(\frac{0.25}{x}\right) = P_a\left(\frac{0.25}{x}\right)$

POSITION 1: $x_1 = 120\ mm = 0.120\ m$, $v_1 = 0$ $T_1 = 0$

POSITION 2: $x_2 = 250\ mm = 0.25\ m$, (MAX v OCCURS AT EQUIL)

$v_2 = v_m$ $T_2 = \frac{1}{2}mv_m^2 = \frac{1}{2}(4\ kg)v_m^2$

$$U_{1-2} = \int_{x_1}^{x_2}(P - P_a)\,dx = \int_{0.12}^{0.25}\left[P_a\left(\frac{0.25}{x}\right) - P_a\right]dx = P_a\left|0.25\ln x - x\right|_{0.12}^{0.25}$$

$$= P_a\left|0.25\ln\frac{0.25}{0.12} - 0.25 + 0.12\right| = P_a(53.492\times10^{-3})$$

$$U_{1-2} = (795.6\ N)(53.492\times10^{-3}\ m) = 42.558\ J$$

$T_1 + U_{1-2} = T_2$: $0 + 42.558 = \frac{1}{2}(4)v_m^2$

$v_m^2 = 21.279$ $v_m = 4.61\ m/s$ ◀

13.36

SEE SOLUTION OF PROB 13.35.

FOR x_{max}, $v_2 = 0$ THUS $T_2 = 0$

SINCE $v_1 = 0$, $T_1 = 0$, $T_1 + U_{1-2} = T_2$ YIELDS $U_{1-2} = 0$

$$U_{1-2} = \int_{x_1}^{x_m}(P - P_a)\,dx = \int_{0.12}^{x_m}\left[P_a\left(\frac{0.25}{x}\right) - P_a\right]dx = P_a\left|0.25\ln x - x\right|_{0.12}^{x_m}$$

$$P_a\left[0.25\ln\frac{x_m}{0.12} - x_m + 0.12\right] = 0$$

$\ln\frac{x_m}{0.12} = \frac{x_m - 0.12}{0.25}$ SOLVE BY TRIAL:

$x_m = 0.451\ m = 451\ mm$ ◀

13.37

POSITION 1 AT SURFACE OF MOON:

$v_1 = 600\ m/s$ $T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m(600\ m/s)^2 = m(180\times10^3)$

POSITION 2: $y = h$ = MAXIMUM ELEVATION, $v = 0$, $T_2 = 0$

(a) UNIFORM GRAVITATIONAL FIELD. $a = 1.62\ m/s^2$

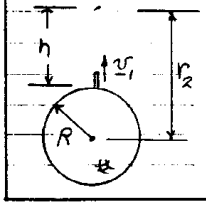
$g = 1.62\ m/s^2$ $W = mg = m(1.62)$

$U_{1-2} = -Wh = -m(1.62)h$

$T_1 + U_{1-2} = T_2$: $m(180\times10^3) - m(1.62)h = 0$

$h = 111.11\times10^3\ m$ $h = 111.1\ km$ ◀

(b) NEWTON'S LAW OF GRAVITATION



POSITION 1: SAME AS ABOVE $T_1 = m(180\times10^3)$

$r_1 = R = 1740\ km = 1740\times10^3\ m$

$W = \frac{GMm}{R^2}$ ALSO $W = mg$

$\therefore \frac{GMm}{R^2} = mg$ $GM = gR^2$

(CONTINUED)

13.37 CONTINUED

h = MAXIMUM ALTITUDE

POSITION 2: $r_2 = R + h$ $v_2 = 0$ $T_2 = 0$

EQ(13.7): $U_{1-2} = \frac{GMm}{r_2} - \frac{GMm}{r_1} = GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$U_{1-2} = gR^2m\left(\frac{1}{r_2} - \frac{1}{R}\right) = gRm\left(\frac{R}{r_2} - 1\right)$

$T_1 + U_{1-2} = T_2$: $m(180\times10^3) + gRm\left(\frac{R}{r_2} - 1\right) = 0$

$180\times10^3 = (1.62\ m/s^2)(1740\times10^3\ m)\left(1 - \frac{R}{r_2}\right)$

$1 - \frac{R}{r_2} = 63.857\times10^{-3}$ $\frac{R}{r_2} = 0.936143$

$r_2 = \frac{R}{0.936143} = \frac{1740\ km}{0.936143} = 1858.69\ km$

$h = r_2 - R = 1858.69\ km - 1740\ km = 118.69\ km$

$h = 118.7\ km$ ◀

13.38

$F = G\frac{Mm}{r^2}$

POSITION 1: $r_1 = \infty$, $v_1 = 0$ $T_1 = 0$

POSITION 2: $r_2 = R$, $v = v_m$, $T_2 = \frac{1}{2}mv_m^2$

$$U_{1-2} = \int_{r_1}^{r_2}F\,dr = \int_{\infty}^{R}-G\frac{Mm}{r^2}\,dr = -GMm\left|-\frac{1}{r}\right|_{\infty}^{R} = +\frac{GMm}{R}$$

$T_1 + U_{1-2} = T_2$: $0 + \frac{GMm}{R} = \frac{1}{2}mv_m^2$

$$v_m^2 = \frac{2GM}{R} \quad (1)$$

(a) FOR EARTH: AT SURFACE, $r = R = 6370\ km$

$F = W = \frac{GMm}{R^2}$ AND $W = mg$ $\therefore GM = gR^2$ (2)

EQ(1): $v_m^2 = \frac{2}{R}(gR^2) = 2gR = 2(9.81\ m/s^2)(6370\times10^3\ m)$

$v_m = 11.179\times10^3\ m/s$ $v_m = 11.18\ km/s$ ◀

(b) FOR THE MOON:

EQUATION (1) IS VALID PROVIDED WE USE GM AND R FOR THE MOON.

$$v_m^2 = 2\frac{GM_m}{R_m} = 2\frac{GM_e}{R_e}\cdot\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)$$

BUT FROM EQ(2): $GM_e = g_eR_e^2$

$$v_m^2 = 2g_eR_e\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right) \quad (3)$$

DATA: $g_e = 9.81\ m/s^2$

$\left(\frac{M_m}{M_e}\right) = 0.01230$

$R_e = 6370\ km$, $R_m = 1740\ km$

EQ(3) $v_m^2 = 2(9.81\ m/s^2)(6370\times10^3\ m)(0.01230)\left(\frac{6370\ km}{1740\ km}\right)$

$v_m = 2372.3\ m/s$ $v_m = 2.37\ km/s$ ◀

13.39

POSITION 1: $r = r_1$, $v = v_1$ $\quad T_1 = \frac{1}{2}mv_1^2$

POSITION 2: $r = r_2$, $v = v_2$ $\quad T_2 = \frac{1}{2}mv_2^2$

EQ(13.7) $\quad U_{1-2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$

$T_1 + U_{1-2} = T_2$: $\quad \frac{1}{2}mv_1^2 + \frac{GMm}{r_2} - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2$

BUT $GM = gR^2$: $\quad v_1^2 + 2gR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = v_2^2$

$$v_1^2 - v_2^2 = 2gR\left(\frac{R}{r_1} - \frac{R}{r_2}\right) \qquad (1)$$

GIVEN: $v_1 = 16{,}000$ ft/s $\quad v_2 = 0$

$R = 3960$ mi $= 3960 \times 5280 = 20.909 \times 10^6$ ft

$r_1 = 3960$ mi $+ 60$ mi $= 4020$ mi

$\frac{R}{r_1} = \frac{3960 \text{ mi}}{4020 \text{ mi}} = 0.985075$

EQ(1): $(16{,}000)^2 - 0 = 2(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})\left(0.985075 - \frac{R}{r_2}\right)$

$\frac{R}{r_2} = 0.985075 - \frac{(16{,}000)^2}{2(32.2)(20.909 \times 10^6)} = 0.985075 - 0.190117$

$\frac{R}{r_2} = 0.794958$ $\quad r_2 = \frac{3960 \text{ mi}}{0.794958} = 4981.4$ mi

MAX ALTITUDE: $h = r_2 - R = 4981.4 \text{ mi} - 3960 \text{ mi} = 1021.4$ mi

$h = 1021$ mi ◀

13.40

SEE PROB 13.39 FOR DERIVATION OF EQ(1):

$$v_1^2 - v_2^2 = 2gR\left(\frac{R}{r_1} - \frac{R}{r_2}\right) \qquad (1)$$

GIVEN: $v_1 = v_B$ $\quad r_1 = 3960 \text{ mi} + 60 \text{ mi} = 4020$ mi

$v_2 = 0$ $\quad r_2 = 3960 \text{ mi} + 900 \text{ mi} = 4860$ mi

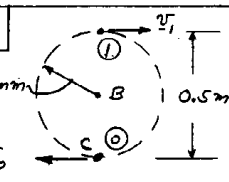
$R = 3960 \text{ mi} = 3960 \times 5280 = 20.909 \times 10^6$ ft

EQ(1): $v_B^2 - 0 = 2(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})\left[\frac{3960 \text{ mi}}{4020 \text{ mi}} - \frac{3960 \text{ mi}}{4860 \text{ mi}}\right]$

$v_B^2 = 1.3465 \times 10^9 \left[0.985075 - 0.814815\right] = 229.26 \times 10^6$

$v_1 = 15.141 \times 10^3$ ft/s $\quad v_1 = 15{,}141$ ft/s ◀

13.41



$W = mg$

$U_{0-1} = -W(0.5\text{m}) = -mg(0.5\text{m})$

POSITION 0: $T_0 = \frac{1}{2}mv_0^2$

POSITION 1: $T_1 = \frac{1}{2}mv_1^2$

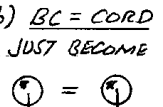
$T_0 + U_{0-1} = T_1$: $\quad \frac{1}{2}mv_0^2 - 0.5mg = \frac{1}{2}mv_1^2$

$v_0^2 = v_1^2 + g$ $\quad v_0^2 = v_1^2 + 9.81$ $\qquad (1)$

(a) BC = SLENDER ROD: $v_1 \approx 0$ WHEN C IS ABOVE B

EQ(1): $v_0^2 = 0 + 9.81$ $\quad v_0 = 3.132$ m/s $\quad v_0 = 3.13$ m/s ◀

(b) BC = CORD: FOR SMALLEST v_0, THE CORD MUST JUST BECOME SLACK WHEN C IS ABOVE B, TENSION = 0



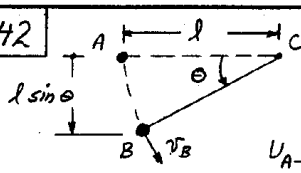
$\downarrow \Sigma F_y = ma_y$: $\quad mg = ma_n = m\frac{v_1^2}{r}$

$v_1^2 = rg = (0.25g)$

EQ(1): $v_0^2 = 0.25g + g = \frac{5}{4}g = 12.263$

$v_0 = 3.502$ m/s $\quad v_0 = 3.50$ m/s ◀

13.42



POSITION A: $v_A = 0$ $\quad T_A = 0$

POSITION B: $v = v_B$

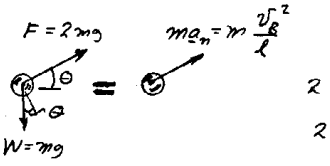
$T_B = \frac{1}{2}mv_B^2$

$U_{A-B} = Wl\sin\theta = mgl\sin\theta$

$T_A + U_{A-B} = T_B$: $\quad 0 + mgl\sin\theta = \frac{1}{2}mv_B^2$

$$v_B^2 = 2gl\sin\theta \qquad (1)$$

FOR TENSION EQUAL TO 2W. $\quad F = 2W = 2mg$



$+\nearrow \Sigma F = ma_n$

$2mg - mg\sin\theta = m\frac{v_B^2}{l}$

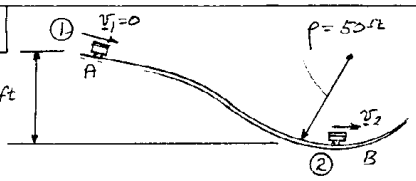
$2 - \sin\theta = \frac{v_B^2}{gl}$ $\qquad (2)$

SUBSTITUTE FOR v_B^2 FROM EQ(1) INTO EQ(2):

$2 - \sin\theta = \frac{2gl\sin\theta}{gl} = 2\sin\theta$

$2 = 3\sin\theta$ $\quad \sin\theta = \frac{2}{3}$ $\quad \theta = 41.8°$ ◀

13.43



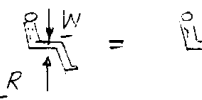
POSITION 1: $v_1 = 0$ $\quad T_1 = 0$

POSITION 2: $v = v_2$ $\quad T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{W}{g}v_2^2$

WORK $\quad U_{1-2} = W(50 - t)$

$T_1 + U_{1-2} = T_2$: $\quad 0 + 50W = \frac{1}{2}\frac{W}{g}v_2^2$ $\quad v_2^2 = 100g$

THE "APPARENT WEIGHT" OF THE 150-lb PASSENGER IS EQUAL TO THE REACTION R EXERTED ON HIM BY THE SEAT



$+\uparrow \Sigma F = \Sigma F_{eff}$: $\quad R - W = 2W$

$R = 3W = 3(150 \text{ lb})$ $\quad R = 450$ lb ◀

13.44

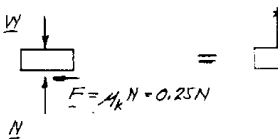
SEE SOLUTION OF PROB 13.43: $v_2^2 = 100g$

NORMAL COMPONENT OF ACCELERATION

$\rho = 50$ ft $\quad a_n = \frac{v_2^2}{\rho} = \frac{100g}{50} = 2g$ $\quad a_n = 64.4$ ft/s^2 ↑ ◀

TANGENTIAL COMPONENT OF ACCELERATION

WE APPLY NEWTON'S SECOND LAW



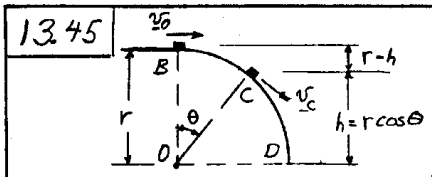
$+\uparrow \Sigma F_n = ma_n$: $\quad N - W = 2W$ $\quad N = 3W$

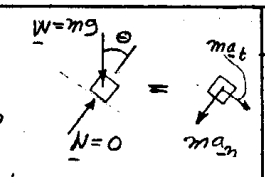
$F = \mu_k N = 0.25(3W) = 0.75W$

$\pm \Sigma F_t = ma_t$: $\quad 0.75W = \frac{W}{g}a_t$

$a_t = 0.75g$ $\quad a_t = 24.2$ ft/s^2 ← ◀

13.45





AT POINT C:
$N = 0, \quad a_n = \frac{v^2}{r}$

$+\swarrow \Sigma F_n = ma_n$

$mg\cos\theta = ma_n = m\frac{v_c^2}{r}$

$v_c^2 = g(r\cos\theta)$

$v_c^2 = gh \qquad (1)$

WORK-ENERGY PRINCIPLE

$T_B = \frac{1}{2}mv_0^2 \qquad T_C = \frac{1}{2}mv_c^2$

$U_{B-C} = W(r-h) = mg(r-h)$

$T_B + U_{B-C} = T_C: \quad \frac{1}{2}mv_0^2 + mg(r-h) = \frac{1}{2}mv_c^2$

$v_0^2 + 2gr - 2gh = v_c^2$

USE EQ(1): $v_0^2 + 2gr - 2gh = gh$

$h = \frac{1}{3}\frac{v_0^2}{g} + \frac{2}{3}r \qquad (2)$

USE EQ(1) $\qquad v_c^2 = \frac{1}{3}v_0^2 + \frac{2}{3}gr \qquad (3)$

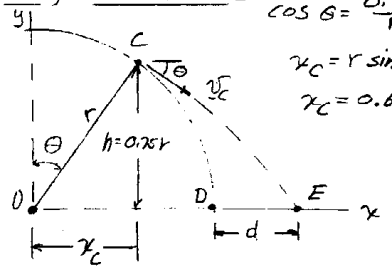
(a) FOR $v_0 = \frac{1}{2}\sqrt{gr}$

EQ(2): $h = \frac{1}{3g}\left(\frac{1}{2}\sqrt{gr}\right)^2 + \frac{2}{3}r = \left(\frac{1}{12} + \frac{2}{3}\right)r \qquad h = 0.75r$ ◀

$v_c^2 = \frac{1}{3}\left(\frac{1}{2}\sqrt{gr}\right)^2 + \frac{2}{3}gr = \frac{3}{4}gr \qquad v_c = \frac{\sqrt{3}}{2}\sqrt{gr} \qquad (4)$

(b) FIRST, LOCATE POINT C:

$\cos\theta = \frac{0.75r}{r} \qquad \theta = 41.41°$



$x_c = r\sin\theta = r\sin 41.41°$

$x_c = 0.66144r$

AT C: $(v_c)_x = +v_c\cos\theta = +\frac{\sqrt{3}}{2}\sqrt{gr}\,(0.75) = +0.64952\sqrt{gr}$

$(v_c)_y = -v_c\sin\theta = -\frac{\sqrt{3}}{2}\sqrt{gr}\,(\sin 41.41) = -0.57283\sqrt{gr}$

$y = h + (v_c)_y t - \frac{1}{2}gt^2 = 0.75r - 0.57283\sqrt{gr}\,t - \frac{1}{2}gt^2$

AT E: $y = 0 \qquad t^2 + 1.14565\sqrt{\frac{r}{g}}\,t - 1.5\frac{r}{g} = 0$

$t = -1.925\sqrt{\frac{r}{g}}$ (IMPOSSIBLE) $\qquad t = 0.77926\sqrt{\frac{r}{g}}$

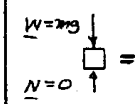
AT E: $x_E = x_c + (v_c)_x t = 0.66144r + (0.64952\sqrt{gr})(0.77926\sqrt{\frac{r}{g}})$

$x_E = 0.66144r + 0.50614r = 1.16758r$

DISTANCE DE $= x_E - r \qquad d = 0.1676r$ ◀

13.46

(a) FOR BLOCK TO LEAVE CYLINDER AT B THE RADIUS OF CURVATURE OF THE PATH MUST BE r AND N MUST EQUAL ZERO

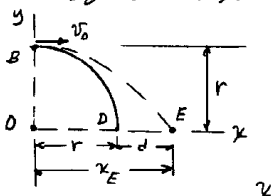


$+\downarrow \Sigma F_y = ma_n \qquad mg = ma_n$

$a_n = \frac{v_0^2}{r} = g \qquad v_0^2 = gr$

FOR $r = 800\text{ mm} = 0.8\text{ m}$

$v_0^2 = (9.81\text{ m/s}^2)(0.8\text{ m}) = 7.848 \qquad v_0 = 2.80\text{ m/s}$ ◀



$y = r - \frac{1}{2}gt^2$

AT E: $y = 0 = r - \frac{1}{2}gt^2$

$t = \sqrt{2}\sqrt{\frac{r}{g}}$

$x_E = v_0 t = (\sqrt{gr})(\sqrt{2}\sqrt{\frac{r}{g}})$

$x_E = \sqrt{2}\,r = \sqrt{2}(0.8\text{ m}) = 1.13137\text{ m}$

DISTANCE DE $= d = x_E - r = 1.13137\text{ m} - 0.8\text{ m}$

$d = 0.33137\text{ m} \qquad d = 331\text{ mm}$ ◀

(b) FOR SMALLEST VALUES OF h AND d, $v_0 = 0$

SEE SOLUTION OF PROB 13.45 FOR DERIVATION OF EQS (2) AND (3)

EQ(2): $h = \frac{1}{3}\frac{v_0^2}{g} + \frac{2}{3}r$

FOR $v_0 = 0 \qquad h = \frac{2}{3}r = \frac{2}{3}(800\text{ mm}) \qquad h = 533\text{ mm}$ ◀

EQ(3) $\qquad v_c^2 = \frac{1}{3}v_0^2 + \frac{2}{3}gr = 0 + \frac{2}{3}(9.81\text{ m/s}^2)(0.8\text{ m})$

$v_c^2 = 5.232 \qquad v_c = 2.2874\text{ m/s}$

LOCATE POINT C:

$\cos\theta = \frac{533.3\text{ mm}}{800\text{ mm}} = \frac{2}{3}$

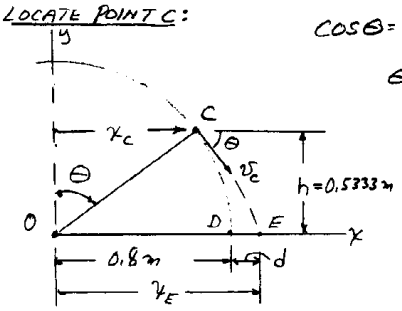
$\theta = 48.19°$

$x_c = r\sin\theta$

$= (0.8\text{ m})\sin 48.19°$

$x_c = 0.59628\text{ m}$

$= 596.3\text{ mm}$



AT C: $(v_c)_x = v_c\cos\theta = (2.2874\text{ m/s})\frac{2}{3} = 1.5247\text{ m/s}$

$(v_c)_y = -v_c\sin\theta = (2.2874\text{ m/s})\sin 48.19° = -1.7049\text{ m/s}$

$y = h + (v_c)_y t - \frac{1}{2}gt^2 = 0.5333 - 1.7049t - \frac{1}{2}(9.81)t^2$

AT E: $y = 0 \qquad 4.905t^2 + 1.7049t - 0.5333 = 0$

$t = -0.54655$ (IMPOSSIBLE) $\qquad t = 0.19849\text{ s}$

AT E: $x_E = x_c + (v_c)_x t = 0.59628\text{ m} + (1.5247\text{ m/s})(0.19849\text{ s})$

$= 0.59628\text{ m} + 0.22754\text{ m}$

$x_E = 0.89892\text{ m}$

DISTANCE DE $= x_E - r = 0.89892\text{ m} - 0.8\text{ m}$

$d = 0.09892\text{ m} \qquad d = 98.9\text{ mm}$ ◀

13.47 $m = 18\text{ g} = 18\times10^{-3}\text{ kg}$ $\Delta t = 2\times10^{-3}\text{ s}$

$t=0$: $v_1 = 0$ $T_1 = 0$

$t = \Delta t = 2\times10^{-3}$ s: $v = 750$ m/s

$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(18\times10^{-3}\text{ kg})(750\text{ m/s})^2 = 5062.5\text{ J}$

$T_1 + U_{1-2} = T_2$: $0 + U_{1-2} = 5062.5\text{ J}$

AVERAGE POWER $= \frac{\Delta U}{\Delta t} = \frac{5062.5\text{ J}}{2\times10^{-3}\text{ s}} = 2.53\times10^6\text{ W} = 2.53\text{ MW}$ ◀

13.48 (a) 70-kg MAN: $W = (70\text{ kg})(9.81\text{ m/s}^2) = 686.7\text{ N}$

POWER $= \frac{\Delta U}{\Delta t} = \frac{(686.7\text{ N})(5\text{ m})}{6\text{ s}} = 572.25\text{ W} = 572\text{ W}$ ◀

(b) 55-kg WOMAN: $W = (55\text{ kg})(9.81\text{ m/s}^2) = 539.55\text{ N}$

POWER = (80% POWER OF MAN) $= 0.8(572.25\text{ W}) = 457.8\text{ W}$

POWER $= \frac{\Delta U}{\Delta t}$: $457.8\text{ W} = \frac{(539.55\text{ N})(4\text{ m})}{\Delta t}$

$\Delta t = 4.714\text{ s}$ $\Delta t = 4.71\text{ s}$ ◀

13.49 INPUT POWER $= \frac{\Delta U}{\Delta t} = \frac{(600\text{ tons/h})(2000\text{ lb/ton})(180\text{ ft})}{(1\text{ h})(3600\text{ s/h})}$

INPUT POWER $= 60000\ \frac{\text{ft·lb}}{\text{s}}\left(\frac{1\text{ hp}}{550\ \frac{\text{ft·lb}}{\text{s}}}\right) = 109.09\text{ hp}$

(a) AND (b)

OUTPUT POWER $= (0.65)$(INPUT POWER$) = 0.65(109.09\text{ hp}) = 70.91\text{ hp}$

$= 70.91\text{ hp}\left(\frac{0.7457\text{ kW}}{1\text{ hp}}\right) = 52.9\text{ kW}$ ◀

NOTE: SOLUTION IS INDEPENDENT OF SPEED

13.50 NOTE: SOLUTION IS INDEPENDENT OF SPEED

(a) AVERAGE POWER $= \frac{\Delta U}{\Delta t} = \frac{(900)(160\text{ lb})(1000\text{ ft})}{3600\text{ s}} = 40{,}000\ \frac{\text{ft·lb}}{\text{s}}$

$1\text{ hp} = 550\ \frac{\text{ft·lb}}{\text{s}}$: AVERAGE POWER $= \frac{40000}{550} = 72.7\text{ hp}$ ◀

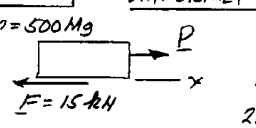
(b) MAX POWER REQUIRED WITH 300% OVERLOAD

$= \frac{100+300}{100}(72.7\text{ hp}) = 290.1\text{ hp}$

REQUIRED MOTOR CAPACITY (85% EFFICIENT)

MOTOR CAPACITY $= \frac{290.1\text{ hp}}{0.85} = 342\text{ hp}$ ◀

13.51 UNIFORMLY ACCELERATED MOTION $(t < 50\text{s})$



$m = 500\text{ Mg}$ $v = 90\text{ km/h} = 25\text{ m/s}$

$v_0 = 0$ $t = 50\text{s}$

$v = v_0 + at$

$25\text{ m/s} = 0 + a(50\text{s})$ $a = 0.5\text{ m/s}^2$

$\Sigma F = ma$: $P - 15\text{ kN} = (500\text{ Mg})(0.5\text{ m/s}^2)$

$P - 15\text{ kN} = 250\text{ kN}$ $P = 265\text{ kN}$

POWER $= Pv = P(at) = (265\text{ kN})(0.5\text{ m/s}^2)t$

$t \le 50\text{s}$: POWER $= (132.5\text{ kW/s})t$ ◀

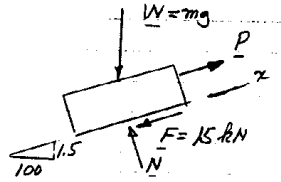
WHEN $t = 50\text{s}$: POWER $= (132.5\text{ kW/s})(50\text{s}) = 6.63\text{ MW}$

UNIFORM MOTION $(t > 50\text{s})$ $a = 0$ $P = F = 15\text{ kN}$

POWER $= Pv = (15\text{ kN})(25\text{ m/s}) = 375\times10^3\text{ W}$

$t > 50\text{s}$: POWER $= 375\text{ kW}$ ◀

13.52 (a) UNIFORMLY ACCELERATED MOTION $(t < 50\text{s})$



$m = 500\text{ Mg}$

$v = 90\text{ km/h} = 25\text{ m/s}$

$v_0 = 0$ $t = 50\text{s}$

$v = v_0 + at$

$25\text{ m/s} = 0 + a(50\text{s})$

$a = 0.5\text{ m/s}^2$

$\nearrow \Sigma F = ma$: $P - 15\text{ kN} - mg\left(\frac{1.5}{100}\right) = ma$ (1)

$P - 15\text{ kN} - (500\text{ Mg})(9.81\text{ m/s}^2)\left(\frac{1.5}{100}\right) = (500\text{ Mg})(0.5\text{ m/s}^2)$

$P - 15\text{ kN} - 73.575\text{ kN} = 250\text{ kN}$

$P = 338.58\text{ kN}$

POWER $= Pv = P(at) = (338.58\text{ kN})(0.5\text{ m/s}^2)t$

$t \le 50\text{s}$: POWER $= (169.3\text{ kW/s})t$ ◀

WHEN $t = 50\text{s}$: POWER $= (169.3\text{ kW/s})(50\text{s}) = 8.46\text{ MW}$ ◀

(b) UNIFORM MOTION $(t > 50\text{s})$ $a = 0$

EQ(1): $P - 15\text{ kN} - mg\left(\frac{1.5}{100}\right) = 0$

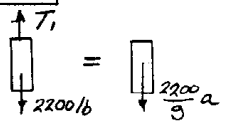
$P - 15\text{ kN} - (500\text{ Mg})(9.81\text{ m/s}^2)\left(\frac{1.5}{100}\right) = 0$

$P - 15\text{ kN} - 73.575\text{ kN} = 0$ $P = 88.58\text{ kN}$

POWER $= Pv = (88.58\text{ kN})(25\text{ m/s}) = 2.21\times10^6\text{ W}$

$t > 50\text{s}$: POWER $= 2.21\text{ MW}$ ◀

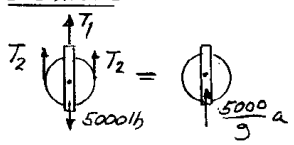
13.53 COUNTERWEIGHT ACCELERATION $= a\downarrow$



$+\downarrow \Sigma F = \Sigma F_{eff}$:

$2200 - T_1 = \frac{2200}{g}a$ (1)

ELEVATOR ACCELERATION $= a\uparrow$



$+\uparrow \Sigma F = \Sigma F_{eff}$:

$2T_2 + T_1 - 5000 = \frac{5000}{g}a$ (2)

ADD EQS (1) AND (2)

$2T_2 - 5000 + 2200 = \frac{7200}{g}a$

$T_2 = 1400 + \frac{3600}{g}a$

T_2 ALSO IS FORCE EXERTED BY MOTOR ON CABLE C WHICH HAS A SPEED $v = 30\text{ ft/s}$

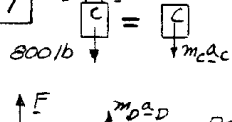
(a) $a = 0$, $T_2 = 1400\text{ lb}$, $P = T_2 v = (1400\text{ lb})(30\text{ ft/s}) = 42000\ \frac{\text{lb·ft}}{\text{s}}$

POWER $= 76.4\text{ hp}$ ◀

(b) $a = 3\text{ ft/s}^2$, $T_2 = 1400 + \frac{3600}{32.2}(3) = 1735.4$

$P = T_2 v = (1735\text{ lb})(30\text{ ft/s}) = 52060\ \frac{\text{lb·ft}}{\text{s}}$ POWER $= 94.7\text{ hp}$ ◀

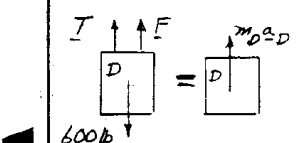
13.54



$a_C = \frac{1}{2}a_D$

$+\downarrow \Sigma F = m_C a_C$: $800 - 2T = \frac{800}{g}\frac{a_D}{2}$

$T = 400 - 200\frac{a_D}{g}$



BODY D:

$+\uparrow \Sigma F = m_D a_D$:

$F + \left(400 - 200\frac{a_D}{g}\right) - 600 = \frac{600}{g}a_D$

$F = 200 + 800\frac{a_D}{g}$ (1)

$v = 6\text{ ft/s}$

POWER $= 3\text{ hp} = (3\text{ hp})\left(\frac{550\text{ ft·lb/s}}{1\text{ hp}}\right) = 1650\text{ ft·lb/s}$

BUT POWER $= Fv$: $1650\text{ ft·lb/s} = F(6\text{ ft/s})$ $F = 275\text{ lb}$

EQ(1): $F = 275 = 200 + 800\frac{a_D}{32.2}$ $a_D = 3.02\text{ ft/s}^2\uparrow$ ◀

13.55

FOR CONSTANT POWER P:

TIME: $P = Fv = (ma)v = m\frac{dv}{dt}v$

THUS: $dt = \frac{m}{P}v\,dv$

$$t = \frac{m}{P}\int_{v_0}^{v_1} v\,dv = \frac{m}{P}\left(\frac{v_1^2}{2} - \frac{v_0^2}{2}\right)$$

$$t = \frac{m}{2P}\left(v_1^2 - v_0^2\right) \quad (1)$$

DISTANCE: $P = Fv = (ma)v = m\frac{v\,dv}{dx}v$

THUS: $dx = \frac{m}{P}v^2\,dv$

$$x = \frac{m}{P}\int_{v_0}^{v_1} v^2\,dv = \frac{m}{P}\left(\frac{v_1^3}{3} - \frac{v_0^3}{3}\right)$$

$$x = \frac{m}{3P}\left(v_1^3 - v_0^3\right) \quad (2)$$

DATA FOR TRUCK: $m = 15\text{ Mg}$ $P = 50\text{ kW}$

(a) $v_0 = 36\text{ km/h} = 10\text{ m/s}$

$v_1 = 54\text{ km/h} = 15\text{ m/s}$

EQ(1): $t = \frac{15\times10^3\text{ kg}}{2(50\times10^3\text{ W})}\left[(15\text{ m/s})^2 - (10\text{ m/s})^2\right]$ $t = 18.75\text{ s}$ ◀

EQ(2): $x = \frac{15\times10^3\text{ kg}}{3(50\times10^3\text{ W})}\left[(15\text{ m/s})^3 - (10\text{ m/s})^3\right]$

$= 237.5\text{ m}$ $x = 238\text{ m}$ ◀

(b) $v_0 = 54\text{ km/h} = 15\text{ m/s}$

$v_1 = 72\text{ km/h} = 20\text{ m/s}$

EQ(1): $t = \frac{15\times10^3\text{ kg}}{2(50\times10^3\text{ W})}\left[(20\text{ m/s})^2 - (15\text{ m/s})^2\right] = 26.25\text{ s}$ $t = 26.25\text{ s}$ ◀

EQ(2): $x = \frac{15\times10^3\text{ kg}}{3(50\times10^3\text{ W})}\left[(20\text{ m/s})^3 - (15\text{ m/s})^3\right]$

$= 462.5\text{ m}$ $x = 462\text{ m}$ ◀

13.56

(a) POWER DEVELOPED BY TUGBOAT @ 5.4 km/h

$v_0 = 4.5\text{ km/h} = 1.25\text{ m/s}$

$F_0 = 300\text{ kN}$

$P = F_0 v_0 = (300\text{ kN})(1.25\text{ m/s})$ $P = 375\text{ kW}$ ◀

(b) MAXIMUM SPEED

POWER REQUIRED TO TOW SHIP AT SPEED v:

$$F = F_0\left(\frac{v}{v_0}\right)^{1.75} \qquad P = Fv = F_0\left(\frac{v}{v_0}\right)^{1.75}v = F_0 v_0\left(\frac{v}{v_0}\right)^{2.75} \quad (1)$$

SINCE WE HAVE TWO TUGBOATS, THE AVAILABLE POWER IS TWICE MAXIMUM POWER $F_0 v_0$ DEVELOPED BY ONE TUGBOAT.

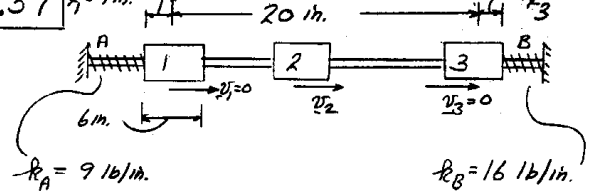
$$2F_0 v_0 = F_0 v_0\left(\frac{v}{v_0}\right)^{2.75}$$

$$\left(\frac{v}{v_0}\right)^{2.75} = 2 \qquad v = v_0(2)^{\frac{1}{2.75}} = v_0(1.2863)$$

RECALLING THAT $v_0 = 4.5\text{ km/h}$

$v = (4.5\text{ km/h})(1.2863) = 5.7902\text{ km/h}$ $v = 5.79\text{ km/h}$ ◀

13.57



$k_A = 9\text{ lb/in.}$ $k_B = 16\text{ lb/in.}$

POSITION 1: $v_1 = 0$, $T_1 = 0$

$x_1 = 4\text{ in.}$, $V_1 = \frac{1}{2}k_A x_1^2 = \frac{1}{2}(9\text{ lb/in})(4\text{ in.})^2 = 72\text{ in.lb} = 6\text{ ft.lb}$

POSITION 3: $v_3 = 0$, $T_3 = 0$

$V_3 = \frac{1}{2}k_B x_3^2 = \frac{1}{2}(16\text{ lb/in.})x_3^2$

$T_1 + V_1 = T_3 + V_3$: $0 + 72\text{ in.lb} = \frac{1}{2}(16\text{ lb/in})x_3^2$

$x_3^2 = 9\text{ in.}$ $x_3 = 3\text{ in.}$

RESULT IS INDEPENDENT OF MASS OF COLLAR, THUS FOR (a) AND (b)

DISTANCE TRAVELED = 4 in. + 20 in. + 3 in. − 6 in. = 21 in. ◀

MAXIMUM VELOCITY OCCURS WHILE COLLAR MOVES BETWEEN SPRINGS: $v_m = v_2$ $T_2 = \frac{1}{2}m v_2^2$, $V_2 = 0$

$T_1 + V_1 = T_2 + V_2$: $0 + 6\text{ lb·ft} = \frac{1}{2}\frac{W}{32.2}v_2^2$ $v_2^2 = \frac{12(32.2)}{W}$

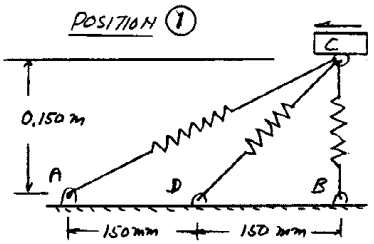
FOR $W = 2$ lb: $v_2^2 = \frac{12(32.2)}{2} = 193.2$ $v_m = 13.90\text{ ft/s}$ ◀

FOR $W = 5$ lb: $v_2^2 = \frac{12(32.2)}{5} = 77.28$ $v_m = 8.79\text{ ft/s}$ ◀

13.58

MAXIMUM VELOCITY OCCURS AT E WHERE COLLAR IS PASSING THROUGH POSITION OF EQUILIBRIUM

POSITION (1) $v_1 = 0$ $T_1 = 0$



NOTE: UNDEFORMED LENGTH OF SPRINGS IS 0.150 m

SPRING AC $L = 335.4\text{ mm}$ $\Delta = 0.3354 - 0.150 = 0.1854\text{ m}$

SPRING CD $L = 212.1\text{ mm}$ $\Delta = 0.2121 - 0.150 = 0.0621\text{ m}$

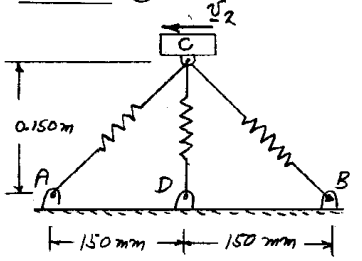
SPRING BD $L = 150\text{ mm}$, $\Delta = 0$

POTENTIAL ENERGY ($k = 400\text{ N/m}$ FOR EACH SPRING)

$V_1 = \sum\frac{1}{2}k\Delta^2 = \frac{1}{2}k\Sigma\Delta^2 = \frac{1}{2}(400\text{ N/m})\left[(0.1854\text{ m})^2 + (0.0621\text{ m})^2 + 0\right]$

$V_1 = 7.6467\text{ J}$

POSITION (2) $m = 1.2\text{ kg}$; $T_2 = \frac{1}{2}m v_2^2 = \frac{1}{2}(1.2\text{ kg})v_2^2$



SPRING AC $L = 212.1\text{ mm}$ $\Delta = 0.2121 - 0.150 = 0.0621\text{ m}$

SPRING CD $L = 150\text{ mm}$ $\Delta = 0$

SPRING BC $L = 212.1\text{ mm}$ $\Delta = 0.2121 - 0.150 = 0.0621\text{ m}$

POTENTIAL ENERGY $V_2 = \sum\frac{1}{2}k\Delta^2 = \frac{1}{2}k\Sigma\Delta^2$

$V_2 = \frac{1}{2}(400\text{ N/m})\left[(0.0621\text{ m})^2 + 0 + (0.0621\text{ m})^2\right] = 1.5426\text{ J}$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + 7.6467\text{ J} = \frac{1}{2}(1.2)v_2^2 + 1.5426\text{ J}$

$v_2^2 = 10.1736$ $v_2 = 3.19\text{ m/s}$ ◀

13.59 SEE SOLUTION OF PROB 13.58 AND REMOVE THE POTENTIAL ENERGY OF SPRING CD.

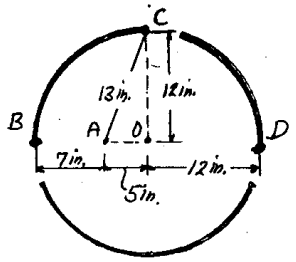
$T_1 = 0$; $V_1 = \frac{1}{2}(400\ N/m)\left[(0.1854\ m)^2 + 0\right] = 6.8746\ J$

$T_2 = \frac{1}{2}(0.2\ kg)v_2^2$: $V_2 = \frac{1}{2}(400)\left[(0.0621)^2 + (0.0621)^2\right] = 1.5426\ J$

$T_1 + V_1 = T_2 + V_2$: $0 + 6.8746\ J = \frac{1}{2}(0.2)v_2^2 + 1.5426\ J$

$v_2^2 = 8.8867$ $\quad \underline{v_2 = 2.98\ m/s \leftarrow}$

13.60



NOTE: THE ROD LIES IN HORIZONTAL PLANE

$v_D = 6\ ft/s$

$k = 1.5\ lb/in$

UNDEFORMED LENGTH OF SPRING = 7 in.

(a) POSITION D: SPRING LENGTH = 5 in. + 12 in. = 17 in.

$x_D = 17\ in. - 7\ in. = 10\ in.$

$V_D = \frac{1}{2}kx_D^2 = \frac{1}{2}(1.5\ lb/in.)(10\ in.)^2 = 75\ lb\cdot in. = 6.25\ lb\cdot ft$

$T_D = \frac{1}{2}mv_D^2 = \frac{1}{2}\frac{3\ lb}{32.2\ ft/s^2}(6\ ft/s)^2 = 1.677\ lb\cdot ft$

POSITION C:

SPRING LENGTH = 13 in. $\quad x_C = 13\ in. - 7\ in. = 6\ in.$

$V_C = \frac{1}{2}kx_C^2 = \frac{1}{2}(1.5\ lb/in.)(6\ in.)^2 = 27\ lb\cdot in = 2.25\ lb\cdot ft$

$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}\frac{3\ lb}{32.2\ ft/s^2}v_C^2 = \frac{1.5}{32.2}v_C^2$

CONSERVATION OF ENERGY

$T_C + V_C = T_D + V_D$: $\frac{1.5}{32.2}v_C^2 + 2.25\ lb\cdot ft = 1.677\ lb\cdot ft + 6.25\ lb\cdot ft$

$\frac{1.5}{32.2}v_C^2 = 5.677$ $\quad v_C^2 = 121.87$ $\quad v_C = 11.04\ ft/s \blacleft$

(b) POSITION B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\frac{3}{32.2}v_B^2 = \frac{1.5}{32.2}v_B^2$

SINCE SPRING IS UNDEFORMED AT B, $V_B = 0$

POSITION D: FROM PART (a):

$V_D = 6.25\ lb\cdot ft$

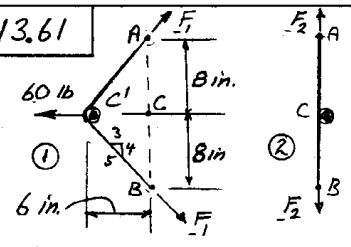
$T_D = 1.677\ lb\cdot ft$

CONSERVATION OF ENERGY

$T_B + V_B = T_D + V_D$: $\frac{1.5}{32.2}v_B^2 + 0 = 1.677\ lb\cdot ft + 6.25\ lb\cdot ft$

$\frac{1.5}{32.2}v_B^2 = 7.927$ $\quad v_B^2 = 170.17$ $\quad v_B = 13.04\ ft/s \blacleft$

13.61



LET ℓ = UNDEFORMED LENGTH OF CORD

POSITION 1: LENGTH AC'B = 20 in.; ELONGATION = $x_1 = 20 - \ell$

$\Sigma F_x = 0$: $2\left(\frac{3}{5}F_1\right) - 60\ lb = 0$ $\quad F_1 = 50\ lb$

POSITION 2: LENGTH ACB = 16 in.; ELONGATION = $x_2 = 16 - \ell$

GIVEN: $F_2 = 10\ lb$

(CONTINUED)

13.61 CONTINUED

$F_1 = kx_1$, $\quad F_2 = kx_2$

$F_1 - F_2 = k(x_1 - x_2)$

$50 - 10 = k\left[(20 - \ell) - (16 - \ell)\right] = 4k$

$k = \frac{40}{4} = 10\ lb/in.$

$x_1 = \frac{F_1}{k} = \frac{50\ lb}{10\ lb/in.} = 5\ in.$ $\quad x_2 = \frac{F_2}{k} = \frac{10\ lb}{10\ lb/in.} = 1\ in.$

POSITION 1: $T_1 = 0$ $\quad V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(10\ lb/in.)(5\ in.)^2 = 125\ in\cdot lb = 10.42\ ft\cdot lb$

POSITION 2: $W = 0.20\ lb$

$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{0.20\ lb}{32.2\ ft/s^2}v_2^2 = 3.106\times10^{-3}v_2^2$

$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(10\ lb/in.)(1\ in.)^2 = 5\ in\cdot lb = 0.4167\ ft\cdot lb$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$

$0 + 10.42\ ft\cdot lb = 3.106\times10^{-3}v_2^2 + 0.4167\ ft\cdot lb$

$10.00 = 3.106\times10^{-3}v_2^2$ $\quad v_2 = 56.745\ ft/s$ $\quad v_2 = 56.7\ ft/s \blacleft$

NOTE: THE HORIZONTAL FORCE APPLIED AT THE MIDPOINT OF THE CORD IS NOT PROPORTIONAL TO THE HORIZONTAL DISTANCE C'C. A SOLUTION BASED ON THE WORK OF THE HORIZONTAL FORCE WOULD BE RATHER INVOLVED.

13.62 AT A: $AF^2 = 500^2 + 350^2 + 400^2$ $\quad AF = 729.7\ mm$

ELONGATION = $\Delta_A = 729.7\ mm - 300\ mm = 429.7\ mm = 0.4297\ m$

$v_A = 0$, $T_A = 0$; $V_A = \frac{1}{2}k\Delta_A^2 = \frac{1}{2}(150\ N/m)(0.4297\ m)^2 = 13.848\ J$

AT B: $BF^2 = 350^2 + 400^2$ $\quad BF = 531.5\ mm$

ELONGATION = $\Delta_B = 531.5\ mm - 300 = 231.5\ mm = 0.2315\ m$

$T_B = \frac{1}{2}mv_B^2$; $V_B = \frac{1}{2}k\Delta_B^2 = \frac{1}{2}(150\ N/m)(0.2315\ m)^2 = 4.020\ J$

AT E: $EF^2 = 500^2 + 350^2$ $\quad EF = 610.3\ mm$

ELONGATION = $\Delta_E = 610.3 - 300 = 310.3\ mm = 0.3103\ m$

$T_E = \frac{1}{2}mv_E^2$; $V_E = \frac{1}{2}k\Delta_E^2 = \frac{1}{2}(150\ N/m)(0.3103\ m)^2 = 7.221\ J$

CONSERVATION OF ENERGY $\quad m = 0.75\ kg$

(a) $T_A + V_A = T_B + V_B$: $0 + 13.848\ J = \frac{1}{2}(0.75\ kg)v_B^2 + 4.020\ J$

$v_B^2 = 26.209$ $\quad v_B = 5.12\ m/s \blacleft$

(b) $T_A + V_A = T_E + V_E$: $0 + 13.848\ J = \frac{1}{2}(0.75\ kg)v_E^2 + 7.221\ J$

$v_E^2 = 17.671$ $\quad v_E = 4.20\ m/s \blacleft$

13.63 UNDEFORMED LENGTH OF SPRING = 450 mm

SEE SOLUTION OF PROB 13.62 FOR:

$AF = 729.7\ mm$ $\quad BF = 531.5\ mm$ $\quad EF = 610.3\ mm$

AT A: ELONGATION = $\Delta_A = 729.7 - 450 = 279.7\ mm = 0.2797\ m$

$v_A = 0$, $T_A = 0$; $V_A = \frac{1}{2}k\Delta_A^2 = \frac{1}{2}(150\ N/m)(0.2797\ m)^2 = 5.867\ J$

AT B: ELONGATION $\Delta_B = 531.5 - 450 = 81.5\ mm = 0.0815\ m$

$T_B = \frac{1}{2}mv_B^2$; $V_B = \frac{1}{2}k\Delta_B^2 = \frac{1}{2}(150\ N/m)(0.0815\ m)^2 = 0.498\ J$

AT E: ELONGATION = $\Delta_E = 610.3 - 450 = 160.3\ mm = 0.1603\ m$

$T_E = \frac{1}{2}mv_E^2$; $V_E = \frac{1}{2}k\Delta_E^2 = \frac{1}{2}(150\ N/m)(0.1603\ m)^2 = 1.927\ J$

CONSERVATION OF ENERGY $\quad m = 0.75\ kg$

(a) $T_A + V_A = T_B + V_B$: $0 + 5.867\ J = \frac{1}{2}(0.75\ kg)v_B^2 + 0.498\ J$

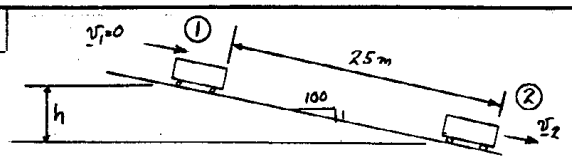
$v_B^2 = 14.317$ $\quad v_B = 3.78\ m/s \blacleft$

(b) $T_A + V_A = T_E + V_E$: $0 + 5.867\ J = \frac{1}{2}(0.75\ kg)v_E^2 + 1.927\ J$

$v_E^2 = 10.507$ $\quad v_E = 3.24\ m/s \blacleft$

THAT CORD BECAME SLACK IS IMMATERIAL.

13.64



$h = \frac{1}{100}(25\text{ m}) = 0.25\text{ m}$

$m = 30\text{ Mg} = 30\times10^3\text{ kg}$

Position 1: $v_1 = 0$, $T_1 = 0$,

$V_1 = Wh = mg(0.2\text{m}) = (30\times10^3\text{kg})(9.81\text{ m/s}^2)(0.2\text{m}) = 58.86\times10^3\text{ J}$

Position 2: $T_2 = \frac{1}{2}mv_2^2$ $\quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$0 + mg(0.2\text{m}) = \frac{1}{2}mv_2^2$ $\quad v_2 = \sqrt{2(9.81)(0.2)}$ $\quad v_2 = 1.981\text{ m/s}$ ◀

(b) **Position 3** CAR AT REST, SPRING COMPRESSED x_m

$T_3 = 0$ $\quad T_3 = 0$ $\quad k = 2400\text{ kN/m}$

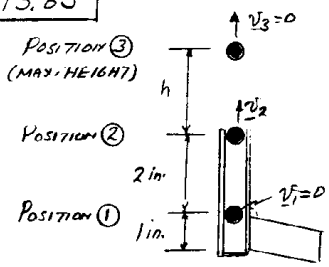
$V_3 = \frac{1}{2}kx_m^2 = \frac{1}{2}(2400\text{ kN/m})x_m^2$

$T_1 + V_1 = T_3 + V_3$

$0 + 58.86\times10^3\text{ J} = 0 + \frac{1}{2}(2.4\times10^6\text{ N/m})x_m^2$

$x_m^2 = 0.04905$ $\quad x_m = 0.22147\text{ m}$ $\quad x_m = 221\text{ mm}$ ◀

13.65



$x_1 = 6\text{ in.} - 1\text{ in.} = 5\text{ in.}$ $\quad F_1 = 10\text{ lb}$

$F_1 = kx_1$: $\quad 10\text{ lb} = k(5\text{ in.})$ $\quad k = 2\text{ lb/in.}$

$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(2\text{ lb/in.})(5\text{ in.})^2 = 25\text{ in.lb} = \frac{25}{12}\text{ ft.lb}$

(a) **Position 2:** BULLET LEAVES GUN

$x_2 = 6\text{ in.} - 3\text{ in.} = 3\text{ in.}$

$W = 1\text{ oz} = 1/16\text{ lb}$ $\quad T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{1/16}{32.2}v_2^2 = 970.5\times10^{-6}v_2^2$

$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(2\text{ lb/in.})(3\text{ in.})^2 = 9\text{ in.lb} = \frac{9}{12}\text{ ft.lb}$

NOTE: CHANGE IN GRAVITATIONAL POTENTIAL ENERGY WILL BE NEGLECTED

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{25}{12} = 970.5\times10^{-6}v_2^2 + \frac{9}{12}$

$v_2^2 = 1373.9$ $\quad v_2 = 37.066$ $\quad v_2 = 37.1\text{ ft/s}$ ◀

(b) **Position 3:** MAXIMUM HEIGHT $\quad v_3 = 0$

$T_1 = T_3 = 0$ $\quad V_1 = \frac{25}{12}\text{ ft-lb}$

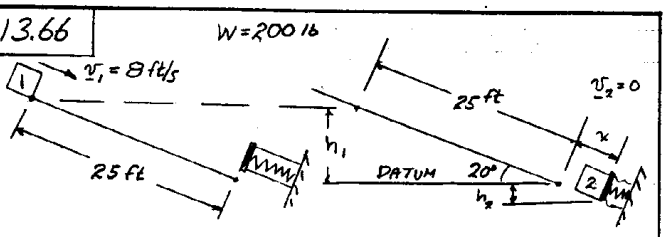
$V_3 = (V_3)_{\text{SPRING}} + (V_3)_{\text{GRAV.}} = \frac{9}{12}\text{ ft-lb} + \left(\frac{1}{16}\text{ lb}\right)h$

$T_1 + V_1 = T_3 + V_3$

$0 + \frac{25}{12}\text{ ft.lb} = 0 + \frac{9}{12}\text{ ft-lb} + \left(\frac{1}{16}\text{ lb}\right)h$

$h = \frac{25-9}{12}(16) = 21.33\text{ ft}$ $\quad h = 21.3\text{ ft}$ ◀

13.66



$h_1 = 25\sin20° = 8.5505\text{ ft}$ $\quad h_2 = x\sin20° = 0.34202x$

$W = 150\text{ lb}$ $\quad k = 125\text{ lb/in.} = 1500\text{ lb/ft}$

Position 1: $T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{200\text{ lb}}{32.2}(8\text{ ft/s})^2 = 198.76\text{ ft.lb}$

$(V_1)_g = Wh_1 = (200\text{lb})(8.5505\text{ ft}) = 1710.1\text{ ft.lb}$

FOR SPRING: $k = 125\text{ lb/in.} = 1500\text{ lb/ft}$, $\Delta_1 = 6\text{ in.} = \frac{1}{2}\text{ ft}$

$(V_1)_e = \frac{1}{2}k\Delta_1^2 = \frac{1}{2}(1500\text{ lb/ft})\left(\frac{1}{2}\text{ ft}\right)^2 = 187.5\text{ ft.lb}$

Position 2: $T_2 = 0$, $(V_2)_g = W(-h_2) = 200\text{lb}(-0.34202x) = -68.404x$

FOR SPRING: TOTAL COMPRESSION $= \frac{1}{2}\text{ ft} + x$

$k = 150\text{ lb/in.} = 1800\text{ lb/ft}$; $(V_2)_e = \frac{1}{2}(1500)\left(\frac{1}{2}+x\right)^2$

CONSERVATION OF ENERGY

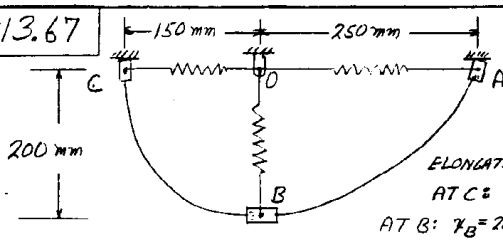
$T_1 + V_1 = T_2 + V_2$: $\quad 198.76 + 1710.1 + 187.5 = \frac{1}{2}(1500)\left(\frac{1}{2}+x\right)^2 - 68.404x$

$2096.36 = 187.5 + 750x + 750x^2 - 68.404x$

$750x^2 + 68.160x - 1908.86 = 0$

$x = -2.113\text{ ft}$ (IMPOSSIBLE) $\quad x = 1.204\text{ ft}$ $\quad x = 14.45\text{ in.}$ ◀

13.67



ELONGATIONS OF SPRING

AT C: $x_C = 0$

AT B: $x_B = 200 - 150 = 50\text{ mm}$

AT A: $x_A = 250 - 150 = 100\text{ mm}$

AT A: $T_A = 0$, $V_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(600\text{ N/m})(0.1\text{m})^2 = 3\text{ J}$

AT B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(2\text{kg})v_B^2$

$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(600\text{ N/m})(0.05\text{m})^2 = 0.75\text{ J}$

$(V_B)_g = -Wh = -(2\text{ kg})(9.81\text{m/s}^2)(0.2\text{m}) = -3.924\text{ J}$

$T_A + V_A = T_B + V_B$: $\quad 0 + 3\text{J} = \frac{1}{2}(2\text{kg})v_B^2 + (0.75\text{J} - 3.924\text{J})$

$v_B^2 = 6.174$ $\quad v_B = 2.48\text{ m/s}$ ◀

AT C: $T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(2\text{kg})v_C^2$ $\quad V_C = 0$

$T_A + V_A = T_C + V_C$: $\quad 0 + 3\text{J} = \frac{1}{2}(2\text{kg})v_C^2 + 0$

$v_C^2 = 3.00$ $\quad v_C = 1.732\text{ m/s}$ ◀

13.68

SEE SKETCH OF SOLUTION OF PROB. 13.67

FOR UNDEFORMED LENGTH = 100 mm,

ELONGATIONS OF SPRING: $x_C = 150 - 100 = 50\text{ mm}$

$x_B = 200 - 100 = 100\text{ mm}$

$x_A = 250 - 100 = 150\text{ mm}$

AT A: $T_A = 0$, $V_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(800\text{ N/m})(0.15\text{m})^2 = 9\text{J}$

AT B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(2\text{kg})v_B^2$

$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(800\text{ N/m})(0.1\text{m})^2 = 4\text{J}$

$(V_B)_g = -Wh = -(2\text{kg})(9.81\text{m/s}^2)(0.2\text{m}) = -3.924$

$T_A + V_A = T_B + V_B$: $\quad 0 + 9\text{ J} = \frac{1}{2}(2\text{kg})v_B^2 + (4\text{J} - 3.924\text{ J})$

$v_B^2 = 8.924$ $\quad v_B = 2.99\text{ m/s}$ ◀

(CONTINUED)

13.68 CONTINUED

AT C: $T_C = \frac{1}{2} m v_C^2 = \frac{1}{2}(2\text{ kg}) v_C^2$

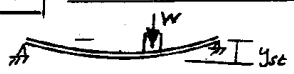
$V_C = \frac{1}{2} k x_C^2 = \frac{1}{2}(800\text{ N/m})(0.05\text{ m})^2 = 1\text{ J}$

$T_A + V_A = T_C + V_C$: $0 + 9\text{ J} = \frac{1}{2}(2\text{ kg}) v_C^2 + 1\text{ J}$

$v_C^2 = 8$ $\qquad v_C = 2.83\text{ m/s}$ ◀

13.69

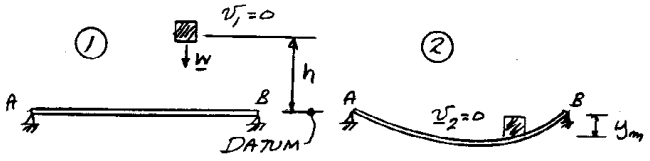
STATIC DEFLECTION OF BEAM



y_{st} = STATIC DEFLECTION CAUSED BY WEIGHT W

DENOTE BY k AN EQUIVALENT SPRING CONSTANT, THEN WE MAY WRITE $W = k y_{st}$ (1)

DROP W FROM HEIGHT h.



$T_1 = 0$; $V_1 = Wh$ $\qquad T_2 = 0$; $V_2 = -W y_m + \frac{1}{2} k y_m^2$

$T_1 + V_1 = T_2 + V_2$: $0 + Wh = -W y_m + \frac{1}{2} k y_m^2$

$W(h + y_m) - \frac{1}{2} k y_m^2 = 0$

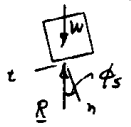
FROM EQ(1): $W = k y_{st}$ $\qquad k y_{st}(h + y_m) - \frac{1}{2} k y_m^2 = 0$

$y_m^2 - 2 y_{st} y_m - 2 y_{st} h = 0$

$$y_m = \frac{+2y_{st} + \sqrt{(2y_{st})^2 + 8 y_{st} h}}{2} \qquad y_m = y_{st}\left(1 + \sqrt{1 + \frac{2h}{y_{st}}}\right) \text{ Q.E.D.}$$

13.70

STATIC COEFFICIENT OF FRICTION

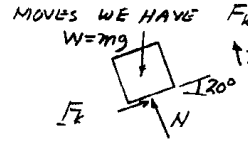


$\mu_s = \tan\phi_s$

BUT $\phi_s = 20°$

$\mu_s = \tan 20°$ $\qquad \mu_s = 0.364$ ◀

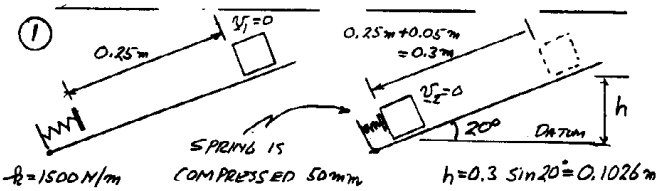
KINETIC COEFFICIENT OF FRICTION AFTER BLOCK MOVES WE HAVE $F_k = \mu_k N$



$+\uparrow \Sigma F$: $N = mg\cos 20° = (6\text{ kg})(9.81)\cos 20°$

$N = 55.31\text{ N}$

$F_k = \mu_k N = 55.31\,\mu_k$



$k = 1500\text{ N/m}$ $\qquad h = 0.3\sin 20° = 0.1026\text{ m}$

$T_1 = 0$; $V_1 = Wh = (6\text{ kg})(9.81\text{ m/s}^2)(0.1026\text{ m}) = 6.039\text{ J}$

ENERGY LOST TO FRICTION IS: $F_k(0.3\text{ m}) = (55.31\,\mu_k)(0.3\text{ m})$

$T_2 = 0$; $V_2 = \frac{1}{2} k \Delta^2 = \frac{1}{2}(1500\text{ N/m})(0.05\text{ m})^2 = 1.875\text{ J}$

$T_1 + V_1 = T_2 + V_2 + \text{ENERGY LOST}$

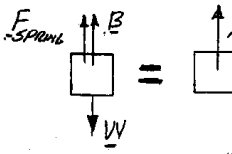
$0 + 6.039\text{ J} = 0 + 1.875\text{ J} + (55.31\,\mu_k)(0.3)$

$4.164 = 16.593\,\mu_k$ $\qquad \mu_k = 0.251$ ◀

13.71

SEE SOLUTION OF PROB 13.67 FOR $v_B = 50\text{ mm}$ $\qquad v_B^2 = 6.174$

AT B: $\qquad$ GIVEN $\rho_B = 200\text{ mm} = 0.2\text{ m}$



$ma_n = m\frac{v_B^2}{\rho_B} = (2\text{ kg})\frac{6.174}{0.2\text{ m}} = 61.74\text{ N}$

$W = mg = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N}$

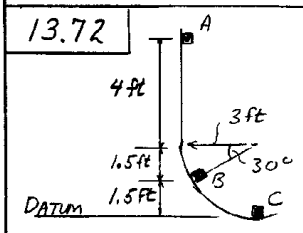
$F_{SPRING} = k y_B = (600\text{ N/m})(0.05\text{ m}) = 30\text{ N}$

$+\uparrow \Sigma F_y = ma_n$: $F_{SPRING} + B - W = ma_n$

$30\text{N} + B - 19.62\text{N} = 61.74\text{N}$

$B = 51.36\text{ N}$ $\qquad \underline{B} = 51.4\text{N}\uparrow$ ◀

13.72



DETERMINATION OF VELOCITIES

$T_A = 0$ $\qquad V_A = W(4+3) = 7W$

$T_B = \frac{1}{2} m v_B^2$ $\qquad V_B = W(1.5) = 1.5W$

$T_A + V_A = T_B + V_B$

$0 + 7W = \frac{1}{2}\frac{W}{g} v_B^2 + 1.5W$

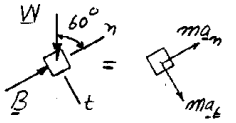
$v_B^2 = 2(7 - 1.5)g = 11g$ ◁

$T_C = \frac{1}{2} m v_C^2$ $\qquad V_C = 0$

$T_A + V_A = T_C + V_C$: $\qquad 0 + 7W = \frac{1}{2}\frac{W}{g} v_C^2 + 0$

$v_C^2 = 2(7)g = 14g$ ◁

(a) AT B: $\qquad v_B^2 = 11g$ $\qquad \rho = 3\text{ ft}$ $\qquad W = 1.25\text{ lb}$



$a_n = \frac{v_B^2}{\rho} = \frac{11g}{3} = 3.667g$

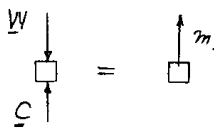
$+\nearrow \Sigma F_n = ma_n$: $\qquad B - W\cos 60° = ma_n$

$B - W(0.5) = \frac{W}{g}(3.667g)$

$B = W(0.5 + 3.667) = (1.25\text{ lb})(4.167)$

$B = 5.208\text{ lb}$ $\qquad \underline{B} = 5.21\text{ lb}\ \angle 30°$ ◀

(b) AT C: $\qquad v_C^2 = 14g$ $\qquad \rho = 3\text{ ft}$ $\qquad W = 1.25\text{ lb}$



$a_n = \frac{v_C^2}{\rho} = \frac{14g}{3} = 4.667g$

$+\uparrow \Sigma F_n = ma_n$: $\qquad C - W = ma_n$

$C - W = \frac{W}{g}(4.667g)$

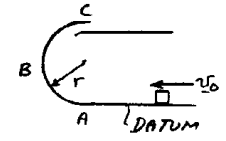
$C = 5.667\,W$

$= 5.667(1.25\text{ lb})$

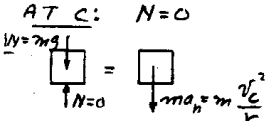
$= 7.083\text{ lb}$

$\underline{C} = 7.08\text{ lb}\uparrow$ ◀

13.73



AT C: $N=0$



$+\downarrow \Sigma F = \Sigma F_{eff}$: $mg = m\frac{v_c^2}{r}$ $\quad v_c^2 = gr$

(a)

$T_A = \frac{1}{2}mv_0^2$; $V_A = 0$

$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}mgr$; $V_C = mg(2r) = 2mgr$

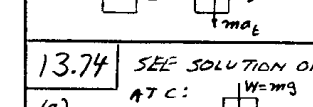
$T_A + V_A = T_C + V_C$: $\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mgr + 2mgr$

$v_0^2 = 5gr$ $\quad \underline{v_0 = \sqrt{5gr}}$ ◀

(b) $T_B = \frac{1}{2}mv_B^2$; $V_B = mg(r)$

$T_A + V_A = T_B + V_B$: $\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_B^2 + mgr$

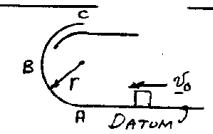
$\frac{1}{2}m(5gr) + 0 = \frac{1}{2}mv_B^2 + mgr$ $\quad v_B^2 = 3gr$ ◁



$+\rightarrow \Sigma F = \Sigma F_{eff}$:

$N_B = 3mg$ $\quad \underline{N_B = 3W}$ ◀

(a)



AT C: $v_C = 0$ $\quad T_C = 0$

$V_C = mg(2r) = 2mgr$

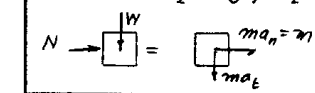
$T_A + V_A = T_C + V_C$:

$\frac{1}{2}mv_0^2 + 0 = 0 + 2mgr$

$v_0^2 = 4gr$ $\quad \underline{v_0 = \sqrt{4gr}}$ ◀

(b) $T_A + V_A = T_B + V_B$: $\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_B^2 + mgr$

$\frac{1}{2}m(4gr) = \frac{1}{2}mv_B^2 + mgr$ $\quad v_B^2 = 2gr$ ◁



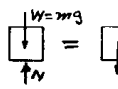
$+\rightarrow \Sigma F = \Sigma F_{eff}$

$N = 2mg$ $\quad \underline{N = 2W}$ ◀

13.74

SEE SOLUTION OF PROB 13.73 $\quad v_c = 1.5 m/s$; $r = 0.3 m$

(a) AT C:



$ma_n = m\frac{v_c^2}{r} = m\frac{1.5^2}{0.3} = 7.5m$

$\downarrow \Sigma F = \Sigma F_{eff}$: $\quad mg - N = 7.5m$ $\quad N = (2.5 - 9.81)m < 0$

IMPOSSIBLE

(b) FOR LOOP 2: $T_A + V_A = T_C + V_C$ $\quad v_c = 1.5 m/s$; $r = 0.3 m$

$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_{c_2}^2 + 2mgr$

FOR $v_0 = 1.5 m/s$: $\frac{1}{2}mv_0^2 = \frac{1}{2}m(1.5)^2 + 2m(9.81)(0.3)$

$v_0^2 = (1.5)^2 + 4(9.81)(0.3)$

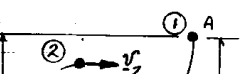
$v_0^2 = 14.022$ $\quad \underline{v_0 = 3.74 m/s}$ ◀

13.75

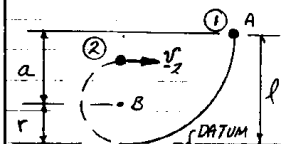
LET $r = l - a$ BE RADIUS OF PATH CENTERED AT B

AT HIGHEST POINT OF CIRCULAR PATH

FOR MINIMUM VELOCITY v_2, TENSION IN CORD $= 0$



$+\downarrow \Sigma F = ma_n$: $\quad mg = m\frac{v_2^2}{r}$ $\quad v_2^2 = gr = g(l-a)$



CONSERVATION OF ENERGY

$T_1 = 0$ $\quad V_1 = mgl$

$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}mg(l-a)$

$V_2 = mg(2r) = 2mg(l-a)$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + mgl = \frac{1}{2}mg(l-a) + 2mg(l-a)$

$l = \frac{1}{2}l - \frac{1}{2}a + 2l - 2a$

$\frac{5}{2}a = \frac{3}{2}l$ $\quad \underline{a = \frac{3}{5}l}$ ◀

*13.76

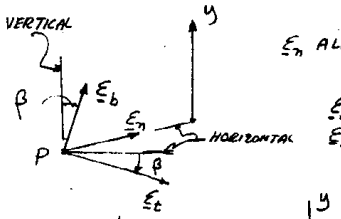
AT POINT A: $v_A = 0$ $\quad T_A = 0$ $\quad V_A = Wh = mgh$

AT ANY POINT P: $T_P = \frac{1}{2}mv^2$ $\quad V_P = W(y) = mgy$

$T_A + V_A = T_P + V_P$: $\quad 0 + mgh = \frac{1}{2}mv^2 + mgy$ $\quad v^2 = 2g(h-y)$

LET β = ANGLE FORMED BY THE CENTERLINE OF THE CHUTE AND THE HORIZONTAL.

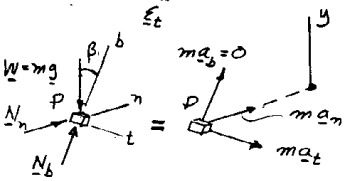
$\beta = \tan^{-1}\frac{h}{2\pi R} = \tan^{-1}\frac{(12 ft)}{2\pi(8 ft)}$ $\quad \beta = 13.427°$



ξ_n ALONG PRINCIPAL NORMAL, HORIZONTAL AND DIRECTED TOWARD y AXIS

ξ_t ALONG CENTERLINE OF CHUTE

ξ_b ALONG BINORMAL



$ma_b = 0$ SINCE $a_b \equiv 0$

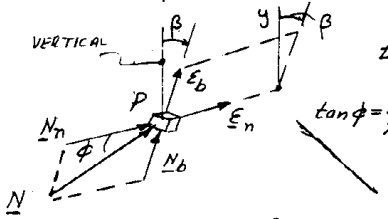
NOTE: FRICTION IS ZERO

$\Sigma F_t = ma_t$: $\quad mg\sin\beta = ma_t$ $\quad a_t = g\sin\beta$

$\Sigma F_b = ma_b$: $\quad N_b - W\cos\beta = 0$ $\quad N_b = W\cos\beta$

$\Sigma F_n = ma_n$: $\quad N_n = m\frac{v^2}{\rho} = m\frac{2g(h-y)}{\rho} = 2W\frac{h-y}{\rho}$

THE TOTAL NORMAL FORCE IS THE RESULTANT OF N_b AND N_n, LIES IN THE b-n PLANE AND FORMS AN ANGLE ϕ WITH THE n AXIS.



$\tan\phi = \frac{N_b}{N_n}$

$\tan\phi = \frac{W\cos\beta}{2W(h-y)/\rho} = \frac{\rho}{2(h-y)}\cos\beta$

GIVEN: $\rho = R\left[1 + \left(\frac{h}{2\pi R}\right)^2\right] = R(1+\tan^2\beta) = \frac{R}{\cos^2\beta}$

THUS:

$\tan\phi = \frac{\rho}{2(h-y)}\cos\beta = \frac{R}{2(h-y)\cos\beta} = \frac{8 ft}{2(12-y)\cos 13.427°}$

(a)

$\tan\phi = \frac{4.113}{12-y}$ $\quad$ OR $\quad \cot\phi = 0.243(12-y)$ ◀

(b) AT POINT B: $y = 0$ $\quad$ FOR x,y,z AXES, WE WRITE $W = 20 lb$

$N_x = N_b\sin\beta = W\cos\beta\sin\beta = (20 lb)\cos 13.427°(\sin 13.427°)$

$N_x = 4.517 lb$

$N_y = N_b\cos\beta = W\cos^2\beta = (20 lb)\cos^2 13.427°$

$N_y = 18.922 lb$

$N_z = -N_n = -2W\frac{h-y}{\rho} = -2W\frac{h-y}{R/\cos^2\beta} = -2W\frac{h-y}{R}\cos^2\beta$

$N_z = -2(20 lb)\frac{(12 ft - 0)}{8 ft}\cos^2 13.427°$ $\quad N_z = -56.765 lb$

$N = \sqrt{(4.517)^2 + (18.922)^2 + (-56.765)^2}$ $\quad N = 60.0 lb$ ◀

$\cos\theta_x = \frac{N_x}{N} = \frac{4.517}{60.0}$ $\quad \theta_x = 85.7°$ ◀

$\cos\theta_y = \frac{N_y}{N} = \frac{18.922}{60.0}$ $\quad \theta_y = 71.6°$ ◀

$\cos\theta_z = \frac{N_z}{N} = -\frac{56.765}{60}$ $\quad \theta_z = 161.1°$ ◀

✱13.77

FOR A CONSERVATIVE FORCE, EQS. (13.22) MUST BE SATISFIED:

$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$$

WE NOW WRITE: $\dfrac{\partial F_x}{\partial y} = -\dfrac{\partial^2 V}{\partial x \partial y}$ $\qquad \dfrac{\partial F_y}{\partial x} = -\dfrac{\partial^2 V}{\partial y \partial x}$

SINCE $\dfrac{\partial^2 V}{\partial x \partial y} = \dfrac{\partial^2 V}{\partial y \partial x}$; $\qquad \dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$ ◀

WE OBTAIN IN A SIMILAR WAY

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \qquad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$ ◀

✱13.78

$$\mathbf{F} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/(x^2+y^2+z^2)^{3/2}$$

(a) $F_x = x/(x^2+y^2+z^2)^{3/2}$ $\qquad F_y = y/(x^2+y^2+z^2)^{3/2}$

$$\frac{\partial F_x}{\partial y} = \frac{x(-\frac{3}{2})2y}{(x^2+y^2+z^2)^{5/2}} \qquad \frac{\partial F_y}{\partial x} = \frac{y(-\frac{3}{2})2x}{(x^2+y^2+z^2)^{5/2}}$$

THUS: $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$

THE OTHER TWO EQUATIONS DERIVED IN PROB 13.77 ARE CHECKED IN A SIMILAR FASHION

(b) RECALLING THAT $F_x = -\dfrac{\partial V}{\partial x}$, $F_y = -\dfrac{\partial V}{\partial y}$, $F_z = -\dfrac{\partial V}{\partial z}$,

WE CONCLUDE THAT $V = -\dfrac{1}{(x^2+y^2+z^2)^{1/2}}$ ◀

WE CHECK THAT

$$F_x = -\frac{\partial V}{\partial x} = -\left(-\frac{1}{2}\right)2x(x^2+y^2+z^2)^{-3/2} = x/(x^2+y^2+z^2)^{3/2}$$

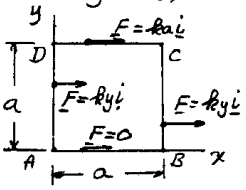
AND THAT THE TWO SIMILAR RELATIONS ARE SATISFIED.

✱13.79

(a) $\mathbf{F} = ky\mathbf{i}$. TO DETERMINE WHETHER FORCE $\mathbf{F}$ IS CONSERVATIVE, WE SHALL CHECK WHETHER THE FIRST OF EQS. DERIVED IN PROB 13.78 IS SATISFIED

$$F_x = ky, \quad \frac{\partial F_x}{\partial y} = k \quad ; \quad F_y = 0, \quad \frac{\partial F_y}{\partial x} = 0$$

THUS: $\dfrac{\partial F_x}{\partial y} \neq \dfrac{\partial F_y}{\partial x}$, $\mathbf{F}$ IS NONCONSERVATIVE ◀



$U = \oint \mathbf{F}\cdot d\mathbf{r}$ AROUND THE SQUARE

WE CHECK THAT $\mathbf{F}\cdot d\mathbf{r} = 0$ ON ALL SIDES EXCEPT CD.

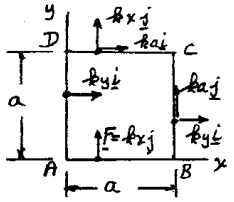
$$U = \int_C^D \mathbf{F}\cdot d\mathbf{r} = ka\mathbf{i}\cdot(-a\mathbf{i}) = -ka^2$$ ◀

(b) $\mathbf{F} = k(y\mathbf{i} + x\mathbf{j})$

$$F_x = ky, \quad \frac{\partial F_x}{\partial y} = k \; ; \qquad F_y = kx, \quad \frac{\partial F_y}{\partial x} = k$$

THUS: $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$ $\mathbf{F}$ IS CONSERVATIVE ◀

SINCE $\mathbf{F}$ IS CONSERVATIVE, ITS WORK AROUND A CLOSED PATH MUST BE ZERO: $U = 0$ ◀



WE CHECK THIS RESULT BY NOTING THAT:

$$U = \int_B^C \mathbf{F}\cdot d\mathbf{r} + \int_C^D \mathbf{F}\cdot d\mathbf{r}$$

$$= (-ka\mathbf{j})\cdot(a\mathbf{j}) + (ka\mathbf{i})\cdot(-a\mathbf{i})$$

$$= -ka^2 - ka^2 = 0$$

✱13.80

$$\mathbf{F} = (x\mathbf{i} + y\mathbf{j})(x^2+y^2)$$

(a) $F_x = x(x^2+y^2)$ $\qquad F_y = y(x^2+y^2)$

$$\frac{\partial F_x}{\partial y} = x(2y) = 2xy \qquad \frac{\partial F_y}{\partial x} = y(2x) = 2xy$$

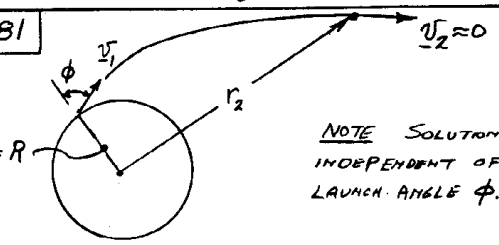
THUS: $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$ FORCE IS CONSERVATIVE

SINCE $F_x = -\dfrac{\partial V}{\partial x}$, WE CONCLUDE: $V = -\frac{1}{4}(x^2+y^2)^2$ ◀

WE CHECK THAT $F_x = -\dfrac{\partial V}{\partial x} = \dfrac{2}{4}(x^2+y^2)(2x) = x(x^2+y^2)$

AND $F_y = -\dfrac{\partial V}{\partial y} = \dfrac{2}{4}(x^2+y^2)(2y) = y(x^2+y^2)$

13.81



NOTE SOLUTION IS INDEPENDENT OF LAUNCH ANGLE ϕ.

AT SURFACE OF EARTH

$v_1 = v_{esc}$ $\qquad r_1 = R = 6370\text{ km} = 6.37\times10^6\text{ m}$

$T_1 = \frac{1}{2}mv_{esc}^2$ $\qquad V_1 = -\dfrac{GMm}{r_1} = -\dfrac{gR^2m}{R} = -gRm$ (cf. Eq 12.21)

WE ASSUME THAT MISSILE WILL REACH $r_2 = \infty$ WITH $v_2 = 0$

$T_2 = 0$ $\qquad V_2 = -\dfrac{GMm}{\infty} = 0$

$T_1 + V_1 = T_2 + V_2$: $\quad \frac{1}{2}mv_{esc}^2 - gRm = 0 + 0$ $\qquad v_{esc} = \sqrt{2gR}$

(a) INITIAL KINETIC ENERGY PER KILOGRAM:

$$T = \frac{T_1}{m} = \frac{1}{2}v_{esc}^2 = \frac{1}{2}(2gR) = gR = (9.81\text{ m/s}^2)(6.37\times10^6\text{ m})$$

$$= 62.49\times10^6\text{ J} \qquad T = 62.5\text{ MJ}$$ ◀

(b) ESCAPE VELOCITY

$$v_{esc} = \sqrt{2gR} = \sqrt{2(9.81\text{ m/s}^2)(6.37\times10^6\text{ m})} \qquad v_{esc} = 11.18\text{ km/s}$$ ◀

13.82

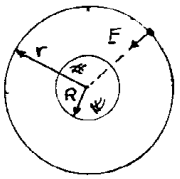
(a) POTENTIAL ENERGY

$$V = -\frac{GMm}{r} + \text{CONSTANT} = -\frac{gR^2m}{r} + \text{CONSTANT} \qquad (\text{cf. Eq } 12.21)$$

CHOOSING CONSTANT SO THAT $V = 0$ FOR $r = R$:

$$V = mgR\left(1 - \frac{R}{r}\right)$$ ◀

(b) KINETIC ENERGY



NEWTON'S SECOND LAW

$F = ma_n$: $\quad \dfrac{GMm}{r^2} = m\dfrac{v^2}{r}$

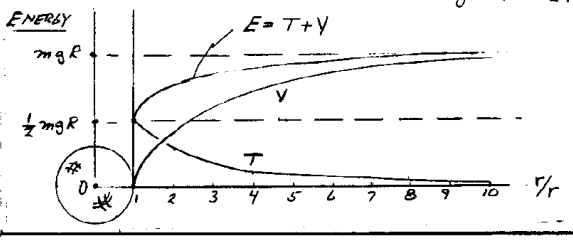
$$v^2 = \frac{GM}{r} = \frac{gR^2}{r}$$

$T = \frac{1}{2}mv^2$ $\qquad T = \frac{1}{2}\dfrac{mgR^2}{r}$ ◀

(c) TOTAL ENERGY

$$E = T + V = \frac{1}{2}\frac{mgR^2}{r} + mgR\left(1 - \frac{R}{r}\right)$$

$$E = mgR\left(1 - \frac{R}{2r}\right)$$ ◀



13.83

EQ (13.17'): $V_g = -\frac{WR^2}{r}$

SETTING $r = R + y$: $V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$

$$V_g = -WR\left(1+\frac{y}{R}\right)^{-1} = -WR\left[1 + \frac{(-1)}{1}\frac{y}{R} + \frac{(-1)(-2)}{1\cdot 2}\left(\frac{y}{R}\right)^2 + \cdots\right]$$

WE ADD THE CONSTANT WR, WHICH IS EQUIVALENT TO CHANGING DATUM FROM $r=\infty$ TO $r=R$:

$$V_g = WR\left[\frac{y}{R} - \left(\frac{y}{R}\right)^2 + \cdots\right]$$

KEEPING ONLY FIRST-ORDER TERM:

$$V_g = WR\left[\frac{y}{R}\right] = Wy \qquad [\text{EQ. (13.16)}]$$ ◀

KEEPING FIRST-ORDER AND SECOND-ORDER TERMS:

$$V_g = WR\left[\frac{y}{R} - \left(\frac{y}{R}\right)^2\right] = Wy - W\frac{y^2}{R}$$

$$V_g = Wy\left(1 - \frac{y}{R}\right)$$ ◀

13.84

NOTE: $GM_{MOON} = 0.0123\, GM_{EARTH}$

BY EQ(12.21): $GM_{MOON} = 0.0123\, g R_E^2$

AT ∞ DISTANCE FROM MOON: $r_2 = \infty$, WE ASSUME $v_2 = 0$

$$E_2 = T_2 + V_2 = 0 - \frac{GM_M m}{\infty} = 0 - 0 = 0$$

(a) ON SURFACE OF MOON: $r_1 = R_M = 1740\ \text{km} = 1.740\times10^6\ \text{m}$

$$E_1 = T_1 + V_1 = 0 - \frac{GM_M m}{r_1} = -\frac{0.0123\, g R_E^2 m}{R_M}$$

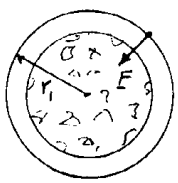
$$E_1 = -\frac{0.0123(9.81\ \text{m/s}^2)(6.370\times10^6\ \text{m})^2 m}{1.740\times10^6\ \text{m}} = -(2.814\times10^6)\,m$$

$$\Delta E = E_1 - E_2 = +(2.814\times10^6\ \text{J/kg})\,m$$

ENERGY PER KILOGRAM:

$$\frac{\Delta E}{m} = 2.81\ \text{MJ/kg}$$ ◀

(b) IN CIRCULAR ORBIT $r_1 = 1740 + 100 = 1840\ \text{km} = 1.840\times10^6\ \text{m}$



NEWTON'S SECOND LAW:

$$F = ma_n: \quad \frac{GM_M m}{r_1^2} = \frac{m v_1^2}{r_1}$$

$$v_1^2 = \frac{GM_M}{r_1}$$

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2} m \frac{GM_M}{r_1}$$

$$E_1 = T_1 + V_1 = \frac{1}{2}\frac{GM_M m}{r_1} - \frac{GM_M m}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{GM_M m}{r_1} = -\frac{1}{2}\frac{0.0123\, g R_E^2 m}{r_1}$$

$$= -\frac{1}{2}\frac{0.0123(9.81\ \text{m/s}^2)(6.37\times10^6\ \text{m})^2 m}{1.840\times10^6\ \text{m}}$$

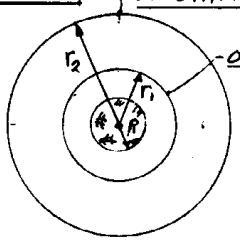
$$E_1 = -(1.330\times10^6\ \text{J/kg})\,m$$

AT ∞ DISTANCE $E_2 = 0$ (SEE ABOVE)

$$\Delta E = E_2 - E_1 = +(1.330\times10^6\ \text{J/kg})\,m$$

ENERGY PER KILOGRAM $= \frac{\Delta E}{m} = 1.330\ \text{MJ/kg}$ ◀

13.85



GEOSYNCHRONOUS ORBIT:
$r_2 = 22{,}230\ \text{mi} + 3960\ \text{mi} = 26{,}190\ \text{mi}$

ORBIT OF SHUTTLE:
$r_1 = 3960\ \text{mi} + 185\ \text{mi} = 4145\ \text{mi}$

RADIUS OF EARTH: $R = 3960\ \text{mi}$

EQ (12.21) $GM = gR^2$

$W = 8000\ \text{lb}$

FOR ANY CIRCULAR ORBIT OF RADIUS r:

NEWTON'S SECOND LAW

$F = ma_n$: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\qquad v^2 = \frac{GM}{r}$

$T = \frac{1}{2} m v^2 = \frac{1}{2} m \frac{GM}{r}$ $\qquad V = -\frac{GMm}{r}$

$$E = T + V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

$$E = -\frac{1}{2}\frac{gR^2 m}{r} = -\frac{1}{2} g m \frac{R^2}{r} = -\frac{1}{2} W \frac{R^2}{r}$$

FOR GEOSYNCHRONOUS ORBIT: $r_2 = 22{,}230 + 3960 = 26{,}190\ \text{mi}$

$$E_2 = -\frac{W}{2}\cdot\frac{R^2}{r_2} = -\frac{8000\ \text{lb}}{2}\cdot\frac{(3960\times5280\ \text{ft})^2}{(26190\times5280\ \text{ft})} = -12.646\times10^9\ \text{ft·lb}$$

FOR ORBIT OF SHUTTLE: $r_1 = 4145\ \text{mi}$

$$E_1 = -\frac{W}{2}\cdot\frac{R^2}{r_1} = -\frac{8000\ \text{lb}}{2}\cdot\frac{(3960\times5280\ \text{ft})^2}{(4145\times5280\ \text{ft})} = -79.902\times10^9\ \text{ft·lb}$$

ON LAUNCHING PAD: $v_0 = 0$ $\qquad R = 3960\ \text{mi}$

$$E_0 = -\frac{GMm}{R} = -\frac{gR^2 m}{R} = -mgR = -WR$$

$$E_0 = -WR = -(8000\ \text{lb})(3960\times5280\ \text{ft}) = -167.27\times10^9\ \text{ft·lb}$$

(a) FROM SHUTTLE ORBIT TO GEOSYNCHRONOUS ORBIT

$$\Delta E = E_2 - E_1 = -12.646\times10^9\ \text{ft·lb} - (-79.902\times10^9\ \text{ft·lb})$$

$$\Delta E = +67.256\times10^9\ \text{ft·lb} \qquad \Delta E = 67.3\times10^9\ \text{ft·lb}$$ ◀

(b) FROM LAUNCH PAD TO GEOSYNCHRONOUS ORBIT

$$\Delta E = E_2 - E_0 = -12.646\times10^9\ \text{ft·lb} - (-167.27\times10^9\ \text{ft·lb})$$

$$\Delta E = +154.62\times10^9\ \text{ft·lb} \qquad \Delta E = 154.6\times10^9\ \text{ft·lb}$$ ◀

13.86

KINETIC ENERGY

WE FOUND IN PROB 12.61: $\delta = eV\ell L/mv_0^2 d$

THUS: $v_0^2 = eV\ell L/md\delta$

AND: $T = \frac{1}{2} m v_0^2 = \frac{1}{2}(eV\ell L/d\delta)$

POTENTIAL ENERGY

DIFFERENCE IN POTENTIAL ENERGY BETWEEN THE PLATES FOR ONE ELECTRON IS

$$Fd = \frac{eV}{d} d = eV$$

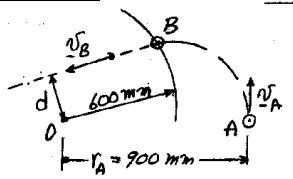
SINCE DATUM IS CHOSEN ON POSITIVE PLATE, POTENTIAL ENERGY HALF-WAY BETWEEN PLATES IS

$$\text{POTENTIAL ENERGY} = \tfrac{1}{2} eV$$

THUS:

$$\frac{\text{POTENTIAL ENERGY}}{\text{KINETIC ENERGY}} = \frac{\frac{1}{2}eV}{\frac{1}{2}(eV\ell L/d\delta)} = \frac{d\delta}{\ell L} \quad \text{(Q.E.D.)}$$

13.87



POSITION A. ELONGATION OF CORD

$x_A = 0.9\text{m} - 0.6\text{m} = 0.3\text{m}$

$V_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(150\text{N/m})(0.3\text{m})^2 = 6.75\text{J}$

$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.2\text{kg})v_A^2 = 0.1v_A^2$

POSITION B. ELONGATION OF CORD = 0

$V_B = \frac{1}{2}kx_B^2 = 0$

$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.2\text{kg})v_B^2 = 0.1v_B^2$

CONSERVATION OF ENERGY

$T_A + V_A = T_B + V_B$: $\frac{1}{2}(0.2)v_A^2 + 6.75 = \frac{1}{2}(0.2)v_B^2$ (1)

CONSERVATION OF ANGULAR MOMENTUM $d = 100\text{mm}$

$mv_A r_A = mv_B d$ $\quad v_A(0.9\text{m}) = v_B(0.1\text{m})$ (2)

SUBSTITUTING FOR v_B FROM (2) INTO (1)

$\frac{1}{2}(0.2)v_A^2 + 6.75 = \frac{1}{2}(0.2)(9v_A)^2$

$6.75 = 8v_A^2$ $\quad v_A = 0.919\text{ m/s}$ ◀

EQ(2) $v_B = 9v_A = 9(0.919)$ $\quad v_B = 8.27\text{ m/s}$ ◀

13.88

SEE SOLUTION OF PROB 13.88 FOR DERIVATION OF EQS. (1) AND (2).

$\frac{1}{2}(0.2)v_A^2 + 6.75 = \frac{1}{2}(0.2)v_B^2$ (1)

$mv_A r_A = mv_B d$ (2)

GIVEN: $v_A = 2.5\text{ m/s}$ $\quad r_A = 0.9\text{ m}$

(a) EQ(1): $\frac{1}{2}(0.2)(2.5)^2 + 6.75 = \frac{1}{2}(0.2)v_B^2$

$v_B = 8.588\text{ m/s}$ $\quad v_B = 8.59\text{ m/s}$ ◀

(b) EQ(2) $m(2.5\text{ m/s})(0.9\text{ m}) = m(8.588\text{ m/s})d$

$d = 0.262\text{ m}$ $\quad d = 262\text{ mm}$ ◀

13.89

SEE SOLUTION OF PROB. 13.87 FOR DERIVATION OF EQS (1) AND (2).

$\frac{1}{2}(0.2)v_A^2 + 6.75 = \frac{1}{2}(0.2)v_B^2$ (1)

$mv_A r_A = mv_B d$ (2)

(a) FOR SMALLEST v_A FOR WHICH CORD STAYS TAUT,

$d = 600\text{ mm} = 0.6\text{ m}$

EQ(2) $mv_A(0.9\text{m}) = mv_B(0.6\text{m})$ $\quad v_B = 1.5v_A$

SUBSTITUTING $v_B = 1.5v_A$ INTO EQ(1)

$\frac{1}{2}(0.2)v_A^2 + 6.75 = \frac{1}{2}(0.2)(1.5v_A)^2$

$6.75 = \frac{0.2}{2}(1.5^2 - 1)v_A^2 = 0.125v_A^2$

(b) $v_A = 7.348\text{ m/s}$ $\quad v_A = 7.35\text{ m/s}$ ◀

CORRESPONDING MAXIMUM SPEED: $v_B = 1.5v_A = 11.02\text{ m/s}$ ◀

13.90

CONSERVATION OF ANGULAR MOMENTUM $r_B = r_m = 1.2\text{m}$

$mr_m v_m = (mv_A \sin 60°)r_A$

$m(1.2\text{m})v_m = (mv_A \frac{\sqrt{3}}{2})(0.5\text{m})$ $\quad v_m = \frac{\sqrt{3}}{4.8}v_A$ (1)

CONSERVATION OF ENERGY (SEE SAMPLE PROB 13.8)

AT POINT A: $T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.6)v_A^2$ $\quad V_A = \frac{1}{2}kr_A = \frac{1}{2}(100)(0.5)^2 = 12.5\text{J}$

AT POINT B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.6\text{kg})v_m^2$

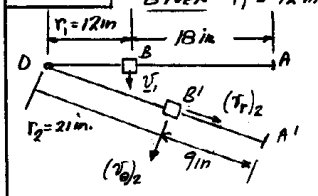
$V_B = \frac{1}{2}kr_B^2 = \frac{1}{2}(100\text{N/m})(1.2\text{m})^2 = 72\text{J}$

$T_A + V_A = T_B + V_B$: $\frac{1}{2}(0.6)v_A^2 + 12.5 = \frac{1}{2}(0.6)v_m^2 + 72$

SUBSTITUTE FOR v_m FROM (1) $\quad 0.3v_A^2 + 12.5 = 0.3\frac{3}{(4.8)^2}v_A^2 + 72$ $\quad v_A = 15.10\text{ m/s}$ ◀

13.91

GIVEN $r_1 = 12\text{ in.}$, $v_1 = 16\text{ ft/s}$, $r_2 = 21\text{ in.}$



CONSERVATION OF ANG. MOMENTUM

$r_1 m v_1 = r_2 m (v_\theta)_2$

$(12\text{in})m(16\text{ft/s}) = (21\text{in})m(v_\theta)_2$

$(v_\theta)_2 = \frac{12}{21}(16\text{ft/s})$

$(v_\theta)_2 = 9.14\text{ ft/s}$ ◀

CONSERVATION OF ENERGY

$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{10}{9}(16)^2 = \frac{1}{2}\frac{2560}{9}$; $\Delta_1 = 0$, $V_1 = \frac{1}{2}k\Delta_1^2 = 0$

$T_2 = \frac{1}{2}m\left[(v_\theta)_2^2 + (v_r)_2^2\right] = \frac{1}{2}\frac{10}{9}\left[(9.14)^2 + (v_r)_2^2\right]$

$\Delta_2 = 18 - 9 = 9\text{in} = 0.75\text{ft}$

$V_2 = \frac{1}{2}k\Delta_2^2 = \frac{1}{2}(50\text{ lb/ft})(0.75\text{ft})^2 = \frac{1}{2}(28.125)$

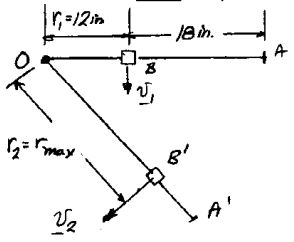
$T_1 + V_1 = T_2 + V_2$: $\frac{1}{2}\frac{2560}{9} = \frac{1}{2}\frac{10}{9}\left[(9.14)^2 + (v_r)_2^2\right] + \frac{1}{2}(28.125)$

$(9.14)^2 + (v_r)_2^2 = 256 - \frac{9}{10}(28.125)$

$(v_r)_2^2 = 82.0$ $\quad (v_r)_2 = 9.05\text{ ft/s}$ ◀

13.92

GIVEN: $r_1 = 12\text{ in.}$, $v_1 = 16\text{ ft/s}$



CONSERVATION OF ANG. MOMENTUM

$r_1 m v_1 = r_2 m v_2$

$(1\text{ft})m(16\text{ft/s}) = r_{max}\, m v_2$

$v_2 = \frac{16\text{ ft}^2/\text{s}}{r_{max}}$ (1)

CONSERVATION OF ENERGY

$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{10}{9}(16)^2 = \frac{1}{2}\frac{2560}{9}$

SPRING: $\Delta_1 = 0$; $V_1 = 0$

$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{10}{9}v_2^2$

SPRING: $\Delta_2 = 18\text{ in.} - (30\text{ in.} - r_{max}) = r_{max} - 12\text{ in.} = r_{max} - 1\text{ ft}$

$V_2 = \frac{1}{2}k\Delta_2^2 = \frac{1}{2}(50\text{ lb/ft})(r_{max} - 1)^2$

$T_1 + V_1 = T_2 + V_2$:

$\frac{1}{2}\frac{2560}{9} = \frac{1}{2}\frac{10}{9}v_2^2 + \frac{1}{2}(50)(r_{max} - 1)^2$ (2)

(a) SUBSTITUTING FOR v_2 FROM (1) INTO (2):

$\frac{1}{2}\frac{2560}{9} = \frac{1}{2}\frac{10}{9}\left(\frac{16}{r_{max}}\right)^2 + \frac{1}{2}(50)(r_{max} - 1)^2$

$g = 32.2\text{ ft/s}^2$ $\quad 39.75 = \frac{39.25}{r_{max}^2} + 25(r_{max} - 1)^2$

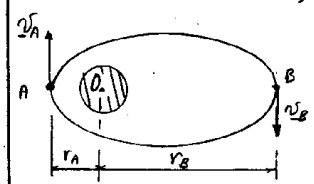
$1 - \frac{1}{r_{max}^2} = 0.6289(r_{max} - 1)^2$

SOLVING BY TRIAL: $r_{max} = 2.11\text{ ft} = 25.3\text{ in.}$ ◀

(b) SUBSTITUTING INTO (1):

$v_2 = \frac{16\text{ ft}^2/\text{s}}{2.11\text{ ft}}$ $\quad v_2 = 7.58\text{ ft/s}$ ◀

13.93 (Prob 12.97)



FOR AN ELLIPTIC TRANSFER ORBIT

CONS. OF ANGULAR MOMENTUM

$$r_A m v_A = r_B m v_B$$

$$v_B = \frac{r_A}{r_B} v_A \qquad (1)$$

CONS. OF ENERGY

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_B} \qquad (2)$$

SUBSTITUTING FOR v_B FROM (1) INTO (2):

$$v_A^2 - \frac{2GM}{r_A} = \frac{r_A^2}{r_B^2}v_A^2 - \frac{2GM}{r_B}$$

$$v_A^2\left(1 - \frac{r_A^2}{r_B^2}\right) = 2GM\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$v_A^2 \frac{r_B^2 - r_A^2}{r_B^2} = 2GM \frac{r_B - r_A}{r_A r_B}$$

$$v_A^2 = 2GM \frac{r_B}{r_A(r_B + r_A)} \qquad (3)$$

ALSO, USING EQ(1)

$$v_B^2 = 2GM \frac{r_A}{r_B(r_B + r_A)} \qquad (4)$$

GIVEN DATA:

$GM = gR^2 = (32.2\ ft/s^2)(3960 \times 5280\ ft)^2 = 14.077 \times 10^{15}$

$h_A = 40$ mi, $r_A = (3960 + 40) = 4000$ mi $= 21.12 \times 10^6$ ft

$h_B = 240$ mi, $r_B = (3960 + 240) = 4200$ mi $= 22.176 \times 10^6$ ft

SUBSTITUTING INTO EQ(3):

(a) $$v_A^2 = 2(14.077 \times 10^{15}) \frac{22.176 \times 10^6}{(21.12 \times 10^6)(22.176 + 21.12)10^6} = 682.781 \times 10^6$$

$v_A = 26.1301 \times 10^3$ ft/s $\quad v_0 = v_A = 26.1 \times 10^3$ ft/s ◀

EQ(1) $v_B = \frac{r_A}{r_B} v_A = \frac{21.12 \times 10^6}{22.176 \times 10^6}(26.1301 \times 10^3)$ $\quad v_B = 24.8858 \times 10^3$ ft/s

(b) FOR CIRCULAR ORBIT THROUGH B: $r_B = 22.176 \times 10^6$ ft

EQ(12.44) $$v_{circ}^2 = \frac{GM}{r_B} = \frac{14.077 \times 10^{15}}{22.176 \times 10^6} = 634.785 \times 10^6 \qquad v_{circ} = 25.1949 \times 10^3 \text{ ft/s}$$

$\Delta v = v_{circ} - v_B = (25.1949 - 24.8858)10^3$ $\quad \Delta = .309$ ft/s ◀

13.94 (Prob 12.98)

SEE SOLUTION OF PROB 13.93 FOR DERIVATION OF EQ(1) THROUGH (4).

GIVEN DATA:

$h_A = 1500$ mi, $r_A = (3960 + 1500) = 5460$ mi $= 28.829 \times 10^6$ ft

$h_B = 6000$ mi, $r_B = (3960 + 6000) = 9960$ mi $= 52.589 \times 10^6$ ft

$GM = gR^2 = (32.2)(3960 \times 5280)^2 = 14.077 \times 10^{15}$

SUBSTITUTE DATA INTO EQ.(3):

(a) $$v_A^2 = 2(14.077 \times 10^{15}) \frac{52.589 \times 10^6}{(28.829 \times 10^6)(28.829 + 52.589)10^6}$$

$v_A^2 = 630.795 \times 10^6$; $v_A = 25,116$ ft/s $\quad v_A = 17,120$ mi/h ◀

(b) INCREASE SPEED AT A BY 8%

$v_A = 1.08(25,116\ ft/s) = 27,125$ ft/s

EQ.(3): $$(27,125)^2 = 2(14.077 \times 10^{15}) \frac{r_B}{(28.829 \times 10^6)(28.829 \times 10^6 + r_B)}$$

$0.753406(28.829 \times 10^6 + r_B) = r_B$

$21.7199 \times 10^6 = 0.246594\, r_B$

$r_B = 88.08 \times 10^6$ ft $= 16,682$ mi $\quad h_B = 12,720$ mi ◀

13.95 (Prob 12.101 a)

SEE SOLUTION OF PROB 13.93 FOR DERIVATION OF EQS (1) THROUGH (4).

GIVEN DATA:

$r_A = 15 \times 10^3$ km $= 15 \times 10^6$ m

$v_A = 6500$ m/s

FOR VENUS: $M_{VENUS} = 0.82\, M_{EARTH}$

$(GM)_{VENUS} = 0.82\,(GM)_{EARTH} = 0.82\, g R_{EARTH}^2$

$= 0.82(398 \times 10^{12})$

$= 326.36 \times 10^{12}$

SUBSTITUTE DATA INTO EQ(3):

$$(6500\ m/s)^2 = 2(326.36 \times 10^{12}) \frac{r_B}{(15 \times 10^6\ m)(r_B + 15 \times 10^6\ m)}$$

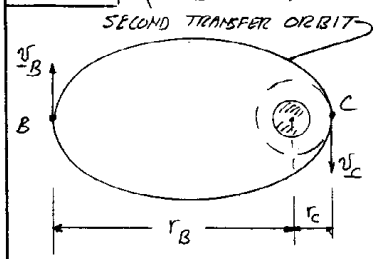
$0.970937(r_B + 15 \times 10^6) = r_B$

$14.569 \times 10^6 = 29.06 \times 10^{-3} r_B$

$r_B = 501.12 \times 10^6$ m

$r_B = 501 \times 10^3$ km ◀

13.96 (Prob 12.100)



SEE SOLUTION OF PROB 13.94 FOR DERIVATION OF EQS. (1) AND (3). WE REWRITE THESE EQUATIONS USING POINTS B AND C.

$$v_C = \frac{r_B}{r_C} v_B \qquad (1)$$

$$v_B^2 = 2GM \frac{r_C}{r_B(r_C + r_B)} \qquad (3)$$

GIVEN DATA:

$r_B = 300 \times 10^3$ km $= 300 \times 10^6$ m

$r_C = 9 \times 10^3$ km $= 9 \times 10^6$ m

FOR VENUS: $M_{VENUS} = 0.82\, M_{EARTH}$

$(GM)_{VENUS} = 0.82(GM)_{EARTH} = 0.82\, g R_E^2$

$= 0.82(398 \times 10^{12}) = 326.4 \times 10^{12}$

(a) VELOCITY AT B:

EQ(3): $$v_B^2 = 2(326.4 \times 10^{12}) \frac{9 \times 10^6\ m}{(300 \times 10^6\ m)(9 + 300)10^6\ m}$$

$v_B^2 = 63.379 \times 10^3$ $\quad v_B = 251.75$ m/s

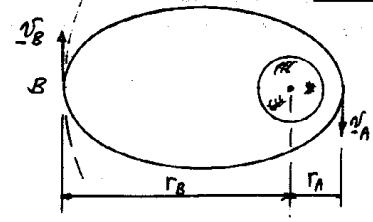
$v_B = 252$ m/s ◀

(b) VELOCITY AT C

EQ(1): $v_C = \frac{300 \times 10^6\ m}{9 \times 10^6\ m}(251.75\ m/s) = 8392$ m/s

$v_C = 8.39$ km/s ◀

13.97



ELLIPTIC TRANSFER ORBIT

SEE PROB 13.93 FOR DERIVATION OF:

$$v_B = \frac{r_A}{r_B} v_A \quad (1)$$

$$v_A^2 = 2GM \frac{r_B}{r_A(r_B + r_A)} \quad (3)$$

GIVEN DATA: AT ENGINE SHUT OFF (POINT A)

$r_A = R_m + h_A = 1740 + 8 = 1748$ km $= 1.748 \times 10^6$ m

AT POINT B: $r_B = R_m + h_B = 1740 + 140 = 1880$ km $= 1.880 \times 10^6$ m

FOR THE MOON: $M_{MOON} = 0.0123\, M_{EARTH}$

$$(GM)_{MOON} = 0.0123\,(GM)_{EARTH} = 0.0123\, g R_E^2$$
$$= 0.0123\,(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2$$
$$= 4.896 \times 10^{12}$$

(a) SPEED AT A:

EQ(3): $$v_A^2 = 2(4.896 \times 10^{12}) \frac{1.880 \times 10^6 \text{ m}}{(1.748 \times 10^6 \text{ m})(1.880 + 1.748) 10^6 \text{ m}}$$

$v_A^2 = 2.9028 \times 10^6$ $\quad v_A = 1703.7$ m/s $\quad v_A = 1704$ m/s ◀

(b) AT POINT B:

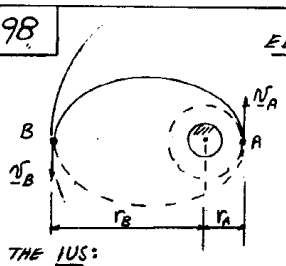
EQ(1): $$v_B = \frac{1.748 \times 10^6 \text{ m}}{1.880 \times 10^6 \text{ m}} (1703.7 \text{ m/s}) \quad v_B = 1584.0 \text{ m/s}$$

COMMAND MODULE IS IN CIRCULAR ORBIT: $r_B = 1.88 \times 10^6$ m

EQ(12.44) $$v_{CIRC} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{4.896 \times 10^{12}}{1.88 \times 10^6}} = 1613.8 \text{ m/s}$$

RELATIVE VELOCITY $= v_{CIRC} - v_B = 1613.8 - 1584.0 = 29.8$ m/s ◀

13.98



ELLIPTIC TRANSFER ORBIT

SEE PROB 13.93 FOR DERIVATION OF:

$$v_B = \frac{r_A}{r_B} v_A \quad (1)$$

$$v_A^2 = 2GM \frac{r_B}{r_A(r_B + r_A)} \quad (3)$$

FOR THE IUS:

GIVEN DATA: $h_A = 185$ mi., $r_A = R + h_A = 3960 + 185 = 4145$ mi $= 21.886 \times 10^6$ ft

$h_B = 22{,}230$ mi., $r_B = R + h_B = 3960 + 22{,}230 = 26{,}190$ mi $= 138.283 \times 10^6$ ft

$GM = g R^2 = 32.2(3960 \times 5280)^2 = 14.077 \times 10^{15}$

EQ(3): $$v_A^2 = 2(14.077 \times 10^{15}) \frac{138.283 \times 10^6 \text{ ft}}{(21.886 \times 10^6 \text{ ft})(138.283 + 21.886) 10^6 \text{ ft}}$$

$v_A^2 = 1.1065 \times 10^9$ $\quad v_A = 33{,}326$ ft/s

EQ(1): $$v_B = \frac{21.886 \times 10^6 \text{ ft}}{138.283 \times 10^6 \text{ ft}} (33{,}326 \text{ ft/s}) \quad v_B = 5274 \text{ ft/s}$$

(a) AT POINT A: SHUTTLE IN CIRCULAR ORBIT: $r_A = 21.886 \times 10^6$ ft

EQ(12.44) $$v_{CIRC} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077 \times 10^{15}}{21.886 \times 10^6}} = 25{,}362 \text{ ft/s}$$

RELATIVE VELOCITY OF IUS WITH RESPECT TO SHUTTLE

$v_A - v_{CIRC} = 33{,}326 - 25{,}362 = 7964$ ft/s

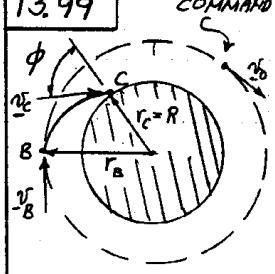
RELATIVE VELOCITY $= 7960$ ft/s ◀

(b) AT POINT B: FOR GEOSYNCHRONOUS ORBIT: $r_B = 138.283 \times 10^6$ ft

EQ(12.44) $$v_{CIRC} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.077 \times 10^{15}}{138.283 \times 10^6}} = 10090 \text{ ft/s}$$

INCREASE IN VELOCITY $= v_{CIRC} - v_B = 10090 - 5274 = 4816$ ft/s $= 4820$ ft/s ◀

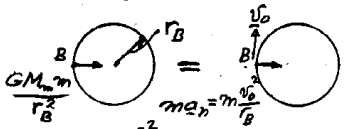
13.99



COMMAND MODULE IN CIRCULAR ORBIT

$r_B = 1740 + 140 = 1880$ km $= 1.88 \times 10^6$ m

$GM_{moon} = 0.01236\, M_{earth}$
$= 0.0123\, g R^2$
$= 0.0123(9.81)(6.37 \times 10^6)^2$
$= 4.896 \times 10^{12}$



$R = 1740$ km $\quad \Sigma F = ma_n$: $\frac{GM_m m}{r_B^2} = m\frac{v_0^2}{r_B}$ $\quad v_0 = \sqrt{GM_m / r_B}$

MOTION OF LEM.

$v_0 = \sqrt{(4.896 \times 10^6)/(1.88 \times 10^6)} = 1614$ m/s; $v_B = 1614 - 200 = 1414$ m/s

CONSERVATION OF ENERGY BETWEEN B AND C

$$\frac{1}{2} m v_B^2 - \frac{GMm}{r_B} = \frac{1}{2} m v_C^2 - \frac{GMm}{R}$$

$$v_C^2 = v_B^2 + \frac{2GM}{r_B}\left(\frac{r_B}{R} - 1\right)$$
$$= (1414 \text{ m/s})^2 + \frac{2(4.896 \times 10^{12})}{1.88 \times 10^6 \text{ m}}\left(\frac{1.88 \times 10^6 \text{ m}}{1.740 \times 10^6 \text{ m}} - 1\right)$$
$$= 1.999 \times 10^6 + 0.4191 \times 10^6 = 2.418 \times 10^6$$

$v_C = 1555.0$ m/s $\quad v_C = 1555$ m/s ◀

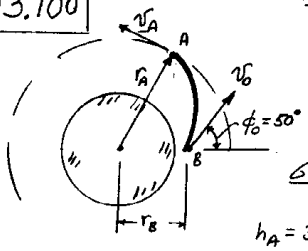
CONSERVATION OF ANGULAR MOMENTUM

$$r_B m v_B = R m v_C \sin\phi$$

$$\sin\phi = \frac{r_B v_B}{R v_C} = \frac{(1.88 \times 10^6 \text{ m})(1414 \text{ m/s})}{(1.74 \times 10^6 \text{ m})(1555 \text{ m/s})} = 0.98249$$

$\phi = 79.26°$ $\quad \phi = 79.3°$ ◀

13.100



CONSERVATION OF ENERGY

$$T_B + V_B = T_A + V_A$$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_B} = \frac{1}{2} m v_A^2 - \frac{GMm}{r_A}$$

$$v_A^2 = v_0^2 - \frac{2GM}{r_B}\left(1 - \frac{r_B}{r_A}\right) \quad (1)$$

GIVEN DATA

$R = 6370$ km $= 6.37 \times 10^6$ m

$h_A = 360$ km

$r_A = R + h_A = 6370 + 360 = 6730$ km $= 6.73 \times 10^6$ m

$h_B = 60$ km

$r_B = R + h_B = 6370 + 60 = 6430$ km $= 6.43 \times 10^6$ m

$GM = g R^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 = 398 \times 10^{12}$

SUBSTITUTING INTO EQ (1):

$$v_A^2 = v_0^2 - \frac{2(398 \times 10^{12})}{6.43 \times 10^6}\left(1 - \frac{6.43 \times 10^6}{6.73 \times 10^6}\right)$$

$$v_A^2 = v_0^2 - 5.518 \times 10^6 \quad (2)$$

CONSERVATION OF ANGULAR MOMENTUM

$$r_B m v_0 \sin\phi = r_A m v_A$$

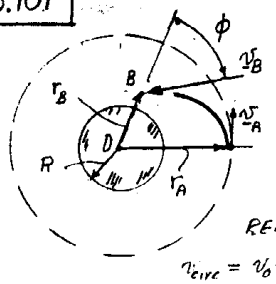
$$v_A = \frac{r_B}{r_A} v_0 \sin\phi = \frac{6.43 \times 10^6 \text{ m}}{6.73 \times 10^6 \text{ m}} (v_0) \sin 50°$$

$v_A = 0.7319\, v_0$

SUBSTITUTE FOR v_A INTO EQ. (2):

$$(0.7319\, v_0)^2 = v_0^2 - 5.518 \times 10^6$$
$$v_0^2 (1 - 0.53567) = 5.518 \times 10^6$$
$$0.46433\, v_0^2 = 5.518 \times 10^6$$
$$v_0^2 = 11.884 \times 10^6$$
$$v_0 = 3447 \text{ m/s} \quad v_0 = 3450 \text{ m/s}$$ ◀

13.101



SHUTTLE IN CIRCULAR ORBIT WITH $h_A = 150$ mi

$r_A = R + h_A = 3960 + 150$

$r_A = 4110$ mi $= 21.7\times10^6$ ft

$GM = gR^2 = 32.2(3960\times5280)^2$

$= 14.077\times10^{15}$

REFERRING TO SOLUTION OF PROB. 13.106

$$v_{circ} = v_0 = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times10^{15}}{21.7\times10^6}}$$

$v_{CIRC} = 25{,}470$ ft/s

FOR SHUTTLE DECENDING AFTER 5% DECREASE IN SPEED

$v_A = 0.95 v_{circ} = 0.95(25{,}470) = 24{,}196$ ft/s

AT B: $h_B = 30$ mi.

$r_B = R + r_B = 3960 + 30 = 3990$ mi $= 21.067\times10^6$ ft

CONSERVATION OF ENERGY

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m v_A^2 - \frac{GMm}{r_A} = \frac{1}{2}m v_B^2 - \frac{GMm}{r_B}$$

$$v_B^2 = v_A^2 + \frac{2GM}{r_A}\left(\frac{r_A}{r_B} - 1\right) \qquad (1)$$

SUBSTITUTE FOR v_A, GM, r_A AND r_B

$$v_B^2 = (24{,}196)^2 + \frac{2(14.077\times10^{15})}{21.7\times10^6}\left(\frac{4110\text{ mi}}{3990\text{ mi}} - 1\right)$$

$= 585.4\times10^6 + 39.02\times10^6 = 624.5\times10^6$

$v_B = 24{,}990$ ft/s $\qquad v_B = 25.0\times10^3$ ft/s ◀

CONSERVATION OF ANGULAR MOMENTUM

$$r_A m v_A = r_B m v_B \sin\phi \qquad (2)$$

$$\sin\phi = \frac{r_A v_A}{r_B v_B} = \frac{(21.7\times10^6\text{ ft})(24{,}196\text{ ft/s})}{(21.067\times10^6\text{ ft})(24{,}990\text{ ft/s})} = 0.9973$$

$\phi = 85.8°$ ◀

13.102

SEE SOLUTION OF PROB. 13.101 FOR DERIVATION OF EQS. (1) AND (2) AND FOR THE FOLLOWING DATA

$r_A = 4110$ mi $= 21.7\times10^6$ ft

$r_B = 3990$ mi $= 21.067\times10^6$ ft

$GM = 14.077\times10^{15}$

EQ(2): $r_A m v_A = r_B m v_B \sin\phi$

FOR $\phi = 80°$, $(4110\text{ mi})v_A = (3990\text{ mi})v_B \sin 80°$

$$v_A = 0.95605 v_B$$

SUBSTITUTING FOR v_A, GM, r_A, r_B INTO EQ(1),

$$v_B^2 = (0.95605 v_B)^2 + \frac{2(14.077\times10^{15})}{21.7\times10^6\text{ ft}}\left(\frac{4110\text{ mi}}{3990\text{ mi}} - 1\right)$$

$(1 - 0.91405)v_B^2 = 39.020\times10^6$

$v_B^2 = 453.93\times10^6$

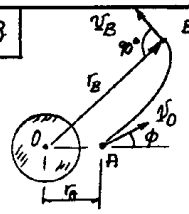
$v_B = 21.306\times10^3$ ft/s $\qquad v_B = 21.3\times10^3$ ft/s ◀

WE MAY NOTE THAT:

$v_A = 0.95605 v_B = 0.95605(21.306\times10^3\text{ ft/s})$

$v_A = 20.37\times10^3$ ft/s

13.103



CONSERVATION OF ANG. MOMENTUM

$$r_A m v_0 \sin\phi = r_B m v_B$$

$$v_B = \frac{r_A}{r_B}\sin\phi\, v_0 \qquad (1)$$

CONSERVATION OF ENERGY

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}m v_0^2 - \frac{GMm}{r_A} = \frac{1}{2}m v_B^2 - \frac{GMm}{r_B}$$

$$v_0^2 - v_B^2 = \frac{2GM}{r_A}\left(1 - \frac{r_A}{r_B}\right)$$

SUBSTITUTING FOR v_A FROM (1):

$$v_0^2\left[1 - \left(\frac{r_A}{r_B}\right)^2\sin^2\phi\right] = \frac{2GM}{r_A}\left(1 - \frac{r_A}{r_B}\right) \qquad (2)$$

GIVEN DATA:

$R = 6370$ km $= 6.37\times10^6$ m

$GM = gR^2 = (9.81\text{ m/s}^2)(6.37\times10^6\text{ m})^2 = 398\times10^{12}$

$h_A = 300$ km, $r_A = R + h_A = 6370 + 300 = 6670$ km $= 6.67\times10^6$ m

$v_0 = 8000$ m/s $\qquad \phi = 40°$

EQ(2): $(8000)^2\left[1 - \left(\frac{r_A}{r_B}\right)^2\sin^2 40°\right] = \frac{2(398\times10^{12})}{6.67\times10^6}\left(1 - \frac{r_A}{r_B}\right)$

$$1 - 0.41318\left(\frac{r_A}{r_B}\right)^2 = 1.86469\left(1 - \frac{r_A}{r_B}\right)$$

$$\left(\frac{r_A}{r_B}\right)^2 - 4.531\,\frac{r_A}{r_B} + 2.093 = 0$$

$\frac{r_A}{r_B} = 0.5247$ $\qquad \frac{r_A}{r_B} = 3.988$

FOR MAXIMUM r_B, WE CHOOSE $\frac{r_A}{r_B} = 0.5247$

$\frac{6.77\times10^6\text{ m}}{r_B} = 0.5247$ $\qquad r_B = 12.901\times10^6$ m

MAX. ALTITUDE: $h_B = r_B - R = 12.901\times10^6 - 6.37\times10^6 = 6.532\times10^6$ m

$h_B = 6530$ km ◀

13.104

SEE SOLUTION OF PROB. 13.103 FOR SKETCH AND DERIVATION OF EQS (1) AND (2).

GIVEN DATA:

$R = 6370$ km $= 6.37\times10^6$ m

$GM = gR^2 = (9.81\text{ m/s}^2)(6.37\times10^6\text{ m})^2 = 398\times10^{12}$

$h_A = 400$ km,

$r_A = 6370 + 400 = 6770$ km $= 6.77\times10^6$ m

$h_B = 3000$ km (MAX ALTITUDE)

$r_B = 6370 + 3000 = 9370$ km $= 9.37\times10^6$ m

$v_0 = 7500$ m/s

EQ(2): $v_0^2\left[1 - \left(\frac{r_A}{r_B}\right)^2\sin^2\phi\right] = \frac{2GM}{r_A}\left(1 - \frac{r_A}{r_B}\right)$

$$(7500\text{ m/s})^2\left[1 - \left(\frac{6770\text{ km}}{9370\text{ km}}\right)^2\sin^2\phi\right] = \frac{2(398\times10^{12})}{6.77\times10^6\text{ m}}\left(1 - \frac{6770\text{ km}}{9370\text{ km}}\right)$$

$$1 - 0.52204\sin^2\phi = 2.0903(1 - 0.72252)$$

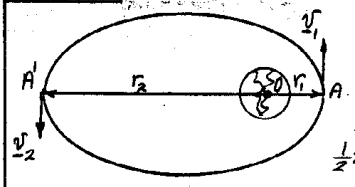
$$1 - 0.52204\sin^2\phi = 0.58001$$

$$\sin^2\phi = 0.80453$$

$$\sin\phi = 0.89695$$

$\phi = 63.76°$ $\qquad \phi = 63.8°$ ◀

13.105



CONS. ANGULAR MOMENTUM

$$r_1 m v_1 = r_2 m v_2$$

$$v_2 = \frac{r_1}{r_2} v_1 \qquad (1)$$

CONS. OF ENERGY

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \qquad (2)$$

SUBSTITUTING FOR v_2 FROM (1) INTO (2):

$$v_1^2 - \frac{2GM}{r_1} = \left(\frac{r_1}{r_2}v_1\right)^2 - \frac{2GM}{r_2}$$

$$\left(1 - \frac{r_1^2}{r_2^2}\right)v_1^2 = 2GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{r_2^2 - r_1^2}{r_2^2}v_1^2 = 2GM\,\frac{r_2 - r_1}{r_1 r_2}$$

OBSERVING THAT $r_2^2 - r_1^2 = (r_2 - r_1)(r_2 + r_1)$

$$v_1^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1} \qquad (3)$$

EXCHANGING THE SUBSCRIPTS 1 AND 2

$$v_2^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_1}{r_2} \qquad \text{(Q.E.D.)}$$

13.106

SEE SOLUTION OF PROB. 13.105 FOR DERIVATION OF EQ(3)

$$v_1^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1} \qquad (3)$$

TOTAL ENERGY AT POINT A IS:

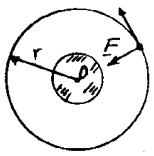
$$E = T_1 + V_1 = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$$

$$= \frac{1}{2}m\left[\frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1}\right] - \frac{GMm}{r_1}$$

$$= GMm\left[\frac{r_2}{r_1(r_1 + r_2)} - \frac{1}{r_1}\right] = GMm\,\frac{r_2 - r_1 - r_2}{r_1(r_1 + r_2)}$$

$$E = -\frac{GMm}{r_1 + r_2} \qquad \blacktriangleleft$$

13.107



(a) FOR A CIRCULAR ORBIT OF RADIUS r

$$F = ma_n: \quad \frac{GMm}{r} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r}$$

$$E = -\frac{1}{2}\frac{GMm}{r} \qquad (1)$$

THUS ΔE REQUIRED TO PASS FROM CIRCULAR ORBIT OF RADIUS r_1 TO CIRCULAR ORBIT OF RADIUS r_2 IS

$$\Delta E = E_2 - E_1 = -\frac{1}{2}\frac{GMm}{r_2} + \frac{1}{2}\frac{GMm}{r_1}$$

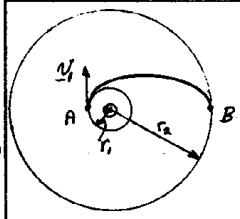
$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \qquad (2) \quad \text{(Q.E.D.)}$$

(b) FOR AN ELLIPTIC ORBIT, WE RECALL EQ(3) DERNED IN PROB 13.106.

$$v_1^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1}$$

(CONTINUED)

13.107 CONTINUED



AT POINT A: INITIALLY SPACECRAFT IS IN CIRCULAR ORBIT OF RADIUS r_1. $v_{CIRC}^2 = \frac{GM}{r_1}$

$$T_{CIRC} = \frac{1}{2}mv_{CIRC}^2 = \frac{1}{2}m\frac{GM}{r_1}$$

AFTER SPACECRAFT ENGINES ARE FIRED AND SPACECRAFT IS PLACED ON SEMIELLIPTIC PATH AB, WE RECALL

$$v_1^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1}$$

AND

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1}$$

THUS AT POINT A THE INCREASE IN ENERGY IS

$$\Delta E_A = T_1 - T_{CIRC} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

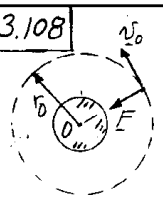
ΔE_A MAY ALSO BE WRITTEN AS

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\left[\frac{GMm(r_2 - r_1)}{2r_1 r_2}\right]$$

OR, RECALLING EQ(2): $\Delta E_A = \frac{r_2}{r_1 + r_2}[\Delta E]$ (Q.E.D.)

A SIMILAR DERIVATION YIELDS: $\Delta E_B = \frac{r_1}{r_1 + r_2}[\Delta E]$ (Q.E.D.)

13.108



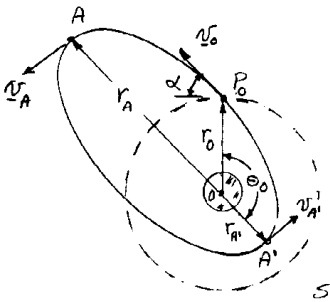
FOR CIRCULAR ORBIT OF RADIUS r_0

$$F = ma_n: \quad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

BUT, v_0 FORMS AN ANGLE α WITH THE INTENDED CIRCULAR PATH.

FOR ELLIPTIC ORBIT:



CONS. OF ANG. MOMENTUM

$$r_0 m v_0 \cos\alpha = r_A m v_A$$

$$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right)v_0 \qquad (1)$$

CONS. OF ENERGY

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

SUBSTITUTE FOR v_A FROM (1)

$$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

BUT $v_0^2 = \frac{GM}{r_0}$, THUS $1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha = 2\left(1 - \frac{r_0}{r_A}\right)$

$$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$$

SOLVING FOR $\frac{r_0}{r_A}$:

$$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha} = \frac{1 \pm \sin\alpha}{(1 + \sin\alpha)(1 - \sin\alpha)}$$

$$r_A = \frac{(1 + \sin\alpha)(1 - \sin\alpha)}{1 \pm \sin\alpha}r_0 = (1 \mp \sin\alpha)r_0$$

{ALSO VALID FOR POINT A'}

THUS: $r_{max} = (1 + \sin\alpha)r_0$, $r_{min} = (1 - \sin\alpha)r_0$ $\blacktriangleleft$

13.109

SEE SOLUTION OF PROB 13.103 FOR DERIVATION OF EQ(2)

$$v_0^2\left[1-\left(\frac{r_A}{r_B}\right)^2\sin^2\phi\right]=\frac{2GM}{r_A}\left(1-\frac{r_A}{r_B}\right) \qquad (2)$$

(a)

SINCE THE MISSILE IS FIRED FROM THE GROUND WE HAVE: $r_A = R$ AND $r_{MAX} = r_B = R + R\alpha = R(1+\alpha)$

OR $\frac{r_A}{r_B} = \frac{R}{R(1+\alpha)} = \frac{1}{1+\alpha}$ ALSO $\phi = \phi_0$

WE ALSO HAVE FROM EQ(12.43): $v_{ESC}^2 = \frac{2GM}{r_A}$

EQ(2): $v_0^2\left[1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right] = v_{ESC}^2\left(1-\frac{1}{1+\alpha}\right)$

$$v_0^2\left[1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right] = v_{ESC}^2\left(\frac{\alpha}{1+\alpha}\right)$$

$$\frac{\sin^2\phi_0}{(1+\alpha)^2} = 1-\left(\frac{v_{ESC}}{v_0}\right)^2\frac{\alpha}{1+\alpha} \qquad (3)$$

$$\sin\phi_0 = (1+\alpha)\sqrt{1-\frac{\alpha}{1+\alpha}\left(\frac{v_{ESC}}{v_0}\right)^2} \qquad (Q.E.D.)$$

(b) ALLOWABLE VALUES OF v_0 (FOR WHICH MAX ALTITUDE = αR)

$0 \le \sin^2\phi_0 \le 1$

FOR $\sin\phi_0 = 0$: $1-\frac{\alpha}{1+\alpha}\left(\frac{v_{ESC}}{v_0}\right)^2 = 0$

$$v_0 = v_{ESC}\sqrt{\frac{\alpha}{1+\alpha}}$$

FOR $\sin\phi_0 = 1$: EQ(3), $\frac{1}{(1+\alpha)^2} = 1-\left(\frac{v_{ESC}}{v_0}\right)^2\frac{\alpha}{1+\alpha}$

$$\left(\frac{v_{ESC}}{v_0}\right)^2 = \frac{1}{\alpha}\left(1+\alpha-\frac{1}{1+\alpha}\right) = \frac{1+2\alpha+\alpha^2-1}{\alpha(1+\alpha)} = \frac{\alpha(2+\alpha)}{\alpha(1+\alpha)} = \frac{2+\alpha}{1+\alpha}$$

$$v_0 = v_{ESC}\sqrt{\frac{1+\alpha}{2+\alpha}} \qquad v_{ESC}\sqrt{\frac{\alpha}{1+\alpha}} \le v_0 \le v_{ESC}\sqrt{\frac{1+\alpha}{2+\alpha}}$$

*13.110

SEE SECOND SKETCH OF SOLUTION OF PROB 13.108. IF THE POINT OF INTERSECTION P_0 OF THE CIRCULAR AND ELLIPTIC ORBITS IS AT AN END OF THE MINOR AXIS, THEN v_0 IS PARALLEL TO THE MAJOR AXIS. THIS WILL BE THE CASE ONLY IF $\alpha + 90° = \theta_0$, THAT IS IF $\cos\theta_0 = -\sin\alpha$. WE MUST THEREFORE PROVE THAT

$$\cos\theta_0 = -\sin\alpha \qquad (1)$$

WE RECALL EQ(12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \qquad (2)$$

WHEN $\theta = 0$, $r = r_A = r_{min}$. BUT, FROM PROB 13.108

$$r_{min} = (1-\sin\alpha)r_0$$

SUBSTITUTE INTO (2): $\frac{1}{(1-\sin\alpha)r_0} = \frac{GM}{h^2} + C \qquad (3)$

FOR $\theta = 180°$, $r = r_A = r_{max} = (1+\sin\alpha)r_0$. SUBSTITUTE INTO 2

$$\frac{1}{(1+\sin\alpha)r_0} = \frac{GM}{h^2} - C \qquad (4)$$

ADDING (3) AND (4) AND DIVIDING BY 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha}+\frac{1}{1+\sin\alpha}\right) = \frac{1}{2r_0}\,\frac{2}{1-\sin^2\alpha} = \frac{1}{r_0\cos^2\alpha}$$

SUBTRACTING (4) FROM (3) AND DIVIDING BY 2:

$$C = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha}-\frac{1}{1+\sin\alpha}\right) = \frac{1}{2r_0}\,\frac{2\sin\alpha}{1-\sin^2\alpha} = \frac{\sin\alpha}{r_0\cos^2\alpha}$$

(CONTINUED)

*13.110 CONTINUED

SUBSTITUTE FOR $\frac{GM}{h^2}$ AND C INTO EQ(2)

$$\frac{1}{r} = \frac{1}{r_0\cos^2\alpha}(1+\sin\alpha\cos\theta) \qquad (5)$$

LETTING $r = r_0$ AND $\theta = \theta_0$ IN EQ(5), WE HAVE

$$\cos^2\alpha = 1+\sin\alpha\cos\theta_0$$

$$\cos\theta_0 = \frac{\cos^2\alpha - 1}{\sin\alpha} = -\frac{\sin^2\alpha}{\sin\alpha} = -\sin\alpha$$

THIS PROVES VALIDITY OF EQ(1) AND THUS P_0 IS AT AN END OF THE MINOR AXIS OF THE ELLIPTIC ORBIT.

*13.111

(a) CONSIDER POINT A IN FIG 13.15:

ANGULAR MOMENTUM PER UNIT MASS:

$$h = \frac{H_0}{m} = \frac{r_{min}\,m\,v_{max}}{m} \qquad h = r_{min}v_{max} \qquad (1)$$

ENERGY PER UNIT MASS:

$$\frac{E}{m} = \frac{1}{m}(T+V) = \frac{1}{m}\left(\frac{1}{2}mv_{max}^2 - \frac{GMm}{r_{min}}\right) = \frac{1}{2}v_{max}^2 - \frac{GM}{r_{min}} \qquad (2)$$

(b) FROM EQ(1): $v_{max} = h/r_{min}$. SUBSTITUTING INTO (2)

$$\frac{E}{m} = \frac{1}{2}\frac{h^2}{r_{min}^2} - \frac{GM}{r_{min}}$$

$$\left(\frac{1}{r_{min}}\right)^2 - \frac{2GM}{h^2}\cdot\frac{1}{r_{min}} - \frac{2(E/m)}{h^2} = 0$$

SOLVING THE QUADRATIC:

$$\frac{1}{r_{min}} = \frac{GM}{h^2} + \sqrt{\left(\frac{GM}{h^2}\right)^2 + \frac{2(E/m)}{h^2}}$$

REARRANGING, WE HAVE

$$\frac{1}{r_{min}} = \frac{GM}{h^2}\left[1+\sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right] \qquad (3)$$

(c) ECCENTRICITY OF THE TRAJECTORY

EQ (12.39') $\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$

WHEN $\theta = 0$, $\cos\theta = 1$ AND $r = r_{min}$, THUS

$$\frac{1}{r_{min}} = \frac{GM}{h^2}(1+\varepsilon) \qquad (4)$$

COMPARING EQS (3) AND (4), WE NOTE THAT

$$\varepsilon = \sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2} \qquad (5)$$

(d) RECALLING DISCUSSION ON PAGE 558, AND IN VIEW OF EQ(5), WE HAVE

1. HYPERBOLA IF $\varepsilon > 1$, THAT IS IF $E > 0$
2. PARABOLA IF $\varepsilon = 1$, THAT IS IF $E = 0$
3. ELLIPSE IF $\varepsilon < 1$, THAT IS IF $E < 0$

NOTE: FOR CIRCLE $\varepsilon = 0$, AND

$$1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2 = 0 \quad \text{OR} \quad E = -\left(\frac{GM}{h}\right)^2\frac{m}{2}$$

BUT FOR CIRCULAR ORBIT $v^2 = GM/r$

$$h^2 = v^2r^2 = GMr$$

THUS $E = -\frac{1}{2}m\frac{(GM)^2}{GMr} = -\frac{1}{2}\frac{GMm}{r}$

WHICH CHECKS EXPRESSION FOUND IN PROB 13.107

13.113 $v_1 = 2.5$ mi/h $= 3.667$ ft/s

$m\underline{v}_1$ + Ft = $m\underline{v}_2 = 0$

$mv_1 - Ft = 0$

$\frac{(40{,}000 \text{ tons})(2000 \text{ lb/ton})}{32.2 \text{ ft/s}^2}(3.667 \text{ ft/s}) - (35{,}000 \text{ lb})t = 0$

$t = 260.3$ s $\quad t = 4$ min 20 s ◀

13.114 $v_1 = 90$ km/h $= 25$ m/s

$m\underline{v}_1$ + Wt, $Nt = Wt$, $Ft = \mu_k Wt$ = $m\underline{v}_2 = 0$

$mv_1 - \mu_k Wt = 0$

$t = \frac{mv_1}{\mu_k W} = \frac{mv_1}{\mu_k mg} = \frac{v_1}{\mu_k g}$

(a) For $\mu_k = 0.75$ $\quad t = \frac{25 \text{ m/s}}{(0.75)(9.81 \text{ m/s}^2)}$ $\quad t = 3.40$ s ◀

(b) For $\mu_k = 0.10$ $\quad t = \frac{25 \text{ m/s}}{(0.10)(9.81 \text{ m/s}^2)}$ $\quad t = 25.5$ s ◀

13.115 ASSUME LOAD DOES NOT SHIFT, USE $\mu_s = 0.40$

$v_1 = 90$ km/h $= 25$ m/s

$m\underline{v}_1$ + Wt, $Nt = Wt$, $Ft = \mu_s Wt$ = $m\underline{v}_2 = 0$

$mv_1 - \mu_s Wt = 0$

$t = \frac{mv_1}{\mu_s W} = \frac{mv_1}{\mu_s mg} = \frac{v_1}{\mu_s g}$

$t = \frac{25 \text{ m/s}}{(0.40)(9.81 \text{ m/s})} = 6.37$ s $\quad t = 6.37$ s ◀

THIS IS SHORTEST TIME IN WHICH LOAD CAN BE BROUGHT TO REST. SINCE LOAD DOES NOT SHIFT ($\mu_s = 0.40$), IT IS ALSO THE TIME REQUIRED TO BRING THE RIG TO REST.

13.116 $m\underline{v}_0 + \int \underline{F}\,dt = m\underline{v}$ (1)

WHERE $\int \underline{F}\,dt = \int_0^t [2t\,\underline{i} + (3-t)\,\underline{j} + t^3\,\underline{k}]\,dt$

$= t^2\,\underline{i} + (3t - \frac{1}{2}t^2)\,\underline{j} + \frac{1}{4}t^4\,\underline{k}$

SUBSTITUTING $m = \frac{10}{9}$ lb AND $\underline{v}_0 = -24\,\underline{i} + 15\,\underline{j} - 60\,\underline{k}$ INTO (1):

$\frac{10}{9}(-24\,\underline{i} + 15\,\underline{j} - 60\,\underline{k}) + t^2\,\underline{i} + (3t - \frac{1}{2}t^2)\,\underline{j} + \frac{1}{4}t^4\,\underline{k} = \frac{10}{9}\underline{v}$

$\underline{v} = (-24 + \frac{9}{10}t^2)\,\underline{i} + [15 + \frac{9}{10}(3t - \frac{1}{2}t^2)]\,\underline{j} + (-60 + \frac{9}{10}\frac{1}{4}t^4)\,\underline{k}$

FOR $t = 4.5$:

$\underline{v} = (-24 + \frac{32.2}{10}4^2)\,\underline{i} + [15 + \frac{32.2}{10}(4\times 3 - \frac{1}{2}4^2)]\,\underline{j} + (-60 + \frac{32.2}{10}\frac{1}{4}4^4)\,\underline{k}$

$\underline{v} = (27.5 \text{ ft/s})\,\underline{i} + (27.9 \text{ ft/s})\,\underline{j} + (146.1 \text{ ft/s})\,\underline{k}$ ◀

13.117 $m\underline{v}_0 + \int \underline{F}\,dt = m\underline{v}$ (1)

WHERE $\int \underline{F}\,dt = \int_0^t [(8-6t)\,\underline{i} + (4-t^2)\,\underline{j} + (4+t)\,\underline{k}]$

$= (8t - 3t^2)\,\underline{i} + (4t - \frac{1}{3}t^3)\,\underline{j} + (4t + \frac{1}{2}t^2)\,\underline{k}$

SUBSTITUTING $m = 2$ kg, $\underline{v}_0 = 150\,\underline{i} + 100\,\underline{j} - 250\,\underline{k}$ INTO (1):

$(2\text{ kg})(150\,\underline{i} + 100\,\underline{j} - 250\,\underline{k}) + (8t - 3t^2)\,\underline{i} + (4t - \frac{1}{3}t^3)\,\underline{j} + (4t + \frac{1}{2}t^2)\,\underline{k} = (2\text{ kg})\underline{v}$

$\underline{v} = (150 + 4t - \frac{3}{2}t^2)\,\underline{i} + (100 + 2t - \frac{1}{6}t^3)\,\underline{j} + (-250 + 2t + \frac{1}{4}t^2)\,\underline{k}$

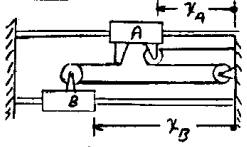
(a) $\underline{v}$ IS PARALLEL TO yz PLANE WHEN $v_x = 0$, THAT IS, WHEN

$150 + 4t - \frac{3}{2}t^2 = 0 \quad t = 11.422$ s $\quad t = 11.42$ s ◀

(b) $\underline{v} = [100 + 2(11.422) - \frac{1}{6}(11.422)^3]\,\underline{j} + [-250 + 2(11.422) + \frac{1}{4}(11.422)^2]\,\underline{k}$

$\underline{v} = -(125.5 \text{ m/s})\,\underline{j} - (194.5 \text{ m/s})\,\underline{k}$ ◀

13.118 (PROB 12.15) KINEMATICS CABLE LENGTH = CONSTANT



$L = 2y_A + (y_B - y_A) + y_B + \text{constant}$

$L = y_A + 2y_B + \text{constant}$

$\frac{dL}{dt} = v_A + 2v_B = 0$

$v_A = -2v_B$ (1)

COLLAR A: $m_A = 10$ kg

$m_A(v_A)_1 = 0$ + $T(1.2\text{ s})$, $2T(1.2\text{ s})$ = $m_A(v_A)_2$

$+\rightarrow \quad 0 - T(1.2) = m_A(v_A)_2$

$0 - 1.2T = 10\,(v_A)_2$ (2)

COLLAR B: $m_B = 15$ kg

$m_B(v_B)_1 = 0$ + $(25\text{ N})(1.2\text{ s})$, $2T(1.2\text{ s})$ = $m_B(v_B)_2$

$+\rightarrow \quad 0 + (25\text{N})(1.2\text{s}) - 2T(1.2\text{s}) = m_B(v_B)_2$

$30 - 2.4T = 15\,(v_B)_2$ (3)

MULTIPLY EQ(2) BY 2 AND SUBTRACT FROM EQ(3):

$30 - \cancel{2.4T} + \cancel{2.4T} = 15\,(v_B)_2 - 20\,(v_A)_2$ (4)

USING EQ(1), WE HAVE $(v_A)_2 = -2(v_B)_2$; SUBSTITUTE INTO (4)

$30 = 15\,(v_B)_2 - 20\,[-2(v_B)_2]$

$30 = 55\,(v_B)_2$; $(v_B)_2 = 0.5454$ $\quad (v_B)_2 = 0.545$ m/s ◀

$(v_A)_2 = -2(v_B)_2 = -2(0.5454) = -1.091$ $\quad (v_A)_2 = 1.091$ m/s ◀

13.119 (a) ENTIRE TRAIN: $v_1 = 30$ mi/h $= 44$ ft/s

$m\underline{v}_1$ + Ft = $m\underline{v}_2 = 0$

$+\rightarrow \quad mv_1 - Ft = 0$

$W = 40 + 50 + 40 = 130$ tons

FOR BRAKES APPLIED ON CARS B AND C: $F = \mu N = (0.35)(50 + 40) = 31.5$ tons

$mv_1 - Ft = 0$: $\frac{(130 \text{ tons})(2000)}{32.2 \text{ ft/s}^2}(44 \text{ ft/s}) - (31.5 \text{ tons})(2000)\,t = 0$

$t = 5.6394$ s $\quad t = 5.64$ s ◀

(b) CAR A: $W = 40$ tons $\quad F_A = \mu N = (0.35)(40) = 14$ tons

$m\underline{v}_1$ + $F_A t$, $F_{AB} t$ = $m\underline{v}_2 = 0$

$+\rightarrow \quad mv_1 - F_A t - F_{AB} t = 0$: $\frac{40 \times 2000}{32.2}(44) - F_{AB}(5.6394) = 0$

$F_{AB} = +19{,}384$ lb $\quad F_{AB} = 19{,}380$ lb T ◀

(CONTINUED)

13.118 CONTINUED TRAIN STOPS IN $t = 5.639$ s

CAR C: $W_C = 40$ tons $F_C = \mu N = (0.35)(40 \times 2000) = 28{,}000$ lb

$m_C v_1$ + $F_{BC}\,t$ (Ft) = $m_C v_2 = 0$

$\pm \quad m_C v_1 + F_{BC}\,t - Ft = 0$

$\frac{40 \times 2000}{32.2}(44) + F_{BC}(5.639\text{ s}) - (28{,}000\text{ lb})(5.639\text{ s}) = 0$

$F_{BC} = +8614$ lb $\qquad F_{BC} = 8610$ lb T ◀

13.119 (a) ENTIRE TRAIN $v_1 = 30$ mi/h $= 44$ ft/s

mv_1 + Ft = $mv_2 = 0$

$W = (40 + 50 + 40) = 130$ tons

FOR BRAKES APPLIED ONLY ON CAR A: $F = \mu N = (0.35)(40) = 14$ tons

$\pm \quad mv_1 - Ft = 0$

$\frac{(130\text{ tons})(2000)}{32.2\text{ ft/s}^2}(44\text{ ft/s}) - (14\text{ tons})(2000)\,t = 0$

$t = 12.689$ s $\qquad t = 12.69$ s ◀

(b) CAR A: $W_A = 40$ tons, $F_A = \mu N = 0.35(40) = 14$ tons

$m_A v_1$ + $F_A t$ + $F_{AB} t$ = $m_A v_2 = 0$

$\pm \quad m_A v_1 - F_A t - F_{AB} t = 0$

$\frac{(40\text{ tons})(2000)}{32.2}(44) - (14\text{ tons})(2000)(12.689\text{ s}) - F_{AB}(12.689\text{ s}) = 0$

$F_{AB} = -19{,}380$ $\qquad F_{AB} = 19{,}380$ lb C ◀

CAR C: $W_C = 40$ tons, $F_C = 0$

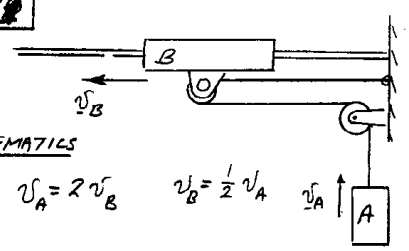
$m_C v_1$ + $F_{BC}\,t$ = $m_C v_2 = 0$

$\pm \quad m_C v_1 + F_{BC}\,t = 0$

$\frac{(40\text{ tons})(2000)}{32.2}(44\text{ ft/s}) + F_{BC}(12.689\text{ s}) = 0$

$F_{BC} = -8{,}615$ lb $\qquad F_{BC} = 8610$ lb C ◀

13.120



KINEMATICS

$v_A = 2v_B \qquad v_B = \frac{1}{2}v_A \qquad v_A$ ↑

COLLAR B $m_B = 8$ kg

$m_B(v_B)_1 = 0$ + 150 t, 2T t = $m_B(v_B)_2$

$\pm \quad 0 + 150\,t - 2T\,t = m_B(v_B)_2$

For $(v_B)_2 = 2.5$ m/s

$150\,t - 2T\,t = (8\text{ kg})(2.5\text{ m/s})$

$150\,t - 2T\,t = 20 \qquad (1)$

(CONTINUED)

13.120 CONTINUED CYLINDER A $m_A = 3$ kg

$m_A(v_A)_1 = 0$ + Tt, $(m_A g)t$ = $m_A(v_A)_2$

$+\uparrow \quad 0 + Tt - (m_A g)t = m_A(v_A)_2$

$Tt - (3\text{ kg})(9.81\text{ m/s}^2)(t) = (3\text{ kg})(v_A)_2$

SINCE $(v_B)_2 = 2.5$ m/s AND $(v_A)_2 = 2(v_B)_2$, $(v_A)_2 = 5$ m/s

$Tt - 29.43t = 3(5) \qquad (2)$

MULTIPLY EQ(2) BY 2 AND ADD TO EQ.(1)

$150\,t - 2Tt + 2Tt - 58.86t = 20 + 30$

$91.14t = 50$

$t = 0.5487$ s $\qquad t = 0.549$ s ◀

EQ(2): $(T - 29.43)(0.5487) = 15$

$T = 56.77$ N $\qquad T = 56.8$ N ◀

13.121 (PROB 12.17b)

(1) $mv_1 = 0$ + $T(2)$, $250(2)$ = $mv_2 = \frac{250}{g}v_2$

BLOCK A

$mv_1 + \Sigma mv_{1-2} = mv_2$

$+\downarrow \quad 0 + 250(2) - T(2) = \frac{250}{g}v_2$

$mv_1 = 0$ + $T(2)$, $150(2)$ = $mv_2 = \frac{150}{g}v_2$

COUNTERWEIGHT

$+\uparrow \quad 0 + T(2) - (150)(2) = \frac{150}{g}v_2$

ADD EQUATIONS: $250(2) - (150)(2) = \frac{250 + 150}{g}v_2$

$v_2 = \frac{200}{400}(32.2) = +16.1$ ft/s $\qquad v_2 = 16.10$ ft/s ↓ ◀

(2) mv_1 + $(150)2$, $(250)(2)$ = $mv_2 = \frac{250}{g}v_2$

BLOCK A: $+\downarrow \quad 0 + (250)(2) - (150)(2) = \frac{250}{g}v_2$

$v_2 = \frac{200}{250}(32.2) = +25.76$ ft/s $\qquad v_2 = 25.8$ ft/s ↓ ◀

(3) BLOCK A $mv_1 = 0$ + $T(2)$, $1250(2)$ = $mv_2 = \frac{1250}{g}v_2$

$+\downarrow \quad 0 + 1250(2) - T(2) = \frac{1250}{g}v_2 \qquad (1)$

COUNTERWEIGHT $mv_1 = 0$ + $T(2)$, $1150(2)$ = $mv_2 = \frac{1150}{g}v_2$

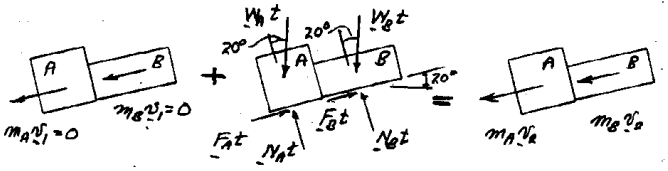
$+\uparrow \quad T(2) - (1150)(2) = \frac{1150}{g}v_2 \qquad (2)$

ADD EQS (1) AND (2): $1250(2) - 1150(2) = \frac{1250 + 1150}{g}v_2$

$v_2 = \frac{200}{2400}(32.2) = 2.683$ ft/s $\qquad v_2 = 2.68$ ft/s ↓ ◀

13.122

(a) ASSUME PACKAGES MOVE AT SAME SPEED



$+\swarrow$ $(W_A + W_B)t \sin 20° - (F_A + F_B)t = (m_A + m_B)v_2$ (1)

NOTE: For $\mu_s = 0.30$ $\phi_s = \tan^{-1} 0.30 = 16.7°$. THUS FOR THE LARGER COEFFICIENT OF STATIC FRICTION THE STATIC ANGLE OF FRICTION IS LESS THAN THE SLOPE ANGLE (20°) OF THE INCLINE. THUS BOTH PACKAGES MOVE,

DATA: $m_A = 6$ kg, $W_A = 6g$, $N_A = W_A \cos 20° = 6g \cos 20°$

$(\mu_A)_k = 0.25$, $F_A = (\mu_A)_k N_A = 0.25(6g \cos 20°) = 1.5g \cos 20°$

$m_B = 9$ kg, $W_B = 9g$, $N_B = W_B \cos 20° = 9g \cos 20°$

$(\mu_B)_k = 0.15$, $F_B = (\mu_B)_k N_B = 0.15(9g) \cos 20° = 1.35g \cos 20°$

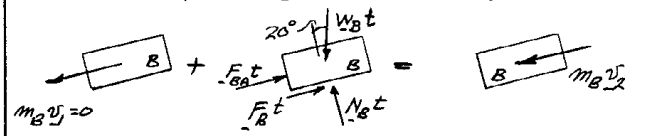
SUBSTITUTE DATA AND $t = 3$ s INTO EQ (1):

$(9g + 6g)(3s) \sin 20° - (1.5g + 1.35g) \cos 20° (3s) = (9+6) v_2$

$150.985 \text{ N} - 78.817 \text{ N} = 15 v_2$

$v_2 = 4.811$ m/s $\quad \underline{v}_A = \underline{v}_B = 4.81$ m/s ◀

(b) CONSIDER PACKAGE B AND ASSUME FORCE EXERTED BY A ON B IS COMPRESSIVE.



$+\swarrow$ $0 + W_B t \sin 20° - F_B t - F_{BA} t = m_B v_2$

SUBSTITUTE DATA AND $v_2 = 4.811$ m/s

$(9g)(3s) \sin 20° - (1.35g) \cos 20° (3s) - F_{BA}(3s) = (9)(4.811)$

$90.59 - 37.33 - 3F_{BA} = 43.3$

$F_{BA} = +3.32$ N $\quad \underline{F}_{BA} = 3.32$ N ◀

WE CONFIRM THAT FORCE IS COMPRESSIVE THUS OUR ASSUMPTION THAT A AND B MOVE AT SAME SPEED IS CORRECT.

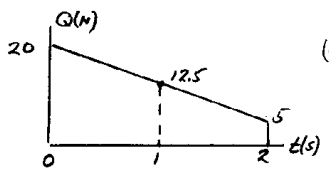
13.123

FRICTION FORCE: $F = \mu_k mg = 0.25(3\text{kg})(9.81 \text{ m/s}^2) = 7.358$ N

INITIAL VELOCITY = ZERO $\quad m = 3$ kg

$m\underline{v}_0 + \int \underline{Q}\,dt - \int \underline{F}\,dt = m\underline{v}$

$0 + \int Q\,dt - \int F\,dt = 3\underline{v}$ (1)



(a) FOR $t = 1$ s:

$\int_0^1 Q\,dt = \frac{1}{2}(20\text{N} + 12.5\text{N})(1\text{s}) = 16.25 \text{ N·s}$

$\int_0^1 F\,dt = (7.358\text{N})(1\text{s}) = 7.358 \text{ N·s}$

EQ(1): $0 + 16.25 \text{ N·s} - 7.358 \text{ N·s} = 3v$

$v = 2.964$ m/s $\quad \underline{v} = 2.96$ m/s ◀

(b) FOR $t = 2$ s:

$\int_0^2 Q\,dt = \frac{1}{2}(20\text{N} + 5\text{N})(2\text{s}) = 25 \text{ N·s}$; $\int_0^2 F\,dt = (7.358)(2) = 14.716 \text{ N·s}$

EQ(2) $0 + 25 \text{ N·s} - 14.716 \text{ N·s} = 3v$

$v = 3.428$ m/s $\quad \underline{v} = 3.43$ m/s ◀

13.124

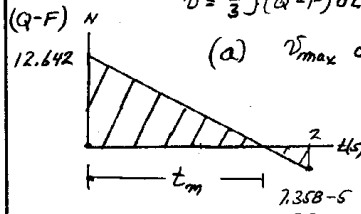
FRICTION FORCE: $F = \mu_k mg = 0.25(3\text{kg})(9.81 \text{ m/s}^2) = 7.358$ N

INITIAL VELOCITY = 0 $\quad m = 3$ kg

$m\underline{v}_0 + \int \underline{Q}\,dt - \int \underline{F}\,dt = m\underline{v}$

$0 + \int (Q - F)\,dt = 3\underline{v}$

$\underline{v} = \frac{1}{3}\int (Q - F)\,dt$



(a) v_{max} OCCURS WHEN $(Q - F) = 0$

SIMILAR TRIANGLES $\quad \frac{t_m}{12.642} = \frac{2 - t_m}{2.358} = \frac{2}{15}$

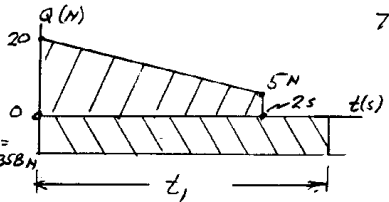
$t_m = \frac{2}{15}(12.642) = 1.6856$ s

$t = 1.686$ s ◀

$v_m = \frac{1}{3}\int_0^{t_m} (Q - F)\,dt = \frac{1}{3}\left[\frac{1}{2}(12.642\text{N})(1.6856\text{s})\right] = 3.5516$ s

$\underline{v}_m = 3.55$ m/s ◀

(b) TIME WHEN $v = 0$ OCCURS WHEN TOTAL IMPULSE IS ZERO, THAT IS, WHEN $\int_0^{t_1} (Q - F)\,dt = 0$



$\int_0^{t_1} Q\,dt - \int_0^{t_1} F\,dt = \frac{1}{2}(20\text{N} + 5\text{N})(2\text{s}) - (7.358\text{N})t_1 = 0$

$t_1 = 3.398$ s $\quad t_1 = 3.40$ s ◀

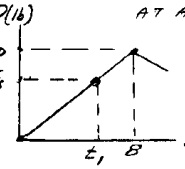
13.125

$m\underline{v}_0 = 0$ + $\int \underline{P}\,dt$ = $m\underline{v}$

$N\underline{t} = W\underline{t} \uparrow$ $\int \underline{F}\,dt$ $\quad W = 125$ lb

$+\rightarrow$ $0 + \int P\,dt - \int F\,dt = mv$

AT ANY TIME: $v = \frac{1}{m}\left[\int P\,dt - \int F\,dt\right]$ (1)



(a) BLOCK STARTS MOVING AT t_1.

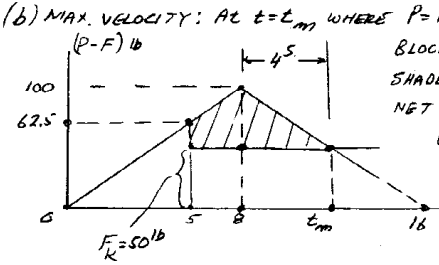
$P = F_s = \mu_s W = (0.50)(125\text{lb}) = 62.5$ lb

$\frac{t_1}{F_s} = \frac{8s}{100\text{lb}}$; $\frac{t_1}{62.5\text{lb}} = \frac{8s}{100\text{lb}}$ $\quad t_1 = 5$ s ◀

(b) MAX. VELOCITY: AT $t = t_m$ WHERE $P = F_k = \mu_k W = 0.4(125) = 50$ lb

BLOCK MOVES @ $t = 5$ s

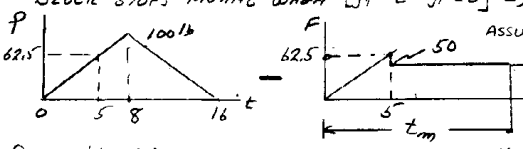
SHADED AREA IS MAXIMUM NET IMPULSE $\int P\,dt - \int F_k\,dt$ WHEN $t = t_m$, $v = v_m$



EQ(1): $v_m = \frac{1}{m}\left[\text{SHADED AREA}\right] = \frac{1}{m}\left[\frac{1}{2}(12.5 + 50)(3) + \frac{1}{2}(50)(4)\right] = \frac{1}{m}(193.75)$

$v_m = \frac{1}{(125\text{lb})/32.2}\left[193.75\right] = 49.91$ ft/s $\quad \underline{v}_m = 49.9$ ft/s → ◀

(c) BLOCK STOPS MOVING WHEN $\left[\int P\,dt - \int F\,dt\right] = 0$; OR $\int P\,dt = \int F\,dt$



ASSUME $t_m > 16$s

$\int P\,dt = \frac{1}{2}(100)(16) = 800$ lb·s $\quad \int F\,dt = \frac{1}{2}(62.5)(5) + (50)(t_m - 5)$

$\int P\,dt - \int F\,dt = 800 - \left[158.25 + 50(t_m - 5)\right] = 0$ $\quad t_m = 17.875$ s

$t_m > 16$s ok $\quad t_m = 17.88$ s ◀

13.126

SEE SOLUTION OF PROB 13.125

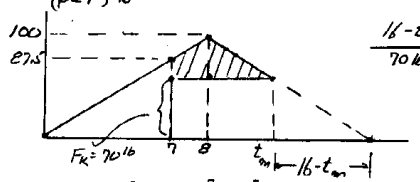
$W = 175\text{ lb}$ $\quad v = \frac{1}{m}\left[\int P\,dt - \int F\,dt\right]$ (1)

(a) BLOCK STARTS MOVING $P = F_s = \mu_s W = (0.50)(175) = 87.5\text{ lb}$

SEE FIRST FIGURE OF PROB 13.125

$\frac{t_1}{F_s} = \frac{8\text{ s}}{100\text{ lb}}$; $\frac{t_1}{87.5\text{ lb}} = \frac{8\text{ s}}{100\text{ s}}$ $\quad t_1 = 7\text{ s}$ ◀

(b) MAXIMUM VELOCITY: $P = F_k = \mu_k W = 0.4(175) = 70\text{ lb}$



$\frac{16 - t_m}{70\text{ lb}} = \frac{8\text{ s}}{100\text{ lb}}$

$16 - t_m = 70\left(\frac{8}{100}\right) = 5.6$

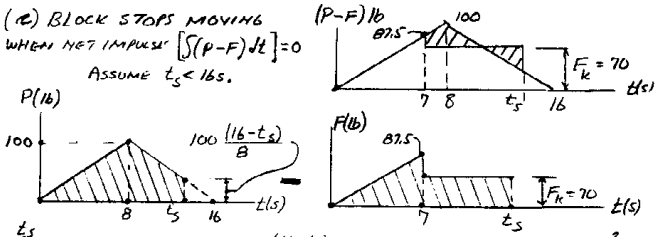
$t_m = 10.4\text{ s}$

EQ(1): $v_m = \frac{1}{m}\left[\begin{matrix}\text{SHADED}\\ \text{AREA}\end{matrix}\right] = \frac{1}{m}\left[\frac{1}{2}(17.5+30)(1s) + \frac{1}{2}(30)(10.4-8)\right] = \frac{1}{m}(59.75)$

$v_m = \frac{1}{(175\text{ lb})/32.2} \left[59.75\right] = 10.994\text{ ft/s}$ $\quad \underline{v_m = 10.99\text{ ft/s}} \rightarrow$ ◀

(c) BLOCK STOPS MOVING WHEN NET IMPULSE $\left[\int (P-F)\,dt\right] = 0$

ASSUME $t_s < 16\text{ s}$.



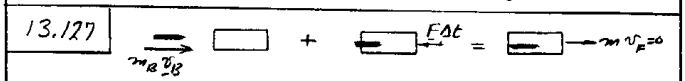
$\int_0^{t_s} P\,dt = \frac{1}{2}(100)(8) + \frac{1}{2}\left[100 + 100\,\frac{(16-t_s)}{8}\right](t_s - 8) = \frac{1}{2}(100)(16) - \frac{1}{2}\left(\frac{100}{8}\right)(16-t_s)^2$

$\int_0^{t_s} F\,dt = \frac{1}{2}(87.5)(7) + (70)(t_s - 7)$

$\therefore \int P\,dt - \int F\,dt = 800 - \frac{100}{16}(16-t_s)^2 - 306.25 - 70(t_s - 7) = 0$

SOLVE FOR $t_s = 13.492\text{ s}$ $\quad t_s = 13.49\text{ s}$ ◀

13.127



$m_B v_B + \Sigma F \Delta t = m v_F$

$\rightarrow$ $(0.025\text{ kg})(600\text{ m/s}) - F(0.75\times10^{-3}\text{ s}) = 0$

$F = 20\times10^3\text{ N}$ $\quad F = 20\text{ kN}$ ◀

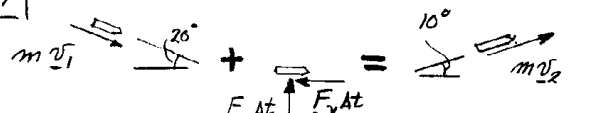
13.128

$mv_1 + \Sigma F \Delta t = mv_2$

$\rightarrow$ $\left(\frac{2000\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(2.5\,\frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ ft}}{\text{mi}} \times \frac{\text{hr}}{3600\text{ s}}\right) - F(0.075\text{ s}) = 0$

$F = 4554.8\text{ lb}$ $\quad F = 4550\text{ lb}$ ◀

13.129



$m\underline{v}_1 + \Sigma \underline{F}\,\Delta t = m\underline{v}_2$

$\pm$ x comps: $m(2000\text{ ft/s})\cos 20^\circ - \underline{F}_x\Delta t = m(1600\text{ ft/s})\cos 10^\circ$

$\underline{F}_x\Delta t = m(1879.38 - 1575.69) = \frac{(1/16\text{ lb})}{32.2\text{ ft/s}^2}(303.69\text{ ft/s})$

$\underline{F}_x\Delta t = 0.58947\text{ lb}\cdot\text{s}$

$+\uparrow$ y comps: $-m(2000\text{ ft/s})\sin 20^\circ + \underline{F}_y\Delta t = m(1600\text{ ft/s})\sin 10^\circ$

$\underline{F}_y\Delta t = m(684.04 + 277.84) = \frac{(1/16\text{ lb})}{32.2\text{ ft/s}^2}(961.87\text{ ft/s})$

$\underline{F}_y\Delta t = 1.867\text{ lb}\cdot\text{s}$

(CONTINUED)

13.129 CONTINUED

$\underline{F}_x\Delta t = 0.58947\text{ lb}\cdot\text{s} \leftarrow$

$\underline{F}_y\Delta t = 1.867\text{ lb}\cdot\text{s} \uparrow$

$\underline{F}\Delta t = 1.9578\text{ lb}\cdot\text{s}\ \angle 72.5^\circ$

SINCE $v_{\text{AVERAGE}} = 1800\text{ ft/s}$, $\Delta x = v_{\text{AVG}}\,\Delta t$

$\left(\frac{2}{12}\text{ ft}\right) = 1800\text{ ft/s}\,(\Delta t)$

$\Delta t = 92.59\times10^{-6}\text{ s}$

THUS $F\Delta t = F(92.59\times10^{-6}\text{ s}) = 1.9578\text{ lb}\cdot\text{s}$

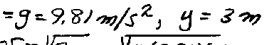
$F = 21{,}144\text{ lb}$ $\quad \underline{F} = 21.1\text{ kips}\ \angle 72.5^\circ$ ◀

13.130

FREE FALL OF MAN FROM TOP OF WALL

$a = g = 9.81\text{ m/s}^2$, $y = 3\text{ m}$

$v_1 = \sqrt{2gy} = \sqrt{2(9.81)(3)} = 7.672\text{ m/s}$



$+\downarrow$ y comps: $m\underline{v}_1 - \underline{R}\Delta t = 0$

$m(7.672\text{ m/s}) - R(0.150\text{ s}) = 0$

$R = \left(\frac{7.672\text{ m/s}}{0.150\text{ s}}\right)m = 51.147(m)$

OR SINCE $m = \frac{W}{g}$,

$R = 51.147\,\frac{W}{9.81\text{ m/s}^2}$ $\quad \underline{R} = 5.21\,W \uparrow$ ◀

NOTE: THE TOTAL REACTION WOULD INCLUDE THE WEIGHT OF THE MAN (NONIMPULSIVE) AND WOULD BE $\underline{R} = 6.21\,W \uparrow$

13.131

(a) SUITCASE + CARRIER (TOTAL MOMENTUM CONSERVED)

$\Sigma m\underline{v}_1 = \Sigma m\underline{v}_2$: $0 + (12\text{ kg})(2.5\text{ m/s}) = (12\text{ kg} + 30\text{ kg})v_2$

$v_2 = 2.5\,\frac{12}{42} = 0.714\text{ m/s}$ $\quad \underline{v}_2 = 0.714\text{ m/s} \leftarrow$ ◀

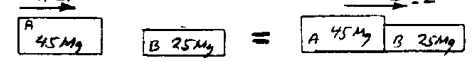
(b) KINETIC ENERGY $\quad T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(12\text{ kg})(2.5\text{ m/s})^2 = 37.5\text{ J}$

$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(12\text{ kg} + 30\text{ kg})(0.714\text{ m/s})^2 = 10.714\text{ J}$

$\frac{T_2}{T_1} = \frac{10.714\text{ J}}{37.5\text{ J}} = \frac{2}{7}$ $\quad \frac{T_2}{T_1} = 0.286$ ◀

13.132

$v_1 = 3\text{ km/h}$



TOTAL MOMENTUM OF THE TWO CARS IS CONSERVED

$\Sigma m\underline{v}_1 = \Sigma m\underline{v}_2$: $(45\text{ Mg})(3\text{ km/h}) = (45\text{ Mg} + 25\text{ Mg})v_2$

$v_2 = (3\text{ km/h})\,\frac{45}{70}$ $\quad v_2 = 1.9286\text{ km/h}$

$\underline{v}_2 = 1.929\text{ km/h} \rightarrow$ ◀

(b) IMPULSE-MOMENTUM PRINCIPLE FOR 25-Mg CAR.

$\pm$ $\quad 0 + F\Delta t = mv_2$

WE SHALL USE THE FOLLOWING:

$m = 25\text{ Mg} = 25\times10^3\text{ kg}$

$\Delta t = 0.3\text{ s}$

$v = 1.9286\text{ km/h} = 0.5357\text{ m/s}$

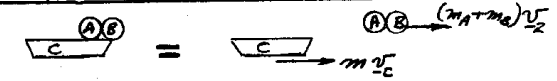
$0 + F(0.3\text{ s}) = (25\times10^3\text{ kg})(0.5357\text{ m/s})$

$F = 44.64\times10^3\text{ N}$ $\quad F = 44.6\text{ kN}$ ◀

13.133

$m = 200\ kg$ = MASS OF BOAT, C.
$m_A = 75\ kg$ = MASS OF SWIMMER A.
$m_B = 50\ kg$ = MASS OF SWIMMER B.

(a) SWIMMERS DIVE SIMULTANEOUSLY



$\Sigma m\underline{v} = \Sigma m\underline{v}$: $\quad 0 = m\underline{v}_C + (m_A+m_B)\underline{v}_2 \qquad (1)$

BUT, RELATIVE VELOCITY OF SWIMMERS WITH RESPECT TO BOAT IS 3 m/s:

$v_2 - v_C = 3\ m/s \qquad v_2 = 3 + v_C$

SUBSTITUTE INTO (1):

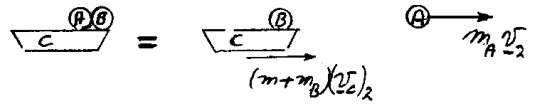
$m v_C + (m_A+m_B)(3+v_C) = 0$

$(m+m_A+m_B)v_C + (m_A+m_B)(3) = 0$

$$v_C = -3\frac{m_A+m_B}{m+m_A+m_B} = -3\frac{75\ kg+50\ kg}{200\ kg+75\ kg+50\ kg}$$

$$v_C = -3\left(\frac{125}{325}\right) = -1.15385 \qquad \underline{v}_C = 1.154\ m/s \leftarrow$$

(b) SWIMMER A DIVES FIRST + THEN SWIMMER B DIVES



$\Sigma m\underline{v} = \Sigma m\underline{v}$: $\quad 0 = (m+m_B)(\underline{v}_C)_2 + m_A\underline{v}_2 \qquad (2)$

RELATIVE VELOCITY OF SWIMMER A WITH RESPECT TO BOAT IS 3 m/s:

$v_2 - (v_C)_2 = 3\ m/s \qquad v_2 = 3 + (v_C)_2$

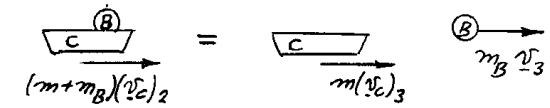
SUBSTITUTE INTO (2):

$(m+m_B)(v_C)_2 + m_A\left[3+(v_C)_2\right] = 0$

$(m+m_A+m_B)(v_C)_2 + m_A(3) = 0$

$$(v_C)_2 = -3\frac{m_A}{m+m_A+m_B} \qquad (3)$$

BOAT AND SWIMMER B



$\Sigma m\underline{v} = \Sigma m\underline{v}$: $\quad (m+m_B)(v_C)_2 = m(v_C)_3 + m_B v_3 \qquad (4)$

RELATIVE VELOCITY OF SWIMMER B WITH RESPECT TO BOAT IS 3 m/s:

$v_3 - (v_C)_3 = 3 \qquad v_3 = 3 + (v_C)_3$

SUBSTITUTE INTO (4):

$(m+m_B)(v_C)_2 = m(v_C)_3 + m_B\left[3+(v_C)_3\right]$

SUBSTITUTE FOR $(v_C)_2$ FROM (3)

$$(m+m_B)\left(-3\frac{m_A}{m+m_A+m_B}\right) = m(v_C)_3 + m_B\left[3+(v_C)_3\right]$$

$$-3\frac{m_A(m+m_B)}{m+m_A+m_B} - 3m_B = (m+m_B)(v_C)_3$$

$$(v_C)_3 = -3\left[\frac{m_A}{m+m_A+m_B} + \frac{m_B}{m+m_B}\right] \qquad (5)$$

$$= -3\left[\frac{75}{200+75+50} + \frac{50}{200+50}\right] = -1.2923\ m/s$$

$$(\underline{v}_C)_3 = 1.293\ m/s \leftarrow$$

(CONTINUED)

13.133 CONTINUED

(c) SWIMMER B DIVES FIRST AND THEN SWIMMER A DIVES

SOLUTION IS SAME AS SOLUTION OF PART (b) WITH m_A AND m_B INTERCHANGED.

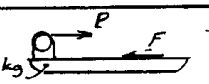
FROM EQ(5): $$(v_C)_2 = (-3)\left[\frac{m_B}{m+m_A+m_B} + \frac{m_A}{m+m_A}\right]$$

$$= (-3)\left[\frac{50}{200+75+50} + \frac{75}{200+75}\right] = -1.2797\ m/s$$

$$(\underline{v}_C)_2 = 1.280\ m/s \leftarrow$$

13.134

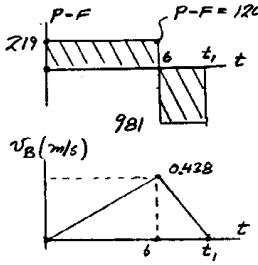
$m = 3M_B = 3000\ kg$



$F = \mu W = \mu mg$
$= 0.2(500)g$
$F = 981\ N$

NET FORCE ON BARGE = P − F

WHERE $P = 1200\ N$ FOR $t < 6\ s$ AND $P = 0$ FOR $t > 6\ s$



(a) IMPULSE = AREA UNDER FORCE-TIME CURVE

v_{max} OCCURS @ $t = 6s$

$m\underline{v}_0 + \Sigma \text{IMP} = m\underline{v}_{max}$

$0 + (219\ N)(6\ s) = (3000\ kg)\,v_{max}$

$$\underline{v}_{max} = 0.438\ m/s \rightarrow$$

IMPULSE, AND THUS v, ARE ZERO AT

$(219\ N)(6s) - (981\ N)(t_1 - 6s) = 0$

$t_1 = 7.339\ s$

(b) DISPLACEMENT OF BARGE = AREA UNDER v-t CURVE

$$x_B = \tfrac{1}{2}(0.438\ m/s)(7.339\ s) = 1.607\ m \qquad \underline{x}_B = 1.607\ m \rightarrow$$

(c) CONSIDER BARGE + CRATE TOGETHER. SINCE THERE ARE NO EXTERNAL FORCES, $\Sigma m\underline{v}_0 = \Sigma m\underline{v}$,

$0 = m_B\underline{v}_B + m_C\underline{v}_C$; $\quad \underline{v}_C = -\frac{m_B}{m_C}\underline{v}_B = -\frac{3000}{500}\underline{v}_B = -6\underline{v}_B$

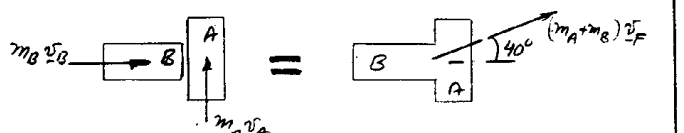
VELOCITY OF CRATE RELATIVE TO BARGE

$\underline{v}_{C/B} = \underline{v}_C - \underline{v}_B = -6\underline{v}_B - \underline{v}_B = -7\underline{v}_B$; $\quad \underline{v}_{C/B} = -7\underline{v}_B$

THUS $x_{C/B} = -7x_B = -7(1.607\ m) = 11.25\ m \qquad \underline{x}_{C/B} = 11.25\ m \leftarrow$

13.135

(a) TOTAL MOMENTUM OF TWO CARS IS CONSERVED.



$\pm$ x comp: $\quad m_B v_B = (m_A+m_B)v_F \cos 40° \qquad (1)$

+↑ y comp: $\quad m_A v_A = (m_A+m_B)v_F \sin 40° \qquad (2)$

DIVIDE (2) BY (1) MEMBER BY MEMBER:

$$\frac{m_A v_A}{m_B v_B} = \tan 40°$$

$$v_A = \frac{m_B}{m_A}\tan 40° = \frac{(3600\ lb/g)}{(2000\ lb/g)}\tan 40°$$

$$v_A = 1.8\tan 40° = 1.5104\,v_B$$

WE FIND THAT CAR A WAS GOING FASTER THAN CAR B, THUS CAR B WAS GOING 35 mi/h

(b) FOR $v_B = 35\ mi/h$

$v_A = 1.5104\,v_B = 1.5104(35\ mi/h)$

$$v_A = 52.9\ mi/h$$

13.136

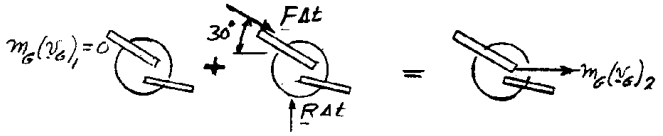
IMPULSE-MOMENTUM PRINCIPLE: SHELL

$$m_s(v_s)_1 + \Sigma \int_{1-2} mp = m_s(v_s)_2$$

$$0 + F\Delta t = \frac{18\ lb}{g}(1500\ ft/s)$$

$$F\Delta t = 27{,}000/g$$

IMPULSE-MOMENTUM PRINCIPLE: GUN



$$m_G(v_G)_1 + \Sigma \int_{1-2} mp = m_G(v_G)_2$$

$\overset{+}{\leftarrow}$ x comp: $\quad 0 - (F\Delta t)\cos 30° = -m_G(v_G)_2$

$$0 - (27{,}000/g)\cos 30° = -(3000/g)(v_G)_2$$

$$(v_G)_2 = \frac{27{,}000}{3000}\cos 30° = 7.794\ ft/s \qquad v_G = 7.79\ ft/s \rightarrow$$

$+\uparrow$ y comp: $\quad 0 + R\Delta t - F\Delta t \sin 30° = 0$

$$R\Delta t - (27{,}000/g)\sin 30° = 0$$

$$R(0.005\ s) = (27{,}000/32.2)\sin 30° = 419.25\ lb\cdot s$$

IMPULSIVE REACTION $\quad R = +83{,}851\ lb \qquad \underline{R} = 83.6\ kips\uparrow$

13.137

BEFORE IMPACT:

$$T_1 = \frac{1}{2} m_H v_H^2 = \frac{1}{2}(0.750\ kg)(6\ m/s)^2 = 13.50\ J$$

AFTER IMPACT:

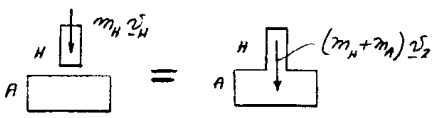
(a) MASS OF ANVIL: $m_A = \infty$, (RIGID SUPPORT): $v_2 = 0 \quad T_2 = 0$

ENERGY ABSORBED: $\Delta E = T_1 - T_2 = 13.50\ J$

IMPULSE $= m_H(v_H - v_2) = (0.750\ kg)(6\ m/s) = 4.50\ N\cdot s$

(b) MASS OF ANVIL $m_A = 4\ kg$:

TOTAL MOMENTUM CONSERVED:



$$m_H v_H = (m_H + m_A)v_2$$

$$(0.750\ kg)(6\ m/s) = (0.750\ kg + 4\ kg)v_2$$

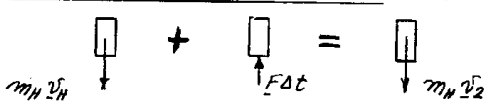
$$v_2 = 0.9474\ m/s$$

$$T_2 = \frac{1}{2}(m_H + m_A)v_2^2 = \frac{1}{2}(0.750 + 4)(0.9474)^2 = 2.1317\ J$$

ENERGY ABSORBED

$$\Delta E = T_1 - T_2 = 13.50\ J - 2.132\ J = 11.37\ J$$

IMPULSE-MOMENTUM PRINCIPLE: HAMMER



$+\downarrow$ $\quad m_H v_H - F\Delta t = m_H v_2$

$$F\Delta t = m_H(v_H - v_2) = (0.750\ kg)(6\ m/s - 0.9474\ m/s)$$

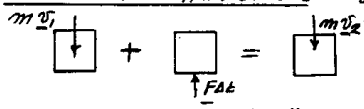
$$F\Delta t = 3.789\ N\cdot s \qquad F\Delta t = 3.79\ N\cdot s$$

13.138

VELOCITY OF HAMMER AS IT HITS MACHINE PART

$$v_1^2 = 2gh = 2(32.2)(5ft) \qquad v_1 = 17.944\ ft/s$$

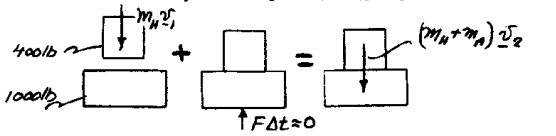
(a) ANVIL RESTS ON HARD GROUND $\quad v_2 = 0$ FOR HAMMER



$+\downarrow$ $\quad m v_1 - F\Delta t = 0$; $\quad \frac{400\ lb}{32.2}(17.944\ ft/s) - F\Delta t$

$$F\Delta t = 222.9\ lb\cdot s \qquad F\Delta t = 223\ lb\cdot s$$

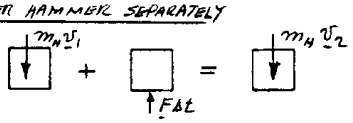
(b) ANVIL RESTS ON SPRINGS SINCE FORCES EXERTED BY SPRINGS ARE NON IMPULSIVE, TOTAL MOMENTUM OF HAMMER AND ANVIL IS CONSERVED



$+\downarrow$ $\quad m_H v_1 = (m_H + m_A)v_2$

$$\frac{400\ lb}{g}(17.944\ ft/s) = \frac{(400\ lb + 1000\ lb)}{g}v_2 \qquad v_2 = 5.127\ ft/s$$

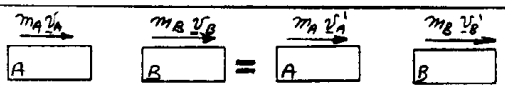
CONSIDER HAMMER SEPARATELY



$+\downarrow$ $\quad m_H v_1 - F\Delta t = m_H v_2$; $\quad \frac{400\ lb}{32.2}(17.944) - F\Delta t = \frac{400\ lb}{32.2}(5.127)$

$$F\Delta t = 159.22\ lb\cdot s \qquad F\Delta t = 159.2\ lb\cdot s$$

13.139



TOTAL MOMENTUM CONSERVED

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$\overset{+}{\rightarrow}$ $\quad \frac{5lb}{g}(6ft/s) + \frac{3lb}{g}(-4ft/s) = \frac{5lb}{g}v_A' + \frac{3lb}{g}v_B'$

$$30 - 12 = 5v_A' + 3v_B' \quad ; \quad 5v_A' + 3v_B' = 18 \qquad (1)$$

RELATIVE VELOCITIES

$$v_B' - v_A' = e(v_A - v_B) = 0.80\left[6 - (-4)\right] = 8$$

$$v_B' = 8 + v_A' \qquad (2)$$

SUBSTITUTE FOR v_B' FROM (2) INTO (1):

$$5v_A' + 3(8 + v_A') = 18 \qquad 8v_A' = 18 - 24$$

$$v_A' = -0.75\ ft/s \qquad \underline{v_A'} = 0.75\ ft/s \leftarrow$$

EQ (2): $\quad v_B' = 8 + (-0.75) = 7.25\ ft/s \qquad v_B' = 7.25\ ft/s \rightarrow$

ENERGY BEFORE IMPACT

$$T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}\frac{5lb}{g}(6\ ft/s)^2 + \frac{1}{2}\frac{3lb}{g}(-4\ ft/s)^2$$

$$= \frac{90}{g} + \frac{24}{g} = \frac{114}{32.2} = 3.5404\ ft\cdot lb$$

ENERGY AFTER IMPACT

$$T_2 = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2 = \frac{1}{2}\frac{5lb}{g}(-0.75\ ft/s)^2 + \frac{1}{2}\frac{3lb}{g}(7.25\ ft/s)^2$$

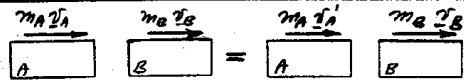
$$= \frac{1.40625}{g} + \frac{78.843}{g} = \frac{80.25}{g} = 2.4922\ ft\cdot lb$$

ENERGY LOSS $= T_1 - T_2 = 3.5404\ ft\cdot lb - 2.4922\ ft\cdot lb$

$$= 1.0481\ ft\cdot lb$$

ENERGY LOSS $= 1.048\ ft\cdot lb$

13.140



TOTAL MOMENTUM CONSERVED

$$m_A\underline{v}_A + m_B\underline{v}_B = m_A\underline{v}_A' + m_B\underline{v}_B'$$

$$\pm\quad \frac{5lb}{g}(6ft/s) + \frac{3lb}{g}(2ft/s) = \frac{5lb}{g}v_A' + \frac{3lb}{g}v_B'$$

$$30+6 = 5v_A' + 3v_B'; \quad 5v_A' + 3v_B' = 36 \qquad (1)$$

RELATIVE VELOCITIES

$$v_B' - v_A' = e(v_A - v_B) = 0.80(6-2) = 3.2$$

$$v_B' = 3.2 + v_A' \qquad (2)$$

SUBSTITUTE FOR v_B' FROM (2) INTO (1)

$$5v_A' + 3(3.2+v_A') = 36; \quad 8v_A' = 36 - 9.6$$

$$v_A' = +3.3\ ft/s \qquad \underline{v}_A' = 3.3\ ft/s \rightarrow$$

EQ.(2): $v_B' = 3.2 + 3.3 = +6.5\ ft/s$ $\qquad \underline{v}_B' = 6.5\ ft/s \rightarrow$

ENERGY BEFORE IMPACT

$$T_1 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 = \frac{1}{2}\frac{5lb}{g}(6ft/s)^2 + \frac{1}{2}\frac{3lb}{g}(2ft/s)^2$$

$$= \frac{90}{g} + \frac{6}{g} = \frac{96}{32.2} = 2.9814\ ft\cdot lb$$

ENERGY AFTER IMPACT

$$T_2 = \frac{1}{2}m_Av_A'^2 + \frac{1}{2}m_Bv_B'^2 = \frac{1}{2}\frac{5lb}{g}(3.3ft/s)^2 + \frac{1}{2}\frac{3lb}{g}(6.5ft/s)^2$$

$$= \frac{27.225}{g} + \frac{63.375}{g} = \frac{90.6}{32.2} = 2.8136\ ft\cdot lb$$

ENERGY LOSS $= T_1 - T_2 = 2.9814 - 2.8136 = 0.1677\ ft\cdot lb$

13.141

TOTAL MOMENTUM CONSERVED

$$m_A\underline{v}_A + m_B\underline{v}_B = m_A\underline{v}_A' + m_B\underline{v}_B'$$

$$\rightarrow\quad (0.6kg)(4m/s) + (0.9kg)(-2m/s) = (0.6kg)v_A' + (0.9kg)v_B'$$

$$0.24 - 0.18 = 0.6v_A' + 0.9v_B'$$

$$0.6v_A' + 0.9v_B' = 0.6 \qquad (1)$$

RELATIVE VELOCITIES

$$v_B' - v_A' = e(v_A - v_B) = 0.75\left[4m/s - (-2m/s)\right] = 4.5$$

$$v_B' = 4.5 + v_A' \qquad (2)$$

SUBSTITUTE FOR v_B' FROM (2) INTO (1):

$$0.6v_A' + 0.9(4.5+v_A') = 0.6; \quad 1.5v_A' = -3.45$$

$$v_A' = -2.3\ m/s \qquad \underline{v}_A' = 2.3\ m/s \leftarrow$$

EQ(2): $v_B' = 4.5 + (-2.3) = 2.2\ m/s$ $\qquad \underline{v}_B' = 2.2\ m/s \rightarrow$

13.142

TOTAL MOMENTUM CONSERVED

$$m_A\underline{v}_A + m_B\underline{v}_B = m_A\underline{v}_A' + m_B\underline{v}_B'$$

$$\pm\quad (0.6kg)(4m/s) + (0.9kg)(-2m/s) = (0.6kg)v_A' + (0.9kg)(2.5m/s)$$

$$2.4 - 1.8 = 0.6v_A' + 2.25; \quad v_A' = -2.75\ m/s$$

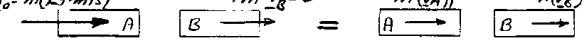
COEFFICIENT OF RESTITUTION

$$e = \frac{v_B' - v_A'}{v_A - v_B} = \frac{2.5 - (-2.75)}{4 - (-2)} = \frac{5.25}{6} \qquad e = 0.875$$

13.143

FIRST COLLISION: CARS A AND B

TOTAL MOMENTUM CONSERVED



$$\pm\quad m(1.5) + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = 1.5 \qquad (1)$$

RELATIVE VELOCITIES: $e = 0.75$

$$(v_B)_1 - (v_A)_1 = e(1.5) = 0.75(1.5) \qquad (v_B)_1 - (v_A)_1 = 1.125 \qquad (2)$$

SOLVING (1) AND (2):

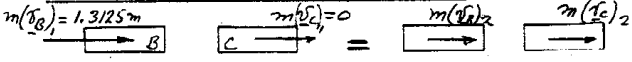
$$(v_A)_1 = 0.1875\ m/s \qquad (\underline{v}_A)_1 = 0.1875\ m/s \rightarrow$$

$$(v_B)_1 = 1.3125\ m/s \qquad (\underline{v}_B)_1 = 1.3125\ m/s \rightarrow$$

(CONTINUED)

13.143 CONTINUED

SECOND COLLISION: CARS B AND C



$$\pm\quad m(1.3125) + m(v_C)_1 = m(v_B)_2 + m(v_C)_2$$

$$(v_B)_2 + (v_C)_2 = 1.3125 \qquad (3)$$

RELATIVE VELOCITIES

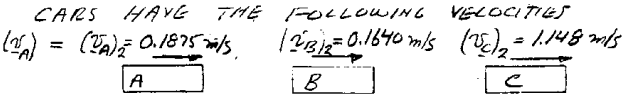
$$(v_C)_2 - (v_B)_2 = e(v_B)_1 = 0.75(1.3125) \qquad (v_C)_2 - (v_B)_2 = 0.9844 \qquad (4)$$

SOLVING (3) AND (4):

$$(v_B)_2 = 0.1640\ m/s \qquad (\underline{v}_B)_2 = 0.1640\ m/s \rightarrow$$

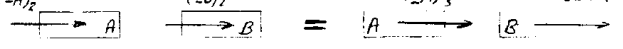
$$(v_C)_2 = 1.148\ m/s \qquad (\underline{v}_C)_2 = 1.148\ m/s \rightarrow$$

AFTER THE SECOND COLLISION THE THREE CARS HAVE THE FOLLOWING VELOCITIES



WE NOTE THAT CAR A WILL OVERTAKE AND COLLIDE WITH CAR B.

THIRD COLLISION: CARS A AND B



$$\pm\quad m(0.1875) + m(0.1640) = m(v_A)_3 + m(v_B)_3$$

$$(v_A)_3 + (v_B)_3 = 0.3516 \qquad (5)$$

RELATIVE VELOCITIES:

$$(v_B)_3 - (v_A)_3 = e\left[(v_A)_2 - (v_B)_2\right] = 0.75\left[0.1875 - 0.1640\right]$$

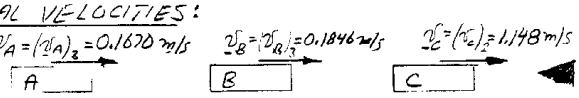
$$(v_B)_3 - (v_A)_3 = 0.01763 \qquad (6)$$

SOLVING (5) AND (6)

$$(v_A)_3 = +0.1670\ m/s \qquad (\underline{v}_A)_3 = 0.1670\ m/s \rightarrow$$

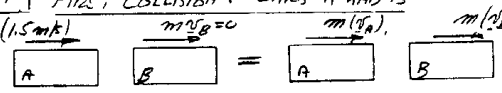
$$(v_B)_3 = +0.1846\ m/s \qquad (\underline{v}_B)_3 = 0.1846\ m/s \rightarrow$$

FINAL VELOCITIES:



13.144

FIRST COLLISION: CARS A AND B



$$\Sigma m\underline{v} = \Sigma m\underline{v}': \quad \pm\quad m(1.5) + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = 1.5 \qquad (1)$$

RELATIVE VELOCITIES $\quad e = 0.5$

$$(v_B)_1 - (v_A)_1 = e(1.5) = 0.5(1.5)$$

$$(v_B)_1 - (v_A)_1 = 0.75 \qquad (2)$$

SOLVING (1) AND (2)

$$(v_A)_1 = +0.375\ m/s \qquad (\underline{v}_A)_1 = 0.375\ m/s \rightarrow$$

$$(v_B)_1 = +1.125\ m/s \qquad (\underline{v}_B)_1 = 1.125\ m/s \rightarrow$$

(CONTINUED)

13.144 CONTINUED

SECOND COLLISION: CARS B AND C $e = 1.00$

$m(v_B)_1 = m(1.125 \text{ m/s})$ $m(v_C)_1 = 0$ $m(v_B)_2$ $m(v_C)_2$

B C = B C

$\Sigma m v = \Sigma m v'$: $\rightarrow$ $m(1.125) + 0 = m(v_B)_2 + m(v_C)_2$

$$(v_B)_2 + (v_C)_2 = 1.125 \quad (3)$$

RELATIVE VELOCITIES $e = 1.00$

$$(v_C)_2 - (v_B)_2 = e(v_B)_1 = 1.00(1.125)$$

$$(v_C)_2 - (v_B)_2 = 1.125 \quad (4)$$

SOLVING (3) AND (4):

$(v_B)_2 = 0$ $(\underline{v}_B)_2 = 0$

$(v_C)_2 = +1.125$ m/s $(\underline{v}_C)_2 = 1.125$ m/s $\rightarrow$

CAR A WILL NOW OVERTAKE AND COLLIDE WITH CAR B.

THIRD COLLISION: CARS A AND B $e = 0.5$

$m(v_A)_2 = m(0.375 \text{ m/s})$ $m(v_B)_2 = 0$ $m(v_A)_3$ $m(v_B)_3$

A B = A B

$\Sigma m v = \Sigma m v'$: $\rightarrow$ $m(0.375) + 0 = m(v_A)_3 + m(v_B)_3$

$$(v_A)_3 + (v_B)_3 = 0.375 \quad (5)$$

RELATIVE VELOCITIES $e = 0.5$

$$(v_B)_3 - (v_A)_3 = e(v_A)_2 = 0.5(0.375)$$

$$(v_B)_3 - (v_A)_3 = 0.1875 \quad (6)$$

SOLVING (5) AND (6)

$(v_A)_3 = +0.09375$ m/s $(v_A)_3 = 0.09375$ m/s $\rightarrow$

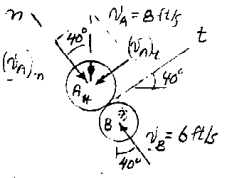
$(v_B)_3 = +0.28125$ m/s $(v_B)_3 = 0.28125$ m/s $\rightarrow$

FINAL VELOCITIES

$\underline{v}_A = (\underline{v}_A)_3 = 0.09375$ m/s $\underline{v}_B = (\underline{v}_B)_3 = 0.28125$ m/s $\underline{v}_C = (\underline{v}_C)_2 = 1.125$ m/s

A B C ◀

13.145



$W_A = 2.5$ lb

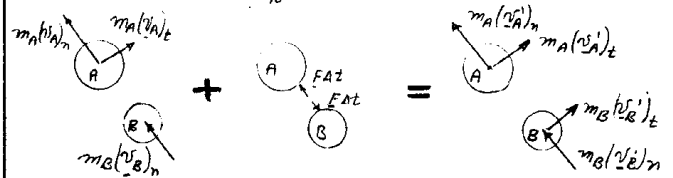
$W_B = 1.5$ lb

$(v_A)_n = -8\cos 40° = -6.128$ ft/s

$(v_A)_t = -8\sin 40° = -5.142$ ft/s

$(v_B)_t = 0$

$(v_B)_n = +6$ ft/s



PRINCIPLE OF IMPULSE & MOMENTUM

BALL A $+\nearrow$ t comp: $m_A(v_A)_t + 0 = m_A(v'_A)_t$

$(v'_A)_t = (v_A)_t = -5.142$ ft/s $(\underline{v}'_A)_t = 5.142$ ft/s $\searrow 40°$ ◁

BALL B $+\nearrow$ t comp: $0 = m_B(v'_B)_t$ $(\underline{v}'_B)_t = 0$ ◁

BALLS A AND B

$+\nwarrow$ n comp: $m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$

$$\frac{2.5}{g}(-6.128) + \frac{1.5}{g}(+6) = \frac{2.5}{g}(v'_A)_n + \frac{1.5}{g}(v'_B)_n$$

$$2.5(v'_A)_n + 1.5(v'_B)_n = -6.32 \quad (1)$$

(CONTINUED

13.145 CONTINUED

RELATIVE VELOCITIES $e = 0.75$

$+\nwarrow$ $(v'_A)_n - (v'_B)_n = e[(v_B)_n - (v_A)_n] = 0.75[+6 - (-6.128)] = +9.096$

$$(v'_A)_n = (v'_B)_n + 9.096 \quad (2)$$

SUBSTITUTE INTO (1):

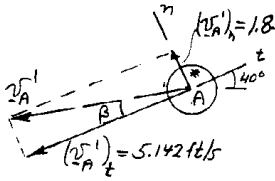
$2.5[(v'_B)_n + 9.096] + 1.5(v'_B)_n = -6.32$; $4(v'_B)_n = -29.06$

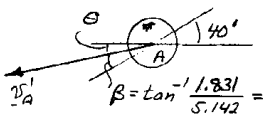
$(v'_B)_n = -7.265$ ft/s $(\underline{v}'_B)_n = 7.265$ ft/s $\searrow 50°$ ◁

EQ(2) $(v'_A)_n = -7.265 + 9.096 = 1.831$ ft/s

$(\underline{v}'_A)_n = 1.831$ ft/s $\nearrow 50°$ ◁

FINAL VELOCITIES

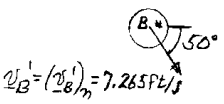




$v'_A = 5.458$ ft/s

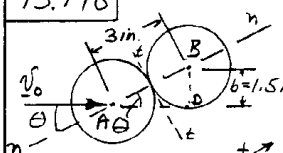
$\theta = 40 - 19.6 = 20.4$

$\underline{v}'_A = 5.46$ ft/s $\searrow 20.4°$ ◀



$v_B = 7.26$ ft/s $\swarrow 50°$ ◀

13.146



(a) FROM ΔABD

$\sin\theta = \frac{BD}{AB} = \frac{1.5}{3}$ $\theta = 30°$

ALONG LINE OF IMPACT n-n

TOTAL MOMENTUM CONSERVED

$+\nearrow$ $m(v_A)_n + m(v_B)_n = m(v'_A)_n + m(v'_B)_n$

$$v_0\cos\theta + 0 = (v'_A)_n + (v'_B)_n \quad (1)$$

RELATIVE VELOCITIES $(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$

$e = 0.80$

$$(v'_B)_n - (v'_A)_n = 0.80\, v_0\cos\theta \quad (2)$$

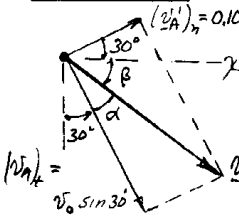
SOLVING (1) AND (2): $(v'_A)_n = 0.10\, v_0\cos\theta$ $(v'_B)_n = 0.90\, v_0\cos\theta$

ALONG LINE t-t PERPENDICULAR TO LINE OF IMPACT

COMPONENT OF VELOCITY OF EACH PUCK IS UNCHANGED

$(v'_A)_t = v_0\sin\theta$ $(v'_B)_t = c$

VELOCITY OF A:



$\tan\alpha = \frac{0.1 v_0\cos 30°}{v_0\sin 30°} = \frac{0.086}{0.5}$

$\alpha = 9.83°$

$\beta = 90° - 30° - 9.83°$ $\beta = 50.17°$

$v'_A = \frac{0.5 v_0}{\cos 9.83°} = 0.507 v_0$

$\underline{v}_A = 0.507 v_0 \swarrow 50.2°$ ◀

VELOCITY OF B: $v_B = (v'_B)_n = 0.9 v_0\cos 30°$ $\underline{v}_B = 0.779 v_0 \angle 30°$ ◀

(b) FOR $e = 1$ WITH ARBITRARY VALUE OF b AND THUS θ

ALONG n-n TOTAL MOMENTUM CONSERVED

$$m v_0\cos\theta = m(v'_A)_n + m(v'_B)_t$$

RELATIVE VELOCITIES: $(v'_B) - (v'_A) = v_0\cos\theta$

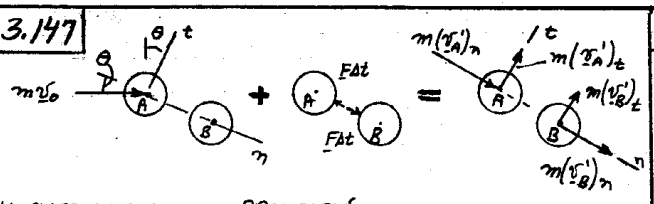
SOLVING SIMULTANEOUSLY: $(v'_A)_n = 0$ $(v'_B)_n = v_0\cos\theta$

ALONG t-t COMPONENT OF VELOCITY OF EACH PUCK REMAINS UNCHANGED

$(v'_A)_t = v_0\sin\theta$ $(v'_B)_t = 0$

THUS $\underline{v}'_A$ IS DIRECTED ALONG t-t AND $\underline{v}'_B$ ALONG n-n. THE VELOCITIES $\underline{v}'_A$ AND $\underline{v}'_B$ FORM A RIGHT ANGLE ◀

13.147



IMPULSE-MOMENTUM PRINCIPLE

BALL A: $+\nearrow t$ comp: $mv_0 \sin\theta = m(v_A')_t$

$(v_A')_t = v_0 \sin\theta \quad \forall\, \theta$ ◁

BALL B: $+\nearrow t$ comp: $0 = m(v_B')_t$ $\quad (v_B')_t = 0$ ◁

BALLS A AND B: $m\underline{v}_0 = m(\underline{v}_A') + m(\underline{v}_B')$

$+\nwarrow n$ comp: $mv_0\cos\theta = m(v_A')_n + m(v_B')_n$

$(v_A')_n + (v_B')_n = v_0\cos\theta \qquad (1)$

RELATIVE VELOCITIES ALONG LINE OF IMPACT (n axis)

$(v_B')_n - (v_A')_n = e\left[(v_A)_n - (v_B)_n\right] = e\left[v_0\cos\theta - 0\right]$

$(v_B')_n - (v_A')_n = e v_0\cos\theta \qquad (2)$

SUBTRACT (2) FROM (1):

$2(v_A')_n = (1-e)v_0\cos\theta \qquad (v_A')_n = v_0\frac{1-e}{2}\cos\theta \;\; \forall\, \theta$ ◁

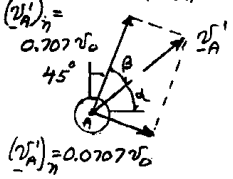
ADD (1) AND (2)

$2(v_B')_n = (1+e)v_0\cos\theta \qquad (v_B')_n = v_0\frac{1+e}{2}\cos\theta \;\; \forall\, \theta$ ◁

FOR $\theta = 45^\circ$ AND $e = 0.80$

BALL A: $(v_A')_t = v_0 \sin 45^\circ = 0.707 v_0 \;\nearrow 45^\circ$

$(v_A')_n = \frac{1-0.80}{2} v_0 \cos 45^\circ = 0.0707 v_0 \;\nwarrow 45^\circ$



$v_A' = 0.7106\, v_0$

$\beta = 5.7^\circ$

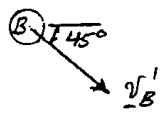
$\alpha = 90^\circ - 45^\circ - \beta = 45^\circ - 5.7^\circ = 39.3^\circ$

$\underline{v}_A' = 0.7106 v_0 \;\angle 39.3^\circ$ ◀

BALL B:

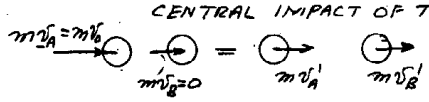
$(v_B')_t = 0 \qquad (v_B')_n = \frac{1+0.8}{2} v_0 \cos 45^\circ = 0.6364 v_0 \;\nwarrow 45^\circ$

$\underline{v}_B = 0.636 v_0 \;\nwarrow 45^\circ$ ◀



13.148

WE FIRST CONSIDER THE PERFECTLY ELASTIC ($e=1$) CENTRAL IMPACT OF TWO EQUAL BALLS



ALONG LINE OF IMPACT

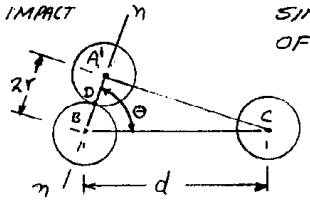
$v_A' + v_B' = v_0 \qquad (1)$

$v_B' - v_A' = e(v_A - v_B) = (1)v_0 \qquad v_B' - v_A' = v_0 \qquad (2)$

SOLVING (1) AND (2) $\quad \underline{v_A' = 0 \quad v_B' = v_0}$

(a)

WE NOW CONSIDER THE THREE BALL A, B, AND C. JUST AFTER IMPACT

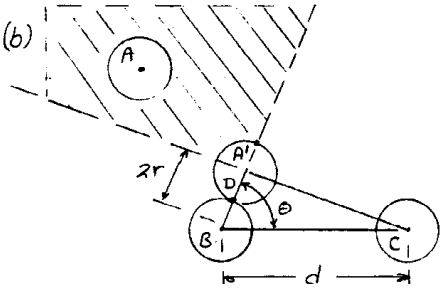


SINCE $e=1$, VELOCITY OF A ALONG n IS ZERO (SEE ABOVE)

THUS LINE nn MUST BE $\perp$ $A'C$, THUS

$\theta = \cos^{-1}\frac{2r}{d}$ ◀

(b)



WE OBSERVE THAT:

FOR BALL A TO STRIKE BALL B AT POINT D, INITIAL POSITION OF BALL A MUST BE ABOVE LINE $A'C$. THAT IS:

$\measuredangle ACB > \measuredangle A'CB = \sin^{-1}\frac{2r}{d}$

ALSO FOR BALL A TO HAVE A FINAL VELOCITY ALONG LINE $A'C$ TOWARD BALL C, INITIAL POSITION OF BALL A MUST BE TO THE LEFT OF LINE BA'. THAT IS

$\measuredangle ABC > \theta = \cos^{-1}\frac{2r}{d}$

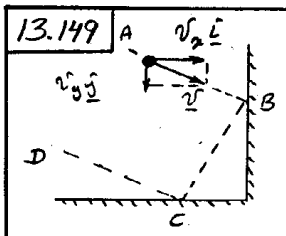
THUS INITIAL POSITION OF A MUST BE LOCATED SO THAT

$\measuredangle ABC > \cos^{-1}\frac{2r}{d}$ ◀

AND

$\measuredangle ACB > \sin^{-1}\frac{2r}{d}$ ◀

13.149



RESOLVE VELOCITIES INTO RECTANGULAR COMPONENTS

$$\underline{v}_{AB} = \underline{v} = v_x\,\underline{i} + v_y\,\underline{j}$$

AFTER REBOUND AT B, x COMP. IS MULTIPLIED BY e AND y COMP IS UNCHANGED

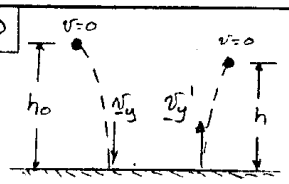
$$\underline{v}_{BC} = -e v_x\,\underline{i} + v_y\,\underline{j}$$

AFTER REBOUND AT C, y COMP IS MULTIPLIED BY e AND x COMP IS UNCHANGED.

$$\underline{v}_{CD} = -e v_x\,\underline{i} - e v_y\,\underline{j} = -e\left(v_x\,\underline{i} + v_y\,\underline{j}\right)$$

THUS $\underline{v}_{CD} = -e\underline{v}$. FINAL VELOCITY IS PARALLEL TO AB ◀

13.150



UNIF. ACCEL. MOTION

$$v_y = \sqrt{2gh_0}$$

$$v_y' = \sqrt{2gh}$$

COEFFICIENT OF RESTITUTION

$$e = \frac{v_y'}{v_y} = \frac{\sqrt{2gh}}{\sqrt{2gh_0}} = \sqrt{\frac{h}{h_0}}$$

HEIGHT OF DROP: $h_0 = 100$ in

HEIGHT OF BOUNCE:

$$53 \text{ in.} \le h \le 58 \text{ in.}$$

WE MUST HAVE:

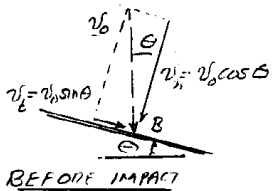
$$\sqrt{\frac{53 \text{ in.}}{100 \text{ in.}}} \le e \le \sqrt{\frac{58 \text{ in.}}{100 \text{ in.}}}$$

$$0.728 \le e \le 0.762$$ ◀

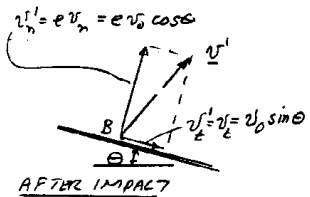
13.151

BALL DROPS FROM HEIGHT h ONTO PLATE. UNIF. ACCEL. MOTION $v_0 = \sqrt{2gh}$

IMPACT AT B

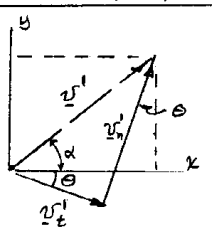


BEFORE IMPACT



AFTER IMPACT

VERTICAL + HORIZONTAL COMPONENTS OF $\underline{v}'$ AT B.



$$v_x' = v_t'\cos\theta + v_n'\sin\theta$$
$$= v_0\sin\theta\cos\theta + e v_0\cos\theta\sin\theta$$

$$v_x' = v_0(1+e)\sin\theta\cos\theta \quad (1)$$

$$v_y' = v_n'\cos\theta - v_t'\sin\theta$$
$$= e v_0\cos^2\theta - v_0\sin^2\theta$$

$$v_y' = v_0\left(e\cos^2\theta - \sin^2\theta\right) \quad (2)$$

FOR GIVEN DATA: $\theta = 20°$, $e = 0.40$

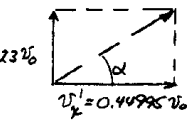
$$v_x' = v_0(1+0.40)\sin 20°\cos 20° \qquad \underline{v}_x' = 0.44995 v_0$$

$$v_y' = v_0\left[0.40\cos^2 20° - \sin^2 20°\right] \qquad v_y' = 0.23623 v_0$$

(CONTINUED)

13.151 CONTINUED

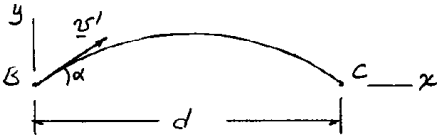
VELOCITY $\underline{v}'$ AT B



$$v' = 0.50819 v_0$$

$$\alpha = 27.70°$$

MOTION OF BALL FROM B TO C:



VERTICAL MOTION: $y = v_y' t - \frac{1}{2} g t^2$

AT C: $0 = (v'\sin\alpha)t - \frac{1}{2} g t^2$

$$t_C = \frac{2v'\sin\alpha}{g}$$

HORIZONTAL MOTION: $x = v_x' t$

AT C: $d = v_x' t_C = v'\cos\alpha\left(\frac{2v'\sin\alpha}{g}\right)$

$$d = \frac{(v')^2}{g}(2\sin\alpha\cos\alpha) = \frac{(v')^2}{g}\sin 2\alpha \quad (3)$$

RECALL THAT $v_0 = \sqrt{2gh}$ $\quad v_0^2 = 2gh$ $\quad g = \frac{v_0^2}{2h}$

THUS $d = \frac{(v')^2}{(v_0)^2/2h}\sin 2\alpha$ $\quad d = 2h\left(\frac{v'}{v_0}\right)^2\sin 2\alpha \quad (4)$

SUBSTITUTE FOR v' AND α,

$$d = 2h\left(\frac{0.50819 v_0}{v_0}\right)^2\sin 2(27.70°) = 0.4252 h \qquad d = 0.425 h$$ ◀

13.152

SEE SOLUTION OF PROB. 13.151

EQ(3): $d = \frac{(v')^2}{g}\sin 2\alpha$ $\quad \sin 2\alpha$ IS MAXIMUM FOR $\alpha = 45°$

THUS $d_{max} = \frac{(v')^2}{g}$ AND $v_x' = v_y'$

FROM EQS (1) AND (2): $v_0(1+e)\sin\theta\cos\theta = v_0(e\cos^2\theta - \sin^2\theta)$

SUBSTITUTE $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$, $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$(1+e)\tfrac{1}{2}\sin 2\theta = \tfrac{1}{2}e(1+\cos 2\theta) - \tfrac{1}{2}(1-\cos 2\theta)$$

$$(1+e)\sin 2\theta = (1+e)\cos 2\theta - (1-e)$$

$$\cos 2\theta - \sin 2\theta = \frac{1-e}{1+e}$$

LET $\varepsilon = \frac{1-e}{1+e}$ AND SUBSTITUTE $\cos 2\theta = \sqrt{1-\sin^2 2\theta}$

$$\sqrt{1-\sin^2 2\theta} - \sin 2\theta = \varepsilon$$

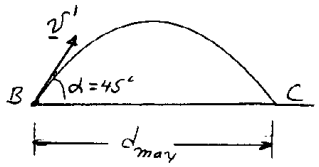
$$1 - \sin^2 2\theta = (\varepsilon + \sin 2\theta)^2$$

$$1 - \sin^2 2\theta = \varepsilon^2 + 2\varepsilon\sin 2\theta + \sin^2 2\theta$$

$$\sin^2 2\theta + \varepsilon\sin 2\theta + \tfrac{1}{2}(\varepsilon^2 - 1) = 0$$

$$\sin 2\theta = \frac{-\varepsilon \pm \sqrt{\varepsilon^2 - 2(\varepsilon^2-1)}}{2} = \frac{1}{2}\left(-\varepsilon \pm \sqrt{2-\varepsilon^2}\right) \quad (1)$$

WITH: $\varepsilon = \frac{1-e}{1+e}$ (2)



$$d_{max} = \frac{(v')^2}{g} = \frac{(v')^2(2h)}{2gh} = \frac{(v')^2}{(v_0)^2}(2h)$$

OR $\quad d_{max} = 2h\left(\frac{v'}{v_0}\right)^2$ (3)

(CONTINUED)

13.152 CONTINUED

(a) FOR $e = 1$

EQ(2) $\varepsilon = \frac{1-e}{1+e} = \frac{0}{2} = 0$

EQ(1) $\sin 2\theta = \frac{1}{2}(0+\sqrt{2}) = \frac{1}{\sqrt{2}}$ $\quad 2\theta = 45°$ $\quad \theta = 22.5°$ ◀

FROM EQ(1) OF PROB. 13.151.

$v_x' = v_0(1+e)\sin\theta\cos\theta = v_0(1+1)\sin 22.5°\cos 22.5°$

$= v_0(0.70711)$

FOR $\alpha = 45°$

$v_x' = v'\cos\alpha$; $0.70711\,v_0 = v'\cos 45°$ $\quad v' = v_0$

EQ(3): $d_{max} = 2h\left(\frac{v'}{v_0}\right)^2 = 2h$ $\quad d_{max} = 2h$ ◀

(b) FOR $e = 0.50$:

EQ(2): $\varepsilon = \frac{1-e}{1+e} = \frac{1-0.5}{1+0.5} = \frac{0.5}{1.5} = \frac{1}{3}$

EQ(1): $\sin 2\theta = \frac{1}{2}\left[-\left(\frac{1}{3}\right)+\sqrt{2-\left(\frac{1}{3}\right)^2}\right] = \frac{1}{2}\left[-\frac{1}{3}+\sqrt{\frac{17}{9}}\right]$

$\sin 2\theta = \frac{1}{2}[-0.3333 + 1.3744] = 0.5205$

$2\theta = 31.367°$ $\quad \theta = 15.683°$ $\quad \theta = 15.7°$ ◀

FROM EQ(1) OF PROB 13.151

$v_x' = v_0(1+e)\sin\theta\cos\theta = v_0\frac{1+e}{2}\sin 2\theta$

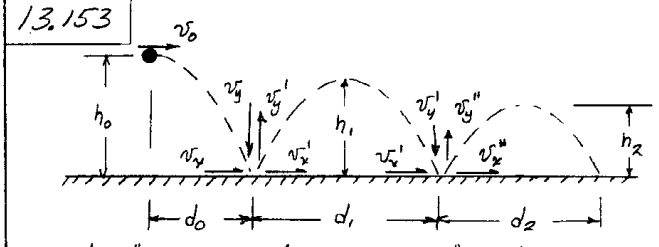
$= v_0\left(\frac{1+0.5}{2}\right)0.5205 = 0.3904\,v_0$

FOR $\alpha = 45°$

$v_x' = v'\cos\alpha$; $0.3904 v_0 = v'\cos 45°$ $\quad v' = 0.5521 v_0$

EQ(3): $d_{max} = 2h\left(\frac{v'}{v_0}\right)^2 = 2h(0.5521 v_0)^2$; $\quad d_{max} = 0.610h$ ◀

13.153



$v_x = v_x' = v_x'' = v_0$ $\quad v_y' = e v_y$ $\quad v_y'' = e v_y'$

VERTICAL MOTION (UNIFORMLY ACCELERATED)

$v_y = \sqrt{2gh_0}$ AND $v_y' = \sqrt{2gh_1}$

BUT $v_y' = e v_y$ THUS: $\sqrt{2gh_1} = e\sqrt{2gh_0}$

$h_1 = e^2 h_0$ (1)

RESULT OBTAINED MAY BE EXTENDED TO ANY TWO SUCCESSIVE BOUNCES

$h_2 = e^2 h_1$ (2) $\quad (n+0)$ $\quad h_{n+1} = e^2 h_n$ (3)

DENOTING BY t_1 THE TIME REQUIRED FOR THE FIRST BOUNCE AND BY t_2 TIME FOR SECOND BOUNCE

$v_y' = g\left(\frac{1}{2}t_1\right)$ AND $v_y'' = g\left(\frac{1}{2}t_2\right)$

BUT $v_y'' = e v_y'$ THUS: $t_2 = e\,t_1$ (4)

FOR ANY TWO SUCCESSIVE BOUNCES: $(n+0)$ $\quad t_{n+1} = e\,t_n$ (5)

(CONTINUED)

13.153 CONTINUED

HORIZONTAL MOTION (UNIF. MOTION)

$d_1 = v_x' t_1 = v_0 t_1$ $\quad$ (SINCE $v_x = v_x' = v_0$)

$d_2 = v_x'' t_2 = v_0 t_2$

DIVIDING MEMBER BY MEMBER AND RECALLING EQ(4):

$\frac{d_2}{d_1} = \frac{t_2}{t_1} = e$ $\quad d_2 = e\,d_1$ (6)

FOR ANY TWO SUCCESSIVE BOUNCES: $(n+0)$ $\quad d_{n+1} = e\,d_n$ (7)

NOTE: FROM POINT A TO THE FIRST BOUNCE WE HAVE t_0 AND d_0 WHICH CORRESPOND TO HALF OF SUCCESSIVE TIMES (t_1, t_2, etc) AND DISTANCES (d_1, d_2, etc). THUS

$t_1 = e(2t_0)$ (4') $\quad d_1 = e(2d_0)$ (6')

GIVEN DATA: $h_0 = 40$ in., $d_0 = 6$ in., $e = 0.85$

(a) EQ(1): $h_1 = e^2 h_0 = (0.85)^2(40\text{ in.})$ $\quad h_1 = 28.9$ in. ◀

EQ(6'): $d_1 = e(2d_0) = (0.85)2(6\text{ in.})$ $\quad d_1 = 10.20$ in. ◀

(b) EQ(3): $h_2 = e^2 h_1 = (0.85)^2(28.9\text{ in.})$ $\quad h_2 = 20.88$ in. ◀

EQ(6): $d_2 = e d_1 = (0.85)(10.20\text{ in.})$ $\quad d_2 = 8.67$ in. ◀

13.154

SEE PROB. 13.153 FOR DERIVATION OF (1) AND (6')

GIVEN DATA: $h_0 = 25$ in., $h_1 = 16$ in., $d_0 = 5$ in.

(a) EQ(1): $h_1 = e^2 h_0$ $\quad 16\text{ in.} = e^2(25\text{ in.})$ $\quad e = 0.80$ ◀

(b) EQ(6'): $d_1 = e(2d_0) = (0.80)(2)(5\text{ in.})$ $\quad d_1 = 8$ in. ◀

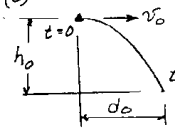
13.155

SEE PROB. 13.153 FOR DERIVATION OF (1) AND (6')

GIVEN DATA: $v_0 = 0.4$ m/s, $d_0 = 80$ mm, $d_1 = 140$ mm

(a) EQ(6'): $d_1 = e(2d_0)$ $\quad 140\text{ mm} = e(2)(80\text{ mm})$ $\quad e = 0.875$ ◀

(b) HORIZONTAL MOTION:



$d_0 = v_0 t_0$

$80\text{ mm} = (400\text{ mm/s})t_0$

$t_0 = 0.20$ s

VERTICAL MOTION

$h_0 = \frac{1}{2}g t_0^2 = \frac{1}{2}(9.81\text{ m/s}^2)(0.20\text{ s})^2$

$h_0 = 0.1962\text{ m} = 196.2\text{ mm}$

EQ(1) $h_1 = e^2 h_0 = (0.875)^2(196.2\text{ mm})$ $\quad h_1 = 150.2$ mm ◀

13.156

SEE PROB. 13.153 FOR DERIVATION OF EQ(7). WE FIRST DETERMINE THE TOTAL HORIZONTAL DISTANCE TRAVELLED BY THE BALL AFTER IT HITS THE FLOOR.

$d = d_1 + d_2 + d_3 + \cdots$

EQ(7) $d = d_1 + e d_1 + e^2 d_1 + \cdots = d_1(1 + e + e^2 + e^3 + \cdots)$

RECALL THAT $\frac{1}{1-e} = 1 + e + e^2 + e^3 + \cdots$

THUS $d = d_1\frac{1}{1-e}$

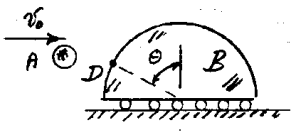
FROM PROB 13.155, WE KNOW THAT $d_1 = 140$ mm AND $e = 0.875$

$d = (140\text{ mm})\frac{1}{1-0.875} = 1120\text{ mm}$

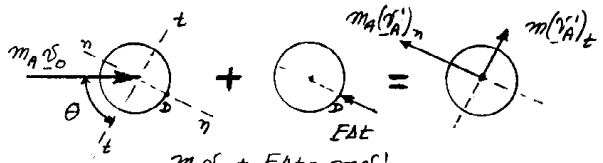
SINCE HORIZONTAL COMPONENT OF VELOCITY IS CONSTANT,

$d = v_0 t$ $\quad t = \frac{d}{v_0} = \frac{1120\text{ mm}}{400\text{ mm/s}} = 2.80$ s $\quad t = 2.80$ s ◀

13.157



CONSIDER SPHERE

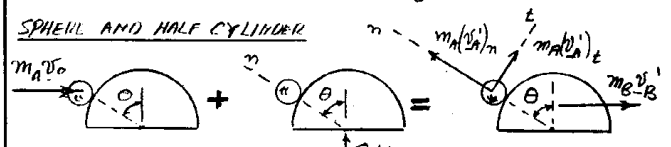


$$m_A \underline{v}_A + F\Delta t = m\underline{v}_A'$$

$+/t$ COMP: $m_A v_0 \cos\theta + 0 = m(v_A')_t$

$$(v_A')_t = v_0 \cos\theta \qquad (1)$$

SPHERE AND HALF CYLINDER



$$m_A \underline{v}_A + \underline{Q}\,\Delta t = m_A \underline{v}_A' + m_B \underline{v}_B'$$

$\rightarrow$ COMP: $m_A v_0 + 0 = -m_A (v_A')_n \sin\theta + m_A (v_A')_t \cos\theta + m_B v_B'$

SUBSTITUTE FOR $(v_A')_t$ FROM (1):

$$m_A v_0 = -m_A (v_A')_n \sin\theta + m_A v_0 \cos^2\theta + m_B v_B' \qquad (2)$$

RELATIVE VELOCITIES IN n DIRECTION

$$(v_A')_n + v_B' \sin\theta = e v_0 \sin\theta \;;\quad (v_A')_n = e v_0 \sin\theta - v_B' \sin\theta \qquad (3)$$

SUBSTITUTE FOR $(v_A')_n$ FROM (3) INTO (2):

$$m_A v_0 = +m_A v_B' \sin^2\theta - e m_A v_0 \sin^2\theta + m_A v_0 \cos^2\theta + m_B v_B'$$

SOLVE FOR v_B':

$$v_B' = v_0 \frac{m_A(1 + e\sin^2\theta - \cos^2\theta)}{m_B + m_A \sin^2\theta} \qquad (4)$$

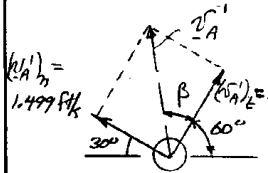
GIVEN DATA: $m_A = \frac{4\,lb}{g}$; $m_B = \frac{10\,lb}{g}$; $v_0 = 5\,ft/s$; $\theta = 60°$; $e = 0.75$

EQ(4): $v_B' = (5\,ft/s)\dfrac{\frac{4lb}{g}(1 + 0.75\sin^2 60° - \cos^2 60°)}{\frac{10lb}{g} + \frac{4lb}{g}\sin^2 60°} = 5\dfrac{(4)(1.3125)}{13.0}$

$v_B' = +2.019\,ft/s$ $\qquad \underline{v}_B' = 2.02\,ft/s \rightarrow$ ◀

EQ.(1): $(v_A')_t = (5\,ft/s)\cos 60° = 2.5\,ft/s \;\angle 60°$

EQ(3): $(v_A')_n = (0.75)(5\,ft/s)\sin 60° - (2.019\,ft/s)\sin 60° = 1.499\,ft/s \;\angle 30°$



$v_A' = 2.914\,ft/s$

$\beta = 30.9°$

$\underline{v}_A' = 2.91\,ft/s \;\angle 89.1°$ ◀

13.158

SEE SOLUTION OF PROB 13.157

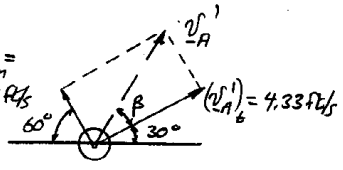
GIVEN DATA: $m_A = \frac{4\,lb}{g}$; $m_B = \frac{10\,lb}{g}$; $v_0 = 5\,ft/s$; $\theta = 30°$; $e = 0.75$

EQ(4): $v_B' = (5\,ft/s)\dfrac{\frac{4lb}{g}(1 + 0.75\sin^2 30° - \cos^2 30°)}{\frac{10lb}{g} + \frac{4lb}{g}\sin^2 30°} = 5\dfrac{(4)(0.4375)}{11.0}$

$v_B' = 0.7954\,ft/s$ $\qquad \underline{v}_B' = 0.795\,ft/s \rightarrow$ ◀

EQ.(1): $(v_A')_t = (5\,ft/s)\cos 30° = 4.33\,ft/s \;\angle 30°$

EQ.(3): $(v_A')_n = (0.75)(5\,ft/s)\sin 30° - (0.7954\,ft/s)\sin 30° = 1.477\,ft/s \;\angle 60°$



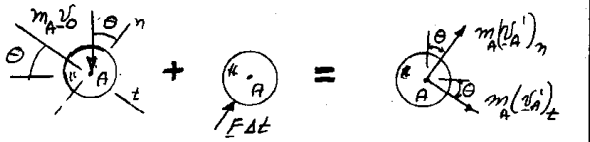
$v_A' = 4.575\,ft/s$

$\beta = 18.84°$

$\underline{v}_A' = 4.58\,ft/s \;\angle 48.8°$ ◀

13.159

SPHERE:

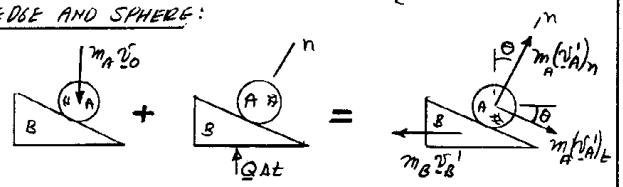


$$m_A \underline{v}_0 + \underline{F}\,\Delta t = m_A \underline{v}_A'$$

$+\searrow t$ comp: $m v_0 \sin\theta + 0 = m(v_A')_t$

$$(v_A')_t = v_0 \sin\theta \qquad (1)$$

WEDGE AND SPHERE:



$$m\underline{v}_0 + \underline{Q}\,\Delta t = m_A \underline{v}_A' + m_B \underline{v}_B'$$

$\rightarrow$ comp: $0 + 0 = m_A (v_A')_n \sin\theta + m_A (v_A')_t \cos\theta - m_B v_B'$

SUBSTITUTE FOR $(v_A')_t$ FROM (1):

$$m_A (v_A')_n \sin\theta - m_B v_B' = -m_A v_0 \sin\theta\cos\theta \qquad (2)$$

RELATIVE VELOCITIES IN THE n DIRECTION

$$(v_A')_n + v_B' \sin\theta = e(v_0 \cos\theta) \qquad (3)$$

MULTIPLY (3) BY $m_A \sin\theta$ AND SUBTRACT (2) FROM (3)

$$m_B v_B' + m_A v_B' \sin^2\theta = +m_A v_0 \sin\theta\cos\theta + e m_A v_0 \sin\theta\cos\theta$$

$$v_B'\left(\frac{m_B}{m_A} + \sin^2\theta\right) = v_0(1+e)\sin\theta\cos\theta \qquad (4)$$

GIVEN DATA: $m_A = 2\,kg$; $m_B = 6\,kg$; $v_0 = 3\,m/s$; $e = 0.80$; $\theta = 30°$

EQ(4): $v_B'\left(\frac{6\,kg}{2\,kg} + \sin^2 30°\right) = (3\,m/s)(1+0.80)\sin 30°\cos 30°$

$$v_B'(3 + 0.25) = (3)(1.8)(0.5)(0.86603)$$

$v_B' = 0.71947\,m/s$ $\qquad \underline{v}_B' = 0.719\,m/s \leftarrow$ ◀

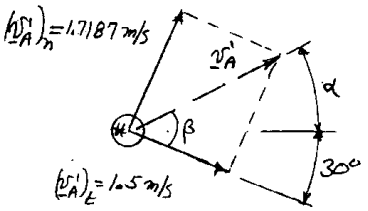
EQ(1): $(v_A')_t = v_0 \sin 30° = (3\,m/s)\sin 30° = 1.5\,m/s$

EQ(3): $(v_A')_n = e(v_0\cos\theta) - v_B' \sin\theta$

$= (0.80)(3\,m/s)\cos 30° - (0.71947\,m/s)\sin 30°$

$= 1.7187\,m/s$

VELOCITY OF SPHERE



$v_A' = 2.2812\,m/s$

$\beta = 48.9°$

$\alpha = \beta - 30° = 18.9°$

$\underline{v}_A' = 2.28\,m/s \;\angle 18.9°$ ◀

13.160 SEE SOLUTION OF PROB 13.159

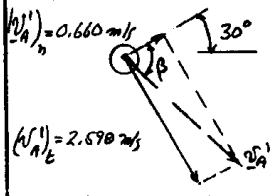
GIVEN DATA: $m_A = 2$ kg; $m_B = 6$ kg; $v_0 = 3$ m/s; $e = 0.80$; $\theta = 60°$

EQ(4): $v_B'\left(\frac{6\text{ kg}}{2\text{ kg}} + \sin^2 60°\right) = (3\text{ m/s})(1+0.80)\sin 60°\cos 60°$

$v_B' = 0.6235$ m/s $\qquad \underline{v}_B' = 0.624\text{ m/s} \leftarrow$ ◀

EQ(1): $(v_A')_t = (3\text{ m/s})\sin 60°$ $\qquad (\underline{v}_A')_t = 2.598\text{ m/s} \searrow 60°$

EQ(3): $(v_A')_n + (0.6235)\sin 60° = 0.80(3\text{ m/s})\cos 60°$; $(\underline{v}_A')_n = 0.660\text{ m/s} \nearrow 30°$

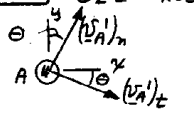


$v_A' = 2.681$ m/s

$\beta = 75.7°$

$\underline{v}_A' = 2.68\text{ m/s} \nwarrow 45.7°$ ◀

13.161 SEE PROB. 13.159 FOR DERIVATION OF (1)(2)(3)+(4)



FOR $\underline{v}_A'$ TO BE HORIZONTAL:

$(v_A')_y = (v_A')_n\cos\theta - (v_A')_t\sin\theta = 0$

SUBSTITUTE FOR $(v_A')_t$ AND $(v_A')_n$ FROM (1) AND (3) RESPECTIVELY:

$$e(v_0\cos\theta)\cos\theta - v_B'\sin\theta\cos\theta = v_0\sin^2\theta \qquad (5)$$

FROM (4) $\quad v_B' = \dfrac{v_0(1+e)\sin\theta\cos\theta}{\frac{m_B}{m_A} + \sin^2\theta}$

SUBSTITUTE INTO (5):

$$e v_0\cos^2\theta - \frac{v_0(1+e)\sin^2\theta\cos^2\theta}{\frac{m_B}{m_A} + \sin^2\theta} = v_0\sin^2\theta$$

$$\frac{m_B}{m_A}e\cos^2\theta + e\cos^2\theta\sin^2\theta - (1+e)\sin^2\theta\cos^2\theta = \frac{m_B}{m_A}\sin^2\theta + \sin^4\theta$$

MULTIPLY BY $\dfrac{m_A}{m_B}\dfrac{1}{\cos^2\theta}$

$$e - \frac{m_A}{m_B}\sin^2\theta = \tan^2\theta + \frac{m_A}{m_B}\sin^2\theta\tan^2\theta$$

$$e - \tan^2\theta = \frac{m_A}{m_B}\sin^2\theta\left(1+\tan^2\theta\right) = \frac{m_A}{m_B}\sin^2\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= \frac{m_A}{m_B}\sin^2\theta\,\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{m_A}{m_B}\tan^2\theta$$

(a) THUS FOR $\underline{v}_A'$ HORIZONTAL: $\quad e = \left(1+\dfrac{m_A}{m_B}\right)\tan^2\theta \qquad (6)$

DATA: $m_A = 2$ kg, $m_B = 6$ kg; $e = 0.80$:

EQ.(6) $\quad 0.80 = \left(1+\dfrac{2\text{ kg}}{6\text{ kg}}\right)\tan^2\theta$

$\tan^2\theta = 0.6 \qquad \theta = 37.8°$ ◀

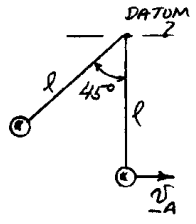
(b) WE MAY SIMULATE THE WEDGE RIGIDLY ATTACHED TO FLOOR BY SETTING $m_B = \infty$

EQ.(6): $0.80 = \left(1+\dfrac{2\text{ kg}}{\infty}\right)\tan^2\theta \qquad \theta = 41.80°$ ◀

13.162 PHASE I

CONSERVATION OF ENERGY FOR A

$T + V = $ CONSTANT



$$0 - W_A l\cos 45° = \frac{1}{2}\frac{W_A}{g}v_A^2 - W_A l$$

$$v_A^2 = 2gl(1-\cos 45°)$$

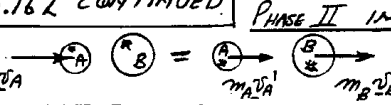
$$v_A^2 = 0.58579\,gl$$

$$v_A = 0.76537\sqrt{gl}$$

(CONTINUED)

13.162 CONTINUED — PHASE II IMPACT

$W_A = 1.5$ lb, $W_B = 3$ lb



TOTAL MOMENTUM CONSERVED

$\rightarrow \quad m_A v_A = m_A v_A' + m_B v_B'$

$$\frac{1.5\text{ lb}}{g}v_A = \frac{1.5}{g}v_A' + \frac{3\text{ lb}}{g}v_B'$$

$$v_A' + 2v_B' = v_A \qquad (2)$$

RELATIVE VELOCITIES

$v_B' - v_A' = e(v_A) = 0.75\,v_A \qquad \underline{v}_B' - \underline{v}_A' = 0.75\,\underline{v}_A \qquad (3)$

MULTIPLY (3) BY 2 AND SUBTRACT FROM (2):

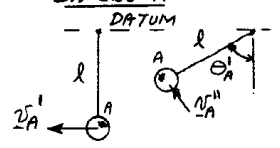
$3v_A' = -0.5\,v_A \qquad v_A' = -0.1667\,v_A \qquad \underline{v}_A' = 0.1667\,v_A \leftarrow$

ADD (2) AND (3):

$3v_B' = 1.75\,v_A \qquad v_B' = 0.5833\,v_A \qquad \underline{v}_B' = 0.5833\,v_A \rightarrow$

PHASE III CONSERVATION OF ENERGY

SPHERE A



FOR MAX θ_A', $v_A'' = 0$

$T + V = $ CONSTANT

$$\frac{1}{2}\frac{W_A}{g}v_A'^2 - W_A l = 0 - W_A l\cos\theta_A'$$

$$v_A'^2 = 2gl(1-\cos\theta_A')$$

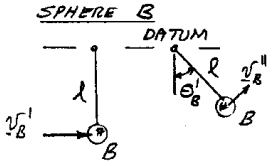
$$(0.1667\,v_A)^2 = 2gl(1-\cos\theta_A')$$

SUBSTITUTE $v_A^2 = 0.58579\,gl$

$$(0.1667)^2(0.58579\,gl) = 2gl(1-\cos\theta_A')$$

$(1-\cos\theta_A') = 0.008156 \qquad \cos\theta_A' = 0.99186 \qquad \theta_A' = 7.3°$ ◀

SPHERE B



FOR MAX θ_B', $v_B'' = 0$

$T + V = $ CONSTANT

$$\frac{1}{2}\frac{W_B}{g}v_B'^2 - W_B l = 0 - W_B l\cos\theta_B'$$

$$v_B'^2 = 2gl(1-\cos\theta_B')$$

$$(0.5833\,v_A)^2 = 2gl(1-\cos\theta_B')$$

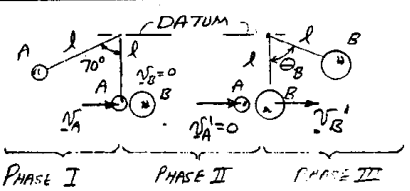
SUBSTITUTE $v_A^2 = 0.58579\,gl$

$$(0.5833)^2(0.58579\,gl) = 2gl(1-\cos\theta_B')$$

$(1-\cos\theta_B') = 0.09967 \qquad \cos\theta_B' = 0.90033 \qquad \theta_B' = 25.8°$ ◀

13.163

$W_A = 1.5$ lb, $W_B = 3$ lb



PHASE I CONSERVATION OF ENERGY FOR A

$T + V = $ CONST. $\qquad 0 - W_A l\cos 70° = \frac{1}{2}\frac{W_A}{g}v_A^2 - W_A l$

$v_A^2 = 2gl(1-\cos 70°) \qquad v_A^2 = 1.3159\,gl \qquad \underline{v}_A = 1.1471\sqrt{gl} \rightarrow$

PHASE II IMPACT NOTE: $v_B = 0$ AND $v_A' = 0$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$\frac{1.5\text{ lb}}{g}v_A + 0 = 0 + \frac{3\text{ lb}}{g}v_B' \qquad v_B' = \frac{1}{2}v_A = 0.57735\sqrt{gl}$$

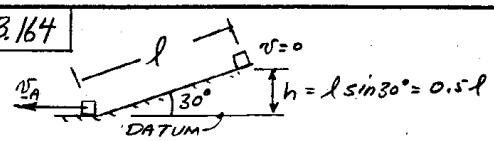
$$e = \frac{v_B'}{v_A} = \frac{\frac{1}{2}v_A}{v_A} \qquad e = 0.50$$ ◀

PHASE III CONSERVATION OF ENERGY FOR B

$T + V = $ CONSTANT $\qquad \frac{1}{2}\frac{W_B}{g}v_B'^2 - W_B l = 0 - W_B l\cos\theta_B$

$(1-\cos\theta_B) = \frac{1}{2gl}v_B'^2 = \frac{1}{2gl}(0.5735\sqrt{gl})^2$; $\cos\theta_B = 0.8355$; $\theta_B = 33.3°$ ◀

13.164

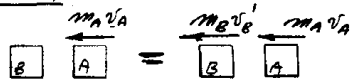


PHASE I CONSERVATION OF ENERGY

$T + V = \text{CONSTANT}$

$0 + m_A g h = \frac{1}{2} m_A v_A^2 + 0 \qquad v_A = \sqrt{2gh}$ (1)

PHASE II IMPACT



$\stackrel{+}{\leftarrow}\quad 0 + m_A v_A = m_B v_B' + m_A v_A'$ (2)

RELATING VELOCITIES

$v_B' - v_A' = e\, v_A \qquad v_B' = v_A' + e\, v_A$ (3)

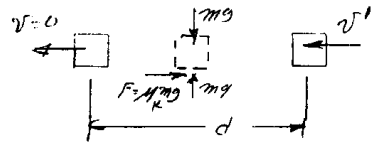
SUBSTITUTE FOR v_B' IN (2): $m_A v_A = m_B(v_A' + e\, v_A) + m_A v_A'$

$v_A' = \frac{m_A - e m_B}{m_A + m_B} v_A$ (4)

SUBSTITUTE FOR v_A' FROM (4) INTO (3):

$v_B' = \frac{m_A + e m_A}{m_A + m_B} v_A$ (5)

PHASE III BLOCKS SLIDE ON HORIZONTAL SURFACE



WORK-ENERGY PRINC. $T_1 + U_{1-2} = T_2$

$\frac{1}{2} m v'^2 - \mu_k mg\, d = 0$

$d = \frac{v'^2}{2\mu_k g}$ (6)

GIVEN DATA $l = 900\text{ mm}$, $h = 0.45\text{ m}$

$m_A = 6\text{ kg}$, $m_B = 3\text{ kg}$, $e = 0.60$, $\mu_k = 0.40$

EQ(1): $v_A = \sqrt{2(9.81\text{ m/s}^2)(0.45\text{ m})} = 2.971\text{ m/s}$

EQ(4): $v_A' = \frac{6\text{ kg} - 0.6(3\text{ kg})}{6\text{ kg} + 3\text{ kg}}(2.971\text{ m/s}) = 1.3866\text{ m/s} \leftarrow$

EQ(5): $v_B' = \frac{6\text{ kg} + (0.6)(6\text{ kg})}{6\text{ kg} + 3\text{ kg}}(2.971\text{ m/s}) = 3.1695\text{ m/s} \leftarrow$

BLOCK A

EQ.(6): $d_A = \frac{(v_A')^2}{2\mu_k g} = \frac{(1.3866)^2}{2(0.4)(9.81)} = 0.245\text{ m}$

$d_A = 245\text{ mm} \leftarrow$

BLOCK B:

EQ.(6) $d_B = \frac{(v_B')^2}{2\mu_k g} = \frac{(3.1695)^2}{2(0.4)(9.81)} = 1.280\text{ m}$

$d_B = 1280\text{ mm} \leftarrow$

13.165

SEE SOLUTION OF PROB 13.164 FOR EQS. (1) THROUGH (6).

GIVEN DATA: $l = 800\text{ mm}$, $h = 0.4\text{ m}$

$m_A = 5\text{ kg}$, $m_B = 3\text{ kg}$, $d_A = 0.24\text{ m}$, $d_B = 1.2\text{ m}$

BLOCK A EQ(6): $d_A = \frac{(v_A')^2}{2\mu_k g}$

BLOCK B EQ(6): $d_B = \frac{(v_B')^2}{2\mu_k g}$

DIVIDE MEMBER BY MEMBER: $\frac{d_A}{d_B} = \frac{(v_A')^2}{(v_B')^2}$ (7)

(CONTINUED)

13.165 CONTINUED

FROM EQ(7) $\frac{v_A'}{v_B'} = \sqrt{\frac{d_A}{d_B}} \qquad \frac{v_A'}{v_B'} = \sqrt{\frac{0.24\text{ m}}{1.2\text{ m}}} = \sqrt{\frac{1}{5}}$ (8)

DIVIDE EQ(4) BY EQ(5), MEMBER BY MEMBER

$\frac{v_A'}{v_B'} = \frac{m_A - e m_B}{m_A + e m_A} = \frac{5\text{ kg} - e(3\text{ kg})}{5\text{ kg} + e(5\text{ kg})} = \sqrt{\frac{1}{5}}$

$5 - 3e = (5 + 5e)\sqrt{\frac{1}{5}} = (5 + 5e)(0.44721)$

$5 - 3e = 2.2361 + 2.2361e$

$2.7639 = 5.2361e \qquad e = 0.5278 \qquad e = 0.528$ ◀

SINCE $h = 0.4\text{ m}$

$v_A = \sqrt{2gh} = \sqrt{2(9.81)(0.4)} = 2.801\text{ m}$

EQ(4) $v_A' = \frac{5 - 0.5278(3)}{5 + 3}(2.801) = 1.196\text{ m/s} \leftarrow$

EQ(5) $v_B' = \frac{5 + 0.5278(5)}{5 + 3}(2.801) = 2.675\text{ m/s} \leftarrow$

BLOCK A:

EQ.(6): $d_A = \frac{(v_A')^2}{2\mu_k g}$; $0.24\text{ m} = \frac{(1.196\text{ m/s})^2}{2\mu_k(9.81)} \qquad \mu_k = 0.304$ ◀

BLOCK B (CHECK)

EQ.(6): $d_B = \frac{(v_B')^2}{2\mu_k g}$; $1.2\text{ m} = \frac{(2.675\text{ m/s})^2}{2\mu_k(9.81)} \qquad \mu_k = 0.304$ (OK)

13.166

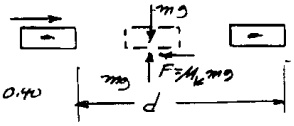
IMPACT CONSERVATION OF MOMENTUM

$\stackrel{+}{\rightarrow}\quad m_{bullet} v_0 = (m_{bullet} + m_{block}) v'$

$(0.02\text{ kg})(600\text{ m/s}) = (0.02\text{ kg} + 4.5\text{ kg})v'$

$v' = 2.6549\text{ m/s}$

(a) BLOCK SLIDES TO REST



PRINC. OF WORK + ENERGY $T_1 + U_{1-2} = T_2$

$\frac{1}{2} m(v')^2 - \mu_k mg\, d = 0$

$d = \frac{(v')^2}{2\mu_k g} = \frac{(2.6549\text{ m/s})^2}{2(0.40)(9.81)}$

$d = 0.898\text{ m} \qquad d = 898\text{ mm}$ ◀

(b) $T_0 = \frac{1}{2} m_{bullet} v_0^2 = \frac{1}{2}(0.02\text{ kg})(600\text{ m/s})^2 = 3600\text{ J}$

$T' = \frac{1}{2}(0.02\text{ kg} + 4.5\text{ kg})(2.6549\text{ m/s})^2 = 15.93\text{ J}$

ENERGY LOSS IN IMPACT $= T_0 - T'$

$= 3600\text{ J} - 15.93\text{ J} = 3584.07\text{ J}$

ENERGY LOST IN FRICTION BETWEEN BLOCK & FLOOR $= T' = 15.93\text{ J}$

% ENERGY LOSS IN FRICTION BETWEEN BLOCK & FLOOR

$\%\text{ LOSS} = (100)\frac{T'}{T_0} = (100)\frac{15.93\text{ J}}{3600\text{ J}} = 0.442\%$ ◀

13.167

PHASE I: "A" DROPS 2.4 m CONS. OF ENERGY

$T+V = T+V$: $0 + mgh = \frac{1}{2}mv_A^2$

$v_A = \sqrt{2gh} = \sqrt{2(9.81\ m/s^2)(2.4\ m)} = 6.862\ m/s$

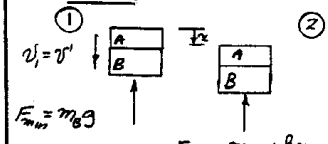
PHASE II: IMPACT ($e=0$) TOTAL MOMENTUM CONSERVED

$m_A v_A + m_B v_B = (m_A + m_B)v'$

$(0.5\ kg)(6.862\ m/s) + 0 = (2.5\ kg + 0.5\ kg)v'$

$v' = 1.1437\ m/s$

(a) **PHASE III:** "A" AND "B" MOVE DOWN THROUGH x



PRINC. OF WORK-ENERGY

$T_1 = \frac{1}{2}(m_A + m_B)v'^2$

$= \frac{1}{2}(0.5 + 2.5)(1.1437\ m/s)^2$

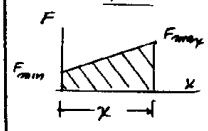
$T_1 = 1.962\ J$

$T_2 = 0$

WORK: GRAVITY $(U_{1-2})_g = (W_A + W_B)x = (0.5\ kg + 2.5\ kg)(9.81\ m/s^2)x$

$= 29.43x$

SPRING $k = 3\ kN/m$



$(U_{1-2})_s = -\frac{1}{2}(F_{min} + F_{max})x$

$= -\frac{1}{2}(m_B g + m_B g + kx)x$

$= -(m_B g x + \frac{1}{2}kx^2)$

$= -(2.5\ kg)(9.81)x - \frac{1}{2}(3000)x^2$

$= -24.525x - 1500x^2$

$U_{1-2} = (U_{1-2})_g + (U_{1-2})_s = 29.43x - 24.525x - 1500x^2 = 4.905x - 1500x^2$

$T_1 + U_{1-2} = T_2$: $1.962 + 4.905x - 1500x^2 = 0$

$x = 0.03784\ m$ $x = 37.8\ mm$ ◀

(b) ENERGY LOSS:

JUST BEFORE IMPACT: $T_0 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(0.5\ kg)(6.862\ m/s)^2 = 11.77\ J$

JUST AFTER IMPACT:

$T_1 = \frac{1}{2}(m_A + m_B)v'^2 = \frac{1}{2}(0.5\ kg + 2.5\ kg)(1.1437\ m/s)^2 = 1.96\ J$

ENERGY LOSS $= T_0 - T_1 = 11.77 - 1.96 = 9.81\ J$ ◀

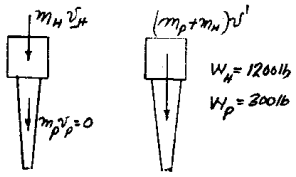
13.168

PHASE I: HAMMER DROPS 3 ft:

$T+V$ = CONSTANT $0 + mg(3\ ft) = \frac{1}{2}m v_H^2$

$v_H^2 = 2(32.2\ ft/s^2)(3\ ft) = 193.2$ $v_H = 13.90\ ft/s$

PHASE II IMPACT ($e=0$) CONS. TOTAL MOMENTUM

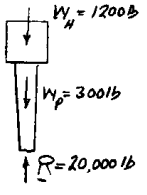


$m_H v_H + m_p v_p = (m_H + m_p)v'$

$\frac{1200}{g}(13.90) + 0 = \frac{1200+300}{g}v'$

$v' = 11.12\ ft/s$

PHASE III HAMMER & PILE MOVE AGAINST GROUND RESISTANCE



x = DISTANCE PILE MOVES ↓ PER BLOW

$T_1 + U_{1-2} = T_2$

$\frac{1}{2}(m_H + m_p)v'^2 + (W_H + W_p - R)x = 0$

$\frac{1}{2}\frac{1200+300}{32.2}(11.12)^2 + (1200+300-20{,}000)x = 0$

$2880 - 18{,}500x = 0$

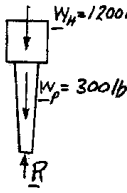
$x = 0.15568\ ft$ $x = 1.868\ in.$ ◀

13.169

PHASE I AND PHASE II SEE SOLUTION OF PROB. 13.168

WE HAVE $v' = 11.12\ ft/s$

PHASE III HAMMER AND PILE MOVE AGAINST GROUND RESISTANCE



PILE IS DRIVEN INTO GROUND

$x = 4\ in. = 0.3333\ ft$

$T_1 + U_{1-2} = T_2$

$\frac{1}{2}(m_H + m_p)v'^2 + (W_H + W_p - R)x = 0$

$\frac{1}{2}\frac{1200+300}{32.2}(11.12)^2 + (1200+300-R)(0.3333) = 0$

$2880 + (1500 - R)(0.3333) = 0$

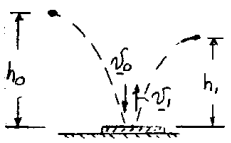
$1500 - R = -8640$

$R = 10{,}140\ lb$ $\underline{R} = 10{,}140\ lb \uparrow$ ◀

13.170

PLATE ON HARD GROUND $v_{PLATE} \equiv 0$

$v_0 = \sqrt{2gh_0}$ $v_1 = \sqrt{2gh_1}$

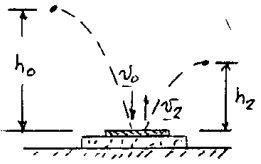


$e = \frac{v_1}{v_0} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$ (1)

$h_1 = e^2 h_0$ (2)

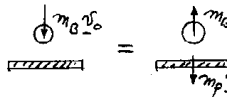
PLATE RESTS ON FOAM-RUBBER MAT

$v_0 = \sqrt{2gh_0}$ $v_2 = \sqrt{2gh_2}$ (3)



v_p = VELOCITY OF PLATE JUST AFTER IMPACT

CONSERVATION OF TOTAL MOMENTUM



$+\downarrow \quad m_B v_0 = m_p v_p - m_B v_2$

$m_B v_2 - m_p v_p = -m_B v_0$ (4)

RELATIVE VELOCITIES $v_p + v_2 = e v_0$ (5)

MULTIPLY (5) BY m_p AND ADD TO (4):

$m_p v_2 + m_p v_2 = -m_B v_0 + e m_p v_0$ (6)

$v_2 = \frac{e m_p - m_B}{m_B + m_p} v_0$ OR: $\frac{v_2}{v_0} = \frac{e m_p - m_B}{m_B + m_p}$

FROM (3): $\left(\frac{v_2}{v_0}\right)^2 = \frac{h_2}{h_0}$ THUS, $\frac{h_2}{h_0} = \left[\frac{e m_p - m_B}{m_B + m_p}\right]^2$ (7)

FROM (2): $h_0 = h_1/e^2$

THUS: $\frac{h_2}{h_1/e^2} = \left[\frac{e m_p - m_B}{m_B + m_p}\right]^2$ $\frac{h_2}{h_1} = \left[\frac{e m_p - m_B}{e(m_B + m_p)}\right]^2$ (8)

GIVEN DATA:

$m_B = 35g = 0.035\ kg$ $m_p = 140g = 0.140\ kg$

$h_1 = 580\ mm$ $h_2 = 160\ mm$

SUBSTITUTE INTO (8): $\frac{160\ mm}{580\ mm} = \left[\frac{e(140g) - 35g}{e(35g + 140g)}\right]^2$

$\left[\frac{140e - 35}{e(175)}\right]^2 = \frac{160}{580}$ $\frac{4e - 1}{5e} = \sqrt{\frac{16}{58}} = 0.52523$

(a) $4e - 1 = 2.6261e$ $e = \frac{1}{1.3739} = 0.7279$ $e = 0.728$ ◀

(b) EQ(2): $h_1 = e^2 h_0$; $580\ mm = (0.7279)^2 h_0$ $h_0 = 1095\ mm$ ◀

13.171

SEE SOLUTION OF PROB 13.170 FOR EQS.(1) THROUGH (7)

GIVEN DATA $m_B = 35\,g$; $h_0 = 600\,mm$; $h_1 = 360\,mm$; $h_2 = 200\,mm$

(a) EQ.(2): $h_1 = e^2 h_0$; $360\,mm = e^2(600\,mm)$

$e^2 = 0.600$ $\quad e = 0.775$

(b) EQ.(7): $\dfrac{h_2}{h_0} = \left[\dfrac{e m_p - m_B}{m_B + m_p}\right]^2$; $\dfrac{200\,mm}{600\,mm} = \left[\dfrac{0.775 m_p - 35}{35 + m_p}\right]^2$

$0.57735 = \dfrac{0.775 m_p - 35}{35 + m_p}$; $20.207 + 0.57735 m_p = 0.775 m_p - 35$

$55.207 = 0.19725 m_p$ $\quad m_p = 279.9\,g$ $\quad m_p = 280\,g$ ◀

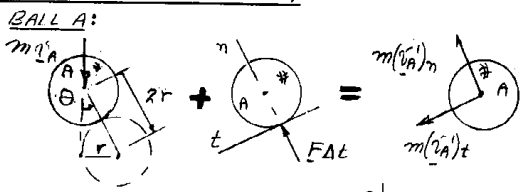
13.172

PHASE I BALL A DROPS $h_A = 200\,mm$

$v_A = \sqrt{2 g h_A} = \sqrt{2(9.81)(0.200)} = 1.9809\ m/s$

PHASE II IMPACT ($e = 1.0$)

BALL A:



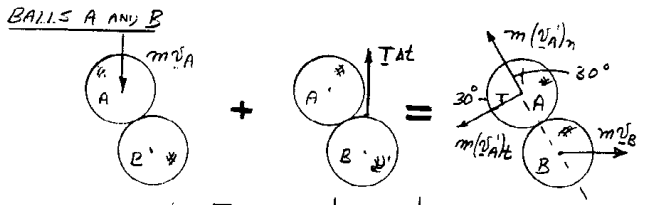
$m\underline{v}_A + \underline{F}\Delta t = m\underline{v}_A'$

COMPONENTS IN t DIRECTION:

$+\swarrow$ $mv_A \sin\theta = m(v_A')_t$ $\quad (v_A')_t = v_A \sin\theta$

BUT $\sin\theta = \frac{r}{2r} = \frac{1}{2}$; $\theta = 30°$; THUS $(v_A')_t = \frac{1}{2} v_A$ (1)

BALLS A AND B

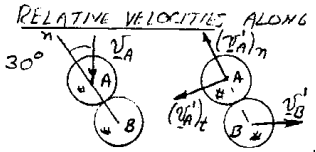


$m\underline{v}_A + \underline{T}\Delta t = m\underline{v}_A' + m\underline{v}_B'$

$\pm$ COMPONENTS: $0 = m(v_A')_n \sin 30° + m(v_A')_t \cos 30° - m v_B'$

BUT $(v_A')_t = \frac{1}{2} v_A$: $0.5(v_A')_n - v_B' + 0.866(v_A')_t = 0$ (2)

RELATIVE VELOCITIES ALONG LINE OF IMPACT: ($e = 1$)



$(v_A')_n - (v_B')_n = (v_B)_n - (v_A)_n$

$(v_A')_n + v_B' \sin 30° = v_A \cos 30°$

$(v_A')_n = 0.866 v_A - 0.5 v_B'$ (3)

SUBSTITUTING FOR $(v_A')_t$ FROM (1) AND FOR $(v_A')_n$ FROM (3) INTO (2),

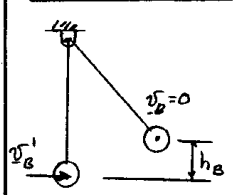
$0.433 v_A - 0.25 v_B' - v_B' + 0.866(\frac{1}{2} v_A) = 0$

$1.25 v_B' = 0.866 v_A$ $\quad \underline{v}_B' = 0.6928 v_A$ ◀

FROM PHASE I, $v_A = 1.9809\ m/s$

THUS $v_B' = 0.6928(1.9809)$ $\quad v_B' = 1.3724\ m/s$ ◀

PHASE III BALL B: CONSERVATION OF ENERGY



$T + V = T + V$

$\frac{1}{2} m_B v_B'^2 + 0 = 0 + m_B g h_B$

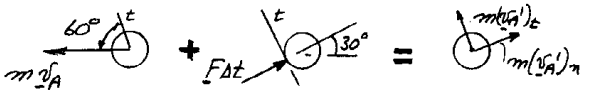
$h_B = \dfrac{v_B'^2}{2g} = \dfrac{(1.3724\ m/s)^2}{2(9.81\ m/s^2)}$

$h_B = 96.0 \times 10^{-3}\ m$ $\quad h_B = 96.0\,mm$ ◀

13.173

PHASE I IMPACT ($e = 0.75$)

SPHERE



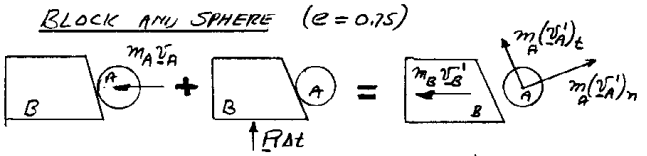
$m\underline{v}_A + \underline{F}\Delta t = m\underline{v}_B$

COMPONENTS IN t DIRECTION:

$+\nwarrow$ $mv_A \cos 60° + 0 = m(v_A')_t$

$(v_A')_t = v_A \cos 60°$ $\quad (v_A')_t = \frac{1}{2} v_A$ (1)

BLOCK AND SPHERE ($e = 0.75$)



$m_A\underline{v}_A + \underline{R}\Delta t = m_A\underline{v}_A' + m_B\underline{v}_B'$

$\pm$ $m_A v_A + 0 = +m_A(v_A')_t \sin 30° - m_A(v_A')_n \cos 30° + m_B v_B'$ (2)

GIVEN DATA:

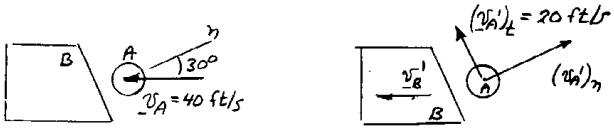
$m_A = (1.5\,lb)/g$; $\quad m_B = (4\,lb)/g$; $\quad v_A = 50\ ft/s$,

EQ (1): $(v_A')_t = \frac{1}{2} v_A = \frac{1}{2}(50) = 25\ ft/s$

EQ(2): $\left(\frac{1.5}{g}\right)(50) = \left(\frac{1.5}{g}\right)(25)(0.5) - \left(\frac{1.5}{g}\right)(v_A')_n (0.866) + \left(\frac{4}{g}\right) v_B'$

$4 v_B' - 1.299 (v_A')_n = 56.25$ (3)

RELATIVE VELOCITIES ALONG LINE OF IMPACT: ($e = 0.75$)



$(v_B')_n + (v_A')_n = e v_A \cos 30°$

$v_B' \cos 30° + (v_A')_n = (0.75)(50)\cos 30°$

$(v_A')_n = 32.476 - v_B'(0.866)$ (4)

SUBSTITUTE FOR $(v_A')_n$ FROM (4) INTO (3):

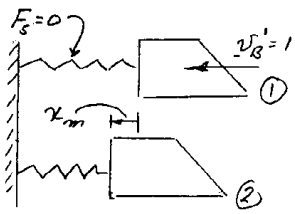
$4 v_B' - 1.299(32.476 - 0.866 v_B') = 56.25$

$4 v_B' - 42.186 + 1.1249 v_B' = 56.25$

$5.1249 v_B' = 98.436$

$\underline{v}_B' = 19.207\ ft/s$ ◀

PHASE II BLOCK MOVES & COMPRESSES SPRING



$W_B = 4\ lb$

$k = 15\ lb/in. = 180\ lb/ft$

$T_1 + V_1 = T_2 + V_2$: $\quad \frac{1}{2}\frac{W_B}{g} v_B'^2 + 0 = 0 + \frac{1}{2} k x_m^2$

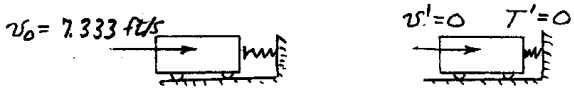
$x_m^2 = \dfrac{W_B v_B'^2}{g k} = \dfrac{(4\,lb)(19.207\ ft/s)^2}{(32.2\ ft/s^2)(180\ lb/ft)} = 0.25461$

$x_m = 0.50458\ ft$ $\quad x_m = 6.06\ in.$ ◀

13.174

(a) ENERGY ABSORBED BY BUMPER

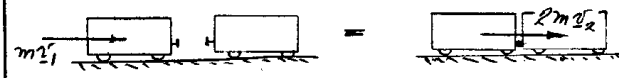
$$v = 5\ \frac{mi}{h} \cdot \frac{h}{3600\ s} \cdot \frac{5280\ ft}{mi} = 7.333\ ft/s$$



$T_0 = \frac{1}{2} m v_0^2$ $T' = 0$ $\Delta E = T_0 - T' = \frac{1}{2} m v_0^2$ (1)

$$\Delta E = \frac{1}{2}\frac{W}{g} v_0^2 = \frac{1}{2}\frac{2500\ lb}{32.2\ ft/s^2}(7.333\ ft/s)^2 \qquad \Delta E = 2088\ ft\cdot lb \blacktriangleleft$$

(b) TOTAL MOMENTUM CONSERVED PLASTIC IMPACT



TOTAL MOMENTUM CONSERVED

$\pm\rightarrow$ $mv_1 = 2mv_2$ $\qquad v_2 = \frac{1}{2} v_1$

ENERGY BEFORE IMPACT: $T_1 = \frac{1}{2} m v_1^2$

ENERGY AFTER IMPACT: $T_2 = \frac{1}{2}(2m) v_2^2 = m v_2^2 = m \frac{v_1^2}{4}$

ENERGY ABSORBED BY BOTH BUMPERS:

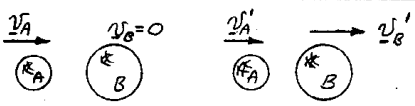
$$(2\Delta E) = T_1 - T_2 = \frac{1}{2} m v_1^2 - \frac{1}{4} m v_1^2 = \frac{1}{4} m v_1^2 \qquad (2)$$

BUT FROM PART (a), EQ(1) WE HAD $\Delta E = \frac{1}{2} m v_0^2$

EQ(2): $2\Delta E = m v_0^2 = \frac{1}{4} m v_1^2$ $\qquad v_1^2 = 4 v_0^2$

$v_1 = 2v_0 = 2(5\ mi/h)$ $\qquad v_1 = 10\ m/h \blacktriangleleft$

*13.175



TOTAL MOMENTUM CONSERVED:

$\pm\rightarrow$ $m_A v_A = m_A v_A' + m_B v_B'$

RELATIVE VELOCITIES: $v_B' - v_A' = e v_A$

SOLVING SIMULTANEOUSLY:

$$v_A' = \frac{m_A - e m_B}{m_A + m_B} v_A \ , \qquad v_B' = \frac{m_A + e m_A}{m_A + m_B} v_A$$

$$T_1 = \frac{1}{2} m_A v_A^2$$

$$T_2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$= \frac{1}{2}\frac{m_A(m_A - e m_B)^2 + m_B(m_A + e m_A)^2}{(m_A + m_B)^2} v_A^2$$

$$= \frac{m_A^3 - 2e m_A^2 m_B + e^2 m_A m_B^2 + m_A^2 m_B + 2 m_A^2 m_B + e^2 m_A^2 m_B}{2(m_A + m_B)^2} v_A^2$$

ENERGY LOSS = $\Delta E = T_1 - T_2$

$$\Delta E = \frac{1}{2} m_A v_A^2 \left[\frac{(m_A + m_B)^2}{(m_A + m_B)^2}\right] - \frac{m_A^3 + e^2 m_A m_B^2 + m_A^2 m_B + e^2 m_A^2 m_B}{2(m_A + m_B)^2} v_A^2$$

$$= \frac{m_A^3 + 2m_A^2 m_B + m_A m_B^2 - m_A^3 - e^2 m_A m_B^2 - m_A^2 m_B - e^2 m_A^2 m_B}{2(m_A + m_B)^2} v_A^2$$

$$= \frac{m_A^2 m_B + m_A m_B^2 - e^2(m_A m_B^2 + m_A^2 m_B)}{2(m_A + m_B)^2} v_A^2$$

(CONTINUED)

*13.175 CONTINUED

$$\Delta E = \frac{(1 - e^2)(m_A m_B)(m_A + m_B)}{2(m_A + m_B)^2} v_A^2$$

$$\Delta E = \frac{1 - e^2}{2} \frac{m_A m_B}{m_A + m_B} v_A^2$$

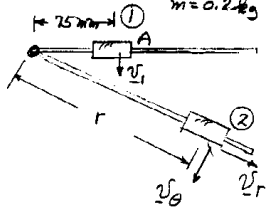
PERCENTAGE OF ENERGY LOST $= 100\ \frac{\Delta E}{T_1}$

$$\%\ \text{LOSS} = 100 \frac{\frac{1 - e^2}{2} \frac{m_A m_B}{m_A + m_B} v_A^2}{\frac{1}{2} m_A v_A^2} = 100(1 - e^2)\left(\frac{m_B}{m_A + m_B}\right)$$

$$\%\ \text{LOSS} = \frac{100(1 - e^2)}{\left(1 + \frac{m_A}{m_B}\right)} \blacktriangleleft$$

13.176

$k = 40\ N/m$ $\qquad m = 0.2\ kg$



POSITION 1 (POINT A)

$r_1 = 75\ mm = 0.075\ m$

$\Delta_1 = 225\ mm - 75\ mm = 150\ mm$

$v_1 = v_A = r_1 \dot{\theta}_0 = (0.075\ m)(12\ rad/s)$

$v_1 = 0.9\ m/s$

$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(0.2\ kg)(0.9\ m/s)^2$

$T_1 = 81 \times 10^{-3}\ J$

$V_1 = \frac{1}{2} k \Delta_1^2 = \frac{1}{2}(40\ N/m)(0.150\ m)^2 = 450 \times 10^{-3}\ J$

POSITION 2 (A GENERAL POSITION)

$\Delta_2 = 0.225\ m - r$

$T_2 = \frac{1}{2}(0.2 kg)(v_r^2 + v_\theta^2)$ $\qquad V_2 = \frac{1}{2} k \Delta_2^2 = \frac{1}{2}(40\ N/m)(0.225 - r)^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$

$$81 \times 10^{-3}\ J + 450 \times 10^{-3}\ J = \frac{1}{2}(0.2)(v_r^2 + v_\theta^2) + \frac{1}{2}(40)(0.225 - r)^2$$

$$v_r^2 + v_\theta^2 = 5.31 + 200(0.225 - r)^2 \qquad (1)$$

CONSERVATION OF ANGULAR MOMENTUM

$m v_1 r_1 = m v_\theta r$

$m(0.9\ m/s)(0.075\ m) = m v_\theta r$ $\qquad v_\theta = \frac{67.5 \times 10^{-3}}{r}$ (2)

SUBSTITUTE FOR v_θ FROM (2) INTO (1)

$$v_r^2 + \left(\frac{67.5 \times 10^{-3}}{r}\right)^2 = 5.31 - 200(0.225 - r)^2$$

AT POINT B $\quad r = 300\ mm = 0.3\ m$

$$v_r^2 + \left(\frac{67.5 \times 10^{-3}}{0.3}\right)^2 = 5.31 - 200(0.225 - 0.3)^2$$

$$v_r^2 + 50.625 \times 10^{-3} = 5.31 - 1.125$$

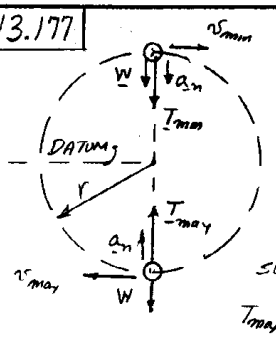
$v_r^2 = 4.134$

$v_r = 2.033\ m/s$ $\qquad v_r = 2.03\ m/s \blacktriangleleft$

EQ.(2): $v_\theta = \frac{67.5 \times 10^{-3}}{r} = \frac{67.5 \times 10^{-3}}{0.3} = 0.225$

$v_\theta = 0.225\ m/s \blacktriangleleft$

13.177



AT TOP OF CIRCLE

$+\uparrow \Sigma F = ma_n$

$T_{min} + W = \frac{W}{g}\frac{v_{min}^2}{r}$ (1)

AT BOTTOM OF CIRCLE

$+\uparrow \Sigma F = ma_n$

$T_{max} - W = \frac{W}{g}\frac{v_{max}^2}{r}$ (2)

SUBTRACT (1) FROM (2)

$T_{max} - W - T_{min} - W = \frac{W}{g}\frac{v_{max}^2 - v_{min}^2}{r}$

$T_{max} - T_{min} = 2W + \frac{W}{g}\frac{v_{max}^2 - v_{min}^2}{r}$ (3)

CONSERVATION OF ENERGY BETWEEN TOP AND BOTTOM

$T+V = T+V$: $\frac{1}{2}\frac{W}{g}v_{min}^2 + Wr = \frac{1}{2}\frac{W}{g}v_{max}^2 - Wr$

$v_{max}^2 - v_{min}^2 = 4gr$

SUBSTITUTE INTO (3):

$T_{max} - T_{min} = 2W + \frac{W}{g}\frac{4gr}{r}$ $\qquad T_{max} - T_{min} = 6W$ ◀

13.178

(a) WORK DONE BY BOY

$\text{WORK} = \frac{1}{2}m v_{\text{RELATIVE}}^2 = \frac{1}{2}(0.6\text{ kg})(5\text{ m/s})^2 = 7.5\text{ J}$ ◀

(b) INCREASE IN KINETIC ENERGY OF STONE

BEFORE THROW: $v_1 = v_{\text{elev}} = 2\text{ m/s}$

$T_1 = \frac{1}{2}m v_1^2 = \frac{1}{2}(0.6\text{ kg})(2\text{ m/s})^2 = 1.2\text{ J}$

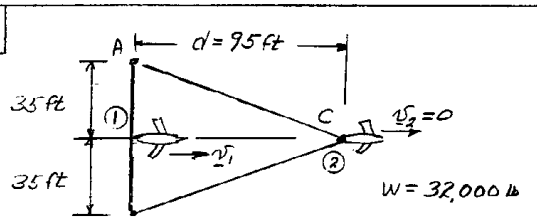
AFTER THROW: $v_2 = 2\text{ m/s} + 5\text{ m/s} = 7\text{ m/s}$

$T_2 = \frac{1}{2}m v_2^2 = \frac{1}{2}(0.6\text{ kg})(7\text{ m/s})^2 = 14.7\text{ J}$

$\Delta T = T_2 - T_1 = 14.7\text{ J} - 1.2\text{ J}$ $\qquad \Delta T = 13.5\text{ J}$ ◀

(c) EXPLANATION: THE INCREASE IN ENERGY FOUND IN (b) INCLUDES BOTH WORK DONE BY BOY AND WORK DONE BY ELEVATOR MOTOR IN MAINTAINING THE CONSTANT SPEED OF THE ELEVATOR WHILE THE BOY IS THROWING THE STONE.

13.179



LENGTH $AC = \sqrt{35^2 + 95^2} = 101.242\text{ ft}$

INCREASE IN TOTAL CABLE LENGTH ABOVE DECK

$\Delta = 2(AC) - AB = 2(101.242\text{ ft}) - 70\text{ ft} = 132.48\text{ ft}$

SINCE CONSTANT TENSION ($Q = 85\text{ kips}$) IS MAINTAINED IN ABOVE-DECK CABLE

$U_{1-2} = -Q\Delta = -(85\times10^3\text{ lb})(132.48\text{ ft}) = -11.261\times10^6\text{ ft·lb}$

PRINCIPLE OF WORK AND ENERGY

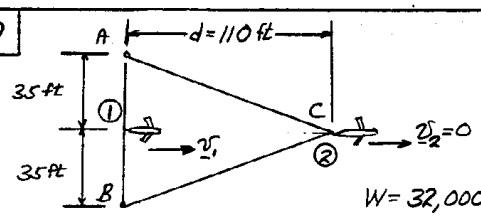
$T_1 + U_{1-2} = T_2$

$\frac{1}{2}\frac{32{,}000}{32.2\text{ ft/s}^2}v_1^2 - 11.261\times10^6\text{ ft·lb} = 0$

$v_1^2 = 22{,}662$

$v_1 = 150.54\text{ ft/s}$ $\qquad v_1 = 102.6\text{ mi/h}$ ◀

13.180



LENGTH $AC = \sqrt{35^2 + 110^2} = 115.433\text{ ft}$

INCREASE IN TOTAL LENGTH OF CABLE ABOVE DECK

$\Delta = 2(AC) - AB = 2(115.433\text{ ft}) - 70\text{ ft} = 160.87\text{ ft}$

SINCE CONSTANT TENSION OF MAGNITUDE Q IS MAINTAINED IN ABOVE-DECK CABLE

$U_{1-2} = -Q\Delta = -Q(160.87\text{ ft})$

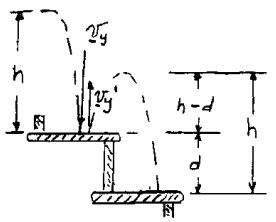
LANDING SPEED: $v_1 = 120\text{ mi/h} = 176\text{ ft/s}$

PRINCIPLE OF WORK AND ENERGY

$T_1 + U_{1-2} = T_2$: $\frac{1}{2}\frac{32{,}000\text{ lb}}{32.2\text{ ft/s}^2}(176\text{ ft/s})^2 - Q(160.87\text{ ft}) = 0$

$Q = 95{,}680\text{ lb}$ $\qquad Q = 95.7\text{ kips}$ ◀

13.181



VERTICAL MOTION (UNIFORMLY ACCELERATED)

$v_y = \sqrt{2gh}$

$v_y' = \sqrt{2g(h-d)}$

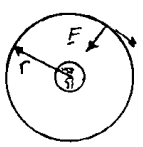
$e = \frac{v_y'}{v_y} = \frac{\sqrt{2g(h-d)}}{\sqrt{2gh}} = \sqrt{\frac{h-d}{h}}$

SOLVING FOR h:

$e^2 h = h - d$ $\qquad h(1-e^2) = d$ $\qquad h = \frac{d}{1-e^2}$ ◀

13.182

(1) FOR CIRCULAR ORBIT OF RADIUS r:

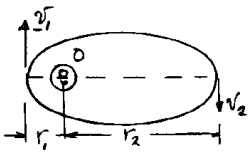


$F = ma_n$

$\frac{GMm}{r^2} = m\frac{v^2}{r}$

$v_{CIRC} = \sqrt{\frac{GM}{r}}$ (1)

(2) ELLIPTIC ORBIT



CONSERVATION OF ANGULAR MOMENTUM

$r_1 m v_1 = r_2 m v_2$ $\qquad v_2 = \frac{r_1}{r_2}v_1$ (2)

CONSERVATION OF ENERGY

$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$ (3)

SUBSTITUTE FOR v_2 FROM (2) INTO (3)

$v_1^2 - \frac{2GM}{r_1} = \left(\frac{r_1}{r_2}v_1\right)^2 - \frac{2GM}{r_2}$

$\left(1 - \frac{r_1^2}{r_2^2}\right)v_1^2 = 2GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\frac{r_2^2 - r_1^2}{r_2^2}v_1^2 = 2GM\frac{r_2 - r_1}{r_1 r_2}$

$v_1^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_2}{r_1}$ $\qquad (v_1)_{ell} = \sqrt{\frac{2GM}{r_1+r_2}\cdot\frac{r_2}{r_1}}$

EXCHANGE SUBSCRIPTS 1 AND 2.

$v_2^2 = \frac{2GM}{r_1 + r_2}\cdot\frac{r_1}{r_2}$ $\qquad (v_2)_{ell} = \sqrt{\frac{2GM}{r_1+r_2}\cdot\frac{r_1}{r_2}}$

(CONTINUED)

13.182 CONTINUED

(3) INCREASE IN VELOCITY ACHIEVED AT A TO MOVE ONTO ELLIPTIC ORBIT

$$\Delta v_A = (v_1)_{ell} - (v_1)_{circ} = \sqrt{\frac{2GM}{r_1+r_2}\cdot\frac{r_2}{r_1}} - \sqrt{\frac{GM}{r_1}}$$

$$\Delta v_A = \sqrt{\frac{GM}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right]$$

SIMILARLY AT B THE DECREASE IN VELOCITY TO MOVE TO CIRCULAR ORBIT IS:

$$\Delta v_B = (v_2)_{circ} - (v_2)_{ell} = \sqrt{\frac{GM}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right]$$

WE NOW WRITE:

$$\frac{\Delta v_B}{\Delta v_A} = \sqrt{\frac{r_1}{r_2}}\cdot\frac{1-\sqrt{\frac{2r_1}{r_1+r_2}}}{\sqrt{\frac{2r_2}{r_1+r_2}} - 1} \qquad (4)$$

GIVEN DATA: $r_1 = 4500$ mi, $r_2 = 8000$ mi

$$\frac{r_1}{r_2} = \frac{4500}{8000} = \frac{9}{16}, \quad \sqrt{\frac{r_1}{r_2}} = \frac{3}{4} = 0.75$$

$$\frac{2r_1}{r_1+r_2} = \frac{9000}{12500} = \frac{18}{25}, \quad \sqrt{\frac{2r_1}{r_1+r_2}} = 0.8485$$

$$\frac{2r_2}{r_1+r_2} = \frac{16000}{12500} = \frac{32}{25}, \quad \sqrt{\frac{2r_2}{r_1+r_2}} = 1.1314$$

EQ.(4): $\frac{\Delta v_B}{\Delta v_A} = (0.75)\frac{1-0.8485}{1.1314-1} = 0.75\,\frac{0.1515}{0.1314} = 0.8648$

IMPULSE AND MOMENTUM IF F IS ENGINE THRUST

$$\Delta v_A = F(\Delta t_A) \qquad \Delta v_B = F(\Delta t_B)$$

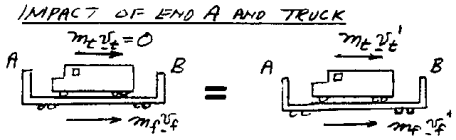
THUS: $\Delta t_B = \Delta t_A \frac{\Delta v_B}{\Delta v_A} = (180\text{ s})(0.8648) = 155.665$

$$\Delta t = 155.7\text{ s}$$ ◀

13.183

IMPACT OF END A AND TRUCK

t = TRUCK
f = FLATCAR
$v_f = 1.4$ m/s



TOTAL MOMENTUM CONSERVED: $m_t v_t + m_f v_f = m_t v_t' + m_f v_f'$

$m_t = 12$ Mg $\quad m_f = 30$ Mg $\quad 0 + (30)(1.4) = 12 v_t' + 30 v_f'$ (1)

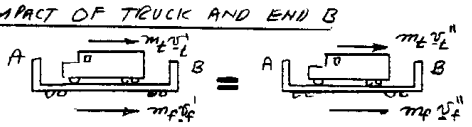
RELATIVE VELOCITIES: $v_t' - v_f' = e(v_f - v_t)$: $\quad v_t' - v_f' = (1)(1.4)$ (2)

SOLVING (1) AND (2) SIMULTANEOUSLY:

$$v_t' = 2.0\text{ m/s} \rightarrow \qquad v_f' = 0.6\text{ m/s} \rightarrow$$ ◀

AFTER THIS IMPACT, THE FLATCAR AND TRUCK MOVE WITH THE ABOVE VELOCITIES UNTIL THE TRUCK STRIKES END B.

IMPACT OF TRUCK AND END B



TOTAL MOMENTUM CONSERVED: $m_t v_t' + m_f v_f' = m_t v_t'' + m_f v_f''$

$(12)(2.0) + (30)(0.6) = (12) v_t'' + (30) v_f''$ (3)

RELATIVE VELOCITIES: $v_f'' - v_t'' = e(v_t' - v_f')$: $\quad v_f'' - v_t'' = (1)(2.0 - 0.6)$ (4)

SOLVING (3) AND (4) SIMULTANEOUSLY:

$$v_t'' = 0 \qquad v_f'' = 1.4\text{ m/s} \rightarrow$$ ◀

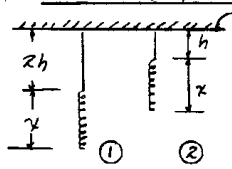
NOTE: THESE VELOCITIES ARE EQUAL TO THE INITIAL VELOCITIES. THE MOTION WILL THUS BE REPEATED INDEFINITELY.

13.184

LET w = WEIGHT OF CHAIN PER UNIT LENGTH. WE OBSERVE THAT THE ELONGATION OF THE RUBBER STRIP UNDER THE LOAD wL IS h.

THUS, $wL = kh$ $\qquad k = \frac{wL}{h}$ (1)

(a) RUBBER STRIP TO BECOME SLACK:



POSITION 1: CHAIN HAS JUST BEEN CUT. $v_1 = 0$ $\quad T_1 = 0$

$$V_1 = -wx\left(2h + \frac{x}{2}\right) + \frac{1}{2}kh^2$$

POSITION 2: RUBBER STRIP HAS BECOME SLACK, v AGAIN IS ZERO $\quad T_2 = 0$

$$V_2 = -wx\left(h + \frac{x}{2}\right) + 0$$

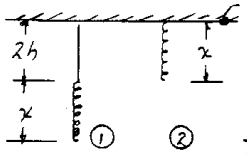
$T_1 + V_1 = T_2 + V_2$: $\quad -wx\left(2h + \frac{x}{2}\right) + \frac{1}{2}kh^2 = -wx\left(h + \frac{x}{2}\right)$

$$x = \frac{1}{2}\frac{kh}{w} \qquad (2)$$

SUBSTITUTE FOR k FROM (1) INTO (2):

$$x = \frac{1}{2}\frac{wL}{h}\cdot\frac{h}{w} \qquad x = \frac{1}{2}L$$ ◀

(b) CHAIN TO TOUCH CEILING:



$T_1 = 0$; $V_1 = -wx\left(2h + \frac{x}{2}\right) + \frac{1}{2}kh^2$

$T_2 = 0$; $V_2 = -wx\left(\frac{x}{2}\right)$

$T_1 + V_1 = T_2 + V_2$

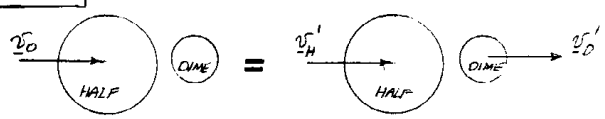
$$0 - wx\left(2h + \frac{x}{2}\right) + \frac{1}{2}kh^2 = 0 - wx\frac{x}{2}$$

$$x = \frac{1}{4}\frac{kh}{w}$$

SUBSTITUTE FOR k FROM (1) INTO (2):

$$x = \frac{1}{4}\frac{wL}{h}\cdot\frac{h}{w} \qquad x = \frac{1}{4}L$$ ◀

13.185



TOTAL MOMENTUM CONSERVED

$$\pm \quad m_H v_0 = m_H v_H' + m_D v_D' \qquad (1)$$

RELATIVE VELOCITIES: $\quad v_D' - v_H' = e v_0 \qquad (2)$

SUBSTITUTING FOR v_0 FROM (1) INTO (2) AND SOLVING FOR e

$$e = \frac{v_D' - v_H'}{v_0} = \frac{v_D' - v_H'}{v_H' + \frac{m_D}{m_H}v_D'} \qquad (3)$$

MOTION AFTER IMPACT

$T_1 + U_{1-2} = T_2$: $\quad \frac{1}{2}mv'^2 - Fx = 0$

BUT, $F = \mu N = \mu W = \mu mg$ THUS:

$$\frac{1}{2}mv'^2 = \mu mgx \qquad v' = \sqrt{2\mu g x}$$

SUBSTITUTING $v_D' = \sqrt{2\mu g x_D}$ AND $v_H' = \sqrt{2\mu g x_H}$ INTO EQ(3) AND REDUCING:

$$e = \frac{\sqrt{x_D} - \sqrt{x_H}}{\sqrt{x_H} + \frac{m_D}{m_H}\sqrt{x_D}} \qquad (4)$$

(CONTINUED)

13.185 CONTINUED

GIVEN DATA: $\frac{m_D}{m_H} = \frac{2.5g}{12.5g} = 0.2$

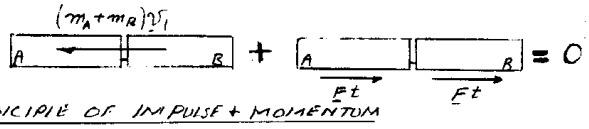
$r_H = 95\ mm \qquad r_D = 480\ mm$

SUBSTITUTE DATA INTO EQ(4):

$$e = \frac{\sqrt{r_D} - \sqrt{r_H}}{\sqrt{r_H} + \frac{m_D}{m_H}\sqrt{r_D}} = \frac{\sqrt{480} - \sqrt{95}}{\sqrt{95} + (0.2)\sqrt{480}} = 0.8608 \qquad e = 0.861$$ ◀

13.186

(a) ENTIRE TRAIN $v_1 = 90\ km/h = 25\ m/s$



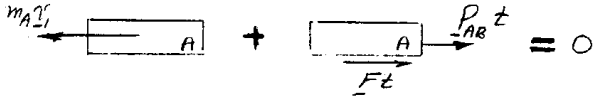
PRINCIPLE OF IMPULSE + MOMENTUM

$$\stackrel{+}{\leftarrow} \quad (m_A+m_B)v_1 - 2Ft = 0$$

$$(25\,Mg + 20\,Mg)(25\ m/s) - 2(30\ kN)t = 0$$

$$t = 18.75\ s$$ ◀

(b) CONSIDER CAR A ALONE:



$$\stackrel{+}{\leftarrow} \quad m_A v_1 - Ft - P_{AB}t = 0$$

$$(25\,Mg)(25\ m/s) - (30\ kN)(18.75\ s) - P_{AB}(18.75\ s) = 0$$

$$67.5 \times 10^3 - P_{AB}(18.75\ s) = 0$$

$$P_{AB} = +3333\ N$$

$$P_{AB} = 3.33\ kN\ T$$ ◀

13.187

PHASE I COLLAR A DROPS 3 ft

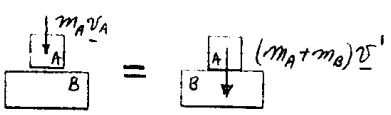
CONSERVATION OF ENERGY

$T + V = \text{CONSTANT}$

$$0 + W_A(3\ ft) = \frac{1}{2}\frac{W_A}{g}v_A^2 + 0$$

$$v_A^2 = 2(32.2)(3) = 193.2 \qquad v_A = 13.90\ ft/s \downarrow$$

PHASE II IMPACT $(e = 0)$



TOTAL MOMENTUM CONSERVED

$$\stackrel{+}{\downarrow} \quad m_A v_A = (m_A+m_B)v'$$

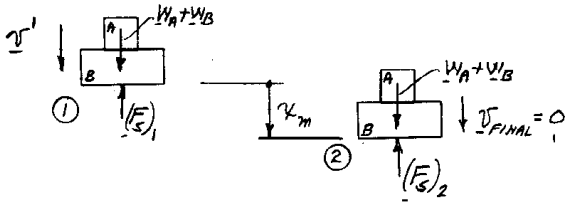
$$\frac{2.5\,lb}{g}(13.90\ ft/s) = \frac{2.5\,lb + 8\,lb}{g}v'$$

$$v' = 3.3094\ ft/s \downarrow$$

(CONTINUED)

13.187 CONTINUED

PHASE III COLLARS A AND B MOVE DOWN THROUGH x_m



PRINCIPLE OF WORK AND ENERGY

$$T_1 = \frac{1}{2}(m_A+m_B)v'^2 = \frac{1}{2}\frac{2.5\,lb + 8\,lb}{32.2\ ft/s^2}(3.3094\ ft/s)^2 = 1.7857\ ft\cdot lb$$

$$T_2 = 0$$

GRAVITY: $(U_{1-2})_g = (W_A+W_B)x_m = (2.5+8)x = 10.5x$

SPRING: $(F_s)_1 = W_B = 8\ lb \qquad k = 5\ lb/in = 60\ lb/ft$

$$(F_s)_2 = (F_s)_1 + kx = 8\ lb + (60\ lb/ft)x_m = 8 + 60x_m$$

$$(F_s)_{AVG} = \frac{1}{2}\left[(F_s)_1 + (F_s)_2\right] = \frac{1}{2}\left[8 + 8 + 60x\right]$$

$$(U_{1-2})_s = -(F_s)_{AVG}x_m = -\frac{1}{2}(16 + 60x_m)x_m$$

$$= -8x_m - 30x_m^2$$

$$U_{1-2} = (U_{1-2})_g + (U_{1-2})_s = 10.5x_m - 8x_m - 30x_m^2$$

$$= 2.5x_m - 30x_m^2$$

(a) PRINCIPLE OF WORK AND ENERGY

$$T_1 + U_{1-2} = T_2$$

$$1.7857 + (2.5x_m - 30x_m^2) = 0$$

$$30x_m^2 - 2.5x_m - 1.7857 = 0$$

$$x_m = 0.2891\ ft$$

$$x_m = 3.47\ in.$$ ◀

(b) ENERGY LOSS

JUST BEFORE IMPACT

$$T_1 = \frac{1}{2}m_A v_A^2 = W_A h = (2.5\,lb)(3\ ft) = 7.50\ ft\cdot lb$$

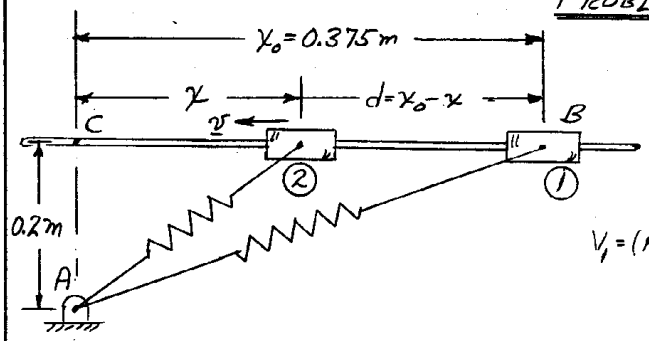
JUST AFTER IMPACT

$$T_2 = \frac{1}{2}(m_A+m_B)v'^2 = 1.7857\ ft\cdot lb \quad \text{(see part a)}$$

$\Delta E = \text{ENERGY LOSS} = T_1 - T_2 = 7.50\ ft\cdot lb - 1.7857\ ft\cdot lb$

$$\Delta E = 5.71\ ft\cdot lb$$ ◀

PROBLEM 13.C1



SPRING CONSTANT: $k_2 = 60\ N/m$

POSITION ①: COLLAR RELEASED FROM REST
$x = x_0$; v_1 = INITIAL VELOCITY = 0, $T_1 = 0$
ELONGATION OF SPRING = AB − 0.125 m
$\Delta_0 = \sqrt{x_0^2 + 0.2^2} - 0.125\,m$

$$V_1 = (PE)_{x=x_0} = \frac{1}{2}k\Delta_0^2 = \frac{1}{2}(60\ N/m)\left[\sqrt{x_0^2+0.2^2} - 0.125\right]^2$$

POSITION ②: v = VELOCITY $T_2 = \frac{1}{2}mv^2$
$\Delta = \sqrt{x^2+0.2^2} - 0.125\,m$

$$V_2 = (PE)_{x=x} = \frac{1}{2}k\Delta^2 = \frac{1}{2}(60\ N/m)\left[\sqrt{x^2+0.2^2} - 0.125\right]^2$$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

$$0 + (PE)_{x=x_0} = \frac{1}{2}mv^2 + (PE)_{x=x}$$

$$v = \sqrt{\frac{2}{m}\left[PE_{x=x_0} - PE_{x=x}\right]}$$ ◁

KINEMATICS: CONSIDER MOTION AS COLLAR MOVES DISTANCE Δx FROM
$x = x_i$ WHERE $v = v_i$
$x = x_{i+1} = x_i + \Delta x$ WHERE $v = v_{i+1}$

IF WE ASSUME ACCELERATION IS CONSTANT OVER Δt, THE AVERAGE VELOCITY IS: $v_{AVG} = \frac{1}{2}(v_i + v_{i+1})$ ◁

AND, THE TIME REQUIRED FOR COLLAR TO TRAVEL Δx IS: $\Delta t = \frac{\Delta x}{v_{AVG}}$ ◁

WE SUM Δt TO FIND TIME REQUIRED FOR COLLAR TO REACH POINT C.

```
10 '*****************************************************************
20 '*                      PROBLEM 13.C1                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '              Definition of Symbols
70 '
80 '               T = time   (seconds)
90 '               X = position coordinate   (meters)
100 '             X0 = initial value of X
110 '              D = X0 - X = distance traveled by collar  (meters)
120 '              V = velocity   (m/s)
130 '             DX = decrement of X
140 '            VIN = value of v at beginning of interval DX
150 '            VFL = value of v at end of interval DX
160 '            VAV = average value of v over interval DX
170 '         FNPE(X) = potential energy as a function of x
180 '              Y = counter
190 '            NBR = number of decrements in a printing interval
200 '
210 '       Given Data:
220 '
230       LET X0 = .375
240       LET NBR = 5
250       LET MASS = 1.8
260       LET DX = .005
270 '
280       PRINT USING "Initial value of X:  X0 = CB = #.### meters";X0
290       PRINT USING "Decrement of X used:  DX = #.### meters";DX
300       PRINT USING "Interval used for printing = #.### meters";DX*NBR
310       PRINT
320       PRINT USING "    !       "";"D","x","t","v"
330       PRINT "  meters      meters      seconds       m/s"
```

CONTINUED

```
C*****************************************************************
C*                      PROBLEM 13.C1                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C                T = time   (seconds)
C                X = position coordinate   (meters)
C               X0 = initial value of X    (meters)
C                D = X0 - X = distance traveled by collar   (meters)
C                V = velocity    (m/s)
C               DX = decrement of X
C              VIN = value of V at beginning of interval DX
C              VFL = value of V at end of interval DX
C              VAV = average value of V over interval DX
C           FNPE(X) = potential energy as a function of X
C                Y = counter
C              NBR = number of decrements in a printing interval
C
C           Given DATA :
C
      REAL NBR, MASS
      FNPE(X) = 0.5*60.*(SQRT(X**2 +0.2**2) - 0.125)**2
      X0 = 0.375
      NBR = 5.
      MASS = 1.8
      DX = 0.005
C
      WRITE (*,10) X0
   10 FORMAT (/,7X,'Initial value of X:   X0 = CB = ',F5.3,' meters')
      WRITE (*,11) DX
   11 FORMAT (7X,'Decrement of X used:       DX = ',F5.3,' meters')
      WRITE (*,12) DX*NBR
   12 FORMAT (7X,'Interval used for printing = ',F5.3,' meters')
      WRITE (*,13)
   13 FORMAT (/,T7,'D',T24,'X',T40,'T',T57,'V')
      WRITE (*,14)
   14 FORMAT (T5,'meters',T22,'meters',T37,'seconds',T56,'m/s')
      WRITE (*,'(/,A\)')
```

CONTINUED

PROBLEM 13.C1 CONTINUED

BASIC VERSION (CONTINUED)

```
340       PRINT
350       DEF FNPE(X) = .5*60*(SQR(X^2 + .2^2) - .125)^2
360 '
370 '       Initial Conditions
380 '
390       LET X = X0
400       LET T = 0
410       LET VIN = 0
420       LET Y = 0
430 '
440       WHILE Y <= X0/DX
450          IF INT(Y/NBR) = Y/NBR THEN 460 ELSE 470
460          PRINT USING "  #.###      "; X0-X, X, T, VIN
470          Y = Y + 1
480          X = X0 - Y * DX
490          LET VFL = SQR((2/MASS)*(FNPE(X0) - FNPE(X)))
500          LET VAV = .5 * (VIN + VFL)
510          LET T = T + (DX/VAV)
520          LET VIN = VFL
530       WEND
540       END
```

```
Initial value of X:  X0 = CB = 0.375 meters
Decrement of X used:  DX = 0.005 meters
Interval used for printing = 0.025 meters
```

D meters	x meters	t seconds	v m/s
0.000	0.375	0.000	0.000
0.025	0.350	0.076	0.649
0.050	0.325	0.108	0.897
0.075	0.300	0.134	1.073
0.100	0.275	0.155	1.208
0.125	0.250	0.175	1.315
0.150	0.225	0.194	1.402
0.175	0.200	0.211	1.473
0.200	0.175	0.228	1.530
0.225	0.150	0.244	1.575
0.250	0.125	0.259	1.609
0.275	0.100	0.275	1.636
0.300	0.075	0.290	1.655
0.325	0.050	0.305	1.667
0.350	0.025	0.320	1.675
0.375	0.000	0.335	1.677

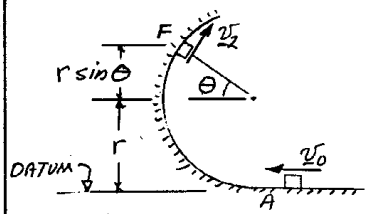
FORTRAN VERSION (CONTINUED)

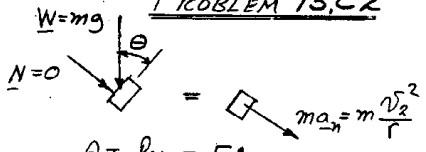
```
C
C     Initial Conditions
C
      X = X0
      T = 0.
      VIN = 0.
      Y = 0.
C
  450 IF (Y.LE.(X0/DX)) THEN
         IF (INT(Y/NBR).EQ.(Y/NBR)) THEN
            GOTO 470
         ELSE
            GOTO 480
         ENDIF
  470    WRITE (*,20) X0-X,X,T,VIN
   20    FORMAT (3X,F5.3,12X,F5.3,11X,F5.3,12X,F5.3)
  480    Y = Y + 1
         X = X0 - Y * DX
         VFL = SQRT((2./MASS)*(FNPE(X0) - FNPE(X)))
         VAV = 0.5*(VIN+VFL)
         T = T + (DX/VAV)
         VIN = VFL
      ELSE
         GOTO 30
      ENDIF
      GOTO 450
   30 CONTINUE
      END
```

```
Initial value of X:   X0 = CB =  .375 meters
Decrement of X used:       DX =  .005 meters
Interval used for printing =  .025 meters
```

D meters	X meters	T seconds	V m/s
.000	.375	.000	.000
.025	.350	.076	.649
.050	.325	.108	.897
.075	.300	.134	1.073
.100	.275	.155	1.208
.125	.250	.175	1.315
.150	.225	.194	1.402
.175	.200	.211	1.473
.200	.175	.228	1.530
.225	.150	.244	1.575
.250	.125	.259	1.609
.275	.100	.275	1.636
.300	.075	.290	1.655
.325	.050	.305	1.667
.350	.025	.320	1.675
.375	.000	.335	1.677

PROBLEM 13.C2





AT POINT F:

$+\nwarrow \Sigma F = ma_n$

$mg\sin\theta = m\frac{v_2^2}{r}$

$v_2^2 = gr\sin\theta$ ◁

CONSERVATION OF ENERGY (BETWEEN A AND F)

$T_A + V_A = T_F + V_F : \quad \frac{1}{2}mv_0^2 + 0 = mg(r + r\sin\theta) + \frac{1}{2}mv_2^2$

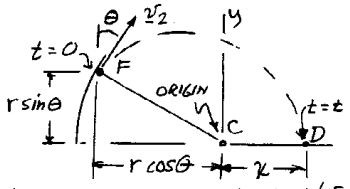
SUBSTITUTE $v_2^2 = gr\sin\theta$:

$\frac{1}{2}mv_0^2 = mgr(1+\sin\theta) + \frac{1}{2}mgr\sin\theta$

$\frac{1}{2}v_0^2 = gr\left(1 + \frac{3}{2}\sin\theta\right)$

$\sin\theta = \frac{1}{3}\left(\frac{v_0^2}{gr} - 2\right)$

MOTION FROM F TO D:



$(v_2)_x = v_2\sin\theta$

$(v_2)_y = v_2\cos\theta$

$a_x = 0$

$a_y = -g$

VERTICAL MOTION: $y = y_0 + (v_2)_y t + \frac{1}{2}a_y t^2$

AT D: $y = 0 = r\sin\theta + (v_2\cos\theta)t - \frac{1}{2}gt^2$

$t^2 - 2\frac{v_2}{g}\cos\theta\, t - 2\frac{r}{g}\sin\theta = 0 \qquad t = \frac{1}{2}\left(-b + \sqrt{b^2 - 4c}\right)$ ◁

(b = coefficient of t, c = constant term)

HORIZONTAL MOTION: AT D, $t = t$

$x = x_0 + (v_2)_x t \;; \qquad x = v_2 t\sin\theta - r\cos\theta$ ◁

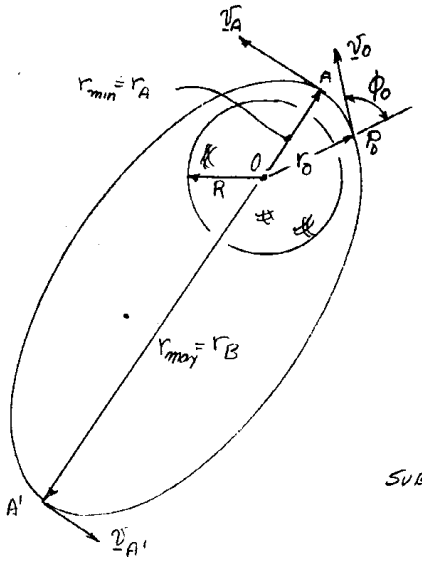
```
10 '*************************************************************
20 '*                      PROBLEM 13.C2                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '              R = radius of loop
90 '            VO2 = (VO)^2 square of initial speed of package
100 '             J = coefficient of G*R in (V0)^2 = J*G*R
110 '            TH = angle theta defining point where
120 '                 package leaves the loop  (degrees)
130 '            V2 = speed of package as it leaves loop
140 '             T = time after package has left the loop  (seconds)
150 '             X = distance from C to D where package hits shelf
160 '
170       'To obtain results in terms of G and R:
180         LET G = 1
190         LET R = 1
200 '
210     PRINT " J            X/R          theta  ":PRINT " "
220     K = 4*ATN(1)/180
230     FOR J = 2 TO 5 STEP .2
240        VO2 = J*G*R
250           'Calculate value of theta.
260        S = (1/3)*(VO2/(G*R)-2)
270        TH = (ATN(S/SQR(1-S^2)))/K
280           'Speed as package leaves the loop.
290        V2 = SQR(G*R*SIN(K*TH))
300           'Motion from F to D: T = 0 at F; calculate T and X at D.
310        B = -2*V2*COS(K*TH)/G: C = -2*R*SIN(K*TH)/G
320        T = .5*(-B +SQR(B^2 -4*C))
330        X = V2*T*SIN(K*TH) - R*COS(K*TH)
340        PRINT USING "#.#          ##.###          ##.#";J,X,TH
350     NEXT
```

```
 J            X/R          theta

2.0          -1.000         0.0
2.2          -0.986         3.8
2.4          -0.943         7.7
2.6          -0.872        11.5
2.8          -0.774        15.5
3.0          -0.649        19.5
3.2          -0.500        23.6
3.4          -0.329        27.8
3.6          -0.137        32.2
3.8           0.073        36.9
4.0           0.296        41.8
4.2           0.530        47.2
4.4           0.767        53.1
4.6           1.002        60.1
4.8           1.225        69.0
5.0           1.414        90.0
```

```
C*************************************************************
C*'                      PROBLEM 13.C2                        *
C*                      \ FORTRAN VERSION                    *
C*************************************************************
C
C           Definition of Symbols
C
C                R = radius of loop
C              VO2 = (V0)**2 = square of initial speed of package
C                J = coefficient of G*R in (VO)**2 = J*G*R
C               TH = angle theta defining point where
C                    package leaves the loop (degrees)
C               V2 = speed of package as it leaves loop
C                T = time after package has left loop (seconds)
C                X = distance from C to D where package hits shelf
C
C          To obtain results in terms of G and R:
      REAL K, J
      G = 1.
      R = 1.
      WRITE (*,'(/,2X,A\)') '      J              X/R            theta'
      WRITE (*,'(/,A\)')
C
      K = 4.*ATAN(1.)/180.
      J = 2.
      STEP = 0.2
C
   10 VO2 = J*G*R
C          Calculate value of theta.
      TH = (ASIN((VO2/(G*R) - 2.)/3.))/K
C          Speed as package leaves the loop.
      V2 = SQRT(G*R*SIN(K*TH))
C          Motion from F to D: T = 0 at F; calculate T and X at D
      B = -2.*V2*COS(K*TH)/G
      C = -2.*R*SIN(K*TH)/G
      T = 0.5*(-B + SQRT(B**2 - 4.*C))
      X = V2*T*SIN(K*TH) - R*COS(K*TH)
      WRITE (*,20) J,X,TH
   20 FORMAT (7X,F3.1,11X,F6.3,11X,F4.1)
      J = J + STEP
      IF (J.LT.5.) GOTO 10
   30 WRITE (*,'(/,A\)')
      END
```

```
          J            X/R          theta

        2.0          -1.000           .0
        2.2           -.986          3.8
        2.4           -.943          7.7
        2.6           -.872         11.5
        2.8           -.774         15.5
        3.0           -.649         19.5
        3.2           -.500         23.6
        3.4           -.329         27.8
        3.6           -.137         32.2
        3.8            .073         36.9
        4.0            .296         41.8
        4.2            .530         47.2
        4.4            .767         53.1
        4.6           1.002         60.1
        4.8           1.225         69.0
        5.0           1.414         90.0
```

PROBLEM 13.C3



AT BURNOUT: VELOCITY $\underline{v}_0$ FORMS ANGLE ϕ_0 WITH VERTICAL.

CONSERVATION OF ANGULAR MOMENTUM ABOUT O.

$$(m v_0 \sin\phi_0) r_0 = m v_A r_A$$

$$v_A = \frac{r_0 v_0 \sin\phi_0}{r_A} \qquad (1)$$

CONSERVATION OF ENERGY: FROM P_0 TO A

$$T_{P_0} + V_{P_0} = T_A + V_A$$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \frac{1}{2} m v_A^2 - \frac{GMm}{r_A}$$

SUBSTITUTING FOR v_A FROM (1): $v_0^2 - 2\frac{GM}{r_0} = \frac{r_0^2 \sin^2\phi_0}{r_A^2} v_0^2 - 2\frac{GM}{r_A}$

$$\sin^2\phi_0 \left(\frac{r_0}{r_A}\right)^2 - \frac{2GM}{r_0 v_0^2}\left(\frac{r_0}{r_A}\right) + \left(\frac{2GM}{r_0 v_0^2} - 1\right) = 0$$

THIS IS A QUADRATIC IN (r_0/r_A). LET $J = r_0/r_A$.

$a J^2 + b J + c = 0$; WHERE $\begin{cases} a = \sin^2\phi_0 \\ b = -(2GM)/(r_0 v_0^2) \\ c = ABS(b) - 1 \end{cases}$

AND, $J_{1,2} = \frac{1}{2a}\left(-b \mp \sqrt{b^2 - 4ac}\right)$ (TWO VALUES)

$r_{max} = r_0/J_1$; $h_{max} = r_{max} - R$. $r_{min} = r_0/J_2$; $h_{min} = r_{min} - R$

```
10 '*****************************************************************
20 '*                        PROBLEM 13.C3                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '            R = radius of the earth   (meters)
90 '            H = altitude at engine burnout   (meters)
100 '          R0 = R + H = initial distance of the satellite
110 '                       from the center of the earth
120 '          V0 = satellite speed at burnout   (m/s)
130 '         PHI = angle formed by V0 and the vertical  (degrees)
140 '   HMAX,HMIN = maximum, minimum altitude of orbit  (km)
150 '                satellite from the center of the earth
160 '            J = R0/MAX or (J = R0/MIN) = ratio of initial and
170 '                and extreme value of the distance of the
180 '                satellite from the center of the earth.
190 '       A,B,C = coefficient in quadratic folumla
200 '
210   R = 6.37*10^6
220   H = 2.4*10^6
230   V0 = 8100
240   G = 9.810001
250   GM = G*R^2
260   R0 = R + H
270
280   K = 4*ATN(1)/180
290   PRINT "  PHI          H min             H max   "
300   PRINT "degrees      kilometers        kilometers  ":PRINT
```

CONTINUED

```
C*****************************************************************
C*'                        PROBLEM 13.C3                          *
C*                         FORTRAN VERSION                        *
C*****************************************************************
C
C           Definition of Symbols
C
C           R = radius of earth  (meters)
C           H = altitude at engine burnout     (meters)
C          R0 = R + H = initial distance of the satellite from
C                                    the center of the earth
C          V0 = satellite speed at burnout
C         PHI = angle formed by V0 and the vertical   (degrees)
C   HMAX,HMIN = max and min distance of the satellite from
C                                    the center of the earth
C           J = R0/MAX of (J = R0/MIN) = ratios of initial and
C                          extreme value of the distance of the
C                          satellite from the center of the earth
C       A,B,C = coefficient in quadratic formula
C
C
      REAL J, K, J1, J2, MAX, MIN

C             PHI = Angle formed by VO and the vertical
C       HMAX,HMIN = Maximum and Minimum altitude of orbit (km.)
C               J = R0/RA   Where RA is max or min,distance from
C                   of satellite from center of earth.
C
      R = 6.37*10**6
      H = 2.4*10**6
      V0 = 8100
      G = 9.810001
      GM = G*R**2
      R0 = R + H
C
      K = 4.*ATAN(1.)/180.
      WRITE (*,'(/,2X,2A\)') '  PHI              H min',
     +'              H max'
      WRITE (*,'(/,2X,2A\)') 'degrees          kilometers',
     +'          kilometers'
      WRITE (*,'(/,A\)')
```

CONTINUED

PROBLEM 13:C3 (CONTINUED)

BASIC VERSION CONTINUED

```
310 '
320   FOR PHI = 60 TO 120       STEP 5
330      A = (SIN(K*PHI))^2
340      B = -(2*GM)/(R0*V0^2)
350      C = ABS(B) - 1
360          'Solve quadratic in J for two values
370      J1 = (-B-SQR(B^2-4*A*C))/(2*A)
380      J2 = (-B+SQR(B^2-4*A*C))/(2*A)
390          'Calculate HMAX in kilometers: let MAX = largest distance
400          'of satellite from center of the earth.
410      MAX = R0/J1
420      HMAX = (MAX - R)/1000
430          'Calculate HMIN in kilometers: let MIN = smallest distance
440          ' of satellite from center of the earth.
450      MIN = R0/J2
460      HMIN = (MIN - R)/1000
470          'Check whether orbit is achieved
480      WHILE HMIN < 300
490         PRINT USING " ###";PHI,
500         PRINT "              *                 *"
510         GOTO 550
520      WEND
530      PRINT USING " ###          ";PHI,
540      PRINT USING "#####             ";HMIN,HMAX
550   NEXT
560     PRINT: PRINT "      *  For these cases:  HMIN < 300 km.
570     PRINT "         A permanent orbit is not achieved.
```

PHI degrees	H min kilometers	H max kilometers
60	*	*
65	*	*
70	896	17997
75	1504	17389
80	1983	16909
85	2293	16600
90	2400	16493
95	2293	16600
100	1983	16909
105	1504	17389
110	896	17997
115	*	*
120	*	*

* For these cases: HMIN < 300 km.
A permanent orbit is not achieved.

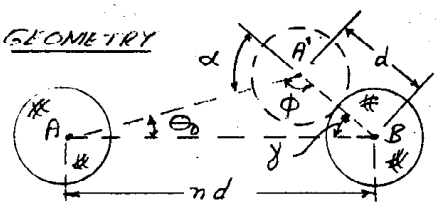
FORTRAN VERSION CONTINUED

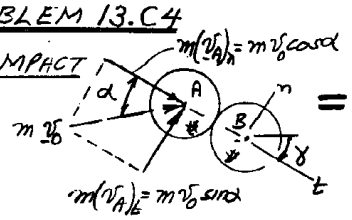
```
C
      DO 450 I = 60,120,5
         PHI = I
         A = (SIN(K*PHI))**2
         B = -(2.*GM)/(R0*V0**2)
         C = ABS(B) - 1.
C             Solve quadratic in J for two values
         J1 = (-B-SQRT(B**2 -4.*A*C))/(2.*A)
         J2 = (-B+SQRT(B**2 -4.*A*C))/(2.*A)
C             Calculate HMAX in kilometers: let MAX = largest
C              distance of satellite from center of the earth.
         MAX = R0/J1
         HMAX = (MAX - R)/1000.
C             Calculate HMIN in kilometers: let MIN = smallest
C              distance of satellite from center of the earth.
         MIN = R0/J2
         HMIN = (MIN - R)/1000.
C             Check whether orbit is achieved
  380    IF (HMIN.LT.300.) THEN
            WRITE (*,20) PHI
    20      FORMAT (4X,F4.0,17X,'*',19X,'*')
            GOTO 450
         ENDIF
         WRITE (*,30) PHI,HMIN,HMAX
    30   FORMAT (4X,F4.0,12X,F6.0,14X,F6.0)
  450 CONTINUE
      WRITE (*,'(/,7X,A\)') '*  For these cases:  HMIN < 300 km.'
      WRITE (*,'(/,7X,A\)') '   A permanent orbit is not achieved.'
      END
```

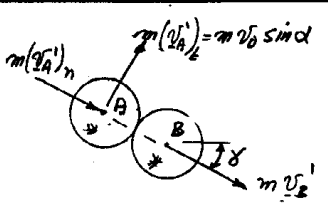
PHI degrees	H min. kilometers	H max. kilometers
60.	*	*
65.	*	*
70.	896.	17997.
75.	1504.	17389.
80.	1983.	16909.
85.	2293.	16600.
90.	2400.	16493.
95.	2293.	16600.
100.	1983.	16909.
105.	1504.	17389.
110.	896.	17997.
115.	*	*
120.	*	*

* For these cases: HMIN < 300 km.
A permanent orbit is not achieved.

PROBLEM 13.C4







LAW OF SINES:

$$\frac{\sin\phi}{nd} = \frac{\sin\theta_0}{d}\ ;\quad \sin\phi = n\sin\theta_0 \quad \triangleleft$$

$+\searrow$ components: $mv_0\cos\alpha = m(v_A')_n + mv_B'$ ①

$$e = \frac{v_B' - (v_A')_n}{v_0\cos\alpha}\ ;\quad v_B' - (v_A')_n = ev_0\cos\alpha \quad ②'$$

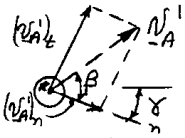
SOLVE EQS. ① AND ② FOR $(v_A')_n$:

$$(v_A')_n = \tfrac{1}{2}(1-e)v_0\cos\alpha \quad \nwarrow\gamma \quad \triangleleft$$

$$(v_A)_t = v_0\sin\alpha \quad \swarrow\gamma \quad \triangleleft$$

$$v_A = \sqrt{(v_A)_t^2 + (v_A')_n^2} \quad \triangleleft$$

DIRECTION OF $\underline{v}_A'$:



$$\tan\beta = \frac{(v_A')_t}{(v_A')_n} \quad \triangleleft$$

$$\psi = \beta - \gamma \quad \triangleleft$$

```
10 '*************************************************************
20 '*'                    PROBLEM 13.C4                          *
30 '*                    BASIC VERSION                          *
40 '*************************************************************
50 '
60 '           Definition of Symbols
70 '
80 '            V0 = initial speed of ball A   (m/s)
90 '            VA = speed of ball A after the impact  (m/s)
100 '       VAN,VAT = normal and tangential components of VA  (m/s)
110 '            VB = speed of ball B after impact   (m/s)
120 '
130 '            TH = angle between V0 and line AB   (degrees)
140 '           PSI = angle between VA and line AB   (degrees)
150 '         GAMMA = angle between VB and line AB   (degrees)
160 '    ALPHA,BETA = angles defined in sketch above
170 '
180 '             N = number of diameters between centers of A and B
190 '             E = coefficient of restitution between A and B
200 '
210 '      Data
220   V0 = 4:   N = 10
230   DATA 0.95, 0.90, 0.70
240   READ E
250 '
260   PI = 4*ATN(1)
270    K = 4*ATN(1)/180
280 '
290   PRINT
300   PRINT USING "Case with coefficient of restitution E = #.##";E
310   PRINT
320   PRINT " THETA       VA          PSI",
330   PRINT "   (PSI = final direction of ball A)"
340   PRINT "degrees     m/s        degrees":PRINT
350 '
360   FOR TH = 0 TO .5 STEP .05
370      GOSUB 480
380   NEXT
390   PRINT
400   FOR TH = .5 TO 5.5 STEP .5
410      GOSUB 480
420   NEXT
430   GOTO 240
440 '
450          'Subroutine to compute magnitude and direction
460          '     of the velocity of ball A after impact.
470 '
480      SINPHI = N*SIN(K*TH)       'sine of the angle PHI
490      PHI = (ATN(SINPHI/SQR(1-SINPHI^2)))/K
500 '        Since PHI must be greater than 90 degrees,we write
510      PHI = 180 - PHI
520      ALPHA = 180 - PHI          'ALPHA is supplement of PHI
530      VAN = .5*(1 - E)*V0*COS(K*ALPHA)
540      VAT = V0*SIN(K*ALPHA)
550      VA = SQR(VAT^2 + VAN^2)
560      BETA = (ATN(VAT/VAN))/K
570      GAMMA = 180 - TH - PHI
580      PSI = BETA - GAMMA
590      PRINT USING " #.##       ";TH,
600      PRINT USING "#.####       ##.##";VA,PSI
610    RETURN
```

CONTINUED

```
C*************************************************************
C*                    PROBLEM 13.C4                          *
C*                  FORTRAN VERSION                          *
C*************************************************************
C
C           Definition of Symbols
```

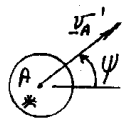
SEE LISTING AT LEFT

```
      REAL N, K
      V0 = 4.
      N = 10.
      E = 0.95
      LOOP = 1
      PI = 4.*ATAN(1.)
      K = 4.*ATAN(1.)/180.
C
  280 WRITE (*,10) E
   10 FORMAT (///,5X,'Case with coefficient of restitution  E = ',F4.2)
      WRITE (*,'(/,2X,A\)') ' THETA              VA                PSI
     +(PSI = final direction of ball A)'
      WRITE (*,'(/,2X,A\)') 'degrees             m/s              degrees'
      WRITE (*,'(/,A\)')
C
      TH = 0.
      STEP = 0.05
   20 CALL BALL(N,K,TH,PHI,V0,E)
      TH = TH + STEP
      IF (TH.LT.0.5) GOTO 20
      WRITE(*,'(/,A\)')
C
      TH = 0.5
      STEP = 0.5
   30 CALL BALL(N,K,TH,PHI,V0,E)
      TH = TH + STEP
      IF (TH.LE.5.5) GOTO 30
C
      LOOP = LOOP + 1
      IF (LOOP.EQ.2) THEN
          E = 0.9
          GOTO 280
      ENDIF
      IF (LOOP.EQ.3) THEN
          E = 0.7
          GOTO 280
      ENDIF
C
      WRITE (*,'(/,A\)')
      END
C               Subroutine to compute magnitude and direction
C                    of the velocity of ball A after impact.
      SUBROUTINE BALL (N,K,TH,PHI,V0,E)
      REAL K, N

      PHI = (ASIN(N*SIN(K*TH)))/K
C           Since PHI must be greater than 90 degrees, we write
      PHI = 180.- PHI
C           Note that ALPHA is supplement of PHI
      ALPHA = 180.- PHI
      VAN = 0.5*(1.- E)*V0*COS(K*ALPHA)
      VAT = V0*SIN(K*ALPHA)
      VA = SQRT(VAT**2 + VAN**2)

      BETA = (ATAN(VAT/VAN))/K
      GAMMA = 180.- TH - PHI
      PSI = BETA - GAMMA
      WRITE (*,11) TH,VA,PSI
   11 FORMAT (3X,F4.2,13X,F6.4,13X,F5.2)
      RETURN
      END
```

CONTINUED

PROBLEM 13.C4 CONTINUED

PROGRAM RUN
BASIC VERSION



Case with coefficient of restitution E = 0.95

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
0.00	0.1000	0.00	
0.05	0.1059	18.79	
0.10	0.1219	34.02	
0.15	0.1448	44.98	
0.20	0.1717	52.61	
0.25	0.2011	57.96	
0.30	0.2320	61.81	
0.35	0.2639	64.63	
0.40	0.2965	66.74	
0.45	0.3296	68.34	
0.50	0.3630	69.57	
1.00	0.7050	72.92	
1.50	1.0515	71.06	
2.00	1.3991	67.73	
2.50	1.7471	63.69	
3.00	2.0952	59.11	
3.50	2.4432	54.02	
4.00	2.7912	48.30	
4.50	3.1390	41.69	
5.00	3.4866	33.55	
5.50	3.8339	21.65	

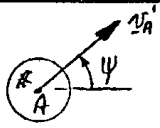
Case with coefficient of restitution E = 0.90

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
0.00	0.2000	0.00	
0.05	0.2030	9.45	
0.10	0.2118	18.35	
0.15	0.2257	26.29	
0.20	0.2438	33.14	
0.25	0.2653	38.89	
0.30	0.2894	43.66	
0.35	0.3155	47.60	
0.40	0.3432	50.85	
0.45	0.3721	53.54	
0.50	0.4019	55.78	
1.00	0.7253	65.20	
1.50	1.0647	65.88	
2.00	1.4085	63.93	
2.50	1.7540	60.75	
3.00	2.1004	56.79	
3.50	2.4471	52.16	
4.00	2.7939	46.83	
4.50	3.1408	40.55	
5.00	3.4876	32.75	
5.50	3.8343	21.22	

Case with coefficient of restitution E = 0.70

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
0.00	0.6000	0.00	
0.05	0.6010	2.88	
0.10	0.6040	5.74	
0.15	0.6089	8.55	
0.20	0.6157	11.31	
0.25	0.6243	13.98	
0.30	0.6347	16.57	
0.35	0.6468	19.04	
0.40	0.6605	21.41	
0.45	0.6756	23.65	
0.50	0.6922	25.78	
1.00	0.9145	40.71	
1.50	1.1965	47.38	
2.00	1.5050	49.64	
2.50	1.8264	49.44	
3.00	2.1550	47.72	
3.50	2.4878	44.86	
4.00	2.8232	41.01	
4.50	3.1603	36.06	
5.00	3.4986	29.54	
5.50	3.8376	19.52	

PROGRAM RUN
FORTRAN VERSION



Case with coefficient of restitution E = .95

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
.00	.1000	.00	
.05	.1059	18.79	
.10	.1219	34.02	
.15	.1448	44.98	
.20	.1717	52.61	
.25	.2011	57.96	
.30	.2320	61.81	
.35	.2639	64.63	
.40	.2965	66.74	
.45	.3296	68.34	
.50	.3630	69.57	
1.00	.7050	72.92	
1.50	1.0515	71.06	
2.00	1.3991	67.73	
2.50	1.7471	63.69	
3.00	2.0952	59.11	
3.50	2.4432	54.02	
4.00	2.7912	48.30	
4.50	3.1390	41.69	
5.00	3.4866	33.55	
5.50	3.8339	21.65	

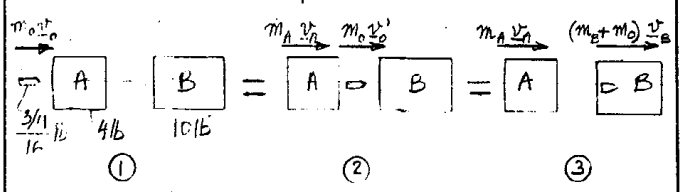
Case with coefficient of restitution E = .90

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
.00	.2000	.00	
.05	.2030	9.45	
.10	.2118	18.34	
.15	.2257	26.29	
.20	.2438	33.14	
.25	.2653	38.89	
.30	.2894	43.66	
.35	.3155	47.60	
.40	.8432	50.85	
.45	.3721	53.54	
.50	.4019	55.78	
1.00	.7253	65.20	
1.50	1.0647	65.88	
2.00	1.4085	63.93	
2.50	1.7540	60.75	
3.00	2.1004	56.79	
3.50	2.4471	52.16	
4.00	2.7939	46.83	
4.50	3.1408	40.55	
5.00	3.4876	32.75	
5.50	3.8343	21.22	

Case with coefficient of restitution E = .70

THETA degrees	VA m/s	PSI degrees	(PSI = final direction of ball A)
.00	.6000	.00	
.05	.6010	2.88	
.10	.6040	5.74	
.15	.6089	8.55	
.20	.6157	11.31	
.25	.6243	13.98	
.30	.6347	16.57	
.35	.6468	19.04	
.40	.6605	21.41	
.45	.6756	23.65	
.50	.6922	25.78	
1.00	.9145	40.71	
1.50	1.1965	47.38	
2.00	1.5050	49.64	
2.50	1.8264	49.44	
3.00	2.1550	47.72	
3.50	2.4878	44.86	
4.00	2.8232	41.01	
4.50	3.1603	36.06	
5.00	3.4986	29.54	
5.50	3.8376	19.52	

14.1 Since there is no external impulsive force, we have conservation of the linear momentum $\underline{L}$ for the entire system:



(a) $\underline{L}_1 = \underline{L}_3$: $m_0\underline{v}_0 = m_A\underline{v}_A + (m_B+m_0)\underline{v}_B$

$$\frac{3}{64g}v_0 = \frac{4}{g}(7.5) + \frac{10+(3/64)}{g}(5)$$

$v_0 = 1711.7$ ft/s $\quad \underline{v}_0 = 1712$ ft/s →

(b) $\underline{L}_1 = \underline{L}_2$: $m_0\underline{v}_0 = m_A\underline{v}_A + m_0\underline{v}_0'$

$$\frac{3}{64g}(1711.7) = \frac{4}{g}(7.5) + \frac{3}{64g}v_0'$$

$v_0' = 1711.7 - \frac{4(7.5)(64)}{3} = 1711.7 - 640$, $\underline{v}_0' = 1072$ ft/s →

14.2 See diagram of solution of Prob. 14.1

(a) $\underline{L}_1 = \underline{L}_3$: $m_0\underline{v}_0 = m_A\underline{v}_A + (m_B+m_0)\underline{v}_B$

$$\frac{3}{64g}(1800) = \frac{4}{g}v_A + \frac{10+(3/64)}{g}(6)$$

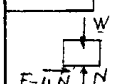
$4m_A = 84.375 - 60.281$, $v_A = 6.0234$, $\underline{v}_A = 6.02$ ft/s →

(b) $\underline{L}_1 = \underline{L}_2$: $m_0\underline{v}_0 = m_A\underline{v}_A + m_0\underline{v}_0'$

$$\frac{3}{64g}(1800) = \frac{4}{g}(6.0234) + \frac{3}{64g}v_0'$$

$v_0' = 1800 - \frac{4(64)}{3}(6.0234) = 1800 - 514$, $\underline{v}_0' = 1286$ ft/s →

14.3 (a) Velocity immediately after coupling



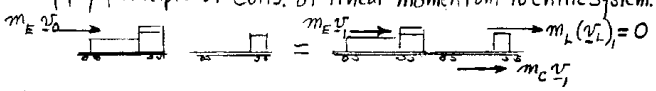
First consider load. We have $F = \mu_k N = 0.20W$

Since coupling occurs in $\Delta t \approx 0$: $F\Delta t \approx 0$

$m_L(v_L)_0 + F\Delta t = m_L(v_L)_1$: $0 + 0 = m_L(v_L)_1$

$(v_L)_1 = 0$

We apply principle of cons. of linear momentum to entire system:



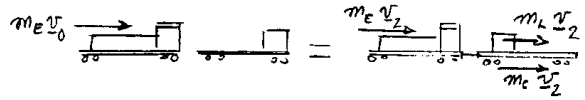
$\underline{L}_0 = \underline{L}_1$: $m_E v_0 = (m_E + m_C)v_1$

$v_1 = \frac{m_E}{m_E+m_C}v_0 = \frac{60}{60+10}(7 \text{ km/h})$, $\underline{v}_1 = 6$ km/h →

(b) Velocity after load has stopped sliding

The engine, car, and load have the same velocity $\underline{v}_2$.

Cons. of linear momentum for entire system:



$\underline{L}_0 = \underline{L}_2$: $m_E v_0 = (m_E + m_C + m_L)v_2$

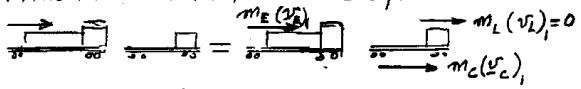
$v_2 = \frac{m_E}{m_E+m_C+m_L}v_0 = \frac{60}{60+10+25}(7 \text{ km/h})$

$v_2 = 4.421$ km/h $\quad \underline{v}_2 = 4.42$ km/h →

14.4 (a) Velocity immediately after impact

From solution of Prob. 14.3: vel. of load $= (\underline{v}_L)_1 = 0$

Cons. of linear momentum for entire system:



$\underline{L}_0 = \underline{L}_1$: $m_E\underline{v}_0 = m_E(\underline{v}_E)_1 + m_C(\underline{v}_C)_1$

$60(7) = 60(v_E)_1 + 10(v_C)_1$, $6(v_E)_1 + (v_C)_1 = 42$ (1)

Relative velocities of engine and car ($e = 1$)

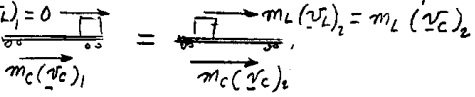
$(v_C)_1 - (v_E)_1 = ev_0$ $\quad (v_C)_1 - (v_E)_1 = 7$ (2)

Multiply (2) by 6 and add (1):

$7(v_C)_1 = 84$ $\quad (\underline{v}_C)_1 = 12$ km/h →

(b) Velocity after load has stopped sliding (after first impact)

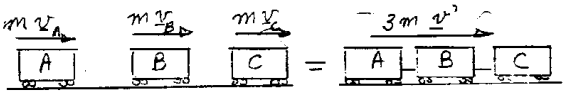
Cons. of linear momentum for car-load system



$\underline{L}_1 = \underline{L}_2$: $m_C(v_C)_1 + m_L(v_L)_1 = (m_C + m_L)(v_C)_2$

$10(12 \text{ km/h}) + 0 = (10+25)(v_C)_2$ $\quad (\underline{v}_C)_2 = 3.43$ km/h →

14.5 (a) All 3 cars automatically coupled:



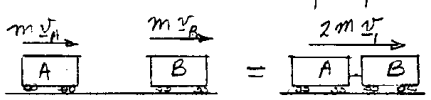
Conservation of linear momentum:

$\pm$ $mv_A + mv_B + mv_C = 3mv'$

$v' = \frac{1}{3}(v_A + v_B + v_C) = \frac{1}{3}(7.5 \text{ km/h} + 0 - 6 \text{ km/h}) = 0.50$ km/h

$\underline{v}_A' = \underline{v}_B' = \underline{v}_C' = \underline{v}' = 0.50$ km/h →

(b) Cars A and B automatically coupled:

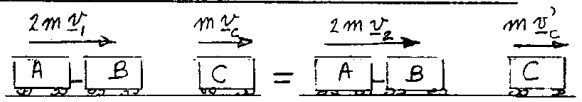


Conservation of linear momentum:

$\pm$ $mv_A + mv_B = 2mv_1$

$v_1 = \frac{1}{2}(v_A + v_B) = \frac{1}{2}(7.5 \text{ km/h} + 0) = 3.75$ km/h

Car C hits and bounces off A and B ($e = 1$):



Conservation of linear momentum.

$\pm$ $2mv_1 + mv_C = 2mv_2 + mv_C'$

$2(3.75) - 6 = 2v_2 + v_C'$

$2v_2 + v_C' = 1.50$ (1)

Relative velocities:

$v_C' - v_2 = e(v_1 - v_C)$ $\quad v_C' - v_2 = (1)(3.75+6)$

$v_C' - v_2 = 9.75$ (2)

Subtract (2) from (1): $3v_2 = 1.50 - 9.75 = -8.25$

$v_A' = v_B' = v_2 = -2.75$ km/h, $\quad \underline{v}_A' = \underline{v}_B' = 2.75$ km/h ←

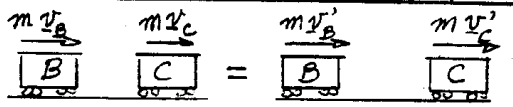
From (2): $v_C' = 9.75 + v_2 = 9.75 - 2.75$, $\quad \underline{v}_C' = 7.00$ km/h →

14.6

(a) All cars automatically coupled:

Same as Prob. 14.5a: $\underline{v}_A' = \underline{v}_B' = \underline{v}_C' = 0.50 \text{ km/h} \rightarrow$ ◀

(b) Cars B and C hit and bounce off ($e = 1$):



Conservation of linear momentum

$\pm m v_B + m v_C = m v_B' + m v_C'$

$0 - 6 = v_B' + v_C'$ $\quad v_B' + v_C' = -6$ (1)

Relative velocities

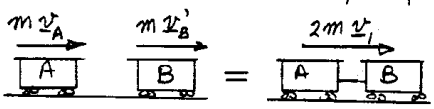
$v_C' - v_B' = e(v_B - v_C)$ $\quad v_C' - v_B' = 0 + 6$ (2)

Adding (1) and (2): $2v_C' = 0$ $\quad \underline{v}_C' = 0$

From (1): $v_B' = -6$ km/h

$\underline{v}_B' = 6 \text{ km/h} \leftarrow$ ◀

Cars A and B get automatically coupled:

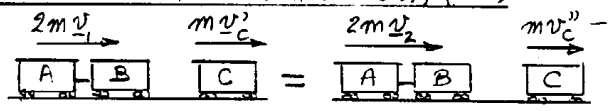


Conservation of linear momentum:

$\pm m v_A + m v_B' = 2m v_1$

$v_1 = \frac{1}{2}(v_A + v_B') = \frac{1}{2}(7.5 \text{ km/h} - 6 \text{ km/h}) = +0.75 \text{ km/h}$

Cars A and B hit car C and bounce off ($e = 1$)



Conservation of linear momentum

$\pm 2m v_1 + m v_C' = 2m v_2 + m v_C''$

$2(0.75) + 0 = 2v_2 + v_C''$ $\quad 2v_2 + v_C'' = 1.50$ (3)

Relative velocities:

$v_C'' - v_2 = e(v_1 - v_C')$ $\quad v_C'' - v_2 = (1)(0.75 - 0)$

$v_C'' - v_2 = 0.75$ (4)

Subtract (4) from (3): $3v_2 = 0.75$ $\quad v_2 = 0.25 \text{ km/h}$

$\underline{v}_A' = \underline{v}_B'' = \underline{v}_2 = 0.25 \text{ km/h} \rightarrow$ ◀

From (4): $v_C'' = 0.75 + v_2 = 0.75 + 0.25 = 1.00 \text{ km/h}$

$\underline{v}_C'' = 1.00 \text{ km/h} \rightarrow$ ◀

14.7

$$\underline{H}_O = \sum \underline{r}_i \times m\underline{v}_i = \sum m_i \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= \frac{5}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 5 & 4 \\ 2 & 3 & -2 \end{vmatrix} + \frac{4}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 4 & 3 \\ v_x & 2 & v_z \end{vmatrix} + \frac{3}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 8 & 6 & 0 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{g}\left[5(-10-12) + 4(4v_z - 6) + 3(6-0)\right]\underline{i}$$
$$+ \frac{1}{g}\left[5(8-0) + 4(3v_x - 4v_z) + 3(0-8)\right]\underline{j}$$
$$+ \frac{1}{g}\left[5(0-10) + 4(8-4v_x) + 3(-16+18)\right]\underline{k}$$

$$\underline{H}_O = \frac{1}{g}\left[(16v_z - 116)\underline{i} + (12v_x - 16v_z + 16)\underline{j} + (-16v_x - 12)\underline{k}\right] \quad (1)$$

(a) For $\underline{H}_O \parallel x$ axis, we must have $H_y = H_z = 0$:

$H_z = 0$: $-16v_x - 12 = 0$ $\quad v_x = -0.75$ ft/s ◀

$H_y = 0$: $12(-0.75) - 16v_z + 16 = 0$ $\quad v_z = 0.4375$ ft/s ◀

(b) Substitute into (1):

$$\underline{H}_O = \frac{1}{g}(16v_z - 116)\underline{i} = \frac{1}{g}\left[16(0.4375) - 116\right]\underline{i} = -\frac{109.0}{32.2}\underline{i}$$

$$\underline{H}_O = -(3.39 \text{ ft·lb·s})\,\underline{i}$$ ◀

14.8

See solution of Prob. 14.7 for derivation of Eq. (1):

$$\underline{H}_O = \frac{1}{g}\left[(16v_z - 116)\underline{i} + (12v_x - 16v_z + 16)\underline{j} + (-16v_x - 12)\underline{k}\right] \quad (1)$$

(a) For $\underline{H}_O \parallel z$ axis, we must have $H_x = H_y = 0$:

$H_x = 0$: $16v_z - 116 = 0$ $\quad v_z = 7.25$ ft/s ◀

$H_y = 0$: $12v_x - 16(7.25) + 16 = 0$ $\quad v_x = 8.33$ ft/s ◀

(b) Substitute into (1):

$$\underline{H}_O = \frac{1}{32.2}\left[-16(8.33) - 12\right]\underline{k} \qquad \underline{H}_O = -(4.51 \text{ ft·lb·s})\,\underline{k}$$ ◀

14.9

(a)

$$\underline{H}_O = \sum \underline{r}_i \times m_i\underline{v}_i = \sum m_i \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= 1\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 1 \\ 3 & -2 & 4 \end{vmatrix} + 2\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 0 & 2.5 \\ 4 & 3 & 0 \end{vmatrix} + 3\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 2 & 1 \\ 2 & 5 & -3 \end{vmatrix}$$

$$= \left[1(12+2) + 2(0-7.5) + 3(-6-5)\right]\underline{i}$$
$$+ \left[1(3-0) + 2(10-0) + 3(2+12)\right]\underline{j}$$
$$+ \left[1(0-9) + 2(9-0) + 3(20-4)\right]\underline{k}$$
$$= -34\underline{i} + 65\underline{j} + 57\underline{k}$$

$$\underline{H}_O = -(34.0 \text{ kg·m}^2/\text{s})\,\underline{i} + (65.0 \text{ kg·m}^2/\text{s})\,\underline{j} + (57.0 \text{ kg·m}^2/\text{s})\,\underline{k}$$ ◀

(b) From answers to Prob. 14.10:

$\underline{r} = 3\underline{i} + 1.5\underline{j} + 1.5\underline{k}$

$m\underline{v} = 17\underline{i} + 19\underline{j} - 5\underline{k}$

$\underline{H}_G = 2\underline{i} + 24.5\underline{j} + 25.5\underline{k}$

Using relation of Prob. 14.22:

$$\underline{H}_O = \underline{r} \times m\underline{v} + \underline{H}_G = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 1.5 & 1.5 \\ 17 & 19 & -5 \end{vmatrix} + 2\underline{i} + 24.5\underline{j} + 25.5\underline{k}$$

$$= -34\underline{i} + 65\underline{j} + 57\underline{k}$$ (Checks with answer to part a)

14.10 (a) Mass Center

$(\Sigma m_i)\bar{r} = \Sigma m_i r_i$:

$(m_A + m_B + m_C)\bar{r} = m_A r_A + m_B r_B + m_C r_C$

$6\bar{r} = (1)(3j + k) + 2(3i + 2.5k) + 3(4i + 2j + k)$

$6\bar{r} = (6+12)i + (3+6)j + (1+5+3)k$

$\bar{r} = (3\text{m})i + (1.5\text{m})j + (1.5\text{m})k$ ◀

(b) Linear Momentum

$L = m\bar{v} = \Sigma m_i v_i = m_A v_A + m_B v_B + m_C v_C$

$= (1)(3i - 2j + 4k) + 2(4i + 3j) + 3(2i + 5j - 3k)$

$= (3+8+6)i + (-2+6+15)j + (4+0-9)k$

$L = m\bar{v} = (17\,\text{kg·m/s})i + (19\,\text{kg·m/s})j - (5\,\text{kg·m/s})k$ ◀

(c) Angular Momentum about G

We determine the position vectors r_i' relative to G:

$r_A' = r_A - \bar{r} = 3j + k - (3i + 1.5j + 1.5k) = -3i + 1.5j - 0.5k$

$r_B' = r_B - \bar{r} = 3i + 2.5k - (3i + 1.5j + 1.5k) = -1.5j + k$

$r_C' = r_C - \bar{r} = 4i + 2j + k - (3i + 1.5j + 1.5k) = i + 0.5j - 0.5k$

Using Eq. (14.24):

$$H_G = \Sigma r_i' \times m_i v_i = 1\begin{vmatrix} i & j & k \\ -3 & 1.5 & -0.5 \\ 3 & -2 & 4 \end{vmatrix} + 2\begin{vmatrix} i & j & k \\ 0 & -1.5 & 1 \\ 4 & 3 & 0 \end{vmatrix} + 3\begin{vmatrix} i & j & k \\ 1 & 0.5 & -0.5 \\ 2 & 5 & -3 \end{vmatrix}$$

$H_G = (2\,\text{kg·m}^2\text{/s})i + (24.5\,\text{kg·m}^2\text{/s})j + (25.5\,\text{kg·m}^2\text{/s})k$ ◀

14.11 Since there is no external force, mass center G moves with constant velocity $v_0 = (500\,\text{m/s})k$.

At $t = 3\text{s}$: $\bar{r} = v_0 t = (500\,\text{m/s})k(3\text{s}) = (1500\,\text{m})k$

From Eq. (14.12): $m\bar{r} = m_A r_A + m_B r_B + m_C r_C$

$240(1500k) = 40 r_A + 80(375i + 825j + 2025k) + 120(-300i - 600j + 1200k)$

$r_A = 9000k - 750i - 1650j - 4050k + 900i + 1800j - 3600k$

$r_A = (150\,\text{m})i + (150\,\text{m})j + (1350\,\text{m})k$ ◀

14.12 $v_A = 60\,\text{mi/h} = 88\,\text{ft/s}$, $v_C = 45\,\text{mi/h} = 66\,\text{ft/s}$

If there had been no collision, positions of cars at time t would have been $r_A = v_A t = (88t)i$, $r_B = v_B t = -v_B t\, j$

$r_C = (r_C)_0 + v_C t = 20i - 63j + (66t)j$

Since all external forces are neglected, motion of mass center of system is unaffected. Thus:

$m_A r_A + m_B r_B + m_C r_C = (m_A + m_B + m_C)\bar{r}$

Substituting for $m_A, \ldots, r_A, \ldots$, and noting that at time t, we have $\bar{r} = (42\text{ft})i - (46.5\text{ft})j$:

$(3000/g)(88t)i + (3600/g)(-v_B t)j + (3000/g)(20i - 63j + 66tj) = (9600/g)(42i - 46.5j)$

Equating coeff. of i: $3000(88t) + 3000(20) = 9600(42)$

(b) $t = 1.300\,\text{s}$ ◀

Equating coeff. of j:

$-3600 v_B(1.300) + 3000(-63) + 3000(66)(1.300) = 9600(-46.5)$

$v_B = 110\,\text{ft/s}$

(a) $v_B = 75.0\,\text{mi/h}$ ◀

14.13 Motion of mass center:

Vertical motion: uniformly accelerated

$h = \frac{1}{2}gt^2$ $\quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(30{,}000\,\text{ft})}{32.2\,\text{ft/s}^2}} = 43.167\,\text{s}$

Position of G when it hits the ground:

$\bar{r} = \bar{v}t = (540 \times \frac{44}{30})i\,(43.167) = (34{,}188\,\text{ft})i$

Positions of fragments:

$m\bar{r} = \Sigma m_i r_i = m_A r_A + m_B r_B + m_C r_C$

$60(34{,}188i) = 30(34{,}500i + 280j) + 20(36{,}000i - 900j) + 10 r_C$

$r_C = (205.13 - 103.50 - 72.00)10^3 i + (-840 + 1800)j$

$r_C = (29{,}600\,\text{ft})i + (960\,\text{ft})j$ ◀

14.14 Motion of mass center is uniform in x direction, uniformly accelerated in y direction, and zero in z direction. Thus, at $t = 1.5\text{s}$:

$\bar{x} = (v_0)_x t = (165\,\text{m/s})(1.5\text{s}) = 247.5\,\text{m}$

$\bar{y} = y_0 - \frac{1}{2}at^2 = 15\,\text{m} - \frac{1}{2}(9.81\,\text{m/s}^2)(1.5\text{s})^2 = 3.964\,\text{m}$

$\bar{z} = 0$

The position of the first cannon ball at that time is

$x_1 = 240\,\text{m}$, $\quad y_1 = 0$, $\quad z_1 = 7\,\text{m}$

We write $m\bar{r} = m_1 r_1 + m_2 r_2$

$30(247.5i + 3.964j) = 15(240i + 7k) + 15 r_2$

$r_2 = 255i + 7.928j - 7k$

The position of the second cannon ball at $t = 1.5\text{s}$ is thus

$x = 255\,\text{m}$, $\quad y = 7.93\,\text{m}$, $\quad z = -7.00\,\text{m}$ ◀

14.15 Use part of solution of Prob. 14.14 above dashed line. We then write

$m\bar{r} = m_1 r_1 + m_2 r_2$

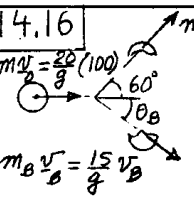
$30(247.5i + 3.964j) = 12(240i + 7k) + 18 r_2$

$r_2 = 252.5i + 6.61j - 4.67k$

The position of the second cannon ball at $t = 1.5\text{s}$ is thus

$x = 253\,\text{m}$, $\quad y = 6.61\,\text{m}$, $\quad z = -4.67\,\text{m}$ ◀

14.16



Conservation of linear momentum:

$m\bar{v}_0 = m_A v_A + m_B v_B$

$\pm\rightarrow$ x components:

$\frac{20}{g}(100) = \frac{5}{g}(250)\cos 60° + \frac{15}{g} v_B \cos\theta_B$

$+\uparrow$ y components:

$0 = \frac{5}{g}(250)\sin 60° - \frac{15}{g} v_B \sin\theta_B$

Rearranging terms in both equations:

$15 v_B \cos\theta_B = 1375$ (1)

$15 v_B \sin\theta_B = 1082.5$ (2)

Dividing (2) by (1): $\tan\theta_B = 0.78730$ $\quad \theta_B = 38.21°$

From (1): $15 v_B \cos 38.21° = 1375$ $\quad v_B = 116.67\,\text{ft/s}$

$v_B = 116.7\,\text{ft/s} \measuredangle 38.2°$ ◀

14.17 Conservation of linear momentum:

$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$$

$$m(10\underline{i}) = m(3.92\underline{i} - 4.56\underline{j}) + mv_B(\sin 3.8°\,\underline{i} + \cos 3.8°\,\underline{j}) + mv_C\underline{i}$$

Equating coeff. of $\underline{j}$: $0 = -4.56 + v_B\cos 3.8°$

$$v_B = 4.57 \text{ ft/s} \blacktriangleleft$$

Equating coeff. of $\underline{i}$: $10 = 3.92 + v_B\sin 3.8° + v_C$

$$10 = 3.92 + 4.57\sin 3.8° + v_C$$

$$v_C = 5.78 \text{ ft/s} \blacktriangleleft$$

14.18 Mass of oxygen nucleus $= m$

Mass of α particle $= \frac{1}{4}m$

Before collisions:

For α particle A: $\underline{v}_0 = -600\underline{i} + 750\underline{j} - 800\underline{k}$

For nuclei B and C: $\underline{v}_0 = 600\underline{j}$

After collisions:

For α particle A: $\underline{v}_A = v_A\dfrac{\overrightarrow{A_1A_2}}{A_1A_2} = v_A\dfrac{80\underline{i} + 80\underline{j} + 40\underline{k}}{120}$

$$\underline{v}_A = \frac{v_A}{3}(2\underline{i} + 2\underline{j} + \underline{k})$$

For nucleus B: $\underline{v}_B = v_B\dfrac{\overrightarrow{B_1B_2}}{B_1B_2} = v_B\dfrac{-33\underline{i} + 70\underline{j} - 10\underline{k}}{78.03}$

$$\underline{v}_B = (-0.4229\underline{i} + 0.8971\underline{j} - 0.1282\underline{k})v_B$$

For nucleus C: $\underline{v}_C = v_C\dfrac{\overrightarrow{C_1C_2}}{C_1C_2} = v_C\dfrac{48\underline{j} - 15\underline{k}}{50.29}$

$$\underline{v}_C = (0.9545\underline{j} - 0.2983\underline{k})v_C$$

Conservation of linear momentum:

$$\left(\frac{1}{4}m\right)\underline{v}_0 + 2m\underline{v}_0 = \left(\frac{1}{4}m\right)\underline{v}_A + m\underline{v}_B + m\underline{v}_C$$

$$-150\underline{i} + 1875\underline{j} - 200\underline{k} + 1200\underline{j} = \left(\frac{1}{6}\underline{i} + \frac{1}{6}\underline{j} + \frac{1}{12}\underline{k}\right)v_A + (-0.4229\underline{i} + 0.8971\underline{j} - 0.1282\underline{k})v_B + (0.9545\underline{j} - 0.2983\underline{k})v_C$$

Equating the coefficients of the unit vectors:

$$0.1667v_A - 0.4229v_B = -150 \quad (1)$$

$$0.1667v_A + 0.8971v_B + 0.9545v_C = 1387.5 \quad (2)$$

$$0.08333v_A - 0.1282v_B - 0.2983v_C = -200 \quad (3)$$

Multiply (2) by 0.2983, (3) by 0.9545, and add:

$$0.12926v_A + 0.14524v_B = 222.99 \quad (4)$$

Multiply (1) by 0.14524, (4) by 0.4229, and add:

$$0.07887v_A = 72.52$$

$$v_A = 919.5 \qquad v_A = 919 \text{ m/s} \blacktriangleleft$$

From (1):

$$0.1667(919.5) - 0.4229v_B = -150$$

$$v_B = 717.1 \qquad v_B = 717 \text{ m/s} \blacktriangleleft$$

From (3):

$$0.08333(919.5) - 0.1282(717.1) - 0.2983v_C = -200$$

$$v_C = 619.1 \qquad v_C = 619 \text{ m/s} \blacktriangleleft$$

14.19 Conservation of linear momentum:

$$m\underline{v}_0 = m_A\underline{v}_A + m_B\underline{v}_B + m_C\underline{v}_C \quad (1)$$

But $\underline{v}_A = v_A\dfrac{\overrightarrow{DA'}}{DA'} = (490 \text{ m/s})\dfrac{-2.4\underline{i} + 3.6\underline{j} + 2.3\underline{k}}{4.9}$

$$\underline{v}_A = -240\underline{i} + 360\underline{j} + 230\underline{k}$$

$$\underline{v}_B = v_B\frac{\overrightarrow{DB'}}{DB'} = v_B\frac{-2.4\underline{i} + 1.2\underline{j} - 2.4\underline{k}}{3.6}$$

$$\underline{v}_C = v_C\frac{\overrightarrow{DC'}}{DC'} = v_C\frac{-2.4\underline{i} - 2.4\underline{j} + 2.3\underline{k}}{4.1}$$

Substituting for $\underline{v}_A$, $\underline{v}_B$, $\underline{v}_C$ into Eq. (1) and recalling that $m_A = 0.3$ kg, $m_B = 0.4$ kg, $m_C = 0.5$ kg:

$$(1.2)(-v_0\underline{i}) = \left[0.3(-240) + 0.4\left(-\frac{2.4}{3.6}v_B\right) + 0.5\left(-\frac{2.4}{4.1}v_C\right)\right]\underline{i} + \left[0.3(360) + 0.4\left(\frac{1.2}{3.6}v_B\right) + 0.5\left(-\frac{2.4}{4.1}v_C\right)\right]\underline{j} + \left[0.3(230) + 0.4\left(-\frac{2.4}{3.6}v_B\right) + 0.5\left(\frac{2.3}{4.1}v_C\right)\right]\underline{k} \quad (2)$$

Equating to zero the coefficients of $\underline{j}$ and $\underline{k}$:

$$108 + 0.13333v_B - 0.29268v_C = 0 \quad (3)$$

$$69 - 0.26667v_B + 0.28049v_C = 0 \quad (4)$$

Multiply (3) by 2 and add (4):

$$285 - 0.30488v_C = 0 \qquad v_C = 934.8 \text{ m/s}$$

Substitute for v_C in (3) and solve for v_B:

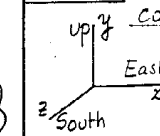
$$108 + 0.13333v_B - 0.29268(934.8) = 0 \qquad v_B = 1242.0 \text{ m/s}$$

Equating the coefficients of $\underline{i}$ in Eq. (2):

$$-1.2v_0 = -72 - 0.26667(1242.0) - 0.29268(934.8)$$

$$v_0 = 564.0 \text{ m/s} \qquad \underline{v}_0 = -v_0\underline{i} = -(564 \text{ m/s})\underline{i} \blacktriangleleft$$

14.20 Velocity of wreckage immediately after collision:



$$\underline{v}' = v_x'\underline{i} + v_y'\underline{j} + v_z'\underline{k}$$

But

$$x = v_x't, \quad v_x' = \frac{x}{t} = \frac{525 \text{ m}}{12 \text{ s}} = 43.75 \text{ m/s}$$

$$z = v_z't, \quad v_z' = \frac{z}{t} = \frac{135 \text{ m}}{12 \text{ s}} = 11.25 \text{ m/s}$$

$$-h = v_y't - \frac{1}{2}gt^2, \quad -800 \text{ m} = v_y'(12 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(12 \text{ s})^2$$

$$v_y' = -7.8067 \text{ m/s}$$

Thus: $\underline{v}' = 43.75\underline{i} - 7.8067\underline{j} + 11.25\underline{k}$

Impact: Conservation of linear momentum

$$m_A\underline{v}_A + m_B\underline{v}_B = (m_A + m_B)\underline{v}'$$

Substituting value found for $\underline{v}'$, $\underline{v}_A' = (120 \text{ km/h})\underline{i} = (33.333 \text{ m/s})\underline{i}$ and values of m_A and m_B:

$$4500(33.333\underline{i}) + 6000\underline{v}_B = 10500(43.75\underline{i} - 7.8067\underline{j} + 11.25\underline{k})$$

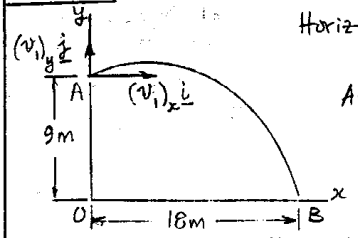
$$\underline{v}_B = (76.563 - 25)\underline{i} - 13.662\underline{j} + 19.688\underline{k}$$

$$= (51.563 \text{ m/s})\underline{i} - (13.662 \text{ m/s})\underline{j} + (19.688 \text{ m/s})\underline{k}$$

$$= (185.6 \text{ km/h})\underline{i} - (49.2 \text{ km/h})\underline{j} + (70.9 \text{ km/h})\underline{k}$$

Components of velocity of B before collision were 185.6 km/h east, 70.9 km/h south, 49.2 km/h down ◀

14.21 Motion of bird and arrow after impact:



Horizontal motion:

$x = (v_1)_x t$

At B: $18\text{m} = (v_1)_x (1.5\text{s})$

$(v_1)_x = 12\ \text{m/s}$

Vertical motion:

$y = y_1 + (v_1)_y t - \frac{1}{2} g t^2$

At B: $0 = (9\text{m}) + (v_1)_y (1.5\text{s}) - \frac{1}{2}(9.81\ \text{m/s}^2)(1.5\text{s})^2$

$(v_1)_y = +1.3575\ \text{m/s}$

Thus, velocity immediately after impact is

$\underline{v}_1 = (12\text{m/s})\underline{i} + (1.3575\ \text{m/s})\underline{j}$

Impact: Conservation of linear momentum:

Let m_a = mass of arrow = 0.040 kg, m_b = mass of bird

$$m_a \underline{v}_a + m_b \underline{v}_b = (m_a + m_b)\underline{v}_1$$

$$m_a (110\ \text{m/s})(\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j}) + m_b v_b \underline{i} = (m_a + m_b)(12\underline{i} + 1.3575\underline{j}) \quad (1)$$

(a) Equating the coefficients of $\underline{j}$ in Eq. (1):

$$m_a (110)\cos 30^\circ = (m_a + m_b)1.3575$$

$$\frac{m_a + m_b}{m_a} = \frac{110\cos 30^\circ}{1.3575} = 70.175$$

$$\frac{m_b}{m_a} = 70.175 - 1 = 69.175$$

$$m_b = 69.175\, m_a = 69.175(0.040\ \text{kg}) = 2.77\ \text{kg} \quad \blacktriangleleft$$

(b) Equating the coefficients of $\underline{i}$ in Eq. (1):

$$m_a (110)\sin 30^\circ + m_b v_b = (m_a + m_b)12$$

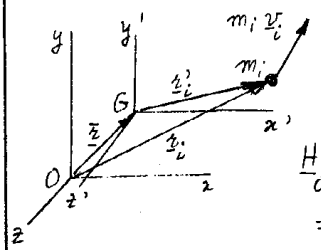
$$110\sin 30^\circ + \frac{m_b}{m_a} v_b = \frac{m_a + m_b}{m_a}(12)$$

$$55 + 69.175\, v_b = 70.175(12)$$

$$v_b = 11.38\ \text{m/s} \quad \blacktriangleleft$$

14.22 Eq. (14.7): $\underline{H}_O = \sum (\underline{r}_i \times m_i \underline{v}_i)$

Eq. (14.24): $\underline{H}_G = \sum (\underline{r}_i' \times m_i \underline{v}_i')$



We note that

$\underline{r}_i = \underline{\bar{r}} + \underline{r}_i'$

Substituting into (14.7):

$$\underline{H}_O = \sum \left[(\underline{\bar{r}} + \underline{r}_i') \times m_i \underline{v}_i \right]$$

$$= \sum (\underline{\bar{r}} \times m_i \underline{v}_i) + \sum (\underline{r}_i' \times m_i \underline{v}_i)$$

$$= \underline{\bar{r}} \times \sum m_i \underline{v}_i + \sum (\underline{r}_i' \times m_i \underline{v}_i)$$

But, by Eq. (14.13), $\sum m_i \underline{v}_i = m\underline{\bar{v}}$

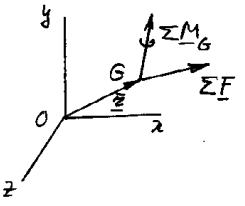
and, by Eq. (14.24), $\sum (\underline{r}_i' \times m_i \underline{v}_i) = \underline{H}_G$

Thus: $\underline{H}_O = \underline{\bar{r}} \times m\underline{\bar{v}} + \underline{H}_G$ ◀

(Q.E.D.)

14.23 Eq. (14.23): $\sum \underline{M}_G = \dot{\underline{H}}_G$

Eq. (14.11): $\sum \underline{M}_O = \dot{\underline{H}}_O$



Since the system of the external forces is equipollent to the vector $\Sigma \underline{F}$ attached at G and the couple $\Sigma \underline{M}_G$, we have

$$\sum \underline{M}_O = \underline{\bar{r}} \times \sum \underline{F} + \sum \underline{M}_G \quad (1)$$

But, from Prob. 14.22: $\underline{H}_O = \underline{\bar{r}} \times m\underline{\bar{v}} + \underline{H}_G$

and, differentiating: $\dot{\underline{H}}_O = \dot{\underline{\bar{r}}} \times m\underline{\bar{v}} + \underline{\bar{r}} \times m\dot{\underline{\bar{v}}} + \dot{\underline{H}}_G$

$$= \underline{\bar{v}} \times m\underline{\bar{v}} + \underline{\bar{r}} \times m\underline{\bar{a}} + \dot{\underline{H}}_G$$

Since $\underline{\bar{v}}$ and $m\underline{\bar{v}}$ are parallel, $\underline{\bar{v}} \times m\underline{\bar{v}} = 0$.

and, by Eq. (14.16), $m\underline{\bar{a}} = \sum \underline{F}$

Thus: $\dot{\underline{H}}_O = \underline{\bar{r}} \times \sum \underline{F} + \dot{\underline{H}}_G \quad (2)$

Substituting for $\sum \underline{M}_O$ and $\dot{\underline{H}}_O$ from (1) and (2) into (14.11):

$$\underline{\bar{r}} \times \sum \underline{F} + \sum \underline{M}_G = \underline{\bar{r}} \times \sum \underline{F} + \dot{\underline{H}}_G$$

or $\sum \underline{M}_G = \dot{\underline{H}}_G$ ◀

(Q.E.D.)

14.24 We start with Eq. (2) of problem statement:

$$\underline{H}_A = \sum \underline{r}_i' \times m_i \underline{v}_i \quad (2)$$

and substitute $\underline{v}_i = \underline{v}_A + \underline{v}_i'$:

$$\underline{H}_A = \sum \underline{r}_i' \times m_i (\underline{v}_A + \underline{v}_i')$$

$$= \sum \underline{r}_i' \times m_i \underline{v}_A + \sum \underline{r}_i' \times m_i \underline{v}_i' \quad (3)$$

But $\sum \underline{r}_i' \times m_i \underline{v}_A = \sum m_i \underline{r}_i' \times \underline{v}_A$

and, by (14.12), $= m\underline{\bar{r}}' \times \underline{v}_A = m\overrightarrow{AG} \times \underline{v}_A$

where G is the mass center of the system.

On the other hand, Eq. (1) of problem statement states that

$$\sum \underline{r}_i' \times m_i \underline{v}_i' = \underline{H}_A'$$

Thus, we may write Eq. (3) as

$$\underline{H}_A = m\overrightarrow{AG} \times \underline{v}_A + \underline{H}_A'$$

and conclude that $\underline{H}_A = \underline{H}_A'$ whenever the cross-product $\overrightarrow{AG} \times \underline{v}_A$ is zero. This will occur if, either

(a) $\underline{v}_A = 0$ (A has zero velocity)

(b) $\overrightarrow{AG} = 0$ (A coincides with G)

(c) $\underline{v}_A \parallel \overrightarrow{AG}$ ($\underline{v}_A$ is directed along AG)

(Q.E.D.)

14.25 We recall Eq. (1) of statement of Prob. 14.24:

$$\underline{H}_A' = \sum \underline{r}_i' \times m_i \underline{v}_i' \qquad (1)$$

Differentiating:

$$\dot{\underline{H}}_A' = \sum \dot{\underline{r}}_i' \times m_i \underline{v}_i' + \sum \underline{r}_i' \times m_i \dot{\underline{v}}_i'$$

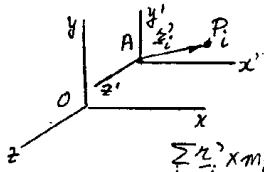
$$= \sum \underline{v}_i' \times m_i \underline{v}_i' + \sum \underline{r}_i' \times m_i \underline{a}_i'$$

The first term is zero, since $\underline{v}_i'$ and $m_i \underline{v}_i'$ are parallel.

Since $\underline{a}_i' = \underline{a}_i - \underline{a}_A$, we have

$$\dot{\underline{H}}_A' = \sum \underline{r}_i' \times m_i \underline{a}_i - \sum \underline{r}_i' \times m_i \underline{a}_A \qquad (3)$$

Since $\underline{a}_i$ is the acceleration of particle P_i relative to the newtonian frame of reference $Oxyz$, we have by Eq. (14.1):



$$m_i \underline{a}_i = \underline{F}_i + \sum_j \underline{f}_{ij}$$

Thus:

$$\sum_i \underline{r}_i' \times m_i \underline{a}_i = \sum_i \underline{r}_i' \times \underline{F}_i + \sum_i \sum_j \underline{r}_i' \times \underline{f}_{ij}$$

$$= \sum \underline{M}_A + 0 \qquad (4)$$

through a reasoning similar to that leading to Eq. (14.3).

On the other hand, by Eq. (14.12),

$$\sum \underline{r}_i' \times m_i \underline{a}_A = \sum m_i \underline{r}_i' \times \underline{a}_A = m \bar{\underline{r}}' \times \underline{a}_A = m \overrightarrow{AG} \times \underline{a}_A \qquad (5)$$

where G is the mass center of the system.

Substituting from (4) and (5) into (3):

$$\dot{\underline{H}}_A' = \sum \underline{M}_A - m \overrightarrow{AG} \times \underline{a}_A$$

We conclude that $\sum \underline{M}_A = \dot{\underline{H}}_A'$ whenever the cross-product $\overrightarrow{AG} \times \underline{a}_A$ is zero. This will occur if either

(a) $\underline{a}_A = 0$ (frame $Ax'y'z'$ is newtonian)

(b) $\overrightarrow{AG} = 0$ (A coincides with G)

(c) $\underline{a}_A \parallel \overrightarrow{AG}$ ($\underline{a}_A$ is directed along AG)

(Q.E.D.)

14.26 (a) Energy lost as bullet moves through block A.
(See statement and solution of Prob. 14.1.)

Before bullet hits A:

$$T_1 = \tfrac{1}{2} m_0 v_0^2 = \tfrac{1}{2} \tfrac{3}{64(32.2)} (1711.7)^2 = 2132.6 \text{ ft·lb}$$

As bullet leaves A (energy of bullet and block A):

$$T_2 = \tfrac{1}{2} m_0 v_0'^2 + \tfrac{1}{2} m_A v_A^2$$

$$= \tfrac{1}{2} \tfrac{3}{64(32.2)} (1071.7)^2 + \tfrac{1}{2} \tfrac{4}{32.2} (7.5)^2 = 836.0 + 3.49 = 839.5 \text{ ft·lb}$$

Energy lost $= T_1 - T_2 = 2132.6 - 839.5 = 1293$ ft·lb ◀

(b) Energy lost as bullet penetrates block B.

Before bullet hits B: $T_2' = \tfrac{1}{2} m_0 v_0'^2 = 836.0$ ft·lb

After bullet has penetrated B (energy of bullet and block B):

$$T_3 = \tfrac{1}{2}(m_0 + m_B) v_B^2 = \tfrac{1}{2} \tfrac{(3/64) + 10}{32.2} (5)^2 = 3.90 \text{ ft·lb}$$

Energy lost $= T_2' - T_3 = 836.0 - 3.90 = 832$ ft·lb ◀

14.27 (a) Energy lost in coupling.
(See statement and solution of Prob. 14.3.)

Before coupling: $v_0 = 7 \text{ km/h} = (7/3.6) \text{ m/s}$

$$T_0 = \tfrac{1}{2} m_E v_0^2 = \tfrac{1}{2} (60 \times 10^3 \text{ kg}) (7/3.6)^2 = 113.426 \text{ kJ}$$

After coupling: $v_1 = 6 \text{ km/h} = (6/3.6) \text{ m/s}$

$$T_1 = \tfrac{1}{2} (m_E + m_C) v_1^2 = \tfrac{1}{2} (60+10) 10^3 (6/3.6)^2 = 97.222 \text{ kJ}$$

Energy lost $= T_0 - T_1 = 113.426 - 97.222 = 16.20$ kJ ◀

(b) Energy lost in friction

After load has stopped sliding:

$$v_2 = 4.421 \text{ km/h} = 1.2281 \text{ m/s}$$

$$T_2 = \tfrac{1}{2} (m_E + m_C + m_L) v_2^2 = \tfrac{1}{2} (60+10+25) 10^3 (1.2281)^2 = 71.637 \text{ kJ}$$

Energy lost $= T_1 - T_2 = 97.222 - 71.637 = 25.6$ kJ ◀

14.28 (a) First coupling. From part b of Prob. 14.5

$$v_A = 7.5 \text{ km/h} = 2.08333 \text{ m/s} \qquad v_B = 0$$

$$v_A' = v_B' = v_1 = 3.75 \text{ km/h} = 1.04167 \text{ m/s}$$

$$T = \tfrac{1}{2} m v_A^2 = \tfrac{1}{2} (36 \times 10^3 \text{ kg}) (2.08333 \text{ m/s})^2 = 78.125 \text{ kJ}$$

$$T' = \tfrac{1}{2} (2m) v_1^2 = \tfrac{1}{2} (2 \times 36 \times 10^3) (1.04167)^2 = 39.063 \text{ kJ}$$

Energy lost in first coupling $= 78.125 - 39.063 = 39.1$ kJ ◀

(b) Second coupling. Consider all 3 cars.

Before coupling: $v_A = v_B = v_1 = 1.04167$ m/s

$v_C = 6 \text{ km/h} = 1.66667 \text{ m/s}$

$$T = \tfrac{1}{2} (2m) v_1^2 + \tfrac{1}{2} m v_C^2$$

$$= \tfrac{1}{2} (2 \times 36 \times 10^3) (1.04167)^2 + \tfrac{1}{2} (36 \times 10^3) (1.66667)^2$$

$$= 39.063 \text{ kJ} + 50.000 \text{ kJ} = 89.063 \text{ kJ}$$

After coupling: From part a of Prob. 14.5:

$$v_A' = v_B' = v_C' = v' = 0.5 \text{ km/h} = 0.13889 \text{ m/s}$$

$$T' = \tfrac{1}{2} (3m) v'^2 = \tfrac{1}{2} (3 \times 36 \times 10^3) (0.13889)^2 = 1.042 \text{ kJ}$$

Energy lost in second coupling $= 89.063 - 1.042 = 88.0$ kJ ◀

14.29 From solution of Prob. 14.19:

$$v_0 = 564.0 \text{ m/s}$$

$v_A = 490$ m/s, $v_B = 1242.0$ m/s, $v_C = 934.2$ m/s

$$T_0 = \tfrac{1}{2} m v_0^2 = \tfrac{1}{2} (1.2 \text{ kg}) (564.0 \text{ m/s})^2 = 190.86 \text{ kJ}$$

$$T_1 = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 + \tfrac{1}{2} m_C v_C^2$$

$$= \tfrac{1}{2} (0.3) (490)^2 + \tfrac{1}{2} (0.4) (1242.0)^2 + \tfrac{1}{2} (0.5) (934.8)^2$$

$$= 36.02 \text{ kJ} + 308.51 \text{ kJ} + 218.46 \text{ kJ}$$

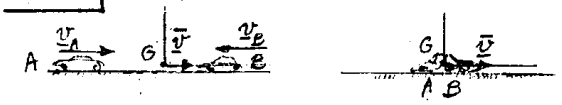
$$= 562.99 \text{ kJ}$$

Work done by interna' forces:

$$= T_1 - T_0 = 562.99 \text{ kJ} - 190.86 \text{ kJ}$$

$$= 372 \text{ kJ} \quad ◀$$

14.30 Before collision / After collision



Before collision

Motion of mass center G:

$$(m_A+m_B)\bar{v} = m_A v_A - m_B v_B$$

$$\bar{v} = \frac{m_A v_A - m_B v_B}{m_A+m_B}$$

Motion relative to mass center:

$$v_{A/G} = v_A - \bar{v} = v_A - \frac{m_A v_A - m_B v_B}{m_A+m_B}$$

$$= \frac{m_A v_A + m_B v_A - m_A v_A + m_B v_B}{m_A+m_B}$$

$$= \frac{m_B}{m_A+m_B}(v_A+v_B)$$

$$v_{B/G} = -v_B - \bar{v} = -v_B - \frac{m_A v_A - m_B v_B}{m_A+m_B}$$

$$= \frac{-m_A v_B - m_B v_B - m_A v_A + m_B v_B}{m_A+m_B}$$

$$= -\frac{m_A}{m_A+m_B}(v_A+v_B)$$

$$T_{A/G} = \frac{1}{2} m_A v_{A/G}^2 = \frac{1}{2}\frac{m_A m_B^2}{(m_A+m_B)^2}(v_A+v_B)^2 \quad (1)$$

$$T_{B/G} = \frac{1}{2} m_B v_{B/G}^2 = \frac{1}{2}\frac{m_A^2 m_B}{(m_A+m_B)^2}(v_A+v_B)^2 \quad (2)$$

After collision

Since there is no external force, mass center G keeps moving with same velocity $\bar{v}$. Since impact is perfectly plastic, $v_A' = v_B' = \bar{v}$ and $v_{A/G}' = v_{B/G}' = 0$. Thus $T_{A/G}' = T_{B/G}' = 0$

It follows that the expressions (1) and (2) represent the energies E_A and E_B absorbed by the two automobiles.

(a) Dividing (1) by (2) member by member:

$$\frac{E_A}{E_B} = \frac{T_{A/G}}{T_{B/G}} = \frac{m_B}{m_A} \qquad \text{(Q.E.D.)}$$ ◀

(b) For given data:

$m_A = 3500/g$, $m_B = 2000/g$, $v_A = 55\text{ mi/h} = 80.67\text{ ft/s}$ and $v_B = 35\text{ mi/h} = 51.33\text{ ft/s}$:

From (1): $E_A = T_{A/G} = \frac{1}{2}\frac{(3500/g)(2000/g)^2}{(\frac{3500+2000}{g})}(80.67+51.33)^2$

$$= \frac{1}{2}\frac{(3500)(2000)^2}{(32.2)(5500)^2}(132)^2$$

$$E_A = 125.2 \times 10^3\text{ ft·lb}$$ ◀

From (2): $E_B = T_{B/G} = \frac{1}{2}\frac{(3500)^2(2000)}{(32.2)(5500)^2}(132)^2$

$$E_B = 219.1 \times 10^3\text{ ft·lb}$$ ◀

We check that

$$\frac{E_A}{E_B} = \frac{125.2\times10^3}{219.1\times10^3} = 0.5714 \text{ and } \frac{m_B}{m_A} = \frac{2000}{3500} = 0.5714 = \frac{E_A}{E_B}$$

14.31 See solution of Prob. 14.30, where it has been established that energies absorbed in collision by automobiles A and B are

$$E_A = T_{A/G} = \frac{1}{2}\frac{m_A m_B^2}{(m_A+m_B)^2}(v_A+v_B)^2 \quad (1)$$

$$E_B = T_{B/G} = \frac{1}{2}\frac{m_A^2 m_B}{(m_A+m_B)^2}(v_A+v_B)^2 \quad (2)$$

On the other hand, the energy absorbed by each automobile in a crash test at speed v_0 into a solid wall is

$$(E_A)_0 = \frac{1}{2} m_A v_0^2 \qquad (E_B)_0 = \frac{1}{2} m_B v_0^2$$

By our definition, the severity of the collision for automobile A is measured by the ratio

$$\frac{E_A}{(E_A)_0} = \frac{\frac{1}{2}\frac{m_A m_B^2}{(m_A+m_B)^2}(v_A+v_B)^2}{\frac{1}{2} m_A v_0^2} = \left(\frac{m_B}{m_A+m_B}\right)^2\left(\frac{v_A+v_B}{v_0}\right)^2 \quad (3)$$

Similarly, the severity of the collision for B is found to be measured by

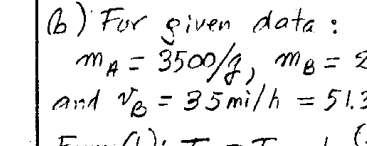
$$\frac{E_B}{(E_B)_0} = \left(\frac{m_A}{m_A+m_B}\right)^2\left(\frac{v_A+v_B}{v_0}\right)^2 \quad (4)$$

Dividing (4) by (3) member by member:

$$\frac{E_B/(E_B)_0}{E_A/(E_A)_0} = \left(\frac{m_A}{m_B}\right)^2$$ ◀

which shows that the collision is $(m_A/m_B)^2$ times more severe for B than for A (Q.E.D.)

14.32 The mass center G of A and B will keep moving to the right at the constant speed $v_0 = 20\text{ ft/s}$ after the two particles fly apart. The system of axes Gx'y' which moves with G in translation is therefore a Newtonian frame of reference. Denoting by v_A' and v_B' the velocities of A and B relative to that frame, we write the following relations:



Conservation of linear momentum (relative to Gx'y'):

$$m_A v_A' + m_B v_B' = 0$$

$$v_B' = -\frac{m_A}{m_B} v_A'$$

Magnitudes: $v_B' = \frac{m_A}{m_B} v_A' = \frac{4}{6} v_A' = \frac{2}{3} v_A'$ (1)

Conservation of energy (relative to Gx'y'):

$T_1' + V_1 = T_2' + V_2$: $\quad 0 + V_1 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 + 0$

Substituting data and recalling (1):

$$40\text{ ft·lb} = \frac{1}{2}\frac{4\text{ lb}}{32.2\text{ ft/s}^2} v_A'^2 + \frac{1}{2}\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{2}{3} v_A'\right)^2$$

$$40 = 0.06211\, v_A'^2 + 0.04141\, v_A'^2$$

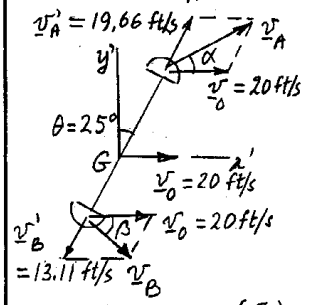
$$v_A'^2 = 386.4 \qquad v_A' = 19.66\text{ ft/s}$$

From (1): $v_B' = \frac{2}{3} v_A' = \frac{2}{3}(19.66) = 13.11\text{ ft/s}$

(CONTINUED)

14.32 CONTINUED

Recalling that the frame $Gx'y'$ moves to the right with the speed $v_0 = 20$ ft/s, we obtain the absolute velocities $\underline{v}_A$ and $\underline{v}_B$ as follows:



$$(v_A)_x = v_0 + v'_A \sin\theta = 20 + 19.66 \sin 25° = 28.309 \text{ ft/s}$$

$$(v_A)_y = v'_A \cos\theta = 19.66 \cos 25° = 17.818 \text{ ft/s}$$

$$\underline{v}_A = 33.4 \text{ ft/s} \measuredangle 32.2°$$

$$(v_B)_x = v_0 - v'_B \sin\theta = 20 - 13.11 \sin 25° = 14.46 \text{ ft/s}$$

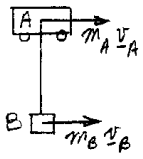
$$(v_B)_y = -v'_B \cos\theta = -13.11 \cos 25° = -11.88 \text{ ft/s}$$

$$\underline{v}_B = 18.71 \text{ ft/s} \measuredangle 39.4°$$

14.33

Conservation of linear momentum:

Since block and cart are initially at rest $L_0 = 0$



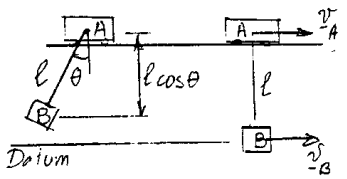
Thus, as B passes under A:

$$\underline{L} = m_A \underline{v}_A + m_B \underline{v}_B = 0$$

$$+\rightarrow \quad m_A v_A + m_B v_B = 0$$

$$v_A = -\frac{m_B}{m_A} v_B \qquad (1)$$

Conservation of energy:



Initially:

$T_0 = 0$

$V_0 = m_B g \ell (1 - \cos\theta)$

As B passes under A:

$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$V = 0$

Thus: $T_0 + V_0 = T + V$: $m_B g \ell (1-\cos\theta) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

Substituting for v_A from (1) and multiplying by 2:

$$2 m_B g \ell (1-\cos\theta) = m_A \left(\frac{m_B^2}{m_A^2} v_B^2\right) + m_B v_B^2$$

$$= \left(\frac{m_B^2}{m_A} + m_B\right) v_B^2 = m_B \frac{m_B + m_A}{m_A} v_B^2$$

$$v_B = \sqrt{\frac{2 m_A}{m_A + m_B} g \ell (1 - \cos\theta)}$$

With given data:

$$v_B = \sqrt{\frac{2(30)}{30+20}(9.81 \text{ m/s}^2)(2\text{m})(1-\cos 25°)} = 1.4852 \text{ m/s}$$

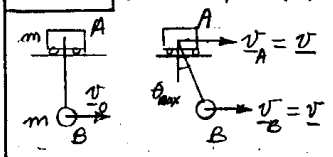
$$\underline{v}_B = 1.485 \text{ m/s} \rightarrow$$

From (1): $v_A = -\frac{20}{30}(1.4852) = -0.9901$ m/s

$$\underline{v}_A = 0.990 \text{ m/s} \leftarrow$$

14.34

(a) Use part (a) of Sample Prob. 14.4 with $m_A = m_B = m$

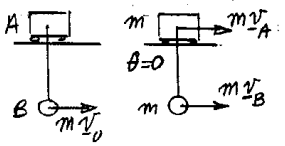


We have

$$v = \frac{m}{m+m} v_0 = \frac{1}{2} v_0$$

$$\underline{v}_A = \underline{v}_B = \frac{1}{2} v_0 \rightarrow$$

(b) Consider initial position and position when $\theta = 0$ again:



Conservation of momentum:

$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B$$

$$\underline{v}_A + \underline{v}_B = \underline{v}_0 \qquad (1)$$

Conservation of energy:

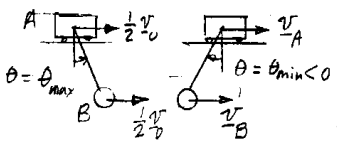
$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

$$v_A^2 + v_B^2 = v_0^2 \qquad (2)$$

Eqs. (1) and (2) are satisfied simultaneously only for $\underline{v}_A = 0$, $\underline{v}_B = \underline{v}_0$ (initial position), or for $\underline{v}_A = \underline{v}_0$, $\underline{v}_B = 0$. Thus, when $\theta = 0$ again,

$$\underline{v}_A = \underline{v}_0 \rightarrow ; \quad \underline{v}_B = 0$$

(c) Consider positions $\theta = \theta_{max}$ and $\theta = \theta_{min}$:



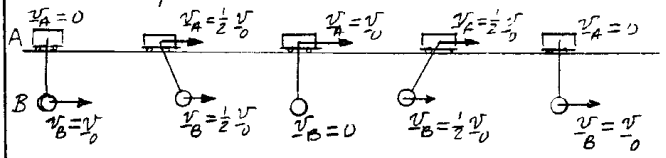
Since $\underline{v}_{B/A} = 0$ for $\theta = \theta_{min}$ we have $\underline{v}_A = \underline{v}_B = \underline{v}'$

Conservation of momentum:

$$2m\left(\frac{1}{2}\underline{v}_0\right) = 2m\underline{v}'$$

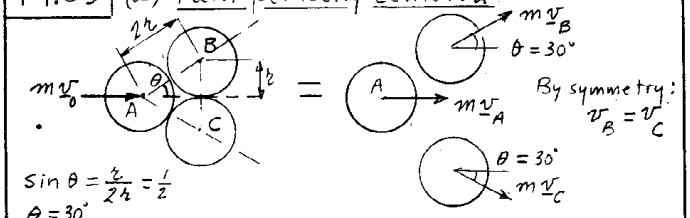
$$\underline{v}_A = \underline{v}_B = \underline{v}' = \frac{1}{2}\underline{v}_0 \rightarrow$$

Cons. of energy would show that elevation of B is the same. Both A and B keep moving to the right, with A and B slipping intermittently:



14.35

(a) Path perfectly centered



By symmetry: $v_B = v_C$

$\sin\theta = \frac{r}{2r} = \frac{1}{2}$

$\theta = 30°$

Conservation of linear momentum:

x comp: $m v_0 = m v_A + m v_B \cos 30° + m v_C \cos 30°$

$$v_0 - v_A = 2 v_B \cos 30°$$

$$v_0 - v_A = v_B \sqrt{3} \qquad (1)$$

Conservation of energy:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2$$

$$v_0^2 - v_A^2 = 2 v_B^2$$

Factoring: $(v_0 + v_A)(v_0 - v_A) = 2 v_B^2 \qquad (2)$

Divide (2) by (1): $v_0 + v_A = \frac{2}{\sqrt{3}} v_B \qquad (3)$

Add (1) and (3): $2 v_0 = \left(\sqrt{3} + \frac{2}{\sqrt{3}}\right) v_B \qquad v_B = \frac{2}{5}\sqrt{3}\, v_0$

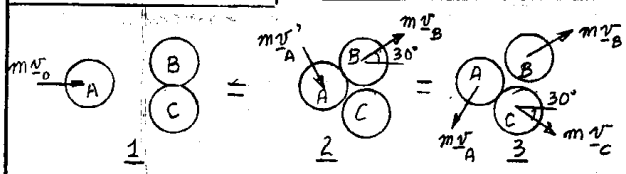
From (1): $v_A = v_0 - v_B\sqrt{3} = v_0\left(1 - \frac{2}{5}\sqrt{3}\sqrt{3}\right), \quad v_A = -\frac{1}{5} v_0$

Thus:

$\underline{v}_A = 0.2\, \underline{v}_0 \leftarrow$; $\underline{v}_B = 0.693 v_0 \measuredangle 30°$; $\underline{v}_C = 0.693 v_0 \measuredangle 30°$

(CONTINUED)

14.35 CONTINUED (6) A strikes B before C



Cons. of linear mom. from 1 to 2:

x comp: $mv_0 = m(v'_A)_x + mv_B\cos 30°$ $\quad (v'_A)_x = v_0 - v_B\cos 30°$ (4)

y comp: $0 = m(v'_A)_y + mv_B\sin 30°$ $\quad (v'_A)_y = v_B\sin 30°$ (5)

Square both members of (4) and (5) and add:

$$v'^2_A = v_0^2 - 2v_0v_B\cos 30° + v_B^2 \quad (6)$$

Cons. of energy from 1 to 2:

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv_B^2$ $\quad v_0^2 = v'^2_A + v_B^2$ (7)

Substitute for v'^2_A from (6) into (7):

$v_0^2 = v_0^2 - 2v_0v_B\cos 30° + v_B^2 + v_B^2$ $\quad v_B = v_0\cos 30°$ (8)

Cons. of linear momentum from 1 to 3:

x comp: $mv_0 = m(v_A)_x + mv_B\cos 30° + mv_C\cos 30°$

y comp: $0 = m(v_A)_y + mv_B\sin 30° - mv_C\sin 30°$

Substitute for v_B from (8) and solve for $(v_A)_x$ and $(v_A)_y$:

$(v_A)_x = v_0 - v_0\cos^2 30° - v_C\cos 30°$ $\quad (v_A)_x = v_0\sin^2 30° - v_C\cos 30°$ (9)

$(v_A)_y = -v_0\sin 30°\cos 30° + v_C\sin 30°$ (10)

Square both members of (9) and (10) and add:

$$v_A^2 = 0.25v_0^2 - 0.866v_0v_C + v_C^2 \quad (11)$$

Cons. of energy from 1 to 3:

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$ $\quad v_0^2 = v_A^2 + v_B^2 + v_C^2$ (12)

Substitute for v_A and v_B from (11) and (8) into (12):

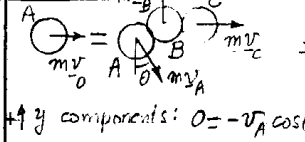
$v_0^2 = 0.25v_0^2 - 0.866v_0v_C + v_C^2 + 0.75v_0^2 + v_C^2$, $\quad v_C = 0.4330v_0$ (13)

From (9): $(v_A)_x = 0.25v_0 - 0.4330v_0\cos 30° = -0.1250v_0$

From (10): $(v_A)_y = -0.4330v_0 + 0.4330v_0\sin 30° = -0.2165v_0$

Thus: $\underline{v}_A = 0.250v_0 \measuredangle 60°$; $\underline{v}_B = 0.866v_0 \measuredangle 30°$; $\underline{v}_C = 0.433v_0 \measuredangle 30°$ ◀

14.36 Conservation of linear momentum:



$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$

$\pm$ x comp: $v_0 = v_A\sin\theta + v_C$

$v_C = v_0 - v_A\sin\theta$ (1)

+↑ y components: $0 = -v_A\cos\theta + v_B = 0$ $\quad v_B = v_A\cos\theta$ (2)

Conservation of energy:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 \quad (3)$$

Substituting for v_C and v_B from (1) and (2) into (3):

$v_0^2 = v_A^2 + v_A^2\cos^2\theta + (v_0 - v_A\sin\theta)^2$

$v_0^2 = v_A^2 + v_A^2\cos^2\theta + v_A^2\sin^2\theta - 2v_0v_A\sin\theta + v_0^2$

$2v_A^2 - 2v_0v_A\sin\theta = 0$ $\quad v_A = v_0\sin\theta$

Substituting for v_A in (2): $\quad v_B = v_0\sin\theta\cos\theta$

Substituting for v_A in (1):

$v_C = v_0 - (v_0\sin\theta)\sin\theta = v_0(1 - \sin^2\theta)$ $\quad v_C = v_0\cos^2\theta$

For given data: $v_0 = 5$ m/s, $\theta = 20°$:

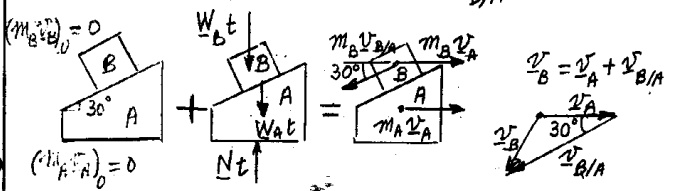
$v_A = (5\text{ m/s})\sin 20°$ $\quad v_A = 1.710$ m/s ◀

$v_B = (5\text{ m/s})\sin 20°\cos 20°$ $\quad v_B = 1.607$ m/s ◀

$v_C = (5\text{ m/s})\cos^2 20°$ $\quad v_C = 4.42$ m/s ◀

14.37 Principle of Impulse and Momentum:

We resolve the velocity $\underline{v}_B$ of block B into its components $\underline{v}_A$ (horizontal) and $\underline{v}_{B/A}$ (// to incline):



$$\Sigma m\underline{v}_0 + \Sigma\underline{F}t = \Sigma m\underline{v}$$

$\pm$ x comp: $0 + 0 = m_Av_A + m_Bv_A - m_Bv_{B/A}\cos 30°$

$v_A = \frac{m_B\cos 30°}{m_A + m_B}v_{B/A} = \frac{15\cos 30°}{25+15}v_{B/A} = 0.32476v_{B/A}$ (1)

Conservation of energy:

Selecting data in final position of B, we have

$T_0 = 0$, $V_0 = (15\text{ lb})(3\text{ ft})\sin 30° = 22.5$ ft·lb

$T = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2$ $\quad V = 0$

$T = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_B(v_A^2 + v_{B/A}^2 - 2v_Av_{B/A}\cos 30°)$

Recalling Eq. (1) and substituting given data,

$T = \frac{1}{2}\frac{25}{g}(0.32476v_{B/A})^2 + \frac{1}{2}\frac{15}{g}[(0.32476)^2 + 1 - 2(0.32476)\cos 30°]v_{B/A}^2$

$= \frac{1}{2(32.2)}[25(0.10547) + 15(0.54297)]v_{B/A}^2 = 0.16741v_{B/A}^2$

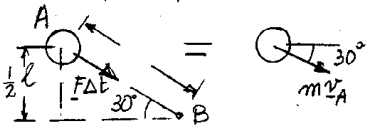
$T_0 + V_0 = T + V$: $\quad 0 + 22.5 = 0.16741v_{B/A}^2 + 0$

(a) $v_{B/A}^2 = 134.40$, $\quad v_{B/A} = 11.593$ ft/s, $\quad \underline{v}_{B/A} = 11.59\text{ ft/s} \measuredangle 30°$ ◀

(b) From (1): $v_A = 0.32476(11.593)$ $\quad \underline{v}_A = 3.76\text{ ft/s} \rightarrow$ ◀

14.38 Effect of constraints on final velocities

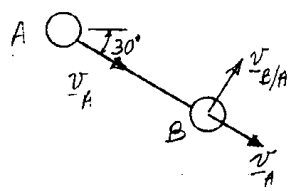
Velocity of sphere A



Since only impulse on A is exerted by cord AB,

$\underline{v}_A = v_A \measuredangle 30°$

Velocity of sphere B

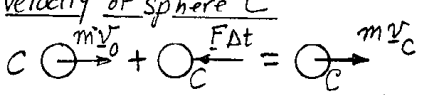


Since cord AB is inextensible, the component of $\underline{v}_B$ along AB must equal $\underline{v}_A$

The component of $\underline{v}_B$ perpendicular to AB represents the velocity $\underline{v}_{B/A}$ of B relative to A:

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = v_A \measuredangle 30° + v_{B/A} \measuredangle 60°$

Velocity of sphere C

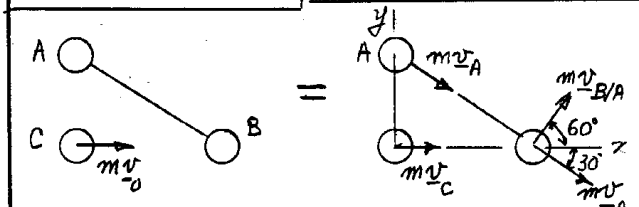


Since the impulse exerted by B on C is horizontal the final velocity of C is

$\underline{v}_C = v_C \rightarrow$

(CONTINUED)

14.38 CONTINUED Conservation of linear momentum:



$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$$
$$= 2m\underline{v}_A + m\underline{v}_{B/A} + m\underline{v}_C$$

+↑ y comp.: $0 = -2mv_A \sin 30^\circ + mv_{B/A}\sin 60^\circ$

$$v_{B/A} = 2\frac{(1/2)}{(\sqrt{3}/2)}v_A \qquad v_{B/A} = \frac{2\sqrt{3}}{3}v_A \qquad (1)$$

+→ x comp.: $mv_0 = 2mv_A\cos 30^\circ + mv_{B/A}\cos 60^\circ + mv_C$

$$v_C = v_0 - 2v_A\left(\frac{\sqrt{3}}{2}\right) - v_{B/A}\left(\frac{1}{2}\right) = v_0 - \sqrt{3}v_A - \frac{1}{2}\left(\frac{2\sqrt{3}}{3}v_A\right)$$
$$v_C = v_0 - \frac{4\sqrt{3}}{3}v_A \qquad (2)$$

Conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$
$$= \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(v_A^2 + v_{B/A}^2\right) + \frac{1}{2}mv_C^2$$
$$v_0^2 = 2v_A^2 + v_{B/A}^2 + v_C^2$$

Substitute for $v_{B/A}$ and v_C from (1) and (2):

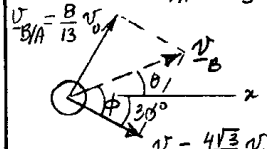
$$v_0^2 = 2v_A^2 + \frac{4}{3}v_A^2 + v_0^2 - \frac{8\sqrt{3}}{3}v_0v_A + \frac{16}{3}v_A^2$$
$$v_A\left(\frac{26}{3}v_A - \frac{8\sqrt{3}}{3}v_0\right) = 0$$

$v_A = 0$ (initially) or $v_A = \frac{4\sqrt{3}}{13}v_0$ (3)

$$\underline{v}_A = 0.533v_0 \measuredangle 30^\circ$$ ◀

Substitute for v_A from (3) into (1):

$$v_{B/A} = \frac{2\sqrt{3}}{3}\left(\frac{4\sqrt{3}}{13}v_0\right) = \frac{8}{13}v_0$$



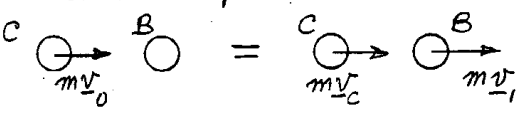
$$v_B = \sqrt{v_A^2 + v_{B/A}^2}$$
$$= \frac{\sqrt{48+64}}{13}v_0 = 0.814v_0$$
$$\tan\phi = \frac{8/13}{4\sqrt{3}/13} = \frac{2}{\sqrt{3}}, \quad \phi = 49.1^\circ$$
$$\theta = \phi - 30^\circ = 49.1^\circ - 30^\circ = 19.1^\circ$$
$$\underline{v}_B = 0.814v_0 \measuredangle 19.1^\circ$$ ◀

Substitute for v_A from (3) into (2):

$$v_C = v_0 - \frac{4\sqrt{3}}{3}\left(\frac{4\sqrt{3}}{13}v_0\right) = v_0 - \frac{16}{13}v_0 = -\frac{3}{13}v_0$$
$$\underline{v}_C = 0.231v_0 \leftarrow$$ ◀

14.39 (a) Impact between B and C

Since both spheres are unconstrained:



Conservation of linear momentum:

$$m\underline{v}_0 = m\underline{v}_C + m\underline{v}_1$$
$$\underline{v}_C + \underline{v}_1 = \underline{v}_0 \qquad (1)$$

Conservation of energy (perfectly elastic impact):

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_C^2 + \frac{1}{2}mv_1^2$$
$$v_C^2 + v_1^2 = v_0^2 \qquad (2)$$

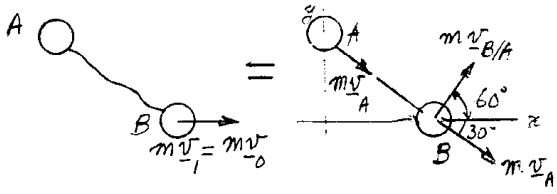
Only values of v_C and v_1 satisfying (1) and (2) simultaneously are either $v_C = v_0$, $v_1 = 0$ (initially) or

$$v_C = 0, \quad v_1 = v_0 \qquad \text{Thus } \underline{v}_C = 0$$ ◀

Cord AB becomes taut

See solution of Prob. 14.38 for discussion of effect of constraints on final velocities of A and B.

We have $\underline{v}_A = v_A \measuredangle 30^\circ$, $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A \measuredangle 30^\circ + \underline{v}_{B/A} \measuredangle 60^\circ$



Conservation of linear momentum:

$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B$$
$$= 2m\underline{v}_A + m\underline{v}_{B/A}$$

+↑ y comp.: $0 = -2mv_A\sin 30^\circ + mv_{B/A}\sin 60^\circ$

$$v_{B/A} = 2\frac{(1/2)}{(\sqrt{3}/2)}v_A \qquad v_{B/A} = \frac{2\sqrt{3}}{3}v_A \qquad (1)$$

+→ x comp.: $mv_0 = 2mv_A\cos 30^\circ + mv_{B/A}\cos 60^\circ$

$$v_0 = \frac{2\sqrt{3}}{2}v_A + \frac{1}{2}v_{B/A} \qquad (2)$$

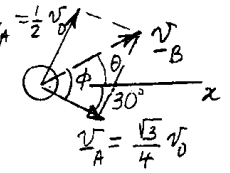
Substitute for $v_{B/A}$ from (1) into (2):

$$v_0 = \sqrt{3}v_A + \frac{1}{2}\left(\frac{2\sqrt{3}}{3}v_A\right) = \frac{4\sqrt{3}}{3}v_A \qquad v_A = \frac{\sqrt{3}}{4}v_0 \quad (3)$$

$$\underline{v}_A = 0.433v_0 \measuredangle 30^\circ$$ ◀

Substitute for v_A from (3) into (1):

$$v_{B/A} = \frac{2\sqrt{3}}{3}\left(\frac{\sqrt{3}}{4}v_0\right) = \frac{1}{2}v_0$$



$$v_B = \sqrt{v_A^2 + v_{B/A}^2}$$
$$= \sqrt{\frac{3}{16} + \frac{1}{4}}\,v_0 = 0.661v_0$$
$$\tan\phi = \frac{1/2}{\sqrt{3}/4} = \frac{2}{\sqrt{3}}, \quad \phi = 49.1^\circ$$
$$\theta = \phi - 30^\circ = 49.1^\circ - 30^\circ = 19.1^\circ$$
$$\underline{v}_B = 0.661v_0 \measuredangle 19.1^\circ$$ ◀

(CONTINUED)

14.39 CONTINUED

(b) Fraction of initial kinetic energy lost when cord becomes taut

Initial K.E. $= T_0 = \frac{1}{2} m v_0^2$

Final K.E. $= T = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$ (since $v_C = 0$)

$= \frac{1}{2} m v_A^2 + \frac{1}{2} m (v_A^2 + v_{B/A}^2)$

$= \frac{1}{2} m \left(2 v_A^2 + v_{B/A}^2\right)$

$= \frac{1}{2} m \left[2\left(\frac{\sqrt{3}}{4} v_0\right)^2 + \left(\frac{v_0}{2}\right)^2\right]$

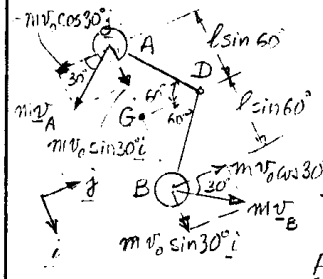
$= \frac{1}{2} m \left(\frac{6}{16} + \frac{1}{4}\right) v_0^2 = \frac{1}{2} m \left(\frac{5}{8} v_0^2\right)$

$T = \frac{5}{8} T_0 \qquad \frac{T}{T_0} = \frac{5}{8}$

Thus, fraction of energy lost $= \frac{T_0 - T}{T_0} = \left(1 - \frac{5}{8}\right) = \frac{3}{8}$ ◀

14.40

Consider the momenta of the system consisting of A and B, just after cord CD breaks:



Linear momentum:

$\underline{L}_0 = m \underline{v}_A + m \underline{v}_B$

$= m v_0 (\sin 30° \underline{i} - \cos 30° \underline{j}) + m v_0 (\sin 30° \underline{i} + \cos 30° \underline{j})$

$= 2 m v_0 \sin 30° \underline{i}$

$\underline{L}_0 = m v_0 \underline{i}$

Angular momentum about G:

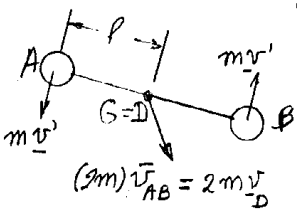
$(\underline{H}_G)_0 = \underline{r}'_A \times m \underline{v}'_A + \underline{r}'_B \times m \underline{v}'_B$

$= (-l \sin 60° \underline{i}) \times (-m v_0 \cos 30° \underline{j}) + (l \sin 60° \underline{i}) \times (m v_0 \cos 30° \underline{j})$

$= 2 m v_0 l \sin 60° \cos 30° \underline{k} = 2 m v_0 l \left(\frac{\sqrt{3}}{2}\right)^2 \underline{k}$

$(\underline{H}_G)_0 = \frac{3}{2} m v_0 l \, \underline{k}$

Consider now the same system (A and B) after the cords again become taut. The momenta of the system may be represented by the linear momentum



$\underline{L} = (2m) \bar{v}_{AB} = 2 m \underline{v}_D$

and the angular momentum

$\underline{H}_G = 2 l m v' \underline{k}$

(a) Conservation of linear momentum

$\underline{L}_0 = \underline{L}$: $\quad m v_0 \underline{i} = (2m) \bar{v}_{AB} \quad v_D = \bar{v}_{AB} = \frac{1}{2} v_0$ ◀

(b) Cons. of ang. mom. about G:

$(\underline{H}_G)_0 = \underline{H}_G$: $\quad \frac{3}{2} m v_0 l \underline{k} = 2 l m v' \underline{k}$, $\quad v' = \frac{3}{4} v_0$ ◀

(c) Fraction of energy of original system dissipated.

Initially, for entire system: $T_0 = 3\left(\frac{1}{2} m v_0^2\right) = \frac{3}{2} m v_0^2$

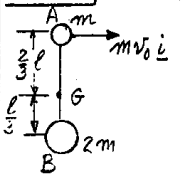
Final energy: We consider the system A+B, and the sphere C separately. (Note that $v_C = v_0$)

$T = T_{AB} + T_C = \left[\frac{1}{2}(m_A + m_B) \bar{v}_{AB}^2 + \frac{1}{2} m_A v'^2 + \frac{1}{2} m_B v'^2\right] + \frac{1}{2} m_C v_C^2$

$= \frac{1}{2}(2m)\left(\frac{1}{2} v_0\right)^2 + 2\left[\frac{1}{2} m \left(\frac{3}{4} v_0\right)^2\right] + \frac{1}{2} m v_0^2 = \frac{21}{16} m v_0^2$

Fraction dissipated $= \frac{T_0 - T}{T_0} = 1 - \frac{T}{T_0} = 1 - \frac{21/16}{3/2} = 1 - \frac{7}{8} = \frac{1}{8}$ ◀

14.41



(a) Linear and angular momentum

$\underline{L} = m_A \underline{v}_A + m_B \underline{v}_B = m v_0 \underline{i} + 0$

$\underline{L} = m v_0 \underline{i}$ ◀

$\underline{H}_G = \frac{2}{3} l \underline{j} \times m v_0 \underline{i} = -\frac{2}{3} m l v_0 \underline{k}$ ◀

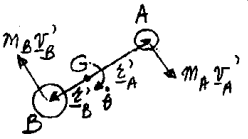
Motion of mass center G

Since there is no external force $\underline{L} = m v_0 \underline{i} = \text{constant}$

But $\underline{L} = (3m) \bar{v}$. Thus

$3m \bar{v} = m v_0 \underline{i} \qquad \bar{v} = \frac{1}{3} v_0 \underline{i} \qquad (1)$

Motion about mass center



We have $\underline{H}_G = \underline{H}'_G = \sum \underline{r}'_i \times m_i \underline{v}'_i$

$= \underline{r}'_A \times m_A \underline{v}'_A + \underline{r}'_B \times m_B \underline{v}'_B$

But $r'_A = \frac{2}{3} l$, $r'_B = \frac{1}{3} l$

$v'_A = \left(\frac{2}{3} l\right)\dot{\theta}$, $\quad v'_B = \left(\frac{1}{3} l\right)\dot{\theta}$

$m_A = m \qquad m_B = 2m$

Thus: $\underline{H}_G = \left[\left(\frac{2}{3} l\right) m \left(\frac{2}{3} l\right)\dot{\theta} + \left(\frac{1}{3} l\right)(2m)\left(\frac{1}{3} l\right)\dot{\theta}\right](-\underline{k})$

$\underline{H}_G = -\left[\left(\frac{4}{9} + \frac{2}{9}\right) m l^2 \dot{\theta}\right] \underline{k} = -\frac{2}{3} m l^2 \dot{\theta} \, \underline{k}$

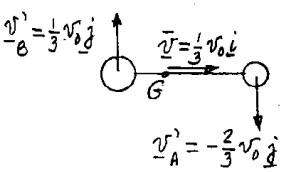
But, from above, $\underline{H}_G = -\frac{2}{3} m l v_0 \underline{k} = \text{constant}$

Thus: $-\frac{2}{3} m l^2 \dot{\theta} = -\frac{2}{3} m l v_0 \qquad \dot{\theta} = \frac{v_0}{l}$

and $v'_A = \frac{2}{3} l \dot{\theta} = \frac{2}{3} l \frac{v_0}{l} = \frac{2}{3} v_0 \qquad (2)$

$v'_B = \frac{1}{3} l \dot{\theta} = \frac{1}{3} l \frac{v_0}{l} = \frac{1}{3} v_0 \qquad (3)$

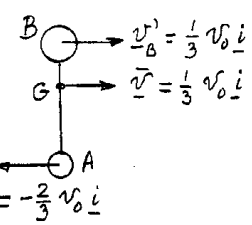
(b) After 90° rotation: Using Eqs. (1), (2), and (3):



$\underline{v}_A = \bar{v} + \underline{v}'_A = \frac{1}{3} v_0 \underline{i} - \frac{2}{3} v_0 \underline{j}$ ◀

$\underline{v}_B = \bar{v} + \underline{v}'_B = \frac{1}{3} v_0 \underline{i} + \frac{1}{3} v_0 \underline{j}$ ◀

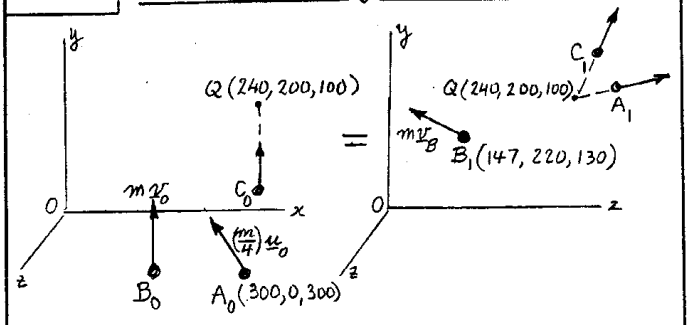
(c) After 180° rotation: Using Eqs. (1), (2), and (3):



$\underline{v}_A = \bar{v} + \underline{v}'_A = \frac{1}{3} v_0 \underline{i} - \frac{2}{3} v_0 \underline{i} = -\frac{1}{3} v_0 \underline{i}$ ◀

$\underline{v}_B = \bar{v} + \underline{v}'_B = \frac{1}{3} v_0 \underline{i} + \frac{1}{3} v_0 \underline{i} = \frac{2}{3} v_0 \underline{i}$ ◀

14.42 Conservation of angular momentum about Q.



We also recall from the statement of Prob. 14.18 that

$$\underline{u}_0 = -(600\ \text{m/s})\underline{i} + (750\ \text{m/s})\underline{j} - (800\ \text{m/s})\underline{k}$$

and $\underline{v}_0 = (600\ \text{m/s})\underline{j}$

We note that the angular momentum about Q of particle A after the collisions is zero and that the angular momentum about Q of the nucleus C is zero before and after the collisions. Thus, the conservation of the angular momentum about Q of the three particles will be expressed as follows:

$$\overrightarrow{QA}_0 \times \left(\frac{m}{4}\right)\underline{u}_0 + \overrightarrow{QB}_0 \times m\underline{v}_0 = \overrightarrow{QB}_1 \times m\underline{v}_B \qquad (1)$$

where $\overrightarrow{QA}_0 = \underline{r}_{A_0} - \underline{r}_Q = 300\underline{i} + 300\underline{k} - (240\underline{i} + 200\underline{j} + 100\underline{k})$

$= 60\underline{i} - 200\underline{j} + 200\underline{k}$

$\overrightarrow{QB}_0 = (\Delta x)\underline{i} + (\Delta y)\underline{j} + (\Delta z)\underline{k}$

$\overrightarrow{QB}_1 = \underline{r}_{B_1} - \underline{r}_Q = 147\underline{i} + 220\underline{j} + 130\underline{k} - (240\underline{i} + 200\underline{j} + 100\underline{k})$

$= -93\underline{i} + 20\underline{j} + 30\underline{k}$

$\underline{u}_0 = -600\underline{i} + 750\underline{j} - 800\underline{k}$ $\qquad \underline{v}_0 = 600\underline{j}$

and, from solution of Prob. 14.18:

$$\underline{v}_B = v_B\underline{\lambda}_B = 717.1\,(-0.4229\underline{i} + 0.8971\underline{j} - 0.1282\underline{k})$$

$$= -303.3\underline{i} + 643.3\underline{j} - 91.93\underline{k}$$

Substituting into Eq. (1), and using determinant form:

$$\frac{m}{4}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 60 & -200 & 200 \\ -600 & 750 & -800 \end{vmatrix} + m\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \Delta x & \Delta y & \Delta z \\ 0 & 600 & 0 \end{vmatrix} = m\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -93 & 20 & 30 \\ -303.3 & 643.3 & -91.93 \end{vmatrix}$$

Equating the coefficients of $\underline{i}$ and $\underline{k}$:

$\underline{i}$: $\frac{1}{4}(10000) - 600\Delta z = -21138$ $\qquad \Delta z = 39.40$

$\underline{k}$: $\frac{1}{4}(-75000) + 600\Delta x = -53761$ $\qquad \Delta x = -58.35$

$x_{B_0} = x_Q + \Delta x = 240 - 58.3$ $\qquad x_{B_0} = 181.7$ mm ◀

$z_{B_0} = z_Q + \Delta z = 100 + 39.40$ $\qquad z_{B_0} = 139.4$ mm ◀

14.43 Conservation of angular momentum about O:

Given data:

$$m_A = \frac{125}{g},\quad m_B = \frac{250}{g},\quad m_C = \frac{375}{g},\quad m = \frac{750}{g}$$

$\underline{r}_A = 240\underline{i} + 240\underline{j} + 2160\underline{k}$, $\quad \underline{v}_A = (v_A)_x\underline{i} + (v_A)_y\underline{j} + (v_A)_z\underline{k}$

$\underline{r}_B = 600\underline{i} + 1320\underline{j} + 3240\underline{k}$, $\quad \underline{v}_B = 500\underline{i} + 1100\underline{j} + 2200\underline{k}$

$\underline{r}_C = -480\underline{i} - 960\underline{j} + 1920\underline{k}$, $\quad \underline{v}_C = -400\underline{i} + (v_C)_y\underline{j} + (v_C)_z\underline{k}$

Since space vehicle passes through O, we have $\underline{H}_O = 0$ or

$$\underline{H}_O = \underline{r}_A \times m_A\underline{v}_A + \underline{r}_B \times m_B\underline{v}_B + \underline{r}_C \times m_C\underline{v}_C = 0$$

Using determinant form:

$$\underline{H}_O = \frac{125}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 240 & 240 & 2160 \\ (v_A)_x & (v_A)_y & (v_A)_z \end{vmatrix} + \frac{250}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 600 & 1320 & 3240 \\ 500 & 1100 & 2200 \end{vmatrix} + \frac{375}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -480 & -960 & 1920 \\ -400 & (v_C)_y & (v_C)_z \end{vmatrix} = 0$$

Equating to zero the coefficients of $\underline{i}$, $\underline{j}$, $\underline{k}$, and dividing by $125/g$:

$\underline{i}$: $240(v_A)_z - 2160(v_A)_y - 1.320\times10^6 - 2880(v_C)_z - 5760(v_C)_y = 0$

$\underline{j}$: $2160(v_A)_x - 240(v_A)_z + 0.600\times10^6 - 2.304\times10^6 + 1440(v_C)_z = 0$

$\underline{k}$: $240(v_A)_y - 240(v_A)_x + 0 - 1440(v_C)_y - 1.152\times10^6 = 0$

or, after reductions:

$$(v_A)_z - 9(v_A)_y - 12(v_C)_z - 24(v_C)_y = 5500 \qquad (1)$$

$$-(v_A)_z + 9(v_A)_x + 6(v_C)_z = 7100 \qquad (2)$$

$$(v_A)_y - (v_A)_x - 6(v_C)_y = 4800 \qquad (3)$$

Conservation of linear momentum:

$$m_A\underline{v}_A + m_B\underline{v}_B + m_C\underline{v}_C = m\underline{v}_0$$

$$\frac{125}{g}\left[(v_A)_x\underline{i} + (v_A)_y\underline{j} + (v_A)_z\underline{k}\right] + \frac{250}{g}\left[500\underline{i} + 1100\underline{j} + 2200\underline{k}\right] + \frac{375}{g}\left[-400\underline{i} + (v_C)_y\underline{j} + (v_C)_z\underline{k}\right] = \frac{750}{g}(1500\underline{k})$$

Equating the coefficients of the unit vectors in the previous and dividing by $125/g$:

$\underline{i}$: $(v_A)_x + 1000 - 1200 = 0$ $\qquad (v_A)_x = 200$ ft/s (4)

$\underline{j}$: $(v_A)_y + 2200 + 3(v_C)_y = 0$ $\qquad (v_A)_y = -2200 - 3(v_C)_y$ (5)

$\underline{k}$: $(v_A)_z + 4400 + 3(v_C)_z = 9000$ $\qquad (v_A)_z = 4600 - 3(v_C)_z$ (6)

Substituting from (4), (5), (6) into (2) and (3):

$-4600 + 3(v_C)_z + 9(200) + 6(v_C)_z = 7100$, $\qquad (v_C)_z = 1100$ ft/s

$-2200 - 3(v_C)_y - 200 - 6(v_C)_y = 4800$ $\qquad (v_C)_y = -800$ ft/s

Substituting for $(v_C)_y$ and $(v_C)_z$ into (5) and (6):

$(v_A)_y = -2200 - 3(-800)$ $\qquad (v_A)_y = 200$ ft/s

$(v_A)_z = 4600 - 3(1100)$ $\qquad (v_A)_z = 1300$ ft/s

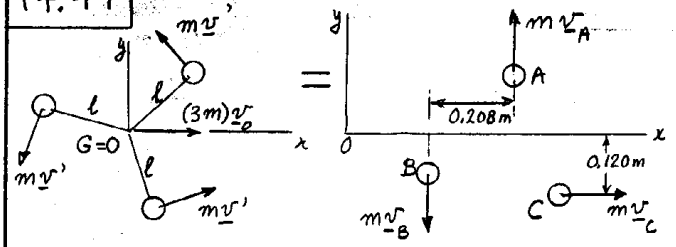
The velocity of part A is therefore

$$\underline{v}_A = (200\ \text{ft/s})\underline{i} + (200\ \text{ft/s})\underline{j} + (1300\ \text{ft/s})\underline{k}$$ ◀

Check: Substituting the values obtained for $(v_A)_z$, $(v_A)_y$, $(v_C)_z$, and $(v_C)_y$ into the left-hand member of Eq. (1), we obtain

$1300 - 9(200) - 12(1100) - 24(-800) = 5500$ (O.K.)

14.44



Given: $\underline{v}_A = (0.732\ \text{m/s})\,\underline{j}$, $\underline{v}_C = (1.2\ \text{m/s})\,\underline{i}$

Conservation of linear momentum:

$$3m\,\underline{v}_0 = m\,\underline{v}_A + m\,\underline{v}_B + m\,\underline{v}_C$$

$\pm\rightarrow$ x comp.: $3mv_0 = m(1.2\ \text{m/s})$ $\quad \underline{v}_0 = 0.4\ \text{m/s} \rightarrow$ ◀

$+\uparrow$ y comp.: $0 = mv_A - mv_B$ $\quad v_B = v_A = 0.732\ \text{m/s}$

Conservation of energy:

Initial: $T_0 = \frac{1}{2}(3m)v_0^2 + \sum_i \frac{1}{2}m_i v_i'^2 = \frac{1}{2}(3m)(0.4)^2 + 3\left(\frac{1}{2}mv'^2\right)$

Final: $T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 = \frac{1}{2}(2m)(0.732)^2 + \frac{1}{2}m(1.2)^2$

$T_0 = T$: $\frac{1}{2}(3m)(0.4)^2 + 3\left(\frac{1}{2}mv'^2\right) = \frac{1}{2}(2m)(0.732)^2 + \frac{1}{2}m(1.2)^2$

$0.240 + 1.5v'^2 = 1.2558$, $v'^2 = 0.6772$, $v' = 0.8229$ m/s

Conservation of angular momentum about O:

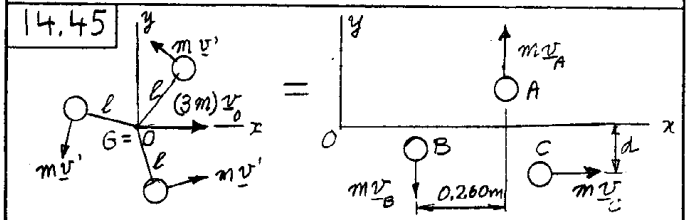
$+\circlearrowleft$ $3mv'\ell = mv_A(0.208) + mv_C(0.120)$

$3m(0.8229)\ell = m(0.732)(0.208) + m(1.2)(0.120)$

$\ell = 0.120\ \text{m}$ $\quad \ell = 120.0\ \text{mm}$ ◀

Rate of rotation $= \frac{v'}{\ell} = \frac{0.8229\ \text{m/s}}{0.120\ \text{m}} = 6.86\ \text{rad/s}$ ◀

14.45



Given: $\ell = 0.150\ \text{m}$, $\underline{v}_0 = (0.3\ \text{m/s})\,\underline{i}$, $v' = 0.6\ \text{m/s}$

Conservation of linear momentum:

$$3m\,\underline{v}_0 = m\,\underline{v}_A + m\,\underline{v}_B + m\,\underline{v}_C$$

$\pm\rightarrow$ x comp.: $3m(0.3\ \text{m/s}) = mv_C$ $\quad \underline{v}_C = 0.900\ \text{m/s} \rightarrow$ ◀

$+\uparrow$ y comp.: $0 = mv_A - mv_B$ $\quad v_B = v_A$ (1)

Conservation of angular momentum about O:

$+\circlearrowleft$ $3mv'\ell = mv_A(0.260) + mv_C d$ [since $v_B = v_A$]

$3m(0.6\ \text{m/s})(0.150\ \text{m}) = mv_A(0.260\ \text{m}) + m(0.900\ \text{m/s})\,d$

$d = 0.3\ \text{m} - 0.289\,v_A$ (2)

Conservation of energy:

Initial: $T_0 = \frac{1}{2}(3m)v_0^2 + \sum_i \frac{1}{2}m_i v_i'^2 = \frac{1}{2}(3m)(0.3)^2 + 3\left[\frac{1}{2}m(0.6)^2\right]$

$= (0.675)m$

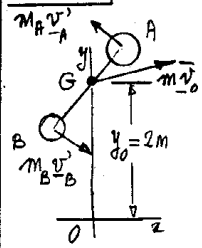
Final: $T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 = \frac{1}{2}(2mv_A^2) + \frac{1}{2}m(0.9)^2$

$T_0 = T$: $0.675m = mv_A^2 + 0.405m$, $v_A^2 = 0.270$, $v_A = 0.520\ \text{m/s}$

$\underline{v}_A = 0.520\ \text{m/s}\uparrow$; $\underline{v}_B = 0.520\ \text{m/s}\downarrow$ ◀

From (2): $d = 0.3\ \text{m} - 0.289(0.520)$, $\quad d = 150\ \text{mm}$ ◀

14.46 Initial conditions



Location of G:

$$\frac{AG}{m_B} = \frac{BG}{m_A} = \frac{AG+GB}{m_B+m_A} = \frac{AB}{m} = \frac{0.6\ \text{m}}{4.5\ \text{kg}}$$

$AG = 1.5\left(\frac{0.6}{4.5}\right) = 0.2\ \text{m}$ $\quad BG = 0.4\ \text{m}$

Linear momentum:

$\underline{L}_0 = m\,\underline{v}_0 = (4.5\ \text{kg})(1.2\,\underline{i} + 0.96\,\underline{j})$

$= 5.4\,\underline{i} + 4.32\,\underline{j}$

Angular momentum:

About G: $(\underline{H}_G)_0 = \overrightarrow{GA} \times m_A\underline{v}_A' + \overrightarrow{GB} \times m_B v_B'$

$= (0.2\ \text{m})(3\ \text{kg})(0.2\ \text{m} \times 10\ \text{rad/s})\,\underline{k}$

$+ (0.4\ \text{m})(1.5\ \text{kg})(0.4\ \text{m} \times 10\ \text{rad/s})\,\underline{k}$

$(\underline{H}_G)_0 = (3.6\ \text{kg}\cdot\text{m}^2/\text{s})\,\underline{k}$

About O: Using formula derived in Prob. 14.22,

$(\underline{H}_O)_0 = \underline{r} \times m\underline{v}_0 + (\underline{H}_G)_0 = 2\,\underline{j} \times (5.4\,\underline{i} + 4.32\,\underline{j}) + 3.6\,\underline{k}$

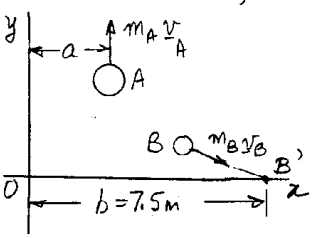
$= -10.8\,\underline{k} + 3.6\,\underline{k} = -(7.2\ \text{kg}\cdot\text{m}^2/\text{s})\,\underline{k}$

Kinetic energy: Using Eq. (14.29):

$T_0 = \frac{1}{2}m\underline{v}_0^2 + \frac{1}{2}\sum_i m_i v_i'^2 = \frac{1}{2}m\underline{v}_0^2 + \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$

$= \frac{1}{2}(4.5)\left[(1.2)^2 + (0.96)^2\right] + \frac{1}{2}(3)(0.2\times 10)^2 + \frac{1}{2}(1.5)(0.4\times 10)^2$

$= 5.3136 + 6 + 12) = 23.314\ \text{J}$



(a) Conservation of linear momentum:

$$\underline{L}_0 = \underline{L}$$

$5.4\,\underline{i} + 4.32\,\underline{j} = m_A\underline{v}_A + m_B\underline{v}_B$

$= 3(v_A\,\underline{j}) + 1.5\left[(v_B)_x\,\underline{i} + (v_B)_y\,\underline{j}\right]$

Equating coeff. of $\underline{i}$: $5.4 = 1.5(v_B)_x$

$(v_B)_x = 3.6\ \text{m/s}$ (1)

Equating coeff. of $\underline{j}$: $4.32 = 3v_A + 1.5(v_B)_y$

$(v_B)_y = 2.88 - 2v_A$ (2)

Conservation of energy

$T_0 = T$: $\quad T_0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$

$23.314\ \text{J} = \frac{1}{2}(3)(v_A^2) + \frac{1}{2}(1.5)\left[(v_B)_x^2 + (v_B)_y^2\right]$

Substituting from (1) and (2):

$23.314 = 1.5v_A^2 + 0.75(3.6)^2 + 0.75(2.88 - 2v_A)^2$

$4.5v_A^2 - 8.64v_A - 7.373 = 0$

$v_A^2 - 1.92v_A - 1.6384 = 0$

$v_A = 0.96 + 1.60 = 2.56\ \text{m/s}$ $\quad \underline{v}_A = 2.56\ \text{m/s}\uparrow$ ◀

and $v_A = 0.96 - 1.60 = -0.64\ \text{m/s}$ (rejected, since $\underline{v}_A$ is shown directed up)

From (1) and (2): $(v_B)_x = 3.6\ \text{m/s}$, $(v_B)_y = 2.88 - 2(2.56) = -2.24\ \text{m/s}$

$\underline{v}_B = 3.6\,\underline{i} - 2.24\,\underline{j}$ $\quad \underline{v}_B = 4.24\ \text{m/s} \searrow 31.9°$ ◀

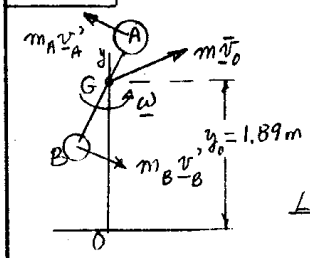
(b) Conservation of angular momentum about O:

$(\underline{H}_O)_0 = \underline{H}_O$: $\quad -7.2\,\underline{k} = a\,\underline{i} \times m_A\underline{v}_A + b\,\underline{i} \times m_B\underline{v}_B$

$-7.2\,\underline{k} = a\,\underline{i} \times 3(2.56\,\underline{j}) + 7.5\,\underline{i} \times 1.5(3.6\,\underline{i} - 2.24\,\underline{j})$

$-7.2\,\underline{k} = 7.68a\,\underline{k} - 25.2\,\underline{k}$ $\quad a = 2.34\ \text{m}$ ◀

14.47 Initial conditions



Location of G:

$$\frac{\overline{AG}}{m_B} = \frac{\overline{BG}}{m_A} = \frac{\overline{AG}+\overline{GB}}{m_B+m_A} = \frac{\ell}{m}$$

$$\overline{AG} = \frac{m_B}{m}\ell = \frac{1}{3}\ell$$

$$\overline{BG} = \frac{m_A}{m}\ell = \frac{2}{3}\ell$$

Linear momentum:

$$\underline{L}_0 = m\underline{\bar{v}}_0 = 3\underline{\bar{v}}_0$$

Angular momentum about G:

$$(\underline{H}_G)_0 = \overrightarrow{GA} \times m_A \underline{v}_A' + \overrightarrow{GB} \times m_B \underline{v}_B'$$

$$= \left(\tfrac{1}{3}\ell\right)(2\,kg)\left(\tfrac{1}{3}\ell\omega\right)\underline{k} + \left(\tfrac{2}{3}\ell\right)(1\,kg)\left(\tfrac{2}{3}\ell\omega\right)\underline{k}$$

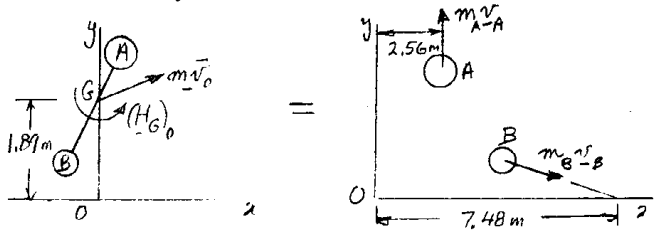
$$= \frac{2}{3}\ell^2\omega\,\underline{k}$$

Kinetic energy: Using Eq. (14.29):

$$T_0 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}\sum_i m_i \bar{v}_i'^2 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

$$= \tfrac{1}{2}(3)\bar{v}_0^2 + \tfrac{1}{2}(2)\left(\tfrac{1}{3}\ell\omega\right)^2 + \tfrac{1}{2}(1)\left(\tfrac{2}{3}\ell\omega\right)^2$$

$$T_0 = \frac{3}{2}\bar{v}_0^2 + \frac{1}{3}\ell^2\omega^2$$



Conservation of linear momentum:

$$m\underline{\bar{v}}_0 = m_A\underline{v}_A + m_B\underline{v}_B$$

$$3\underline{\bar{v}}_0 = (2)(5\underline{j}) + (1)(7.2\underline{i} - 4.6\underline{j}) = 7.2\underline{i} + 5.4\underline{j}$$

$$\underline{\bar{v}}_0 = (2.4\,m/s)\underline{i} + (1.8\,m/s)\underline{j}$$

Conservation of angular momentum about O:

$$+\circlearrowleft (1.89\underline{j}) \times m\underline{\bar{v}}_0 + (\underline{H}_G)_0 = (2.56\underline{i}) \times m_A\underline{v}_A + (7.48\underline{i}) \times m_B\underline{v}_B$$

Substituting for $\underline{\bar{v}}_0$, $(\underline{H}_G)_0$, $\underline{v}_A$, $\underline{v}_B$ and the masses:

$$(1.89\underline{j}) \times 3(2.4\underline{i} + 1.8\underline{j}) + \frac{2}{3}\ell^2\omega\,\underline{k} =$$

$$= (2.56\underline{i}) \times 2(5\underline{j}) + (7.48\underline{i}) \times (7.2\underline{i} - 4.6\underline{j})$$

$$-13.608\,\underline{k} + \frac{2}{3}\ell^2\omega\,\underline{k} = 25.6\,\underline{k} - 34.408\,\underline{k}$$

$$\frac{2}{3}\ell^2\omega = 4.80 \qquad \ell^2\omega = 7.20 \qquad (1)$$

Conservation of energy:

$$T_0 = T: \quad \frac{3}{2}\bar{v}_0^2 + \frac{1}{3}\ell^2\omega^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\frac{3}{2}\left[(2.4)^2 + (1.8)^2\right] + \frac{1}{3}\ell^2\omega^2 = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)\left[(7.2)^2 + (4.6)^2\right]$$

$$13.5 + \frac{1}{3}\ell^2\omega^2 = 25 + 36.5 \qquad \ell^2\omega^2 = 144.0 \qquad (2)$$

Dividing (2) by (1), member by member.

$$\omega = \frac{144.0}{7.20} = 20.0\,rad/s$$

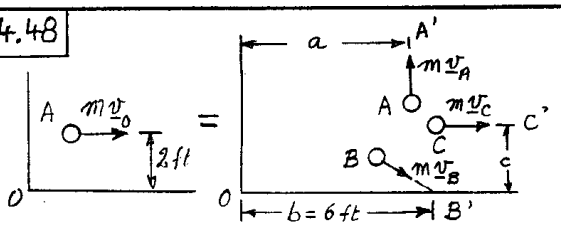
Original rate of spin = 20.0 rad/s ◀

Substituting for ω into Eq. (1):

$$\ell^2(20.0) = 7.20 \qquad \ell^2 = 0.360 \qquad \ell = 0.600\,m$$

Length of cord = 600 mm ◀

14.48



Conservation of linear momentum:

$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$$

$\pm\rightarrow$ comp: $\quad v_0 = (v_B)_x + v_C \qquad (1)$

$+\uparrow$ comp: $\quad 0 = v_A + (v_B)_y \qquad (2)$

Conservation of angular momentum about O:

$$2\underline{j} \times m\underline{v}_0\underline{i} = a\underline{i} \times m\underline{v}_A\underline{j} + 6\underline{i} \times m\left[(v_B)_x\underline{i} + (v_B)_y\underline{j}\right] + c\underline{j} \times m\underline{v}_C\underline{i}$$

$$-2v_0\underline{k} = av_A\underline{k} + 6(v_B)_y\underline{k} - cv_C\underline{k}$$

$$av_A + 6(v_B)_y - cv_C + 2v_0 = 0 \qquad (3)$$

Conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$v_A^2 + (v_B)_x^2 + (v_B)_y^2 + v_C^2 = v_0^2 \qquad (4)$$

Given data: $v_0 = 10\,ft/s$, $v_A = 4\,ft/s$, $a = 7\,ft$

Eq. (1): $10 = (v_B)_x + v_C \qquad (v_B)_x = 10 - v_C$

Eq. (2): $0 = 4 + (v_B)_y \qquad (v_B)_y = -4\,ft/s$

Eq. (4): $(4)^2 + (10 - v_C)^2 + (-4)^2 + v_C^2 = (10)^2$

$$16 + 100 - 20v_C + v_C^2 + 16 + v_C^2 = 100$$

$$2v_C^2 - 20v_C + 32 = 0$$

$$2(v_C - 2)(v_C - 8) = 0$$

Assume $v_C = 2$. Then Eq. (3) yields

$$7(4) + 6(-4) - c(2) + 2(10) = 0$$

$c = 12\,ft$ (Impossible since side = 10 ft)

Thus: $v_C = 8\,ft/s$ and Eq. (3) yields

$$7(4) + 6(-4) - c(8) + 2(10) = 0 \qquad c = 3.00\,ft$$ ◀

Eq. (1): $(v_B)_x = 10 - 8 = 2\,ft/s$, $\underline{v}_B = (2\,ft/s)\underline{i} - (4\,ft/s)\underline{j}$

$v_B = 4.47\,ft/s \searrow 63.4°$; $\underline{v}_C = 8.00\,ft/s \rightarrow$ ◀

14.49

See solution of Prob. 14.48 for diagram and derivation of Eqs. (1), (2), (3), and (4).

Given data: $v_0 = 12\,ft/s$, $v_C = 7\,ft$, $c = 3\,ft$

Eq. (1): $12 = (v_B)_x + 7 \qquad (v_B)_x = 5\,ft/s$

Eq (2): $0 = v_A + (v_B)_y \qquad (v_B)_y = -v_A$

Eq. (4): $v_A^2 + (5)^2 + v_A^2 + (7)^2 = (12)^2$

$$v_A^2 = 35 \qquad v_A = 5.916 \qquad \underline{v}_A = 5.92\,ft/s \uparrow$$ ◀

Eq. (2): $(v_B)_y = -5.916 \qquad \underline{v}_B = (5\,ft/s)\underline{i} - (5.916\,ft/s)\underline{j}$

$$v_B = 7.75\,ft/s \searrow 49.8°$$ ◀

Eq. (3): $a(5.916) + 6(-5.916) - 3(7) + 2(12) = 0$

$$a = 5.49\,ft$$ ◀

14.50 Eq.(14.38):

$$(\Delta m)\underline{v}_A + \Sigma \underline{F}\,\Delta t = (\Delta m)\underline{v}_B$$

where $\underline{v}_A = 0$, $\underline{v}_B = 35\text{ m/s} \measuredangle 30°$

$$\frac{\Delta m}{\Delta t} = (12\text{ m}^3/\text{min})(1000\text{ kg/m}^3)\frac{1\text{ min}}{60\text{ s}} = 200\text{ kg/s}$$

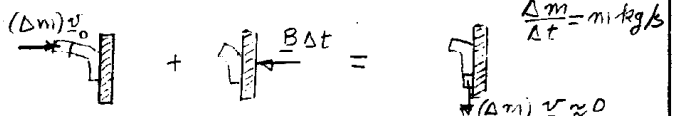
Thus, the force exerted on the water stream is

$$\Sigma \underline{F} = \frac{\Delta m}{\Delta t}\underline{v}_B = (200\text{ kg/s})(35\text{ m/s}) = 7.00\times10^3\text{ N} \measuredangle 30°$$

The force exerted by the water on the boat is

7.00 kN ⦜ 30° ◀

14.51 Plate B and sand stream

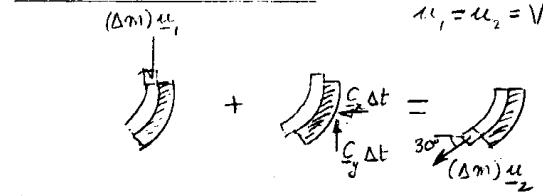


$\pm$ x comp.: $(\Delta m)v_0 - B\Delta t = 0$

$$B = \frac{\Delta m}{\Delta t}v_0 \qquad \underline{B} = m v_0 \leftarrow$$ ◀

Plate C and sand stream

$$v_1 = u_2 = \sqrt{2gh}$$



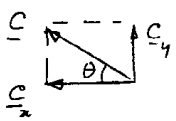
$\pm$ x comp.: $0 + C_x\Delta t = (\Delta m)u_2\cos 30°$

$$C_x = \frac{\Delta m}{\Delta t}u_2\cos 30° = m\sqrt{2gh}\,\frac{\sqrt{3}}{2} = \frac{1}{2}m\sqrt{6gh}$$

$+\downarrow$ y comp.: $(\Delta m)u_1 - C_y\Delta t = (\Delta m)u_2\sin 30°$

$$C_y = \frac{\Delta m}{\Delta t}(u_1 - u_2\sin 30°) = m\sqrt{2gh}\,(1-\tfrac{1}{2})$$

$$C_y = \frac{1}{2}m\sqrt{2gh}$$



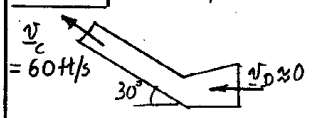
$$C^2 = C_x^2 + C_y^2 = \left(\frac{m}{2}\right)^2(6gh + 2gh) = 2m^2gh$$

$$C = m\sqrt{2gh}$$

$$\tan\theta = \frac{C_y}{C_x} = \frac{\sqrt{2gh}}{\sqrt{6gh}} = \frac{1}{\sqrt{3}}, \qquad \theta = 30°$$

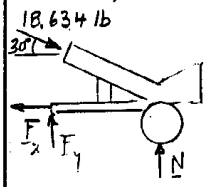
$$\underline{C} = m\sqrt{2gh} \measuredangle 30°$$ ◀

14.52 Eq.(14.38): $(\Delta m)\underline{v}_D + \Sigma \underline{F}\,\Delta t = (\Delta m)\underline{v}_C$



$$\Sigma \underline{F} = \left(\frac{\Delta m}{\Delta t}\right)\underline{v}_C = \frac{10\text{ lb/s}}{32.2\text{ ft/s}^2}(60\text{ ft/s} \measuredangle 30°)$$

Force exerted on wood chips: $\Sigma \underline{F} = 18.634\text{ lb} \measuredangle 30°$



Free Body: Shredder

$\pm \Sigma F_x = 0$: $(18.634\text{ lb})\cos 30° - F_x = 0$

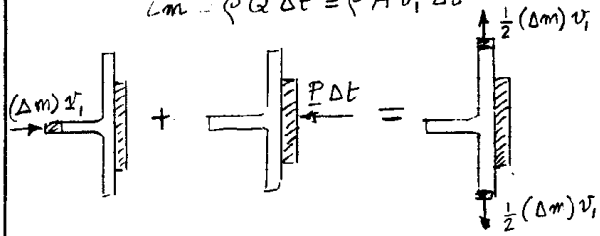
$$\underline{F}_x = 16.14\text{ lb} \leftarrow$$

Force exerted by shredder on truck:

$$= -\underline{F}_x = 16.14\text{ lb} \rightarrow$$ ◀

14.53 The mass entering the system and leaving it in time Δt is

$$\Delta m = \rho Q\,\Delta t = \rho A v_1\,\Delta t$$



Principle of impulse and momentum:

$\pm$ x comp.: $(\Delta m)v_1 - P\Delta t = 0$

$$P = \frac{\Delta m}{\Delta t}v_1 = (\rho A v_1)v_1 = \rho A v_1^2$$

With given data, $\rho = 1000\text{ kg/m}^3$, $A = 500\times10^{-6}\text{ m}^2$, $v_1 = 25\text{ m/s}$:

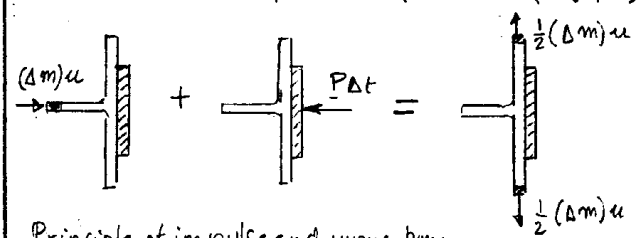
$$P = (1000\text{ kg/m}^3)(500\times10^{-6}\text{ m}^2)(25\text{ m/s})^2$$

$$P = 312\text{ N}$$ ◀

14.54 We measure velocities with respect to the plate. Velocity of stream relative to plate $= u = v_1 - V$

The mass entering the system and leaving it in time Δt is

$$\Delta m = \rho Q\,\Delta t = \rho A u\,\Delta t = \rho A(v_1 - V)\Delta t$$



Principle of impulse and momentum:

$\pm$ x comp.: $(\Delta m)u - P\Delta t = 0$

$$P = \frac{\Delta m}{\Delta t}u = (\rho A u)u = \rho A u^2 = \rho A(v_1 - V)^2$$

With given data, $\rho = 1000\text{ kg/m}^3$, $A = 600\times10^{-6}\text{ m}^2$, $v_1 = 30\text{ m/s}$, $P = 400\text{ N}$:

$$400\text{ N} = (1000\text{ kg/m}^3)(600\times10^{-6}\text{ m}^2)(30\text{ m/s} - V)^2$$

$$(30 - V)^2 = 666.67\text{ m}^2/\text{s}^2 \qquad 30 - V = 25.82\text{ m/s}$$

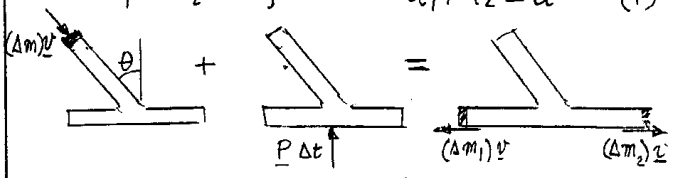
$$V = 4.18\text{ m/s}$$ ◀

14.55 The mass of water entering the system in the time Δt is

$$\Delta m = \rho Q \Delta t$$

This mass is split into $\Delta m_1 = \rho Q_1 \Delta t$ and $\Delta m_2 = \rho Q_2 \Delta t$.

Since $\Delta m_1 + \Delta m_2 = \Delta m$, we have $Q_1 + Q_2 = Q$ (1)



Principle of impulse and momentum:

(a) $\pm\rightarrow$ x comp.: $(\Delta m)v\sin\theta = (\Delta m_2)v - (\Delta m_1)v$

$$(\rho Q \Delta t)v\sin\theta = (\rho Q_2 \Delta t)v - (\rho Q_1 \Delta t)v$$

$$Q\sin\theta = Q_2 - Q_1 \quad (2)$$

Substituting for Q from (1) and solving for $\sin\theta$:

$$\sin\theta = \frac{Q_2 - Q_1}{Q_2 + Q_1} = \frac{150-30}{150+30} = 0.6667 \qquad \theta = 41.8°$$ ◀

(b) $+\uparrow$ y comp.: $(\Delta m)v\cos\theta - P\Delta t = 0$

$$P = \frac{\Delta m}{\Delta t} v\cos\theta = \rho Q v\cos\theta = \frac{\gamma}{g} Q v\cos\theta \quad (3)$$

$$P = \frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}(180\ \text{gal/min})\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\frac{1\ \text{min}}{60\ \text{s}}(90\ \text{ft/s})\cos 41.8°$$

$$P = 52.1\ \text{lb}\uparrow$$

Force exerted by streams on plates: $-P = 52.1\ \text{lb}\downarrow$ ◀

14.56 See solution of Prob. 14.55 for derivation of Eqs. (1), (2), and (3).

Eq. (3): $P = \frac{\gamma}{g} Q v\cos\theta$

$$Q = \frac{g}{\gamma}\frac{P}{v\cos\theta} = \frac{32.2\ \text{ft/s}^2}{62.4\ \text{lb/ft}^3}\frac{90\ \text{lb}}{(120\ \text{ft/s})\cos 25°} = 0.42703\ \text{ft}^3\text{/s}$$

$$= (0.42703\ \text{ft}^3\text{/s})\frac{7.48\ \text{gal}}{1\ \text{ft}^3}\frac{60\ \text{s}}{1\ \text{min}} = 191.65\ \text{gal/min}$$

Recalling Eqs. (1) and (2):

$$Q_2 + Q_1 = Q \quad (1)$$

$$Q_2 - Q_1 = Q\sin\theta \quad (2)$$

Adding: $2Q_2 = Q(1+\sin\theta)$ $\quad Q_2 = \frac{1}{2}Q(1+\sin\theta)$

Subtracting: $2Q_1 = Q(1-\sin\theta)$ $\quad Q_1 = \frac{1}{2}Q(1-\sin\theta)$

Substituting $\theta = 25°$ and $Q = 191.65$ gal/min:

$$Q_1 = \tfrac{1}{2}Q(1-\sin\theta) = \tfrac{1}{2}(191.65)(1-\sin 25°)$$

$$Q_1 = 55.3\ \text{gal/min}$$ ◀

$$Q_2 = \tfrac{1}{2}Q(1+\sin\theta) = \tfrac{1}{2}(191.65)(1+\sin 25°)$$

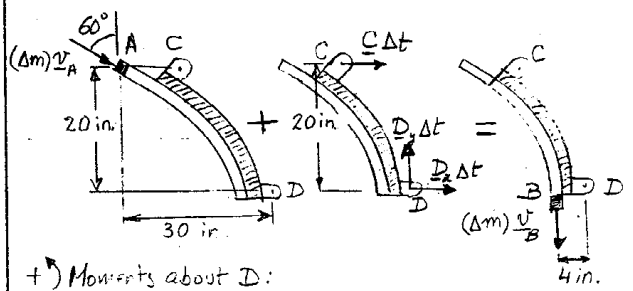
$$Q_2 = 136.3\ \text{gal/min}$$ ◀

14.57 The mass of water entering the system and leaving it in the time Δt is

$$\Delta m = \rho Q \Delta t = \frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}(250\ \text{gal/min})\frac{1\ \text{min}}{60\ \text{s}}\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\Delta t$$

$$\Delta m = 1.0795\ \Delta t \quad (1)$$

We apply the principle of impulse and momentum to the system formed by the blade, the particles in the portion of jet shown, and those striking the blade in the time Δt:



$+\circlearrowleft$ Moments about D:

$$-(\Delta m)v_A\sin 60°(20\ \text{in.}) + (\Delta m)v_A\cos 60°(30\ \text{in.}) - C\Delta t(20\ \text{in.}) = (\Delta m)v_B(4\ \text{in.})$$

$$20C = \frac{\Delta m}{\Delta t}(30v_A\cos 60° - 20v_A\sin 60° - 4v_B)$$

Substituting for Δm from (1), and recalling that $v_A = v_B = 90$ ft/s,

$$20C = 1.0795(90)(30\cos 60° - 20\sin 60° - 4) = -614.06$$

$$C = -30.7\ \text{lb} \qquad \underline{C} = 30.7\ \text{lb} \leftarrow$$ ◀

$\pm\rightarrow$ x components:

$$(\Delta m)v_A\sin 60° + C\Delta t + D_x\Delta t = 0$$

$$D_x = -\frac{\Delta m}{\Delta t}v_A\sin 60° - C = -1.0795(90)\sin 60° + 30.7$$

$$D_x = -53.4\ \text{lb} \qquad \underline{D}_x = 53.4\ \text{lb} \leftarrow$$ ◀

$+\uparrow$ y components:

$$-(\Delta m)v_A\cos 60° + D_y\Delta t = -(\Delta m)v_B$$

$$D_y = \frac{\Delta m}{\Delta t}(v_A\cos 60° - v_B) = 1.0795(90)(\cos 60° - 1)$$

$$D_y = -48.6\ \text{lb} \qquad \underline{D}_y = 48.6\ \text{lb}\downarrow$$ ◀

14.58 The mass of water entering the system and leaving it in the time Δt is

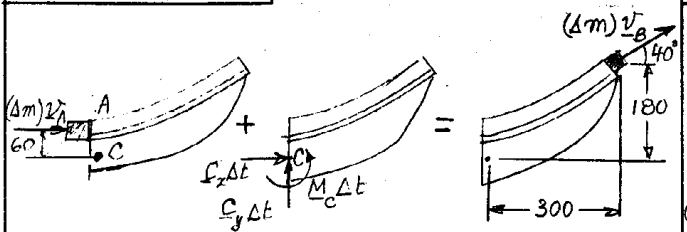
$$\Delta m = \rho Q \Delta t = (1000\ \text{kg/m}^3)(1.2\ \text{m}^3\text{/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\Delta t$$

$$\Delta m = (20\ \text{kg/s})\Delta t \quad (1)$$

We apply the principle of impulse and momentum to the system formed by the blade, the particles of water in contact with the blade, and those striking the blade in the time Δt.

(CONTINUED)

14.58 CONTINUED Dimensions in mm



$\pm\rightarrow$ x comp: $(\Delta m)v_A + C_x\Delta t = (\Delta m)v_B\cos 40°$ (2)

$+\uparrow$ y comp: $0 + C_y\Delta t = (\Delta m)v_B\sin 40°$ (3)

$+\circlearrowleft$ mom. about C: $-(\Delta m)v_A(60) + M_C\Delta t = (\Delta m)v_B(300\sin 40° - 180\cos 40°)$ (4)

Recalling Eq. (1) and the fact that $v_A = v_B = 50$ m/s, we solve Eqs. (2), (3), (4) for C_x, C_y, M_C, respectively:

From (2): $C_x = \frac{\Delta m}{\Delta t}(v_B\cos 40° - v_A)$

$= (20\ \text{kg/s})(50\ \text{m/s})(\cos 40° - 1)$

$= -233.96$ N $\quad \underline{C_x = 234\ \text{N}} \leftarrow$ ◀

From (3): $C_y = \frac{\Delta m}{\Delta t}v_B\sin 40° = (20\ \text{kg/s})(50\ \text{m/s})\sin 40°$

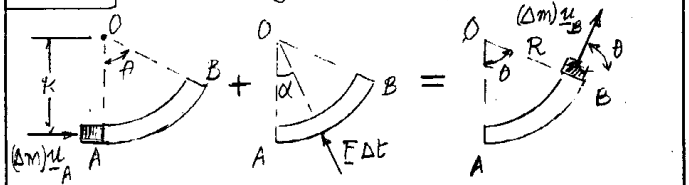
$= 642.8$ N $\quad \underline{C_y = 643\ \text{N}} \uparrow$ ◀

From (4): $M_C = \frac{\Delta m}{\Delta t}[v_B(300\sin 40° - 180\cos 40°) + 60v_A]$

$= (20\ \text{kg/s})(50\ \text{m/s})(300\sin 40° - 180\cos 40° + 60)\text{mm}$

$= 114.9\times 10^3$ N·mm $\quad \underline{M_C = 114.9\ \text{N·m}}$ ◀

14.59 Surfaces of jet are in shape of arcs of circle



We recall that $u_A = v_B = u$

$+\circlearrowleft$ Moments about O: $R(\Delta m)u$ + Mom. of $\underline{F}$ = $R(\Delta m)u$

Thus: Mom. of $\underline{F}$ about O = 0; line of action of $\underline{F}$ passes through O.

$\pm\rightarrow$ x comp.: $(\Delta m)u - (F\Delta t)\sin\alpha = (\Delta m)u\cos\theta$

or $(F\Delta t)\sin\alpha = (\Delta m)u(1-\cos\theta)$ (1)

$+\uparrow$ y comp.: $0 + (F\Delta t)\cos\alpha = (\Delta m)u\sin\theta$ (2)

Dividing (1) by (2), member by member:

$$\tan\alpha = \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

Thus: $\tan\alpha = \tan\frac{\theta}{2}$, $\quad \alpha = \frac{\theta}{2}$

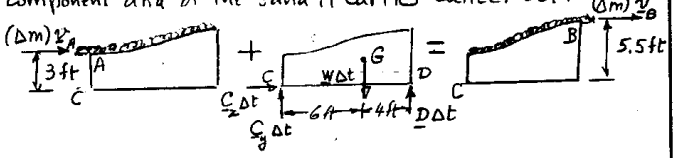
The line of action of $\underline{F}$ bisects $\angle AOB$; $\underline{F}$ is applied at midpoint C of arc AB ◀

(Q.E.D.)

14.60 The mass of sand entering and leaving the system per second is

$$\frac{\Delta m}{\Delta t} = \frac{240}{32.2} = 7.453$$

We apply the principle of impulse and momentum to the system formed by the final component, the sand it carries, and the sand it receives in the time Δt. We note that the initial and final momenta of the moving parts of the component and of the sand it carries cancel out.



$\pm\rightarrow$ x comp.: $(\Delta m)v_A + C_x\Delta t = (\Delta m)v_B$

$C_x = \frac{\Delta m}{\Delta t}(v_B - v_A) = 7.453(15-15) = 0$

$+\circlearrowleft$ moments about C: $-3(\Delta m)v_A + 10D\Delta t - 6W\Delta t = -5.5(\Delta m)v_B$

$10D = 6W + \frac{\Delta m}{\Delta t}(15)(3-5.5) = 6(900) + (7.453)(15)(-2.5)$

$= 5120$ $\quad \underline{D = 512\ \text{lb}} \uparrow$ ◀

$+\uparrow$ y comp.: $0 + C_y\Delta t - W\Delta t + D\Delta t = 0$

$C_y - 900 + 512 = 0$ $\quad C_y = 388$ lb

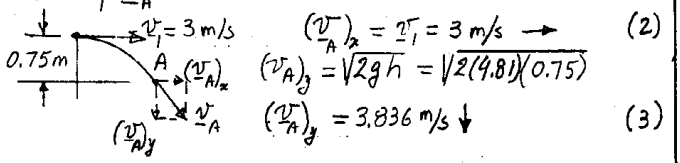
Since $C_x = 0$, $\quad \underline{C = 388\ \text{lb}} \uparrow$ ◀

Note: If component was stopped, we would have $\underline{C = 360\ \text{lb}} \uparrow$, $\underline{D = 540\ \text{lb}} \uparrow$

14.61 Mass of coal entering system and leaving it in time interval Δt:

$\Delta m = (150\ \text{kg/s})\Delta t$ (1)

Velocity $\underline{v_A}$ with which coal hits second belt:

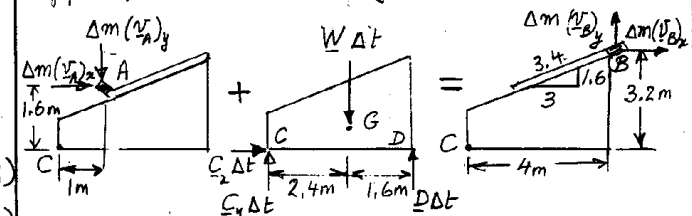


$(v_A)_x = v_1 = 3$ m/s → (2)

$(v_A)_y = \sqrt{2gh} = \sqrt{2(4.81)(0.75)}$

$(v_A)_y = 3.836$ m/s ↓ (3)

We apply the principle of impulse and momentum to the second conveyor belt assembly, the coal it supports, and the coal hitting it in time Δt:



$+\circlearrowleft$ mom. about C:

$(1.6)\Delta m(v_A)_x + (1)\Delta m(v_A)_y + (2.4)W\Delta t - (4)D\Delta t = (3.2)\Delta m(v_B)_x - (4)\Delta m(v_B)_y$

Recalling (1), (2), (3) and solving for D:

$4D = 2.4W + \frac{\Delta m}{\Delta t}[(1.6)(v_A)_x + (1)(v_A)_y - (3.2)(v_B)_x + (4)(v_B)_y]$

$= 2.4W + 150[(1.6)(3) + (1)(3.836) - (3.2)(\frac{3}{3.4}v_B) + (4)(\frac{1.6}{3.4}v_B)]$

$D = 0.6W + 323.9 - \frac{120}{3.4}v_B$ (4)

But $W = (500\ \text{kg})(9.81\ \text{m/s}^2) = 4905$ N, $\quad v_B = 4.25$ m/s

Thus: $D = 0.6(4905) + 323.9 - 150 = 3117$ N, $\quad \underline{D = 3120\ \text{N}} \uparrow$ ◀

(CONTINUED)

14.61 CONTINUED

$\pm$ x comp: $\Delta m(v_A)_x + C_x \Delta t = \Delta m(v_B)_x = \Delta m\left(\frac{3}{3.4} v_B\right)$

$C_x = \frac{\Delta m}{\Delta t}\left[\frac{3}{3.4} v_B - (v_A)_x\right] = 150\left(\frac{3}{3.4} v_B - 3\right)$

$$C_x = \frac{450}{3.4} v_B - 450 \qquad (5)$$

Since $v_B = 4.25$ m/s: $C_x = \frac{450}{3.4}(4.25) - 450 = 112.5$ N

$\underline{C_x = 112.5 \text{ N} \rightarrow}$ ◀

$+\uparrow$ y comp: $-\Delta m (v_A)_y + C_y \Delta t - W\Delta t + D\Delta t = \Delta m (v_B)_y$

$C_y = W - D + \frac{\Delta m}{\Delta t}\left[(v_A)_y + \frac{1.6}{3.4} v_B\right] = W - D + 150\left(3.836 + \frac{1.6}{3.4} v_B\right)$

$$C_y = W - D + 575.4 + \frac{240}{3.4} v_B \qquad (6)$$

Making $W = 4905$ N, $D = 3117$ N, and $v_B = 4.25$ m/s:

$C_y = 4905 - 3117 + 575.4 + \frac{240}{3.4}(4.25) = 2663$ N, $\underline{C_y = 2660 \text{ N} \uparrow}$ ◀

Note: At rest, $C_x = 0$, $C_y = 1962$ N $\uparrow$, $D = 2940$ N $\uparrow$

14.62 See solution of Prob. 14.61 for diagram and derivation of Eqs. (4), (5), and (6):

$$D = 0.6W + 323.9 - \frac{120}{3.4} v_B \qquad (4)$$

$$C_x = \frac{450}{3.4} v_B - 450 \qquad (5)$$

$$C_y = W - D + 575.4 + \frac{240}{3.4} v_B \qquad (6)$$

We now have $W = (500 \text{ kg} + 50 \text{ kg})(9.81 \text{ m/s}^2) = 5396$ N

$v_B = 2.55$ m/s

Substituting in Eqs. (4) and (5):

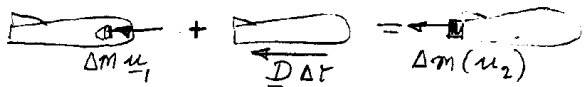
$D = 0.6(5396) + 323.9 - \frac{120}{3.4}(2.55)$, $\underline{D = 3470 \text{ N} \uparrow}$ ◀

$C_x = \frac{450}{3.4}(2.55) - 450 = -112.5$ N $\underline{C_x = 112.5 \text{ N} \leftarrow}$ ◀

Substituting for W, D, and v_B in Eq. (6):

$C_y = 5396 - 3470 + 575.4 + \frac{240}{3.4}(2.55)$ $\underline{C_y = 2680 \text{ N} \uparrow}$ ◀

14.63 We apply the principle of impulse and momentum, measuring velocities with respect to the airplane.



$\pm$ $\quad \Delta m u_1 + D\Delta t = \Delta m u_2$

$$D = \frac{\Delta m}{\Delta t}(u_2 - u_1) \qquad (1)$$

Data: $u_1 = v = 900$ km/h $= 250$ m/s

$u_2 = 600$ m/s

$D = 35$ kN $= 35 \times 10^3$ N

Substituting into Eq. (1):

$35 \times 10^3 = \frac{\Delta m}{\Delta t}(600 - 250)$

$\underline{\frac{\Delta m}{\Delta t} = 100.0 \text{ kg/s}}$ ◀

14.64 See sketch and derivation of Eq. (1) in Solution of Prob. 14.63:

$$D = \frac{\Delta m}{\Delta t}(u_2 - u_1) \qquad (1)$$

Data: $u_1 = v = 540$ mi/h $= 792$ ft/s, $u_2 = 2000$ ft/s

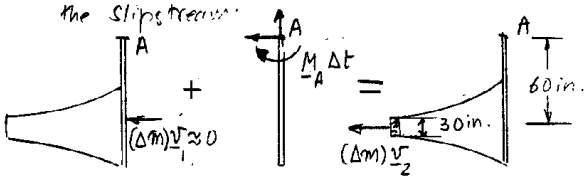
$\frac{\Delta m}{\Delta t} = \frac{150 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 4.6584$ lb·s/ft

Substituting in Eq. (1):

$D = (4.6584 \text{ lb·s/ft})(2000 - 792)$ ft/s $= 5627$ lb

$\underline{D = 5.63 \text{ kips}}$ ◀

14.65 We apply the principle of impulse and momentum to the fan and the air in the slipstream:



$+\circlearrowleft$ moments about A:

$0 + M_A \Delta t = (\Delta m) v_2 \left(\frac{60}{12} \text{ ft}\right)$

$$M_A = \frac{\Delta m}{\Delta t} v_2 (5) \qquad (1)$$

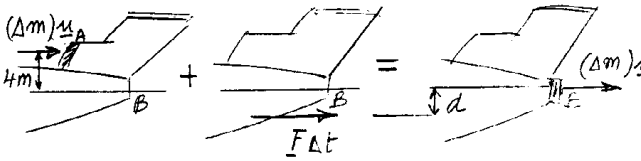
But $\frac{\Delta m}{\Delta t} = \rho Q = \frac{\gamma}{g} A v_2 = \frac{0.076}{32.2} \pi \left(\frac{15}{12}\right)^2 v_2 = 0.011586 v_2$

Substituting for $\frac{\Delta m}{\Delta t}$ in Eq. (1) and making $M_A = 80$ lb·ft,

$80 = (0.011586 v_2) v_2 (5)$

$v_2^2 = 1381$ $\qquad v = 37.2$ ft/s ◀

14.66 We apply the principle of impulse and momentum, measuring velocities with respect to the airplane:



$\pm$ x comp: $(\Delta m) u_A + F\Delta t = (\Delta m) u_B$

$$F = \frac{\Delta m}{\Delta t}(u_B - u_A) = (90 \text{ kg/s})(750 - u_A) \qquad (1)$$

$+\circlearrowleft$ mom. about B: $-4(\Delta m) u_A + (F\Delta t) d = 0$

$d = \frac{\Delta m}{\Delta t} \cdot \frac{4u_A}{F} = (90)\frac{4u_A}{90(750 - u_A)}$

$$d = \frac{4u_A}{750 - u_A} \qquad (2)$$

(a) $u_A = v = 500$ km/h $= 138.89$ m/s

Eq. (1): $F = 90(750 - 138.89)$ $\qquad F = 55.0$ kN ◀

Eq. (2): $d = \frac{4(138.89)}{750 - 138.89} = 0.9091$ m

Line of action 0.909 m below B ◀

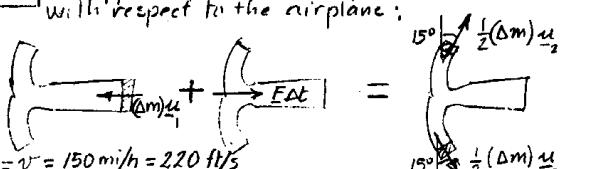
(b) $u_A = v = 1000$ km/h $= 277.78$ m/s

Eq. (1): $F = 90(750 - 277.78)$ $\qquad F = 42.5$ kN ◀

Eq. (2): $d = \frac{4(277.78)}{750 - 277.78} = 2.35$ m below B ◀

Note: Propulsive thrust is equal and opposite to force F.

14.67 We apply the principle of impulse and momentum with respect to the airplane:



$u_1 = v = 150 \text{ mi/h} = 220 \text{ ft/s}$

$\pm\rightarrow x$ comp.: $-(\Delta m)u_1 + F\Delta t = 2\left[\frac{1}{2}(\Delta m)u_2 \sin 15°\right]$

$$F = \frac{\Delta m}{\Delta t}(u_1 + u_2 \sin 15°) = \frac{250 \text{ lb/s}}{32.2 \text{ ft/s}^2}(220 + 2000 \sin 15°)$$

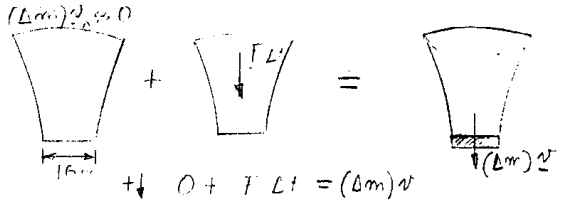
$$= (7.764 \text{ lb·s/ft})(737.6 \text{ ft/s}) = 5727 \text{ lb} \qquad F = 5730 \text{ lb} \blacktriangleleft$$

The reversed thrust is equal and opposite to the force F exerted on the stream.

14.68 Since the helicopter is in equilibrium, the lift is equal and opposite to the gross weight W. Since it is also equal and opposite to the force F exerted by the propeller on the slipstream, we have

$$F = W \qquad (1)$$

Applying the principle of impulse and momentum to the slipstream as the mass Δm entering and leaving in the interval Δt, we have



$$+\downarrow \quad 0 + F\Delta t = (\Delta m)v$$

$$F = \left(\frac{\Delta m}{\Delta t}\right)v = \rho Q v = \rho(Av)v = \rho A v^2$$

Recalling (1): $W = \rho A v^2 \qquad (2)$

Data: $v = 30 \text{ m/s}$, $\rho = 1.21 \text{ kg/m}^3$, $A = \pi(8 \text{ m})^2 = 64\pi \text{ m}^2$

Thus: $W = (1.21 \text{ kg/m}^3)(64\pi \text{ m}^2)(30 \text{ m/s})^2 = 218.96 \text{ kN}$

Gross mass $= \frac{W}{g} = \frac{218.16 \text{ kN}}{9.81 \text{ m/s}^2} = 22.32 \text{ Mg}$

Mass of max. payload and fuel load $= 22.32 - 12 = 10.32 \text{ Mg}$ ◀

14.69 We recall Eq. (1) derived for Prob. 14.63:

$$D = \frac{\Delta m}{\Delta t}(u_2 - u_1) \qquad (1)$$

where $u_1 = v$ = velocity of airplane

$u_2 = 800 \text{ m/s}$ = exhaust velocity.

Substituting $D = kv^2$ in Eq. (1), we write

$$\frac{\Delta m}{\Delta t}(800 - v) = kv^2 \qquad (2)$$

With 3 engines, we know that $v_0 = 900 \text{ km/h} = 250 \text{ m/s}$. Substituting into Eq. (2):

$$\left(\frac{\Delta m}{\Delta t}\right)_0 (800 - 250) = k(250)^2, \quad \left(\frac{\Delta m}{\Delta t}\right)_0 = 113.64k$$

(a) With one engine lost (only 2 remaining), we make

$$\frac{\Delta m}{\Delta t} = \frac{2}{3}\left(\frac{\Delta m}{\Delta t}\right)_0 = \frac{2}{3}(113.64k) = 75.758k \text{ in Eq. (2)}$$

$$(75.758k)(800 - v_2) = kv_2^2$$

$$v_2^2 + 75.758v_2 - 60.606 \times 10^3 = 0$$

$$v_2 = 211.20 \text{ m/s}. \qquad v_2 = 760 \text{ km/h} \blacktriangleleft$$

(CONTINUED)

14.69 CONTINUED

(b) With 2 engines lost (only one remaining), we make

$$\frac{\Delta m}{\Delta t} = \frac{1}{3}\left(\frac{\Delta m}{\Delta t}\right)_0 = \frac{1}{3}(113.64k) = 37.88k \text{ in Eq. (2)}:$$

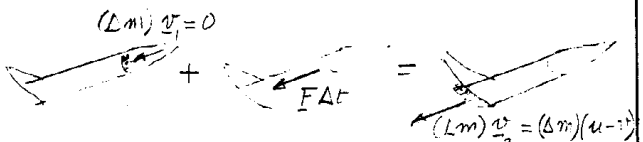
$$(37.88k)(800 - v_1) = kv_1^2$$

$$v_1^2 + 37.88v_1 - 30.304 \times 10^3 = 0$$

$$v_1 = 156.17 \text{ m/s} \qquad v_1 = 562 \text{ km/h} \blacktriangleleft$$

14.70 Determination of thrust

Since the airplane is accelerated in horizontal flight, we shall use a frame of reference at rest with respect to the atmosphere (cf. footnote, page 680). The air particles are thus observed to enter the engine with no velocity and to leave it with a velocity $\underline{v}_2$ of magnitude $u - v$, where u = discharge velocity relative to the plane, and v = velocity of plane.



Principle of impulse and momentum:

$$0 + F\Delta t = (\Delta m)(u - v)$$

$$F = \frac{\Delta m}{\Delta t}(u - v) \qquad (1)$$

The force exerted on the plane is equal and opposite to the force F exerted by the plane on the air.

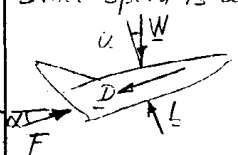
Airplane climbing:

Data: $u = 2250 \text{ ft/s}$, $v = 480 \text{ mi/h} = 704 \text{ ft/s}$

$$\frac{\Delta m}{\Delta t} = \frac{600 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 18.634$$

From Eq. (1): $F = 18.634(2250 - 704) = 28,807 \text{ lb}$

Since speed is constant, plane is in equilibrium:



$$+\nearrow \sum F_x = 0: \quad F - D - W\sin\alpha = 0$$

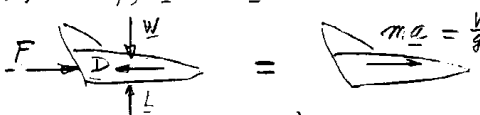
$$D = F - W\sin\alpha$$

$$= 28,807 - 35,000 \sin 15°$$

$$D = 19,749 \text{ lb}$$

Horizontal flight:

(a) Initially, F and D have the values found above



$$\pm\rightarrow \sum F_x = ma: \quad F - D = \frac{W}{g}a \qquad (2)$$

$$28,807 - 19,749 = \frac{35,000}{32.2}a \qquad a = 8.33 \text{ ft/s}^2 \blacktriangleleft$$

(b) For max. horizontal speed v_m, we have $a = 0$. Making $a = 0$ in Eq. (2) and recalling (1):

$$D_m = F_m = \frac{\Delta m}{\Delta t}(u - v_m) = 18.634(2250 - v_m) \qquad (3)$$

Since $D \propto v^2$, we also have $\frac{D_m}{D_0} = \left(\frac{v_m}{v_0}\right)^2$ or

$$D_m = D_0\left(\frac{v_m}{v_0}\right)^2 = \frac{19,749}{(704)^2}v_m^2 = 0.03985v_m^2$$

Substituting in (3): $0.03985v_m^2 = 18.634(2250 - v_m)$

$$v_m^2 + 467.6v_m - 1.0521 \times 10^6 = 0$$

$$v_m = 818.2 \text{ ft/s} \qquad v_m = 558 \text{ mi/h} \blacktriangleleft$$

14.71 (a) Kinetic energy per second

$$= \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}\rho Q v^2 = \frac{1}{2}\rho(Av)v^2 = \frac{1}{2}\rho A v^3$$

$$= \frac{1}{2}(1.21\ \text{kg/m}^3)\pi(4.10\ \text{m})^2\left(\frac{45}{3.6}\ \text{m/s}\right)^3 = 62.40\ \text{kJ/s}$$

$$= 62.4\ \text{kJ/s}$$ ◀

(b) Efficiency

Eq. (13.15): $\eta = \dfrac{\text{Power output}}{\text{Power input}} = \dfrac{20\ \text{kW}}{62.40\ \text{kW}} = 0.320$ ◀

14.72 (a) Kinetic energy per second

Eq. (13.20): $\eta = \dfrac{\text{Power output}}{\text{Power input}}$ $\qquad 0.30 = \dfrac{15\ \text{kW}}{\text{K.E. per second}}$

$$\text{K.E. per second} = \frac{15\ \text{kW}}{0.30} = 50.0\ \text{kJ/s}$$ ◀

(b) Wind speed

$$\text{K.E. per second} = \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}\rho Q v^2 = \frac{1}{2}\rho(Av)v^2 = \frac{1}{2}\rho A v^3$$

$$50.0\times10^3\ \text{J/s} = \frac{1}{2}(1.21\ \text{kg/m}^3)\pi(4.10\ \text{m})^2 v^3$$

$$v^3 = 1.565\times10^3 \qquad v = 11.61\ \text{m/s} = 41.8\ \text{km/h}$$ ◀

14.73 We follow the same procedure as in Sample Prob. 14.7 and write

$$(\Delta m)u - F_x\Delta t = (\Delta m)u\cos\theta$$

$$F_x = \frac{\Delta m}{\Delta t}u(1-\cos\theta)$$

But we now have

$$\frac{\Delta m}{\Delta t} = A\rho v_A \qquad u = v_A - V$$

Thus: $F_x = A\rho v_A(v_A - V)(1-\cos\theta)$

$$\text{Output power} = F_x V = A\rho v_A(v_A - V)(1-\cos\theta)V \qquad (1)$$

$$= A\rho v_A(v_A V - V^2)(1-\cos\theta)$$

(a) Differentiate and set derivative equal to zero:

$$\frac{d(\text{Power})}{dV} = A\rho v_A(v_A - 2V)(1-\cos\theta) = 0$$

$$V = \frac{1}{2}v_A \rightarrow$$ ◀

(b) Maximum power

Substitute $V = \frac{1}{2}v_A$ into Eq. (1):

$$\text{Max. power} = A\rho v_A\left(v_A - \frac{1}{2}v_A\right)(1-\cos\theta)\frac{v_A}{2}$$

$$\text{Max. power} = \frac{1}{4}A\rho(1-\cos\theta)v_A^3$$ ◀

(c) Efficiency

We first compute the input power:

$$\text{Input power} = \frac{1}{2}\frac{\Delta m}{\Delta t}v_A^2 = \frac{1}{2}(\rho Q)v_A^2 = \frac{1}{2}\rho A v_A^3 \qquad (2)$$

Dividing (1) by (2):

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{A\rho v_A(v_A - V)(1-\cos\theta)V}{\frac{1}{2}\rho A v_A^3}$$

$$\eta = 2\frac{V}{v_A}\left(1 - \frac{V}{v_A}\right)(1-\cos\theta)$$ ◀

Note: The maximum efficiency is obtained for $\theta = 180°$ and $V = \frac{1}{2}v_A$:

$$\eta_{max} = 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1+1) = 1$$

14.74 Mass of air scooped in per second

$$\frac{\Delta m}{\Delta t} = \frac{240\ \text{lb/s}}{32.2\ \text{ft/s}^2} = 7.453\ \text{lb}\cdot\text{s/ft}$$

(a) Power used to propel plane.

See solution of Prob. 14.70 for derivation of Eq. (1) for thrust:

$$F = \frac{\Delta m}{\Delta t}(u - v) \qquad (1)$$

where $u = 2200\ \text{ft/s}$ = discharge velocity relative to plane

$v = 570\ \text{mi/h} = 836\ \text{ft/s}$ = speed of plane

Thus $\quad \text{Thrust} = F = (7.453\ \text{lb}\cdot\text{s/ft})(2200 - 836)\ \text{ft/s}$

$$= 10{,}166\ \text{lb}$$

$$\text{Power} = P_{prop.} = Fv = (10{,}166\ \text{lb})(836\ \text{ft/s}) = 8.499\times10^6\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$= \frac{8.499\times10^6}{550}\ \text{hp} = 15{,}452\ \text{hp}$$

$$P_{prop} = 15{,}450\ \text{hp}$$ ◀

(b) Total power developed by engine

Power dissipated in exhaust flow

$$P_{exhaust} = \frac{1}{2}\left(\frac{\Delta m}{\Delta t}\right)v_1^2$$

where v_1 = absolute discharge velocity $= u - v$. Thus

$$P_{exhaust} = \frac{1}{2}(7.453)(2200 - 836)^2 = 6.933\times10^6\ \text{ft}\cdot\text{lb/s}$$

$$= \frac{6.933\times10^6}{550}\ \text{hp} = 12{,}606\ \text{hp}$$

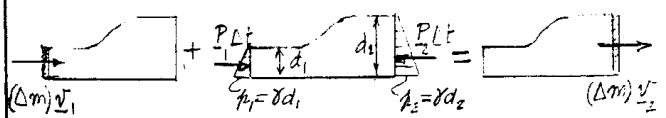
$$P_{total} = P_{prop.} + P_{exhaust} = 15{,}452 + 12{,}606 = 28{,}058\ \text{hp}$$

$$P_{total} = 28{,}060\ \text{hp}$$ ◀

(c) Mechanical efficiency of plane

$$\eta = \frac{\text{Useful power}}{\text{Total power}} = \frac{P_{prop}}{P_{total}} = \frac{15{,}452\ \text{hp}}{28{,}058\ \text{hp}} = 0.551$$ ◀

***14.75** We apply the principle of impulse and momentum to the section of water shown and to the mass of water Δm entering and leaving in the time interval Δt.



$\pm\rightarrow$ x comp: $\quad (\Delta m)v_1 + P_1\Delta t - P_2\Delta t = (\Delta m)v_2$

$$\frac{\Delta m}{\Delta t}(v_1 - v_2) = P_2 - P_1 \qquad (1)$$

But $\quad \frac{\Delta m}{\Delta t} = \rho Q$

Also: $\quad Q = A_1v_1 = A_2v_2 \qquad v_1 = \dfrac{Q}{A_1} = \dfrac{Q}{bd_1} \qquad v_2 = \dfrac{Q}{bd_2}$

On the other hand: $\quad P_1 = \frac{1}{2}p_1A_1 = \frac{1}{2}(\gamma d_1)(bd_1) = \frac{1}{2}\gamma b d_1^2$

$$P_2 = \frac{1}{2}\gamma b d_2^2$$

Substituting in (1):

$$\rho Q\left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b d_2^2 - \frac{1}{2}\gamma b d_1^2$$

$$\rho Q^2\frac{d_2 - d_1}{b d_1 d_2} = \frac{1}{2}\gamma b(d_2 + d_1)(d_2 - d_1)$$

Dividing through by $d_2 - d_1$, recalling that $\gamma = \rho g$, and solving for Q:

$$Q = b\sqrt{\tfrac{1}{2}g d_1 d_2(d_1 + d_2)}$$ ◀

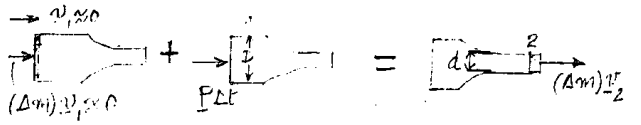
***14.76** See solution of Prob. 14.75 for derivation of

$$Q = b\sqrt{\tfrac{1}{2}g\,d_1 d_2 (d_1 + d_2)}$$

Substituting $d_1 = 1.2\text{ m}$, $d_2 = 1.5\text{ m}$, $b = 4\text{ m}$, $g = 9.81\text{ m/s}^2$:

$$Q = 4\sqrt{\tfrac{1}{2}(9.81)(1.2)(1.5)(1.2+1.5)} = 19.53\text{ m}^3/\text{s} \quad \blacktriangleleft$$

14.77 We apply the principle of impulse and momentum to the section of water indicated and to the mass of water entering and leaving the section in the time interval Δt:



$+\rightarrow$ x comp.: $0 + P\Delta t = (\Delta m)v_2$

$$P = \left(\frac{\Delta m}{\Delta t}\right)v_2 \qquad (1)$$

But $P = pA_1 = (\rho g h)\dfrac{\pi D^2}{4}$

and $v_2 = v = \sqrt{2gh}$

Also: $\dfrac{\Delta m}{\Delta t} = \rho Q = \rho A_2 v_2 = \rho\dfrac{\pi d^2}{4}\sqrt{2gh}$

Substituting into Eq. (1):

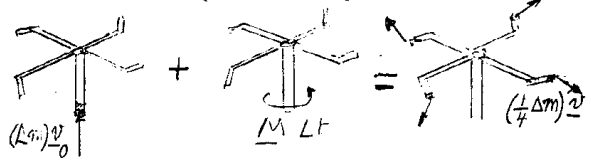
$$\rho g h\frac{\pi D^2}{4} = \rho\frac{\pi d^2}{4}\sqrt{2gh}\sqrt{2gh}$$

$$gh D^2 = d^2(2gh)$$

$$D^2 = 2d^2$$

$$d = \frac{D}{\sqrt{2}} \quad \blacktriangleleft$$

14.78 We apply the principle of impulse and momentum to the sprinkler, the water it contains, and the mass Δm of water entering and leaving in the time Δt:



Equating moments about the axis of rotation:

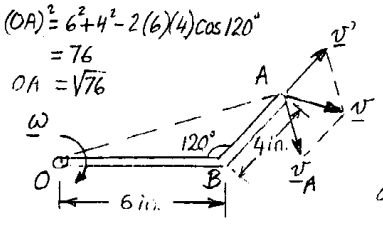
$$0 + M\Delta t = 4[\text{Moment of } \tfrac{1}{4}(\Delta m)\underline{v}]$$

$$M\Delta t = \text{Moment of } (\Delta m)\underline{v} \qquad (1)$$

$(OA)^2 = 6^2 + 4^2 - 2(6)(4)\cos 120°$

$= 76$

$OA = \sqrt{76}$



The absolute velocity $\underline{v}$ of water particles is $\underline{v} = \underline{v}_A + \underline{v}'$ where $\underline{v}_A = (OA)\omega$ and $\underline{v}'$ = velocity of water particles relative to arm.

(CONTINUED)

14.78 CONTINUED

$+\circlearrowleft$ Mom. of $\underline{v}$ about O = Mom. of $\underline{v}'$ + Mom. of $\underline{v}_A$

$$= (OB)v'\cos 30° - (OA)v_A = (OB)v'\cos 30° - (OA)^2\omega$$

$$= \left(\frac{6}{12}\text{ ft}\right)(60\text{ ft/s})\cos 30° - \left(\frac{76}{144}\text{ ft}^2\right)\omega$$

$$= 25.981 - 0.52778\omega$$

Substitute into Eq. (1), making $M = 0.275\text{ lb·ft}$:

$$0.275\,\Delta t = \Delta m\,(25.981 - 0.52778\omega)$$

$$0.275 = \frac{\Delta m}{\Delta t}(25.981 - 0.52778\omega) \qquad (2)$$

But $\dfrac{\Delta m}{\Delta t} = \rho Q = \dfrac{\gamma}{g}Q = \dfrac{62.4\text{ lb/ft}^3}{32.2\text{ ft/s}^2}\left(5\dfrac{\text{gal}}{\text{min}}\right)\dfrac{1\text{ min}}{60\text{ s}}\dfrac{1\text{ ft}^3}{7.48\text{ gal}}$

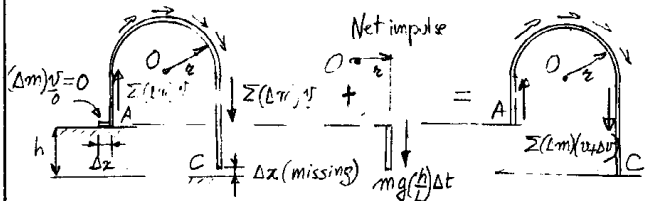
$$= 0.08636\text{ lb·s/ft}$$

Substitute into (2):

$$0.275 = 0.08636(25.981 - 0.52778\omega)$$

$$\omega = 43.19\text{ rad/s} = 412\text{ rpm} \quad \blacktriangleleft$$

14.79 We apply the principle of impulse and momentum to the portion of chain of mass m in motion at $t+\Delta t$



Let L be the length and m be the mass of the portion of chain in motion at $t+\Delta t$. Of this portion of chain, an element at A of length Δx and mass $\Delta m = \dfrac{m}{L}\Delta x$ is not in motion at t. (The extra element at C is not part of the system considered here.)

Equating moments about O:

$+\circlearrowleft$ $r(m - \Delta m)v + r\,mg\dfrac{h}{L}\Delta t = r\,m(v + \Delta v)$

$$-(\Delta m)v + mg\frac{h}{L}\Delta t = m\,\Delta v$$

Substituting $\Delta m = \dfrac{m}{L}\Delta x$ and dividing by $\dfrac{m}{L}\Delta t$,

$$-v\frac{\Delta x}{\Delta t} + gh = L\frac{\Delta v}{\Delta t}$$

Letting $\Delta t \to 0$, and noting that $\dfrac{dx}{dt} = v$,

$$gh - v^2 = L\frac{dv}{dt} \qquad (1)$$

If the chain is to keep moving at its initial speed, we must have $dv/dt = 0$ and Eq. (1) yields

$$h = v^2/g \quad \blacktriangleleft$$

Note: The mass of the pulley does not intervene, since it rotates at a constant speed.

14.80 See solution of Prob. 14.79 for derivation of Eq. (1):

$$gh - v^2 = L\frac{dv}{dt} \qquad (1)$$

Solving for dt and integrating,

$$t = \int_0^v \frac{L\,dv}{gh - v^2}$$

Setting $gh = c^2$:

$$t = \int_0^v \frac{L\,dv}{c^2 - v^2} = \frac{L}{2c}\left[\ln\frac{c+v}{c-v}\right]_0^v = \frac{L}{2c}\ln\frac{c+v}{c-v}$$

or

$$\frac{c+v}{c-v} = e^{2ct/L}$$

$$c + v = ce^{2ct/L} - ve^{2ct/L}$$

$$v = \frac{ce^{2ct/L} - c}{e^{2ct/L} + 1} = c\,\frac{e^{2ct/L} - 1}{e^{2ct/L} + 1}$$

or, recalling that $c^2 = gh$:

$$v = \sqrt{gh}\,\frac{e^{(2\sqrt{gh}/L)t} - 1}{e^{(2\sqrt{gh}/L)t} + 1}$$

Note: The answer may also be written in the form

$$v = \sqrt{gh}\tanh(\sqrt{gh}/L)t$$

14.81 See solution of Prob. 14.79 for derivation of Eq. (1):

$$gh - v^2 = L\frac{dv}{dt} \qquad (1)$$

Making $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ in Eq. (1):

$$gh - v^2 = Lv\frac{dv}{dx}$$

Solving for dx and integrating:

$$x = \int_0^v \frac{Lv\,dv}{gh - v^2} = -\frac{L}{2}\left[\ln(gh - v^2)\right]_0^v$$

$$x = -\frac{L}{2}\ln\frac{gh - v^2}{gh} \qquad (2)$$

Using exponential form:

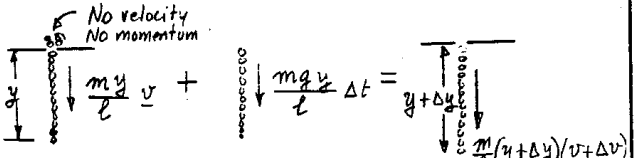
$$\frac{gh - v^2}{gh} = e^{-2x/L}$$

$$gh - v^2 = gh\,e^{-2x/L}$$

$$v^2 = gh\left(1 - e^{-2x/L}\right)$$

$$v = \sqrt{gh\left(1 - e^{-2x/L}\right)}$$

14.82 Case 1. We apply the principle of impulse and momentum to the portion of chain which has already passed through the hole at time t and to the portion which will pass in interval Δt.



$$+\downarrow \quad \frac{m}{L}yv + \frac{m}{L}gy\,\Delta t = \frac{m}{L}(yv + y\,\Delta v + v\,\Delta y + \Delta y\,\Delta v)$$

Divide by Δt and let $\Delta t \to 0$:

$$gy = y\frac{dv}{dt} + v\frac{dy}{dt}$$

$$= \frac{d}{dt}(yv)$$

Multiply both sides by $yv\,dt$ and, since $v\,dt = dy$:

$$gy^2\,dy = yv\,d(yv)$$

Setting $yv = u$ and integrating:

$$\int_0^y gy^2\,dy = \int_0^{yv} u\,du$$

$$\frac{1}{3}gy^3 = \frac{1}{2}(yv)^2 \qquad v^2 = \frac{2}{3}gy \qquad (1)$$

(a) Differentiate (1) with respect to t:

$$2v\frac{dv}{dt} = \frac{2}{3}g\frac{dy}{dt}$$

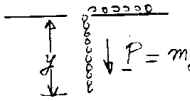
Since $\frac{dv}{dt} = a$ and $\frac{dy}{dt} = v$,

$$2va = \frac{2}{3}gv \qquad a = \frac{1}{3}g \text{ (constant)}$$

(b) As the last link passes through the hole, we have $y = L$ and Eq. (1) yields

$$v^2 = \frac{2}{3}gL \qquad v = \sqrt{\frac{2}{3}gL}$$

Case 2. We consider the entire chain as a single system. At time t, each link has the same acceleration a and the net force causing the acceleration is



$$P = mg\left(\frac{y}{L}\right)$$

(a) We have $P = ma$

$$mg\left(\frac{y}{L}\right) = ma$$

$$a = \frac{g}{L}y$$

(b) Setting $a = v\frac{dv}{dy}$, we have

$$v\,dv = \frac{g}{L}y\,dy$$

$$\int_0^v v\,dv = \frac{g}{L}\int_0^y y\,dy$$

$$\frac{1}{2}v^2 = \frac{1}{2}\frac{g}{L}y^2$$

$$v^2 = \frac{g}{L}y^2 \qquad (2)$$

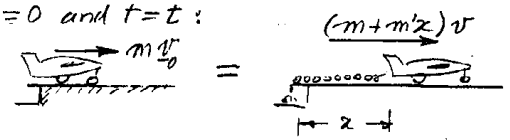
As the last link passes through the hole, we have $y = L$ and Eq. (2) yields

$$v^2 = \frac{g}{L}L^2 = gL \qquad v = \sqrt{gL}$$

14.83 Let m' = mass of chain per unit length
x = distance traveled at time t

Thus; $m'x$ = mass of chain moving at time t

Conservation of linear momentum between $t=0$ and $t=t$:



$$\pm\rightarrow \quad m v_0 = (m+m'x)v \qquad (1)$$

a) We want $v=\beta v_0$ for $x=\ell$. Substituting:

$$m v_0 = (m+m'\ell)\beta v_0$$

$$m v_0(1-\beta) = m'\ell\,\beta v_0$$

Mass of chain $= m'\ell = \dfrac{1-\beta}{\beta} m \qquad (2)$ ◀

b) Solve Eq. (1) for v:

$$v = \frac{m v_0}{m+m'x} \qquad (3)$$

$$a = \frac{dv}{dt} = -\frac{m v_0}{(m+m'x)^2} m'\frac{dx}{dt} = -\frac{m m' v_0 v}{(m+m'x)^2}$$

Recalling (3): $$a = -\frac{m^2 m' v_0^2}{(m+m'x)^3} \qquad (4)$$

Deceleration is maximum for $x=0$. We have

$$(-a)_{max} = \frac{m'}{m} v_0^2 \qquad (5)$$

Writing $|F|_{max} = m|a|_{max}$ and recalling (2):

$$|F|_{max} = m' v_0^2 \qquad |F'|_{max} = \frac{1-\beta}{\beta}\frac{m v_0^2}{\ell}$$ ◀

14.84 See solution of Prob. 14.83 for derivation of Eqs. (1), (3), (4), and (5).

Data: $m = (12{,}000\ \text{lb})/g$, $m' = (24\ \text{lb/ft})/g$

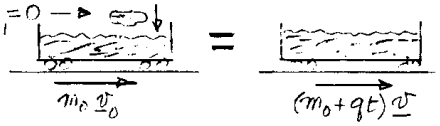
$$v_0 = 105\ \text{mi/h} = 154\ \text{ft/s}$$

Eq. (5): $$(-a)_{max} = \frac{m'}{m} v_0^2 = \frac{24\ \text{lb/ft}}{12{,}000\ \text{lb}} (154\ \text{ft/s})^2$$

$$(-a)_{max} = 47.4\ \text{ft/s}^2$$ ◀

14.85 We consider the system consisting of the mass m_0 of the car and its contents at $t=0$ and of the additional mass qt which falls into the car in the time interval t.

Conservation of linear momentum in horizontal direction



$$\pm\rightarrow \quad m_0 v_0 = (m_0+qt)v \qquad v = \frac{m_0 v_0}{m_0+qt} \qquad (1)$$

Letting $v = \dfrac{dx}{dt}$ in (1):

$$dx = \frac{m_0 v_0\,dt}{m_0+qt} \qquad x = m_0 v_0 \int_0^t \frac{dt}{m_0+qt}$$

$$x = \frac{m_0 v_0}{q}\left[\ln(m_0+qt)\right]_0^t = \frac{m_0 v_0}{q} \ln\frac{m_0+qt}{m_0}$$

(CONTINUED)

14.85 CONTINUED Using the exponential form:

$$m_0+qt = m_0 e^{qx/m_0 v_0}$$

Where m_0+qt represents the mass at time t and after the car has moved through x.

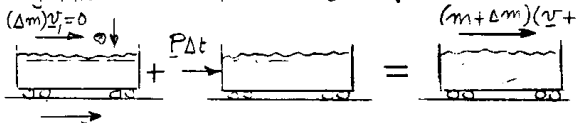
(a) Making $x=L$, we obtain the final mass:

$$m_f = m_0+qt_f = m_0 e^{qL/m_0 v_0}$$ ◀

(b) Making $t=t_f$ in (1), we obtain the final velocity

$$v = \frac{m_0 v_0}{m_0+qt_f} = \frac{m_0}{m_f} v_0 = v_0 e^{-qL/m_0 v_0}$$ ◀

14.86 We apply the principle of impulse and momentum to the car and its contents and to the mass of sand Δm falling into the car in the time Δt.



$$\pm\rightarrow x\text{ comp:} \quad mv + P\Delta t = (m+\Delta m)(v+\Delta v)$$

$$mv + P\Delta t = mv + m\Delta v + (\Delta m)v + (\Delta m)\Delta v$$ (2nd order)

$$P = m\frac{\Delta v}{\Delta t} + \frac{\Delta m}{\Delta t} v$$

But $\dfrac{\Delta m}{\Delta t} = q$ and $\dfrac{\Delta v}{\Delta t} = a$ Thus

$$P = ma + qv \qquad (1)$$

(a) If $v=v_0$, then $a=0$ and (1) reduces to

$$P = q v_0$$ ◀

(b) If $a=$ constant, then $v = v_0 + at$. On the other hand, $m = m_0+qt$. Substituting in (1):

$$P = (m_0+qt)a + q(v_0+at)$$

$$P = m_0 a + q(v_0+2at)$$ ◀

14.87 From Eq. (14.43), for each engine

$$P = \frac{dm}{dt} u = \frac{750\ \text{lb/s}}{32.2\ \text{ft/s}^2} (12{,}500\ \text{ft/s})$$

$$= 291\ \text{kips}$$

For 3 engines: Thrust $= 3(291\ \text{kips}) = 873\ \text{kips}$ ◀

14.88 Thrust of each engine $= P = \frac{1}{3}(1200\ \text{kips}) = 400\ \text{kips}$

Eq. (14.43): $$P = \frac{dm}{dt} u$$

$$400\times10^3\ \text{lb} = \frac{dm}{dt}(12{,}500\ \text{ft/s})$$

$$\frac{dm}{dt} = \frac{400\times10^3\ \text{lb}}{12{,}500\ \text{ft/s}} = 32.0\ \text{lb·s/ft}$$

$$\frac{dv}{dt} = g\frac{dm}{dt} = (32.2\ \text{ft/s}^2)(32.0\ \text{lb·s/ft}) = 1030\ \text{lb/s}$$ ◀

14.89 Eq. (14.43): $$P = \frac{dm}{dt} u = (18\ \text{kg/s})(3000\ \text{m/s}) = 54\times10^3\ \text{N}$$

$F=ma_t$: $$P = ma_t \qquad a_t = \frac{P}{m} = \frac{54\times10^3\ \text{N}}{m}$$

(a) As engine is fired: $m = m_0 = 600\ \text{kg}$

$$a_t = \frac{54\times10^3\ \text{N}}{600\ \text{kg}} \qquad a_t = 90.0\ \text{m/s}^2$$ ◀

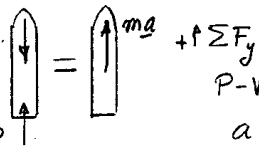
(b) Max. speed of capsule relative to vehicle is obtained from expression derived in Sample Prob. 14.8:

$$v_{C/V} = u\ln\frac{m_0}{m_0-qt} = 3000\ln\frac{600}{600-400} = 3295.8\ \text{m/s} = 11.865\times10^3\ \text{km/h}$$

$$v_C = v_V + v_{C/V} = 24\times10^3\ \text{km/h} + 11.865\times10^3\ \text{km/h} = 35.9\times10^3\ \text{km/h}$$ ◀

14.90 Eq. (14.43):

$$P = \frac{dm}{dt}u = (12.5\ kg/s)(4000\ m/s)$$
$$= 50\times10^3\ N$$



$+\uparrow \Sigma F_y = ma$:

$$P - W = ma$$
$$a = \frac{P}{m} - g \qquad (1)$$

(a) As rocket is fired: $m = 1200\ kg$

Eq. (1): $a = \frac{50\times10^3\ N}{1200\ kg} - g = 41.67 - 9.81$
$= 31.86\ m/s^2$

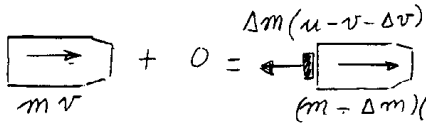
$\underline{a} = 31.9\ m/s^2 \uparrow$ ◀

(b) As last particle of fuel is consumed: $m = 200\ kg$

Eq. (1): $a = \frac{50\times10^3\ N}{200\ kg} - g = 250 - 9.81 = 240.2\ m/s^2$

$\underline{a} = 240\ m/s^2 \uparrow$ ◀

14.91 Principle of impulse and momentum is applied to satellite and fuel expelled in time Δt:



$\pm\rightarrow \quad mv = (m-\Delta m)(v+\Delta v) - \Delta m(u - v - \Delta v)$

$mv = mv - v\,\Delta m + m\,\Delta v - u\,\Delta m + v\,\Delta m + 2^{nd}\text{ order}$

$$m\,\Delta v = u\,\Delta m$$

But $\Delta m = q\,\Delta t$ and $m = m_0 - qt$.

Thus: $$(m_0 - qt)\Delta v = uq\,\Delta t$$

As $\Delta t \to 0$: $$\frac{dv}{dt} = \frac{uq}{m_0 - qt}$$

$$v = \int_0^t \frac{uq\,dt}{m_0 - qt} = -u\left[\ln(m_0 - qt)\right]_0^t$$

$$v = u \ln \frac{m_0}{m_0 - qt} \qquad (1)$$

or, in exponential form:

$$\frac{m_0}{m_0 - qt} = e^{v/u} \qquad (2)$$

Making $v = 2430\ m/s$, $u = 4200\ m/s$, $m_0 = 5000\ kg$:

$$\frac{5000}{5000 - qt} = e^{\frac{2430}{4200}} = 1.7835$$

$$5000 - qt = \frac{5000}{1.7835} = 2803\ kg$$

Fuel expanded $= qt = 5000 - 2803 = 2200\ kg$ ◀

14.92 See solution of Prob. 14.91 for derivation of Eq. (1):

$$v = u \ln \frac{m_0}{m_0 - qt} \qquad (1)$$

Making $u = 4200\ m/s$, $m_0 = 5000\ kg$, $qt = 1500\ kg$:

$$v = 4200 \ln \frac{5000}{5000 - 1500} = 4200 \ln 1.4286$$

$$v = 1498\ m/s$$ ◀

14.93 See Sample Prob. 14.8 for derivation of

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

Data: $u = 12{,}000\ ft/s$, $q = \frac{500}{g}$, $m_0 = \frac{42{,}600 + 1200}{g} = \frac{43{,}800}{g}$

$m_f = qt = 40{,}000/g$. Thus, $t = \frac{40{,}000/g}{500/g} = 80\ s$

Substituting into (1):

$$v = (12{,}000\ ft/s) \ln \frac{43{,}800}{43{,}800 - 40{,}000} - (32.2\ ft/s^2)(80\ s)$$
$$= (12{,}000\ ft/s) \ln 11.526 - 2{,}576\ ft/s = 26{,}760\ ft/s$$

$$v = 26{,}800\ ft/s$$ ◀

14.94 See Sample Prob. 14.8 for derivation of

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

(a) First stage

$u = 12{,}000\ ft/s$, $q = \frac{500}{g}$, $m_0 = \frac{2(21{,}300) + 1200}{g} = \frac{43{,}800}{g}$

$m_f = qt_1 = 20{,}000/g$. Thus: $t_1 = \frac{20{,}000/g}{500/g} = 40\ s$

Substituting into (1):

$$v_1 = (12{,}000\ ft/s) \ln \frac{43{,}800}{43{,}800 - 20{,}000} - (32.2\ ft/s^2)(40\ s)$$
$$= (12{,}000\ ft/s) \ln 1.8403 - 1288\ ft/s = 7319 - 1288$$

$$v_1 = 6031\ ft/s \qquad v_1 = 6030\ ft/s$$ ◀

(b) Second stage

$u = 12{,}000\ ft/s$, $q = \frac{500}{g}$, $m_1 = \frac{21{,}300 + 1200}{g} = \frac{22{,}500}{g}$

$m_f = qt_2 = 20{,}000/g$. Thus: $t_2 = \frac{20{,}000/g}{500/g} = 40\ s$

Replacing v by $v_2 - v_1$ and m_0 by m_1 in Eq. (1):

$$v_2 - v_1 = u \ln \frac{m_1}{m_1 - qt_2} - gt_2$$
$$= (12{,}000\ ft/s) \ln \frac{22{,}500}{22{,}500 - 20{,}000} - (32.2\ ft/s^2)(40\ s)$$
$$= (12{,}000\ ft/s) \ln 9 - 1{,}288\ ft/s = 26{,}367 - 1{,}288$$
$$= 25{,}079\ ft/s$$

$v_2 = v_1 + 25{,}079 = 6031 + 25{,}079 = 31{,}110\ ft/s$

$$v_2 = 31{,}100\ ft/s$$ ◀

14.95 See Prob. 14.93 for Eq. (1) and data. Setting $v = dy/dt$ in Eq. (1) and integrating:

$$h = \int_0^h dy = \int_0^t \left[u \ln \frac{m_0}{m_0 - qt} - gt\right] dt = -u\int_0^t \ln \frac{m_0 - qt}{m_0}\,dt - \frac{1}{2}gt^2$$

Setting $\frac{m_0 - qt}{m_0} = z$, $dt = -\frac{m_0}{q}\,dz$, we have

$$h = +\frac{m_0}{q}u\int_1^z \ln z\,dz - \frac{1}{2}gt^2 = \frac{m_0 u}{q}\left[z\ln z - z\right]_1^z - \frac{1}{2}gt^2$$
$$= \frac{m_0}{q}u\,(z\ln z - z + 1) - \frac{1}{2}gt^2$$
$$= \frac{m_0}{q}u\,\frac{m_0 - qt}{m_0}\ln\frac{m_0 - qt}{m_0} - \frac{m_0}{q}u\,\frac{m_0 - qt}{m_0} + \frac{m_0 u}{q} - \frac{1}{2}gt^2$$
$$= u\,\frac{m_0 - qt}{q}\ln\frac{m_0 - qt}{m_0} - u\,\frac{m_0 - qt}{q} + \frac{m_0 u}{q} - \frac{1}{2}gt^2$$

$$h = u\left[t - \frac{m_0 - qt}{q}\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

With given data, $u = 12{,}000$, $q = 500/g$, $m_0 = 43{,}800/g$, $t = 80\ s$ and noting that $m_0 - qt = (43{,}800 - 40{,}000)/g = 3800/g$,

$$h = 12{,}000\left[80 - \frac{3800}{500}\ln\frac{43{,}800}{3800}\right] - \frac{1}{2}(32.2)(80)^2 = 634{,}000\ ft$$ ◀

(Note that g has been assumed constant)

14.96 See solution of Prob. 14.95 for derivation of Eq. (2):

$$h = u\left[t - \frac{m_0 - qt}{q}\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

(a) First stage. From Prob. 14.94a we have

$u = 12{,}000$ ft/s, $q = 500/g$, $m_0 = 43{,}800/g$, $t_1 = 40$ s,

$m_1 - qt_1 = (43{,}800 - 20{,}000)/g = 23{,}800/g$. Substitute in (2):

$$h_1 = 12{,}000\left[40 - \frac{23{,}800}{500}\ln\frac{43{,}800}{23{,}800}\right] - \frac{1}{2}(32.2)(40)^2$$

$$= 12{,}000(40 - 47.6\ln 1.8403) - 25{,}760 = 105{,}849 \text{ ft}$$

$h_1 = 105{,}800$ ft ◀

(b) Second stage. From Prob. 14.94b, we have

$u = 12{,}000$ ft/s, $q = 500/g$, $m_1 = 22{,}500/g$, $t_2 = 40$ s,

$m_1 - qt_2 = (22{,}500 - 20{,}000)/g = 2500/g$.

Since the second stage has an initial velocity v_1 and an altitude h_1 when fired, the altitude h_2 reached at the end of the stage is obtained by adding the terms h_1 and $v_1 t_2$ to the right-hand member of Eq. (2), where $h_1 = 105{,}849$ ft and (see Prob. 14.94a): $v_1 = 6031$ ft/s.

$$h_2 = h_1 + v_1 t_2 + u\left[t_2 - \frac{m_1 - qt_2}{q}\ln\frac{m_1}{m_1 - qt_2}\right] - \frac{1}{2}gt_2^2 \qquad (3)$$

$$= 105{,}849 + (6031)(40) + 12{,}000\left[40 - \frac{2500}{500}\ln\frac{22{,}500}{2500}\right] - \frac{1}{2}(32.2)(40)^2$$

$$= 105{,}849 + 241{,}240 + 12{,}000(40 - 5\ln 9) - 25{,}760 = 669{,}496 \text{ ft}$$

$h_2 = 669{,}500$ ft ◀

(Note that g has been assumed constant.)

14.97 See solution of Prob. 14.91 for derivation of Eq. (1):

$$v = u\ln\frac{m_0}{m_0 - qt} \qquad (1)$$

Setting $v = -\frac{dx}{dt}$ and integrating:

$$x = \int_0^t u\ln\frac{m_0}{m_0 - qt}\,dt = -u\int_0^t \ln\frac{m_0 - qt}{m_0}\,dt$$

Setting $\frac{m_0 - qt}{m_0} = z$, $dt = -\frac{m_0}{q}dz$, we have

$$x = +\frac{m_0 u}{q}\int_1^z \ln z\,dz = \frac{m_0 u}{q}\left[z\ln z - z\right]_1^z$$

$$= \frac{m_0 u}{q}(z\ln z - z + 1)$$

$$= \frac{m_0 u}{q}\cdot\frac{m_0 - qt}{m_0}\ln\frac{m_0 - qt}{m_0} - \frac{m_0 u}{q}\cdot\frac{m_0 - qt}{m_0} + \frac{m_0 u}{q}$$

$$= u\frac{m_0 - qt}{q}\ln\frac{m_0 - qt}{m_0} - u\frac{m_0 - qt}{q} + \frac{m_0 u}{q}$$

$$x = u\left[t - \frac{m_0 - qt}{q}\ln\frac{m_0}{m_0 - qt}\right] \qquad (2)$$

Given data: $u = 4200$ m/s, $t = 80$ s, $q = 18.75$ kg/s

$m_0 = 5000$ kg

Therefore: $m_0 - qt = 5000 \text{ kg} - (18.75 \text{ kg/s})(80 \text{ s})$

$= 3500$ kg

Substituting into Eq. (2):

$$x = 4200\left(80 - \frac{3500}{18.75}\ln\frac{5000}{3500}\right)$$

$$= 4200(80 - 186.67\ln 1.4286) = 56\,367 \text{ m}$$

$x = 56.4$ km ◀

Note that the initial velocity of the satellite was not taken into account since it is zero relative to the shuttle.

14.98 See Sample Prob. 14.8 for derivation of

$$v = u\ln\frac{m_0}{m_0 - qt} - gt \qquad (1)$$

and the solution of Prob. 14.95 for the derivation of

$$h = u\left[t - \frac{m_0 - qt}{q}\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

From the statement of Prob. 14.90 we have $u = 4000$ m/s,

$m_0 = 1200$ kg, $m_f = qt = 1000$ kg, $q = 12.5$ kg/s

Thus $t = \frac{qt}{q} = \frac{1000 \text{ kg}}{12.5 \text{ kg/s}} = 80$ s. Also, we have

$m_0 - qt = 1200 \text{ kg} - 1000 \text{ kg} = 200$ kg

(a) Substituting the data into Eq. (2):

$$h = 4000\left[80 - \frac{200}{12.5}\ln\frac{1200}{200}\right] - \frac{1}{2}(9.81)(80)^2$$

$$= 4000(80 - 16\ln 6) - 31\,392 = 205\,327 - 31\,392$$

$h = 173.9 \times 10^3$ m $= 173.9$ km ◀

(b) Substituting the data into Eq. (1):

$$v = 4000\ln\frac{1200}{200} - (9.81)(80) = 4000\ln 6 - 784.8$$

$v = 6382$ m/s $\qquad v = 6.38$ km/s ◀

(Note that g has been assumed constant.)

14.99 We recall Eq. (14.43), page 682 for the thrust:

$$P = \frac{dm}{dt}u$$

Thus:

Useful power $= Pv = \left(\frac{dm}{dt}\right)uv$

Wasted power = K.E. per second imparted to expelled fuel (absolute velocity $= u - v$)

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(u - v)^2$$

Total power = Useful power + Wasted power

$$= \left(\frac{dm}{dt}\right)uv + \frac{1}{2}\left(\frac{dm}{dt}\right)(u - v)^2$$

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(2uv + u^2 - 2uv + v^2)$$

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(u^2 + v^2)$$

Mechanical efficiency

$$= \eta = \frac{\text{Useful Power}}{\text{Total Power}} = \frac{\left(\frac{dm}{dt}\right)uv}{\frac{1}{2}\left(\frac{dm}{dt}\right)(u^2 + v^2)}$$

$$\eta = \frac{2uv}{u^2 + v^2} \quad \text{(Q.E.D.)} \quad ◀$$

When $u = v$, the absolute velocity $u - v$ of the expelled fuel is zero. Thus, no energy is imparted to the expelled fuel and no power is wasted.

14.100 Recalling that the thrust $\underline{F}$ is equal and opposite to the force exerted by the airplane on the jet, we apply formula (14.39), page 679 measuring the velocities relative to the airplane, and write

$$F = \frac{dm}{dt}(v_B - v_A)$$

where $v_B = u$ = exhaust velocity relative to the plane
$v_A = v$ = airplane speed

Thus: $F = \frac{dm}{dt}(u - v)$

Useful power $= Fv = \frac{dm}{dt}(u-v)v$

Wasted power = K.E. per second imparted to the expelled gases (absolute velocity $= u - v$).

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(u-v)^2$$

Total power = Useful power + Wasted power

$$= \left(\frac{dm}{dt}\right)(u-v)v + \frac{1}{2}\left(\frac{dm}{dt}\right)(u-v)^2$$

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(2uv - 2v^2 + u^2 - 2uv + v^2)$$

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)(u^2 - v^2)$$

Mechanical efficiency

$$= \eta = \frac{\text{Useful Power}}{\text{Total power}} = \frac{\left(\frac{dm}{dt}\right)(u-v)v}{\frac{1}{2}\left(\frac{dm}{dt}\right)(u^2-v^2)} = \frac{2(u-v)v}{(u-v)(u+v)}$$

$$\eta = \frac{2v}{u+v} \quad \text{(Q.E.D.)}$$

When $u = v$, the absolute velocity $u - v$ of the expelled gases is zero. Thus, no energy is imparted to the expelled gases and no power is wasted.

The formula derived here may be checked by applying it to the solution of Prob. 14.74. The data given in that problem was

$u = 2200$ ft/s, $v = 570$ mi/h $= 836$ ft/s

Substituting:

$$\eta = \frac{2v}{u+v} = \frac{2(836)}{2200+836} = 0.551$$

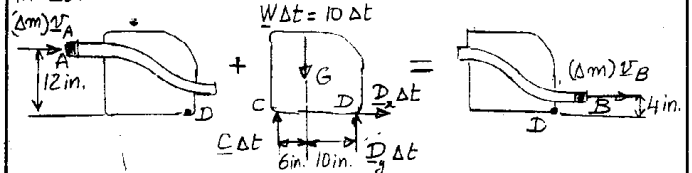
which checks with the answer to Prob. 14.74c.

14.101 The mass of water entering the vanes and leaving it in the time interval Δt is

$$\Delta m = \rho Q \Delta t = \frac{\gamma}{g} A v \Delta t = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\left(\frac{1.2}{144}\text{ ft}^2\right)(60 \text{ ft/s})\Delta t$$

$$\Delta m = 0.96894\,\Delta t \qquad (1)$$

We apply the principle of impulse and momentum to the plate, vanes, water in contact with the system, and water entering it in Δt:



+↻ Moments about D:

$$(\Delta m)v_A(12\text{ in.}) + C\Delta t(16\text{ in}) - 10\Delta t(10\text{ in.}) = (\Delta m)v_B(4\text{ in.})$$

(CONTINUED)

14.101 CONTINUED Solving for C, substituting from (1), and recalling that $v_A = v_B = 60$ ft/s:

$$16C = 100 + \frac{\Delta m}{\Delta t}(4v_B - 12v_A) = 100 + 0.96894(4-12)(60)$$

$$= -365.1 \qquad C = -22.82 \text{ lb} \qquad \underline{C = 22.8 \text{ lb}} \downarrow$$

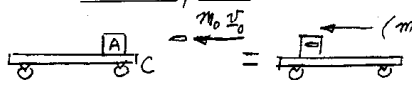
+→ x comp: $(\Delta m)v_A + D_x \Delta t = (\Delta m)v_B$

$$D_x = \frac{\Delta m}{\Delta t}(v_B - v_A) = 0$$

+↑ y comp: $0 + D_y \Delta t + C\Delta t - 10\Delta t = 0$

$$D_y = -C + 10 = +22.82 + 10 = 32.82 \text{ lb}, \quad \underline{D = 32.8 \text{ lb}} \uparrow$$

14.102 (a) Conservation of Linear Momentum for entire system

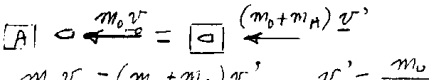


(v_f = final velocity)

$$v_f = \frac{m_0}{m_0 + m_A + m_C} v_0 = \frac{1/16}{(1/16) + 10 + 8}(1600 \text{ ft/s})$$

$$v_f = 5.536 \text{ ft/s} \qquad \underline{v_f = 5.54 \text{ ft/s}}$$

(b) Cons. of Linear Momentum during impact



$$m_0 v_0 = (m_0 + m_A)v' \qquad v' = \frac{m_0}{m_0 + m_A} v_0$$

$$v' = \frac{1/16}{(1/16) + 10}(1600 \text{ ft/s}) = 9.938 \text{ ft/s}$$

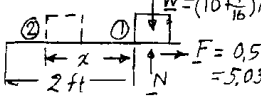
Energy after impact $= T' = \frac{1}{2}(m_0 + m_A)v'^2 = \frac{1}{2g}\left[\frac{1}{16} + 10\right](9.938)^2$

$$= 15.432 \text{ ft·lb}$$

Final energy $= T_f = \frac{1}{2}(m_0 + m_A + m_C)v_f^2 = \frac{1}{2g}\left[\frac{1}{16} + 10 + 8\right](5.536)^2$

$$= 8.596 \text{ ft·lb}$$

Work-Energy Principle for entire system during sliding



Initial KE $= T' = 15.432$

Final KE $= T_f = 8.596$

Only force to do work is friction force $\underline{F}$

$$T' + U_{1\to 2} = T_f: \quad 15.432 - 5.0312x = 8.596$$

$$x = 1.359 \text{ ft}$$

Distance of block from B $= 2 - 1.359 = 0.641$ ft

14.103 Conservation of Linear Momentum

$$m\underline{v}_0 = m_A\underline{v}_A + m_B\underline{v}_B + m_C\underline{v}_C$$

$$\frac{500}{g}(1500\underline{i}) = \frac{120}{g}(2000\underline{i} - 200\underline{j} + 300\underline{k}) + \frac{180}{g}(1000\underline{i} + 300\underline{j} - 500\underline{k}) + \frac{200}{g}\underline{v}_C$$

$$200\underline{v}_C = (750 - 240 - 180)10^3\underline{i} + (24 - 54)10^3\underline{j} + (-36 + 90)10^3\underline{k}$$

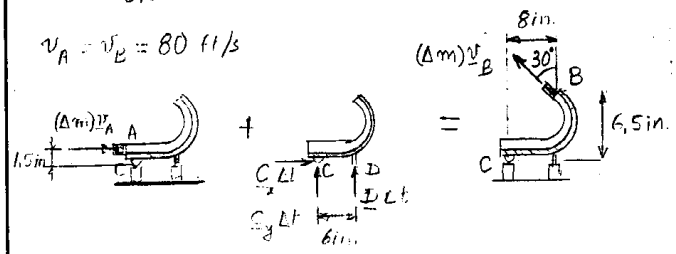
$$\underline{v}_C = (1650 \text{ ft/s})\underline{i} - (150 \text{ ft/s})\underline{j} + (270 \text{ ft/s})\underline{k}$$

14.104 We apply the principle of impulse and momentum to the water in contact with the vane and to the mass Δm of water entering and leaving the system in the interval Δt. We have

$$\Delta m = \rho Q \Delta t = \frac{\gamma}{g} Q \Delta t = \frac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\left(200\frac{\text{gal}}{\text{min}}\right)\left(\frac{1 \text{ ft}^3}{7.48 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\Delta t$$

$$\Delta m = 0.8636\,\Delta t$$

$v_A = v_B = 80$ ft/s



$+\circlearrowleft \sum M_C$ = moments about C:

$$-(\Delta m)v_A(1.5\text{ in.}) + (D\Delta t)(6\text{ in.}) = (\Delta m)v_B \sin 30°(6.5\text{ in.}) + (\Delta m)v_B \cos 30°(8\text{ in.})$$

$$-(0.8636\,\Delta t)(80)(1.5) + (D\Delta t)(6) = (0.8636\,\Delta t)(80)(6.5\sin 30° + 8\cos 30°)$$

$$6D = (0.8636)(80)(1.5 + 6.5\sin 30° + 8\cos 30°)$$

$$6D = 806.81 \qquad D = 134.47 \text{ lb} \qquad \underline{D} = 134.5 \text{ lb} \uparrow$$

$\pm\Sigma F_x$ comp.: $(\Delta m)v_A + C_x \Delta t = -(\Delta m)v_B \sin 30°$

$$C_x = -(0.8636)(80)(1 + \sin 30°) = -103.6 \text{ lb}$$

$$\underline{C}_x = 103.6 \text{ lb} \leftarrow$$

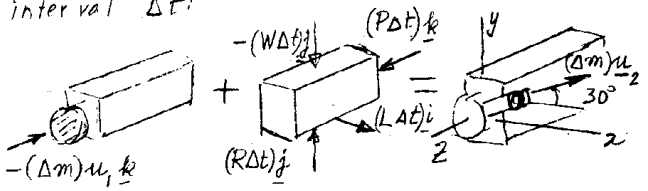
$+\uparrow \Sigma F_y$ comp.: $0 + C_y \Delta t + D\Delta t = (\Delta m)v_B \cos 30°$

$$C_y = -D + (0.8636)(80\cos 30°)$$

$$C_y = -134.47 + 59.83 = -74.64 \text{ lb}$$

$$\underline{C}_y = 74.6 \text{ lb} \downarrow$$

14.105 Measuring all velocities with respect to the plow car, we apply the principle of impulse and momentum to the plow car, the snow it contains, and the snow entering in the time interval Δt:



$$-(\Delta m)u_1\underline{k} + (L\Delta t)\underline{i} + (P\Delta t)\underline{k} + (R\Delta t)\underline{j} - (W\Delta t)\underline{j} = (\Delta m)u_2(\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$$

Equating the coefficients of the unit vectors:

$\underline{k}$: $-(\Delta m)u_1 + P\Delta t = 0$ $\qquad P = \frac{\Delta m}{\Delta t}u_1$ (1)

$\underline{i}$: $L\Delta t = (\Delta m)u_2 \cos 30°$ $\qquad L = \frac{\Delta m}{\Delta t}u_2 \cos 30°$ (2)

(CONTINUED)

14.105 CONTINUED Given data:

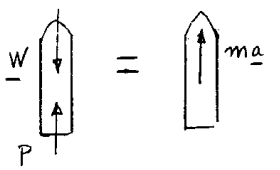
$u_1 = v = 15 \text{ mi/h} = 22 \text{ ft/s}$, $u_2 = 50 \text{ ft/s}$

$$\frac{\Delta m}{\Delta t} = \frac{(150 \text{ tons})(2000 \text{ lb/ton})}{(60 \text{ s})(32.2 \text{ ft/s}^2)} = 155.28 \text{ lb}\cdot\text{s/ft}$$

Eq. (1): $P = (155.28 \text{ lb}\cdot\text{s/ft})(22 \text{ ft/s})$, $P = 3420 \text{ lb}$ ◀

Eq. (2): $L = (155.28 \text{ lb}\cdot\text{s/ft})(50 \text{ ft/s})\cos 30°$, $L = 6720 \text{ lb}$ ◀

14.106



$+\uparrow \Sigma F_y = ma$: $\quad P - W = ma$

$$P = m(g + a)$$

But, by Eq. (14.43): $\quad P = \frac{dm}{dt}u$

Thus: $\quad \frac{dm}{dt}u = m(g + a)$ (1)

At $t = 0$: $\frac{dm}{dt} = 10 \text{ kg/s}$, $m = m_0$, $a = 30 \text{ m/s}^2$

Eq. (1): $\quad 10u = m_0(9.81 + 30)$ (2)

At $t = 80$ s: $\frac{dm}{dt} = 10 \text{ kg/s}$, $m = m_0 - (10 \text{ kg/s})(80 \text{ s})$

$a = 350 \text{ m/s}^2$ $\qquad m = m_0 - 800$

Eq. (1): $\quad 10u = (m_0 - 800)(9.81 + 350)$ (3)

(a) Subtract (2) from (3):

$$359.81 m_0 - 39.81 m_0 - 800(359.81) = 0$$

$$320 m_0 = 800(359.81)$$

$$m_0 = 899.53 \qquad m_0 = 900 \text{ kg}$$ ◀

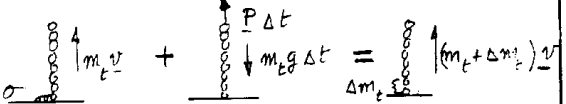
(b) Eq. (2): $\quad 10u = 899.53(39.81)$

$$u = 3580 \text{ m/s}$$ ◀

14.107 (a) At time t the mass of chain already off the floor is $m_t = m(vt/\ell)$. The additional mass Δm_t set in motion during time interval Δt is

$$\Delta m_t = \frac{mv}{\ell}\Delta t$$

Applying the principle of impulse and momentum to the portion of chain off the floor at $t + \Delta t$:



$+\uparrow$ y comp.: $\quad m_t v + P\Delta t - m_t g \Delta t = m_t v + \Delta m_t v$

$$P = m_t g + \frac{\Delta m_t}{\Delta t}v = m_t g + \frac{mv}{\ell}v$$

When $m_t = \frac{1}{2}m$: $\qquad P = \frac{1}{2}mg + \frac{mv^2}{\ell}$ ◀

(b) The reaction $\underline{R}$ is equal to the weight of the chain still on the floor. Thus $\qquad R = \frac{1}{2}mg$ ◀

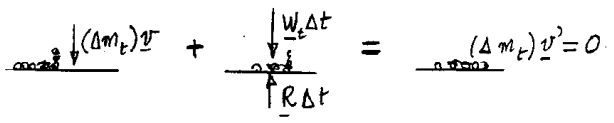
14.108 (a) The required force $\underline{P}$ is equal to the weight of the chain still off the floor.

Thus: $P = \frac{1}{2} mg$ ◀

(b) At time t, the mass of chain already on the floor is $m_t = m(vt/\ell)$. The additional mass Δm_t hitting the floor in Δt is

$$\Delta m_t = \frac{mv}{\ell}\Delta t$$

Applying the principle of impulse and momentum to the portion of chain on the floor at time $t + \Delta t$:



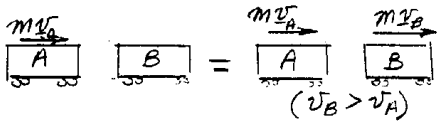
$+\downarrow$ y comp: $(\Delta m_t)v + W_t\Delta t - R\Delta t = 0$

$$R = \frac{\Delta m_t}{\Delta t}v + m_t g = \frac{mv}{\ell}v + m_t g$$

When $m_t = \frac{1}{2}m$: $\quad R = \frac{1}{2}mg + \frac{mv^2}{\ell}$ ◀

14.109 Case (a)

A hits B:



$(v_B > v_A)$

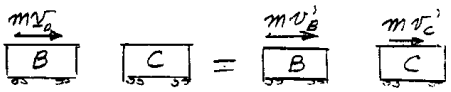
Cons. of momentum: $mv_0 = mv_A + mv_B$

Cons. of energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$

or $\quad v_0 = v_A + v_B \quad$ and $\quad v_0^2 = v_A^2 + v_B^2$

Only values of v_A and v_B satisfying both equations and $v_B > v_A$ are $\quad v_A = 0, \quad v_B = v_0$

B hits C:



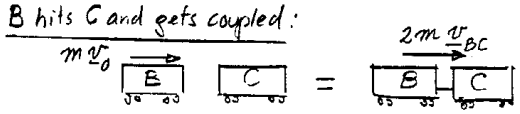
Same situation as above, with B and C replacing A and B. Therefore $\quad v_B' = 0, \quad v_C' = v_0$

Final velocities: $\quad \underline{v}_A = \underline{v}_B = 0 \; ; \; \underline{v}_C = v_0 \rightarrow$ ◀

Case (b)

A hits B: Same as for case (a): $\quad v_A = 0, \quad v_B = v_0$

B hits C and gets coupled:



Cons. of momentum: $\quad mv_0 = 2mv_{BC} \quad v_{BC} = \frac{1}{2}v_0$

Final velocities: $\quad \underline{v}_A = 0 \; ; \; \underline{v}_B = \underline{v}_C = \frac{1}{2}v_0 \rightarrow$ ◀

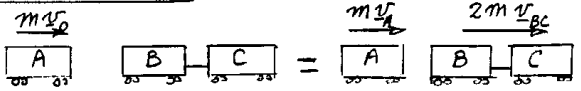
Note: No conservation of energy when B hits C

(CONTINUED)

14.109 CONTINUED

Case (c)

A hits B and C



Cons. of momentum: $\quad mv_0 = mv_A + 2mv_{BC}$

Cons. of energy ($e = 1$): $\quad \frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}(2m)v_{BC}^2$

$$v_A + 2v_{BC} = v_0 \qquad (1)$$
$$v_A^2 + 2v_{BC}^2 = v_0^2 \qquad (2)$$

or
$$2v_{BC} = v_0 - v_A \qquad (1')$$
$$2v_{BC}^2 = v_0^2 - v_A^2 \qquad (2')$$

Dividing (2') by (1') member by member:

$$\frac{2v_{BC}^2}{2v_{BC}} = \frac{v_0^2 - v_A^2}{v_0 - v_A} \qquad v_{BC} = v_0 + v_A$$

$$v_{BC} - v_A = v_0 \qquad (3)$$

Adding (1) and (3): $\quad 3v_{BC} = 2v_0 \quad v_{BC} = \frac{2}{3}v_0$

From (3): $\quad v_A = v_{BC} - v_0 = \frac{2}{3}v_0 - v_0 = -\frac{1}{3}v_0$

Final velocities: $\quad \underline{v}_A = \frac{1}{3}v_0 \leftarrow \; ; \; \underline{v}_B = \underline{v}_C = \frac{2}{3}v_0 \rightarrow$ ◀

Note: Eq. (3) could have been obtained directly from relative-velocity equation $v_{BC} - v_A = e(v_0 - 0)$ with $e = 1$.

14.110 (a) We apply the principle of impulse and momentum to the gravel on the belt and to the mass Δm of gravel hitting and leaving the belt in the time interval Δt.

$\pm\rightarrow$ x comp: $\quad mv + P\Delta t = mv + (\Delta m)v$

$$P = \frac{\Delta m}{\Delta t}v = qv, \qquad P = qv$$ ◀

(b) Kinetic energy acquired per unit time

$$= \frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}qv^2$$

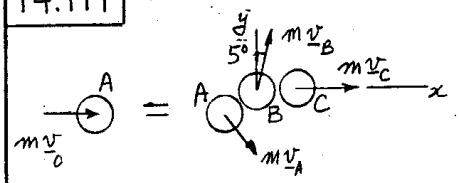
Work done per unit time

$$= \frac{\Delta U}{\Delta t} = \frac{P\Delta x}{\Delta t} = Pv = (qv)v = qv^2$$

Thus: $\quad \frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta U}{\Delta t} \qquad$ (Q.E.D.) ◀

The other half of the work of $\underline{P}$ is dissipated into heat (by friction) as the gravel slips on the belt before reaching the speed v.

14.111



$v_0 = 3\ m/s$
$v_B = 1.375\ m/s$
$v_C = 1.700\ m/s$

(a) Conservation of linear momentum

$$m\underline{v}_0 = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$$

$$m(3\underline{i}) = m\underline{v}_A + m(1.375)(\sin 5°\,\underline{i} + \cos 5°\,\underline{j}) + m(1.700\,\underline{i})$$

$$3\underline{i} = \underline{v}_A + 0.11984\,\underline{i} + 1.3698\,\underline{j} + 1.700\,\underline{i}$$

$$\underline{v}_A = (3 - 0.11984 - 1.700)\underline{i} - 1.3698\,\underline{j}$$

$$= 1.1802\,\underline{i} - 1.3698\,\underline{j}$$

$$\underline{v}_A = 1.808\ m/s \measuredangle 49.3°$$ ◀

(b) Percentage of energy lost

Initially: $T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} m (3)^2 = 4.5m$

Final state: $T = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2$

$$= \frac{1}{2} m \left[(1.808)^2 + (1.375)^2 + (1.700)^2 \right]$$

$$= 4.025\,m$$

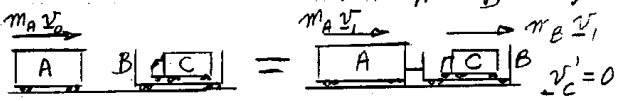
$$\text{Percent lost} = \frac{T_0 - T}{T_0}(100) = \frac{4.5 - 4.025}{4.5}(100)$$

$$= 10.56\%$$ ◀

14.112

(a) Cars A and B get automatically coupled

We note that truck C remains at rest ($\underline{v}_C = 0$) Immediately after coupling and that A and B have the same velocity ($\underline{v}_A' = \underline{v}_B' = \underline{v}_1$)



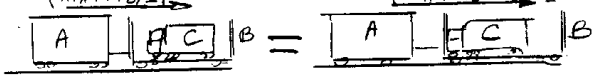
Conservation of linear momentum:

$$\pm\rightarrow \quad m_A v_0 = (m_A + m_B) v_1$$

$$v_1 = \frac{m_A}{m_A + m_B} v_0 = \frac{40}{40+20}(4.5\,mi/h) = 3.00\,mi/h$$

$$\underline{v}_A' = \underline{v}_B' = \underline{v}_1 = 3.00\,mi/h \rightarrow ; \quad \underline{v}_C' = 0$$ ◀

(b) Flatcar hits truck (perfectly plastic impact)



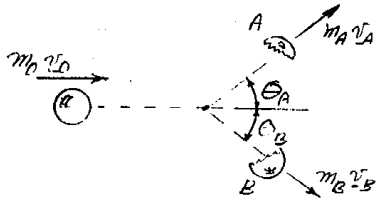
Conservation of linear momentum:

$$\pm\rightarrow \quad (m_A + m_B) v_1 = (m_A + m_B + m_C) v_2$$

$$v_2 = \frac{m_A + m_B}{m_A + m_B + m_C} v_1 = \frac{40+20}{40+20+30}(3.00\,mi/h) = 2.00\,mi/h$$

$$\underline{v}_A'' = \underline{v}_B'' = \underline{v}_C'' = \underline{v}_2 = 2.00\,mi/h \rightarrow$$ ◀

PROBLEM 14.C1



$\pm$ comps: $m_0 v_0 = m_A v_A \cos\theta_A + m_B v_B \cos\theta_B$ (1)

$+\uparrow$ comps: $0 = m_A v_A \sin\theta_A - m_B v_B \sin\theta_B$ (2)

From (2): $v_B = \frac{m_A}{m_B} \cdot \frac{\sin\theta_A}{\sin\theta_B} v_A$ $\qquad v_B = (COEF.)\, v_A$ ◁

WHERE, $COEF. = \left(\frac{m_A}{m_B}\right)\left(\frac{\sin\theta_A}{\sin\theta_B}\right)$ ◁

From (1): $m_0 v_0 = m_A v_A \cos\theta_A + m_B (COEF) v_A \cos\theta_B$

$m_0 v_0 = \left(m_A \cos\theta_A + m_B (COEF) \cos\theta_B\right) v_A$

LET: $DENOM = m_A \cos\theta_A + m_B (COEF) \cos\theta_B$ ◁

$v_A = \frac{m_0 v_0}{DENOM}$ ◁

```
10 '*****************************************************************
20 '*                        PROBLEM 14.C1                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '             M0 = initial mass of projectile  (kg)
90 '             V0 = initial velocity of projectile  (m/s)
100 '
110 '            MA = mass of fragment A   (kg)
120 '            VA = final velocity of fragment A   (m/s)
130 '           THA = angle theta sub A   (degrees)
140 '
150 '            MB = mass of fragment B  (kg)
160 '            VB = final velocity of fragment B   (m/s)
170 '           THB = angle theta sub B   (degrees)
180 '
190 '    COEF,DENOM = functions as defined above
200 '
210     K = 4*ATN(1)/180
220     M0 = 12
230     V0 = 40
240     MA = 3
250     MB = 9
260     THB = 30
270 '
280     PRINT "theta sub A    Vel of A      Vel of B"
290     PRINT"  degrees        m/s           m/s":PRINT
300 '
310     FOR THA = 30 TO 120 STEP 5
320        COEF = (MA/MB)*(SIN(K*THA)/SIN(K*THB))
330        DENOM = MA*COS(K*THA) + MB*COEF*COS(K*THB)
340        VA = (M0*V0)/DENOM
350        VB = COEF*VA
360        PRINT USING "   ###    ";THA,
370        PRINT USING "      ###.##  "; VA,VB
380     NEXT
```

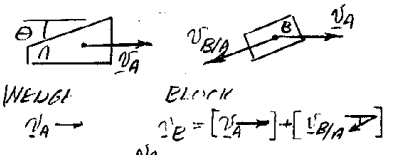
theta sub A degrees	Vel of A m/s	Vel of B m/s
30	92.38	30.79
35	88.27	33.75
40	85.13	36.48
45	82.82	39.04
50	81.23	41.49
55	80.31	43.85
60	80.00	46.19
65	80.31	48.52
70	81.23	50.89
75	82.82	53.33
80	85.13	55.89
85	88.27	58.62
90	92.38	61.58
95	97.66	64.86
100	104.43	68.56
105	113.14	72.85
110	124.46	77.97
115	139.48	84.27
120	160.00	92.38

```
C*****************************************************************
C*                        PROBLEM 14.C1                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C            Definition of Symbols
C
C                M0 = initial mass of projectile (kg)
C                V0 = initial velocity of projectile  (m/s)
C
C                MA = mass of fragment A   (kg)
C                VA = final velocity of fragment A  (m/s)
C               THA = angle theta sub A  (degrees)
C
C                MB = mass of fragment B   (kg)
C                VB = final velocity of fragment B  (m/s)
C               THB = angle theta sub B  (degrees)
C
C       COEF,DENOM = functions as defined above
C
      REAL  K, M0, MA, MB
      K = 4.*ATAN(1.)/180.
      M0 = 12.
      V0 = 40.
      MA = 3.
      MB = 9.
      THB = 30.
C
      WRITE (*,'(/,2X,2A\)') 'theta sub A          Vel of A',
     +'         Vel of B'
      WRITE (*,'(/,2X,2A\)') '  degrees               m/s  ',
     +'          m/s'
      WRITE (*,'(/,A\)')
C
      DO 10 I = 30,120,5
           THA = I
           COEF = (MA/MB)*(SIN(K*THA)/SIN(K*THB))
           DENOM = MA*COS(K*THA) + MB*COEF*COS(K*THB)
           VA = (M0*V0)/DENOM
           VB = COEF*VA
           WRITE (*,20) THA,VA,VB
   20      FORMAT (6X,F4.0,13X,F6.2,12X,F5.2)
   10 CONTINUE
      END
```

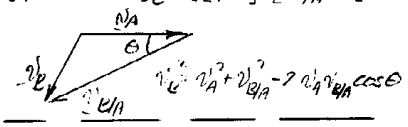
theta sub A degrees	Vel of A m/s	Vel of B m/s
30.	92.38	30.79
35.	88.27	33.75
40.	85.13	36.48
45.	82.82	39.04
50.	81.23	41.49
55.	80.31	43.85
60.	80.00	46.19
65.	80.31	48.52
70.	81.23	50.89
75.	82.82	53.33
80.	85.13	55.89
85.	88.27	58.62
90.	92.38	61.58
95.	97.66	64.86
100.	104.43	68.56
105.	113.14	72.85
110.	124.46	77.97
115.	139.48	84.27
120.	160.00	92.38

PROBLEM 14.C2

KINEMATICS

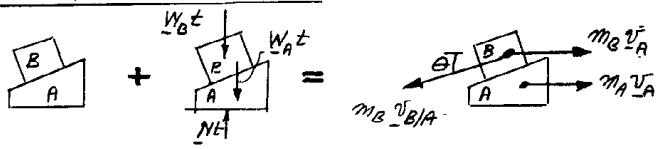


$\vec{v}_A \rightarrow$ $\quad \vec{v}_B = [\vec{v}_A \rightarrow] + [\vec{v}_{B/A}]$



$v_B^2 = v_A^2 + v_{B/A}^2 - 2 v_A v_{B/A} \cos\theta$

PRINCIPLE OF IMPULSE AND MOMENTUM

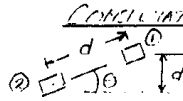


$\pm \rightarrow$ comps: $0 + 0 = m_A v_A + m_B v_A - m_B v_{B/A} \cos\theta$

$$v_A = \left(\frac{m_B \cos\theta}{m_A + m_B}\right) v_{B/A}, \quad \text{LET: } K = \frac{m_B \cos\theta}{m_A + m_B}$$

$$v_A = K v_{B/A}$$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$



① $T_1 = 0$; $V_1 = W_B (d \sin\theta)$

② $T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$; $V_2 = 0$

$$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(v_A^2 + v_{B/A}^2 - 2 v_A v_{B/A} \cos\theta\right)$$

SUBSTITUTE $v_A = K v_{B/A}$:

$$T_2 = \frac{1}{2} m_A (K v_{B/A})^2 + \frac{1}{2} m_B \left(K^2 + 1 - 2K\cos\theta\right) v_{B/A}^2$$

$T_1 + V_1 = T_2 + V_2$:

$$0 + W_B d \sin\theta = \frac{1}{2}\left[\frac{W_A}{g} K^2 + \frac{W_B}{g}\left(K^2 + 1 - 2K\cos\theta\right)\right] v_{B/A}^2$$

$$\text{LET: } C = \left(W_A + W_B\right) K^2 + \left(1 - 2K\cos\theta\right) W_B$$

WE HAVE: $W_B d \sin\theta = \frac{C}{2g} v_{B/A}^2$; $\quad v_{B/A} = \sqrt{\frac{2gd\sin\theta}{C} W_B}$

GIVEN, $d \sin\theta = (3\text{ft}) \sin 30^\circ = 1.5\text{ft}$ $\quad v_{B/A} = \sqrt{\frac{3 W_B g}{C}}$

```
10 '*****************************************************************
20 '*                      PROBLEM 14.C2                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '            WA = weight of wedge A   (pounds)
90 '            WB = weight of block B   (pounds)
100 '            G = acceleration of gravity   (ft/s^2)
110 '           TH = angle theta between inclined edge of the
120 '                   wedge and the horizontal   (degrees)
130 '          C,K = functions as defined above
140 '
150     WA = 25:  G = 32.2
160     J = 4*ATN(1)/180
170     TH = 30
180 '
190     PRINT "Weight of B      vel B/A           vel A"
200     PRINT "   lb             ft/s             ft/s":PRINT
210     FOR WB = 5 TO 20 STEP 1
220         K = (WB*COS(J*TH))/(WA + WB)
230         C = (WA + WB)*K^2 + (1 -2*K*COS(J*TH))*WB
240         VBA = SQR(3*WB*G/C)
250         VA = K*VBA
260         PRINT USING "   ##        ";WB,
270         PRINT USING "    ##.###       ";VBA,VA
280     NEXT
```

```
 Weight of B      vel B/A          vel A
    lb              ft/s            ft/s

     5             10.507          1.517
     6             10.630          1.782
     7             10.750          2.036
     8             10.866          2.281
     9             10.979          2.517
    10             11.088          2.744
    11             11.195          2.962
    12             11.298          3.173
    13             11.399          3.377
    14             11.497          3.574
    15             11.593          3.765
    16             11.686          3.950
    17             11.777          4.128
    18             11.866          4.302
    19             11.953          4.470
    20             12.037          4.633
```

```
C*****************************************************************
C*                      PROBLEM 14.C2                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C            Definition of Symbols
C
C               WA = weight of wedge A  (pounds)
C               WB = weight of block B  (pounds)
C                G = acceleration of gravity  (ft/s**2)
C               TH = angle theta between inclined edge of the
C                       wedge and the horizontal  (degrees)
C              C,K = functions as defined above
C
      REAL J, WB, K
      WA = 25.
      G = 32.2
      J = 4.*ATAN(1.)/180.
      TH = 30
      WRITE (*,'(/,2X,2A\)') 'Weight of B          vel B/A',
     +'              vel A'
      WRITE (*,'(/,2X,2A\)') '    lb                 ft/s ',
     +'               ft/s'
      WRITE (*,'(/,A\)')
      DO 260 I = 5,20,1
          WB = I
          K = (WB*COS(J*TH))/(WA+WB)
          C = (WA+WB)* K**2 +(1.-2.*K*COS(J*TH))*WB
          VBA = SQRT(3.*WB*G/C)
          VA = K*VBA
          WRITE (*,30) WB,VBA,VA
   30     FORMAT (6X,F3.0,14X,F6.3,14X,F5.3)
  260 CONTINUE
      END
```

```
 Weight of B          vel B/A              vel A
    lb                  ft/s                ft/s

     5.                10.507              1.517
     6.                10.630              1.782
     7.                10.750              2.036
     8.                10.866              2.281
     9.                10.979              2.517
    10.                11.088              2.744
    11.                11.195              2.962
    12.                11.298              3.173
    13.                11.399              3.377
    14.                11.497              3.574
    15.                11.593              3.765
    16.                11.686              3.950
    17.                11.777              4.128
    18.                11.866              4.302
    19.                11.953              4.470
    20.                12.037              4.633
```

PROBLEM 14.C3

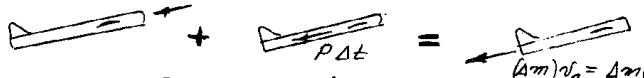
PRINCIPLE OF IMPULSE AND MOMENTUM CHOOSE FRAME AT REST WITH RESPECT TO ATMOSPHERE.

SEE SOLUTION OF PROB 14.67

v = VELOCITY OF AIRPLANE

BY NEWTON'S THIRD LAW, P = PROPULSIVE FORCE

$(\Delta m)v_1 = 0$



$(\Delta m)v_2 = \Delta m(u - v)$

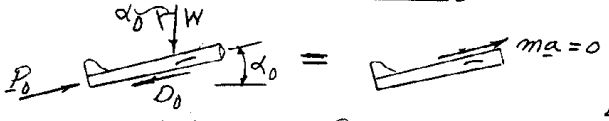
$\pm$ comps. $P\Delta t = \Delta m(u - v)$; $P = \frac{\Delta m}{\Delta t}(u - v)$

AIRPLANE CLIMBING AT INITIAL ANGLE α_0. ACCELERATION = 0

Let R = WEIGHT OF AIR SCOOPED IN PER SECOND

$\therefore R = g\left(\frac{\Delta m}{\Delta t}\right)_g$, THUS $P = (R/g)(u - v)$ ◁

$P_0 = \frac{R}{g}(u - v_0)$ ◁

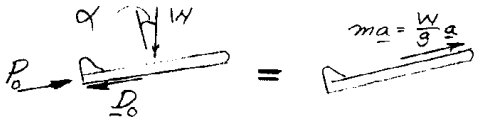


$\pm \rightarrow \Sigma F = 0$ $P_0 - D_0 - W\sin\alpha_0 = 0$

DRAG AT α_0 AND v_0

$D_0 = P_0 - W\sin\alpha_0$ ◁

AIRPLANE CLIMBING AT ANGLE α AND SPEED v_0 WITH ACCELERATION a.



$\pm \rightarrow \Sigma F = ma$: $P_0 - D_0 - W\sin\alpha = \frac{W}{g}a$

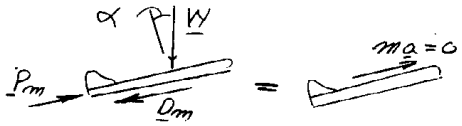
$a = \frac{P_0 - D_0 - W\sin\alpha}{(W/g)}$ ◁

MAXIMUM SPEED WHILE CLIMBING AT ANGLE α

LET v = MAXIMUM SPEED

$D_m = P_m = \frac{R}{g}(u - v)$; BUT $D \sim v^2$; $\frac{D}{D_0} = \left(\frac{v}{v_0}\right)^2$; $D = D_0\left(\frac{v}{v_0}\right)^2$ ◁

FOR MAXIMUM SPEED v, ACCELERATION = 0



$\pm \rightarrow \Sigma F = 0$: $P_m - D_m - W\sin\alpha = 0$

$\frac{R}{g}(u - v) - D_0\left(\frac{v}{v_0}\right)^2 - W\sin\alpha = 0$

$\frac{D_0}{v_0^2}v^2 + \frac{R}{g}v - \left(\frac{R}{g}u - W\sin\alpha\right) = 0$

$v^2 + \underbrace{\frac{R/g}{(D_0/v_0^2)}}_{b}v - \underbrace{\frac{(R/g)u - W\sin\alpha}{(D_0/v_0^2)}}_{c} = 0$

MAX VELOCITY: $v = \frac{1}{2}\left(-b + \sqrt{b^2 - 4c}\right)$ ◁

SUMMARY:

Definition of Symbols

```
      W = weight of jet airplane  (pounds)
      A = acceleration of airplane (ft/s**2)
      R = rate at which air is scooped in airplane (lb/s)
      G = acceleration of gravity (ft/s**2)
      U = velocity of discharge relative to airplane (ft/s)
  ALPHA = angle of climb (degrees)
 ALPHA0 = initial angle of climb (degrees)
      V = maximum speed attained (mi/h)
     V0 = constant initial speed of airplane (mi/h)
      D = drag force acting on airplane at speed VMAX (lb)
     D0 = drag force acting on airplane at speed V0 (lb)
      P = propulsive force developed at speed VMAX (lb)
     P0 = initial propulsive force developed at speed V0 (lb)
    B,C = coefficient in quadratic formula
```

FOR PROGRAMS SEE NEXT PAGE

PROBLEM 14.C3 (CONTINUED)

```
10 '*****************************************************************
20 '*                      PROBLEM 14.C3                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         W = weight of jet airplane  (pounds)
90 '         A = acceleration of airplane  (ft/s^2)
100 '        R = rate at which air is scooped in airplane  (lb/s)
110 '        G = acceleration of gravity   (ft/s^2)
120 '        U = velocity of discharge relative to airplane  (ft/s)
130 '    ALPHA = angle of climb   (degrees)
140 '   ALPHA0 = initial angle of climb  (degrees)
150 '        V = maximum speed attained  (mi/h)
160 '       V0 = constant initial speed of airplane   (mi/h)
170 '        D = drag force acting on airplane at speed VMAX  (lb)
180 '       D0 = drag force acting on airplane at speed V0  (lb)
190 '        P = propulsive force developed at speed VMAX  (lb)
200 '       P0 = initial propulsive force developed at speed V0 (lb)
210 '      B,C = coefficient in quadratic formula
220 '
230     'Data
240 '
250     LET ALPHA0 = 15   'degrees
260     LET V0 = 480      'mi/h
270         V0 = V0*88/60 'ft/s
280     LET W = 35000!     'lb
290     LET G = 32.2      'ft/s^2
300     LET R = 600       'lb/s
310     LET U = 2250      'ft/s
320         K = 4*ATN(1)/180
330 '
340       'Calculate initial drag force D0
350     P0 = (R/G)*(U - V0)
360     D0 = P0 - W*SIN(K*ALPHA0)
370 '
380     PRINT " alpha      acceleration        max v"
390     PRINT "degrees       ft/s^2            mi/h ";PRINT
400 '
410     FOR ALPHA = 0 TO 20
420           'Calculate initial acceleration
430         A = (P0 - D0 - W*SIN(K*ALPHA))/(W/G)
440 '
450           'Calculate propulsive force and drag for maximum speed
460         P = (R/G)*(U - V)
470         D = D0*(V/V0)^2
480 '
490           'Calculate maximum velocity V for angle of climb ALPHA.
500             'Solve resulting quadratic equation for V.
510           B = (R/G)/(D0/V0^2)
520           C = -(((R/G)*U) - W*SIN(K*ALPHA))/(D0/V0^2)
530           V = .5*(-B +SQR(B^2 - 4*C))
540           V = V*(60/88)     'return units of V to (mi/h)
550         PRINT USING "  ##    ";ALPHA,
560         PRINT USING "     ##.###       ";A,
570         PRINT USING "      ###.#";V
580     NEXT
```

alpha degrees	acceleration ft/s^2	max v mi/h
0	8.334	557.9
1	7.772	552.9
2	7.210	547.9
3	6.649	542.8
4	6.088	537.8
5	5.528	532.6
6	4.968	527.5
7	4.410	522.3
8	3.853	517.1
9	3.297	511.9
10	2.743	506.6
11	2.190	501.4
12	1.639	496.1
13	1.091	490.7
14	0.544	485.4
15	0.000	480.0
16	-0.542	474.6
17	-1.080	469.2
18	-1.616	463.8
19	-2.149	458.3
20	-2.679	452.9

```
C*****************************************************************
C*                      PROBLEM 14.C3                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C             W = weight of jet airplane  (pounds)
C             A = acceleration of airplane (ft/s**2)
C             R = rate at which air is scooped in airplane (lb/s)
C             G = acceleration of gravity (ft/s**2)
C             U = velocity of discharge relative to airplane (ft/s)
C         ALPHA = angle of climb (degrees)
C        ALPHA0 = initial angle of climb (degrees)
C             V = maximum speed attained (mi/h)
C            V0 = constant initial speed of airplane (mi/h)
C             D = drag force acting on airplane at speed VMAX (lb)
C            D0 = drag force acting on airplane at speed V0 (lb)
C             P = propulsive force developed at speed VMAX (lb)
C            P0 = initial propulsive force developed at speed V0 (lb)
C           B,C = coefficient in quadratic formula
C
C               DATA

      REAL  K, ALPHA
      ALPHA0 = 15.
C        Initial velocity in mi/h
      V0 = 480.
C        Initial velocity in ft/s
      V0 = V0*88./60.
      W = 35000.
      G = 32.2
      R = 600.
      U = 2250.
      K = 4.*ATAN(1.)/180.
C
C        Calculate initial drag force D0
      P0 = (R/G)*(U - V0)
      D0 = P0 - W*SIN(K*ALPHA0)
C
C
      WRITE (*,'(/,2X,2A\)') ' Alpha           Acceleration',
     +'           Max V'
      WRITE (*,'(/,2X,2A\)') 'degrees             ft/s**2    ',
     +'           mi/h'
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,20
C       Calculate initial acceleration
        ALPHA = I
          A = (P0 - D0 - W*SIN(K*ALPHA))/(W/G)
C
C       Calculate propulsive force and drag for maximum speed
          P = (R/G)*(U - V)
          D = D0*(V/V0)**2
C
C       Calculate maximum velocity V for angle of climb ALPHA.
C          Solve resulting quardatic equation for V.
          B = (R/G)/(D0/V0**2)
          C = -(((R/G)*U) - W*SIN(K*ALPHA))/(D0/V0**2)
          V = 0.5*(-B + SQRT(B**2 - 4.*C))
C       Return units of V to (mi/h)
          V = V*(60./88.)
          WRITE (*,11) ALPHA,A,V
   11     FORMAT (3X,F3.0,15X,F6.3,15X,F5.1)
   10 CONTINUE
      END
```

Alpha degrees	Acceleration ft/s**2	Max V mi/h
0.	8.334	557.9
1.	7.772	552.9
2.	7.210	547.9
3.	6.649	542.8
4.	6.088	537.8
5.	5.528	532.6
6.	4.968	527.5
7.	4.410	522.3
8.	3.853	517.1
9.	3.297	511.9
10.	2.743	506.6
11.	2.190	501.4
12.	1.639	496.1
13.	1.091	490.7
14.	.544	485.4
15.	.000	480.0
16.	-.542	474.6
17.	-1.080	469.2
18.	-1.616	463.8
19.	-2.149	458.3
20.	-2.679	452.9

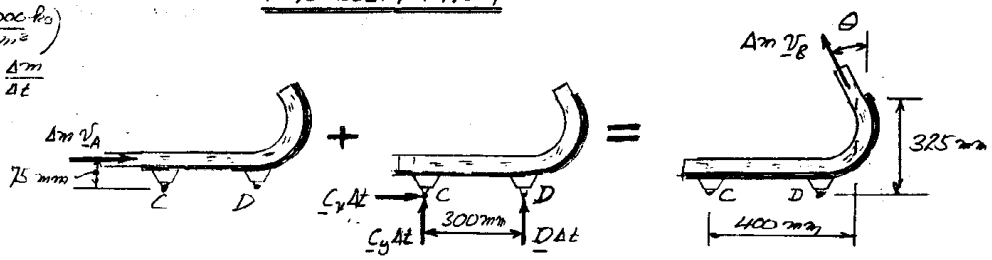
MAKRON Books
Marketing Universitário
(0xx11) 3865-3005
(0xx11) 3862-6729
fax (0xx11) 3865-6095

PROBLEM 14.C4

q = FLOW IN m^3/min

$Q = q\,\frac{m^3}{min}\left(\frac{min}{Acc}\right)\left(\frac{1000\,kg}{m^3}\right)$

Q = FLOW IN $kg/s = \frac{\Delta m}{\Delta t}$



$+\circlearrowleft \Sigma M_C$: $-(\Delta m)v_A(75\,mm)+(D\Delta t)(300\,mm) = (\Delta m)v_B\cos\theta(400\,mm)+(\Delta m)v_B\sin\theta(325\,mm)$

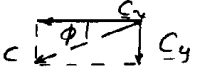
FOR STEADY FLOW: $v = v_A = v_B$: $D = \frac{\Delta m}{\Delta t}\frac{v}{300}(325\sin\theta + 400\cos\theta + 75)$

SUBSTITUTE $Q = \frac{\Delta m}{\Delta t}$: $D = (Qv/300)(325\sin\theta + 400\cos\theta + 75)$ ◁

$\xrightarrow{+}$ x comps: $\Delta m v_A + C_x\Delta t = -(\Delta m)v_B\sin\theta$ $\quad C_x = -(Qv)(1+\sin\theta)$ ◁

$+\uparrow$ y comps: $C_y\Delta t + D\Delta t = (\Delta m)v_B\cos\theta$ $\quad C_y = -D+(Qv)\cos\theta$ ◁

WE NOTE THAT $C_x < 0$ AND THUS IS DIRECTED TO LEFT. IT MAY BE CHECKED THAT $C_y < 0$.




```
10 '*****************************************************************
20 '*                      PROBLEM 14.C4                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '            Q = rate of flow of water  (cubic meters per minute)
90 '            V = velocity of water   (m/s)
100 '          TH = angle theta     (degrees)
110 '           C = magnitude of reaction at C  (N)
120 '         PHI = direction of C  (degrees)
130 '           D = magnitude of vertical reaction at D  (N)
140 '
150    'Data
160     LET V = 40          '(m/s)
170     LET Q = 1.25        '(cubic meters per minute)
180         Q = Q*1000/60   'express Q in kg/s
190 '
200     K = 4*ATN(1)/180
210 '
220     PRINT "  THETA          C          PHI            D"
230     PRINT " degrees       newtons     degrees        newtons":PRINT
240 '
250       'Calculate reactions
260     FOR TH = 0 TO 90 STEP 10
270        D = (Q*V/300)*(325*SIN(K*TH) + 400*COS(K*TH) +75)
280        CX = -(Q*V)*(1 + SIN(K*TH))
290        CY = -D + (Q*V)*COS(K*TH)
300        C = SQR(CX^2 + CY^2)
310        PHI = (ATN(CY/CX))/K
320        PRINT USING "  ###          ####.#       ";TH,C,
330        PRINT USING "###.##         ####.#";PHI,D
340     NEXT
350     PRINT
360     PRINT "  Since CX is to the left and CY is downward, the
370     PRINT "  angle PHI is measured down to the left from the
380     PRINT "  horizontal. Reaction at D is vertical upward.
Ok
RUN
  THETA          C          PHI            D
 degrees       newtons     degrees        newtons

    0           964.8       30.26         1319.4
   10          1168.1       33.14         1459.3
   20          1362.4       34.83         1561.2
   30          1540.5       35.76         1622.0
   40          1696.2       36.19         1639.8
   50          1824.5       36.23         1614.1
   60          1921.7       35.98         1545.7
   70          1984.7       35.47         1436.7
   80          2012.0       34.71         1290.3
   90          2003.1       33.69         1111.1

 Since CX is to the left and CY is downward, the
 angle PHI is measured down to the left from the
 horizontal. Reaction at D is vertical upward.
```

```
C*****************************************************************
C*                      PROBLEM 14.C4                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C                Q = rate of flow of water (cubic meters per minute)
C                V = velocity of water (m/s)
C               TH = angle theta (degree)
C                C = magnitude of reaction at C  (N)
C              PHI = direction of C  (degrees)
C                D = magnitude of vertical reaction at D  (N)
C
      REAL K, J
      V = 40.
C           Q = Cubic meter per minute
      Q = 1.25
C           Express Q in kg/s
      Q = Q*1000./60.
C
      K = 4.*ATAN(1.)/180.
      WRITE (*,'(/,2X,A\)') ' THETA              C              PHI
     +       D'
      WRITE (*,'(/,2X,A\)') 'degrees          newtons          degrees
     +    newtons'
      WRITE (*,'(/,A\)')
C
C           Calculate reactions
      DO 10 I = 0,90,10
           TH = I
            D = (Q*V/300.)*(325.*SIN(K*TH) + 400.*COS(K*TH) +75.)
           CX = -(Q*V)*(1. + SIN(K*TH))
           CY = -D + (Q*V)*COS(K*TH)
            C = SQRT(CX**2 + CY**2)
          PHI = (ATAN(CY/CX))/K
          WRITE (*,20) TH,C,PHI,D
   20     FORMAT (4X,F4.0,10X,F6.1,10X,F6.2,10X,F6.1)
   10 CONTINUE
      WRITE (*,'(/,5X,A\)')'Since CX is to the left and CY is downward,'
      WRITE (*,'(/,5X,A\)')'angle PHI is measured down to the left'
      WRITE (*,'(/,5X,A\)')'from the horizontal. Reaction at D is'
      WRITE (*,'(/,5X,A\)')'vertical and upward.'
      END
```

```
  THETA          C          PHI            D
 degrees       newtons     degrees        newtons

    0.          964.8       30.26         1319.4
   10.         1168.1       33.14         1459.3
   20.         1362.4       34.83         1561.2
   30.         1540.5       35.76         1622.0
   40.         1696.2       36.19         1639.8
   50.         1824.5       36.23         1614.1
   60.         1921.7       35.98         1545.7
   70.         1984.7       35.47         1436.7
   80.         2012.0       34.71         1290.3
   90.         2003.1       33.69         1111.1

 Since CX is to the left and CY is downward,
 angle PHI is measured down to the left
 from the horizontal. Reaction at D is
 vertical and upward.
```

15.1

$\theta = 0.75\cos(2\pi t/1.20)$

$\omega = d\theta/dt = -0.75(2\pi/1.20)\sin(2\pi t/1.2)$

$\alpha = d\omega/dt = -0.75(2\pi/1.20)^2\cos(2\pi t/1.2)$

$\omega = -3.927\sin(2\pi t/1.2)$

$\omega_{max} = 3.93$ rad/s ◀

$\alpha = -20.562\cos(2\pi t/1.2)$

$\alpha_{max} = 20.56$ rad/s^2 ◀

15.2

$\theta = 1.20(1 - e^{-t/3})$

$\omega = d\theta/dt = +\frac{1}{3}(1.20)e^{-t/3} = 0.40\,e^{-t/3}$

$\alpha = d\omega/dt = -\frac{1}{3}(0.40)e^{-t/3} = -0.1333\,e^{-t/3}$

(a) $t = 0$: $\theta = 1.20$ rad, $\omega = 0.40$ rad/s, $\alpha = -0.1333$ rad/s^2 ◀

(b) $t = 3$ s: $\theta = 1.20(1 - e^{-1})$ $\quad \theta = 0.759$ rad ◀

$\omega = 0.40e^{-1}$ $\quad \omega = 1.472$ rad/s ◀

$\alpha = -0.1333e^{-1}$ $\quad \alpha = -0.0491$ rad/s^2 ◀

(c) $t = \infty$: $\theta = 1.20$ rad; $\quad \omega = \alpha = 0$ ◀

15.3

UNIF. ACCEL. MOTION $\quad \omega = \omega_0 + \alpha t$ (1)

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (2)

(a) $\omega_0 = 0$; AT $t = 5$ s, $\omega = 3600$ rpm $= 60$ rev/s

EQ(1): 60 rev/s $= 0 + \alpha(5\text{ s})$ $\quad \alpha = 12$ rev/s^2

EQ(2): $\theta = 0 + \frac{1}{2}(12\text{ rev/s}^2)(5\text{ s})^2$ $\quad \theta = 150$ rev ◀

(b) $\omega_0 = 60$ rev/s; AT $t = 70$ s, $\omega = 0$

EQ(1): $0 = 60\text{ rev/s} + \alpha(70\text{ s})$ $\quad \alpha = -\frac{6}{7}$ rev/s^2

EQ(2): $\theta = (60\text{ rev/s})(70\text{ s}) + \frac{1}{2}(-\frac{6}{7}\text{ rev/s}^2)(70\text{ s})^2$

$\theta = 2100$ rev ◀

15.4

UNIF. ACCEL. MOTION $\quad \omega = \omega_0 + \alpha t$ (1)

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (2)

(a) $\omega_0 = 0$, $\theta_0 = 0$

AT $t = 0.3$ s $\quad \theta = 2.5$ rev $= 2.5(2\pi) = 5\pi$ rad

EQ(2): 5π rad $= 0 + 0 + \frac{1}{2}\alpha(0.3\text{ s})^2$

$\alpha = 349.06$ rad/s^2 $\quad \alpha = 349$ rad/s^2 ◀

(b) EQ(1): $\omega = 0 + (349.06\text{ rad/s}^2)(0.3\text{ s})$

$\omega = 104.71$ rad/s $\quad \omega = 1000$ rpm ◀

15.5

$\alpha = kt$; $\quad \omega = \int_0^t \alpha\,dt = \int_0^t kt\,dt = \frac{1}{2}kt^2$

$\theta = \int_0^t \omega\,dt = \int_0^t \frac{1}{2}kt^2\,dt = \frac{1}{6}kt^3$

AT $t = 10$ s: 20 rev $= \frac{1}{6}k(10\text{ s})^3$ $\quad k = 0.120$ rev/s^3 ◀

THUS $\theta = \frac{1}{6}(0.120)t^3$ $\quad \theta = 0.020t^3$ (rev) ◀

$\omega = d\theta/dt = 0.060t^2$ (rev/s); $\alpha = d\omega/dt = 0.120t$ (rev/s^2) ◀

(a) $t = 20$ s: $\omega = 0.060(20)^2 = 24$ rev/s $= 1440$ rpm ◀

(b) $\theta = 40$ rev: $40 = 0.020t^3$; $t^3 = 2000$ $\quad t = 12.60$ s ◀

15.6

$\vec{DA} = -(200\text{ mm})\underline{i} + (120\text{ mm})\underline{j} + (90\text{ mm})\underline{k}$

$= 250$ mm

$\omega = 75$ rad/s

$\underline{\lambda}_{DA} = \frac{\vec{DA}}{DA} = \frac{-200\underline{i} + 120\underline{j} + 90\underline{k}}{250} = \frac{1}{25}(-20\underline{i} + 12\underline{j} + 9\underline{k})$

$\underline{\omega} = \omega\underline{\lambda}_{AD} = (75\text{ rad/s})\frac{1}{25}(-20\underline{i} + 12\underline{j} + 9\underline{k})$

$\underline{\omega} = -(60\text{ rad/s})\underline{i} + (36\text{ rad/s})\underline{j} + (27\text{ rad/s})\underline{k}$

$\alpha = 0$

(CONTINUED)

15.6 CONTINUED

POINT B: $\underline{r}_{B/A} = (200\text{ mm})\underline{i} = (0.2\text{ m})\underline{i}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -60 & 36 & 27 \\ 0.2 & 0 & 0 \end{vmatrix} = 5.4\underline{j} - 7.2\underline{k}$

$\underline{v}_B = (5.4\text{ m/s})\underline{j} - (7.2\text{ m/s})\underline{k}$ ◀

$\underline{a}_F = \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{B/A}) = 0 + \underline{\omega} \times \underline{v}_B$

$\underline{a}_F = \underline{\omega} \times \underline{v}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -60 & 36 & 27 \\ 0 & 5.4 & -7.2 \end{vmatrix} = (-259.2 - 145.8)\underline{i} - 432\underline{j} - 324\underline{k}$

$\underline{a}_B = -(405\text{ m/s}^2)\underline{i} - (432\text{ m/s}^2)\underline{j} - (324\text{ m/s}^2)\underline{k}$ ◀

15.7

SEE PROBLEM 15.6 FOR $\underline{\lambda}_{AD}$ AND $\underline{\omega}_{AD}$

$\alpha = -600$ rad/s^2; $\quad \underline{\alpha} = \alpha\underline{\lambda}_{AD} = (-600)\frac{1}{25}(-20\underline{i} + 12\underline{j} + 9\underline{k})$

$\underline{\alpha} = (480\text{ rad/s}^2)\underline{i} - (288\text{ rad/s}^2)\underline{j} - (216\text{ rad/s}^2)\underline{k}$

POINT B: $\underline{r}_{B/A} = (200\text{ mm})\underline{i} = (0.2\text{ m})\underline{i}$

$\underline{v}_B$ IS SAME AS FOR PROB 15.6: $\underline{v}_B = (5.4\text{ m/s})\underline{j} - (7.2\text{ m/s})\underline{k}$ ◀

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{B/A}) = \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times \underline{v}_B$

$\underline{a}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 480 & -288 & -216 \\ 0.2 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -60 & 36 & 27 \\ 0 & 5.4 & -7.2 \end{vmatrix}$

$= -43.2\underline{j} + 57.6\underline{k} + (-259.2 - 145.8)\underline{i} - 432\underline{j} - 324\underline{k}$

$\underline{a}_B = -(405\text{ m/s}^2)\underline{i} - (475\text{ m/s}^2)\underline{j} - (266\text{ m/s}^2)\underline{k}$ ◀

15.8

$\vec{BA} = -(5\text{ in.})\underline{j} + (12\text{ in.})\underline{k}$ $\quad BA = 13$ in.

$\underline{\lambda}_{BA} = \frac{\vec{BA}}{BA} = \frac{-5\underline{j} + 12\underline{k}}{13} = \frac{1}{13}(-5\underline{j} + 12\underline{k})$, $\alpha = 0$

$\underline{\omega} = \omega\underline{\lambda}_{BA} = (26\text{ rad/s})(\frac{1}{13})(-5\underline{j} + 12\underline{k}) = -(10\text{ rad/s})\underline{j} + (24\text{ rad/s})\underline{k}$

POINT E: $\underline{r}_{E/A} = (6\text{ in.})\underline{i} - (4\text{ in.})\underline{j} - (6\text{ in.})\underline{k}$

$\underline{v}_E = \underline{\omega} \times \underline{r}_{E/A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -10 & 24 \\ 6 & -4 & -6 \end{vmatrix} = (60 + 96)\underline{i} + 144\underline{j} + 60\underline{k}$

$\underline{v}_E = +(156\text{ in./s})\underline{i} + (144\text{ in./s})\underline{j} + (60\text{ in./s})\underline{k}$ ◀

$\underline{a}_E = \underline{\alpha} \times \underline{r}_{E/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{E/A}) = 0 + \underline{\omega} \times \underline{v}_B$

$\underline{a}_E = \underline{\omega} \times \underline{v}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -10 & 24 \\ 156 & 144 & 60 \end{vmatrix} = (-600 - 3456)\underline{i} + 3744\underline{j} + 1560\underline{k}$

$= -(4056\text{ in./s}^2)\underline{i} + (3744\text{ in/s}^2)\underline{j} + (1560\text{ in/s}^2)\underline{k}$

$\underline{a}_E = -(338\text{ ft/s}^2)\underline{i} + (312\text{ ft/s}^2)\underline{j} + (130\text{ ft/s}^2)\underline{k}$ ◀

15.9

$\vec{AC} = (350\text{ mm})\underline{i} - (200\text{ mm})\underline{j} + (200\text{ mm})\underline{k}$ $\quad AC = 450$ mm

$\underline{\lambda}_{AC} = \frac{\vec{AC}}{AC} = \frac{350\underline{i} - 200\underline{j} + 200\underline{k}}{450} = \frac{1}{9}(7\underline{i} - 4\underline{j} + 4\underline{k})$

$\underline{\omega} = \omega\underline{\lambda}_{AC} = (9\text{ rad/s})\frac{1}{9}(7\underline{i} - 4\underline{j} + 4\underline{k})$

$\underline{\omega} = (7\text{ rad/s})\underline{i} - (4\text{ rad/s})\underline{j} + (4\text{ rad/s})\underline{k}$ $\quad \underline{\alpha} = 0$

CORNER F: $\underline{r}_{F/B} = (-175\text{ mm})\underline{i} + (100\text{ mm})\underline{k} = -(0.175\text{ m})\underline{i} + (0.100\text{ m})\underline{k}$

$\underline{v}_F = \underline{\omega} \times \underline{r}_{F/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -4 & 4 \\ -0.175 & 0 & 0.100 \end{vmatrix} = -0.4\underline{i} + (-0.7 - 0.7)\underline{j} - 0.7\underline{k}$

$\underline{v}_F = -(0.4\text{ m/s})\underline{i} - (1.4\text{ m/s})\underline{j} - (0.7\text{ m/s})\underline{k}$ ◀

(CONTINUED)

15.9 CONTINUED $\alpha = 0$

$$\underline{a}_F = \underline{\alpha} \times \underline{r}_{F/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{F/B}) = 0 + \underline{\omega} \times \underline{v}_F$$

$$\underline{a}_F = \underline{\omega} \times \underline{v}_F = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -4 & 4 \\ -0.4 & -1.4 & -0.7 \end{vmatrix}$$

$$= (2.8 + 5.6)\underline{i} + (-1.6 + 4.9)\underline{j} + (-9.8 - 1.6)\underline{k}$$

$$\underline{a}_F = (8.4\ m/s^2)\underline{i} + (3.3\ m/s^2)\underline{j} - (11.4\ m/s^2)\underline{k}$$

15.10

SEE PROB 15.9 FOR $\underline{a}_{AC}$ AND $\underline{\omega}$

$$\underline{a}_{AC} = \frac{1}{9}(7\underline{i} - 4\underline{j} + 4\underline{k})$$

$$\underline{\omega} = (7\ rad/s)\underline{i} - (4\ rad/s)\underline{j} + (4\ rad/s)\underline{k}$$

$\alpha = -18\ rad/s^2$; $\underline{\alpha} = \alpha \underline{a}_{AC} = (-18\ rad/s^2)\frac{1}{9}(7\underline{i} - 4\underline{j} + 4\underline{k})$

$$\underline{\alpha} = -(14\ rad/s^2)\underline{i} + (8\ rad/s^2)\underline{j} - (8\ rad/s^2)\underline{k}$$

CORNER H: $\underline{r}_{H/B} = (175\ mm)\underline{i} + (100\ mm)\underline{k} = (0.175\ m)\underline{i} + (0.100\ m)\underline{k}$

$$\underline{v}_H = \underline{\omega} \times \underline{r}_{H/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -4 & 4 \\ 0.175 & 0 & 0.100 \end{vmatrix} = -0.4\underline{i} + (0.7 - 0.7)\underline{j} + 0.7\underline{k}$$

$$\underline{v}_H = -(0.4\ m/s)\underline{i} + (0.7\ m/s)\underline{k}$$

$$\underline{a}_H = \underline{\alpha} \times \underline{r}_{H/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{H/B}) = \underline{\alpha} \times \underline{r}_{H/B} + \underline{\omega} \times \underline{v}_H$$

$$\underline{a}_H = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -14 & 8 & -8 \\ 0.175 & 0 & 0.100 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -4 & +4 \\ -0.4 & 0 & 0.7 \end{vmatrix}$$

$$= 0.8\underline{i} + (-1.40 + 1.40)\underline{j} - 1.40\underline{k} - 2.8\underline{i} + (-1.6 - 4.9)\underline{j} - 1.6\underline{k}$$

$$\underline{a}_H = -(2\ m/s^2)\underline{i} - (6.5\ m/s^2)\underline{j} - (3\ m/s^2)\underline{k}$$

15.11

SEE PROB 15.8 FOR $\underline{a}_{BA}$ AND $\underline{\omega}$

$$\underline{a}_{BA} = \frac{1}{13}(-5\underline{j} + 12\underline{k})$$

$$\underline{\omega} = -(10\ rad/s)\underline{j} + (24\ rad/s)\underline{k}$$

$\alpha = +65\ rad/s^2$; $\underline{\alpha} = \alpha \underline{a}_{BA} = (65\ rad/s^2)\left(\frac{1}{13}\right)(-5\underline{j} + 12\underline{k})$

$$\underline{\alpha} = -(25\ rad/s^2)\underline{j} + (60\ rad/s^2)\underline{k}$$

POINT E $\quad \underline{r}_{E/A} = (6\ in.)\underline{i} - (4\ in.)\underline{j} - (6\ in.)\underline{k}$

$$\underline{v}_E = \underline{\omega} \times \underline{r}_{E/A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -10 & 24 \\ 6 & -4 & -6 \end{vmatrix} = (60 + 96)\underline{i} + 144\underline{j} + 60\underline{k}$$

$$\underline{v} = (156\ in./s)\underline{i} + (144\ in./s)\underline{j} + (60\ in./s)\underline{k}$$

$$\underline{a}_D = \underline{\alpha} \times \underline{r}_{E/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{E/A}) = \underline{\alpha} \times \underline{r}_{E/A} + \underline{\omega} \times \underline{v}_E$$

$$\underline{a}_D = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -25 & 60 \\ 6 & -4 & -6 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -10 & 24 \\ 156 & 144 & 60 \end{vmatrix}$$

$$= (150 + 240)\underline{i} + 360\underline{j} + 150\underline{k} + (-600 - 3456)\underline{i} + 3744\underline{j} - 1560\underline{k}$$

$$= -(3666\ in./s^2)\underline{i} + (4104\ in./s^2)\underline{j} - (1410\ in./s^2)\underline{k}$$

$$\underline{a}_B = -(306\ ft/s^2)\underline{i} + (342\ in./s^2)\underline{j} - (117.5\ ft/s^2)\underline{k}$$

15.12

$$\omega = \frac{2\pi\ rad}{(365.24\ days)\left(\frac{24\ h}{day}\right)\left(\frac{3600\ s}{h}\right)} = 199.11 \times 10^{-9}\ rad/s$$

$$v = r\omega = (93 \times 10^6\ mi)\left(\frac{5280\ ft}{mi}\right)(199.11 \times 10^{-9}\ rad/s) = 97770\ ft/s$$

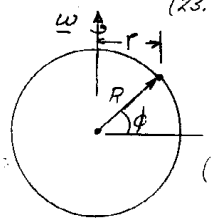
$$v = 66{,}700\ mi/h$$

$$a = a_n = r\omega^2 = (93 \times 10^6)(5280)(199.11 \times 10^{-9})^2$$

$$a = 0.01947\ ft/s^2$$

15.13

$$\omega = \frac{2\pi\ rad}{(23.93\ h)\left(\frac{3600\ s}{h}\right)} = 72.935 \times 10^{-6}\ rad/s$$



$$R = (3960\ mi)\left(\frac{5280\ ft}{mi}\right) = 20.91 \times 10^6\ ft$$

r = RADIUS OF PATH $= R\cos\phi$

(a) AT EQUATOR: LATITUDE, $\phi = 0$

$$v = r\omega = (R\cos 0^\circ)\omega$$

$$= (20.91 \times 10^6\ ft)(72.935 \times 10^{-6}\ rad/s)$$

$$v = 1525\ ft/s$$

$$a = r\omega^2 = (R\cos 0^\circ)\omega^2 = (20.91 \times 10^6\ ft)(72.935 \times 10^{-6}\ rad/s)^2$$

$$a = 0.1112\ ft/s^2$$

(b) AT PHILADELPHIA: LATITUDE, $\phi = 40^\circ$

$$v = r\omega = (R\cos\phi)\omega = (20.91 \times 10^6\ ft)\cos 40^\circ (72.935 \times 10^{-6}\ rad/s)$$

$$v = 1168\ ft/s$$

$$a = r\omega^2 = (R\cos\phi)\omega^2 = (20.91 \times 10^6\ ft)\cos 40^\circ (72.935 \times 10^{-6}\ rad/s)^2$$

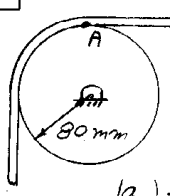
$$a = 0.0852\ ft/s^2$$

(c) AT NORTH POLE: LATITUDE, $\phi = 90^\circ$ $\quad r = R\cos\phi = 0$

$$v = a = 0$$

15.14

80-mm-RADIUS PULLEY



$a = 600\ mm/s^2$

$v = 400\ mm/s$

$$v_A = r\omega_A$$

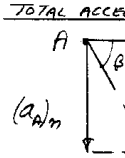
$$400\ mm/s = (80\ mm)\omega_A$$

$$\omega_A = 5\ rad/s \qquad \underline{\omega}_A = 5\ rad/s\ \circlearrowleft$$

$$(a_A)_t = r\alpha_A\ ; \quad 600\ mm/s^2 = (80\ mm)\alpha_A$$

$$\alpha_A = 7.5\ rad/s^2 \qquad \underline{\alpha}_A = 7.5\ rad/s^2\ \circlearrowleft$$

TOTAL ACCEL. OF A:



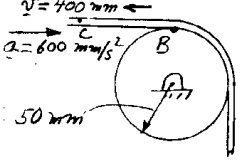
$$(a_A)_n = r\omega_A^2 = (80\ mm)(5\ rad/s)^2 = 2000\ mm/s^2$$

$$(a_A)_t = 600\ mm/s^2$$

$$a_A = 2088\ mm/s^2 = 2.09\ m/s^2\ ; \quad \beta = 73.3^\circ$$

$$\underline{a}_A = 2.09\ m/s^2\ \angle\ 73.3^\circ$$

50-mm-RADIUS PULLEY



$$v_B = r\omega_B$$

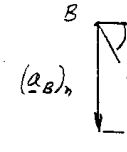
$$400\ mm/s = (50\ mm)\omega_B$$

$$\omega_B = 8\ rad/s \qquad \underline{\omega}_B = 8\ rad/s\ \circlearrowleft$$

$$(a_t)_B = r\alpha_B\ ; \quad 600\ mm/s^2 = (50\ mm)\alpha_B$$

$$\alpha_B = 12\ rad/s^2 \qquad \underline{\alpha}_B = 12\ rad/s^2\ \circlearrowleft$$

TOTAL ACCEL. OF B:



$$(a_B)_n = r\omega_B^2 = (50\ mm)(8\ rad/s)^2 = 3200\ mm/s^2$$

$$(a_B)_t = 600\ mm/s^2$$

$$a_B = 3256\ mm/s^2 = 3.26\ m/s^2$$

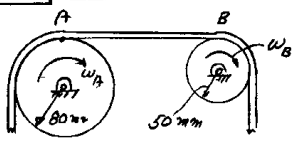
$$\beta = 79.4^\circ$$

$$\underline{a}_B = 3.26\ m/s^2\ \angle\ 79.4^\circ$$

15.15

VELOCITIES

GIVEN: $\omega_A = 5$ rad/s ↻



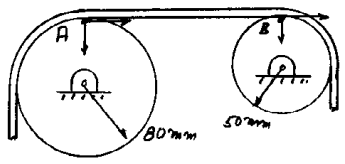
$v_A = v_B$

$r_A\omega_A = r_B\omega_B$

$(80\text{ mm})(5\text{ rad/s}) = (50\text{ mm})\omega_B$

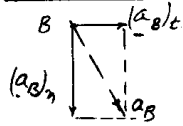
$\omega_B = 8$ rad/s ↻

ACCELERATIONS



$(a_A)_t = (a_B)_t$ (1)

TOTAL ACCELERATION OF B: $a_B = 3.5\text{ m/s}^2 = 3500\text{ mm/s}^2$



$(a_B)_n = r_B\omega_B^2 = (50)(8)^2 = 3200\text{ mm/s}^2$ ↓

$a_B^2 = (a_B)_t^2 + (a_B)_n^2$; $(3500)^2 = (a_B)_t^2 + (3200)^2$

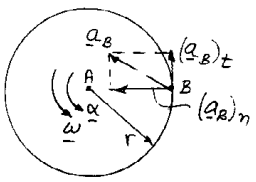
$(a_B)_t^2 = 2.01\times10^6$ $\quad (a_B)_t = 1417.7$ rad/s² ↻

EQ.(1): $(a_A)_t = (a_B)_t = 1417.7$ rad/s² ↻ $\quad \alpha_A = \dfrac{(a_A)_t}{r_A} = \dfrac{1417.7}{80} = 17.72$ rad/s² ↻

15.16

UNIFORMLY ACCEL. MOTION

$\omega_0 = 0$ $\quad \omega = \alpha t$



$(a_B)_t = r\alpha$

$(a_B)_n = r\omega^2 = r\alpha^2t^2$

$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2}$

$= \sqrt{(r\alpha)^2 + (r\alpha^2t^2)^2}$

$a_B = r\alpha\sqrt{1+\alpha^2t^4}$ (1)

GIVEN DATA: $r = 250\text{ mm} = 0.25$ m, $\alpha = 5$ rad/s², $a_B = 4$ m/s²

EQ(1): $4\text{ m/s}^2 = (0.25\text{ m})(5\text{ rad/s}^2)\sqrt{1+(5\text{ rad/s}^2)^2t^4}$

SQUARING: $16 = 1.5625(1+25t^4)$

$t = 0.7797$ s $\quad t = 0.780$ s ◀

$\omega = \alpha t = (5\text{ rad/s}^2)(0.7797\text{ s})$ $\quad \omega = 3.90$ rad/s ◀

15.17

SEE PROB 15.16 FOR DERIVATION OF EQ(1).

$a_B = r\alpha\sqrt{1+\alpha^2t^4}$ (1)

GIVEN DATA: $r = 250\text{ mm} = 0.25$ m, $\alpha = 0.4$ rad/s²

EQ(1): $a_B = (0.25\text{ m})(0.4\text{ rad/s}^2)\sqrt{1+(0.5\text{ rad/s}^2)^2t^4}$

$a_B = (0.1)\sqrt{1+0.25t^4}$ (2)

(a) FOR $t = 0$: $a_B = 0.100$ m/s² $\quad a_B = 100$ mm/s² ◀

(b) FOR $t = 1$ s: $a_B = 0.1\sqrt{1+0.25} = 0.1118$ m/s²

$a_B = 111.8$ mm/s² ◀

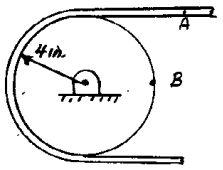
(c) FOR $t = 3$ s: $a_B = 0.1\sqrt{1+0.25(3)^4} = 0.4610$ m/s²

$a_B = 461$ mm/s² ◀

15.18

$\alpha = 90$ rad/s² ↻

AT $t = 3$s: $\omega = \alpha t = 90(3) = 270$ rad/s ↻

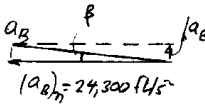


$a_A = r\alpha = (4\text{ in.})(90\text{ rad/s}^2) = 360$ in./s²

$a_A = 30$ ft/s² ← ◀

POINT B: $(a_B)_t = r\alpha = (4\text{ in.})(90\text{ rad/s}^2)$

AT $t = 3$s $\quad (a_B)_t = 360$ in./s² ↑

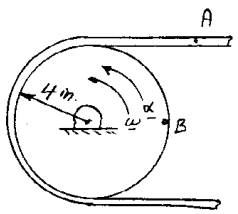


$(a_B)_n = r\omega^2 = (4\text{ in.})(270\text{ rad/s})^2$

$(a_B)_n = 291{,}600$ in./s² ←

NOTE. SINCE WHEEL ROTATES ABSOLUTE DIRECTION OF B IS NOT KNOWN. MAGNITUDE OF $a_B = 24{,}300$ ft/s² ◀

15.19



INITIAL SPEED

$\omega_0 = 600$ rpm $= 62.83$ rad/s ↻

UNIFORMLY ACCEL. MOTION

+↻ $\omega = \omega_0 + \alpha t$

AT $t = 4$s, $\omega = 0$

$0 = 62.83\text{ rad/s} + \alpha(4\text{ s})$

$\alpha = -15.708$ rad/s

$\alpha = 15.708$ rad/s ↻

(a) IMMEDIATELY BEFORE POWER IS TURNED OFF

$\alpha = 0$, $\omega_0 = 62.83$ rad/s

$(a_B)_t = r\alpha = 0$; $(a_B)_n = r\omega_0^2 = (4\text{ in.})(62.83\text{ rad/s})^2$

$(a_B)_n = 15{,}791$ in./s² ←

$a_B = 15{,}791$ in./s² ← ; $a_B = 1316$ ft/s² ← ◀

$v_B = r\omega_0 = (4\text{ in.})(62.83\text{ rad/s}) = 251.3$ in./s ↑; $v_B = 20.9$ ft/s ↑ ◀

(b) WHEN $t = 2.5$s: $\alpha = 15.708$ rad/s² ↻

+↻ $\omega = \omega_0 + \alpha t = 62.83\text{ rad/s} + (-15.708\text{ rad/s}^2)(2.5\text{ s})$

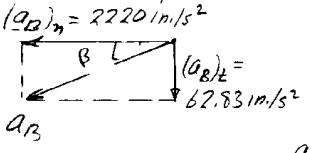
$\omega = 23.558$ rad/s ↻

$v_B = r\omega = (4\text{ in.})(23.558\text{ rad/s}) = 94.23$ in./s ↑

$v_B = 7.85$ ft/s ↑ ◀

$(a_B)_t = r\alpha = (4\text{ in})(15.708\text{ rad/s}^2) = 62.83$ in./s² ↓

$(a_B)_n = r\omega^2 = (4\text{ in.})(23.558\text{ rad/s}^2) = 2220$ in./s² ←

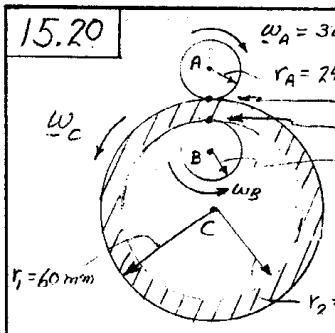


$a_B = 2221$ in./s² ∠

$\beta = 1.6°$

$a_B = 185.1$ ft/s² ∠ 1.6° ◀

15.20



$\omega_A = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 31.416 \text{ rad/s}$

[WE ASSUME SENSES OF ROTATION SHOWN FOR OUR COMPUTATIONS.]

(a) VELOCITIES: POINT 1 (POINT OF CONTACT OF A AND C)

$v_1 = r_A\omega_A = r_1\omega_C$

$\omega_C = \frac{r_A}{r_1}\omega_A = \frac{24 \text{ mm}}{60 \text{ mm}}(300 \text{ rpm}) = 120 \text{ rpm}$

$\underline{\omega}_C = 120 \text{ rpm}$ ◀

POINT 2 (POINT OF CONTACT OF B AND C)

$v_2 = r_B\omega_B = r_2\omega_C$

$\omega_B = \frac{r_2}{r_B}\omega_C = \frac{r_2}{r_B}\left(\frac{r_A}{r_1}\right)\omega_A = \frac{55 \text{ mm}}{24 \text{ mm}}\left(\frac{24 \text{ mm}}{60 \text{ mm}}\right)300 \text{ rpm}$

$\omega_B = 275 \text{ rpm}$ $\quad \underline{\omega}_B = 275 \text{ rpm}$ ◀

(b) ACCELERATIONS:

POINT ON RIM OF A: $r_A = 24 \text{ mm} = 0.024 \text{ m}$

$a_A = r_A\omega_A^2 = (0.024 \text{ m})(31.416 \text{ rad/s})^2 = 23.687 \text{ m/s}^2$

$\underline{a}_A = 23.7 \text{ m/s}^2 \uparrow$ ◀

POINT ON RIM OF B:

$\omega_B = 275 \text{ rpm}\left(\frac{2\pi}{60}\right) = 28.798 \text{ rad/s}$

$a_B = r_B\omega_B^2 = (0.024 \text{ m})(28.798 \text{ rad/s})^2 = 17.904 \text{ m/s}^2$

$\underline{a}_B = 17.90 \text{ m/s}^2 \downarrow$ ◀

15.21

TANGENTIAL COMPONENT OF ACCELERATION OF THE TAPE. UNIF. ACCEL. MOTION

$v_0 = 2 \text{ ft/s}$ $\quad v_1 = 6 \text{ ft/s}$ $\quad t_1 = 4 \text{ s}$

$v_1 = v_0 + a_t t_1$: $\quad 6 \text{ ft/s} = 2 \text{ ft/s} + a_t(4 \text{ s})$

$a_t = 1 \text{ ft/s}^2 = 12 \text{ in./s}^2$

DRUM A: $r_A = 1.5 \text{ in}$

(a) $a_t = \alpha_A r_A$: $\quad 12 \text{ in./s}^2 = \alpha_A(1.5 \text{ in.})$

$\alpha_A = 8 \text{ rad/s}^2$ $\quad \underline{\alpha}_A = 8 \text{ rad/s}^2 \curvearrowright$ ◀

(b) $v_0 = 2 \text{ ft/s} = 24 \text{ in./s}$ $\quad \omega_0 = \frac{v_0}{r_A} = \frac{24 \text{ in./s}}{1.5 \text{ in.}} = 16 \text{ rad/s}$

$v_1 = 6 \text{ ft/s} = 72 \text{ in./s}$ $\quad \omega_1 = \frac{v_1}{r_A} = \frac{72 \text{ in./s}}{1.5 \text{ in}} = 48 \text{ rad/s}$

$\omega_1^2 = \omega_0^2 + 2\alpha_A\theta_A$

$(48 \text{ rad/s})^2 = (16 \text{ rad/s})^2 + 2(8 \text{ rad/s}^2)\theta_A$

$\theta_A = 128 \text{ rad}$

$\theta_A = 128 \text{ rad}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right) = 20.37 \text{ rev}$

$\theta_A = 20.4 \text{ rev}$ ◀

15.22

TANGENTIAL COMPONENT OF ACCELERATION OF THE TAPE. UNIF. ACCEL. MOTION

$v_0 = 2 \text{ ft/s} = 24 \text{ in./s}$

$v_1 = 6 \text{ ft/s} = 72 \text{ in./s}$ $\quad t_1 = 4 \text{ s}$

$v_1 = v_0 + a_t t_1$: $\quad (72 \text{ in./s}) = (24 \text{ in./s}) + a_t(4 \text{ s})$

$a_t = 12 \text{ in./s}^2$

DRUM B: $r_B = 0.9 \text{ in.}$

(a) $a_t = \alpha_B r_B$: $\quad (12 \text{ in./s}^2) = \alpha_B(0.9 \text{ in.})$

$\alpha_B = 13.333 \text{ rad/s}^2$ $\quad \underline{\alpha}_B = 13.33 \text{ rad/s}^2 \curvearrowright$ ◀

(b) $v_0 = 24 \text{ in./s}$ $\quad \omega_0 = \frac{v_0}{r_B} = \frac{24 \text{ in./s}}{0.9 \text{ in}} = 26.667 \text{ rad/s}$

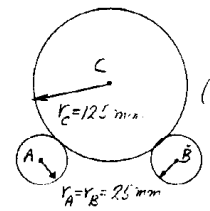
$v_1 = 72 \text{ in./s}$ $\quad \omega_1 = \frac{v_1}{r_B} = \frac{72 \text{ in./s}}{0.9 \text{ in.}} = 80 \text{ rad/s}$

$\omega_1^2 = \omega_0^2 + 2\alpha_B\theta_B$

$(80 \text{ rad/s})^2 = (26.667 \text{ rad/s})^2 + 2(13.333 \text{ rad/s}^2)\theta_B$

$\theta_B = 213.3 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)$ $\quad \underline{\theta}_B = 34.0 \text{ rev} \curvearrowright$ ◀

15.23



FOR DRUM C: UNIF. ACCEL. MOTION

$\omega_0 = 20 \text{ rpm} = 20\left(\frac{2\pi}{60}\right) = 2.0944 \text{ rad/s}$

$\omega_1 = 50 \text{ rpm} = 50\left(\frac{2\pi}{60}\right) = 5.2360 \text{ rad/s}$

$\theta = 15 \text{ rev} = 15(2\pi) = 94.248 \text{ rad}$

(a) $\omega_1^2 = \omega_0^2 + 2\alpha_C\theta$

$(5.2360)^2 = (2.0944)^2 + 2\alpha_C(94.248)$

$\alpha_C = 0.12217 \text{ rad/s}^2$

$(a_C)_t = (a_B)_t$: $\quad r_C\alpha_C = r_B\alpha_B$

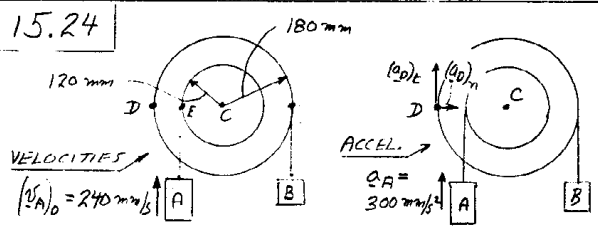
CASTER B: $\quad \alpha_B = \frac{r_C}{r_B}\alpha_C = \frac{125 \text{ mm}}{25 \text{ mm}}(0.12217 \text{ rad/s}^2)$

$\alpha_A = \alpha_B = 0.611 \text{ rad/s}^2$ ◀

(b) DRUM C: $\quad \omega_1 = \omega_0 + \alpha_C t$

$5.2360 = 2.0944 + (0.12217)t$ $\quad t = 25.7 \text{ s}$ ◀

15.24



(a) MOTION OF PULLEY: $(\underline{v}_E)_0 = (\underline{v}_A)_0 = 240 \text{ mm/s} \uparrow$

$(\underline{a}_E)_t = \underline{a}_A = 300 \text{ mm/s}^2 \uparrow$

$(v_E)_0 = r\omega_0$ $\quad 240 \text{ mm/s} = (120 \text{ mm})\omega_0$ $\quad \underline{\omega}_0 = 2 \text{ rad/s} \curvearrowleft$

$(a_E)_t = r\alpha$ $\quad 300 \text{ mm/s}^2 = (120 \text{ mm})\alpha$ $\quad \underline{\alpha} = 2.5 \text{ rad/s}^2 \curvearrowleft$

FOR $t = 3 \text{ s}$: $\quad \omega = \omega_0 + \alpha t = 2 \text{ rad/s} + (2.5 \text{ rad/s}^2)(3 \text{ s}) = 9.5 \text{ rad/s}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (2 \text{ rad/s})(3 \text{ s}) + \frac{1}{2}(2.5 \text{ rad/s}^2)(3 \text{ s})^2$

$\theta = 17.25 \text{ rad}$ $\quad \theta = 17.25\left(\frac{1}{2\pi}\right)$ $\quad \theta = 2.75 \text{ rev}$ ◀

(b) LOAD B: $\quad r = 180 \text{ mm}$ $\quad t = 3 \text{ s}$

$v_B = r\omega = (0.180 \text{ m})(9.5 \text{ rad/s}) = 1.710 \text{ m/s}$ $\quad \underline{v}_B = 1.710 \text{ m/s} \downarrow$ ◀

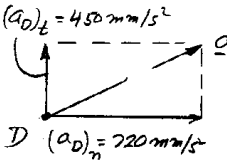
$\Delta y_B = r\theta = (0.180 \text{ m})(17.25 \text{ rad}) = 3.105 \text{ m}$ $\quad \Delta\underline{y}_B = 3.11 \text{ m} \downarrow$ ◀

(c) POINT D: $\quad r = 180 \text{ mm}$ $\quad t = 0$

$(a_D)_t = r\alpha = (180 \text{ mm})(2.5 \text{ rad/s}^2) = 450 \text{ mm/s}^2$

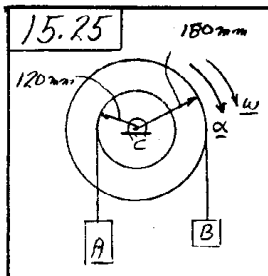
$(a_D)_n = r\omega_0^2 = (180 \text{ mm})(2 \text{ rad/s})^2 = 720 \text{ mm/s}^2$

$(a_D)_t = 450 \text{ mm/s}^2$ $\quad (\underline{a}_D)_n = 720 \text{ mm/s}^2 \rightarrow$



$\underline{a}_D = 849 \text{ mm/s}^2 \angle 32.0°$ ◀

15.25



UNIF. ACCEL. MOTION $\omega_0 = 0$

$\alpha = 2.4 \text{ rad/s}^2 \curvearrowright$

AT $t = 4$ s:

$\omega = \omega_0 + \alpha t = 0 + (2.4 \text{ rad/s}^2)(4 \text{ s})$

$\omega = 9.6 \text{ rad/s} \curvearrowright$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + 0 + \frac{1}{2}(2.4 \text{ rad/s}^2)(4 \text{ s})^2$

$\theta = 19.2 \text{ rad} \curvearrowright$

(a) LOAD A AT $t = 4$ s: $r_A = 120$ mm

$v_A = r_A \omega = (120 \text{ mm})(9.6 \text{ rad/s}) = 1152 \text{ mm/s}$

$\underline{v_A = 1.152 \text{ m/s} \uparrow}$ ◀

$y_A = r_A \theta = (120 \text{ mm})(19.2 \text{ rad}) = 2304 \text{ mm/s}$

$\underline{-y_A = 2.30 \text{ m/s} \uparrow}$ ◀

(b) LOAD B AT $t = 4$ s: $r_B = 180$ mm

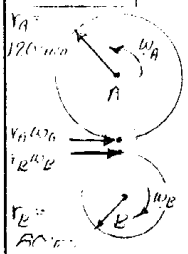
$v_B = r_B \omega = (180 \text{ mm})(9.6 \text{ rad/s}) = 1728 \text{ mm/s}$

$\underline{v_B = 1.728 \text{ m/s} \downarrow}$ ◀

$y_B = r_B \theta = (180 \text{ mm})(19.2 \text{ rad}) = 3456 \text{ mm/s}$

$\underline{-y_B = 3.46 \text{ m/s} \downarrow}$ ◀

15.26



DISK B: $(\omega_B)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s} \curvearrowright$

WHEN $t = 6$ s: $\omega_B = 150 \text{ rpm} = 15.708 \text{ rad/s} \curvearrowright$

$+\curvearrowright$ $\omega_B = (\omega_B)_0 + \alpha_B t$

$15.708 \text{ rad/s} = 52.36 \text{ rad/s} + \alpha_B (6 \text{ s})$

$\alpha_B = -4.5815 \text{ rad/s}^2$ $\underline{\alpha_B = 4.58 \text{ rad/s}^2 \curvearrowleft}$ ◀

DISK A. WHEN $t = 8$ s.

$r_A \omega_A = r_B \omega_B$ $\omega_A = \frac{r_B}{r_A}\omega_B$

$\omega_A = \frac{80 \text{ mm}}{120 \text{ mm}}(15.708 \text{ rad/s})$ $\underline{\omega_A = 10.472 \text{ rad/s} \curvearrowleft}$

$+\curvearrowleft$ $\omega_A = (\omega_A)_0 + \alpha_A t$: $10.472 \text{ rad/s} = 0 + \alpha_A (8 \text{ s})$

$\alpha_A = +1.309 \text{ rad/s}^2$ $\underline{\alpha_A = 1.309 \text{ rad/s}^2 \curvearrowleft}$ ◀

15.27

SAME FIGURE OF SOLUTION OF PROB. 15.26

WE NOTE THAT DISKS MAY BE BROUGHT TOGETHER WHEN $r_A \omega_A = r_B \omega_B$.

DISK B: $(\omega_B)_0 = 500 \text{ rpm} = 52.360 \text{ rad/s} \curvearrowright$

DISK B WILL COAST TO REST IN 60 s.

$\omega_B = (\omega_B)_0 + \alpha_B t$: $0 = 52.360 \text{ rad/s} + \alpha_B (60 \text{ s})$

$\alpha_B = -0.87266 \text{ rad/s}^2$

AT TIME t:

$\omega_B = (\omega_B)_0 + \alpha_B t$ $\omega_B = 52.360 - 0.87266 t$

DISK A: $\alpha_A = 3 \text{ rad/s}^2 \curvearrowleft$ $(\omega_A)_0 = 0$

AT TIME t: $\omega_A = (\omega_A)_0 + \alpha_A t$ $\omega_A = 3t$

(a) BRING DISKS TOGETHER: $r_A \omega_A = r_B \omega_B$

$(120 \text{ mm})(3t) = (80 \text{ mm})(52.360 - 0.87266 t)$

$360 t = 4188.8 - 69.81 t$

$429.8 t = 4188.8$

$t = 9.746 \text{ s}$ $t = 9.75 \text{ s}$ ◀

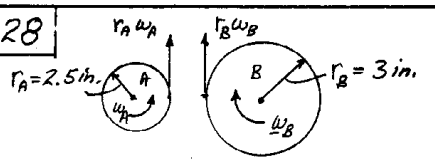
(b) WHEN CONTACT IS MADE

$\omega_A = 3t = 3(9.746) = 29.24 \text{ rad/s}$ $\underline{\omega_A = 279 \text{ rpm} \curvearrowleft}$ ◀

$\omega_B = 52.360 - 0.87266 t$

$= 52.360 - 0.87266(9.746) = 43.86 \text{ rad/s}$ $\underline{\omega_B = 419 \text{ rpm} \curvearrowright}$ ◀

15.28



DISK B: $(\omega_B)_0 = 500 \text{ rpm} = 52.360 \text{ rad/s} \curvearrowright$

DISK B WILL COAST TO REST IN 60 s.

$\omega_B = (\omega_B)_0 + \alpha_B t$: $0 = 52.360 \text{ rad/s} + \alpha_B (60 \text{ s})$

$\alpha_B = -0.87266 \text{ rad/s}^2$

AT TIME t:

$\omega_B = (\omega_B)_0 + \alpha_B t$: $\omega_B = 52.360 - 0.87266 t$

DISK A: $\alpha_A = 3 \text{ rad/s}^2 \curvearrowleft$ $(\omega_A)_0 = 0$

AT TIME t: $\omega_A = (\omega_A)_0 + \alpha_A t$ $\omega_A = 3t$

(a) BRING DISKS TOGETHER WHEN $r_A \omega_A = r_B \omega_B$

$(2.5 \text{ in.})(3t) = (3 \text{ in.})(52.36 - 0.87266 t)$

$7.5 t = 157.08 - 2.618 t$

$10.118 t = 157.08$

$t = 15.5248 \text{ s}$ $t = 15.52 \text{ s}$ ◀

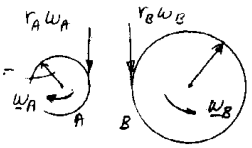
(b) WHEN CONTACT IS MADE

$\omega_A = 3t = 3(15.525 \text{ s}) = 46.574 \text{ rad/s}$ $\underline{\omega_A = 445 \text{ rpm} \curvearrowleft}$ ◀

$\omega_B = 52.36 - 0.87266 t$

$= 52.36 - (0.87266)(15.525) = 38.812 \text{ rad/s}$ $\underline{\omega_B = 371 \text{ rpm} \curvearrowright}$ ◀

15.29



(a)

WHEEL B: [POSITIVE $\curvearrowleft$]

$(\omega_B)_0 = 300 \text{ rpm} = 31.416 \text{ rad/s}$

WHEN $t = 12$ s, $\omega_B = 75 \text{ rpm} = 7.854 \text{ rad/s}$

$+\curvearrowleft$ $\omega_B = (\omega_B)_0 + \alpha_B t$:

$7.854 \text{ rad/s} = 31.416 \text{ rad/s} + \alpha_B (12 \text{ s})$

$-23.562 = 12 \alpha_B$

$\alpha_B = -1.9635 \text{ rad/s}^2$ $\underline{\alpha_B = 1.963 \text{ rad/s}^2 \curvearrowright}$ ◀

WHEEL A: [POSITIVE $\curvearrowright$]

$(\omega_A)_0 = -300 \text{ rpm} = -31.416 \text{ rad/s}$

WHEN $t = 12$ s: $r_A \omega_A = r_B \omega_B$

$(2.5 \text{ in.}) \omega_A = (3 \text{ in.})(7.854 \text{ rad/s})$

$\omega_A = 9.4248 \text{ rad/s}$

$+\curvearrowright$ $\omega_A = (\omega_A)_0 + \alpha_A t$:

$9.4248 \text{ rad/s} = -31.416 \text{ rad/s} + \alpha_A (12 \text{ s})$

$40.841 = 12 \alpha_A$

$\alpha_A = 3.4034 \text{ rad/s}^2$ $\underline{\alpha_A = 3.40 \text{ rad/s}^2 \curvearrowright}$ ◀

(b) TIME WHEN ω_A IS ZERO.

$\omega_A = (\omega_A)_0 + \alpha_A t$

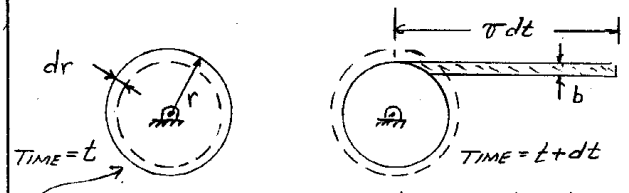
$0 = -31.416 \text{ rad/s} + (3.4034 \text{ rad/s}^2) t$

$t = 9.231 \text{ s}$ $t = 9.23 \text{ s}$ ◀

***15.30** WE HAVE $\omega = v/r$: SINCE v = CONSTANT,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v}{r}\right) = v\frac{d}{dt}\left(\frac{1}{r}\right)\frac{dr}{dt} = -\frac{v}{r^2}\frac{dr}{dt} \qquad (1)$$

WE NOTE THAT THE VOLUME OF PAPER UNROLLED IN TIME dt (ASSUMING UNIT WIDTH) IS:



$d(\text{VOLUME}) = -2\pi r\,dr$ $\qquad$ $d(\text{VOLUME}) = b\,v\,dt$

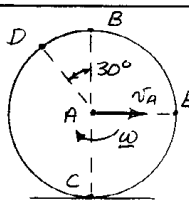
$$\therefore \quad -2\pi r\,dr = b\,v\,dt$$

$$\frac{dr}{dt} = -\frac{bv}{2\pi r} \qquad (2)$$

SUBSTITUTING FOR dr/dt FROM (2) INTO (1),

$$\alpha = -\frac{v}{r^2}\left(-\frac{bv}{2\pi r}\right) \qquad \alpha = \frac{bv^2}{2\pi r^3} \circlearrowright$$

15.31



$v_A = 90\ \text{km/h} = 25\ \text{m/s}$

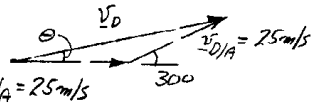
$\omega = \frac{v_A}{r} = \frac{25}{r}$ ↻

$r = 11$ in.

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = [25 \rightarrow] + [r(\tfrac{25}{r}) \rightarrow] = 50\ \text{m/s} \rightarrow$

$\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = [25 \rightarrow] + [r(\tfrac{25}{r}) \leftarrow] = 0$

$\underline{v}_D = \underline{v}_A + \underline{v}_{D/A} = [25 \rightarrow] + [r(\tfrac{25}{r}) \angle 30^\circ]$



$2\theta = 30^\circ \quad \theta = 15^\circ$

$v_D = 2\,v_A \cos 15^\circ$

$= 2(25)\cos 15^\circ$

$\underline{v}_D = 48.3\ \text{m/s} \angle 15^\circ$

$\underline{v}_E = \underline{v}_A + \underline{v}_{E/A} = [25 \rightarrow] + [r(\tfrac{25}{r}) \downarrow]$

$v_E = 25\sqrt{2} = 35.36\ \text{m/s}$ $\qquad \underline{v}_E = 35.4\ \text{ft/s} \searrow 45^\circ$

NOTE: RESULTS ARE INDEPENDENT OF RADIUS OF TIRE

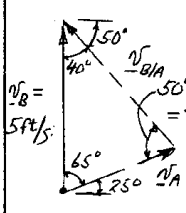
15.32 $\theta = 50^\circ$, $AB = 4$ ft

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$[5\ \text{ft/s} \uparrow] = [v_A \angle 25^\circ] + [v_{B/A} \angle 50^\circ]$

LAW OF SINES

$$\frac{v_A}{\sin 40^\circ} = \frac{v_{B/A}}{\sin 65^\circ} = \frac{5\ \text{ft/s}}{\sin 75^\circ}$$



$\underline{v}_A = 3.33\ \text{ft/s} \angle 25^\circ$

$\underline{v}_{B/A} = 4.69\ \text{ft/s} \angle 50^\circ$

$v_{B/A} = r\omega = (AB)\omega$

$4.69\ \text{ft/s} = (4\ \text{ft})\omega \qquad \omega = 1.173\ \text{rad/s}$ ↻

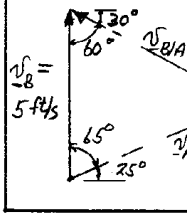
15.33 $\theta = 30^\circ$, $AB = 4$ ft

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$[5\ \text{ft/s} \uparrow] = [v_A \angle 25^\circ] + [v_{B/A} \angle 30^\circ]$

LAW OF SINES

$$\frac{v_A}{\sin 60^\circ} = \frac{v_{B/A}}{\sin 65^\circ} = \frac{5\ \text{ft/s}}{\sin 55^\circ}$$



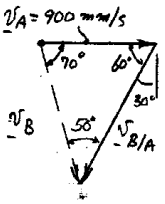
$\underline{v}_A = 5.29\ \text{ft/s} \angle 25^\circ$

$\underline{v}_{B/A} = 5.53\ \text{ft/s} \angle 30^\circ$

$v_{B/A} = (AB)\omega$: $5.53 = 4\omega \qquad \omega = 1.383\ \text{rad/s}$ ↻

15.34 $AB = 300$ mm; $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$(v_B \angle 70^\circ) = (900\ \text{mm/s} \rightarrow) + (v_{B/A} \angle 30^\circ)$



LAW OF SINES

$$\frac{v_B}{\sin 60^\circ} = \frac{v_{B/A}}{\sin 70^\circ} = \frac{900\ \text{mm/s}}{\sin 50^\circ}$$

$v_B = 1017\ \text{mm/s}$; $\underline{v}_B = 1.017\ \text{m/s} \angle 70^\circ$

$v_{B/A} = 1104\ \text{mm/s}$; $\underline{v}_{B/A} = 1.104\ \text{m/s} \angle 30^\circ$

$v_{B/A} = (AB)\omega$: $1104\ \text{mm/s} = (300\ \text{mm})\omega \qquad \omega = 3.68\ \text{rad/s}$ ↻

15.35 $\underline{r}_{C/A} = (200\ \text{mm})\underline{i} - (150\ \text{mm})\underline{j}$ $\qquad \underline{\omega} = \omega\underline{k}$

VELOCITY OF C: $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{C/A}$

$-(500\ \text{mm/s})\underline{i} + (v_C)_y\underline{j} = (250\ \text{mm/s})\underline{i} + (v_A)_y\underline{j} + \omega\underline{k} \times [(200\ \text{mm})\underline{i} - (150\ \text{mm})\underline{j}]$

$-500\,\underline{i} + (v_C)_y\underline{j} = 250\,\underline{i} + (v_A)_y\underline{j} + 200\,\omega\,\underline{j} + 150\,\omega\,\underline{i}$

COEFFICIENTS OF $\underline{i}$: $-500 = 250 + 150\omega$

$-750 = 150\omega$

$\omega = -5\ \text{rad/s} \qquad \underline{\omega} = 5\ \text{rad/s}$ ↻

COEFFICIENTS OF $\underline{j}$: $(v_C)_y = (v_A)_y + 200\omega$

$(v_C)_y = (v_A)_y + (200)(-5)$

$(v_C)_y = (v_A)_y - 1000 \qquad (1)$

VELOCITY OF B: $\underline{r}_{B/A} = (150\ \text{mm})\underline{i}$

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}$

$(v_B)_x\underline{i} - (450\ \text{mm/s})\underline{j} = (250\ \text{mm/s})\underline{i} + (v_A)_y\underline{j} + (-5\ \text{rad/s})\underline{k} \times (150\ \text{mm})\underline{i}$

$(v_B)_x\underline{i} - 450\,\underline{j} = 250\,\underline{i} + (v_A)_y\underline{j} - 750\,\underline{j}$

COEFFICIENTS OF $\underline{i}$: $(v_B)_x = +250\ \text{mm/s} \qquad (2)$

COEFFICIENTS OF $\underline{j}$: $-450 = (v_A)_y - 750$

$(v_A)_y = 300\ \text{mm/s}$

EQ(1): $(v_C)_y = 300\ \text{mm/s} - 1000\ \text{mm/s} = -700\ \text{mm/s}$

$\underline{v}_A = (250\ \text{mm/s})\underline{i} + (300\ \text{mm/s})\underline{j}$

$\underline{v}_C = -(500\ \text{mm/s})\underline{i} - (700\ \text{mm/s})\underline{j}$

15.36 $\underline{r}_{A/B} = -(2\ \text{in.})\underline{i} + (4\ \text{in.})\underline{j} \qquad \underline{\omega} = \omega\underline{k}$

VELOCITY OF A: $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$(12\ \text{in./s})\underline{i} + (v_A)_y\underline{j} = -(4\ \text{in./s})\underline{i} + (v_B)_y\underline{j} + \omega\underline{k} \times [-(2\ \text{in.})\underline{i} + (4\ \text{in.})\underline{j}]$

$12\,\underline{i} + (v_A)_y\underline{j} = -4\,\underline{i} + (v_B)_y\underline{j} - 2\omega\,\underline{j} - 4\omega\,\underline{i}$

COEFFICIENTS OF $\underline{i}$: $12 = -4 - 4\omega$

$\omega = -4\ \text{rad/s} \qquad \underline{\omega} = 4\ \text{rad/s}$ ↻

COEFFICIENTS OF $\underline{j}$: $(v_A)_y = (v_B)_y - 2\omega = (v_B)_y - 2(-4)$

$(v_A)_y = (v_B)_y + 8 \qquad (1)$

VELOCITY OF C: $\underline{r}_{C/B} = (6\ \text{in.})\underline{i} - (2\ \text{in.})\underline{j}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$(v_C)_x\underline{i} - (24\ \text{in./s})\underline{j} = -(4\ \text{in./s})\underline{i} + (v_B)_y\underline{j} + (-4\ \text{rad/s})\underline{k} \times [(6\ \text{in.})\underline{i} - (2\ \text{in.})\underline{j}]$

$(v_C)_x\underline{i} - 24\,\underline{j} = -4\,\underline{i} + (v_B)_y\underline{j} - 24\,\underline{j} - 8\,\underline{i}$

COEFFICIENTS OF $\underline{i}$: $(v_C)_x = -4 - 8 = -12$

COEFFICIENTS OF $\underline{j}$: $-24 = (v_B)_y - 24 \qquad (v_B)_y = 0$

EQ(1): $(v_A)_y = 0 + 8 = +8\ \text{in./s}$

$\underline{v}_A = (12\ \text{in./s})\underline{i} + (8\ \text{in./s})\underline{j}$

$\underline{v}_B = -(4\ \text{in./s})\underline{i}$

15.37 SEE SOLUTION OF PROB 15.36 FOR:

$\underline{v}_A = (12\text{ in./s})\underline{i} + (8\text{ in./s})\underline{j}$ ◀

AND FOR: $\underline{\omega} = -(4\text{ rad/s})\underline{k}$

DENOTE BY $P(x,y)$ AN ARBITRARY POINT.

$\underline{r}_{P/A} = x\underline{i} + (y-6\text{ in})\underline{j}$

VELOCITY OF P: $\underline{v}_P = \underline{v}_A + \underline{v}_{P/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{P/A}$

$\underline{v}_P = [(12\text{ in./s})\underline{i} + (8\text{ in./s})\underline{j}] + (-4\text{ rad/s})\underline{k} \times [x\underline{i} + (y-6)\underline{j}]$

$\underline{v}_P = 12\underline{i} + 8\underline{j} - 4x\underline{j} + (4y-24)\underline{i}$

$\underline{v}_P = [12 + 4y - 24]\underline{i} + [8 - 4x]\underline{j}$

FOR $\underline{v}_P = 0$: $[12 + 4y - 24]\underline{i} = 0$ $\quad y = 3\text{ in.}$ ◀

$[8 - 4x]\underline{j} = 0$ $\quad x = 2\text{ in.}$ ◀

15.38 SEE SOLUTION OF PROB 15.35 FOR

$\underline{\omega} = -(5\text{ rad/s})\underline{k}$

AND FOR $P(x,y)$: $(v_P)_x = +250\text{ mm/s}$

SINCE $(v_P)_y = -450\text{ mm/s}$ WAS PART OF GIVEN DATA,

$\underline{v}_P = +(250\text{ mm/s})\underline{i} - (450\text{ mm/s})\underline{j}$ ◀

DENOTE BY $P(x,y)$ AN ARBITRARY POINT

$\underline{r}_{P/E} = (x - 150\text{ mm})\underline{i} + (y - 150\text{ mm})\underline{j}$

$\underline{v}_P = \underline{v}_E + \underline{v}_{P/E} = \underline{v}_E + \underline{\omega} \times \underline{r}_{P/E}$

$\underline{v}_P = (250\text{ mm/s})\underline{i} - (450\text{ mm/s})\underline{j} + (-5\text{ rad/s})\underline{k} \times [(x-150\text{ mm})\underline{i} + (y-150\text{ mm})\underline{j}]$

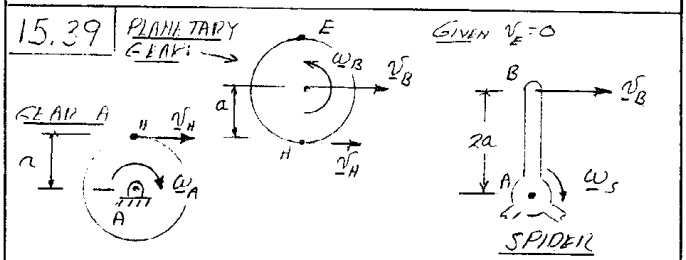
$\underline{v}_P = 250\underline{i} - 450\underline{j} - 5(x-150)\underline{j} + 5(y-150)\underline{i}$

$\underline{v}_P = [250 + 5(y-150)]\underline{i} + [-450 - 5(x-150)]\underline{j}$

FOR $\underline{v}_P = 0$: $250 + 5y - 750 = 0$ $\quad y = 100\text{ mm}$ ◀

$-450 - 5x + 750 = 0$ $\quad x = 60\text{ mm}$ ◀

15.39 PLANETARY GEAR:



GEAR A $\quad \underline{v}_H = a\omega_A \rightarrow$

(a) PLANETARY GEAR: $\underline{v}_H = \underline{v}_E + \underline{v}_{H/E}$

$+\rightarrow \quad a\omega_A = 0 + (2a)\omega_B$

$\omega_B = +\frac{1}{2}\omega_A$ $\quad \underline{\omega}_B = \frac{1}{2}\omega_A$ ◀

$\underline{v}_B = \underline{v}_E + \underline{v}_{B/E}$

$+\downarrow \quad v_B = 0 + a(\omega_B) = a\left(\frac{1}{2}\omega_A\right)$ $\quad \underline{v}_B = \frac{1}{2}a\omega_A \rightarrow$ (1)

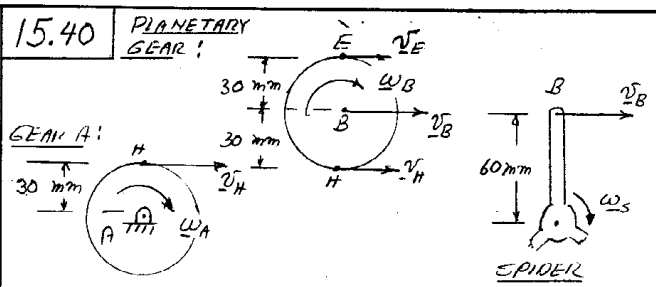
(b) SPIDER:

$\underline{v}_B = (2a)\underline{\omega}_S \rightarrow$

SUBSTITUTE FOR v_B FROM (1):

$+\rightarrow \quad \frac{1}{2}a\omega_A = 2a\omega_S$ $\quad \omega_S = \frac{1}{4}\omega_A$ $\quad \underline{\omega}_S = \frac{1}{4}\omega_S$ ◀

15.40 PLANETARY GEAR:



OUTER GEAR E: RADIUS = 90 mm $\quad \underline{\omega}_E = 180\text{ rpm}$

$\underline{v}_E = (90\text{ mm})\omega_E \rightarrow$

GEAR A: RADIUS = 30 mm $\quad \underline{\omega}_A = 240\text{ rpm}$

$\underline{v}_H = (30\text{ mm})\omega_A \rightarrow$

PLANETARY GEAR: RADIUS = 30 mm $\quad \underline{\omega}_B = \omega_B$

$\underline{v}_H = \underline{v}_E + \underline{v}_{H/E}$

$[(30\text{ mm})\omega_A \rightarrow] = [(90\text{ mm})\omega_E \rightarrow] + [(60\text{ mm})\omega_B \leftarrow]$

$+\rightarrow \quad (30\text{ mm})\omega_A = (90\text{ mm})\omega_E - (60\text{ mm})\omega_B$

$+\circlearrowright \quad \omega_B = \frac{90\omega_E - 30\omega_A}{60}$

FOR $\omega_A = 240$ rpm AND $\omega_E = 180$ rpm:

$\omega_B = \frac{90(180) - 30(240)}{60}$ $\quad \underline{\omega}_B = 150\text{ rpm}$ ◀

$\underline{v}_B = \underline{v}_H + \underline{v}_{B/H} = [(30\text{ mm})\omega_A \rightarrow] + [(30\text{ mm})\omega_B \rightarrow]$

$v_B = 30\omega_A + 30\omega_B$

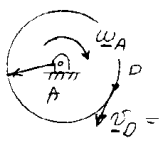
SPIDER: $v_B = (60\text{ mm})\omega_S$

$30\omega_A + 30\omega_B = 60\omega_S$ $\quad \omega_S = \frac{1}{2}(\omega_A + \omega_B)$

SUBSTITUTE $\omega_A = 240$ rpm AND $\omega_B = 150$ rpm

$\omega_S = \frac{1}{2}(240 + 150) = 195\text{ rpm}$ $\quad \underline{\omega}_S = 195\text{ rpm}$ ◀

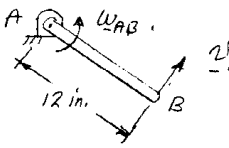
15.41



GEAR A: $\underline{\omega}_A = 60\text{ rpm}$

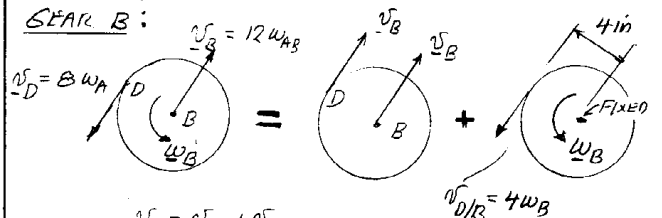
$\underline{v}_D = (8\text{ in.})\omega_A$

(a) ARM AB ROTATES COUNTERCLOCKWISE: $\underline{\omega}_{AB} = 40\text{ rpm}$



$\underline{v}_B = (12\text{ in.})\omega_{AB}$

GEAR B:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$+\swarrow \quad 8\omega_A = -12\omega_{AB} + 4\omega_B$

$\omega_B = 2\omega_A + 3\omega_{AB} = 2(60\text{ rpm}) + 3(40\text{ rpm}) = 240\text{ rpm}$

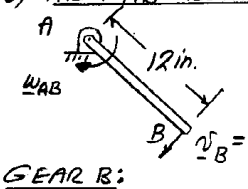
$\underline{\omega}_B = 240\text{ rpm}$ ◀

(CONTINUED)

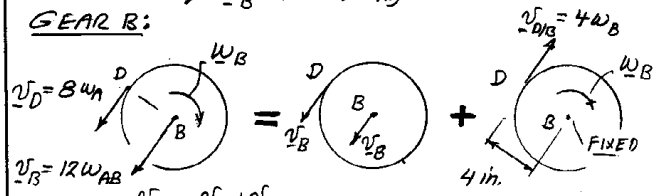
15.41 CONTINUED

WE RECALL THAT $\underline{v}_D = (8\text{ in.})\,\omega_A$ ↙ $\underline{\omega}_A = 60$ rpm ↻

(b) ARM AB ROTATES CLOCKWISE: $\underline{\omega}_{AB} = 40$ rpm ↻



GEAR B:



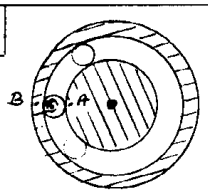
$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

+↙ $8\omega_A = 12\omega_{AB} - 4\omega_B$

$\omega_B = 3\omega_{AB} - 2\omega_A = 3(40\text{ rpm}) - 2(60\text{ rpm}) = 0$

MOTION OF GEAR B IS A CURVILINEAR TRANSLATION: $\underline{\omega}_B = 0$ ◀

15.42



$\underline{\omega}_A = 3600\text{ rpm} = 377\text{ rad/s}$ ↻

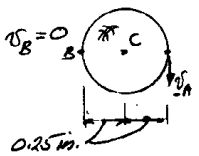
$\omega_B = 0$

$r_A = 1.25$ in., $r_B = 1.75$ in.

BALL BEARING: $r = 0.25$ in.

[WE ASSUME ω_A ↻ FOR COMPUTATIONS]

CONSIDER BALL:



$v_A = r_A\omega_A = (1.25\text{ in.})(377\text{ rad/s}) = 471.2\text{ in/s}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$471.2\text{ in/s}\downarrow = 0 + (0.5\text{ in})\,\omega_{BALL}$

$\underline{\omega}_{BALL} = 942.4\text{ rad/s}$ ↻

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

$v_C = 0 + (0.25\text{ in.})(\omega_{BALL})$

(a) $v_C = (0.25\text{ in.})(942.4\text{ rad/s}) = 235.6\text{ in/s}$ $\quad \underline{v}_C = 236\text{ in/s}$ ◀

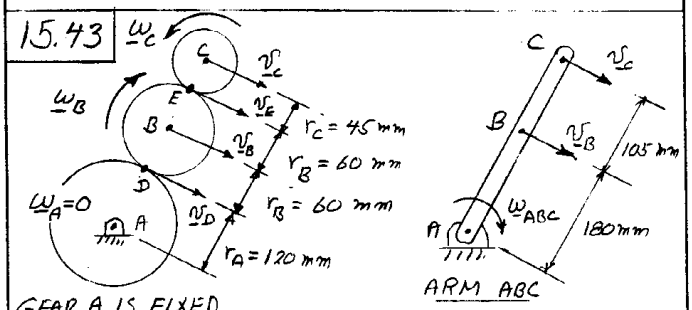
(b) $\omega_{BALL} = 942.4\text{ rad/s}\left(\frac{60}{2\pi}\right) = 8999\text{ rpm}$ $\quad \underline{\omega}_{BALL} = 9000$ rpm ◀

(c) CIRCUMFERENCE OF CIRCLE DESCRIBED BY C:

$= 2\pi(1.25\text{ in.} + 0.25\text{ in.}) = 9.425$ in.

n = NUMBER OF TIMES/MIN. C DESCRIBES CIRCLE $= \frac{236\text{ in/s}}{9.425\text{ in.}}\left(\frac{60\text{ s}}{\text{m}}\right) = 1502$ ◀

15.43



GEAR A IS FIXED

ARM ABC $\quad \underline{\omega}_{ABC} = 75$ rpm ↻

$\underline{v}_B = (AB)\,\omega_{ABC} = (180\text{ mm})\,\omega_{ABC}$ → (1)

$\underline{v}_C = (AC)\,\omega_{ABC} = (285\text{ mm})\,\omega_{ABC}$ → (2)

(CONTINUED)

15.43 CONTINUED

GEAR A: FIXED, $\underline{v}_D = 0$

GEAR B: $\underline{v}_B = \underline{v}_D + \underline{v}_{B/D} = \underline{v}_D + r_B\omega_B$ ↘

$v_B = (60\text{ mm})\,\omega_B$ ↘

SUBSTITUTE FROM (1): $(180\text{ mm})\,\omega_{ABC} = (60\text{ mm})\,\omega_B$

$\omega_B = 3\omega_{ABC} = 3(75\text{ rpm})$ $\quad \underline{\omega}_B = 225$ rpm ↻ ◀

$\underline{v}_E = \underline{v}_D + \underline{v}_{E/D} = \underline{v}_D + 2r_B\omega_B$ ↘

$v_E = 0 + (2)(60\text{ mm})\,\omega_B = 120\,\omega_B$

BUT, $\omega_B = 3\omega_{ABC}$, THUS: $v_E = 120(3\omega_{ABC}) = 360\,\omega_{ABC}$ (3)

GEAR C:

$\underline{v}_E = \underline{v}_C + \underline{v}_{E/C} = \underline{v}_C + r_C\omega_C$ ↘

SUBSTITUTE FOR v_C FROM (2) AND FOR v_E FROM (3):

$360\,\omega_{ABC} = 285\,\omega_{ABC} + (45)\omega_C$

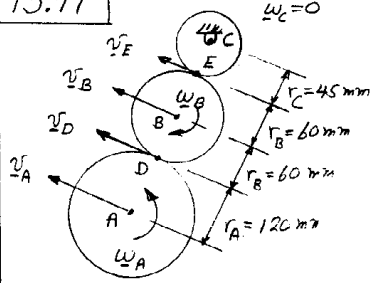
$\omega_C = 8\,\omega_{ABC} - 6.333\,\omega_{ABC} = 1.667\,\omega_{ABC}$ ↻

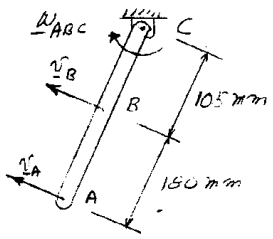
FOR $\omega_{ABC} = 75$ rpm: $\omega_C = 1.667(75\text{ rpm})$ $\quad \underline{\omega}_C = 125$ rpm ↻ ◀

15.44

GEAR C IS FIXED

$\omega_C = 0$





ARM ABC: $\quad \underline{\omega}_{ABC} = 80$ rpm ↻

$\underline{v}_B = (CB)\,\omega_{ABC} = (105\text{ mm})\,\omega_{ABC}$ ↖ (1)

$\underline{v}_A = (CA)\,\omega_{ABC} = (285\text{ mm})\,\omega_{ABC}$ ↖ (2)

GEAR C: FIXED, $\underline{v}_E = 0$

GEAR B: $\underline{v}_B = \underline{v}_E + \underline{v}_{B/E} = \underline{v}_E + r_B\omega_B$ ↖

$v_B = (60\text{ mm})\,\omega_B$

SUBSTITUTE FOR v_B FROM (1): $(105\text{ mm})\,\omega_{ABC} = (60\text{ mm})\,\omega_B$

$\omega_B = 1.75\,\omega_{ABC}$

BUT, $\omega_{ABC} = 80$ rpm:

$\omega_B = 1.75(80\text{ rpm}) = 140\text{ rpm}$ $\quad \underline{\omega}_B = 140$ rpm ↻ ◀

$\underline{v}_D = \underline{v}_E + \underline{v}_{D/E} = \underline{v}_E + 2r_B\omega_B$ ↖

$v_D = 0 + 2(60\text{ mm})\,\omega_B = 120\,\omega_B$

BUT, $\omega_B = 1.75\,\omega_{ABC}$ $\quad v_D = 120(1.75\,\omega_{ABC}) = 210\,\omega_{ABC}$ (3)

GEAR A: $\quad \underline{v}_D = \underline{v}_A + \underline{v}_{D/A} = \underline{v}_A + r_A\omega_A$ ↖

$v_D = v_A + (120\text{ mm})\,\omega_A$

SUBSTITUTE FOR v_A FROM (2) AND FOR v_D FROM (3):

$210\,\omega_{ABC} = 285\,\omega_{ABC} + (120)\,\omega_A$

$\omega_A = 1.75\,\omega_{ABC} - 2.375\,\omega_{ABC} = -0.625\,\omega_{ABC}$

FOR $\omega_{ABC} = 80$ rpm: $\omega_A = -0.625(80\text{ rpm}) = -50$ rpm

THE ASSUMED DIRECTION OF ω_A WAS COUNTERCLOCKWISE, THUS THE MINUS SIGN INDICATES THAT: $\quad \underline{\omega}_A = 50$ rpm ↻ ◀

15.45

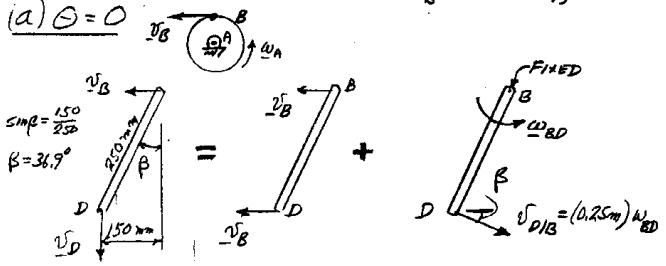
DISK

$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$

$v_B = (AB)\omega_A = (0.05 \text{ m})(52.36 \text{ rad/s})$

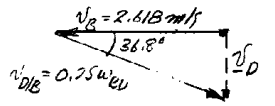
$v_B = 2.618 \text{ m/s}$

(a) $\theta = 0$



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \downarrow] = [2.618 \text{ m/s}] + [0.25\omega_{BD} \measuredangle 36.9°]$



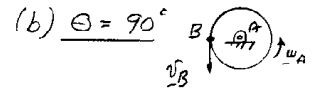
$v_D = (2.618)\tan 36.8°$

$v_D = 1.964 \text{ m/s} \downarrow$ ◀

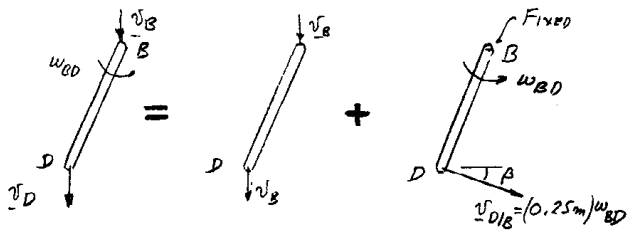
$v_{D/B} = 0.25\omega_{BD} = (2.618)/\cos 36.8°$

$\omega_{BD} = 13.09 \text{ rad/s} \curvearrowright$ ◀

(b) $\theta = 90°$



$\underline{v}_B = 2.618 \text{ m/s} \downarrow$



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \downarrow] = [v_B \downarrow] + [0.25\omega_{BD} \measuredangle \beta]$

$\rightarrow \quad 0 = 0 + 0.25\omega_{BD}\cos\beta \qquad \omega_{BD} = 0$ ◀

$\downarrow \quad v_D = 2.618 \text{ m/s} + 0 \qquad v_D = 2.62 \text{ m/s} \downarrow$ ◀

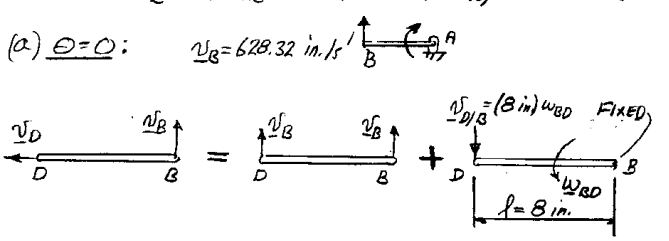
15.46

CRANK AB $\quad b = 3 \text{ in.}$

$\omega_{AB} = 2000 \text{ rpm} = 209.44 \text{ rad/s}$

$v_B = (b)\omega_{AB} = (2 \text{ in.})(209.44 \text{ rad/s}) = 628.32 \text{ in./s}$

(a) $\theta = 0$: $\quad \underline{v}_B = 628.32 \text{ in./s}$



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \leftarrow] = [628.32 \text{ in./s} \uparrow] + [(8 \text{ in.})\omega_{BD} \downarrow]$

$\rightarrow \quad v_D = 0 \qquad \underline{v}_D = \underline{v}_P = 0$ ◀

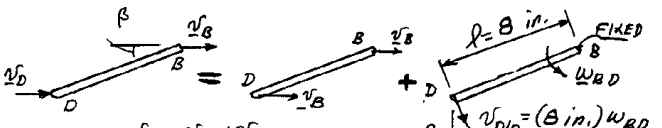
$\uparrow \quad 0 = 628.32 - 8\omega_{BD}$

$\omega_{BD} = 78.54 \text{ rad/s} = 750 \text{ rpm} \qquad \underline{\omega}_{BD} = 750 \text{ rpm} \curvearrowright$ ◀

(CONTINUED)

15.46 CONTINUED

(b) $\theta = 90°$: $\quad \underline{v}_B = 628.32 \text{ in./s}$



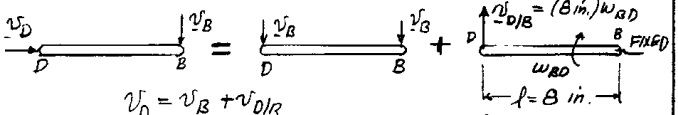
$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \rightarrow] = [628.32 \text{ in./s} \rightarrow] + [(8 \text{ in.})\omega_{BD} \measuredangle \beta]$

$+\downarrow \quad 0 = 0 + 8\omega_{BD}\cos\beta \qquad \omega_{BD} = 0$ ◀

$\rightarrow \quad v_D = 628.32 \text{ in./s} \qquad \underline{v}_D = \underline{v}_P = 52.4 \text{ ft/s} \rightarrow$ ◀

(c) $\theta = 180°$: $\quad \underline{v}_B = 638.32 \text{ in./s}$



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \rightarrow] = [628.32 \text{ in/s} \downarrow] + [(8 \text{ in.})(\omega_{BD}) \curvearrowright]$

$\rightarrow \quad v_D = 0 \qquad \underline{v}_D = \underline{v}_P = 0$ ◀

$+\downarrow \quad 0 = 628.32 \text{ in./s} - (8 \text{ in.})\omega_{BD}$

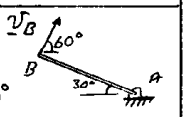
$\omega_{BD} = 78.54 \text{ rad/s} = 750 \text{ rpm} \qquad \underline{\omega}_{BD} = 750 \text{ rpm} \curvearrowright$ ◀

15.47

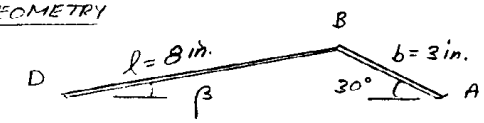
CRANK AB: $\quad b = 3 \text{ in.}$

$\omega_{AB} = 2000 \text{ rpm} = 209.44 \text{ rad/s} \curvearrowright$

$v_B = (b)\omega_{AB} = (3 \text{ in.})(209.44 \text{ rad/s}) = 628.32 \text{ in./s} \measuredangle 60°$



GEOMETRY

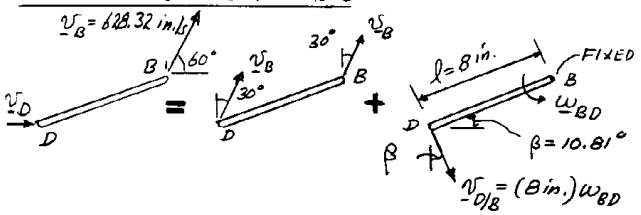


LAW OF SINES:

$\frac{\sin\beta}{3 \text{ in.}} = \frac{\sin 30°}{8 \text{ in.}} \qquad \beta = 10.81°$

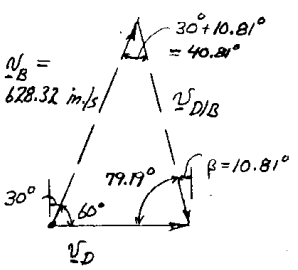
MOTION OF CONNECTING ROD

$\underline{v}_B = 628.32 \text{ in./s}$



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \rightarrow] = [628.32 \text{ in./s} \measuredangle 60°] + [v_{D/B} \measuredangle 10.81°]$



LAW OF SINES

$\frac{v_D}{\sin 40.81°} = \frac{v_{D/B}}{\sin 60°} = \frac{628.32 \text{ in/s}}{\sin 79.19°}$

$v_D = 418.05 \text{ in./s}$

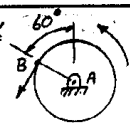
$\underline{v}_D = \underline{v}_P = 34.84 \text{ ft/s} \rightarrow$ ◀

$v_{D/B} = 553.96 \text{ in./s}$

$(8 \text{ in.})\omega_{BD} = 553.96 \text{ in/s}$

$\omega_{BD} = 69.246 \text{ rad/s} \qquad \underline{\omega}_{BD} = 69.2 \text{ rad/s} \curvearrowright$ ◀

15.48 DISK



$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$

$v_B = (AB)\omega_A = (0.05\text{m})(52.36 \text{ rad/s})$

$v_B = 2.618 \text{ m/s}$

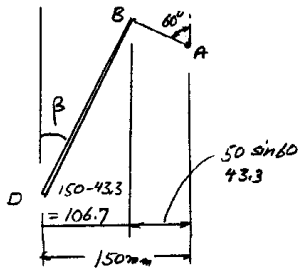
GEOMETRY

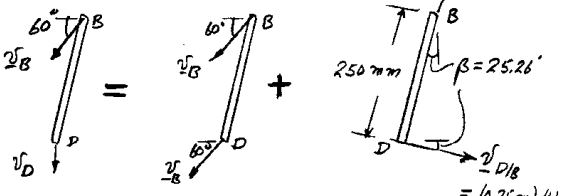
$AB = 50 \text{ mm}$

$BD = 250 \text{ mm}$

$\sin\beta = \frac{106.7 \text{ mm}}{250 \text{ mm}}$

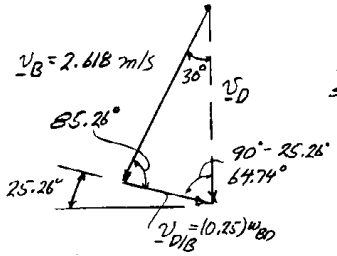
$\beta = 25.26°$





$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \downarrow] = [2.618 \text{ m/s} \searrow 60°] + [v_{D/B} \swarrow 25.26°]$



LAW OF SINES

$\frac{v_D}{\sin 85.26°} = \frac{v_{D/B}}{\sin 30°} = \frac{2.618 \text{ m/s}}{\sin 64.74°}$

$v_D = 2.88 \text{ m/s} \downarrow$ ◀

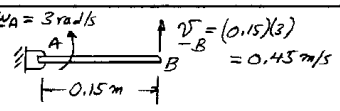
$v_{D/B} = 1.4474 \text{ m/s}$

$(0.25)\omega_{BD} = 1.4474 \text{ m/s}$

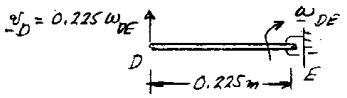
$\omega_{BD} = 5.79 \text{ rad/s} \circlearrowright$ ◀

15.49

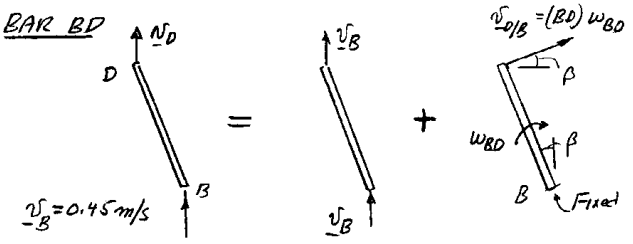
BAR AB



BAR DE



BAR BD



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \uparrow] = [0.45 \text{ m/s}] + [v_{D/B} \nearrow]$

$\xrightarrow{+} \quad 0 = v_{D/B}\cos\beta \qquad v_{D/B} = (BD)\omega_{BD} = 0 \qquad \omega_{BD} = 0$ ◀

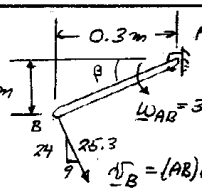
$+\uparrow \quad v_D = 0.45 \text{ m/s}$

$v_D = (0.225\text{m})\omega_{DE} = 0.45 \text{ m/s}$

$\omega_{DE} = 2 \text{ rad/s} \circlearrowright$ ◀

15.50

BAR AB:



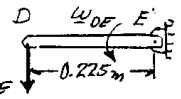
$AB = \sqrt{(0.1)^2 + (0.3)^2} = 0.31623 \text{ m}$

$\beta = 18.43°$

$\underline{v}_B = (AB)\omega_{AB} = (0.31623\text{m})(3 \text{ rad/s})$

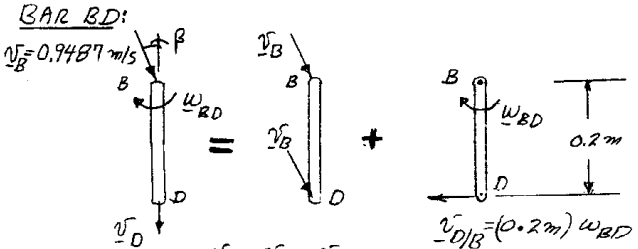
$= 0.9487 \text{ m/s}$

BAR DE:



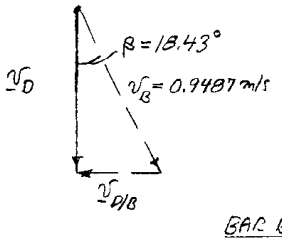
$\underline{v}_D = 0.225\,\omega_{DE}$

BAR BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \downarrow] = [0.9487 \text{ m/s} \swarrow \beta] + [(0.2\text{m})\,\omega_{BD} \leftarrow]$



$v_D = 0.9487 \cos 18.43°$

$= 0.900 \text{ m/s}$

BAR DE:

$v_D = (0.225\text{m})\,\omega_{DE} = 0.900 \text{ m/s}$

$\omega_{DE} = 4.00 \text{ rad/s} \circlearrowright$ ◀

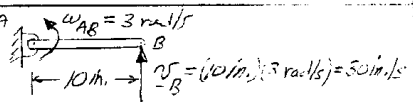
$v_{D/B} = (0.9487)\sin 18.43° = 0.300 \text{ m/s}$

BAR BD: $v_{D/B} = (0.2\text{m})\omega_{BD} = 0.300 \text{ m/s}$

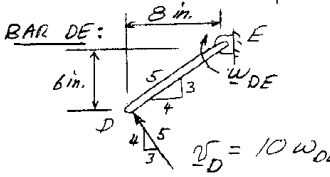
$\omega_{BD} = 1.5 \text{ rad/s} \circlearrowright$ ◀

15.51

BAR AB:



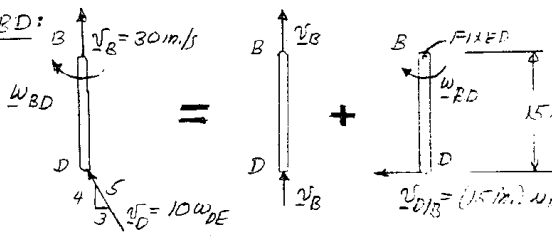
BAR DE:



$DE = \sqrt{(6)^2 + (8)^2} = 10 \text{ in.}$

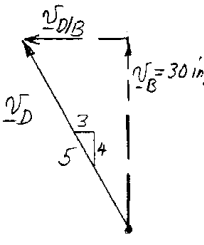
$\underline{v}_D = 10\,\omega_{DE}$

BAR BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \swarrow 3/4] = [30 \text{ in/s} \uparrow] + [v_{D/B} \leftarrow]$



SIMILAR TRIANGLES

$\frac{v_D}{5} = \frac{v_{D/B}}{3} = \frac{30 \text{ in/s}}{4}$

$v_D = 37.5 \text{ in/s} = (10 \text{ in.})\,\omega_{DE}$

$\omega_{DE} = 3.75 \text{ rad/s} \circlearrowright$ ◀

$v_{D/B} = 22.5 \text{ in/s} = (15 \text{ in.})\,\omega_{BD}$

$\omega_{BD} = 1.5 \text{ rad/s} \circlearrowright$ ◀

15.52

BAR AB

$v_B = (13\text{ in})(3\text{ rad/s}) = 39\text{ in./s}$

$AB = 13\text{ in.}$ $\quad \beta = 22.62°$

$\omega_{AB} = 3\text{ rad/s}$

5 in.

12 in.

BAR DE

$\gamma = 36.87°$

8 in.

6 in.

$v_D = 10\,\omega_{DE}$

BAR BD

$v_B = 39\text{ in./s}$

$v_D = 8\,\omega_{DE}$

13 in.

$v_{D/B} = 13\,\omega_{BD}$

$\beta = 22.62°$ $\quad \gamma = 36.87°$

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$\left[v_D \angle \gamma\right] = \left[39\text{ in./s} \nwarrow \beta\right] + \left[v_{D/B} \rightarrow\right]$

LAW OF SINES

$\dfrac{v_D}{\sin 67.38°} = \dfrac{v_{D/B}}{\sin 75.75°} = \dfrac{39\text{ in./s}}{\sin 36.87°}$

$v_D = 60\text{ in./s} = 10\,\omega_{DE}$

$\omega_{DE} = 6\text{ rad/s}$ ◀

$v_{D/B} = 63\text{ in./s} = 13\,\omega_{BD}$

$\omega_{BD} = 4.85\text{ rad/s}$ ◀

15.53

(a) $\beta = 0$: $\quad AB = 0.9\text{ m}$

WHEEL: $v_C = 0$ $\quad \omega_{BC} = v_O/0.25\text{m} = (1.5\text{ m/s})/(0.25\text{ m}) = 6\text{ rad/s}$

$\underline{v}_B = \underline{v}_O + \underline{v}_{B/C}$

$= (1.5\text{ m/s} \rightarrow) + (0.25\text{m})(6\text{ rad/s}) = 1.5 + 1.5 = 3\text{ m/s} \rightarrow$

ROD AB: $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\left[v_A \rightarrow\right] = \left[v_B \rightarrow\right] + \left[(AB)\omega \nwarrow \theta\right]$

$+\downarrow \quad 0 = 0 + (AB)\omega\cos\theta \qquad \omega = 0$ ◀

$+\rightarrow \quad v_A = v_B = 3\text{ m/s} \qquad \underline{v}_A = 3\text{ m/s} \rightarrow$ ◀

(b) $\beta = 90°$:

WHEEL: $\omega_{BC} = 6\text{ rad/s}$ (SEE PART c)

$v_O = 1.5\text{ m/s}$

$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}$

$\underline{v}_B = \left[1.5\text{ m/s} \rightarrow\right] + \left[(0.25\text{m})(6\text{ rad/s}) \downarrow\right]$

$= \left[1.5\text{ m/s} \rightarrow\right] + \left[1.5\text{ m/s} \downarrow\right]$

$\underline{v}_B = (1.5\text{ m/s})\sqrt{2} \;\angle 45°$

(CONTINUED)

15.53 CONTINUED

GEOMETRY $\quad AB = 0.9\text{ m}$

ROD AB:

$\sin\theta = \dfrac{0.25\text{ m}}{0.9\text{ m}}$ $\quad \theta = 16.128°$

$v_{A/B} = (0.9\text{ m})\omega$ FIXED

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\left[v_A \rightarrow\right] = \left[(1.5\text{ m/s})\sqrt{2} \;\angle 45°\right] + \left[v_{A/B} \perp \theta\right]$

$+\downarrow \quad 0 = (1.5\sqrt{2})\sin 45° - v_{A/B}\cos 16.128°$

$0 = 1.5 - v_{A/B}(0.96065)$

$v_{A/B} = 1.5614\text{ m/s}$

$(0.9\text{ m})\omega = 1.5614\text{ m/s}$

$\omega = 1.735\text{ rad/s} \qquad \underline{\omega} = 1.735\text{ rad/s}\,\circlearrowleft$ ◀

$+\rightarrow \quad v_A = (1.5\sqrt{2})\cos 45° - v_{A/B}\sin 16.128°$

$v_A = 1.5 - (1.5614\text{ m/s})(0.27778)$

$\underline{v}_A = 1.066\text{ m/s} \rightarrow$ ◀

15.54

$\beta = 30°$: WHEEL $\quad \omega_{BC} = (1.5\text{ m/s})/0.25\text{m} = 6\text{ rad/s}\,\circlearrowleft$

$\omega_{BC} = 6\text{ rad/s}$ $\quad$ RADIUS $= 0.3\text{ m}$

$v_O = 1.8\text{ m/s}$

$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}$

$\underline{v}_B = \left[1.5\text{ m/s} \rightarrow\right] + \left[(0.25\text{m})(6\text{ rad/s}) \;\angle 30°\right]$

$\underline{v}_B = \left[1.5\text{ m/s} \rightarrow\right] + \left[1.5\text{ m/s} \;\angle 30°\right]$

ROD AB: GEOMETRY

RADIUS $= 0.25\text{ m}$

$AB = 0.9\text{ m}$

$(0.25)\cos 30° = 0.2165\text{ m}$

0.4665 m

0.25 m

$\sin\theta = \dfrac{0.4665\text{ m}}{0.9\text{ m}}$

$\theta = 31.22°$

$v_{A/B} = (0.9\text{m})\omega$ FIXED

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\left[v_A \rightarrow\right] = \left[1.5\text{ m/s} \rightarrow + 1.5\text{ m/s} \;\angle 30°\right] + \left[v_{A/B} \perp \theta\right]$

$v_{A/B} = 0.9\omega$

$\theta = 31.22°$

$+\downarrow \quad 0 = 1.5\sin 30° - (0.9\,\omega)\cos 31.22°$

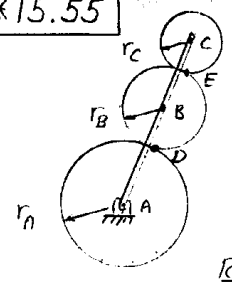
$0 = 0.75 - 0.76966\,\omega$; $\omega = 0.97446\text{ rad/s} \qquad \underline{\omega} = 0.974\text{ rad/s}\,\circlearrowleft$ ◀

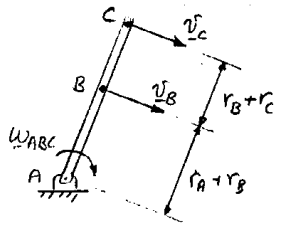
$+\rightarrow \quad \underline{v}_A = 1.5 + 1.5\cos 30° - (0.9)(0.9744)\sin 31.22°$

$= 1.5 + 1.299 - 0.4546 = 2.344\text{ m/s}$

$\underline{v}_A = 2.34\text{ m/s} \rightarrow$ ◀

*15.55

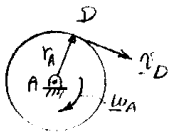




ROD ABC: ω_{ABC} ↷

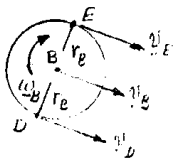
$v_B = (r_A + r_B)\omega_{ABC}$ ↘ (1)

$v_C = (r_A + 2r_B + r_C)\omega_{ABC}$ ↘ (2)



GEAR A: ω_A ↷

$v_D = r_A \omega_A$ ↘ (3)



GEAR B: ASSUME ω_B ↷

$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$

$\vec{v}_D = \vec{v}_B + r_B \omega_B$ ↖

SUBSTITUTING FROM (1) AND (3):

+↘ $r_A \omega_A = (r_A + r_B)\omega_{ABC} - r_B \omega_B$

SOLVING FOR ω_B: $\omega_B = \left(\frac{r_A}{r_B} + 1\right)\omega_{ABC} - \frac{r_A}{r_B}\omega_A$ (4)

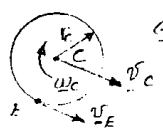
ALSO: $\vec{v}_E = \vec{v}_B + \vec{v}_{E/B} = \vec{v}_B + r_B \omega_B$ ←

SUBSTITUTING FROM (1): +← $v_E = (r_A + r_B)\omega_{ABC} + r_B \omega_B$

NOW SUBSTITUTE FOR ω_B FROM (4):

$v_E = (r_A + r_B)\omega_{ABC} + r_B\left[\left(\frac{r_A}{r_B} + 1\right)\omega_{ABC} - \frac{r_A}{r_B}\omega_A\right]$

$v_E = 2(r_A + r_B)\omega_{ABC} - r_A \omega_A$ (5)



GEAR C ASSUME ω_C ↷

$\vec{v}_E = \vec{v}_C + \vec{v}_{E/C} = \vec{v}_C + r_C \omega_C$ ↖

SUBSTITUTING FROM (2) AND (5):

$2(r_A + r_B)\omega_{ABC} - r_A \omega_A = (r_A + 2r_B + r_C)\omega_{ABC} - r_C \omega_C$

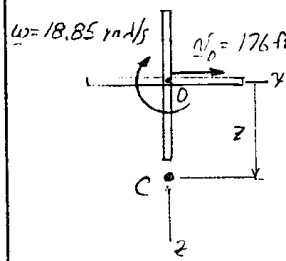
SOLVING FOR ω_C: $\omega_C = \frac{r_A}{r_C}\omega_A + \left(1 - \frac{r_A}{r_C}\right)\omega_{ABC}$ ◀

RESULT IS INDEPENDENT OF r_B ◀

15.56

$v_D = 120$ mi/h $= 176$ ft/s →

$\omega = 180$ rpm $= 18.85$ rad/s ↷



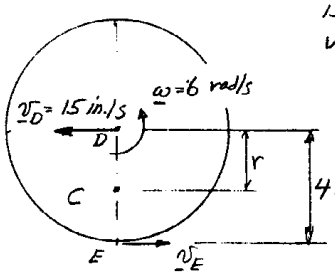
INSTANTANEOUS AXIS IS VERTICAL LINE LOCATED AT A DISTANCE r FROM THE y AXIS SUCH THAT

$\vec{v}_C = \vec{v}_D + \vec{v}_{C/D} = 0$

$0 = [176 \text{ ft/s} \rightarrow] + [z(18.85 \text{ rad/s}) \leftarrow]$

$z = 9.34$ ft ◀

15.57



INSTANTANEOUS CENTER IS LOCATED ON LINE DE WHICH IS ⊥ TO v_D AT

$r = \frac{v_D}{\omega} = \frac{15 \text{ in./s}}{6 \text{ rad/s}}$

$r = 2.5$ in ◀

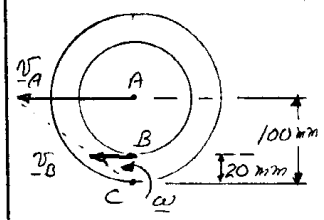
$v_E = (CE)\omega$

$= (4 \text{ in.} - 2.5 \text{ in.})\, 6 \text{ rad/s}$

$v_E = 9$ in./s → ◀

15.58

SINCE DISK ROLLS WITHOUT SLIDING $v_C = 0$



(a) $v_B = v_D = 120$ mm/s ←

$v_B = (BC)\omega$

120 mm/s $= (20 \text{ mm})\omega$

$\omega = 6$ rad/s ↻ ◀

(b) $v_A = (CA)\omega = (100 \text{ mm})(6 \text{ rad/s})$

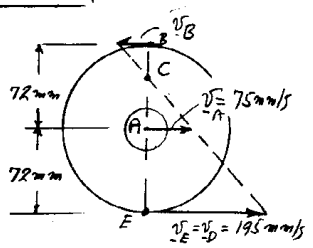
$v_A = 600$ mm/s ← ◀

(c) SINCE $v_A > v_D$, DRUM GAINS ON CORD, AND CORD IS WOUND AT THE RATE OF

$v_A - v_D = (600 \text{ mm/s}) - (120 \text{ mm/s}) = 480$ mm/s

CORD WOUND PER SECOND = 480 mm ◀

15.59



(a) SINCE v_A AND v_E ARE PARALLEL, THE INSTANTANEOUS CENTER IS AT INTERSECTION C OF LINE JOINING END POINTS OF v_B AND v_E WITH THE VERTICAL DIAMETER BE

SIMILAR TRIANGLES

$\frac{AC}{75} = \frac{EC}{195} = \frac{EC - AC}{120}$

BUT $EC - AC = 72$ mm

THUS $\frac{AC}{75} = \frac{72}{120}$ $AC = 45$ mm ◀

$v_A = (AC)\omega$; $75 \text{ mm/s} = (45 \text{ mm})\omega$

$\omega = 1.6667$ rad/s ↻

(b) VELOCITY OF POINT B $BC = 72 - AC = 72 - 45 = 27$ mm

$v_B = (BC)\omega = (27 \text{ mm})(1.6667 \text{ rad/s})$

$v_B = 45$ mm/s $\quad \vec{v}_B = 45$ mm/s ← ◀

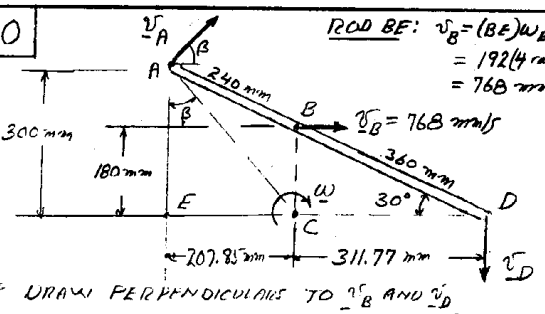
(c) SINCE $v_D > v_A$, THE PAPER AT D MOVES AWAY FROM THE CENTER A OF THE ROLL

THUS PAPER IS UNWOUND

$v_D - v_A = 195 \text{ mm/s} - 75 \text{ mm/s} = 120$ mm/s

LENGTH OF PAPER UNWOUND PER SECOND = 120 mm ◀

15.60



ROD BE: $v_B = (BE)\omega_{BE}$
$= 192(4 \text{ rad/s})$
$= 768 \text{ mm/s}$

WE DRAW PERPENDICULARS TO v_B AND v_D TO LOCATE THE INSTANTANEOUS CENTER C.

(a) ANGULAR VELOCITY:

$v_B = (BC)\omega$ $\quad 768 \text{ mm/s} = (180 \text{ mm})\omega$

$\omega = 4.2667 \text{ rad/s}$ $\quad \underline{\omega = 4.27 \text{ rad/s}}$ ◀

(b) $v_D = (CD)\omega = (311.77 \text{ mm})(4.2667 \text{ rad/s})$

$v_D = 1330.2 \text{ mm/s}$ $\quad \underline{v_D = 1.330 \text{ m/s}}$ ↓ ◀

(c) $v_A = (AC)\omega$

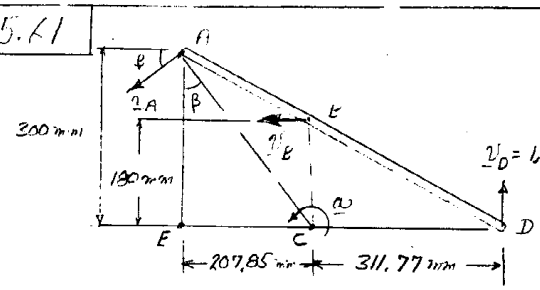
IN TRIANGLE ACE: $\tan\beta = \frac{207.85 \text{ mm}}{300 \text{ mm}}$ $\quad \beta = 34.72°$

$AC = \sqrt{(207.85)^2 + (300)^2}$ $\quad AC = 364.97 \text{ mm}$

$\therefore v_A = (364.97 \text{ mm})(4.2667 \text{ rad/s}) = 1557.2 \text{ mm/s}$

$\underline{v_A = 1.557 \text{ m/s} \angle 34.7°}$ ◀

15.61



WE DRAW PERPENDICULARS TO v_B AND v_D TO LOCATE INSTANTANEOUS CENTER C.

(a) ANGULAR VELOCITY:

$v_D = (CD)\omega$ $\quad 1.6 \text{ m/s} = (0.31177 \text{ m})\omega$

$\omega = 5.132 \text{ rad/s}$ $\quad \underline{\omega = 5.13 \text{ rad/s}}$ ◀

(b) $v_B = (BC)\omega = (180 \text{ mm})(5.132 \text{ rad/s})$

$v_B = 923.76 \text{ mm/s}$ $\quad \underline{v_B = 0.924 \text{ m/s}}$ ← ◀

(c) $v_A = (AC)\omega$

IN TRIANGLE ACE: $\tan\beta = \frac{207.85 \text{ mm}}{300 \text{ mm}}$ $\quad \beta = 34.72°$

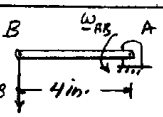
$AC = \sqrt{(207.85)^2 + (300)^2}$ $\quad AC = 364.97 \text{ mm}$

$v_A = (364.97 \text{ mm})(5.132 \text{ rad/s}) = 1873.0 \text{ mm/s}$

$\underline{v_A = 1.870 \text{ m/s} \angle 34.7°}$ ◀

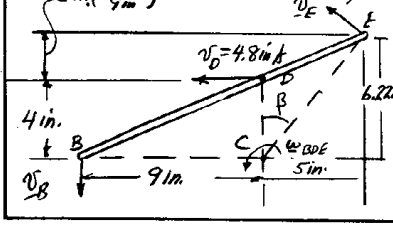
15.62

CRANK AB



$v_B = (AB)\omega_{AB} = (4 \text{ in.})\omega_{AB}$

(a)



DRAW ⊥ LINES TO v_B AND v_D TO LOCATE C.

$CE = \sqrt{(5)^2 + (6.222)^2}$

$CE = 7.982 \text{ in.}$

$\beta = 38.78°$

(CONTINUED)

15.62 CONTINUED

ROD BDE:

$v_D = (CD)\omega_{BDE}$; $\quad 4.8 \text{ in./s} = (4 \text{ in.})\omega_{BDE}$

$\omega_{BDE} = 1.200 \text{ rad/s}$ $\quad \underline{\omega_{BDE} = 1.200 \text{ rad/s}}$ ◀

$v_E = (CD)\omega_{BDE} = (7.982 \text{ in.})(1.200 \text{ rad/s}) = 9.579 \text{ in./s}$

$\underline{v_E = 9.58 \text{ in./s} \angle 38.8°}$ ◀

$v_B = (BC)\omega_{BDE} = (9 \text{ in.})(1.2 \text{ rad/s}) = 10.8 \text{ in./s}$

CRANK AB WE RECALL: $v_B = (4)\omega_{AB}$; $\quad (10.8 \text{ in./s}) = (4)\omega_{AB}$

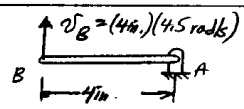
$\omega_{AB} = 2.700 \text{ rad/s}$ $\quad \underline{\omega_{AB} = 2.700 \text{ rad/s}}$ ◀

15.63

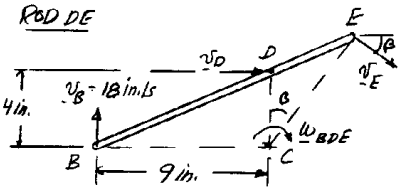
CRANK AB

$\omega_{AB} = 4.5 \text{ rad/s}$

$v_B = 18 \text{ in/s}$ ↑



ROD DE



SEE SOLUTION OF PROB 15.62 FOR LOCATION OF C AND $CE = 7.982 \text{ in.}$, $\beta = 38.78°$

$v_B = (BC)\omega_{BDE}$; $\quad 18 \text{ in./s} = (9 \text{ in.})\omega_{BDE}$ $\quad \underline{\omega_{BDE} = 2 \text{ rad/s}}$ ◀

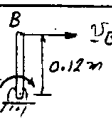
$v_D = (CD)\omega_{BDE} = (4 \text{ in.})(2 \text{ rad/s}) = 8 \text{ in./s}$ $\quad \underline{v_D = 8 \text{ in./s}} \rightarrow$ ◀

$v_E = (CE)\omega_{BDE} = (7.982 \text{ in.})(2 \text{ rad/s}) = 15.964 \text{ in./s}$

$\underline{v_E = 15.96 \text{ in./s} \angle 38.8°}$ ◀

15.64

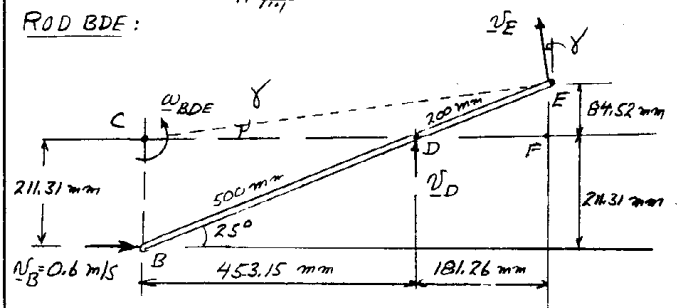
CRANK AB:



$v_B = (0.12 \text{ m})(5 \text{ rad/s}) = 0.6 \text{ m/s}$

$\omega_{AB} = 5 \text{ rad/s}$

ROD BDE:



WE DRAW LINES ⊥ TO v_B AND v_D TO LOCATE INST. CENTER C.

(a) $v_B = (BC)\omega_{BDE}$; $\quad 0.6 \text{ m/s} = (0.21131 \text{ m})\omega_{BDE}$

$\omega_{BDE} = 2.839 \text{ rad/s}$ $\quad \underline{\omega_{BDE} = 2.84 \text{ rad/s}}$ ◀

(b) $v_E = (CE)\omega_{BDE}$

IN TRIANGLE CFE:

$(CE)^2 = (453.15 + 181.26)^2 + (84.52)^2$

$(CE)^2 = (634.41)^2 + (84.52)^2$ $\quad CE = 640.01 \text{ mm}$

$\gamma = \tan^{-1}\frac{84.52 \text{ mm}}{634.41 \text{ mm}}$ $\quad \gamma = 7.59°$

$v_E = (CE)\omega_{BDE} = (640.01 \text{ mm})(2.839 \text{ rad/s})$

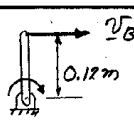
$= 1816.7 \text{ mm/s}$

$v_E = 1.817 \text{ m/s} \angle 7.59°$

$\underline{v_E = 1.817 \text{ m/s} \angle 82.4°}$ ◀

15.65

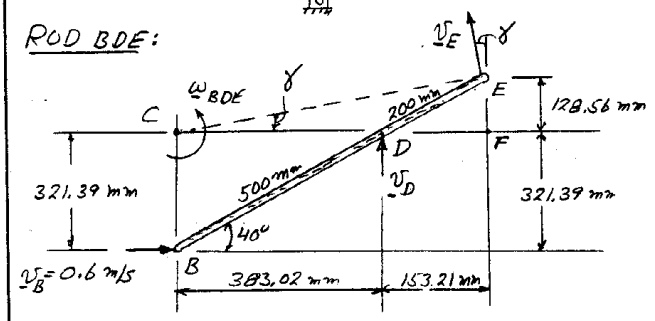
CRANK AB: $\omega_{AB} = 5$ rad/s



$\underline{v}_B = (0.12\text{ m})(5\text{ rad/s})$

$\underline{v}_B = 0.6$ m/s →

ROD BDE:



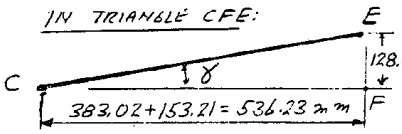
WE DRAW LINES ⊥ TO $\underline{v}_B$ AND $\underline{v}_D$ TO LOCATE INST. CENTER C

(a) $v_B = (BC)\omega_{BDE}$; $0.6\text{ m/s} = (0.32139\text{ m})\,\omega_{BDE}$

$\omega_{BDE} = 1.8669$ rad/s $\quad \underline{\omega}_{BDE} = 1.867$ rad/s ↻ ◀

(b) $v_E = (CE)\,\omega_{BDE}$

IN TRIANGLE CFE:



$CF = 551.4$ mm

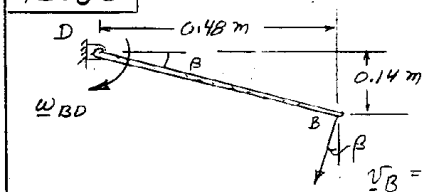
$\gamma = 13.5°$

$v_E = (CE)\,\omega_{BDE} = (551.4\text{ mm})(1.8669\text{ rad/s})$

$\underline{v}_E = 1029$ mm/s ∢ 13.5° $\quad \underline{v}_E = 1.029$ m/s ∠ 76.5° ◀

15.66

CYLINDER D IS STATIONARY; LINK BD



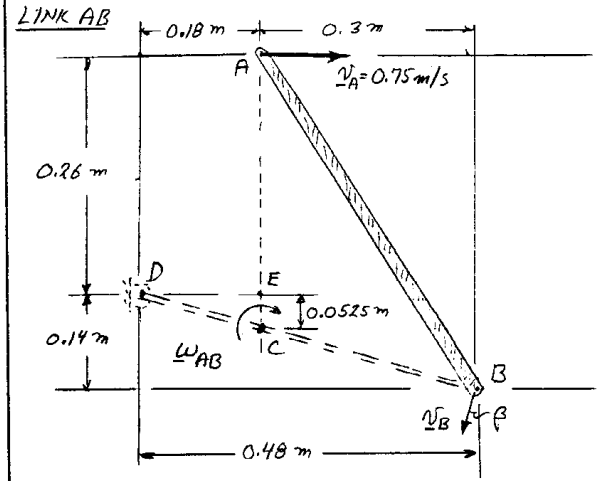
ASSUME ω_{BD} ↻

$BD = 0.5$ m

$\beta = 16.26°$

$\underline{v}_B = (0.5\text{ m})\,\omega_{BD}$ (1)

LINK AB



WE DRAW LINES ⊥ TO $\underline{v}_B$ AND $\underline{v}_A$ TO LOCATE INST. CENTER C.

$AC = AE + EC = 0.26\text{ m} + 0.14\text{ m}\,\frac{0.18\text{ m}}{0.48\text{ m}} = 0.26 + 0.0525 = 0.3125$ m

$BC = BD\,\frac{0.3\text{ m}}{0.48\text{ m}} = (0.5\text{ m})\,\frac{0.3\text{ m}}{0.48\text{ m}} = 0.3125$ m

$v_A = (AC)\,\omega_{AB} : (0.75\text{ m/s}) = (0.3125\text{ m})\,\omega_{AB}$

$\omega_{AB} = 2.4$ rad/s $\quad \underline{\omega}_{AB} = 2.4$ rad/s ↺ ◀

(CONTINUED)

15.66 CONTINUED

LINK AB: $v_B = (BC)\,\omega_{AB} = (0.3125\text{ m})(2.4\text{ rad/s}) = 0.75$ m/s

SINCE $\beta = 16.26°$

$\underline{v}_B = 0.75$ m/s ∠ 16.3° OR $\underline{v}_B = 0.75$ m/s ∇ 73.7° ◀

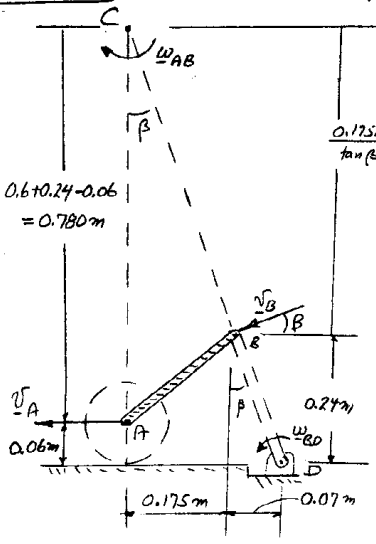
RETURN TO LINK BD: FROM EQ (1):

$v_B = (0.5\text{ m})\,\omega_{BD}$; $0.75\text{ m/s} = (0.5\text{ m})\,\omega_{BD}$

$\underline{\omega}_{BD} = 1.5$ rad/s ↻ ◀

15.67

GIVEN: $\underline{v}_A = 300$ mm/s ←



$DB^2 = 0.24^2 + 0.07^2$

$DB = 0.25$ m

$\beta = 16.26°$

$\frac{0.175\text{ m}}{\tan\beta} = \frac{0.175\text{ m}}{7/24} = 0.6$ m

LINK BD

$\underline{v}_B = (DB)\,\omega_{BD} = 0.25\,\omega_{BD}$ ↙

DRAW ⊥ LINES TO $\underline{v}_A$ AND $\underline{v}_B$ TO LOCATE C.

LINK AB $\quad v_A = (AC)\,\omega_{AB}$

$0.3\text{ m/s} = (0.780\text{ m})\,\omega_{AB}$

$\omega_{AB} = 0.3846$ rad/s

$\underline{\omega}_{AB} = 0.385$ rad/s ↺ ◀

$CB^2 = 0.175^2 + 0.600^2$ $\quad CB = 0.625$ m

$v_B = (CB)\,\omega_{AB}$

$= (0.625\text{ m})(0.3846\text{ rad/s})$

$= 0.2404$ m/s

$\underline{v}_B = 0.240$ m/s ∇ 16.3° ◀

LINK DB $\quad v_B = (DB)\,\omega_{BD}$

$0.2404\text{ m/s} = (0.25\text{ m})\,\omega_{BD}$

$\omega_{BD} = 0.9616$ rad/s

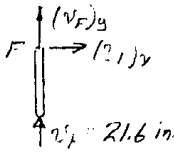
$\underline{\omega}_{BD} = 0.962$ rad/s ↻ ◀

15.68

CRANK DE: $v_E = (DE)\omega_{DE} = (3.6\text{in})(6\text{ rad/s})$

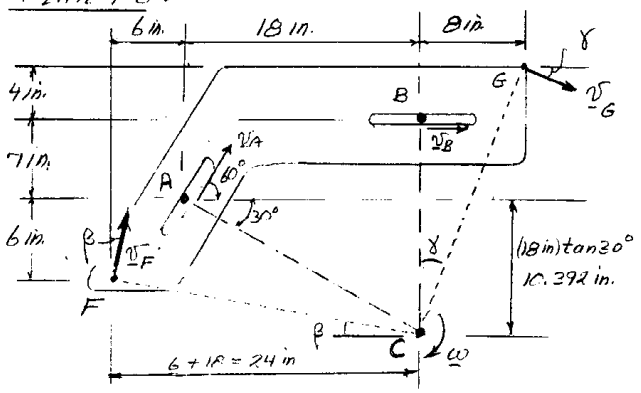
$v_E = 21.6$ in./s ↑

ROD EF:



$(v_F)_y = v_E = 21.6$ in./s ↑

PLATE FG:



v_A AND v_B ARE VELOCITIES OF POINTS ON THE PLATE NEXT TO THE PINS A AND B. WE DRAW LINES ⊥ TO v_A AND v_B TO LOCATE THE INSTANTANEOUS CENTER C

(a) VELOCITY OF POINT F:

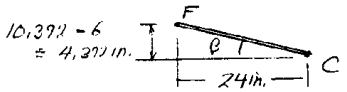
$v_F = (CF)\omega$

$(v_F)_y = [(CF)\omega]\cos\beta = [(CF)\cos\beta]\omega$

BUT $(v_F)_y = 21.6$ in./s AND $(CF)\cos\beta = 24$ in.:

21.6 in./s $= (24$ in.$)\omega$

$\omega = 0.9$ rad/s $\quad \underline{\omega} = 0.9$ rad/s ↻



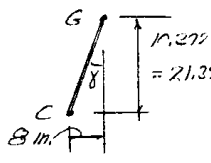
$CF = 24.399$ in.

$\beta = 10.39°$

$v_F = (CF)\omega = (24.399\text{ in.})(0.9\text{ rad/s}) = 21.96$ in./s

$\underline{v}_F = 22.0$ in./s ∠ 10.4° $\quad \underline{v}_F = 22.0$ in./s ∠ 79.6° ◀

(b) VELOCITY OF POINT G:



$CG = 22.838$ in.

$\gamma = 20.50°$

$v_G = (CG)\omega = (22.838\text{ in.})(0.9\text{ rad/s})$

$v_G = 20.554$ in./s $\quad \underline{v}_G = 20.6$ in./s ∠ 20.5° ◀

15.69

SEE SOLUTION OF PROB 15.68 FOR SKETCH AND DETERMINATION OF $\underline{\omega} = 0.9$ rad/s ↻

VELOCITY OF POINT OF PLATE AT A:

$AC = (18\text{ in.})/\cos 30° = 20.785$ in.

$v_A = (AC)\omega = (20.785\text{ in.})(0.9\text{ rad/s}) = 18.707$ in/s

$\underline{v}_A = 18.71$ in./s ∠ 60° ◀

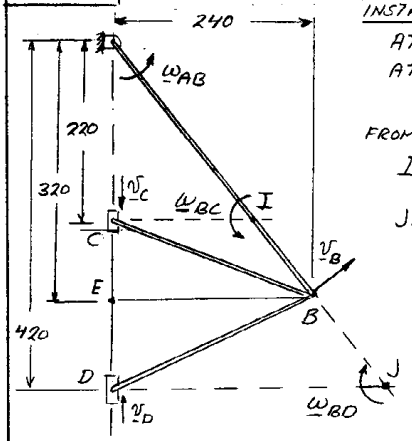
VELOCITY OF POINT OF PLATE AT B:

$BC = 10.392 + 7 = 17.392$ in

$v_B = (BC)\omega = (17.392\text{ in.})(0.9\text{ rad/s})$ $\quad \underline{v}_B = 15.65$ in./s → ◀

15.70

DIMENSIONS IN mm $\quad AB = 400$ mm



INSTANTANEOUS CENTERS:

AT I FOR MEMBER BC,

AT J FOR MEMBER BD.

FROM SIMILAR TRIANGLES:

$IC = \frac{220}{320}(240) = 165$ mm

$JD = \frac{420}{320}(240) = 315$ mm

$AI = \frac{220}{320}(400) = 275$ mm

$BI = AB - AI$

$= 400 - 275 = 125$ mm

$BJ = IB = 125$ mm

MEMBER BC $\quad v_C = 660$ mm/s ↓ (GIVEN)

$\omega_{BC} = \frac{v_C}{CI} = \frac{660\text{ mm/s}}{165\text{ mm}} = 4$ rad/s

$v_B = (BI)\omega_{BC} = (125\text{ mm})(4\text{ rad/s}) = 500$ mm/s

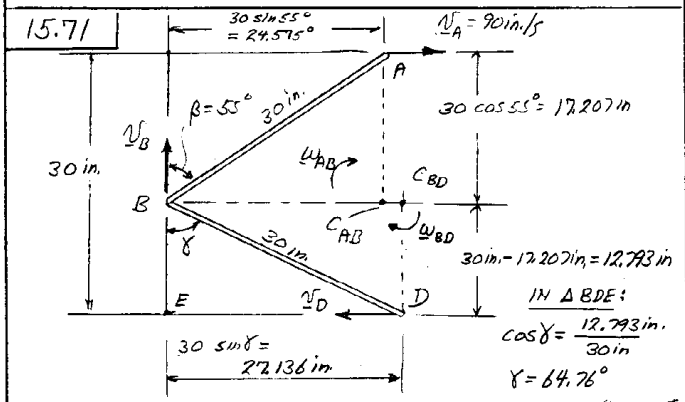
MEMBER BD. $\quad \omega_{BD} = \frac{v_B}{BJ} = \frac{500\text{ mm/s}}{125\text{ mm/s}} = 4$ rad/s

(a) $v_D = (JD)\omega_{BD} = (315\text{ mm})(4\text{ rad/s})$ $\quad \underline{v}_D = 1260$ mm/s ↑ ◀

MEMBER AB

(b) $\omega_{AB} = \frac{v_B}{AB} = \frac{500\text{ mm/s}}{400\text{ mm}}$ $\quad \underline{\omega}_{AB} = 1.25$ rad/s ↻ ◀

15.71



IN △ BDE:

$\cos\gamma = \frac{12.793\text{ in.}}{30\text{ in}}$

$\gamma = 64.76°$

$\gamma = 64.8°$ ◀

WE DRAW LINES ⊥ TO v_A AND v_B TO LOCATE INST. CENTER C_{AB} AND LINES ⊥ TO v_B AND v_D TO LOCATE INST CENTER C_{BD}.

LINK AB:

$v_A = (AC_{AB})\omega_{AB}$

90 in./s $= (17.207\text{ in.})\omega_{AB}$ $\quad \underline{\omega}_{AB} = 5.230$ rad/s ↻

$v_B = (BC_{AB})\omega_{AB} = (24.575\text{ in.})(5.230\text{ rad/s})$

$\underline{v}_B = 128.54$ in./s ↑

LINK BD:

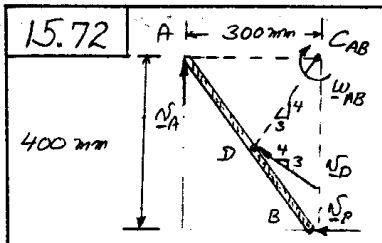
$v_B = (BC_{BD})\omega_{BD}$

128.54 in./s $= (27.136\text{ in.})\omega_{BD}$ $\quad \underline{\omega}_{BD} = 4.737$ rad/s ↻

$v_D = (DC_{BD})\omega_{BD} = (12.793\text{ in.})(4.737\text{ rad/s})$

$v_D = 60.60$ in/s $\quad \underline{v}_D = 60.6$ in./s ← ◀

15.72



ROD AB: DRAW LINES ⊥ TO $\underline{v}_A$ AND $\underline{v}_B$ TO LOCATE INST. CTR. C_{AB}

$v_B = (BC_{AB})\omega_{AB}$

$360 \text{ mm/s} = (400 \text{ mm})\omega_{AB}$

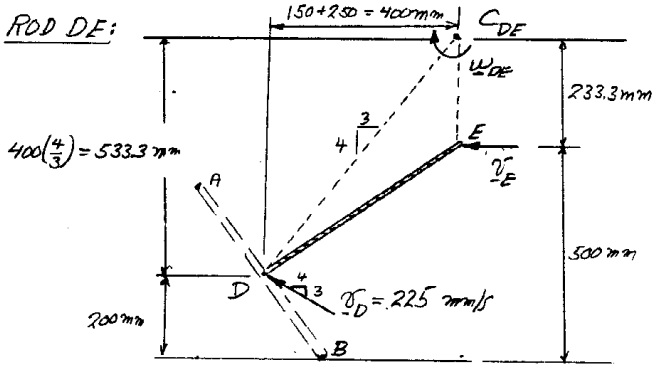
$\underline{\omega}_{AB} = 0.9 \text{ rad/s}$ ↺

VELOCITY OF D: $DC_{AB} = 250$ mm

$v_D = (DC_{AB})\omega_{AB} = (250 \text{ mm})(0.9 \text{ rad/s})$

$\underline{v}_D = 225 \text{ mm/s}$ ↙ (4–3)

ROD DE:



DRAW LINES ⊥ TO $\underline{v}_D$ AND $\underline{v}_E$ TO LOCATE INST. CTR. C_{DE}

$DC_{DE} = \sqrt{(400)^2 + (533.3)^2} = 666.6$ mm

$v_D = (DC_{DE})\omega_{DE}$

$225 \text{ mm/s} = (666.6 \text{ mm})\omega_{DE}$; $\omega_{DE} = 0.3375$ rad/s $\quad \underline{\omega}_{DE} = 0.338 \text{ rad/s}$ ↻

$v_E = (EC_{DE})\omega_{DE} = (233.3 \text{ mm})(0.3375 \text{ rad/s})$

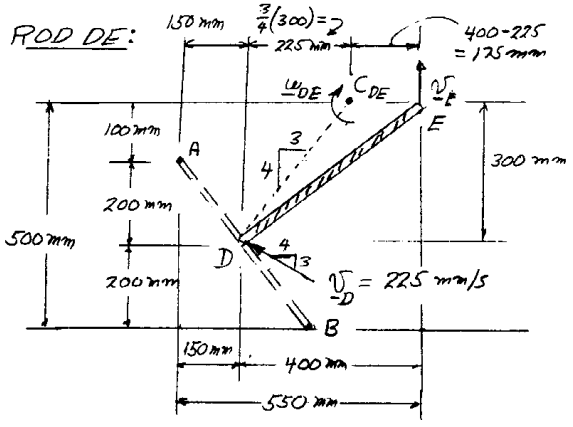
$v_E = 78.75 \text{ mm/s} \qquad \underline{v}_E = 78.8 \text{ mm/s}$ ←

15.73

SEE SOLUTION OF PROB 15.72 FOR MOTION OF ROD AB: $\underline{\omega}_{AB} = 0.9 \text{ rad/s}$ ↻

$\underline{v}_D = 225 \text{ mm/s}$ ↖ (4–3)

ROD DE:



DRAW LINES ⊥ TO $\underline{v}_D$ AND $\underline{v}_E$ TO LOCATE INST. CENTER C_{DE}

$DC_{DE} = \sqrt{(225)^2 + (300)^2} = 375$ mm

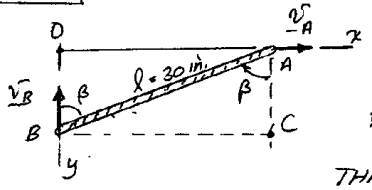
$v_D = (DC_{DE})\omega_{DE}$

$225 \text{ mm/s} = (375 \text{ mm})\omega_{DE}$ $\quad \underline{\omega}_{DE} = 0.60 \text{ rad/s}$ ↻

$v_E = (EC_{DE})\omega_{DE} = (175 \text{ mm})(0.60 \text{ rad/s})$

$\underline{v}_E = 105 \text{ mm/s}$ ↓

15.74



SPACE CENTRODE: INST. CENTER AT C

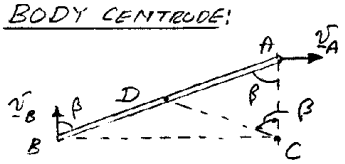
$x = l \sin\beta$

$y = l \cos\beta$

$x^2 + y^2 = l^2(\sin^2\beta + \cos^2\beta) = l^2$

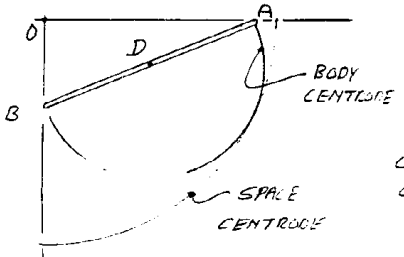
THIS IS THE EQUATION OF A CIRCLE OF RADIUS $l = 30$ in. WITH CENTER AT INTERSECTION OF THE RODS ON WHICH COLLARS A AND B MOVE

BODY CENTRODE:



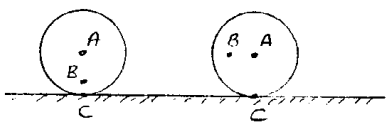
INST. CENTER AT C. WE DRAW CD WHICH CONNECTS C WITH MIDPOINT D OF LINK AB.

$BD = \frac{1}{2}l$, $BD = DC = \frac{1}{2}l$ = CONSTANT. THUS BODY CENTRODE IS CIRCLE OF RADIUS $\frac{1}{2}l = 15$ in. CENTER AT THE MIDPOINT D OF LINK AB



NOTE: BODY CENTRODE ROLLS ON SPACE CENTRODE

15.75

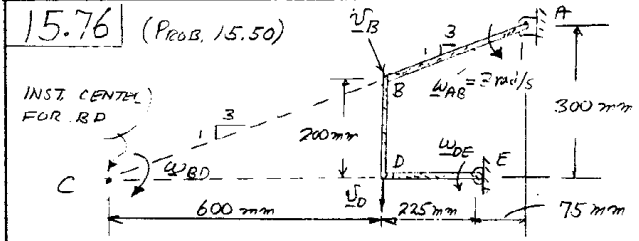


SINCE CYLINDER ROLLS ON HORIZONTAL SURFACE, INSTANTANEOUS CENTER IS ALWAYS AT POINT OF CONTACT

SPACE CENTRODE: THE HORIZONTAL SURFACE

BODY CENTRODE: CIRCUMFERENCE OF CYLINDER

15.76 (PROB. 15.50)



NOTE: $BC = 2(AB)$

BAR AB: $v_B = (AB)\omega_{AB}$

BAR BD:

$v_B = (BC)\omega_{BD} = (AB)\omega_{AB}$

$\omega_{BD} = \frac{(AB)}{(BC)}\omega_{BD} = \frac{1}{2}(3 \text{ rad/s})$

$\underline{\omega}_{BD} = 1.5 \text{ rad/s}$ ↻

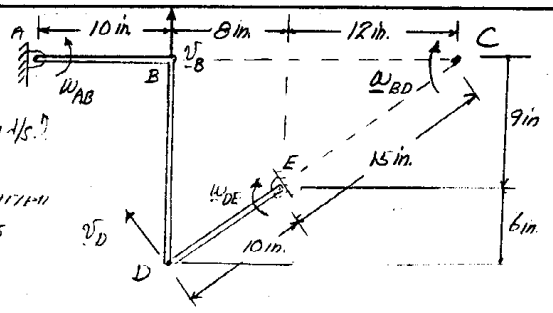
$\underline{v}_D = (CD)\omega_{BD} = (600 \text{ mm})(1.5 \text{ rad/s}) = 900 \text{ mm/s}$ ↓

BAR DE: $v_D = (DE)\omega_{DE}$

$900 \text{ mm/s} = (225 \text{ mm})\omega_{DE}$ $\quad \underline{\omega}_{DE} = 4 \text{ rad/s}$ ↻

15.77

(Prob. 15.51)



$\omega_{AB} = 3$ rad/s ↻

INST. CENTER OF BD IS AT C

BAR AB: $v_B = (AB)\omega_{AB} = (10\text{ in.})(3\text{ rad/s}) = 30$ in./s ↑

BAR BD:

$\omega_{BD} = \frac{v_B}{BC} = \frac{30\text{ in./s}}{20\text{ in.}}$ $\quad \underline{\omega_{BD} = 1.5\text{ rad/s}}$ ↻ ◀

$v_D = (DC)\omega_{BD} = (25\text{ in.})(1.5\text{ rad/s}) = 37.5$ in./s

BAR DE:

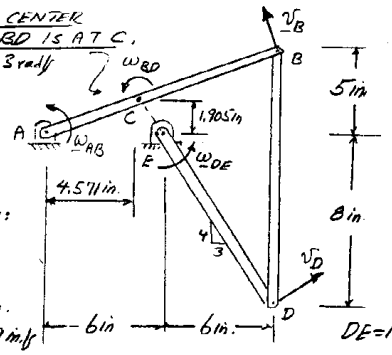
$\omega_{DE} = \frac{v_D}{DE} = \frac{37.5\text{ in./s}}{10\text{ in.}}$ $\quad \underline{\omega_{DE} = 3.75\text{ rad/s}}$ ↻ ◀

15.78

(Prob. 15.52)

INST. CENTER OF BD IS AT C.

$\omega_{AB} = 3$ rad/s



DE = 10 in.

GEOMETRY. ORIGIN AT A

AB: $y = \frac{5}{17}x$

CD: $y = -\frac{8}{6}(x-6)$

SET $y = y$, SOLVE FOR x:

$x = 4.571$ in.

$y = 1.905$ in.

BAR AB: AB = 13 in.

$v_B = (13\text{ in.})(3\text{ rad/s}) = 39$ in./s

BAR BD

$AC^2 = (4.571)^2 + (1.905)^2$ $\quad AC = 4.952$ $\quad BC = 13 - AC = 8.048$ in.

$(CD)_y = 8 + 1.905 = 9.905$ in. $\quad CD = (9.905)\frac{5}{4} = 12.381$ in.

$\omega_{BD} = v_B/BC = (39\text{ in/s})/8.048\text{ in}$ $\quad \underline{\omega_{BD} = 4.846\text{ rad/s}}$ ↻ ◀

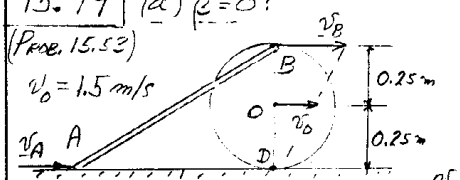
$v_D = (CD)\omega_{BD} = (12.381\text{ in})(4.846\text{ rad/s}) = 60$ in./s

BAR DE: $\omega_{DE} = \frac{v_D}{DE} = \frac{60\text{ in/s}}{10\text{ in}}$ $\quad \underline{\omega_{DE} = 6\text{ rad/s}}$ ↻ ◀

15.79

(a) $\beta = 0$:

(Prob. 15.53)



INST. CENTER OF WHEEL IS AT D

$\frac{v_O}{0.25\text{ m}} = \frac{v_B}{0.50\text{ m}}$

$v_B = 2v_O = 2(1.5) = 3$ m/s →

ROD AB: BOTH v_A AND v_B ARE HORIZONTAL, THUS INSTANTANEOUS CENTER OF AB IS AT ∞.

$\underline{\omega_{AB} = 0}$ ◀

SINCE INST. CENTER OF AB IS AT ∞, WE HAVE

$\underline{v_A = v_B = 3\text{ m/s}}$ → ◀

(CONTINUED)

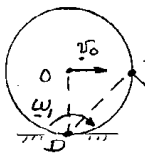
15.79 CONTINUED

(b) $\beta = 90°$:

WHEEL:

INST. CENTER IS AT D.

$v_O = 1.5$ m/s →



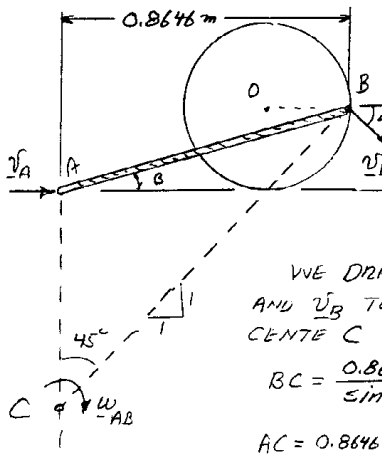
r = RADIUS = 0.25 m

OD = r

$BD = \sqrt{2}\,r$

$\omega_1 = \frac{v_O}{r}$

$v_B = (BD)\omega_1 = (\sqrt{2}\,r)\frac{v_O}{r} = \sqrt{2}\,v_O = \sqrt{2}(1.5) = 2.121$ m/s ↘ 45°



ROD AB:

AB = 0.9 m

$\sin\beta = \frac{0.25\text{ m}}{0.9\text{ m}}$ $\quad \beta = 16.128°$

$(AB)_x = (0.9)\cos\beta = 0.8646$ m

WE DRAW LINES ⊥ TO v_A AND v_B TO LOCATE INSTANTANEOUS CENTER C OF ROD AB.

$BC = \frac{0.8646\text{ m}}{\sin 45°} = 1.2227$ m

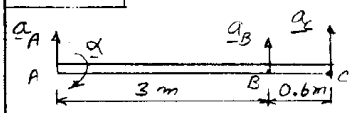
$AC = 0.8646\text{ m} - 0.25\text{ m} = 0.6146$ m

$\omega_{AB} = \frac{v_B}{BC} = \frac{2.121\text{ m/s}}{1.2227\text{ m}}$ $\quad \underline{\omega_{AB} = 1.735\text{ rad/s}}$ ↻ ◀

$v_A = (AC)\omega_{AC} = (0.6146\text{ m})(1.735\text{ rad/s})$ $\quad \underline{v_A = 1.066\text{ m/s}}$ → ◀

15.80

$\omega = 0$



(a)

$\underline{a_B} = \underline{a_A} + \underline{a}_{B/A}$

$1.5\text{ m/s}^2 \uparrow = 4.2\text{ m/s}^2 \uparrow + (AB)\alpha \downarrow$

$+\uparrow \quad -2.7\text{ m/s}^2 = -(3\text{ m})\alpha$

$\alpha = +0.9\text{ rad/s}^2$

$\underline{\alpha = 0.9\text{ rad/s}^2}$ ↻ ◀

(b)

$\underline{a_C} = \underline{a_A} + \underline{a}_{C/A}$

$= [4.2\text{ m/s}^2 \uparrow] + [(AC)\alpha \downarrow] = [4.2\text{ m/s}^2 \uparrow] + [(3.6\text{ m})(0.9\text{ rad/s}^2) \downarrow]$

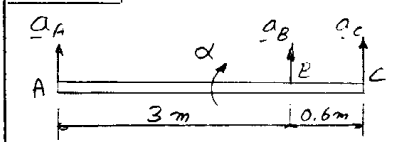
$\underline{a_C} = [4.2\text{ m/s}^2 \uparrow] + [3.24\text{ m/s}^2 \downarrow]$ $\quad \underline{a_C = 0.96\text{ m/s}^2 \uparrow}$ ◀

15.81

$\omega = 0$

$\underline{\alpha = 0.8\text{ rad/s}^2}$ ↻

$\underline{a_C = 0.3\text{ m/s}^2 \downarrow}$



$\underline{a_A} = \underline{a_C} + \underline{a}_{A/C} = [0.4\text{m/s}^2 \downarrow] + [(AC)\alpha \uparrow]$

$= [0.3\text{m/s}^2 \downarrow] + [(3.6\text{m})(0.8\text{ rad/s}^2) \uparrow] = [0.3\text{m/s}^2 \downarrow] + [2.88\text{m/s}^2 \uparrow]$

$\underline{a_A = 2.58\text{ m/s}^2 \uparrow}$ ◀

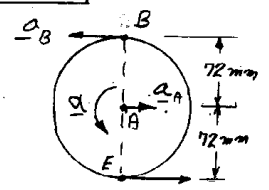
$\underline{a_B} = \underline{a_C} + \underline{a}_{B/C} = [0.3\text{m/s}^2 \downarrow] + [(BC)\alpha \uparrow]$

$= [0.3\text{m/s}^2 \downarrow] + [(0.6\text{m})(0.8\text{ rad/s}^2) \uparrow] = [0.3\text{m/s}^2 \downarrow] + [0.48\text{m/s}^2 \uparrow]$

$\underline{a_B = 0.18\text{ m/s}^2 \uparrow}$ ◀

15.82

$\underline{a}_A = 300\ mm/s^2 \rightarrow$



$\underline{a}_E = \underline{a}_D = 780\ mm/s^2 \rightarrow$

(a) $\underline{a}_E = \underline{a}_A + \underline{a}_{E/A} = a_A + (AE)\alpha$

$[780\ mm/s^2 \rightarrow] = [300\ mm/s^2 \rightarrow] + [(AE)\alpha \rightarrow]$

$+\rightarrow \quad 780 = 300 + 72\alpha$

$\alpha = 480/72 = 6.667\ rad/s^2$

$\underline{\alpha} = 6.67\ rad/s^2$ ◀

(b) $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = a_A + (AB)\alpha \leftarrow$

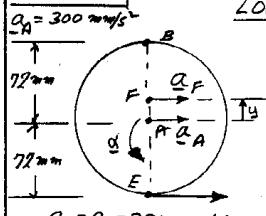
$= [300\ mm/s^2 \rightarrow] + [(72\ mm)(6.667\ rad/s^2) \leftarrow]$

$= [300\ mm/s^2 \rightarrow] + [480\ mm/s^2 \leftarrow]$

$\underline{a}_B = 180\ mm/s^2 \leftarrow$ ◀

15.83

LET F BE POINT ON DIAMETER LOCATED AT A DISTANCE y ABOVE A.



$\underline{a}_E = \underline{a}_D = 780\ mm/s^2 \rightarrow$

$a_F = a_A + a_{F/A} = a_A + (AE)\alpha$

$[780\ mm/s^2 \rightarrow] = [300\ mm/s^2 \rightarrow] + [(72\ mm)\alpha \rightarrow]$

$+\rightarrow \quad 780 = 300 + 72\alpha$

$\alpha = 480/72 = 6.667\ rad/s$

$a_F = a_A + a_{F/A} = \underline{a}_A + y\alpha \leftarrow$

$= [300 \rightarrow] + [6.667y \leftarrow]$

$+\rightarrow \quad a_F = 300 - 6.667y$

(a) FOR $a_F = 0$: $300 - 6.667y = 0$; $y = 45$

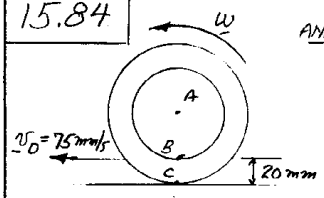
POINT OF ZERO ACCELERATION IS 45 mm ABOVE A. ◀

(b) FOR $\underline{a}_F = 500\ mm/s^2 \rightarrow$, $a_F = a_A + y\alpha$

$[500\ mm/s^2 \rightarrow] = [300\ mm/s^2 \rightarrow] + [6.667y \leftarrow]$

$+\rightarrow \quad 500 = 300 - 6.667y$; $y = -30$ mm: 30 mm BELOW A ◀

15.84



ANGULAR VELOCITY OF DRUM

INST. CENTER AT C

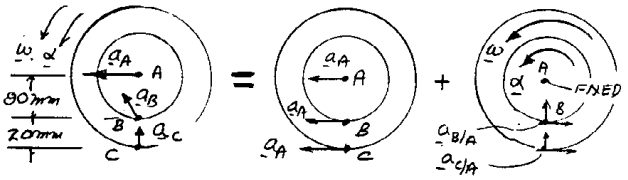
$\omega = \frac{v_D}{BC} = \frac{75\ mm/s}{20\ mm}$

$\underline{\omega} = 3.75\ rad/s$)

ACCELERATIONS: SINCE THE DISK ROLLS, WE HAVE

$v_A = (CA)\omega = (100\ mm)\omega \leftarrow$

DIFFERENTIATING, WE FIND: $a_A = (CA)\alpha = (100\ mm)\alpha \leftarrow$



POINT B: $(\underline{a}_B)_x = \underline{a}_D = 400\ mm/s^2 \leftarrow$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = [a_A \leftarrow] + [(AB)\alpha \rightarrow] + [(AB)\omega^2 \uparrow]$

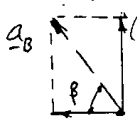
$[400\ mm/s^2 \leftarrow] + [(a_B)_y \uparrow] = [(100\ mm)\alpha \leftarrow] + [(80\ mm)\alpha \rightarrow] + [(80\ mm)(3.75\ rad/s)^2 \uparrow]$

$\pm \rightarrow x$ COMPONENTS: $400 = 20\alpha$ $\quad \alpha = 20\ rad/s^2$ $\quad \alpha = 20\ rad/s^2$)

POINT A: $a_A = (120\ mm)\alpha = (100\ mm)(20\ rad/s^2) = 2000\ mm/s^2$ $\quad \underline{a}_A = 2\ m/s^2 \leftarrow$ ◀

$+\uparrow y$ COMPONENTS: $(\underline{a}_B)_y = 80(3.75)^2 = 1125\ mm/s^2 \uparrow$

POINT B:



$(a_B)_y = 400\ mm/s^2$

$\underline{a}_B = 1.194\ m/s^2 \nearrow 70.4°$ ◀

(CONTINUED)

15.84 CONTINUED

POINT C:

$\underline{a}_A = \underline{a}_C + \underline{a}_{A/C} = [a_C \uparrow] + [(AC)\alpha \leftarrow] + [(AC)\omega^2 \downarrow]$

$[a_A \leftarrow] = [a_C \uparrow] + [(100\ mm)(20\ rad/s^2) \leftarrow] + [(100\ mm)(3.75\ rad/s)^2 \downarrow]$

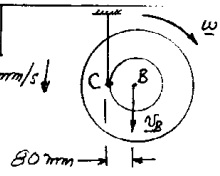
$[a_A \leftarrow] = [a_C \uparrow] + [2000\ mm/s^2 \leftarrow] + [1406.3\ mm/s^2 \downarrow]$

$+\uparrow y$ COMPONENTS: $0 = a_C - 1406.3\ mm/s^2$

$a_C = 1406.3\ mm/s^2$

$\underline{a}_C = 1.406\ m/s^2 \uparrow$ ◀

15.85



$\underline{v}_B = 600\ mm/s \downarrow$

VELOCITY OF PULLEY:

$v_C = 0$, INST. CTR. AT C

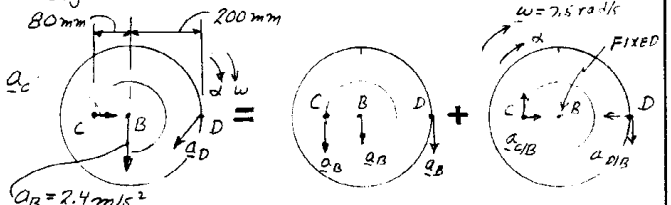
$v_B = (CB)\omega$

$0.6\ m/s = (0.08\ m)\omega$

$\omega = 7.5\ rad/s$)

ACCELERATIONS: CENTER B MOVES IN STRAIGHT VERTICAL LINE, $\underline{a}_B = \downarrow$ SINCE CORD IS INEXTENSIBLE

$(a_C)_y = 0$ SINCE CORD IS FIXED AT A.



$\underline{a}_B = 2.4\ m/s^2 \downarrow$

$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = [a_B \downarrow] + [(BC)\alpha \uparrow] + [(BC)\omega^2 \rightarrow]$

$\underline{a}_C = [2.4\ m/s^2 \downarrow] + [(0.08\ m)\alpha \uparrow] + [(0.08\ m)(7.5\ rad/s)^2 \rightarrow]$

$\pm \rightarrow x$ COMPONENTS: $\underline{a}_C = (0.08)(7.5)^2 = 4.5\ m/s^2 \rightarrow$

$+\downarrow y$ COMPONENTS: $0 = 2.4 - 0.08\alpha$ $\quad \underline{\alpha} = 30\ rad/s^2$)

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = [a_B \downarrow] + [(BD)\alpha \downarrow] + [(BD)\omega^2 \leftarrow]$

$\underline{a}_D = [2.4\ m/s^2 \downarrow] + [(0.2\ m)(30\ rad/s^2) \downarrow] + [(0.2\ m)(7.5\ rad/s)^2 \leftarrow]$

$= 2.4\ m/s^2 \downarrow + 6\ m/s^2 \downarrow + 11.25\ m/s^2 \leftarrow$

$(a_D)_x = 11.25\ m/s$

$(a_D)_y = 8.4\ m/s^2$

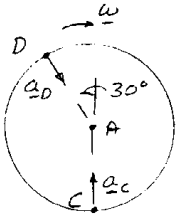
$\underline{a}_D$

$\underline{a}_D = 14.04\ m/s^2 \swarrow 36.7°$ ◀

15.86

SINCE SPEED IS CONSTANT $a_A = 0$, $\underline{\alpha} = 0$

$\underline{v}_A = 90\ km/h = 25\ m/s \rightarrow$



RADIUS $= \frac{1}{2}(550\ mm) = 0.275\ m$

$\underline{\omega} = \frac{v_A}{r} = \frac{25\ m/s}{0.275\ m} = 90.91\ rad/s$)

(a) POINT C:

$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = 0 + r\omega^2 \uparrow$

$= (0.275\ m)(90.91\ rad/s)^2 \uparrow$

$a_C = 2273\ m/s^2$ $\quad \underline{a}_C = 2270\ m/s^2 \uparrow$ ◀

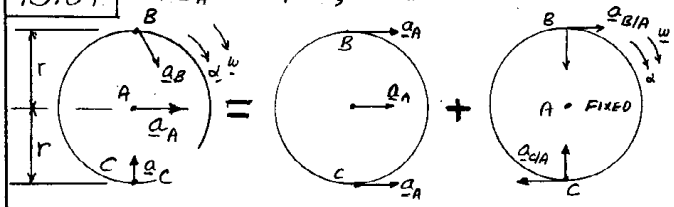
(b) POINT D: $\underline{a}_D = \underline{a}_A + \underline{a}_{D/A} = 0 + r\omega^2 \searrow 30°$

$= (0.275\ m)(90.91\ rad/s)^2 \searrow 30°$

$a_C = 2273\ m/s^2$ $\quad \underline{a}_C = 2270\ m/s^2 \swarrow 60°$ ◀

15.87

$\underline{a}_A = 2.4\ m/s^2 \rightarrow$, $r = \frac{1}{2}(550\ mm) = 0.275\ m$



POINT C: SINCE WHEEL ROLLS, C HAS NO HORIZ. COMPONENT OF ACCEL.

$$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = [a_A \rightarrow] + [r\alpha \leftarrow] + [r\omega^2 \uparrow]$$

$$[a_C \uparrow] = [2.4\ m/s^2 \rightarrow] + [(0.275\ m)\alpha \leftarrow] + [r\omega^2 \uparrow]$$

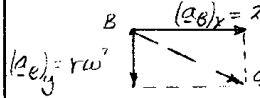
$\pm\rightarrow$ x COMPONENTS: $0 = 2.4\ m/s^2 - (0.275\ m)\alpha$

$\alpha = 8.727\ rad/s^2$ $\qquad \underline{\alpha} = 8.727\ rad/s^2\ \circlearrowright$

POINT B:

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = [2.4\ m/s^2 \rightarrow] + [r\alpha \rightarrow] + [r\omega^2 \downarrow]$$

$$= [2.4\ m/s^2 \rightarrow] + [(0.275\ m)(8.727\ rad/s^2) \rightarrow] + [r\omega^2 \downarrow]$$



$16^2 = 4.8^2 + (r\omega^2)^2$ $\qquad r\omega^2 = 15.263\ m/s^2$

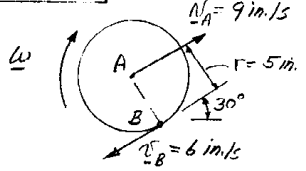
FOR $r = 0.275\ m$: $(0.275\ m)\omega^2 = 15.263$; $\qquad \underline{\omega} = 7.45\ rad/s\ \circlearrowright$

SINCE INST. CENTER IS AT C: $v_A = r\omega = (0.275\ m)(7.45\ rad/s)$

$v_A = 2.05\ m/s$ ◀

15.88

VELOCITIES:



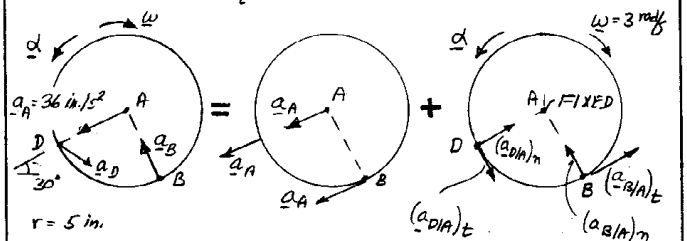
$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + r\omega$$

$$[9\ in./s \nearrow] = [6\ in./s \swarrow] + [(5\ in.)\omega \nearrow]$$

$\pm\nearrow$ COMPONENTS: $9 = -6 + 5\omega$

$\underline{\omega} = 3\ rad/s\ \circlearrowright$

ACCELERATIONS: BELT MOVES AT CONSTANT SPEED, THUS $(\underline{a}_B)_t = 0$ AND $\underline{a}_B$ IS $\uparrow$.



$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = [a_A \leftarrow] + [(a_{B/A})_t \nearrow] + [(a_{B/A})_n \nwarrow]$$

$$[a_B \uparrow] = [36\ in./s^2 \leftarrow] + [(5\ in.)\alpha \nearrow] + [r\omega^2 \nwarrow]$$

$+\nearrow$ COMPONENTS: $0 = -36 + 5\alpha$

$\alpha = +7.2\ rad/s^2$

$\underline{\alpha} = 7.2\ rad/s^2\ \circlearrowright$

(CONTINUED)

15.88 CONTINUED

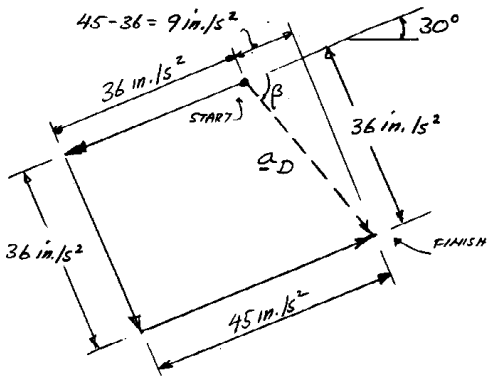
POINT D:

$$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A} = [a_A \leftarrow] + [(a_{D/A})_t \searrow] + [(a_{D/A})_n \rightarrow]$$

$$\underline{a}_D = [a_A \leftarrow] + [r\alpha \searrow] + [r\omega^2 \rightarrow]$$

$$= [36\ in./s^2 \leftarrow] + [(5\ in.)(7.2\ rad/s^2) \searrow] + [(5\ in.)(3\ rad/s)^2 \rightarrow]$$

$$= [36\ in./s^2 \leftarrow] + [36\ in./s^2 \searrow] + [45\ in./s^2 \rightarrow]$$



$a_D = \sqrt{(36)^2 + (9)^2} = 37.108\ in./s^2$

$\beta = \tan^{-1}\frac{36\ in./s^2}{9\ in./s^2} = 75.96°$

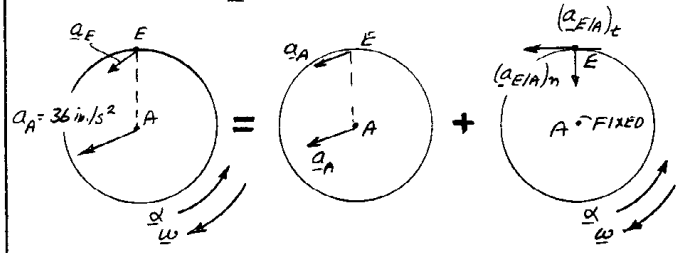
$75.96° - 30° = 45.96°$ $\qquad \underline{a}_D = 37.1\ in./s^2\ \measuredangle\ 46.0°$ ◀

15.89

SEE SOLUTION OF PROB. 15.88 FOR

$\underline{\alpha} = 7.2\ rad/s^2\ \circlearrowright$

$\underline{\omega} = 3\ rad/s\ \circlearrowright$

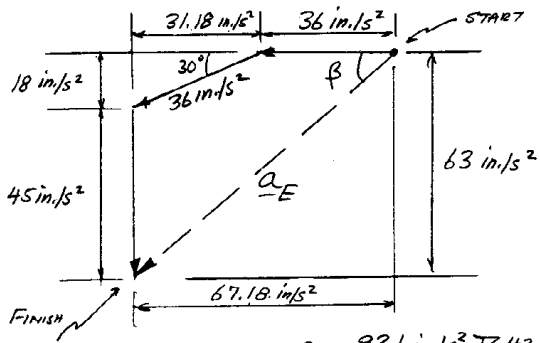


$$\underline{a}_E = \underline{a}_A + \underline{a}_{E/A} = [a_A \leftarrow] + [(a_{E/A})_t \leftarrow] + [(a_{E/A})_n \downarrow]$$

$$= [a_A \leftarrow] + [r\alpha \leftarrow] + [r\omega^2 \downarrow]$$

$$= [36\ in./s^2 \leftarrow] + [(5\ in.)(7.2\ rad/s^2) \leftarrow] + [(5\ in.)(3\ rad/s)^2 \downarrow]$$

$$= [36\ in./s^2 \leftarrow] + [36\ in./s^2 \leftarrow] + [45\ in./s^2 \downarrow]$$



$\underline{a}_E = 92.1\ in./s^2\ \measuredangle\ 43.2°$ ◀

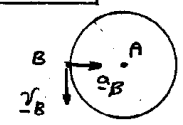
15.90 (a) $\theta = 90°$ DISK

$\omega_{DISK} = 500$ rpm $= 52.36$ rad/s

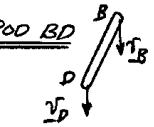
$AB = 0.05$ m

$a_B = (AB)\omega_{DISK}^2 = (0.05\text{m})(52.36\text{ rad/s})^2$

$\underline{a}_B = 137.08\text{ m/s}^2 \rightarrow$



ROD BD



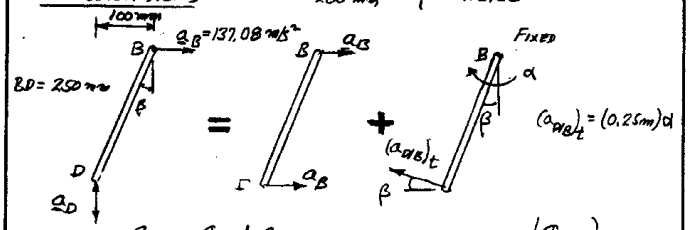
VELOCITIES

v_B AND v_D ARE PARALLEL

$\therefore \underline{\omega}_{BD} = 0$

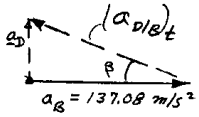
ACCELERATIONS

$\sin\beta = \frac{100\text{ mm}}{250\text{ mm}}$ $\beta = 23.58°$



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$

$[a_D \uparrow] = [a_B \rightarrow] + [0.25\alpha \angle \beta]$

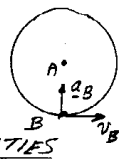


$a_D = a_B \tan\beta = (137.08\text{ m/s}^2)\tan 23.57° = 59.83\text{ m/s}^2$

$\underline{a}_D = 59.8\text{ m/s}^2 \uparrow$ ◀

(b) $\theta = 180°$

$AB = 0.05$ m



$\omega_{DISK} = 500$ rpm $= 52.36$ rad/s

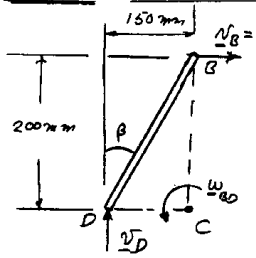
$v_B = (AB)\omega_{DISK} = (0.05\text{m})(52.36\text{ rad/s})$

$v_B = 2.618$ m/s $\rightarrow$

$a_B = (AB)\omega_{DISK}^2 = (0.05\text{m})(52.36\text{ rad/s})^2$

$\underline{a}_B = 137.07\text{ m/s}^2 \uparrow$

ROD BD VELOCITIES



BD = 250 mm

$BC^2 = 250^2 - 150^2$ $BC = 200$

$\tan\beta = \frac{150}{200}$ $\beta = 36.9°$

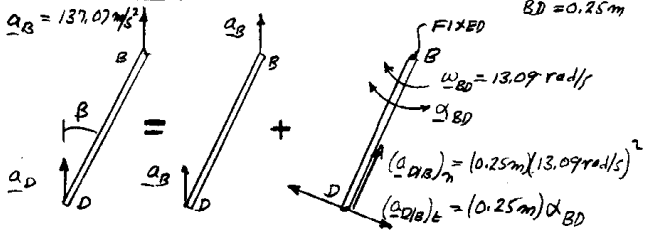
INSTANTANEOUS CENTER IS AT C.

$v_B = (BC)\omega_{BD}$

$2.618\text{ m/s} = (0.2\text{ m})\omega_{BD}$

$\underline{\omega}_{BD} = 13.09\text{ rad/s}$ ↻

ACCELERATIONS

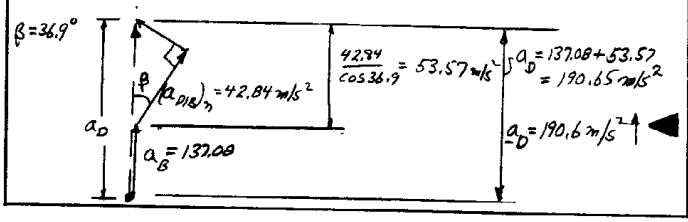


BD = 0.25 m

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$

$[a_D \uparrow] = [137.08\text{ m/s}^2 \uparrow] + [(0.25\text{m})(13.09\text{ rad/s})^2 \angle \beta] + [(0.25)\alpha_{BD} \angle \beta]$

$= [137.08\text{ m/s}^2 \uparrow] + [42.84\text{ m/s} \angle 36.9°] + [0.25\alpha_{BD} \angle 36.9°]$

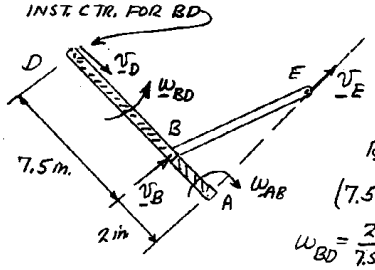


$\underline{a}_D = 190.6\text{ m/s}^2 \uparrow$ ◀

15.91 $\theta = 0$ $\omega_{AB} = 1500$ rpm $= 157.08$ rad/s $\alpha_{AB} = 0$

VELOCITIES:

INST. CTR. FOR BD



ROD BE:

SINCE v_B & v_E ARE PARALLEL, $\omega_{BE} = 0$

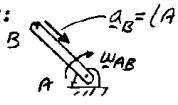
ROD BD: $v_B = v_B$

$(7.5\text{ in.})\omega_{BD} = (2\text{ in.})\omega_{AB}$

$\omega_{BD} = \frac{2\text{ in.}}{7.5\text{ in.}}(157.08\text{ rad/s}) = 41.888\text{ rad/s}$

ACCELERATIONS:

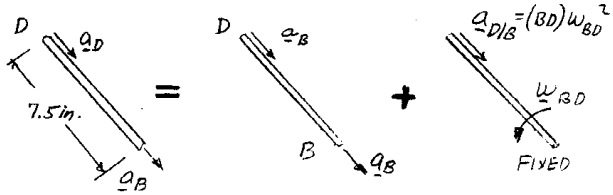
CRANK AB:



$\underline{a}_B = (AB)\omega_{AB}^2 = \left(\frac{2}{12}\text{ ft}\right)(157.08\text{ rad/s})^2$

$\underline{a}_B = 4112\text{ ft/s}^2 \angle 45°$

ROD BD:

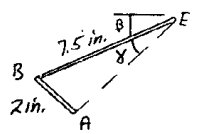


$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = [4112\text{ ft/s}^2 \angle 45°] + [(\frac{7.5}{12}\text{ ft})(41.888\text{ rad/s})^2 \angle 45°]$

$= [4112\text{ ft/s}^2 \angle 45°] + [1097\text{ ft/s}^2 \angle 45°]$

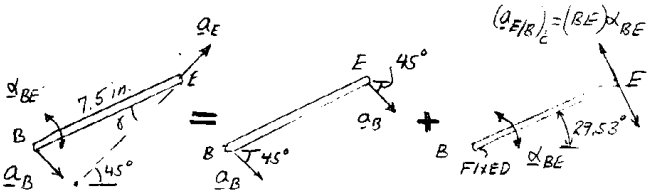
$= 5209\text{ ft/s}^2 \angle 45°$ $\underline{a}_D = 5210\text{ ft/s}^2 \angle 45°$ ◀

ROD BE: $\omega_{BE} = 0$



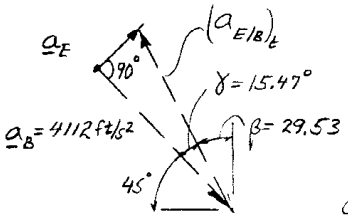
$\gamma = \sin^{-1}\frac{2\text{ in.}}{7.5\text{ in.}}$ $\gamma = 15.47°$

$\beta = 45° - \gamma = 29.53°$



$\underline{a}_E = \underline{a}_B + \underline{a}_{E/B} = [4112\text{ ft/s}^2 \angle 45°] + [(\frac{7.5}{12}\text{ ft})(\alpha_{BE}) \angle 29.53°]$

$[a_E \angle 45°] = [4112\text{ ft/s}^2 \angle 45°] + [(\frac{7.5}{12})\alpha_{BE} \angle 29.53°]$



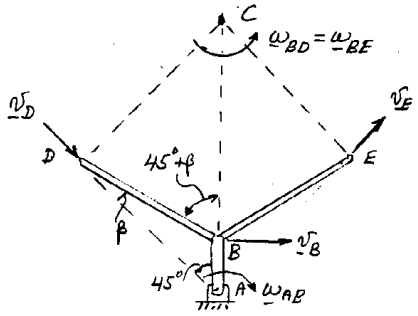
$a_E = a_B \tan\gamma$

$= (4112\text{ ft/s}^2)\tan 15.47°$

$\underline{a}_E = 1138\text{ ft/s}^2 \angle 45°$ ◀

15.92

$\theta = 45°$ $\omega_{AB} = 1500$ rpm $= 157.08$ rad/s, $\alpha_{AB} = 0$



AB = 2 in.
BD = 7.5 in.

$v_B = (2\text{in.})\,\omega_{AB} = (2)(157.08)$
$\underline{v}_B = 314.16$ in./s →

IN A DRA: AB = 2 in., BD = 7.5 in.

LAW OF SINES $\dfrac{2\text{ in.}}{\sin\beta} = \dfrac{7.5\text{ in}}{\sin 45°} = \dfrac{AD}{\sin[180° - (45° + \beta)]}$

$\sin\beta = \frac{2}{7.5}\sin 45°$ $\beta = 10.869°$

$AD = 7.5\text{in.}\dfrac{\sin(45° + \beta)}{\sin 45°} = 7.5\dfrac{\sin 55.869°}{\sin 45°} = 8.780$ in.

VELOCITIES

DRAW LINES ⊥ TO $\underline{v}_D$, $\underline{v}_B$ AND $\underline{v}_E$ AND LOCATE C WHICH (FOR $\theta = 45°$) IS INST. CENTER FOR BD AND BE.

NOTE: ADCE IS A SQUARE,

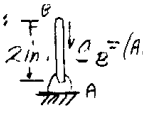
∴ $CA = \sqrt{2}\,(AD) = \sqrt{2}\,(8.780\text{ in.}) = 12.416$ in.

AND, $CB = CA - 2\text{in} = 12.416\text{ in.} - 2\text{in.} = 10.416$ in

$\omega_{BD} = \omega_{BE} = \dfrac{v_B}{CB} = \dfrac{314.16\text{ in./s}}{10.416\text{ in.}} = 30.16$ rad/s ↻

ACCELERATIONS:

CRANK AB:

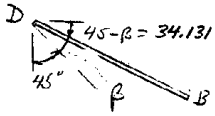


$\underline{a}_B = (AB)\omega_{AB}^2 = \left(\frac{2}{12}\text{ ft}\right)(157.08\text{ rad/s})^2$

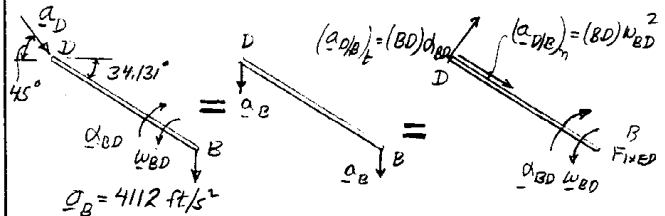
$\underline{a}_B = 4112$ ft/s² ↓

ROD BD:

$\beta = 10.869°$



BD = 7.5 in.



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$

$[a_D \angle 45°] = [4112\text{ ft/s}^2 \downarrow] + \left[\left(\frac{7.5}{12}\text{ft}\right)\alpha_{BD} \angle 34.131°\right]$

$+ \left[\left(\frac{7.5}{12}\text{ft}\right)(30.16\text{ rad/s})^2 \angle 34.131°\right]$

$[a_D \angle 45°] = [4112\text{ ft/s}^2 \downarrow] + \left[\frac{7.5}{12}\alpha_{BD} \angle 34.131°\right] + [568.5\text{ ft/s}^2 \angle 34.131°]$

$\pm$ → x COMPONENTS: $0.707\,a_D = 0.625\,\alpha_{BD}(\sin 34.131°) + 568.5\cos 34.131°$

$0.707\,a_D = 0.3507\,\alpha_{BD} + 470.6$ (1)

+↓ y COMPONENTS: $0.707\,a_D = 4112 - (0.625)\alpha_{BD}\cos 34.131° + 568.5\sin 34.131°$

$0.707\,a_D = 4112 - 0.5174\,\alpha_{BD} + 319.0$ (2)

(CONTINUED)

15.92 CONTINUED

EQ(1): $0.707\,a_D - 0.3507\,\alpha_{BD} = 470.6$

EQ(2): $0.707\,a_D + 0.5174\,\alpha_{BD} = 4431$

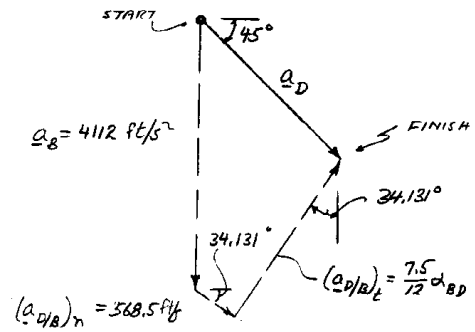
EQ(2) − EQ(1): $0.8681\,\alpha_{BD} = 3960$

$\alpha_{BD} = 4562$ rad/s² ↻

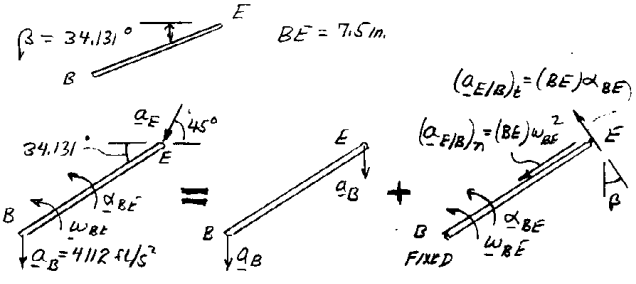
EQ(1): $0.707\,a_D - (0.3507)(4562) = 470.6$

$a_D = +2928$ ft/s² $\quad \underline{a}_D = 2930$ ft/s² ∠ 45° ◀

CORRESPONDING VECTOR DIAGRAM IS:



ROD BE: $\underline{a}_B = 4112$ ft/s² ↓ $\quad \omega_{BE} = 30.16$ rad/s ↻

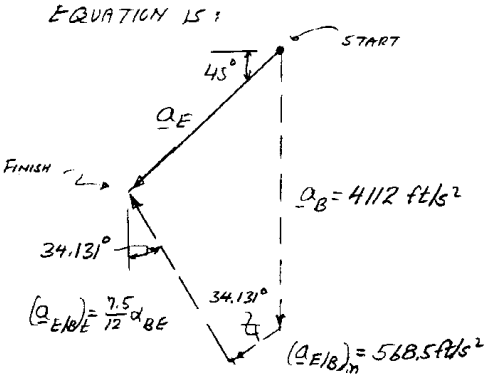


$\underline{a}_E = \underline{a}_B + \underline{a}_{E/B} = \underline{a}_B + (\underline{a}_{E/B})_t + (\underline{a}_{E/B})_n$

$(a_{E/B})_n = \left(\frac{7.5}{12}\text{ft}\right)(30.16\text{ rad/s})^2 = 568.5\text{ ft/s}^2 \angle 34.131°$

$[a_E \angle 45°] = [4112\text{ ft/s}^2 \downarrow] + \left[\frac{7.5}{12}\alpha_{BE} \angle 34.131°\right] + [568.5\text{ ft/s}^2 \angle 34.131°]$

THE VECTOR DIAGRAM CORRESPONDING TO THIS EQUATION IS:



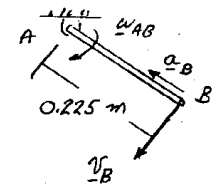
WE OBSERVE THAT THE SIDES AND ANGLES OF THIS DIAGRAM ARE THE SAME MAGNITUDE AS THE CORRESPONDING PARTS OF THE VECTOR DIAGRAM FOUND FOR ROD BD. THUS

$\underline{a}_E = 2930$ ft/s² ∠ 45° ◀

15.93

ROD AB: ANG. VELOCITIES ARE CONSTANT

$\omega_{AB} = 60 \text{ rpm} = 6.283 \text{ rad/s}$



$v_B = (AB)\omega_{AB} = (0.225\text{m})(6.283 \text{ rad/s})$

$\underline{v}_B = 1.4137 \text{ m/s} \downarrow$

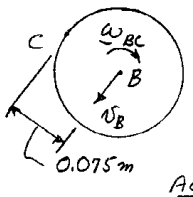
$a_B = (AB)\omega_{AB}^2 = (0.225\text{m})(6.283 \text{ rad/s})^2$

$\underline{a}_B = 8.883 \text{ m/s}^2 \nwarrow$

GEAR B:

VELOCITY:

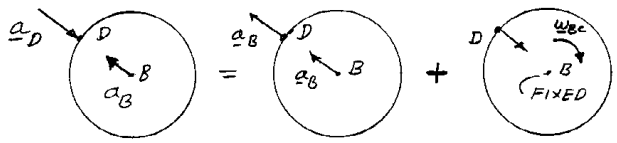
SINCE GEAR ROLLS ON FIXED GEAR A, $v_C = 0$ (C IS INST. CTR.)



$\omega_{BC} = \frac{v_B}{BC} = \frac{1.4137 \text{ m/s}}{0.075\text{m}} = 18.849 \text{ rad/s}$

ACCELERATION:

$a_{D/B} = (BC)\omega_{BC}^2$



$a_D \searrow = a_B \nwarrow + a_{D/B} \searrow = 8.883 \text{ m/s}^2 \nwarrow + (0.075\text{m})(18.849 \text{ rad/s})^2 \searrow$

$= 8.883 \text{ m/s}^2 \nwarrow + 26.646 \text{ m/s}^2 \searrow$

$\underline{a}_D = 17.764 \text{ m/s}^2 \swarrow$ $\quad \underline{a}_D = 17.76 \text{ m/s}^2 \swarrow$ ◀

15.94

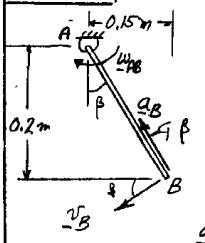
LINK AB

$\omega_{AB} = 4 \text{ rad/s}$ ↻

$\alpha_{AB} = 0$

$AB = 0.25\text{m}$

$\beta = 36.87$



$v_B = (0.25\text{m})(4 \text{ rad/s})$

$\underline{v}_B = 1 \text{ m/s} \swarrow 36.87°$

$a_B = (0.25\text{m})(4 \text{ rad/s})^2$

$\underline{a}_B = 4 \text{ m/s}^2 \swarrow 36.87°$

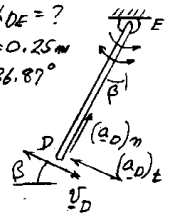
LINK DE

$\omega_{DE} = ?$

$\alpha_{DE} = ?$

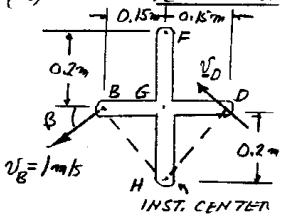
$DE = 0.25\text{m}$

$\beta = 36.87°$



CROSS

(a) VELOCITIES:



DRAW LINES ⊥ TO $\underline{v}_B$ AND $\underline{v}_D$ TO LOCATE INSTANTANEOUS CENTER WHICH IS AT H.

LET ω = ω OF CROSS

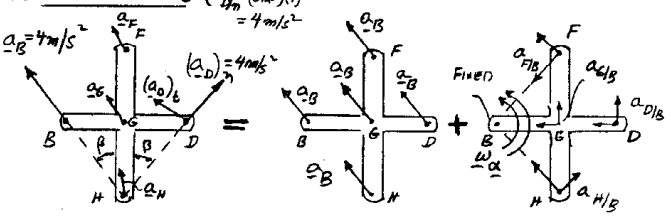
$BH = DH = 0.25\text{m}$

$\therefore v_B = v_D = 1 \text{ m/s}$

$\omega = \frac{v_B}{BH} = \frac{1\text{m/s}}{0.25\text{m}} = 4 \text{ rad/s}$ ↻ ◀

$\omega_{DE} = \frac{v_D}{DE} = \frac{1\text{m/s}}{0.25\text{m}} = 4 \text{ rad/s}$ ↻

(b) ACCELERATIONS: $(a_G)_n = (0.25)(4)^2 = 4 \text{ m/s}^2$



PLANE MOTION = TRANSLATION WITH B + ROTATION ABOUT B

$BG = 0.15\text{m}$; $BD = 0.3\text{m}$

$BF = BH = 0.25\text{m}$ $\quad \beta = 36.87°$

(CONTINUED)

15.94 CONTINUED

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$

$[(a_D)_t \angle \beta] + [4 \text{ m/s}^2 \swarrow \beta] = [4 \text{ m/s}^2 \swarrow \beta] + [(0.3\text{m})\alpha \uparrow] + [(0.3\text{m})(4 \text{ rad/s})^2 \leftarrow]$

$\pm \quad (a_D)_t \cos\beta - 4\sin\beta = 4\sin\beta + 4.8$ $\qquad \sin\beta = \sin 36.87 = 0.6$, $\cos\beta = \cos 36.87 = 0.8$

$(a_D)_t(0.8) - 4(0.6) = 4(0.6) + 4.8$

$(a_D)_t = 12 \text{ m/s}^2$ $\quad (\underline{a}_D)_t = 12 \text{ m/s}^2 \angle \beta$

$+\uparrow \quad (12)\sin\beta + 4\cos\beta = 4\cos\beta + 0.3\alpha$

$12(0.6) + 4(0.8) = 4(0.8) + 0.3\alpha$ $\qquad \alpha = 24 \text{ rad/s}^2$ ↻ ◀

(c) $\underline{a}_H = \underline{a}_B + \underline{a}_{H/B} = \underline{a}_B + (\underline{a}_{H/B})_t + (\underline{a}_{H/B})_n$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(BH)\alpha \angle \beta] + [(BH)\omega^2 \swarrow \beta]$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(0.25\text{m})\alpha \angle \beta] + [(0.25\text{m})(4 \text{ rad/s})^2 \swarrow \beta]$

$\pm \quad (a_H)_x = 4\sin\beta - (0.25)(24)\cos\beta + 4\sin\beta = 4(0.6) - 4.8 + 4(0.6) = 0$

$+\uparrow \quad (a_H)_y = 4\cos\beta + (0.25)(24)\sin\beta + 4\cos\beta = 3.2 + 3.6 + 3.2 = 10$

SEE VECTOR DIAGRAM BELOW. $\qquad \underline{a}_H = 10 \text{ m/s}^2 \uparrow$ ◀

15.95

SEE SOLUTION OF PROBS. 15.94.

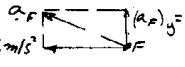
(a) $\underline{a}_F = \underline{a}_B + \underline{a}_{F/B} = \underline{a}_B + (\underline{a}_{F/B})_t + (\underline{a}_{F/B})_n$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(BF)\alpha \angle \beta] + [(BF)\omega^2 \swarrow \beta]$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(0.25\text{m})(24 \text{ rad/s}^2) \angle \beta] + [(0.25\text{m})(4 \text{ rad/s})^2 \swarrow \beta]$

$\pm \quad (a_F)_x = 4\sin\beta + (0.25)(24)\cos\beta + 4\sin\beta = 4(0.6) + 6(0.8) + 4(0.6) = 9.6$

$+\uparrow \quad (a_F)_y = 4\cos\beta + (0.25)(24)\sin\beta - 4\cos\beta = 4(0.8) + 6(0.6) - 4(0.8) = 3.6$



$\underline{a}_F = 10.25 \text{ m/s}^2 \angle 20.5°$ ◀

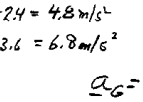
(b) $\underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + (\underline{a}_{G/B})_t + (\underline{a}_{G/B})_n$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(BG)\alpha \uparrow] + [(BG)\omega^2 \leftarrow]$

$= [4 \text{ m/s}^2 \swarrow \beta] + [(0.15\text{m})(24 \text{ rad/s}^2) \uparrow] + [(0.15\text{m})(4 \text{ rad/s})^2 \leftarrow]$

$\leftarrow \quad (a_G)_x = 4\sin\beta + 2.4 = 4(0.6) + 2.4 = 4.8 \text{ m/s}^2$

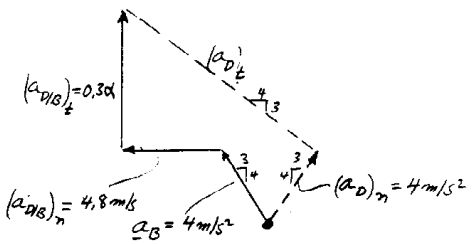
$+\uparrow \quad (a_G)_y = 4\cos\beta + 3.6 = 4(0.8) + 3.6 = 6.8 \text{ m/s}^2$



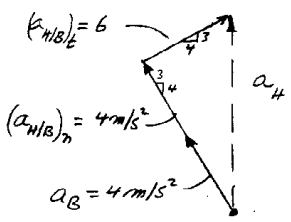
$\underline{a}_G = 8.32 \text{ m/s}^2 \angle 54.8°$ ◀

VECTOR DIAGRAMS FOR PROB 15.94

PART (b)



PART (c)

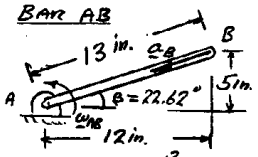


15.96

FROM PROB 15.52: $\alpha_{AB} = 0$

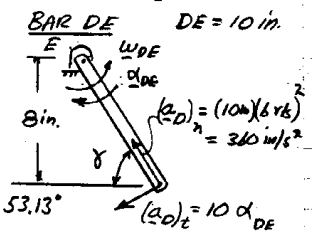
$\omega_{AB} = 3$ rad/s ↻ ; $\omega_{BD} = 4.85$ rad/s ↻ ; $\omega_{DE} = 6$ rad/s ↺

BAR AB



$a_B = (13\text{ in})(3\text{ rad/s})^2 = 117\text{ in./s}^2$

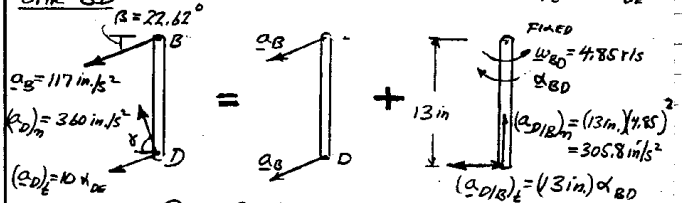
BAR DE DE = 10 in.



$(a_D)_n = (10\text{in})(6\text{ r/s})^2 = 360\text{ in/s}^2$

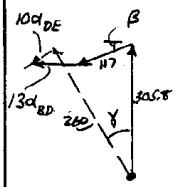
$(a_D)_t = 10\,\alpha_{DE}$

BAR BD



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$

$[360\text{ in/s}^2 \angle 53.13^\circ] + [10\alpha_{DE} \angle 53.13^\circ] = [117\text{ in/s}^2 \angle 22.62^\circ] + [305.8\text{ in/s}^2 \uparrow] + [13\alpha_{BD} \leftarrow]$



+↑ $360\sin 53.13 - 10\alpha_{DE}\cos 53.13^\circ = -117\sin 22.62^\circ + 305.8$

$288 - 6\alpha_{DE} = -45 + 305.8$

$\alpha_{DE} = +4.533$ rad/s² $\quad \underline{\alpha}_{DE} = 4.53$ rad/s² ↺

+→ $360\cos 53.13^\circ + (10)(4.533)\sin 53.13^\circ = 117\cos 22.62^\circ + 13\alpha_{BD}$

$216 + 32.26 = 108 + 13\alpha_{BD}$

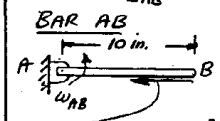
$\alpha_{BD} = 11.097$ rad/s² $\quad \alpha_{BD} = 11.10$ rad/s² ↻

15.97

FROM PROB 15.51: $\alpha_{AB} = 0$

$\omega_{AB} = 3$ rad/s ↻ ; $\omega_{BD} = 1.5$ rad/s ↻ ; $\omega_{DE} = 3.75$ rad/s ↻

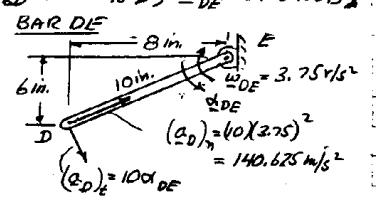
BAR AB



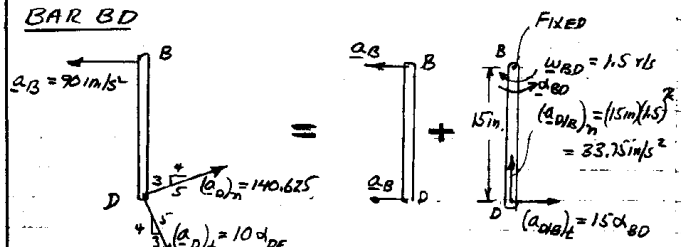
$a_B = (10\text{in.})(3\text{ rad/s})^2$

$a_B = 90\text{ in./s}^2 \leftarrow$

BAR DE

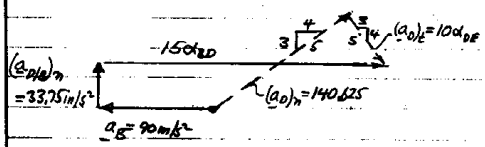


BAR BD



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$

$[140.625 \angle 53.13^\circ] + [10\alpha_{DE} \angle 53.13^\circ] = [90 \leftarrow] + [33.75 \uparrow] + [15\alpha_{BD} \rightarrow]$



+↑ $\frac{3}{5}(140.625) - \frac{4}{5}(10\alpha_{DE}) = 33.75$

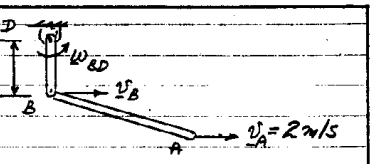
$84.375 - 8\alpha_{DE} = 33.75$; $\quad \alpha_{DE} = 6.328$ r/s² $\quad \underline{\alpha}_{DE} = 6.33$ rad/s² ↺

+→ $\frac{4}{5}(140.625) + \frac{3}{5}(10)(6.328) = -90 + 15\alpha_{BD}$

$112.5 + 37.97 = -90 + 15\alpha_{BD}$; $\quad \alpha_{BD} = 16.03$ r/s² $\quad \underline{\alpha}_{BD} = 16.03$ rad/s² ↻

15.98

VELOCITIES



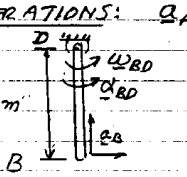
SINCE $\underline{v}_A$ & $\underline{v}_B$ ARE BOTH HORIZONTAL, INST. CTR. OF BD IS AT ∞.

$\omega_{AB} = 0$; $\underline{v}_B = \underline{v}_A = 2$ m/s →

$\omega_{BD} = \frac{v_B}{BD} = \frac{2\text{ m/s}}{1.25\text{ m}} = 1.6$ rad/s ↻

ACCELERATIONS: $\underline{a}_A = 0$ (GIVEN)

ROD BD

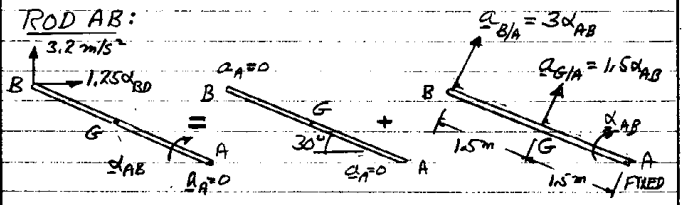


$a_B = [(BD)\omega_{BD}^2 \uparrow] + [(BD)\alpha_{BD} \rightarrow]$

$= [(1.25)(1.6)^2 \uparrow] + [(1.25)\alpha_{BD} \rightarrow]$

$\underline{a}_B = 3.2\text{ m/s}^2 \uparrow + 1.25\,\alpha_{BD} \rightarrow$

ROD AB:



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$[3.2\text{ m/s}^2 \uparrow] + [1.25\alpha_{BD} \rightarrow] = 0 + [3\alpha_{AB} \angle 60^\circ]$

+↑ y COMPS. $3.2 = 3\alpha_{AB}\sin 60^\circ$ $\quad \alpha_{AB} = 1.2317$ rad/s² ↻

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 0 + (1.5\text{m})(1.232\text{ rad/s}) = 1.8475\text{ m/s}^2 \angle 60^\circ$

$\underline{a}_G = 1.848\text{ m/s}^2 \angle 60^\circ$

15.99

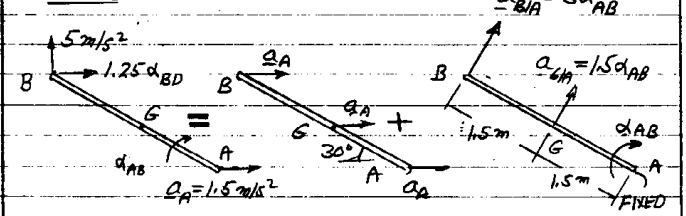
SEE SOLUTION OF PROB 15.98 FOR $\omega_{AB} = 0$

$v_B = v_A = 2.5$ m/s → $\quad \omega_{BD} = v_B/1.25 = 2.5\text{m/s}/1.25\text{m} = 2$ rad/s

ROD BD $\quad (a_B)_n = (BD)\omega_{BD}^2 = (1.25\text{m})(2\text{ rad/s})^2 = 5\text{ m/s}^2 \uparrow$

$(a_B)_t = (BD)\alpha_{BD} = (1.25\text{m})\alpha_{BD} \rightarrow$

ROD AB



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$[5\text{ m/s}^2 \uparrow] + [1.25\alpha_{AB} \rightarrow] = [1.5\text{ m/s}^2 \rightarrow] + [3\alpha_{AB} \angle 60^\circ]$

+y COMPS: $5 = 3\alpha_{AB}\sin 60^\circ$

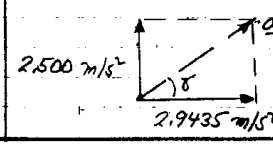
$\alpha_{AB} = 1.925$ rad/s² $\quad \underline{\alpha}_{AB} = 1.925$ rad/s² ↻

ACCELERATION OF G:

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = [1.5\text{ m/s}^2 \rightarrow] + [1.5(1.925) \angle 60^\circ]$

$= [1.5 \rightarrow] + [2.887 \angle 60^\circ]$

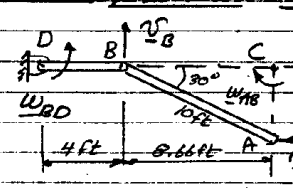
$= [1.5 \rightarrow] + [1.4435 \rightarrow] + [2.500 \uparrow] = [2.9435 \rightarrow] + [2.500 \uparrow]$



$\underline{a}_G = 3.86\text{ m/s}^2 \angle 40.3^\circ$

15.100

VELOCITIES ROD AB INST. CENTER OF AB AT C



$\omega_{AB} = \frac{v_A}{CA} = \frac{2.5\ ft/s}{5\ ft}$

$\omega_{AB} = 0.5\ rad/s$ ↻

$v_B = (BC)\omega_{AB}$

$= (8.66)(0.5)$

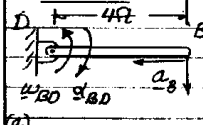
$v_B = 4.33\ ft/s$

ROD BD

$\omega_{BD} = \frac{v_B}{BD} = \frac{4.33\ ft/s}{4} = 1.0825\ rad/s$ ↺

ACCELERATIONS.

ROD BD

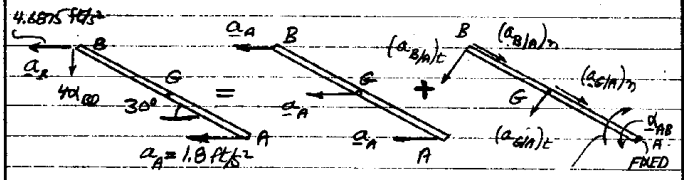


$a_B = [(BD)\omega_{BD}^2 \leftarrow] + [(BD)\alpha_{BD} \downarrow]$

$= [(4ft)(1.0825)^2 \leftarrow] + [(4ft)\alpha_{BD} \downarrow]$

$\underline{a}_B = [4.6875\ ft/s^2 \leftarrow] + [4\alpha_{BD} \downarrow]$

(a) ROD AB $a_A = 1.8\ ft/s^2 \leftarrow$ (GIVEN)



$\omega_{AB} = 0.5\ rad/s$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A + (\underline{a}_{B/A})_t + (\underline{a}_{B/A})_n$ (1)

$[4.6875\ ft/s^2 \leftarrow] + [4\alpha_{BD} \downarrow] = [1.8\ ft/s^2 \leftarrow] + [(AB)\alpha_{AB} \perp 30°] + [(AB)\omega_{AB}^2 \angle 30°]$

$[4.6875 \leftarrow] + [4\alpha_{BD} \downarrow] = [1.8 \leftarrow] + [(10ft)\alpha_{AB} \perp 30°]$

$+ [(10ft)(0.5\ rad/s)^2 \angle 30°]$

$[4.6875 \leftarrow] + [4\alpha_{BD} \downarrow] = [1.8 \leftarrow] + [10\alpha_{AB} \perp 30°] + [2.5 \angle 30°]$ (2)

$\pm$ x COMPONENTS: $4.6875 = 1.8 + 10\alpha_{AB}\sin 30° - 2.5\cos 30°$

$\alpha_{AB} = +1.0105\ rad/s^2$ $\quad \underline{\alpha}_{AB} = 1.011\ rad/s^2$ ↻ ◀

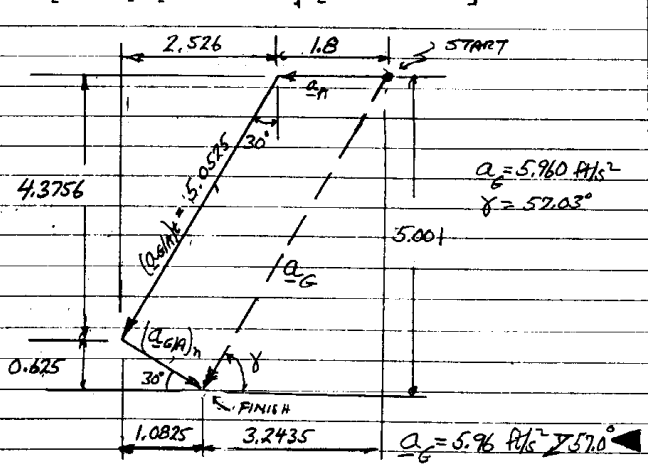
(b) ACCELERATION OF G:

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = a_A + (a_{G/A})_t + (a_{G/A})_n$

$\underline{a}_G = [1.8\ ft/s^2 \leftarrow] + [(GA)\alpha_{AB} \perp 30°] + [(GA)\omega_{AB}^2 \angle 30°]$

$= [1.8 \leftarrow] + [(5ft)(1.0105\ rad/s) \perp 30°] + [(5ft)(0.5\ rad/s)^2 \angle 30°]$

$= [1.8 \leftarrow] + [5.0525 \perp 30°] + [1.25 \angle 30°]$



$a_G = 5.960\ ft/s^2$

$\gamma = 57.03°$

$\underline{a}_G = 5.96\ ft/s^2 \angle 57.0°$ ◀

15.101

SEE SOLUTION OF PROB 15.100.

WE STILL HAVE THE SAME VELOCITIES

$\underline{\omega}_{AB} = 0.5\ rad/s$ ↻

(a) WE NOW HAVE $\underline{a}_A = 0$, THUS EQUATION (1)

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_t + (\underline{a}_{B/A})_n$ (1)

YIELDS:

$[4.6875 \leftarrow] + [4\alpha_{BD} \downarrow] = 0 + [10\alpha_{AB} \perp 30°] + [2.5 \angle 30°]$ (2)

$\pm$ x COMPONENTS: $+4.6875 = 10\alpha_{AB}\sin 30° - 2.5\cos 30°$

$\alpha_{AB} = +1.3705\ rad/s^2$ $\quad \underline{\alpha}_{AB} = 1.371\ rad/s^2$ ↻ ◀

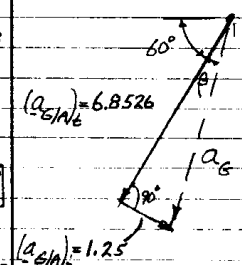
(b) ACCELERATION OF G: (SEE SKETCH OF PROB. 15.100)

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = \underline{a}_A + (\underline{a}_{G/A})_t + (\underline{a}_{G/A})_n$

$\underline{a}_G = 0 + [(GA)\alpha_{AB} \perp 30°] + [(GA)\omega_{AB}^2 \angle 30°]$

$= + [(5ft)(1.3705\ rad/s^2) \perp 30°] + [(5ft)(0.5\ rad/s)^2 \angle 30°]$

$= + [6.8526\ ft/s^2 \perp 30°] + [1.25\ ft/s^2 \angle 30°]$



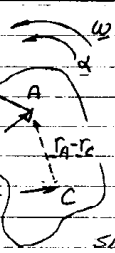
$\tan\beta = \frac{1.25}{6.8526}$

$\beta = 10.34°$

$a_G = \frac{6.8526}{\cos\beta} = 6.9657\ ft/s^2$

$\underline{a}_G = 6.97\ ft/s^2 \angle 70.3°$ ◀

15.102



(a) LET C = INST. CENTER OF SLAB.

THEN: $\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}$

$= \underline{v}_C + (\underline{\omega}) \times (\underline{r}_A - \underline{r}_C)$

BUT $v_C = 0$: $\underline{v}_A = \underline{\omega} \times (\underline{r}_A - \underline{r}_C)$

CROSS MULTIPLY EACH MEMBER BY $\underline{\omega}$

$\underline{\omega} \times \underline{v}_A = \underline{\omega} \times [\underline{\omega} \times (\underline{r}_A - \underline{r}_C)]$

SINCE $\underline{\omega}$ IS $\perp$ TO PLANE CONTAINING $(\underline{r}_A - \underline{r}_C)$, CROSS MULTIPLYING $(\underline{r}_A - \underline{r}_C)$ TWICE BY $\underline{\omega}$ IS EQUIVALENT TO MULTIPLYING $(\underline{r}_A - \underline{r}_C)$ BY ω^2 AND ROTATING IT THROUGH 180°. THUS,

$\underline{\omega} \times \underline{v}_A = -\omega^2(\underline{r}_A - \underline{r}_C)$

SOLVING FOR $\underline{r}_C$: $\underline{r}_C = \underline{r}_A + \frac{\underline{\omega} \times \underline{v}_A}{\omega^2}$ (Q.E.D.)

(b) SINCE WE WANT $a_C = 0$, WE SHALL WRITE

$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = 0$ (1)

USING EQ 15.11, page 707: $\underline{a}_{C/A} = \underline{\alpha} \times \underline{r}_{C/A} - \omega^2 \underline{r}_{C/A}$ (2)

FROM Part a: $\underline{r}_{C/A} = \underline{r}_C - \underline{r}_A = \frac{\underline{\omega} \times \underline{v}_A}{\omega^2}$

EQ(2): $\underline{a}_{C/A} = \underline{\alpha} \times \frac{\underline{\omega} \times \underline{v}_A}{\omega^2} - (\underline{\omega} \times \underline{v}_A)$

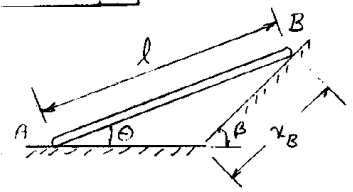
BUT $\underline{\alpha} = \alpha \underline{k}$ AND $\underline{\omega} = \omega \underline{k}$, AND SINCE $\underline{k} \perp \underline{v}_A$

$\underline{a}_{C/A} = \frac{\alpha}{\omega}\left[\underline{k} \times (\underline{k} \times \underline{v}_A)\right] - \underline{\omega} \times \underline{v}_A = -\frac{\alpha}{\omega}\underline{v}_A - \underline{\omega} \times \underline{v}_A$

SUBSTITUTE INTO (1) AND SOLVE FOR $\underline{a}_A$

$\underline{a}_A = \frac{\alpha}{\omega}\underline{v}_A + \underline{\omega} \times \underline{v}_A$ (Q.E.D)

*15.103



LAW OF SINES

$$\frac{x_B}{\sin\theta} = \frac{l}{\sin\beta}$$

$$x_B = \frac{l}{\sin\beta}\sin\theta$$

$$v_B = \frac{dx_B}{dt} = \frac{l}{\sin\beta}\cos\theta\frac{d\theta}{dt} = \frac{l}{\sin\beta}\cos\theta\,\omega$$

$$\omega = \frac{v_B}{l}\frac{\sin\beta}{\cos\theta}\curvearrowright$$

*15.104

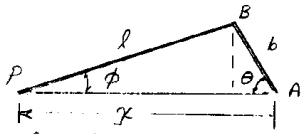
WE DIFFERENTIATE THE EXPRESSION FOR ω FOUND IN PROB. 15.103. (NOTE THAT $dv_B/dt = 0$)

$$\alpha = \frac{d\omega}{dt} = \frac{v_B\sin\beta}{l}\cdot\frac{\sin\theta}{\cos^2\theta}\cdot\frac{d\theta}{dt}$$

$$\alpha = \frac{v_B\sin\beta\sin\theta}{l\cos^2\theta}\cdot\frac{v_B\sin\beta}{l\cos\theta}$$

$$\alpha = \left[\frac{v_B\sin\beta}{l}\right]^2\frac{\sin\theta}{\cos^3\theta}\curvearrowright$$

*15.105



$$x = b\cos\theta + l\cos\phi \qquad \dot\theta = \omega$$

$$v_P = \frac{dx}{dt} = -b\sin\theta\,\dot\theta - l\sin\phi\,\dot\phi \qquad (1)$$

LAW OF SINES: $\frac{\sin\phi}{b} = \frac{\sin\theta}{l}$ $\qquad \sin\phi = \frac{b}{l}\sin\theta$

DIFFERENTIATING: $\cos\phi\,\dot\phi = \frac{b}{l}\cos\theta\,\dot\theta$

OR: $\dot\phi = \frac{b\dot\theta}{l}\frac{\cos\theta}{\cos\phi} = \frac{b\dot\theta}{l}\frac{\cos\theta}{\sqrt{1-\sin^2\phi}} = \frac{b\dot\theta}{l}\frac{\cos\theta}{\sqrt{1-\frac{b^2}{l^2}\sin^2\theta}}$

SUBSTITUTING FOR $\sin\phi$ AND $\dot\phi$ INTO (1):

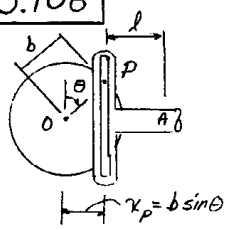
$$v_P = -b\omega\sin\theta\left[1+\frac{b\cos\theta}{l\sqrt{1-\frac{b^2}{l^2}\sin^2\theta}}\right]$$

OR, SINCE $\theta = \omega t$

$$v_P = -b\omega\sin\omega t\left[1+\frac{b\cos\omega t}{l\sqrt{1-\frac{b^2}{l^2}\sin^2\omega t}}\right]$$

$$\underline{v_P = b\omega\sin\omega t\left[1+\frac{b\cos\omega t}{l\sqrt{1-\frac{b^2}{l^2}\sin^2\omega t}}\right]}\rightarrow$$

*15.106



(a) $x_A = l + x_P = l + b\sin\theta$

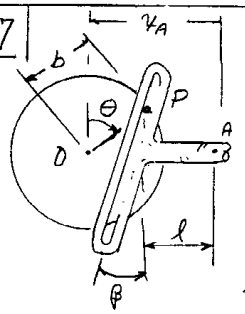
$$v_A = \dot x_A = b\cos\theta\,\dot\theta$$

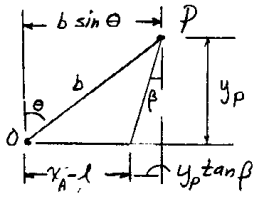
$$\underline{v_A = b\omega\cos\theta}$$

(b) $a_A = \ddot x_A = -b\sin\theta\,\dot\theta^2 + b\cos\theta\,\ddot\theta$

$$\underline{a_A = b\alpha\cos\theta - b\omega^2\sin\theta}$$

*15.107





$$y_P = b\cos\theta$$

$$x_A - l = b\sin\theta - y_P\tan\beta$$

$$= b\sin\theta - (b\cos\theta)\tan\beta$$

$$= b(\sin\theta - \cos\theta\tan\beta)$$

(a) $x_A = l + b(\sin\theta - \cos\theta\tan\beta)$

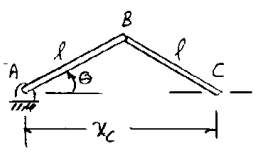
$$v_A = \dot x_A = b(\cos\theta + \sin\theta\tan\beta)\dot\theta$$

$$\underline{v_A = b\omega(\cos\theta + \sin\theta\tan\beta)}$$

(b) $a_A = \ddot x_A = b(-\sin\theta + \cos\theta\tan\beta)\dot\theta^2 + b(\cos\theta + \sin\theta\tan\beta)\ddot\theta$

$$\underline{a_A = b\left[(\cos\theta + \sin\theta\tan\beta)\alpha + (-\sin\theta + \cos\theta\tan\beta)\omega^2\right]}$$

*15.108



$$x_C = 2l\cos\theta$$

$$v_C = \frac{dx_C}{dt} = -2l\sin\theta\,\dot\theta$$

$$\underline{v_C = -2l\omega\sin\theta}$$

$$a_C = \frac{dv_C}{dt} = -2l\sin\theta\,\ddot\theta - 2l\cos\theta\,\dot\theta^2$$

$$\underline{a_C = -2l\alpha\sin\theta - 2l\omega^2\cos\theta}$$

*15.109

WE HAVE $v_D = v_C =$ CONSTANT AND $a_C = 0$

FROM PROB 15.108:

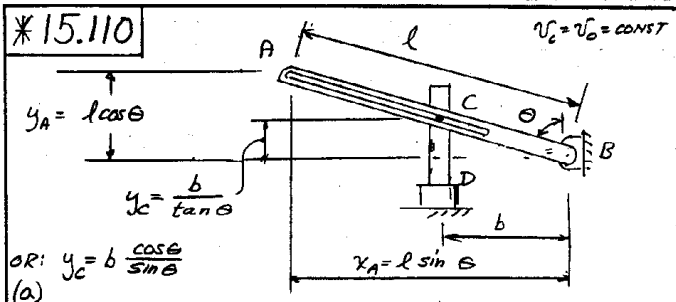
$$v_C = v_D = -2l\omega\sin\theta \qquad \omega = -\frac{v_D}{2l\sin\theta} \qquad \underline{\omega = \frac{v_D}{2l\sin\theta}}$$

AGAIN FROM PROB 15.108:

$$a_C = 0 = -2l\alpha\sin\theta - 2l\left[-\frac{v_D}{2l\sin\theta}\right]^2\cos\theta$$

SOLVE FOR α: $\alpha = -\left(\frac{v_D}{2l}\right)^2\frac{\cos\theta}{\sin^3\theta}$ $\qquad \underline{\alpha = \left(\frac{v_D}{2l}\right)^2\frac{\cos\theta}{\sin^3\theta}}$

*15.110



OR: $y_C = b\frac{\cos\theta}{\sin\theta}$

(a)

$$v_C = v_0 = \frac{dy_C}{dt} = b\frac{d}{dt}\left(\frac{\cos\theta}{\sin\theta}\right) = b\frac{-\sin^2\theta-\cos^2\theta}{\sin^2\theta}\dot\theta = -\frac{b\dot\theta}{\sin^2\theta}$$

BUT $\dot\theta = \omega$

$$v_0 = -\frac{b\omega}{\sin^2\theta} \qquad \omega = -\frac{v_0}{b}\sin^2\theta$$

$$\underline{\omega = \frac{v_0}{b}\sin^2\theta\ \circlearrowright}$$

(b) $(v_A)_x = \dot x_A = \ell\cos\theta\,\dot\theta = \ell\omega\cos\theta$

$$(v_A)_x = -\frac{v_0\ell}{b}\sin^2\theta\cos\theta \qquad (\underline{v_A})_x = \frac{v_0\ell}{b}\sin^2\theta\cos\theta \rightarrow$$

$(v_A)_y = \dot y_A = -\ell\sin\theta\,\dot\theta = -\ell\omega\sin\theta$

$$(v_A)_y = +\frac{v_0\ell}{b}\sin^3\theta \qquad (\underline{v_A})_y = \frac{v_0\ell}{b}\sin^3\theta\ \uparrow$$

ALTERNATE: $\underline{v_A} = \ell\omega\ \angle\theta$

$(v_A)_x = \ell\omega\cos\theta \rightarrow$ $\quad (v_A)_y = \ell\omega\sin\theta\ \uparrow$

SUBSTITUTE FOR ω AND OBTAIN SAME RESULTS.

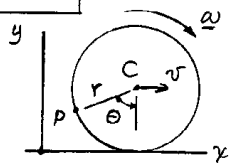
*15.111

$$\omega = -\frac{v_0}{b}\sin^2\theta$$

$$\alpha = \frac{d\omega}{dt} = -2\frac{v_0}{b}\sin\theta\cos\theta\,\dot\theta = -2\frac{v_0}{b}\omega\sin\theta\cos\theta$$

$$\alpha = +2\frac{v_0}{b}\left(\frac{v_0}{b}\sin^2\theta\right)\sin\theta\cos\theta \qquad \underline{\alpha = 2\left(\frac{v_0}{b}\right)^2\sin^3\theta\cos\theta\ \circlearrowleft}$$

*15.112



$$x_P = x_C - r\sin\theta$$
$$y_P = y_C - r\cos\theta$$

SINCE DISK ROLLS →

$x_C = vt$

$y_C = r$

$\omega = v/r$

$$\theta = \omega t = \frac{v}{r}t$$

THUS: $\quad x_P = vt - r\sin\left(\frac{vt}{r}\right)$

$$y_P = r - r\cos\left(\frac{vt}{r}\right)$$

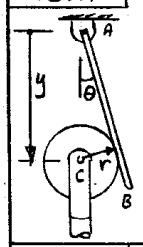
$$v_x = \frac{dx_P}{dt} = v - \frac{v}{r}r\cos\left(\frac{vt}{r}\right)$$

$$\underline{v_x = v\left[1-\cos\left(\frac{vt}{r}\right)\right]}$$

$$v_y = \frac{dy_P}{dt} = \frac{v}{r}r\sin\left(\frac{vt}{r}\right)$$

$$\underline{v_y = v\sin\left(\frac{vt}{r}\right)}$$

*15.113



$$y = \frac{r}{\sin\theta} \qquad \frac{dy}{dt} = -r\frac{\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

BUT: $\frac{dy}{dt} = -v_0$; $\frac{d\theta}{dt} = \omega$;

$$-v_0 = -r\frac{\cos\theta}{\sin^2\theta}\omega \qquad \omega = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta} \quad (1)$$

$$\underline{\omega = \frac{v_0}{r}\sin\theta\tan\theta}$$

*15.114

FROM EQ(1) OF PROB 15.113 AND NOTE v_0 = CONSTANT

$$\alpha = \frac{d\omega}{d\theta} = \frac{v_0}{r}\frac{\cos\theta(2\sin\theta\cos\theta) - \sin^2\theta(-\sin\theta)}{\cos^2\theta}\frac{d\theta}{dt}$$

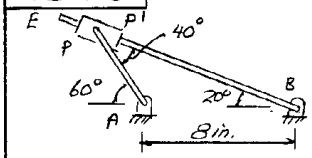
$$\alpha = \frac{v_0}{r}\frac{\sin\theta\left[2\cos^2\theta+\sin^2\theta\right]}{\cos^2\theta}\cdot\frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{v_0^2}{r^2}\frac{\sin\theta\left[1+\cos^2\theta\right]\sin^2\theta}{\cos^3\theta} = \frac{v_0^2}{r^2}\left(\frac{\sin\theta}{\cos\theta}\right)^3(1+\cos^2\theta)$$

$$\underline{\alpha = \frac{v_0^2}{r^2}\tan^3\theta\,(1+\cos^2\theta)}$$

15.115

DIMENSIONS: LAW OF SINES



$$\frac{AP}{\sin 20^\circ} = \frac{BP}{\sin 120^\circ} = \frac{8\text{ in.}}{\sin 40^\circ}$$

$AP = 4.2567$ in.

$BP = 10.7784$ in.

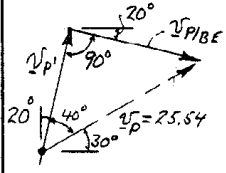
VELOCITIES $\qquad \omega_{AP} = 6$ rad/s ↻

NOTE: P' = POINT OF BE COINCIDING WITH P.

$v_P = (AP)\omega_{AP} = (4.2567\text{ in})(6\text{ rad/s}) = 25.540$ in./s $\angle 30^\circ$

$$\underline{v_P} = \underline{v_{P'}} + \underline{v_{P/BE}}$$

$$\left[25.540\ \angle 30^\circ\right] = \left[v_{P'}\ \angle 70^\circ\right] + \left[v_{P/BE}\ \angle 20^\circ\right]$$



(a) $v_{P'} = (25.54)\cos 40^\circ = 19.565$ in/s

$$\omega_{BE} = \frac{v_{P'}}{BP} = \frac{19.565\text{ in/s}}{10.7784\text{ in}} = 1.8152\text{ rad/s}$$

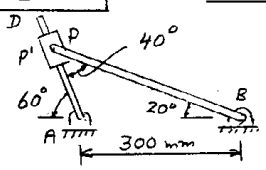
$\underline{\omega_{BE} = 1.815\text{ rad/s}\ \circlearrowright}$

(b) $v_{P/BE} = (25.54)\sin 40^\circ = 16.417$ in./s

$\underline{v_{P/BE} = 16.42\text{ in./s}\ \angle 20^\circ}$

15.116

DIMENSIONS: LAW OF SINES



$$\frac{AP}{\sin 20^\circ} = \frac{BP}{\sin 120^\circ} = \frac{300\text{ mm}}{\sin 40^\circ}$$

$AP = 159.63$ mm

$BP = 404.19$ mm

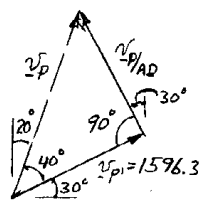
$\underline{\omega_{AD} = 10\text{ rad/s}\ \circlearrowright}$

VELOCITIES: NOTE: P' = POINT OF AD COINCIDING WITH P.

$\underline{v_{P'}} = (AP')\omega_{AD} = (159.63\text{ mm})(10\text{ rad/s}) = 1596.3$ mm/s $\angle 30^\circ$

$$\underline{v_P} = \underline{v_{P'}} + \underline{v_{P/AD}}$$

$$\left[v_P\ \angle 70^\circ\right] = \left[1596.3\ \angle 30^\circ\right] + \left[v_{P/AD}\ \angle 60^\circ\right]$$



(a) $v_P = (1596.3)/\cos 40^\circ = 2083.8$ mm/s

$$\omega_{BP} = \frac{v_P}{BP} = \frac{2083.8\text{ mm/s}}{404.19\text{ mm}} = 5.155\text{ rad/s}$$

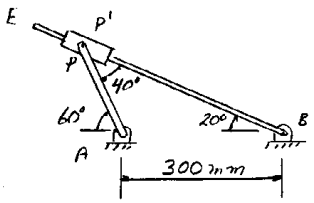
$\underline{\omega_{BD} = 5.16\text{ rad/s}\ \circlearrowright}$

(b) $v_{P/AD} = (1596.3)\tan 40^\circ = 1339.5$ mm/s

$\underline{v_{P/AD} = 1.339\text{ m/s}\ \angle 60^\circ}$

15.117

DIMENSIONS: LAW OF SINES

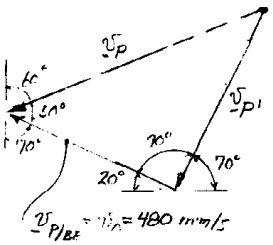


$$\frac{AP}{\sin 20^\circ} = \frac{BP}{\sin 60^\circ} = \frac{300\text{ mm}}{\sin 40^\circ}$$

$AP = 159.63$ mm

$BP = 404.19$ mm

VELOCITIES: NOTE: P' = POINT OF BE COINCIDING WITH P.



$$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/BE}$$

$$[v_P \angle 30^\circ] = [v_{P'} \angle 70^\circ] + [480\text{ mm/s} \angle 20^\circ]$$

$$v_P = \frac{v_{P/BE}}{\cos 50^\circ} = \frac{480}{\cos 60^\circ} = 746.7\text{ mm/s}$$

$$\omega_{AP} = \frac{v_P}{AP} = \frac{746.7\text{ mm/s}}{159.63\text{ mm}} = 4.678\text{ rad/s}$$

$\underline{\omega}_{AP} = 4.68$ rad/s ↻ ◀

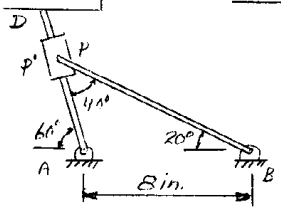
$$v_{P'} = v_{BE} \tan 50^\circ = (480)\tan 50^\circ = 572.04\text{ mm/s}$$

$$\omega_{BE} = \frac{v_{P'}}{BP} = \frac{572.04\text{ mm/s}}{404.19\text{ mm}} = 1.4153\text{ rad/s}$$

$\underline{\omega}_{BE} = 1.415$ rad/s ↻ ◀

15.118

DIMENSIONS LAW OF SINES



$$\frac{AP}{\sin 20^\circ} = \frac{BP}{\sin 60^\circ} = \frac{8\text{ in.}}{\sin 40^\circ}$$

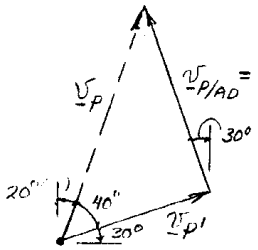
$AP = 4.2567$ in.

$BP = 10.7726$ in.

VELOCITIES NOTE: P' = POINT OF AD COINCIDING WITH P

$$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/AD}$$

$$[v_P \angle 70^\circ] = [v_{P'} \angle 30^\circ] + [9\text{ in./s} \angle 60^\circ]$$



$$v_{P'} = \frac{v_{P/AD}}{\tan 40^\circ} = \frac{9\text{ in./s}}{\tan 40^\circ} = 10.726\text{ in./s}$$

$$\omega_{AD} = \frac{v_{P'}}{AP} = \frac{10.726\text{ in./s}}{4.2567\text{ in.}} = 2.5197\text{ rad/s}$$

$\underline{\omega}_{AD} = 2.52$ rad/s ↻ ◀

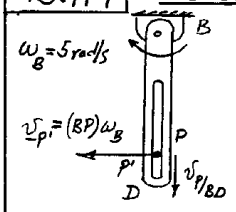
$$v_P = \frac{v_{P/AD}}{\sin 40^\circ} = \frac{9\text{ in./s}}{\sin 40^\circ} = 14.002\text{ in./s}$$

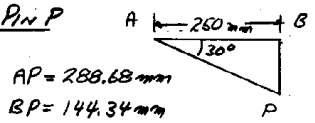
$$\omega_{BP} = \frac{v_P}{BP} = \frac{14.002\text{ in./s}}{10.7726\text{ in.}} = 1.2990\text{ rad/s}$$

$\underline{\omega}_{BP} = 1.299$ rad/s ↻ ◀

15.119

ROD BD AND PIN P





$AP = 288.68$ mm

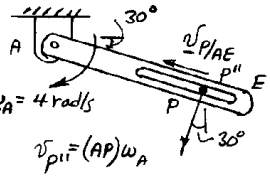
$BP = 144.34$ mm

$$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/BD}$$

$$v_P = [(144.34\text{ mm})(5\text{ rad/s}) \leftarrow] + [v_{P/BD} \downarrow]$$

$$v_P = [721.69\text{ mm/s} \leftarrow] + [v_{P/BD} \downarrow] \quad (1)$$

ROD AE AND PIN P



$$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/AE}$$

$$v_P = [(288.68\text{ mm})(4\text{ rad/s}) \angle 30^\circ] + [v_{P/AE} \angle 30^\circ]$$

$$v_P = [1154.7\text{ mm/s} \nwarrow 30^\circ] + [v_{P/AE} \searrow 30^\circ] \quad (2)$$

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2)

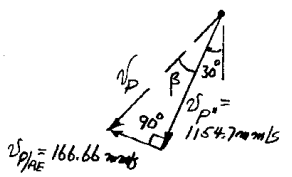
$$[721.69\text{ mm/s} \leftarrow] + [v_{P/BD} \downarrow] = [1154.7\text{ mm/s} \nwarrow 30^\circ] + [v_{P/AE} \searrow 30^\circ]$$

$\pm$ X COMPONENTS: $721.69 = 1154.7 \sin 30^\circ + v_{P/AE} \cos 30^\circ$

$v_{P/AE} = 166.66\text{ mm/s} \swarrow 30^\circ$

VECTOR DIAGRAM

$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/AE}$



$v_P = 1166.7$

$\beta = 8.21°$

$\underline{v}_P = 1167\text{ mm/s} \swarrow 51.8°$ ◀

FOR USE IN PROB 15.139

EQ(1) $+\downarrow \underline{v}_{P/BD} = (\underline{v}_P)_y = 916.7\text{ mm/s} \downarrow$

15.120

ROD BD AND PIN P THIS IS THE SAME AS IN PROB 15.119 EXCEPT ω_B IS REVERSED

$\underline{\omega}_B = 5$ rad/s ↻ EQ(1) BECOMES

$$\underline{v}_P = [721.69\text{ mm/s} \rightarrow] + [v_{P/BD} \downarrow] \quad (1)$$

ROD AE AND PIN P THIS IS THE SAME AS IN PROB 15.119.

EQ(2) IS:

$$v_P = [1154.7\text{ mm/s} \nwarrow 30^\circ] + [v_{P/AE} \searrow 30^\circ] \quad (2)$$

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2).

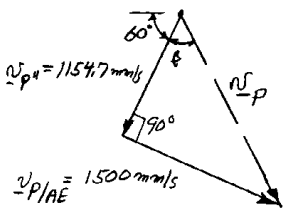
$$[721.69\text{ mm/s} \rightarrow] + [v_{P/BD} \downarrow] = [1154.7\text{ mm/s} \nwarrow 30^\circ] + [v_{P/AE} \searrow 30^\circ]$$

$\pm$ X COMPONENTS: $721.69 = -1154.7 \sin 30 - v_{P/AE} \cos 30^\circ$

$v_{P/AE} = -1500$ mm/s $\quad \underline{v}_{P/AE} = 1500\text{ mm/s} \searrow 30^\circ$

VECTOR DIAGRAM

$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/AE}$



$v_P = 1892.97$ mm/s

$\beta = 52.41°$

$\underline{v}_P = 1893\text{ mm/s} \swarrow 67.6°$ ◀

FOR USE IN PROB 15.140:

EQ(1)

$+\downarrow \underline{v}_{P/BD} = (\underline{v}_P)_y = 1750\text{ mm/s} \downarrow$

15.121

ANGULAR VELOCITY = $\underline{\omega}$ ↻

WE HAVE: $\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_C$

ACCELERATION OF COINCIDING POINT P':

FOR EACH PIN: $a_{P'} = r\omega^2$ TOWARD O

ACCELERATION OF PIN WITH RESPECT TO PLATE.

FOR P_1, P_2, P_4: $a_{P/\mathcal{F}} = 0$

FOR P_3: $a_{P/\mathcal{F}} = \frac{u^2}{r}$ TOWARD O

CORIOLIS ACCELERATION FOR EACH PIN $a_C = 2u\omega$, WITH $\underline{a}_C$ IN A DIRECTION OBTAINED FROM THAT OF $\underline{u}$ BY ROTATING 90° IN THE SENSE OF $\underline{\omega}$.

$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \downarrow]$ $\quad \underline{a}_1 = r\omega^2 \underline{i} - 2u\omega \underline{j}$ ◀

$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \rightarrow]$ $\quad \underline{a}_2 = 2u\omega \underline{i} - r\omega^2 \underline{j}$ ◀

$\underline{a}_3 = [r\omega^2 \leftarrow] + [\frac{u^2}{r} \leftarrow] + [2u\omega \leftarrow]$ $\quad \underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} + 2u\omega)\underline{i}$ ◀

$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \uparrow]$ $\quad \underline{a}_4 = (r\omega^2 + 2u\omega)\underline{j}$ ◀

15.122

ANGULAR VELOCITY = $\underline{\omega}$ ↺

SAME AS PROB 15.121 EXCEPT FOR $\underline{a}_C$ THE DIRECTION OF WHICH IS NOW OBTAINED FROM THAT OF $\underline{u}$ BY ROTATING THROUGH 90° ↺.

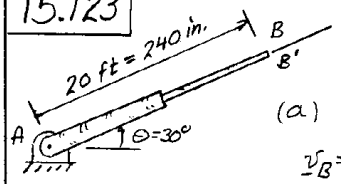
$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \uparrow]$ $\quad \underline{a}_1 = r\omega^2 \underline{i} + 2u\omega \underline{j}$ ◀

$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \leftarrow]$ $\quad \underline{a}_2 = -2u\omega \underline{i} - r\omega^2 \underline{j}$ ◀

$\underline{a}_3 = [r\omega^2 \leftarrow] + [\frac{u^2}{r} \leftarrow] + [2u\omega \rightarrow]$ $\quad \underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} - 2u\omega)\underline{j}$ ◀

$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \downarrow]$ $\quad \underline{a}_4 = (r\omega^2 - 2u\omega)\underline{j}$ ◀

15.123



$\underline{\omega} = 0.075$ rad/s ↺

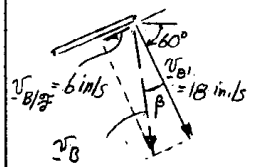
$\underline{v}_{B/\text{FRAME}} = \underline{v}_{B/\mathcal{F}} = 6$ in./s ↙

(a) $\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}}$

$\underline{v}_B = [r\omega \searrow] + [6 \text{ in./s} \swarrow]$

$= [(240)(0.075) \searrow] + [6 \text{ in./s} \swarrow]$

$\underline{v}_B = [18 \text{ in./s} \searrow 60°] + [6 \text{ in./s} \swarrow 30°]$



$\beta = 18.43°$ $\quad v_B = 18.974$ in./s

$\underline{v}_B = 18.97$ in./s ∠ 78.4° ◀

(b) SINCE $\omega = 0.075$ rad/s = CONSTANT, $\alpha = 0$

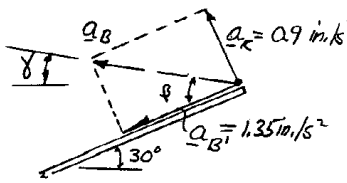
ALSO, SINCE $v_{B/\mathcal{F}} = 6$ in./s = CONSTANT, $a_{B/\mathcal{F}} = 0$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$

$= [r\omega^2 \swarrow] + 0 + [2\omega v_{B/\mathcal{F}} \nwarrow]$

$= [(240)(0.075)^2 \swarrow] + [2(0.075)(6) \nwarrow]$

$\underline{a}_B = [1.35 \text{ in./s}^2 \angle 30°] + [0.9 \text{ in./s}^2 \angle 60°]$



$a_B = 1.6225$ in./s²

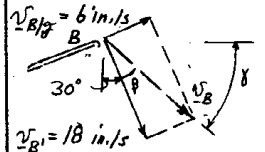
$\beta = 33.7°$

$\gamma = 33.7° - 30° = 3.7°$

$\underline{a}_B = 1.622$ in./s² ∠ 3.7° ◀

15.124

SAME AS PROB 15.123 EXCEPT FOR DIRECTION OF $\underline{v}_{B/\mathcal{F}}$ WHICH IS NOW ↗. THUS



(a) $\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}}$

$\underline{v}_B = [18 \text{ in./s} \searrow 60°] + [6 \text{ in./s} \angle 30°]$

$\beta = 18.43°$ $\quad \gamma = 60 - \beta = 41.6°$

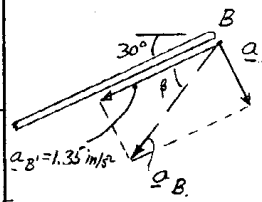
$v_B = 18.974$ in./s

$\underline{v}_B = 18.97$ in./s ∠ 41.6° ◀

(b) SINCE DIRECTION OF $\underline{v}_{B/\mathcal{F}}$ IS ↗ AND $\underline{\omega}$ IS ↻, THE DIRECTION OF $\underline{a}_C$ IS NOW ↘. FROM PROB. 15.123 WE NOW WRITE

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$

$\underline{a}_B = [1.35 \text{ in./s}^2 \angle 30°] + 0 + [0.9 \text{ in./s}^2 \searrow 60°]$



$\beta = 33.7°$

$a_B = 1.6225$ in./s²

$\underline{a}_B = 1.622$ in./s² ∠ 63.7° ◀

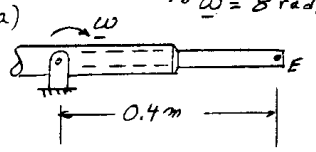
15.125

$\mathcal{F}$: ROTATING FRAME = CYLINDER AB

$\underline{v}_{E/\mathcal{F}} = 2.5$ m/s → $\quad \underline{\omega} = 8$ rad/s ↻

$\underline{a}_{E/\mathcal{F}} = 0$ $\quad \underline{\alpha} = 0$

(a)



$\underline{a}_E = \underline{a}_{E'} + \underline{a}_{E/\mathcal{F}} + \underline{a}_C$ (1)

$a_{E'} = (0.4 \text{ m})(8 \text{ rad/s})^2$

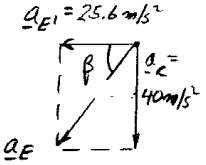
$\underline{a}_{E'} = 25.6$ m/s² ←

$\underline{a}_{E/\mathcal{F}} = 0$

$\underline{a}_C = 2 v_{E/\mathcal{F}} \omega$

$= 2(2.5 \text{ m/s})(8 \text{ rad/s})$

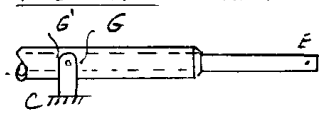
$\underline{a}_C = 40$ m/s² ↓



EQ(1): $\underline{a}_E = [25.6 \text{ m/s}^2 \leftarrow] + 0 + [40 \text{ m/s}^2 \downarrow]$

$\underline{a}_E = 47.8$ m/s² ∠ 57.4° ◀

(b) ACCELERATION OF POINT OF ROD COINCIDING WITH C. DENOTE POINT COINCIDING WITH C BY LETTER G



$\underline{v}_{G/\mathcal{F}} = 2.5$ m/s →

$\underline{a}_G = \underline{a}_{G'} + \underline{a}_{G/\mathcal{F}} + \underline{a}_C$ (2)

$\underline{a}_{G'} = (0)\omega^2 = 0$

$\underline{a}_{G/\mathcal{F}} = 0$

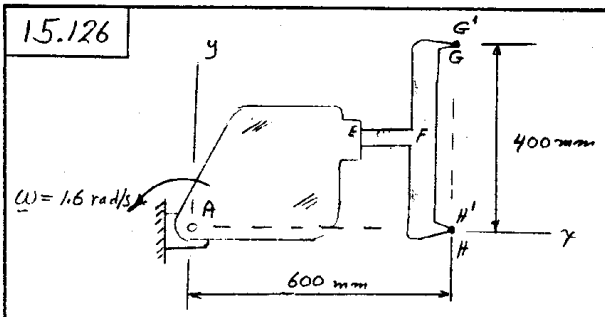
$a_C = 2 v_{G/\mathcal{F}} \omega$

$= 2(2.5 \text{ m/s})(8 \text{ rad/s})$

$= 40$ m/s ↓

EQ(2) $\underline{a}_G = 0 + 0 + 40$ m/s ↓ $\quad \underline{a}_G = 40$ m/s² ↓ ◀

15.126



DISTANCE EF IS INCREASING AT CONSTANT RATE OF 300 mm/s

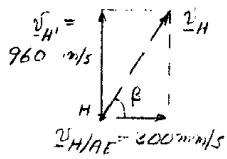
THUS: $\underline{v}_{G/AE} = \underline{v}_{H/AE} = 300 \text{ mm/s} \rightarrow$

AND: $\underline{a}_{G/AE} = \underline{a}_{H/AE} = 0$

(a) VELOCITY OF H: H' IS POINT OF AE COINCIDING WITH H

$\underline{v}_{H'} = (AH)\omega \uparrow = (600 \text{ mm})(1.6 \text{ rad/s}) \uparrow = 960 \text{ mm/s} \uparrow$

$\underline{v}_{H/AE} = 300 \text{ mm/s} \rightarrow$



$\underline{v}_H = \underline{v}_{H'} + \underline{v}_{H/AE}$

$\underline{v}_H = [960 \text{ mm/s} \uparrow] + [300 \text{ mm/s} \rightarrow]$

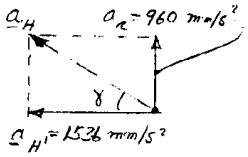
$\beta = 72.6^\circ \qquad v_H = 1005.8 \text{ mm/s}$

$\underline{v}_H = 1006 \text{ mm/s} \measuredangle 72.6^\circ$ ◀

(b) ACCELERATION OF H:

$a_{H'} = (AH)\omega^2 = (600 \text{ mm})(1.6 \text{ rad/s})^2 = 1536 \text{ mm/s}^2 \leftarrow$

$a_c = 2\omega v_{H/AE} = 2(300 \text{ mm/s})(1.6 \text{ rad/s}) = 960 \text{ mm/s}^2 \uparrow$



$\underline{a}_H = \underline{a}_{H'} + \underline{a}_{H/AE} + \underline{a}_c$

$\underline{a}_H = [1536 \text{ mm/s}^2 \leftarrow] + 0 + [960 \text{ mm/s}^2 \uparrow]$

$\gamma = 32.0^\circ \qquad a_H = 1811.3 \text{ mm/s}^2$

$\underline{a}_H = 1811 \text{ mm/s}^2 \measuredangle 32.0^\circ$ ◀

15.127

SEE FIGURE AND SOLUTION OF PROB. 15.126

WE FIRST DETERMINE $\underline{r}_{G/A}$

$\underline{r}_{G/A} = (600 \text{ mm})\,\underline{i} + (400 \text{ mm})\,\underline{j}$

$\underline{\omega} = (1.6 \text{ rad/s})\,\underline{k}$

(a) VELOCITY OF G. G' IS POINT OF AE COINCIDING WITH G

$\underline{v}_{G'} = \underline{\omega} \times \underline{r}_{G/A} = 1.6\,\underline{k} \times (600\,\underline{i} + 400\,\underline{j})$

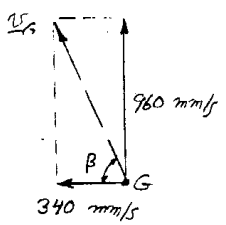
$\underline{v}_{G'} = +(960 \text{ mm/s})\,\underline{j} - (640 \text{ mm/s})\,\underline{i}$

$\underline{v}_{G/AE} = 300 \text{ mm/s} \rightarrow = (300 \text{ mm/s})\,\underline{i}$

$\underline{v}_G = \underline{v}_{G'} + \underline{v}_{G/AE}$

$\underline{v}_G = [(960 \text{ mm/s})\,\underline{j} - (640 \text{ mm/s})\,\underline{i}] + (300 \text{ mm/s})\,\underline{i}$

$\underline{v}_G = -(340 \text{ mm/s})\,\underline{i} + (960 \text{ mm/s})\,\underline{j}$



$\beta = 70.5^\circ$

$v_G = 1018.4 \text{ mm/s}$

$\underline{v}_G = 1018 \text{ mm/s} \measuredangle 70.5^\circ$ ◀

(CONTINUED)

15.127 CONTINUED

(b) ACCELERATION OF G.

$\underline{a}_{G'} = -\omega^2 \underline{r}_{G/A} = -(1.6 \text{ rad/s})^2 [(600 \text{ mm})\,\underline{i} + (400 \text{ mm})\,\underline{j}]$

$\underline{a}_{G'} = -(1536 \text{ mm/s}^2)\,\underline{i} - (1024 \text{ mm/s}^2)\,\underline{j}$

$\underline{a}_{G/AE} = 0$ (SINCE $\underline{v}_{G/AE}$ IS CONSTANT)

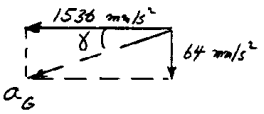
$\underline{a}_c = 2\underline{\omega} \times \underline{v}_{G/AE} = 2(1.6 \text{ rad/s})\,\underline{k} \times (300 \text{ mm/s})\,\underline{i}$

$\underline{a}_c = (960 \text{ mm/s}^2)\,\underline{j}$

$\underline{a}_G = \underline{a}_{G'} + \underline{a}_{G/AE} + \underline{a}_c$

$= [-(1536 \text{ mm/s}^2)\,\underline{i} - (1024 \text{ mm/s}^2)\,\underline{j}] + 0 + [(960 \text{ mm/s}^2)\,\underline{j}]$

$\underline{a}_G = -(1536 \text{ mm/s}^2)\,\underline{i} - (64 \text{ mm/s}^2)\,\underline{j}$

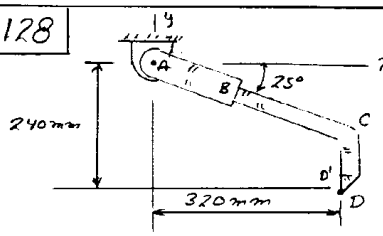


$\gamma = 2.38^\circ$

$a_G = 1537.3 \text{ mm/s}^2$

$\underline{a}_G = 1537 \text{ mm/s}^2 \measuredangle 2.4^\circ$ ◀

15.128



$v_{D/AB} = 250 \text{ mm/s} \leftarrow$

$\omega = 1.8 \text{ rad/s} \curvearrowright$

$\underline{\omega} = -(1.8 \text{ rad/s})\,\underline{k}$

$\alpha = 0$

$\underline{r}_{D/A} = (320 \text{ mm})\,\underline{i} - (240 \text{ mm})\,\underline{j}$

$\underline{v}_{D/AB} = v_{D/AB} \measuredangle 25^\circ = (250 \text{ mm/s})(-\cos 25^\circ\,\underline{i} + \sin 25^\circ\,\underline{j})$

$\underline{v}_{D/AB} = -(226.58 \text{ mm/s})\,\underline{i} + (105.65 \text{ mm/s})\,\underline{j}$ $\qquad \underline{a}_{D/AB} = 0$

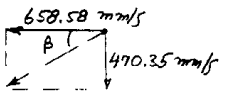
(a) VELOCITY OF D:

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/AB} = \underline{\omega} \times \underline{r}_{D/A} + \underline{v}_{D/AB}$

$= -(1.8 \text{ rad/s})\,\underline{k} \times [(320 \text{ mm})\,\underline{i} - (240 \text{ mm})\,\underline{j}] + \underline{v}_{D/AB}$

$= -(576 \text{ mm/s})\,\underline{j} - (432 \text{ mm/s})\,\underline{i} - (226.58 \text{ mm/s})\,\underline{i} + (105.65 \text{ mm/s})\,\underline{j}$

$\underline{v}_D = -(658.58 \text{ mm/s})\,\underline{i} - (470.35 \text{ mm/s})\,\underline{j}$



$\beta = 35.53^\circ$

$v_D = 809.29 \text{ mm/s}$

$\underline{v}_D = 809 \text{ mm/s} \measuredangle 35.5^\circ$ ◀

(b) ACCELERATION OF D:

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/AB} + \underline{a}_c$

$\underline{a}_{D'} = -\omega^2 \underline{r}_{D/A} = -(1.8 \text{ rad/s})^2 [(320 \text{ mm})\,\underline{i} - (240 \text{ mm})\,\underline{j}]$

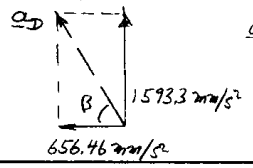
$\underline{a}_{D'} = -(1036.8 \text{ mm/s}^2)\,\underline{i} + (777.6 \text{ mm/s}^2)\,\underline{j}$

$\underline{a}_{D/AB} = 0$

$\underline{a}_c = 2\underline{\omega} \times \underline{v}_{D/AB} = 2[-(1.8 \text{ rad/s})\,\underline{k}] \times [-(226.58 \text{ mm/s})\,\underline{i} + (105.65 \text{ mm/s})\,\underline{j}]$

$\underline{a}_c = (815.69 \text{ mm/s}^2)\,\underline{j} + (380.34 \text{ mm/s}^2)\,\underline{i}$

$\underline{a}_D = [-(1036.8 \text{ mm/s}^2)\,\underline{i} + (777.6 \text{ mm/s}^2)\,\underline{j}] + 0 + [(815.69 \text{ mm/s}^2)\,\underline{j} + (380.34 \text{ mm/s}^2)\,\underline{i}]$



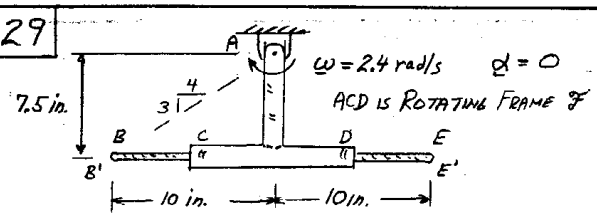
$\underline{a}_D = -(656.46 \text{ mm/s}^2)\,\underline{i} + (1593.3 \text{ mm/s}^2)\,\underline{j}$

$\beta = 67.61^\circ$

$a_D = 1723.2 \text{ mm/s}^2$

$\underline{a}_D = 1.723 \text{ m/s}^2 \measuredangle 67.6^\circ$ ◀

15.129



$\underline{v}_{BC/F} = \underline{v}_{B/F} = \underline{v}_{E/F} = 15$ in./s → $\underline{a}_{B/F} = \underline{a}_{E/F} = 0$

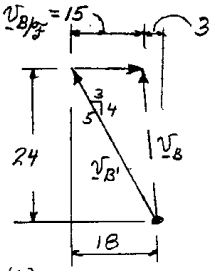
$AB = 12.5$ in. ◢ 3, 4

(a) VELOCITY OF B.

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/F} = (AB)\omega$ ◣ 3, 4 $+ \underline{v}_{B/F}$

$= (12.5\text{ in.})(2.4\text{ rad/s})$ ◣ 3, 4 $+ 15$ in./s →

$\underline{v}_B = 30$ in./s ◣ 3, 4 $+ 15$ in./s →



$\underline{v}_B = [3\text{ in/s} \leftarrow] + [24\text{ in./s} \uparrow]$

$\underline{v}_B = 24.2$ in./s ◺ 82.9° ◀

(b) ACCELERATION OF B.

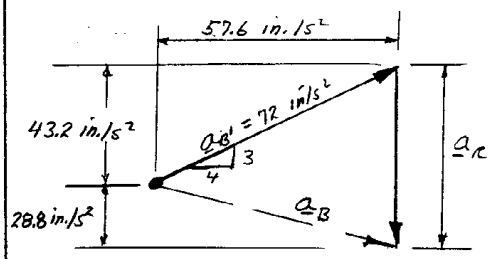
$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/F} + \underline{a}_C$

$\underline{a}_{B'} = (AB)\omega^2$ ◢ 3, 4 $= (12.5\text{ in.})(2.4\text{ rad/s})^2$ ◢ 3, 4 $= 72$ in./s² ◢ 3, 4

$a_{B/F} = 0$

$a_C = 2\omega v_{B/F}$ ↓ $= 2(2.4\text{ rad/s})(15\text{ in./s})$ ↓ $= 72$ in./s² ↓

$\underline{a}_B = 72$ in./s² ◢ 3, 4 $+ 72$ in./s² ↓



$\underline{a}_B = [57.6\text{ in./s}^2 \rightarrow] + [28.8\text{ in./s}^2 \downarrow]$

$\underline{a}_B = 64.4$ in./s² ◹ 26.6° ◀

15.130

SEE SKETCH OF PROB. 15.129

$\omega = 2.4$ rad/s ↻ $\alpha = 0$ $\underline{v}_{E/F} = 15$ in./s →, $\underline{a}_{E/F} = 0$

$AE = 12.5$ in. ◢ 4, 3

(a) VELOCITY OF E.

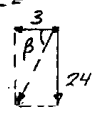
$\underline{v}_E = \underline{v}_{E'} + \underline{v}_{E/F} = (AE)\omega$ ◣ 3, 4 $+ \underline{v}_{E/F}$

$= (12.5\text{ in.})(2.4\text{ rad/s})$ ◣ 3, 4 $+ 15$ in./s →

$= 30$ in/s ◣ 3, 4 $+ 15$ in./s →

$= 18$ in/s ← $+ 24$ in./s ↓ $+ 15$ in./s →

$\underline{v}_E = 3$ in./s ← $+ 24$ in./s ↓



$\beta = 82.87°$ $v_E = 24.19$ in./s

$\underline{v}_E = 24.2$ in./s ◹ 82.9° ◀

(b) ACCELERATION OF E.

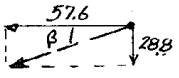
$\underline{a}_E = \underline{a}_{E'} + \underline{a}_{E/F} + \underline{a}_C = (AE)\omega^2$ ◢ 4, 3 $+ 0 + 2\omega v_{E/F}$ ↓

$= (12.5\text{ in.})(2.4\text{ rad/s})^2$ ◢ 4, 3 $+ 2(2.4\text{ rad/s})(15\text{ in./s})$ ↓

$= 72$ in./s² ◢ 4, 3 $+ 72$ in./s² ↓

$= 57.6$ in./s² ← $+ 43.2$ in./s² ↑ $+ 72$ in./s² ↓

$\underline{a}_E = 57.6$ in./s² ← $+ 28.8$ in./s² ↓

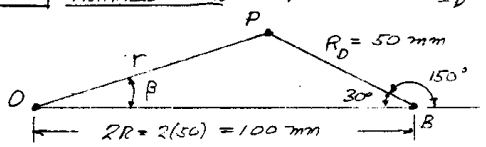


$\beta = 26.56°$ $a_E = 64.4$ in./s²

$\underline{a}_E = 64.4$ in./s² ◹ 26.6° ◀

15.131

TRIANGLE OPB: $\phi = 150°$ $\omega_D = 10$ rad/s ↻ $\alpha_D = 0$



$r^2 = (50)^2 + (100)^2 - 2(50)(100)\cos 30°$ $r = 61.966$ mm

$\frac{\sin\beta}{50\text{ mm}} = \frac{\sin 30°}{61.966\text{ mm}}$ $\beta = 23.79°$

ANGULAR VELOCITY.

$\underline{v}_P = (BP)\omega_D$ ◹ 60° $= (50\text{ mm})(10\text{ rad/s}) = 500$ mm/s ◹ 60

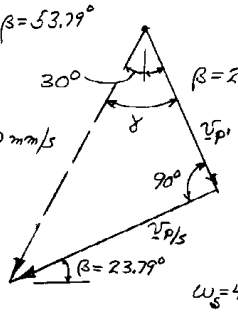
$\underline{v}_{P/S} = v_{P/S}$ ◹ β

$\underline{v}_{P'} = v_{P'}$ ◣ β WHERE P' = COINCIDING POINT ON DISK S.

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/S}$

$[500\text{ mm/s} \nearrow 60°] = [v_{P'} \swarrow \beta] + [v_{P/S} \searrow \beta]$

$\gamma = 30° + \beta = 53.79°$



$v_{P/S} = (500\text{ mm/s})\sin 53.79°$

$\underline{v}_{P/S} = 403.4$ mm/s ↙

$v_{P'} = (500\text{ mm/s})\cos 53.79°$

$\underline{v}_{P'} = 295.4$ mm/s ↘

$\omega_S = \frac{v_{P'}}{r} = \frac{295.4\text{ mm/s}}{61.966\text{ mm}}$

$\omega_S = 4.769$ rad/s $\underline{\omega}_S = 4.77$ rad/s ↺ ◀

(CONTINUED)

15.131 CONTINUED

ANGULAR ACCELERATION:

$\underline{a}_P = (BP)\omega_D^2 \measuredangle 30° = (50\text{ mm})(10\text{ rad/s})^2 \measuredangle 30°$

$= 5000\text{ mm/s}^2 \measuredangle 30°$

$(\underline{a}_{P'})_n = r\omega_S^2 \downarrow \beta = (61.966\text{ mm})(4.767\text{ rad/s})^2 \downarrow \beta$

$= 1408\text{ mm/s}^2 \downarrow 23.79°$

$(\underline{a}_{P'})_t = r\alpha_S \perp \beta = (61.966\text{ mm})\alpha_S \perp 23.79°$

$\underline{a}_{P/S} = a_{P/S} \measuredangle \beta = a_{P/S} \measuredangle 23.79°$

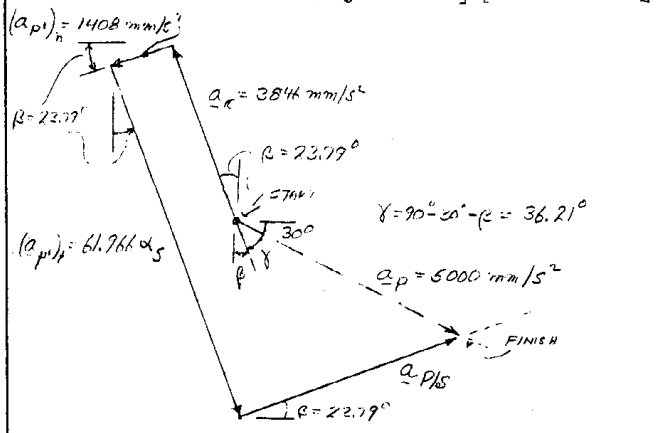
$\underline{a}_c = 2\omega_S v_{P/S} \uparrow \beta = 2(4.767\text{ rad/s})(403.4\text{ mm/s}) \uparrow \beta$

$= 3846\text{ mm/s}^2 \uparrow 23.79°$

$\underline{a}_P = (\underline{a}_{P'})_n + (\underline{a}_{P'})_t + \underline{a}_{P/S} + \underline{a}_c$

$[5000 \measuredangle 30°] = [1408 \downarrow 23.79°] + [61.966\alpha_S \perp 23.79°]$

$+ [a_{P/S} \measuredangle 23.79°] + [3846 \uparrow 23.79°]$



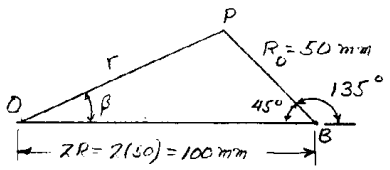
SUM COMPONENTS $\perp$ TO SLOT $\perp \beta = 23.79°$

$+\perp 23.79°$: $5000\cos 36.21° = 61.966\alpha_S - 3846$

$\alpha_S = 127.17\text{ rad/s}^2$ $\quad \underline{\alpha}_S = 127.2\text{ rad/s}^2$ ↻ ◀

15.132

TRIANGLE OPB: $\phi = 135°$ $\quad \underline{\omega}_D = 10\text{ rad/s}$ ↺ $\quad \underline{\alpha}_D = 0$



$r^2 = 50^2 + 100^2 - 2(50)(100)\cos 45°$ $\quad r = 73.68\text{ mm}$

$\frac{\sin\beta}{50\text{ mm}} = \frac{\sin 45°}{73.68\text{ mm}}$ $\quad \beta = 28.68°$

ANGULAR VELOCITY

$\underline{v}_P = (BP)\omega_D \downarrow 45° = (50\text{ mm})(10\text{ rad/s}) = 500\text{ mm/s} \downarrow 45°$

$\underline{v}_{P/S} = v_{P/S} \downarrow \beta$

$\underline{v}_{P'} = v_{P'} \perp \beta$ WHERE P' = COINCIDING POINT ON DISK S

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/S}$

(CONTINUED)

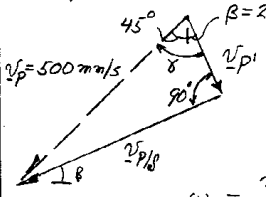
15.132 CONTINUED

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/S}$

$[500\text{ mm/s} \downarrow 45°] = [v_{P'} \perp \beta] + [v_{P/S} \downarrow \beta]$

$\gamma = 45° + \beta = 45° + 28.68°$

$\gamma = 73.68°$



$v_{P/S} = (500\text{ mm/s})\sin 73.68°$

$v_{P/S} = 479.9\text{ mm/s}$ ↙

$v_{P'} = (500\text{ mm/s})\cos 73.68°$

$\underline{v}_{P'} = 140.50\text{ mm/s}$ ↘

$\omega_S = \frac{v_{P'}}{r} = \frac{140.50\text{ mm/s}}{73.68\text{ mm}} = 1.9069\text{ rad/s}$

$\underline{\omega}_S = 1.907\text{ rad/s}$ ↻ ◀

ANGULAR ACCELERATION

$\underline{a}_P = (BP)\omega_D^2 \measuredangle 45° = (50\text{ mm})(10\text{ rad/s})^2 \measuredangle 45°$

$= 5000\text{ mm/s}^2 \measuredangle 45°$

$(\underline{a}_{P'})_n = r\omega_S^2 \downarrow \beta = (73.68\text{ mm})(1.9069\text{ rad/s})^2 \downarrow \beta$

$= 267.9\text{ mm/s}^2 \downarrow 28.68°$

$(\underline{a}_{P'})_t = r\alpha_S \perp \beta = (73.68\text{ mm})\alpha_S \perp 28.68°$

$\underline{a}_{P/S} = a_{P/S} \measuredangle \beta = a_{P/S} \measuredangle 28.68°$

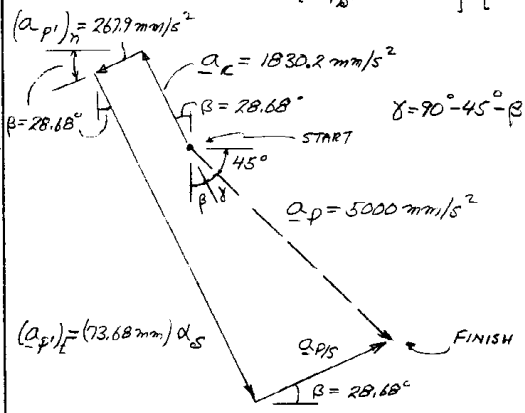
$\underline{a}_c = 2\omega_S v_{P/S} \uparrow \beta = 2(1.9069\text{ rad/s})(479.9\text{ mm/s}) \uparrow \beta$

$= 1830.2\text{ mm/s}^2 \uparrow 28.68°$

$\underline{a}_P = (\underline{a}_{P'})_n + (\underline{a}_{P'})_t + \underline{a}_{P/S} + \underline{a}_c$

$[5000 \measuredangle 45°] = [267.9 \downarrow 28.68°] + [73.68\alpha_S \perp 28.68°]$

$+ [a_{P/S} \measuredangle 28.68°] + [1830.2 \uparrow 28.68°]$



SUM COMPONENTS $\perp$ TO SLOT $\perp \beta = 28.68°$

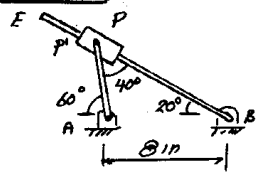
$+\perp 28.68°$: $5000\cos 16.32° = 73.68\alpha_S - 1830.2$

$\alpha_S = 89.97\text{ rad/s}^2$

$\underline{\alpha}_S = 90.0\text{ rad/s}^2$ ↻ ◀

15.133

FROM PROB 15.115:



$AP = 4.2567$ in.
$BP = 10.7784$ in.
$\omega_{AP} = 6$ rad/s ↻
$\omega_{BE} = 1.8152$ rad/s ↻
$v_{P/BE} = 16.417$ in./s ∠ 20°

SINCE ω_{AP} = CONSTANT, $\alpha_{AP} = 0$

$\underline{a}_P = (AP)\omega_{AP}^2 \angle 60° = (4.2567\text{ in})(6\text{ rad/s})^2 = 153.24\text{ in./s}^2 \angle 60°$

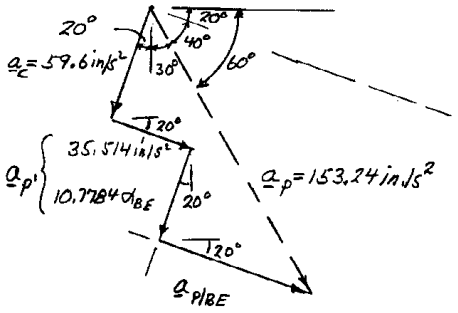
$\underline{a}_{P'} = (BP)\omega_{BE}^2 \angle 20° + (BP)\alpha_{BE} \perp 20°$
$= (10.7784\text{ in.})(1.8152\text{ rad/s})^2 \angle 20° + (10.7784\text{ in})\alpha_{BE} \perp 20°$
$= 35.514\text{ in./s}^2 \angle 20° + 10.7784\,\alpha_{BE} \perp 20°$

$\underline{a}_{P/BE} = a_{P/BE} \angle 20°$

$\underline{a}_c = 2\omega_{BE} v_{P/BE} \perp 20° = 2(1.8152\text{ rad/s})(16.417\text{ in./s}) \perp 20°$
$= 59.60\text{ in./s}^2 \perp 20°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/BE} + \underline{a}_c$

$[153.24 \angle 60°] = [35.514 \angle 20° + 10.7784\,\alpha_{BE} \perp 20°] + [a_{P/BE} \angle 20°] + [59.60 \perp 20°]$



WE SUM COMPONENTS ⊥ TO ROD BE, THAT IS ⊥ 20°.

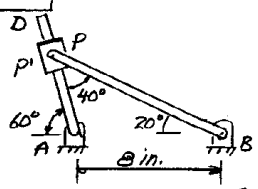
+⊥ 20° COMPONENTS

$(153.24\text{ in/s}^2)\sin 40° = 59.6\text{ in./s}^2 + 10.7784\,\alpha_{BE}$

$\alpha_{BE} = 3.609$ rad/s² $\quad \underline{\alpha}_{BE} = 3.61$ rad/s² ↻ ◀

15.134

FROM PROB 15.118:



$AP = 4.2567$ in.
$BP = 10.7784$ in.
$\omega_{AD} = 2.52$ rad/s ↻
$\omega_{BP} = 1.299$ rad/s ↻
$v_{P/AD} = v_0 = 9$ in./s ∠ 60°

SINCE $v_{P/AD}$ IS CONSTANT, $\underline{a}_{P/AD} = 0$

$\underline{a}_P = (BP)\omega_{BP}^2 \angle 20° + (BP)\alpha_{BP} \perp 20°$
$= (10.7784\text{ in.})(1.299\text{ rad/s})^2 \angle 20° + (10.7784\text{ in.})\alpha_{BP} \perp 20°$
$= 18.187\text{ in./s}^2 \angle 20° + 10.7784\,\alpha_{BP} \perp 20°$

$\underline{a}_{P'} = (AP)\omega_{AD}^2 \angle 60° + (AP)\alpha_{AD} \angle 30°$
$= (4.2567\text{ in.})(2.52\text{ rad/s})^2 \angle 60° + (4.2567\text{ in.})\alpha_{AD} \angle 30°$
$= 27.032\text{ in./s}^2 \angle 60° + 4.2567\,\alpha_{AD} \angle 30°$

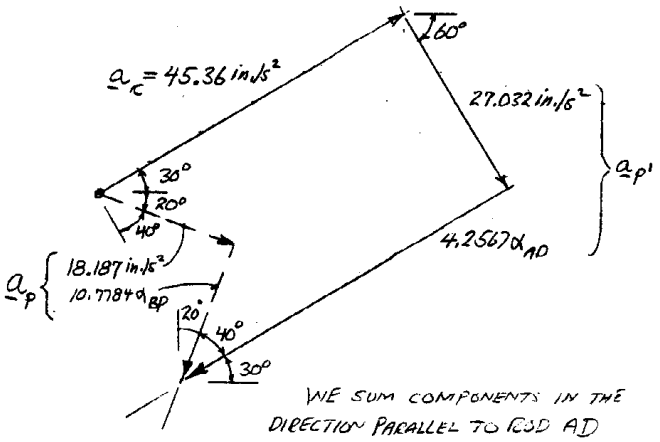
$\underline{a}_c = 2\omega_{AD} v_{P/AD} \perp 30° = 2(2.52\text{ rad/s})(9\text{ in/s}) \perp 30°$
$= 45.36\text{ in./s}^2 \perp 30°$

(CONTINUED)

15.134 CONTINUED

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/AD} + \underline{a}_c$

$[18.187 \angle 20° + 10.7784\,\alpha_{BP} \perp 20°] = [27.032 \angle 60° + 4.2567\,\alpha_{AD} \angle 30°] + 0 + [45.36\text{ in./s}^2 \perp 30°]$



WE SUM COMPONENTS IN THE DIRECTION PARALLEL TO ROD AD

+ ∠ 60° COMPONENTS:

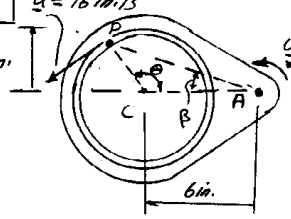
$(18.187\text{ in./s}^2)\cos 40° + (10.7784\,\alpha_{BP})\sin 40° = 27.032\text{ in./s}^2$

$\alpha_{BP} = 1.8909$ rad/s² $\quad \underline{\alpha}_{BP} = 1.891$ rad/s² ↻ ◀

15.135

$v = 16$ in./s

FOR $\theta = 90°$



$\omega = 6$ rad/s ↻
$\alpha = 20$ rad/s² ↻
$AP^2 = 4\text{ in.}^2 + 6\text{ in.}^2$
$AP = 7.211$ in.
$\beta = 33.69°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/O} + \underline{a}_c$ (1)

$(a_{P'})_n = (AP)\omega^2 = (7.211\text{ in})(6\text{ rad/s})^2 = 259.6\text{ in./s}^2 \angle \beta$
$(a_{P'})_t = (AP)\alpha = (7.211\text{ in.})(20\text{ rad/s}^2) = 144.22\text{ in./s}^2 \perp \beta$
$\underline{a}_{P/O} = v^2/(OP) = (16\text{ in./s})^2/4\text{ in.} = 64\text{ in/s}^2 \downarrow$
$\underline{a}_c = 2\omega v_{P/O} = 2(6\text{ rad/s})(16\text{ in./s}) = 192\text{ in./s}^2 \downarrow$

FROM EQ(1): $+\rightarrow (a_P)_x = 259.6\cos\beta - 144.22\sin\beta$
$= 216 - 80 \qquad (\underline{a}_P)_x = 136\text{ in./s}^2 \rightarrow$

$+\downarrow (a_P)_y = 259.6\sin\beta + 144.22\cos\beta + 64 + 192$
$= 144 + 120 + 64 + 192 \qquad (\underline{a}_P)_y = 520\text{ in./s}^2 \downarrow$

$\underline{a}_P = 537\text{ in./s}^2 \angle 75.3°$ ◀

15.136

SEE SKETCH OF PROB 15.135. WE NOW HAVE $\theta = 120°$

$AP^2 = (6 + 4\sin 30°)^2 + (4\cos 30°)^2 \quad AP = 8.718$ in. $\quad \beta = 23.41°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/O} + \underline{a}_c$ (1)

$(\underline{a}_{P'})_n = (AP)\omega^2 = (8.718\text{ in.})(6\text{ rad/s})^2 = 313.8\text{ in./s}^2 \angle \beta$
$(\underline{a}_{P'})_t = (AP)\alpha = (8.718\text{ in.})(20\text{ rad/s}^2) = 174.36\text{ in./s}^2 \perp \beta$
$\underline{a}_{P/O} = v^2/(OP) = (16\text{ in./s})^2/4\text{ in} = 64\text{ in./s}^2 \angle 60°$
$\underline{a}_c = 2\omega v_{P/O} = 2(6\text{ rad/s})(16\text{ in./s}) = 192\text{ in./s}^2 \angle 60°$

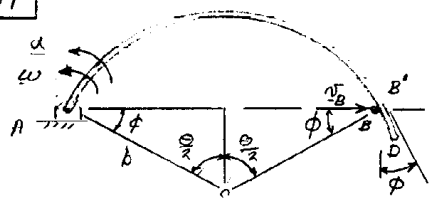
FROM EQ(1):

$+\rightarrow (a_P)_x = 313.8\cos\beta - 174.36\sin\beta + 64\cos 60° + 192\cos 60°$
$= 288.0 - 69.3 + 32.0 + 96 \qquad (\underline{a}_P)_x = 346.7\text{ in./s}^2 \rightarrow$

$+\downarrow (a_P)_y = 313.8\sin\beta + 174.36\cos\beta + 64\sin 60° + 192\sin 60°$
$= 124.7 + 160.0 + 55.4 + 166.3 \qquad (\underline{a}_P)_y = 506.4\text{ in./s}^2 \downarrow$

$\underline{a}_P = 614\text{ in./s}^2 \angle 55.6°$ ◀

15.137



v_B = CONSTANT, $a_B = 0$ $\quad \phi = 90° - \frac{1}{2}\theta$

$AB = 2b\cos\phi$

B' IS POINT OF ROD COINCIDING WITH PIN B

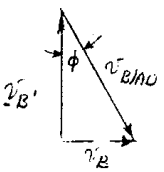
VELOCITY

$\underline{v}_B = v_B \rightarrow$

$\underline{v}_{B'} = (AB)\omega \uparrow = (2b\cos\phi)\omega \uparrow = 2b\omega\cos\phi \uparrow$

$\underline{v}_{B/AD}$ $\nwarrow \phi$

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/AD}$



$v_B = v_{B/AD}\sin\phi$

$v_{B/AD} = \frac{v_B}{\sin\phi}$

$v_{B'} = \frac{v_B}{\tan\phi} = v_B\frac{\cos\phi}{\sin\phi}$

$2b\omega\cos\phi = v_B\frac{\cos\phi}{\sin\phi}$ $\qquad \omega = \frac{v_B}{2b\sin\phi}$) ◁

ACCELERATION $\quad \underline{a}_B = 0$ (GIVEN)

$\underline{a}_{B'} = \left[(AB)\omega^2 \leftarrow\right] + \left[(AB)\alpha \uparrow\right]$

$= \left[(2b\cos\phi)\omega^2 \leftarrow\right] + \left[(2b\cos\phi)\alpha \uparrow\right]$

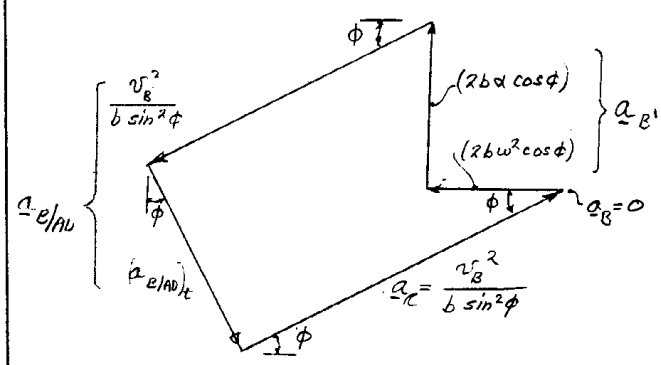
$\underline{a}_{B/AD} = \left[\frac{(v_{B/AD})^2}{b} \nearrow \phi\right] + \left[(a_{B/AD})_t \nwarrow \phi\right]$

$= \left[\frac{1}{b}\left(\frac{v_B}{\sin\phi}\right)^2 \nearrow \phi\right] + \left[(a_{B/AD})_t \nwarrow \phi\right]$

$\underline{a}_c = \left[2\omega v_{B/AD} \measuredangle \phi\right] = \left[2\left(\frac{v_B}{2b\sin\phi}\right)\left(\frac{v_B}{\sin\phi}\right) \measuredangle \phi\right]$

$= \frac{v_B^2}{b}\left(\frac{1}{\sin^2\phi}\right) \measuredangle \phi$

$\underline{a}_B = 0 = \underline{a}_{B'} + \underline{a}_{B/AD} + \underline{a}_c$



$+\measuredangle\phi$ COMPONENTS: $\quad$ CANCEL

$-(2b\omega^2\cos\phi)\cos\phi + (2b\alpha\cos\phi)\sin\phi - \frac{v_B^2}{b\sin^2\phi} + \frac{v_B^2}{b\sin^2\phi} = 0$

(CONTINUED)

15.137 CONTINUED

$-2b\omega^2\cos^2\phi + 2b\alpha\cos\phi\sin\phi = 0$

$\alpha = \omega^2\frac{\cos\phi}{\sin\phi} = \left(\frac{v_B}{2b\sin\phi}\right)^2\frac{\cos\phi}{\sin\phi}$

$\alpha = \left(\frac{v_B}{2b}\right)^2\frac{\cos\phi}{\sin^3\phi}$ ◁

GIVEN DATA: $\theta = 110°$, $\frac{1}{2}\theta = 55°$

$\phi = 90° - \frac{1}{2}\theta = 90° - 55°$ $\qquad \phi = 35°$

$v_B = 75$ mm/s $\qquad b = 150$ mm

$\omega = \frac{v_B}{2b\sin\phi} = \frac{75 \text{ mm/s}}{2(150 \text{ mm})\sin 35} = 0.4359$ rad/s

$\underline{\omega} = 0.436$ rad/s ↺ ◀

$\alpha = \left(\frac{v_B}{2b}\right)^2\frac{\cos\phi}{\sin^3\phi} = \left(\frac{75 \text{ mm/s}}{2 \times 150 \text{ mm}}\right)^2\frac{\cos 35°}{\sin^3 35°} = 0.2713$ rad/s²

$\underline{\alpha} = 0.271$ rad/s² ↺ ◀

15.138

SEE SOLUTION OF PROB 15.137 FOR EXPRESSIONS FOR ω AND α.

GIVEN DATA:

$\theta = 90°$ $\quad \frac{1}{2}\theta = 45°$ $\quad \phi = 90 - \frac{1}{2}\theta = 45°$

$v_B = 75$ mm/s $\qquad b = 150$ mm

$\omega = \frac{v_B}{2b\sin\phi} = \frac{75 \text{ mm/s}}{2(150 \text{ mm})\sin 45°} = 0.3536$ rad/s

$\underline{\omega} = 0.354$ rad/s ↺ ◀

$\alpha = \left(\frac{v_B}{2b}\right)^2\frac{\cos\phi}{\sin^2\phi} = \left(\frac{75 \text{ mm/s}}{2 \times 150 \text{ mm}}\right)^2\frac{\cos 45°}{\sin^3 45°} = 0.125$ rad/s

$\underline{\alpha} = 0.125$ rad/s² ↺ ◀

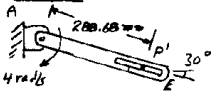
*15.139

FROM PROB 15.119:

ROD AE: $\underline{\omega}_{AE} = 4$ rad/s ↺, $\underline{\alpha}_{AE} = 0$, $\underline{v}_{P/AE} = 166.67$ mm/s $\measuredangle 30°$

ROD BD: $\underline{\omega}_{BD} = 5$ rad/s ↺, $\underline{\alpha}_{BD} = 0$, $\underline{v}_{P/BD} = 916.7$ mm/s ↓

ROD AE:



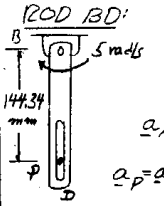
$\underline{a}_{P'} = (288.68 \text{ mm})(4 \text{ rad/s})^2 = 4619$ mm/s² $\measuredangle 30°$

$\underline{a}_{P/AE} = a_{P/AE} \measuredangle 30°$

$\underline{a}_c = 2\omega_{AE}v_{P/AE} = 2(4 \text{ rad/s})(166.67 \text{ mm/s}) = 1333$ mm/s² $\measuredangle 60°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/AE} + \underline{a}_c = 4619 \text{ mm/s}^2 \measuredangle 30° + \underline{a}_{P/AE} \measuredangle 30° + 1333 \text{ mm/s}^2 \measuredangle 60°$ (1)

ROD BD:



$\underline{a}_{P'} = (144.34 \text{ mm})(5 \text{ rad/s})^2 = 3609$ mm/s² ↑

$\underline{a}_{P/BD} = a_{P/BD}$ ↑

$\underline{a}_c = 2\omega_{BD}v_{P/BD} = 2(5 \text{ rad/s})(916.7 \text{ mm/s}) = 9167$ mm/s² ←

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/BD} + \underline{a}_c = 3609 \text{ mm/s}^2 \uparrow + a_{P/BD} \uparrow + 9167 \text{ mm/s}^2 \leftarrow$ (2)

EQUATE RIGHT-HAND MEMBERS OF (1) AND (2):

$4619 \measuredangle 30° + a_{P/AE} \measuredangle 30° + 1333 \measuredangle 60° = 3609 \uparrow + a_{P/BD} \uparrow + 9167 \leftarrow$

$\stackrel{+}{\leftarrow} \Sigma$ COMPS: $\quad 4619\cos 30° + a_{P/AE}\cos 30° - 1333\cos 60° = 9167$

$4000 + 0.86603\,a_{P/AE} - 666.5 = 9167$

$\underline{a}_{P/AE} = 6735 \text{ mm/s}^2 \measuredangle 30°$

RETURN TO EQ(1): $\underline{a}_P = 4619 \measuredangle 30° + 6735 \measuredangle 30° + 1333 \measuredangle 60°$

$\stackrel{+}{\leftarrow} (a_P)_x = 4000 + 5833 - 667 = 9166$ mm/s²

$+\uparrow (a_P)_y = 2310 + 3368 + 1155 = 6833$ mm/s²

$\underline{a}_P = 11\,433 \text{ mm/s}^2 \measuredangle 36.7° = 11.43 \text{ m/s}^2 \measuredangle 36.7°$ ◀

SEE NEXT PAGE FOR VECTOR DIAGRAM

*15.139 CONTINUED

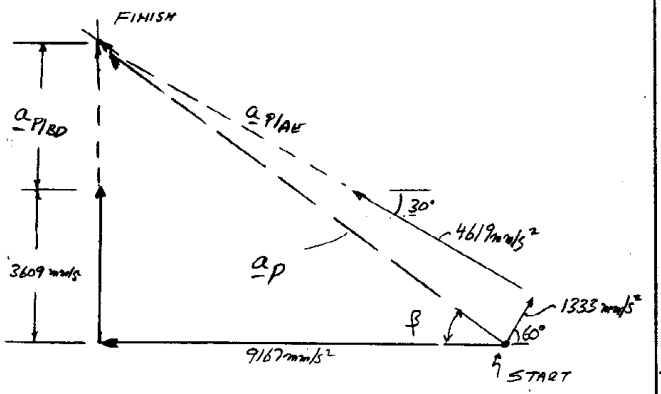
ALTERNATE SOLUTION USING VECTOR DIAGRAM

ROD AE: EQ(1):

$\underline{a}_P = 4619\ mm/s^2 \angle 30° + a_{P/AE} \angle 30° + 1333\ mm/s^2 \angle 60°$

ROD BD EQ(2):

$\underline{a}_P = 3609\ mm/s^2 \uparrow + a_{P/BD} \uparrow + 9167\ mm/s^2 \leftarrow$



$(a_P)_x = 9167\ mm/s^2 \qquad (a_P)_y = 6833\ mm/s^2$

$a_P = 11\,433\ mm/s^2 \qquad \beta = 36.7°$

$\underline{a}_P = 11.43\ m/s^2 \angle 36.7°$ ◀

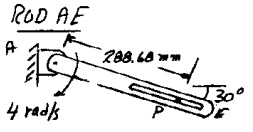
*15.140

FROM SOLUTION OF PROB. 15.120.

ROD AE: $\omega_{AE} = 4\ rad/s \circlearrowright$, $\alpha_{AE} = 0$, $\underline{v}_{P/AE} = 1500\ mm/s \angle 30°$

ROD BD: $\omega_{BD} = 5\ rad/s \circlearrowleft$, $\alpha_{BD} = 0$, $\underline{v}_{P/BD} = 1750\ mm/s \downarrow$

ROD AE



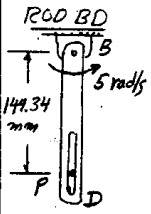
$\underline{a}_{P'} = (288.68\ mm)(4\ rad/s)^2 = 4619\ mm/s^2 \angle 30°$

$\underline{a}_{P/AE} = a_{P/AE} \angle 30°$

$\underline{a}_C = 2\omega_{AE} v_{P/AE} = 2(4)(1500) = 12\,000\ mm/s^2 \angle 30°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/AE} + \underline{a}_C = 4619\ mm/s^2 \angle 30° + a_{P/AE} \angle 30° + 12000\ mm/s^2 \angle 30°$ (1)

ROD BD



$a_{P'} = (144.34)(5)^2 = 3609\ mm/s^2 \uparrow$

$\underline{a}_{P/BD} = a_{P/BD} \uparrow$

$\underline{a}_C = 2\omega_{BD} v_{P/BD} = 2(5)(1750) = 17\,500\ mm/s^2 \rightarrow$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/BD} + \underline{a}_C$

$\underline{a}_P = 3609\ mm/s^2 \uparrow + a_{P/BD} \uparrow + 17\,500\ mm/s^2 \rightarrow$ (2)

EQUATE RIGHT-HAND MEMBERS OF (1) AND (2):

$4619\ mm/s^2 \angle 30° + a_{P/AE} \angle 30° + 12000 \angle 30° = 3609\ mm/s^2 \uparrow + a_{P/BD} \uparrow + 17500\ mm/s^2 \rightarrow$

$+\rightarrow$ X COMP: $-4000 + 0.86603\, a_{P/AE} - 6000 = +17500$

$0.86603\, a_{P/AE} = +27\,500$

$\underline{a}_{P/AE} = 31\,754\ mm/s^2 = 31.754\ m/s^2 \angle 30°$

(CONTINUED)

*15.140 CONTINUED

RETURN TO EQ(1) WITH $\underline{a}_{P/AE} = 31.754\ m/s^2 \angle 30°$

$\underline{a}_P = 4.619\ m/s^2 \angle 30° + 31.754\ m/s^2 \angle 30° + 12.00\ m/s^2 \angle 30°$

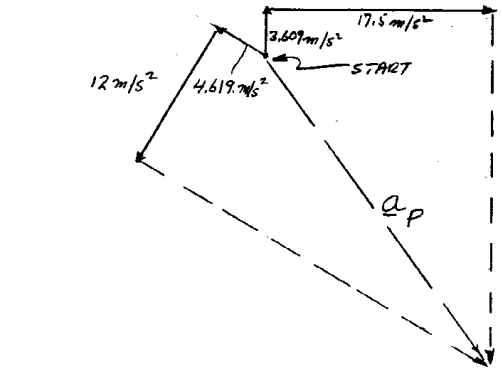
$+\rightarrow (a_P)_x = -4.000 + 27.50 - 6.00 = +17.5\ m/s^2$

$+\downarrow (a_P)_y = -2.310 + 15.88 + 10.392 = +23.96\ m/s^2$

$\underline{a}_P = 29.67\ m/s^2 \angle 53.86°$

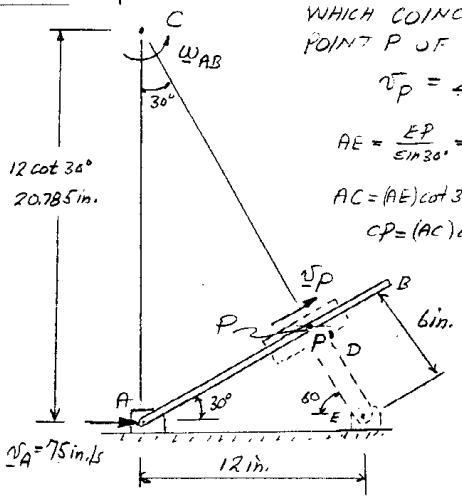
$\underline{a}_P = 29.7\ m/s^2 \angle 53.9°$ ◀

CORRESPONDING VECTOR DIAGRAM



*15.141

VELOCITIES: P' IS POINT OF COLLAR DE WHICH COINCIDES WITH POINT P OF ROD AB.



$\underline{v}_P = 30°$

$AE = \frac{EP}{\sin 30°} = \frac{6\ in}{\sin 30°} = 12\ in.$

$AC = (AE)\cot 30° = 20.785\ in.$

$CP = (AC)\cos 30° = (20.785\ in)\cos 30° = 18\ in.$

$AP = (6\ in)\tan 60° = 10.392\ in.$

WE DRAW LINES ⊥ TO $\underline{v}_A$ AND $\underline{v}_P$ TO LOCATE INSTANTANEOUS CENTER C OF ROD AB.

(a) $\omega_{AB} = \frac{v_A}{AC} = \frac{75\ in/s}{20.785\ in} = 3.6084\ rad/s$

$\underline{\omega}_{AB} = 3.61\ rad/s \circlearrowleft$ ◀

$v_P = (CP)\omega_{AB} = (18\ in.)(3.6084\ rad/s)$

$\underline{v}_P = 64.95\ in/s \angle 30°$

(b) COLLAR-LINK DE (THIS IS CHOSEN AS ROTATING FRAME)

$\underline{\omega}_{DE} = \underline{\omega}_{AB} = 3.6084\ rad/s \circlearrowleft$

$\underline{v}_{P'} = (EP)\omega_{DE} = (6\ in.)(3.6084\ rad/s) = 21.65\ in/s \nearrow$

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/DE}$

$64.95\ in/s \angle 30° = 21.65\ in/s \nearrow 30° + \underline{v}_{P/DE}$

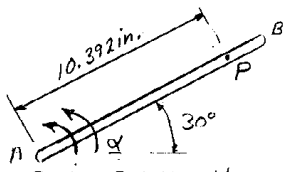
$\underline{v}_{P/DE} = 86.6\ in./s \angle 30°$ ◀

(CONTINUED)

*15.141 CONTINUED (C) ACCELERATIONS

NOTE $\alpha_{AB} = \alpha_{DE} = \alpha$

ROD AB P IS POINT OF ROD AB. P' WILL BE COINCIDING POINT OF COLLAR DE.



$\omega = 3.6084$ rad/s

$a_A = 0$ (GIVEN)

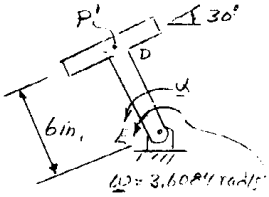
$\underline{a}_P = (AP)\omega^2 \angle 30° + (AP)\alpha \angle 60°$

$= (10.392)(3.6084)^2 \angle 30°$

$+ (10.392)\alpha \angle 60°$

$\underline{a}_P = 135.31 \text{ in./s}^2 \angle 30° + 10.392\alpha \angle 60°$

COLLAR LINK DE (THIS IS CHOSEN AS ROTATING FRAME)



$\underline{a}_{P'} = (P'E)\omega^2 \angle 60° + (P'E)\alpha \angle 30°$

$= (6 \text{ in.})(3.6084 \text{ rad/s})^2 \angle 60°$

$+ (6 \text{ in.})\alpha \angle 30°$

$\underline{a}_{P'} = 78.12 \text{ in./s}^2 \angle 60° + 6\alpha \angle 30°$

$\underline{a}_{P/DE} = a_{P/DE} \angle 30°$

$\underline{a}_c = 2\omega_{P/DE} \angle 90° = 2(3.6084 \text{ rad/s})(86.6 \text{ in./s}) \angle 60°$

$\underline{a}_c = 624.97 \text{ in./s}^2 \angle 60°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/DE} + \underline{a}_c$ (1)

$[135.31 \text{ in./s}^2 \angle 30°] + [10.392\alpha \angle 60°] = [78.12 \text{ in./s}^2 \angle 60°]$

$+ [6\alpha \angle 30°] + [a_{P/DE} \angle 30°] + [624.97 \text{ in./s}^2 \angle 60°]$

+ $\angle 60°$ COMPONENTS:

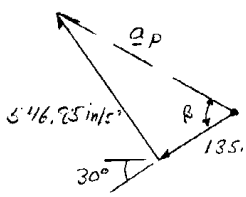
$10.392\alpha = -78.12 + 624.97 = 546.85$

$\alpha = +52.622 \text{ rad/s}^2$ $\quad \underline{\alpha} = 52.622 \text{ rad/s}^2$)

ROD AB.

$\underline{a}_P = 135.31 \text{ in/s}^2 \angle 30° + (10.392 \text{ in.})(52.622 \text{ rad/s}^2) \angle 60°$

$= 135.31 \text{ in/s}^2 \angle 30° + 546.85 \text{ in/s}^2 \angle 60°$



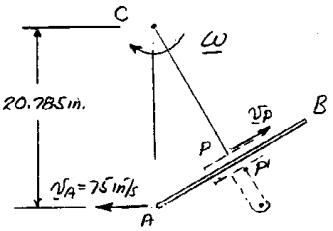
$\beta = 76.1°$ $\quad a_P = 563.3 \text{ in./s}^2$

$\underline{a}_P = 563 \text{ in./s}^2 \angle 46.1°$ ◀

SEE NEXT COLUMN FOR SKETCH OF THE VECTOR DIAGRAM FOR EQ(1).

*15.142 VELOCITIES. SEE SOLUTION OF PROB 15.141.

ALL VELOCITIES ARE NOW OPPOSITE TO THOSE OF PROB 15.141



$\underline{\omega} = 3.61 \text{ rad/s}$) ◀

$\underline{v}_{P/DE} = 86.6 \text{ in./s} \angle 30°$ ◀

ACCELERATIONS:

SEE SOLUTION OF PROB 15.141 AND EXAMINE EACH ACCELERATION SEPARATELY AS WE RECALL THAT EACH VELOCITY IS REVERSED.

1. NORMAL COMPONENTS OF ACCELERATIONS INVOLVE A SQUARE OF VELOCITY AND ARE NOT CHANGED.
2. TANGENTIAL COMPONENTS OF ACCELERATION DO NOT INVOLVE VELOCITIES.
3. CORIOLIS ACCELERATION INVOLVES THE PRODUCT OF TWO VELOCITIES AND IS NOT CHANGED.

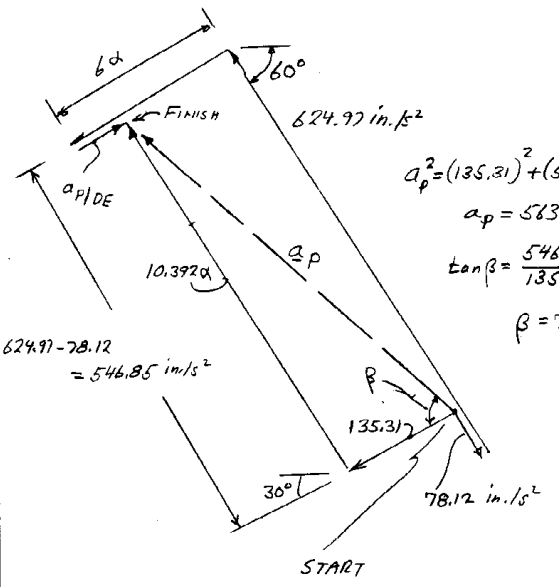
THUS WE FIND THAT NO TERMS OF

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/DE} + \underline{a}_c$ (1)

ARE CHANGED FROM PROB 15.140 AS THE VELOCITIES ARE REVERSED, THE ACCELERATION OF P IS THE SAME AS IN PROB 15.140.

$\underline{a}_P = 563 \text{ in./s}^2 \angle 41.6°$ ◀

THE VECTOR DIAGRAM OF EQ(1) FOR BOTH PROB 15.141 AND 15.142 IS:



$a_P^2 = (135.31)^2 + (546.85)^2$

$a_P = 563 \text{ in./s}^2$

$\tan\beta = \frac{546.85}{135.31}$

$\beta = 76.1°$

15.143

$\omega_x = 1.5$ rad/s $\underline{\omega} = (1.5 \text{ rad/s})\underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$

$\underline{r}_A = -(8 \text{ in.})\underline{i} + (6 \text{ in.})\underline{j} + (4 \text{ in.})\underline{k}$

$\underline{r}_D = +(8 \text{ in.})\underline{i} + (6 \text{ in.})\underline{j} - (4 \text{ in.})\underline{k}$

(a)

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.5 & \omega_y & \omega_z \\ -8 & +6 & +4 \end{vmatrix}$$

$\underline{v}_A = (4\omega_y - 6\omega_z)\underline{i} + (-8\omega_z - 6)\underline{j} + (9 + 8\omega_y)\underline{k}$

BUT, WE ARE GIVEN:

$\underline{v}_A = (3 \text{ in./s})\underline{i} + (14 \text{ in./s})\underline{j} + (v_A)_z \underline{k}$

$(v_A)_x$: $4\omega_y - 6\omega_z = 3$ (1)

$(v_A)_y$: $-8\omega_z - 6 = 14 \rightarrow \omega_z = -2.5$ rad/s (2)

$(v_A)_z$: $9 + 8\omega_y = (v_A)_z$ (3)

SUBSTITUTE $\omega_z = -2.5$ rad/s INTO (1):

$4\omega_y - 6(-2.5) = 3 \quad \omega_y = -3$ rad/s

SUBSTITUTE $\omega_y = -3$ rad/s INTO (3):

$9 + 8(-3) = (v_A)_z \quad (v_A)_z = -15$ in./s

WE HAVE: $\underline{\omega} = (1.5 \text{ rad/s})\underline{i} - (3 \text{ rad/s})\underline{j} - (2.5 \text{ rad/s})\underline{k}$ ◀

(b) VELOCITY OF D.

$$\underline{v}_D = \underline{\omega} \times \underline{r}_D = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.5 & -3 & -2.5 \\ +8 & +6 & -4 \end{vmatrix} = (12 + 15)\underline{i} + (-20 + 6)\underline{j} + (9 + 24)\underline{k}$$

$\underline{v}_D = (27 \text{ in./s})\underline{i} - (14 \text{ in./s})\underline{j} + (33 \text{ in/s})\underline{k}$ ◀

15.144

$\omega_x = -1.5$ rad/s

SEE SOLUTION OF PROB. 15.143 FOR $\underline{r}_A$ AND $\underline{r}_D$

(a)

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1.5 & \omega_y & \omega_z \\ -8 & +6 & +4 \end{vmatrix}$$

$\underline{v}_A = (4\omega_y - 6\omega_z)\underline{i} + (-8\omega_z + 6)\underline{j} + (-9 + 8\omega_y)\underline{k}$

BUT, WE ARE GIVEN:

$\underline{v}_A = (3 \text{ in./s})\underline{i} + (14 \text{ in./s})\underline{j} + (v_A)_z \underline{k}$

$(v_A)_x$: $4\omega_y - 6\omega_z = 3$ (1)

$(v_A)_y$: $-8\omega_z + 6 = 14 \quad \omega_z = -1$ rad/s (2)

$(v_A)_z$: $-9 + 8\omega_y = (v_A)_z$ (3)

SUBSTITUTE $\omega_z = -1$ rad/s INTO (1):

$4\omega_y - 6(-1) = 3 \quad \omega_y = -0.75$ rad/s

SUBSTITUTE $\omega_y = -0.75$ rad/s INTO (3):

$-9 + (8)(-0.75) = (v_A)_z \quad (v_A)_z = -15$ in./s

WE HAVE: $\underline{\omega} = -(1.5 \text{ rad/s})\underline{i} - (0.75 \text{ rad/s})\underline{j} - (1 \text{ rad/s})\underline{k}$ ◀

(b) VELOCITY OF D:

$$\underline{v}_D = \underline{\omega} \times \underline{r}_D = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1.5 & -.75 & -1 \\ 8 & 6 & -4 \end{vmatrix} = (3 + 6)\underline{i} + (-8 - 6)\underline{j} + (-9 + 6)\underline{k}$$

$\underline{v}_D = (9 \text{ in./s})\underline{i} - (14 \text{ in./s})\underline{j} - (3 \text{ in./s})\underline{k}$ ◀

15.145

$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$

$\underline{r}_A = -(100 \text{ mm})\underline{i} + (250 \text{ mm})\underline{j} - (300 \text{ mm})\underline{k}$

$$\underline{v}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -100 & 250 & -300 \end{vmatrix} = (-300\omega_y - 250\omega_z)\underline{i} + (-100\omega_z + 300\omega_x)\underline{j} + (250\omega_x + 100\omega_y)\underline{k}$$

$(v_A)_x = -300\omega_y - 250\omega_z = 10$ mm/s (1)

$(v_A)_y = 300\omega_x - 100\omega_z = (v_A)_y$ (2)

$(v_A)_z = 250\omega_x + 100\omega_y = 80$ mm/s (3)

MULTIPLY (3) BY 3.00 AND ADD (1) AND (3)

$+750\omega_x - 250\omega_z = 10 + 240$ (4)

MULTIPLY (2) BY 2.5 AND WRITE

$+750\omega_x - 250\omega_z = 2.5(v_A)_y$ (5)

EQUATE R.H. SIDES OF (4) AND (5): $250 = 2.5(v_A)_y \quad (v_A)_y = 100$ mm/s ◀

NOTE: COMPONENTS OF $\underline{\omega}$ ARE INDETERMINATE. THE INDETERMINACY CORRESPONDS TO AN UNKNOWN ROTATION ABOUT LINE OA.

15.146

$\omega_z = +3$ rad/s

$\underline{r}_A = -(100 \text{ mm})\underline{i} + (250 \text{ mm})\underline{j} - (300 \text{ mm})\underline{k}$

$\underline{r}_B = (100 \text{ mm})\underline{i} + (250 \text{ mm})\underline{j} - (300 \text{ mm})\underline{k}$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & 3 \\ -100 & 250 & -300 \end{vmatrix} = (-300\omega_y - 750)\underline{i} + (-300 + 300\omega_x)\underline{j} + (250\omega_x + 100\omega_y)\underline{k}$$

$(v_A)_x = -300\omega_y - 750$ (1)

$(v_A)_y = +300\omega_x - 300$ (2)

$(v_A)_z = 250\omega_x + 100\omega_y = 500$ mm/s (3)

$$\underline{v}_B = \underline{\omega} \times \underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & 3 \\ 100 & 250 & -300 \end{vmatrix} = (-300\omega_y - 750)\underline{i} + (300 + 300\omega_x)\underline{j} + (250\omega_x - 100\omega_y)\underline{k}$$

$(v_B)_x = -300\omega_y - 750$ (4)

$(v_B)_y = 300\omega_x + 300$ (5)

$(v_B)_z = 250\omega_x - 100\omega_y = 100$ mm/s (6)

ADD EQS (3) AND (6): $500\omega_x = 600 \quad \omega_x = 1.2$ rad/s

EQ (3): $250(1.2) + 100\omega_y = 500 \quad \omega_y = 2$ rad/s

EQ (1): $(v_A)_x = -300(2) - 750 = -1350$ mm/s; EQ (2): $(v_A)_y = 300(1.2) - 300 = 60$ mm/s

EQ (4): $(v_B)_x = -300(2) - 750 = -1350$ mm/s; EQ (5): $(v_B)_y = (300)(1.2) + 300 = 660$ mm/s

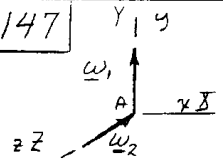
SUMMARY OF RESULTS

$\underline{\omega} = (1.2 \text{ rad/s})\underline{i} + (2 \text{ rad/s})\underline{j} + (3 \text{ rad/s})\underline{k}$ ◀

$\underline{v}_A = -(1350 \text{ mm/s})\underline{i} + (60 \text{ mm/s})\underline{j} + (500 \text{ mm/s})\underline{k}$ ◀

$\underline{v}_B = -(1350 \text{ mm/s})\underline{i} + (660 \text{ mm/s})\underline{j} + (100 \text{ mm/s})\underline{k}$ ◀

15.147



$\underline{\omega}_1 = (4800 \text{ rpm})\underline{j} = (480.77 \text{ rad/s})\underline{j}$

$\underline{\omega}_2 = -(0.5 \text{ rad/s})\underline{k}$

WITH RESPECT TO Axyz:

$(\dot{\omega}_1)_{Axyz} = 0$

$(\dot{\omega}_2)_{Axyz} = -(2.5 \text{ rad/s}^2)\underline{k}$

$\underline{Q} = \underline{\omega}_1 + \underline{\omega}_2$

$\underline{\Omega} = \underline{\omega}_2 =$ ROTATION OF FRAME Axyz (USE EQ 15.31)

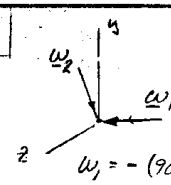
$\underline{\alpha} = (\dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2) = (\dot{\omega}_1 + \dot{\omega}_2)_{Axyz} + \underline{\omega}_2 \times (\underline{\omega}_1 + \underline{\omega}_2)$

$= (\dot{\omega}_2)_{Axyz} + \underline{\omega}_2 \times \underline{\omega}_1$

$= -(2.5 \text{ rad/s}^2)\underline{k} + (-0.5 \text{ rad/s})\underline{k} \times (480.77 \text{ rad/s})\underline{j}$

$\underline{\alpha}_{DISK} = (230 \text{ rad/s}^2)\underline{i} - (2.5 \text{ rad/s}^2)\underline{k}$ ◀

15.148



$\underline{\alpha} = \frac{d\underline{\omega}}{dt} = \underline{\omega}_2 \times \underline{\omega}_1$

$\underline{\alpha} = \underline{\omega}_2 \times (\underline{\omega}_1 + \underline{\omega}_2)$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1$

$\underline{\omega}_1 = -(9000 \text{ rpm})\underline{i} = -(942.48 \text{ rad/s})\underline{i}$

(a) FOR $\underline{\omega}_2 = -(2.4 \text{ rad/s})\underline{j}$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = -(2.4 \text{ rad/s})\underline{j} \times (-942.48 \text{ rad/s})\underline{i}$

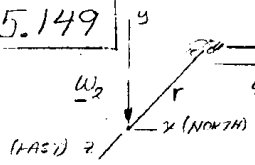
$= -(2262 \text{ rad/s}^2)\underline{k}$ $\quad \underline{\alpha} = -(2260 \text{ rad/s}^2)\underline{k}$ ◀

(b) FOR $\underline{\omega}_2 = -(2.4 \text{ rad/s}^2)\underline{k}$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = -(2.4 \text{ rad/s})\underline{k} \times (-942.48 \text{ rad/s})\underline{i}$

$= +(2262 \text{ rad/s}^2)\underline{j}$ $\quad \underline{\alpha} = (2260 \text{ rad/s}^2)\underline{j}$ ◀

15.149



$v_{AUTO} = 12 \text{ mi/h} = 17.6 \text{ ft/s}$

$\underline{\omega}_1 = -(2750 \text{ rpm})\underline{i} = 288.0 \text{ rad/s})\underline{i}$

$\omega_2 = \frac{v_{AUTO}}{r} = \frac{17.6 \text{ ft/s}}{50 \text{ ft}}$

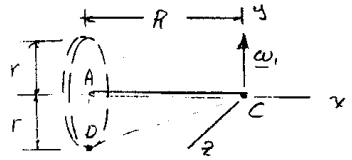
$\underline{\omega}_2 = -(0.352 \text{ rad/s})\underline{j}$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = -(0.352 \text{ rad/s})\underline{j} \times (288.0 \text{ rad/s})\underline{i}$

$\underline{\alpha} = (101.37 \text{ rad/s}^2)\underline{k}$

$\underline{\alpha} = 101.4 \text{ rad/s}^2$ TOWARD THE EAST ◀

15.150



$\underline{r}_{D/C} = -r\underline{i} - r\underline{j}$

CONSIDER WHEEL A (IN ROTATION ABOUT C)

(a) $\omega_y = \omega_1$ $\quad \underline{\omega} = \omega_x\underline{i} + \omega_1\underline{j} + \omega_z\underline{k}$

$\underline{v}_D = 0$

$\underline{v}_D = \underline{\omega} \times \underline{r}_{D/C} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_1 & \omega_z \\ -r & -r & 0 \end{vmatrix} = (r\omega_z)\underline{i} - (r\omega_z)\underline{j} + (-r\omega_x + r\omega_1)\underline{k}$

SINCE $\underline{v}_D = 0$, $r\omega_z = 0$ $\quad \omega_z = 0$

AND $(-r\omega_x + r\omega_1) = 0$ $\quad \omega_x = +\frac{R}{r}\omega_1$

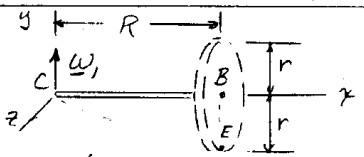
$\underline{\omega} = \frac{R}{r}\omega_1\underline{i} + \omega_1\underline{j}$ ◀

(b) ANGULAR ACCELERATION

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1\underline{j} \times \left(\frac{R}{r}\omega_1\underline{i} + \omega_1\underline{j}\right)$

$\underline{\alpha} = -\left(\frac{R\omega_1^2}{r}\right)\underline{k}$ ◀

15.151



$\underline{r}_{E/C} = R\underline{i} - r\underline{j}$

CONSIDER WHEEL E (IN ROTATION ABOUT C)

(a) $\omega_y = \omega_1$ $\quad \underline{\omega} = \omega_x\underline{i} + \omega_1\underline{j} + \omega_z\underline{k}$, $\underline{v}_E = 0$

$\underline{v}_E = \underline{\omega} \times \underline{r}_{E/C} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_1 & \omega_z \\ r & -r & 0 \end{vmatrix} = (r\omega_z)\underline{i} + (R\omega_z)\underline{j} + (-r\omega_x - R\omega_1)\underline{k}$

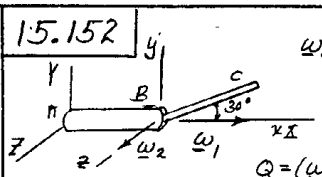
SINCE $\underline{v}_E = 0$, $r\omega_z = 0$, $\omega_z = 0$ AND $(-r\omega_x - R\omega_1) = 0$ $\quad \omega_x = -\frac{R}{r}\omega_1$

$\underline{\omega} = -\frac{R}{r}\omega_1\underline{i} + \omega_1\underline{j}$ ◀

ANGULAR ACCELERATION

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1\underline{j} \times \left(-\frac{R}{r}\omega_1\underline{i} + \omega_1\underline{j}\right)$ $\quad \underline{\alpha} = \left(\frac{R\omega_1^2}{r}\right)\underline{k}$ ◀

15.152



$\underline{\omega}_1 = (3 \text{ rad/s})\underline{i}$ $\quad \underline{\omega}_2 = (4 \text{ rad/s})\underline{k}$

WITH RESPECT TO Bxyz

$(\dot{\omega}_1)_{Bxyz} = 0$ $\quad (\dot{\omega}_2)_{Bxyz} = (5 \text{ rad/s}^2)\underline{k}$

$\underline{\Omega} = (\underline{\omega}_1 + \underline{\omega}_2)$

(use Eq 15.31) $\quad \underline{\Omega} = \underline{\omega}_1 =$ ROTATION OF FRAME Bxyz

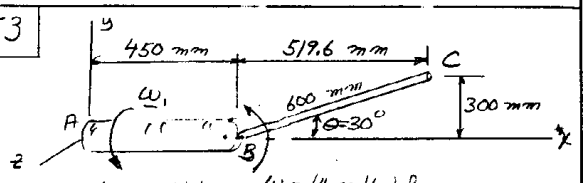
$\underline{\alpha} = (\dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2) = (\dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2)_{Bxyz} + \underline{\omega}_1 \times (\underline{\omega}_1 + \underline{\omega}_2)$

$= (\dot{\underline{\omega}}_2)_{Bxyz} + \underline{\omega}_1 \times \underline{\omega}_2$

$= (5 \text{ rad/s}^2)\underline{k} + (3 \text{ rad/s})\underline{i} \times (4 \text{ rad/s})\underline{k}$

$\underline{\alpha} = -(12 \text{ rad/s}^2)\underline{j} + (5 \text{ rad/s}^2)\underline{k}$ ◀

15.153



$\underline{\omega}_1 = (3 \text{ rad/s})\underline{i}$ $\quad \underline{\omega}_2 = (4 \text{ rad/s})\underline{k}$

$\underline{r}_{C/B} = (519.6 \text{ mm})\underline{i} + (300 \text{ mm})\underline{j}$

$\underline{\omega} = (3 \text{ rad/s})\underline{i} + (4 \text{ rad/s})\underline{k}$

FROM PROB 15.152: $\quad \underline{\alpha} = -(12 \text{ rad/s}^2)\underline{j} + (5 \text{ rad/s}^2)\underline{k}$

VELOCITY OF C:

$\underline{v}_C = \underline{\omega} \times \underline{r}_{C/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 0 & 4 \\ 519.6 & 300 & 0 \end{vmatrix} = -1200\underline{i} + 2078\underline{j} + 900\underline{k}$

$\underline{v}_C = -(1200 \text{ mm/s})\underline{i} + (2078 \text{ mm/s})\underline{j} + (900 \text{ mm/s})\underline{k}$ ◀

ACCELERATION OF C:

$\underline{a}_C = \underline{\alpha} \times \underline{r}_{C/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C/B}) = \underline{\alpha} \times \underline{r}_{C/B} + \underline{\omega} \times \underline{v}_C$

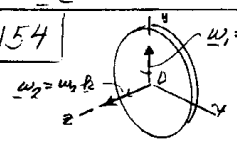
$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -12 & +5 \\ 519.6 & 300 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 0 & 4 \\ -1200 & 2078 & 900 \end{vmatrix}$

$\underline{a}_C = -1500\underline{i} + 2598\underline{j} + 6235\underline{k} - 8312\underline{i} + (-4800 - 2700)\underline{j} + 6234\underline{k}$

$\underline{a}_C = -(9812 \text{ mm/s}^2)\underline{i} - (4902 \text{ mm/s}^2)\underline{j} + (12470 \text{ mm/s}^2)\underline{k}$

$\underline{a}_C = -(9.81 \text{ m/s}^2)\underline{i} - (4.90 \text{ m/s}^2)\underline{j} + (12.47 \text{ m/s}^2)\underline{k}$ ◀

15.154

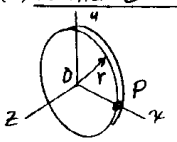


$\underline{\omega} = \omega_1\underline{j} + \omega_2\underline{k}$

(a) $\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1\underline{j} \times (\omega_1\underline{j} + \omega_2\underline{k})$

$\underline{\alpha} = \omega_1\omega_2\underline{i}$ ◀

(b) POSITION $\theta = 0$: $\quad \underline{r}_P = r\underline{i}$



$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (\omega_1\underline{j} + \omega_2\underline{k}) \times r\underline{i}$

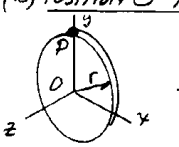
$\underline{v}_P = r\omega_2\underline{j} - r\omega_1\underline{k}$

$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times (\underline{\omega} \times \underline{r}_P) = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

$= (\omega_1\omega_2)\underline{i} \times r\underline{i} + (\omega_1\underline{j} + \omega_2\underline{k}) \times (r\omega_2\underline{j} - r\omega_1\underline{k})$

$= -r\omega_1^2\underline{i} - r\omega_2^2\underline{i}$ $\quad \underline{a}_P = -r(\omega_1^2 + \omega_2^2)\underline{i}$ ◀

(c) POSITION $\theta = 90°$: $\quad \underline{r}_P = r\underline{j}$



$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (\omega_1\underline{j} + \omega_2\underline{k}) \times r\underline{j}$ $\quad \underline{v}_P = -r\omega_2\underline{i}$

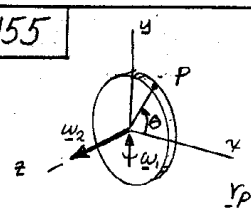
$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times (\underline{\omega} \times \underline{r}_P) = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

$= \omega_1\omega_2\underline{i} \times r\underline{j} + (\omega_1\underline{j} + \omega_2\underline{k}) \times (-r\omega_2\underline{i})$

$= r\omega_1\omega_2\underline{k} + r\omega_1\omega_2\underline{k} - r\omega_2^2\underline{j}$

$\underline{a}_P = -r\omega_2^2\underline{j} + 2r\omega_1\omega_2\underline{k}$ ◀

15.155



$\underline{\omega} = \omega_1 \underline{j} + \omega_2 \underline{k}$

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1 \underline{j} \times (\omega_1 \underline{j} + \omega_2 \underline{k})$

$\underline{\alpha} = \omega_1 \omega_2 \underline{i}$

$\underline{r}_P = r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j}$

VELOCITY OF P: $\underline{v}_P = \underline{\omega} \times \underline{r}_P = (\omega_1 \underline{j} + \omega_2 \underline{k}) \times (r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j})$

$\underline{v}_P = -r\omega_2 \sin\theta\, \underline{i} + r\omega_2 \cos\theta\, \underline{j} - r\omega_1 \cos\theta\, \underline{k}$

ACCELERATION OF P: $\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times (\underline{\omega} \times \underline{r}_P) = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

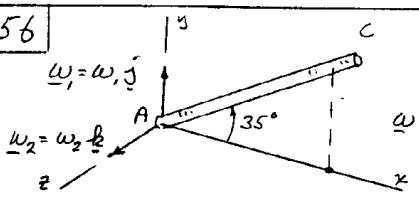
$$\underline{a}_P = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_1\omega_2 & 0 & 0 \\ r\cos\theta & r\sin\theta & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & \omega_1 & \omega_2 \\ -r\omega_2\sin\theta & r\omega_2\cos\theta & -r\omega_1\cos\theta \end{vmatrix}$$

$$\underline{a}_P = r\omega_1\omega_2 \sin\theta\, \underline{k} + (-r\omega_1^2\cos\theta - r\omega_2^2\cos\theta)\underline{i} - r\omega_2^2 \sin\theta\, \underline{j} + r\omega_1\omega_2 \sin\theta\, \underline{k}$$

$$\underline{a}_P = -r(\omega_1^2 + \omega_2^2)\cos\theta\, \underline{i} - r\omega_2^2 \sin\theta\, \underline{j} + 2r\omega_1\omega_2 \sin\theta\, \underline{k}$$ ◀

NOTE: THE ANSWERS TO PROB 15.154 MAY BE CONFIRMED BY SUBSTITUTING $\theta = 0$ AND $\theta = 90°$

15.156



$\underline{r}_C = (32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}$

GIVEN: $\omega_1 = 0.15$ rad/s $\quad \omega_2 = 0.25$ rad/s

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = (0.15 \text{ rad/s})\underline{j} + (0.25 \text{ rad/s})\underline{k}$

(a)

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \underline{\omega}_1 \times (\underline{\omega}_1 + \underline{\omega}_2) = \underline{\omega}_1 \times \underline{\omega}_2$ (1)

$\underline{\alpha} = (0.15 \text{ rad/s})\underline{j} \times (0.25 \text{ rad/s})\underline{k}$

$\underline{\alpha} = (0.0375 \text{ rad/s}^2)\underline{i}$ ◀

(b) VELOCITY OF C:

$$\underline{v}_C = \underline{\omega} \times \underline{r}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.15 & 0.25 \\ 32.77 & 22.94 & 0 \end{vmatrix} = -5.7358\underline{i} + 8.1915\underline{j} - 4.9149\underline{k}$$

$\underline{v}_C = -(5.74 \text{ in./s})\underline{i} + (8.19 \text{ in./s})\underline{j} - (4.91 \text{ in./s})\underline{k}$ ◀

(c) ACCELERATION OF C: $\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C) = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$

$$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.0375 & 0 & 0 \\ 32.77 & 22.94 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.15 & 0.25 \\ -5.736 & 8.191 & -4.915 \end{vmatrix}$$

$\underline{a}_C = 0.8604\underline{k} + (-0.7373 - 2.0478)\underline{i} + (-1.4340)\underline{j} + (0.8604)\underline{k}$

$= -(2.7851 \text{ in/s}^2)\underline{i} - (1.434 \text{ in./s}^2)\underline{j} + (1.7208 \text{ in/s}^2)\underline{k}$

$\underline{a}_C = -(2.79 \text{ in/s}^2)\underline{i} - (1.434 \text{ in./s}^2)\underline{j} + (1.721 \text{ in/s}^2)\underline{k}$ ◀

15.157

SEE SOLUTION OF PROB 15.156

$\underline{r}_C = (32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}$

WE NOW HAVE $\underline{\omega}_1 = (0.25 \text{ rad/s})\underline{j} \quad \underline{\omega}_2 = (0.15 \text{ rad/s})\underline{k}$

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = (0.25 \text{ rad/s})\underline{j} + (0.15 \text{ rad/s})\underline{k}$

(a) EQ(1) $\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_2 = (0.25 \text{ rad/s})\underline{j} \times (0.15 \text{ rad/s})\underline{k}$

$\underline{\alpha} = (0.0375 \text{ rad/s}^2)\underline{i}$ ◀

(b) VELOCITY OF C:

$$\underline{v}_C = \underline{\omega} \times \underline{r}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.25 & 0.15 \\ 32.77 & 22.94 & 0 \end{vmatrix} = (-3.4415)\underline{i} + (4.9149)\underline{j} - (8.1915)\underline{k}$$

$\underline{v}_C = -(3.44 \text{ in./s})\underline{i} + (4.91 \text{ in./s})\underline{j} - (8.19 \text{ in./s})\underline{k}$ ◀

(c) ACCELERATION OF C:

$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C) = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$

$$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.0375 & 0 & 0 \\ 32.77 & 22.94 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.25 & 0.15 \\ -3.441 & 4.915 & -8.191 \end{vmatrix}$$

$\underline{a}_C = 0.8604\underline{k} + (-2.0478 - 0.7373)\underline{i} - (0.5162)\underline{j} + (0.8604)\underline{k}$

$= -(2.7851 \text{ in./s}^2)\underline{i} - (0.5162 \text{ in./s}^2)\underline{j} + (1.7208 \text{ in./s}^2)\underline{k}$

$\underline{a}_C = -(2.79 \text{ in./s}^2)\underline{i} - (0.516 \text{ in./s}^2)\underline{j} + (1.721 \text{ in/s}^2)\underline{k}$ ◀

15.158

SEE SOLUTION OF PROB 15.156

$\underline{r}_C = (32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}$

ANGULAR VELOCITIES AND ANGULAR ACCELERATIONS

GIVEN: $\underline{\omega}_1 = (0.15 \text{ rad/s})\underline{j} \quad ; \quad \dot{\underline{\omega}}_1 = -(0.02 \text{ rad/s}^2)\underline{j}$

$\underline{\omega}_2 = (0.25 \text{ rad/s})\underline{k} \quad ; \quad \dot{\underline{\omega}}_2 = -(0.02 \text{ rad/s}^2)\underline{k}$

(a)

$\underline{\alpha} = \dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2 + \underline{\omega}_1 \times (\underline{\omega}_1 + \underline{\omega}_2) = \dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2 + \underline{\omega}_1 \times \underline{\omega}_2$

$= -(0.02 \text{ rad/s}^2)\underline{j} - (0.02 \text{ rad/s}^2)\underline{k} + (0.15 \text{ rad/s})\underline{j} \times (0.25 \text{ rad/s})\underline{k}$

$\underline{\alpha} = (0.0375 \text{ rad/s}^2)\underline{i} - (0.02 \text{ rad/s}^2)\underline{j} - (0.02 \text{ rad/s}^2)\underline{k}$ ◀

(b) VELOCITY OF C

SINCE VELOCITIES ARE SAME AS IN PROB 15.156 THE VELOCITY OF C IS NOT CHANGED

$\underline{v}_C = -(5.74 \text{ in./s})\underline{i} + (8.19 \text{ in./s})\underline{j} - (4.91 \text{ in./s})\underline{k}$ ◀

(c) ACCELERATION OF C

$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C) = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$

$$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.0375 & -0.02 & -0.02 \\ 32.77 & 22.94 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.15 & 0.25 \\ -5.736 & 8.191 & -4.915 \end{vmatrix}$$

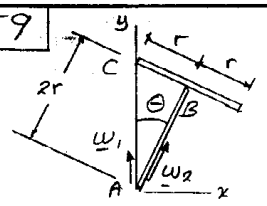
$= +(0.4589)\underline{i} - (0.6553)\underline{j} + (0.8604 + 0.6553)\underline{k}$

$+(-0.7373 - 2.0478)\underline{i} - (1.4340)\underline{j} + 0.8604\underline{k}$

$= -(2.3262 \text{ in/s}^2)\underline{i} - (2.0893 \text{ in./s}^2)\underline{j} + (2.3761 \text{ in./s}^2)\underline{k}$

$\underline{a}_C = -(2.33 \text{ in./s}^2)\underline{i} - (2.091 \text{ in./s}^2)\underline{j} + (2.38 \text{ in./s}^2)\underline{k}$ ◀

15.159



$CA = r/\sin\theta$

$\underline{r}_{C/A} = \frac{r}{\sin\theta}\,\underline{j}$

$\sin\theta = \frac{1}{\sqrt{5}} \quad \cos\theta = \frac{2}{\sqrt{5}}$

$\underline{\omega}_1 = \omega_1\,\underline{j}$

$\underline{\omega}_2 = \omega_2 \sin\theta\,\underline{i} + \omega_2\cos\theta\,\underline{j}$

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2$; $\underline{\omega} = \omega_2\sin\theta\,\underline{i} + (\omega_1 + \omega_2\cos\theta)\,\underline{j}$

(a) $\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = \omega_1\,\underline{j} \times [\omega_2\sin\theta\,\underline{i} + (\omega_1 + \omega_2\cos\theta)\,\underline{j}]$

$\underline{\alpha} = -\omega_1\omega_2\sin\theta\,\underline{k}$; BUT $\sin\theta = \frac{1}{\sqrt{5}}$; $\underline{\alpha} = -\frac{\omega_1\omega_2}{\sqrt{5}}\,\underline{k}$ ◀

(b) VELOCITY OF C.

$$\underline{v}_C = \underline{\omega} \times \underline{r}_{C/A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_2\sin\theta & (\omega_1+\omega_2\cos\theta) & 0 \\ 0 & \frac{r}{\sin\theta} & 0 \end{vmatrix} = r\omega_2\,\underline{k}$$

$\underline{v}_C = r\omega_2\,\underline{k}$ ◀

(c) ACCELERATION OF C.

$\underline{a}_C = \underline{\alpha} \times \underline{r}_{C/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C/A}) = \underline{\alpha} \times \underline{r}_{C/A} + \underline{\omega} \times \underline{v}_C$

$$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -\omega_1\omega_2\sin\theta \\ 0 & \frac{r}{\sin\theta} & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_2\sin\theta & (\omega_1+\omega_2\cos\theta) & 0 \\ 0 & 0 & r\omega_2 \end{vmatrix}$$

$\underline{a}_C = r\omega_1\omega_2\,\underline{i} + r\omega_2(\omega_1+\omega_2\cos\theta)\,\underline{i} - r\omega_2^2\sin\theta\,\underline{j}$

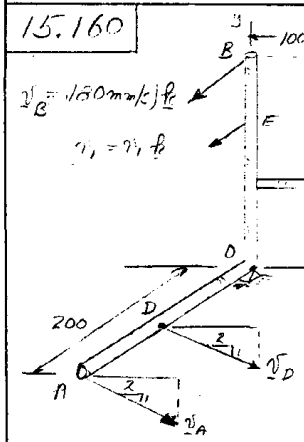
$\underline{a}_C = (2r\omega_1\omega_2 + r\omega_2^2\cos\theta)\,\underline{i} - r\omega_2^2\sin\theta\,\underline{j}$

BUT: $\sin\theta = \frac{1}{\sqrt{5}}$ AND $\cos\theta = \frac{2}{\sqrt{5}}$

$\underline{a}_C = (2r\omega_1\omega_2 + r\omega_2^2\frac{2}{\sqrt{5}})\,\underline{i} - r\omega_2^2\frac{1}{\sqrt{5}}\,\underline{j}$

$\underline{a}_C = 2r(\omega_1\omega_2 + \frac{\omega_2^2}{\sqrt{5}})\,\underline{i} - r\frac{\omega_2^2}{\sqrt{5}}\,\underline{j}$ ◀

15.160



DIMENSIONS IN mm.

$\underline{v}_B = (180\text{ mm/s})\,\underline{k}$

$\underline{r}_1 = r_1\,\underline{k}$

SINCE ROD AT D SLIDES IN SLOT WHICH IS OF 1:2 SLOPE,

$(v_D)_x = -2(v_D)_y$

AND $(v_A)_x = -2(v_A)_y$ (1)

AND FOR USE IN PROB 15.161 WE RECORD: $(a_A)_x = -2(a_A)_y$ (2)

(a) ANGULAR VELOCITY: $\underline{\omega} = \omega_x\,\underline{i} + \omega_y\,\underline{j} + \omega_z\,\underline{k}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_B$: $(180\text{ mm/s})\,\underline{k} = (\omega_x\,\underline{i} + \omega_y\,\underline{j} + \omega_z\,\underline{k}) \times (240\text{ mm})\,\underline{j}$

$180\,\underline{k} = 240\,\omega_x\,\underline{k} - 240\,\omega_z\,\underline{i}$

COEF. OF $\underline{k}$: $180 = 240\,\omega_x$ $\quad \omega_x = 0.75$ rad/s

COEF. OF $\underline{i}$: $0 = -240\,\omega_z$ $\quad \omega_z = 0$

(CONTINUED)

15.160 CONTINUED

$\underline{v}_A = \underline{\omega} \times \underline{r}_A$

$\underline{v}_A = (0.75\,\underline{i} + \omega_y\,\underline{j}) \times (200\,\underline{k})$

$(v_A)_x\,\underline{i} + (v_A)_y\,\underline{j} + (v_A)_z\,\underline{k} = -150\,\underline{j} + 200\,\omega_y\,\underline{i}$

COEF. OF $\underline{j}$: $(v_A)_y = -150$

COEF. OF $\underline{i}$: $(v_A)_x = 200\,\omega_y$

COEF. OF $\underline{k}$: $(v_A)_z = 0$

RECALL EQ(1): $(v_A)_x = -2(v_A)_y$

$200\,\omega_y = -2(-150)$

$\omega_y = 1.5$ rad/s AND $(v_A)_x = 300$ mm/s

$\underline{\omega} = (0.75\text{ rad/s})\,\underline{i} + (1.5\text{ rad/s})\,\underline{j}$ ◀

(b) VELOCITY OF A:

$\underline{v}_A = (300\text{ mm/s})\,\underline{i} - (150\text{ mm/s})\,\underline{j}$ ◀

(c) VELOCITY OF C: $\underline{r}_C = 100\,\underline{i} + 80\,\underline{j} + 40\,\underline{k}$

$$\underline{v}_C = \underline{\omega} \times \underline{r}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.75 & 1.5 & 0 \\ 100 & 80 & 40 \end{vmatrix} = 60\,\underline{i} - 30\,\underline{j} + (60 - 150)\,\underline{k}$$

$\underline{v}_C = (60\text{ mm/s})\,\underline{i} - (30\text{ mm/s})\,\underline{j} - (90\text{ mm/s})\,\underline{k}$ ◀

15.161

GIVEN: $\underline{v}_B = (180\text{ mm/s})\,\underline{k}$; $\underline{r}_B = (240\text{ mm})\,\underline{j}$

$(a_B)_z = 0$; $(a_B)_x = 0$ (IMPOSED BY GUIDE)

THUS: $\underline{a}_B = a_B\,\underline{j} = (a_B)_y\,\underline{j}$

SEE SOLUTION OF PROB 15.160 FOR:

$\underline{\omega} = (0.75\text{ rad/s})\,\underline{i} + (1.5\text{ rad/s})\,\underline{j}$

$\underline{v}_A = (300\text{ mm/s})\,\underline{i} - (150\text{ mm/s})\,\underline{j}$

$\underline{v}_C = (60\text{ mm/s})\,\underline{i} - (30\text{ mm/s})\,\underline{j} - (90\text{ mm/s})\,\underline{k}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}_B) = \underline{\alpha} \times \underline{r}_B + \underline{\omega} \times \underline{v}_B$

$$\underline{a}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 0 & 240 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.75 & 1.5 & 0 \\ 0 & 0 & 180 \end{vmatrix} = -240\alpha_z\,\underline{i} + 240\alpha_x\,\underline{k} + 270\,\underline{i} - 135\,\underline{j}$$

$\underline{a}_B = (270 - 240\alpha_z)\,\underline{i} - 135\,\underline{j} + 240\,\alpha_x\,\underline{k}$ (1)

$(a_B)_x = 270 - 240\,\alpha_z = 0$ $\quad \alpha_z = 1.125$ rad/s^2

$(a_B)_y = -135$ $\quad (a_B)_y = -135$ mm/s^2

$(a_B)_z = 240\,\alpha_x = 0$ $\quad \alpha_x = 0$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_A + \underline{\omega} \times (\underline{\omega} \times \underline{r}_A) = \underline{\alpha} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A$

$$\underline{a}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & \alpha_y & 1.125 \\ 0 & 0 & 200 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.75 & 1.5 & 0 \\ 300 & -150 & 0 \end{vmatrix} = 200\alpha_y\,\underline{i} + (-112.5 - 450)\,\underline{k}$$

$\underline{a}_A = 200\,\alpha_y\,\underline{i} - 562.5\,\underline{k}$

THUS: $(a_A)_x = 200\,\alpha_y$ $\quad (a_A)_y = 0$ $\quad (a_A)_z = -562.5$ mm/s^2

BUT FROM EQ(2) OF PROB 15.160: $(a_A)_x = -2(a_A)_y = 0$

THEREFORE: $(a_A)_x = 200\,\alpha_y = 0$ $\quad \alpha_y = 0$

(a) ANGULAR ACCELERATION: $\underline{\alpha} = (1.125\text{ rad/s}^2)\,\underline{k}$ ◀

(b) ACCELERATION OF C:

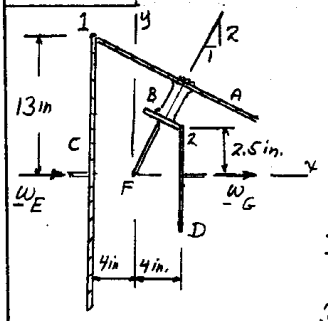
$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C) = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$

$\underline{r}_C = (100\text{ mm})\,\underline{i} + (80\text{ mm})\,\underline{j} + (40\text{ mm})\,\underline{k}$

$$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 1.125 \\ 100 & 80 & 40 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.75 & 1.5 & 0 \\ 60 & -30 & -90 \end{vmatrix} = -90\,\underline{i} + 112.5\,\underline{j} - 135\,\underline{i} + 67.5\,\underline{j} + (-22.5 - 90)\,\underline{k}$$

$\underline{a}_C = -(225\text{ mm/s}^2)\,\underline{i} + (180\text{ mm/s}^2)\,\underline{j} - (112.5\text{ mm/s}^2)\,\underline{k}$ ◀

15.162



PLACE ORIGIN AT F.

POINT 1: $\underline{r}_1 = -(4\text{in.})\underline{i} + (13\text{in.})\underline{j}$

POINT 2: $\underline{r}_2 = +(4\text{in.})\underline{i} + (2.5\text{in.})\underline{j}$

$\underline{\omega}_E = +(30\ \text{rad/s})\underline{i}$

$\underline{\omega}_G = +(20\ \text{rad/s})\underline{i}$

$\underline{v}_1 = \underline{\omega}_E \times \underline{r}_1 = (30\underline{i}) \times (-4\underline{i} + 13\underline{j})$

$\underline{v}_1 = (390\ \text{in./s})\underline{k}$ (1)

$\underline{v}_2 = \underline{\omega}_G \times \underline{r}_2 = (20\underline{i}) \times (4\underline{i} + 2.5\underline{j})$

$\underline{v}_2 = (50\ \text{in./s})\underline{k}$ (2)

MOTION OF GEAR UNIT AB:

$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$

$$\underline{v}_1 = \underline{\omega} \times \underline{r}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -4 & +13 & 0 \end{vmatrix} = -13\omega_z \underline{i} - 4\omega_z \underline{j} + (13\omega_x + 4\omega_y)\underline{k}$$

RECALL FROM EQ(1) THAT $\underline{v}_1 = 390\underline{k}$.

$390\underline{k} = -13\omega_z \underline{i} - 4\omega_z \underline{j} + (13\omega_x + 4\omega_y)\underline{k}$

EQUATE COEFFICIENTS OF UNIT VECTORS:

$\omega_z = 0$

$390 = 13\omega_x + 4\omega_y$ (3)

$$\underline{v}_2 = \underline{\omega} \times \underline{r}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & 0 \\ 4 & 2.5 & 0 \end{vmatrix} = (2.5\omega_x - 4\omega_y)\underline{k}$$

RECALL FROM EQ(2) THAT $\underline{v}_2 = 50\underline{k}$, AND WRITE

$50 = 2.5\omega_x - 4\omega_y$ (4)

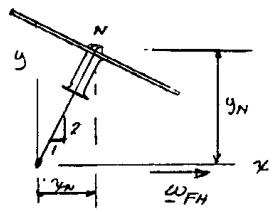
ADD (3) AND (4): $440 = 15.5\,\omega_x$ $\quad \omega_x = 28.387$ rad/s

EQ(4): $50 = 2.5(28.387) - 4\omega_y$ $\quad \omega_y = 5.242$ rad/s

(a) COMMON VELOCITY OF UNIT AB. $\underline{\omega} = (28.387\text{rad/s})\underline{i} + (5.242\ \text{rad/s})\underline{j}$

$\underline{\omega} = (28.4\ \text{rad/s})\underline{i} + (5.24\ \text{rad/s})\underline{j}$ ◀

(b) ANGULAR VELOCITY OF SHAFT FH. (SEE FIG IN TEXT)



POINT N IS AT NUT, WHICH IS A PART OF UNIT AB AND ALSO IS A PART OF SHAFT GH.

$x_N = \frac{1}{2} y_N$

$\underline{r}_N = x_N \underline{i} + y_N \underline{j}$

$\underline{r}_N = \frac{1}{2} y_N \underline{i} + y_N \underline{j}$

NUT N AS A PART OF UNIT AB: $\underline{\omega} = 28.387\text{rad/s})\underline{i} + (5.242\text{r/s})\underline{j}$

$\underline{v}_N = \underline{\omega} \times \underline{r}_N = (28.387\underline{i} + 5.242\underline{j}) \times \left(\frac{1}{2} y_N \underline{i} + y_N \underline{j}\right)$

$\underline{v}_N = (28.387 y_N - 2.621 y_N)\underline{k} = +(25.766 y_N)\underline{k}$ (5)

NUT N AS A PART OF SHAFT FH. $\underline{\omega}_{FH} = \omega_{FH}\underline{i}$

$\underline{v}_N = \underline{\omega}_{FH} \times \underline{r}_N = (\omega_{FH}\underline{i}) \times \left(\frac{1}{2} y_N \underline{i} + y_N \underline{j}\right) = \omega_{FH} y_N \underline{k}$ (6)

EQUATING EXPRESSIONS FOR $\underline{v}_N$ FROM (5) AND (6),

$+(25.766\, y_N)\underline{k} = \omega_{FH} y_N \underline{k}$

$\omega_{FH} = 25.8\ \text{rad/s}$ $\quad \underline{\omega}_{FH} = (25.766\ \text{rad/s})\underline{i}$ ◀

15.163

SEE SOLUTION OF PROB 15.162 FOR

(a) ANGULAR ACCELERATION OF GEAR UNIT AB

ANGULAR VELOCITY OF UNIT IS

$\underline{\omega} = (28.387\ \text{rad/s})\underline{i} + (5.242\ \text{rad/s})\underline{j}$

VECTOR $\underline{\omega}$ ROTATES ABOUT x AXIS WITH

$\underline{\omega}_{FH} = (25.766\ \text{rad/s})\underline{i}$

$\underline{\alpha} = \underline{\omega}_{FH} \times \underline{\omega} = (25.766\underline{i}) \times (28.387\underline{i} + 5.242\underline{j})$

$\underline{\alpha} = (135.07\ \text{rad/s}^2)\underline{k}$ $\quad \underline{\alpha} = (135.1\ \text{rad/s}^2)\underline{k}$ ◀

(b) ACCELERATION OF TOOTH AT POINT 1 OF GEAR A

$\underline{a}_1 = \underline{\alpha} \times \underline{r}_1 + \underline{\omega} \times (\underline{\omega} \times \underline{r}_1) = \underline{\alpha} \times \underline{r}_1 + \underline{\omega} \times \underline{v}_1$

$\underline{a}_1 = (135.07\ \text{rad/s}^2)\underline{k} \times \left[-(4\text{in.})\underline{i} + (13\text{in.})\underline{j}\right]$

$+ \left[(28.387\ \text{rad/s})\underline{i} + (5.242\ \text{rad/s})\underline{j}\right] \times (390\ \text{in./s})\underline{k}$

$\underline{a}_1 = -540.3\underline{j} - 1756.9\underline{i} - 11{,}070.9\underline{j} + 2044.4\underline{i}$

$= (288\ \text{in./s}^2)\underline{i} - (11{,}611\ \text{in./s}^2)\underline{j}$

$\underline{a}_1 = (24\ \text{ft/s}^2)\underline{i} - (968\ \text{ft/s}^2)\underline{j}$ ◀

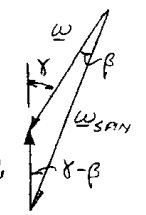
15.164

SINCE SMALL CONE ROLLS ON THE FIXED LARGE CONE, $\underline{v}_O = \underline{v}_A = 0$.

THUS OA IS THE INSTANTANEOUS AXIS OF ROTATION AND DIRECTION OF TOTAL ANGULAR VELOCITY $\underline{\omega}$ IS ⬀.

WE RESOLVE $\underline{\omega}$ INTO COMPONENTS $\underline{\omega}_1$ (ABOUT y AXIS) AND $\underline{\omega}_{SPIN}$ (ABOUT AB)

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_{SPIN}$



LAW OF SINES:

$$\frac{\omega_{SPIN}}{\sin\gamma} = \frac{\omega}{\sin(\gamma-\beta)} = \frac{\omega_1}{\sin\beta}$$

(a) $\omega_{SPIN} = \dfrac{\sin\gamma}{\sin\beta}\,\omega_1$ ◀

(b) $\underline{\omega} = \dfrac{\sin(\gamma-\beta)}{\sin\beta}\,\omega_1$ ⬀ ◀

(c) ANGULAR ACCELERATION $\underline{\omega}$ ROTATES ABOUT y AT THE RATE ω_1

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (\omega_1 \omega_2 \sin\gamma)\underline{k}$

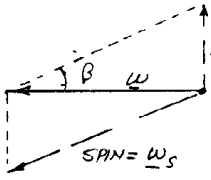
$\underline{\alpha} = \dfrac{\sin(\gamma-\beta)}{\sin\beta} \sin\gamma\ \omega_1^2\,\underline{k}$ ◀

15.165

SINCE CONE ROLLS ON ZX PLANE, $\underline{v}_O = \underline{v}_A = 0$

THUS, OA IS INSTANTANEOUS AXIS OF ROTATION

WE RESOLVE $\underline{\omega} = -\omega\underline{i}$ INTO ω_1 AND ω_S (SPIN).



(a) RATE OF SPIN: $\omega_S = \dfrac{\omega_1}{\sin\beta}$ ◀

(b) TOTAL ANGULAR VELOCITY

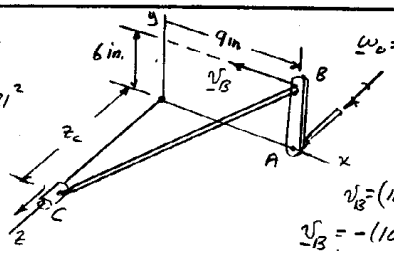
$\omega = -\dfrac{\omega_1}{\tan\beta}$ $\quad \underline{\omega} = -\dfrac{\omega_1}{\tan\beta}\,\underline{i}$ ◀

(c) ANGULAR ACCELERATION WE NOTE THAT THE VECTOR $\underline{\omega}$ ROTATES ABOUT y AXIS AT RATE ω_1

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (\omega_1 \underline{j}) \times \left(-\dfrac{\omega_1}{\tan\beta}\,\underline{i}\right)$

$\underline{\alpha} = \dfrac{\omega_1^2}{\tan\beta}\,\underline{k}$ ◀

15.166



$BC = 21$ in.

$z_c^2 + 6^2 + 9^2 = 21^2$

$z_c = 18$ in.

$\underline{v}_C = v_C\,\underline{k}$

$\underline{\omega} = (18\ \text{rad/s})\,\underline{k}$

$\underline{r}_{B/A} = (6\text{ in.})\,\underline{j}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/A}$

$\underline{v}_B = (18\,\underline{k}) \times (6\,\underline{j})$

$\underline{v}_B = -(108\text{ in./s})\,\underline{i}$

ROD BC $\quad \underline{r}_{C/B} = -(9\text{ in.})\,\underline{i} - (6\text{ in.})\,\underline{j} + (18\text{ in.})\,\underline{k}$

$\underline{\omega} = \omega_x\,\underline{i} + \omega_y\,\underline{j} + \omega_z\,\underline{k}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$(v_C)\,\underline{k} = -108\,\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -9 & -6 & 18 \end{vmatrix}$$

$$v_C\,\underline{k} = -108\,\underline{i} + (18\omega_y + 6\omega_z)\,\underline{i} + (-9\omega_z - 18\omega_x)\,\underline{j} + (-6\omega_x + 9\omega_y)\,\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}$, $\underline{j}$, $\underline{k}$:

$\underline{i}$: $\quad +108 = 18\omega_y + 6\omega_z \qquad (1)$

$\underline{j}$: $\quad 0 = -18\omega_x - 9\omega_z \qquad \omega_z = -2\omega_x \qquad (2)$

$\underline{k}$: $\quad v_C = -6\omega_x + 9\omega_y \qquad (3)$

SUBSTITUTE $\omega_z = -2\omega_x$ INTO EQ (1)

$108 = 18\omega_y - 12\omega_x \qquad (4)$

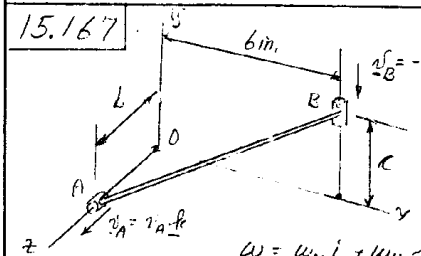
DIVIDE EQ (4) BY 2.0: $\quad 54 = -6\omega_x + 9\omega_y \qquad (5)$

SUBTRACT EQ (3) FROM EQ (2): $\quad v_C - 54 = 0$

$v_C = 54$ in./s $\qquad \underline{v}_C = (54\text{ in./s})\,\underline{k}$ ◀

ANGULAR VELOCITY IS INDETERMINATE. THIS INDETERMINACY CORRESPONDS TO AN ARBITRARY SPIN OF ROD ABOUT ITS AXIS

15.167



$AB = 11$ in. $\quad c = 2$ in.

$(11)^2 = b^2 + 2^2 + b^2$

$b = 9$ in.

$\underline{\omega} = \omega_x\,\underline{i} + \omega_y\,\underline{j} + \omega_z\,\underline{k}$

$\underline{r}_{A/B} = -(6\text{ in.})\,\underline{i} - (2\text{ in.})\,\underline{j} + (9\text{ in.})\,\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A\,\underline{k} = -54\,\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -6 & -2 & 9 \end{vmatrix}$$

$$v_A\,\underline{k} = -54\,\underline{j} + (9\omega_y + 2\omega_z)\,\underline{i} + (-6\omega_z - 9\omega_x)\,\underline{j} + (-2\omega_x + 6\omega_y)\,\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}$, $\underline{j}$, $\underline{k}$:

$\underline{i}$: $\quad 0 = +9\omega_y + 2\omega_z \qquad \omega_z = -4.5\omega_y \qquad (1)$

$\underline{j}$: $\quad -54 = -9\omega_x - 6\omega_z \qquad (2)$

$\underline{k}$: $\quad v_A = -2\omega_x + 6\omega_y \qquad (3)$

SUBSTITUTE $\omega_z = -4.5\omega_y$ INTO (2)

$-54 = -9\omega_x + 27\omega_y \qquad (4)$

MULTIPLY (4) BY $\frac{2}{9}$: $\quad 12 = -2\omega_x + 6\omega_y \qquad (5)$

SUBTRACT (5) FROM (3): $\quad +v_A - 12 = 0$

$v_A = (12\text{ in./s})\,\underline{k}$ ◀

NOTE: ANGULAR VELOCITY IS INDETERMINATE. ROD MAY FREELY SPIN ABOUT ITS AXIS WITHOUT CHANGING v_A

ALTERNATE SOLUTION

FOR ROD BC: $\quad 11^2 = b^2 + c^2 + b^2$ THUS $b^2 + c^2 = \text{CONSTANT}$

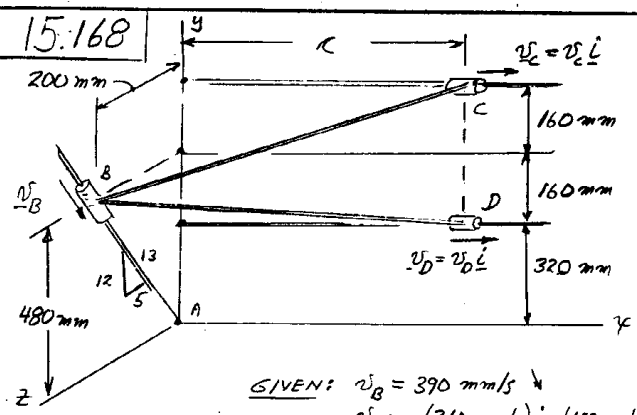
$\frac{d}{dt}$: $\quad 2b\,\frac{db}{dt} + 2c\,\frac{dc}{dt} = 0 \qquad (6)$

BUT $\frac{db}{dt} = v_A$ AND $\frac{dc}{dt} = v_B$ $\quad \therefore \quad v_A = -\frac{c}{b}\,v_B$

DATA: $\quad b = 9$ in.; $\quad c = 2$ in.; $\quad v_B = -54$ in/s

$v_A = -\frac{2\text{ in.}}{9\text{ in.}}(-54\text{ in./s}) \qquad \underline{v}_A = (12\text{ in./s})\,\underline{k}$ ◀

15.168



GIVEN: $\quad \underline{v}_B = 390$ mm/s ↘

$\underline{v}_B = -(360\text{ mm/s})\,\underline{j} - (150\text{ mm/s})\,\underline{k}$

$AB = 520$ mm $\qquad BC = BD = 840$ mm

$(BC)^2 = (BD)^2 = (840)^2 = (160)^2 + (200)^2 + c^2 \qquad c = 800$ mm

$\underline{r}_{C/B} = (800\text{ mm})\,\underline{i} + (160\text{ mm})\,\underline{j} - (200\text{ mm})\,\underline{k}$

FOR USE IN PROB 15.169: $\underline{r}_{D/C} = (800\text{ mm})\,\underline{i} - (160\text{ mm})\,\underline{j} - (200\text{ mm})\,\underline{k}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$v_C\,\underline{i} = -360\,\underline{j} - 150\,\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 800 & +160 & -200 \end{vmatrix}$$

$$v_C\,\underline{i} = -360\,\underline{j} - 150\,\underline{k} + (-200\omega_y - 160\omega_z)\,\underline{i} + (800\omega_z + 200\omega_x)\,\underline{j} + (160\omega_x - 800\omega_y)\,\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}$, $\underline{j}$, $\underline{k}$:

$\underline{i}$: $\quad v_C = -200\omega_y - 160\omega_z \qquad (1)$

$\underline{j}$: $\quad 360 = +200\omega_x + 800\omega_z \qquad (2)$

$\underline{k}$: $\quad 150 = 160\omega_x - 800\omega_y \qquad (3)$

MULTIPLY (3) BY 1.25:

$187.5 = +200\omega_x - 1000\omega_y \qquad (4)$

SUBTRACT (4) FROM (2): $\quad 172.5 = +1000\omega_y + 800\omega_z \qquad (5)$

MULTIPLY (5) BY 0.2: $\quad 34.5 = +200\omega_y + 160\omega_z \qquad (6)$

ADD (1) AND (6): $\quad v_C + 34.5 = 0 \qquad \underline{v}_C = -(34.5\text{ mm/s})\,\underline{i}$ ◀

ALTERNATE SOLUTION. LET $AB = s$ NOTE $y_C = 640$ mm

$(AB)_z = \frac{5}{13}s \qquad (AB)_y = \frac{12}{13}s$

ROD BC:

$(BC)_x = c; \quad (BC)_y = (640 - \frac{12}{13}s), \quad (BC)_z = \frac{5}{13}s$

$(BC)^2 = c^2 + (640 - \frac{12}{13}s)^2 + (\frac{5}{13}s)^2 = \text{CONSTANT} = (630\text{ mm})^2$

$\frac{d}{dt}$: $\quad 0 = 2c\,\frac{dc}{dt} + 2(640 - \frac{12}{13}s)(-\frac{12}{13})\frac{ds}{dt} + 2(\frac{5}{13})^2 s\,\frac{ds}{dt}$

$c\,\frac{dc}{dt} = -\left[(640 - \frac{12}{13}s)(-\frac{12}{13}) + (\frac{5}{13})^2 s\right]\frac{ds}{dt}$

GIVEN DATA: $\quad c = 800$ mm, $\quad s = AB = 520$ mm

$\frac{ds}{dt} = v_B = -390$ mm/s

$800\,\frac{dc}{dt} = -\left[\{640 - \frac{12}{13}(520)\}(-\frac{12}{13}) + (\frac{5}{13})^2 520\right](-390)$

$800\,\frac{dc}{dt} = \left[-\{640 - 480\}\frac{12}{13} + \frac{25}{13}40\right]390$

$800\,\frac{dc}{dt} = \left[-\{160\}12 + 1000\right]30 = -27{,}600$

$\frac{dc}{dt} = -34.5$, BUT, $\frac{dc}{dt} = v_C$

$\underline{v}_C = -(34.5\text{ mm/s})\,\underline{i}$ ◀

15.169

SEE SOLUTION OF PROB 15.168

WE NOW USE: $\underline{r}_{D/B} = (800\text{ mm})\underline{i} - (160\text{ mm})\underline{j} - (200\text{ mm})\underline{k}$

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{BD}$

$$\underline{v}_D = -360\underline{j} - 150\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 800 & -160 & -200 \end{vmatrix}$$

$$\underline{v}_D = -360\underline{j} - 150\underline{k} + (-200\omega_y + 160\omega_z)\underline{i} + (800\omega_z + 200\omega_x)\underline{j} + (-160\omega_x - 800\omega_y)\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

$\underline{i}$: $v_D = -200\omega_y + 160\omega_z$ (1)

$\underline{j}$: $360 = 200\omega_x + 800\omega_z$ (2)

$\underline{k}$: $150 = -160\omega_x - 800\omega_y$ (3)

MULTIPLY (3) BY 1.25:

$187.5 = -200\omega_x - 1000\omega_y$ (4)

ADD (2) AND (4): $547.5 = -1000\omega_y + 800\omega_z$ (5)

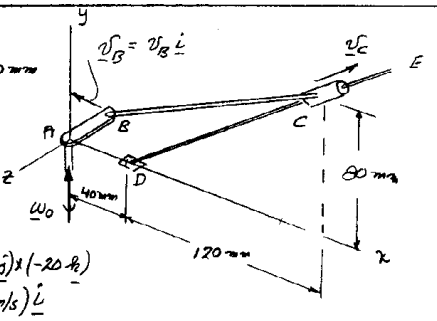
MULTIPLY (5) BY 0.2: $109.5 = -200\omega_y + 160\omega_z$ (6)

SUBTRACT (6) FROM (1): $v_D - 109.5 = 0$

$$\underline{v}_D = (109.5\text{ mm/s})\underline{i}$$ ◀

AN ALTERNATE SOLUTION SIMILAR TO THAT SHOWN IN PROB 15.168 MAY BE USED.

15.170



ARM AB AB = 20 mm

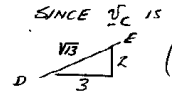
$\underline{r}_{B/A} = -(20\text{ mm})\underline{k}$

$\underline{\omega}_0 = (24\text{ rad/s})\underline{j}$

$\underline{v}_B = \underline{\omega}_0 \times \underline{r}_{B/A} = (24\underline{j}) \times (-20\underline{k})$

$\underline{v}_B = -(480\text{ mm/s})\underline{i}$

SINCE $\underline{v}_C$ IS DIRECTED ALONG ROD DE,



$(v_C)_y = \frac{2}{3}(v_C)_x$; $v_C = \frac{\sqrt{13}}{3}(v_C)_x$ ∠ 33.7°

ROD BC $\underline{r}_{C/B} = (160\text{ mm})\underline{i} + (80\text{ mm})\underline{j} + (20\text{ mm})\underline{k}$

$\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$(v_C)_x\underline{i} + (v_C)_y\underline{j} = -480\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 160 & 80 & 20 \end{vmatrix}$$

$$(v_C)_x\underline{i} + (v_C)_y\underline{j} = -480\underline{i} + (20\omega_y - 80\omega_z)\underline{i} + (160\omega_z - 20\omega_x)\underline{j} + (80\omega_x - 160\omega_y)\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$

$\underline{i}$: $(v_C)_x + 480 = 20\omega_y - 80\omega_z$ (1)

$\underline{j}$: $(v_C)_y = -20\omega_x + 160\omega_z$ (2)

$\underline{k}$: $0 = 80\omega_x - 160\omega_y$ $\omega_x = 2\omega_y$ (3)

SUBSTITUTE $\omega_x = 2\omega_y$ AND $(v_C)_y = \frac{2}{3}(v_C)_x$ INTO EQ (2):

$\frac{2}{3}(v_C)_x = -40\omega_y + 160\omega_z$ (4)

MULTIPLY EQ (4) BY 0.5:

$\frac{1}{3}(v_C)_x = -20\omega_y + 80\omega_z$ (5)

ADD EQ (1) AND EQ (5)

$(v_C)_x + 480 + \frac{1}{3}(v_C)_x = 0$

(CONTINUED)

15.170 CONTINUED

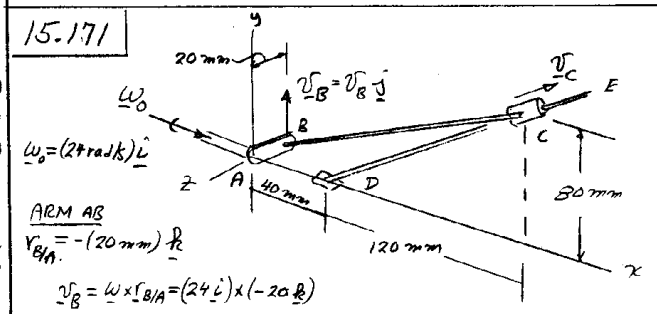
$\frac{4}{3}(v_C)_x = -480$; $(v_C)_x = -360\text{ mm/s}$

RECALL THAT $(v_C)_y = \frac{2}{3}(v_C)_x = \frac{2}{3}(-360\text{ mm/s}) = -240\text{ mm/s}$

WE NOTE VELOCITY OF COLLAR IS DIRECTED TOWARD D

$$\underline{v}_C = -(360\text{ mm/s})\underline{i} - (240\text{ mm/s})\underline{j}$$ ◀

15.171



$\underline{\omega}_0 = (24\text{ rad/s})\underline{i}$

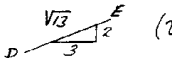
ARM AB

$\underline{r}_{B/A} = -(20\text{ mm})\underline{k}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/A} = (24\underline{i}) \times (-20\underline{k})$

$\underline{v}_B = (480\text{ mm/s})\underline{j}$

SINCE $\underline{v}_C$ IS DIRECTED ALONG ROD DE,



$(v_C)_y = \frac{2}{3}(v_C)_x$; $v_C = \frac{\sqrt{13}}{3}(v_C)_x$ ∠ 33.7°

ROD BC $\underline{r}_{C/B} = (160\text{ mm})\underline{i} + (80\text{ mm})\underline{j} + (20\text{ mm})\underline{k}$

$\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$(v_C)_x\underline{i} + (v_C)_y\underline{j} = 480\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 160 & 80 & 20 \end{vmatrix}$$

$$(v_C)_x\underline{i} + (v_C)_y\underline{j} = 480\underline{j} + (20\omega_y - 80\omega_z)\underline{i} + (160\omega_z - 20\omega_x)\underline{j} + (80\omega_x - 160\omega_y)\underline{k}$$

EQUATE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$.

$\underline{i}$: $(v_C)_x = 20\omega_y - 80\omega_z$ (1)

$\underline{j}$: $(v_C)_y - 480 = -20\omega_x + 160\omega_z$ (2)

$\underline{k}$: $0 = 80\omega_x - 160\omega_y$ (3)

FROM EQ (3): $\omega_x = 2\omega_y$

SUBSTITUTE $\omega_x = 2\omega_y$ AND $(v_C)_y = \frac{2}{3}(v_C)_x$ INTO EQ (2):

$\frac{2}{3}(v_C)_x - 480 = -40\omega_x + 160\omega_z$ (4)

MULTIPLY EQ (4) BY 0.5:

$\frac{1}{3}(v_C)_x - 240 = -20\omega_x + 80\omega_z$ (5)

ADD EQ (1) AND EQ (5):

$(v_C)_x + \frac{1}{3}(v_C)_x - 240 = 0$

$\frac{4}{3}(v_C)_x = 240$ $(v_C)_x = 180\text{ mm/s}$

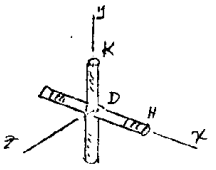
RECALL THAT

$(v_C)_y = \frac{2}{3}(v_C)_x = \frac{2}{3}(180\text{ mm/s})$; $(v_C)_y = 120\text{ mm/s}$

$$\underline{v}_C = (180\text{ mm/s})\underline{i} + (120\text{ mm/s})\underline{j}$$ ◀

15.172

PLACE ORIGIN AT CENTER OF CROSS PIECE AND DENOTE BY l THE LENGTH OF EACH ARM.



$\underline{r}_K = l\,\underline{j}$

$\underline{r}_H = l\,\underline{i}$

$\underline{\omega}_1 = \omega_1\,\underline{k}$

$\underline{\omega}_2 = -\omega_2 \sin 20^\circ\,\underline{j} + \omega_2 \cos 20^\circ\,\underline{k}$

SHAFT AC. $\underline{v}_K = \underline{\omega}_1 \times \underline{r}_K = (\omega_1\underline{k}) \times (l\underline{j}) = -l\omega_1\,\underline{i}$ (1)

SHAFT EG. $\underline{v}_H = \underline{\omega}_2 \times \underline{r}_H = (-\omega_2 \sin 20^\circ\,\underline{j} + \omega_2 \cos 20^\circ\,\underline{k}) \times (l\underline{i})$

$\underline{v}_H = +l\omega_2 \sin 20^\circ\,\underline{k} + l\omega_2 \cos 20^\circ\,\underline{j}$ (2)

CROSSPIECE: $\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_K = \underline{\omega} \times \underline{r}_K = (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) \times (l\underline{j})$

$\underline{v}_K = l\omega_x\underline{k} - l\omega_z\underline{i}$ (3)

$\underline{v}_H = \underline{\omega} \times \underline{r}_H = (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) \times (l\underline{i})$

$\underline{v}_H = -l\omega_y\underline{k} + l\omega_z\underline{j}$ (4)

EQS (1) AND (3): $\underline{v}_K = \underline{v}_K$: $-l\omega_1\underline{i} = l\omega_x\underline{k} - l\omega_z\underline{i}$

EQUATE COEFFICIENTS OF $\underline{i}$ AND $\underline{k}$:

$\omega_x = 0$ $\omega_z = \omega_1$ (5)

EQS (2) AND (4): $\underline{v}_H = \underline{v}_H$:

$l\omega_2 \sin 20^\circ\,\underline{k} + l\omega_2 \cos 20^\circ\,\underline{j} = -l\omega_y\underline{k} + l\omega_z\underline{j}$

EQUATE COEFFICIENTS OF $\underline{j}$ AND $\underline{k}$:

$\underline{k}$: $l\omega_2 \sin 20^\circ = -l\omega_y$ $\omega_y = -\omega_2 \sin 20^\circ$

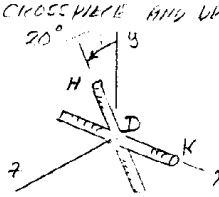
$\underline{j}$: $l\omega_2 \cos 20^\circ = l\omega_z$ $\omega_z = \omega_2 \cos 20^\circ$ (6)

BUT, FROM (5): $\omega_z = \omega_1$

EQ (6): $\omega_1 = \omega_2 \cos 20^\circ$ $\underline{\omega}_2 = \dfrac{\omega_1}{\cos 20^\circ}$ ◀

15.173

PLACE ORIGIN AT CENTER OF CROSSPIECE AND DENOTE BY l THE LENGTH OF EACH ARM.



$\underline{r}_K = l\,\underline{i}$

$\underline{r}_H = l\cos 20^\circ\,\underline{j} + l\sin 20^\circ\,\underline{k}$

$\underline{\omega}_1 = \omega_1\,\underline{k}$

$\underline{\omega}_2 = -\omega_2 \sin 20^\circ\,\underline{j} + \omega_2 \cos 20^\circ\,\underline{k}$

SHAFT AC: $\underline{v}_K = \underline{\omega}_1 \times \underline{r}_K = \omega_1\underline{k} \times l\underline{i} = l\omega_1\,\underline{j}$ (1)

SHAFT EG: $\underline{v}_H = \underline{\omega}_2 \times \underline{r}_H$

$= (-\omega_2 \sin 20^\circ\,\underline{j} + \omega_2 \cos 20^\circ\,\underline{k}) \times (l\cos 20^\circ\,\underline{j} + l\sin 20^\circ\,\underline{k})$

$\underline{v}_H = -\omega_2 l(\sin 20^\circ)^2\,\underline{i} - \omega_2 l(\cos 20^\circ)^2\,\underline{i}$

$\underline{v}_H = -\omega_2 l(\sin^2 20^\circ + \cos^2 20^\circ)\,\underline{i}$ $\underline{v}_H = -l\omega_2\,\underline{i}$ (2)

CROSSPIECE $\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_K = \underline{\omega} \times \underline{r}_K = (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) \times (l\underline{i})$

$\underline{v}_K = -l\omega_y\underline{k} + l\omega_z\underline{j}$ (3)

EQS (1) AND (3): $\underline{v}_K = \underline{v}_K$: $l\omega_1\underline{j} = -l\omega_y\underline{k} + l\omega_z\underline{j}$

COEFFICIENTS OF $\underline{j}$: $l\omega_1 = l\omega_z$ $\omega_z = \omega_1$ (4)

COEFFICIENTS OF $\underline{k}$: $0 = -l\omega_y$ $\omega_y = 0$ (5)

(CONTINUED)

15.173 CONTINUED

CROSSPIECE $\underline{v}_H = \underline{\omega} \times \underline{r}_H$

$$\underline{v}_H = \underline{\omega} \times \underline{r}_H = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & l\cos 20^\circ & l\sin 20^\circ \end{vmatrix}$$

$\underline{v}_H = (l\omega_y \sin 20^\circ - l\omega_z \cos 20^\circ)\,\underline{i} - (l\omega_x \sin 20^\circ)\,\underline{j} + (l\omega_x \cos 20^\circ)\,\underline{k}$

RECALLING FROM (2) THAT $\underline{v}_H = -l\omega_2\,\underline{i}$ AND EQUATING COEFFICIENTS OF $\underline{i}$, $\underline{j}$, $\underline{k}$, WE WRITE

$\underline{i}$: $-l\omega_2 = l\omega_y \sin 20^\circ - l\omega_z \cos 20^\circ$ (6)

$\underline{j}$: $0 = -l\omega_x \sin 20^\circ$

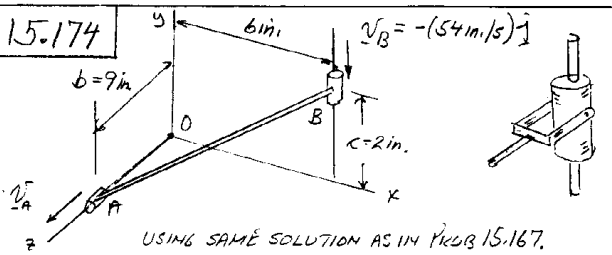
$\underline{k}$: $0 = +l\omega_x \cos 20^\circ$ } $\omega_x = 0$

FROM (4) AND (5): $\omega_y = 0$ $\omega_z = \omega_1$

SUBSTITUTE INTO (6): $-l\omega_2 = 0 - l\omega_1 \cos 20^\circ$

$\omega_2 = \omega_1 \cos 20^\circ$ ◀

15.174



USING SAME SOLUTION AS IN PROB 15.167. WE AGAIN HAVE

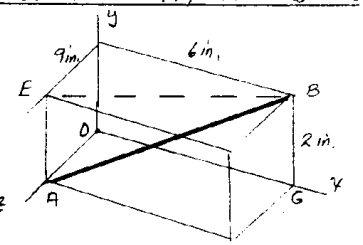
$0 = -9\omega_y - 2\omega_z$ (1)

$-54 = 9\omega_x + 6\omega_z$ (2)

$-v_A = 2\omega_x - 6\omega_y$ (3)

WE AGAIN FIND $v_A = 12$ $\underline{v}_A = (12\text{ in./s})\,\underline{k}$ ◀

ANGULAR VELOCITY OF RODAB



CLEVIS PREVENTS ROTATION OF AB ABOUT AXIS AE WHICH IS ⊥ TO VERTICAL ROD AT E AND WHICH LIES IN PLANE CONTAINING BG AND AB

$\underline{r}_{E/B} = -(6\text{ in.})\,\underline{i} + (9\text{in.})\,\underline{k}$

$\underline{\omega} \cdot \underline{r}_{E/B} = 0$: $(\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) \cdot (-6\,\underline{i} + 9\,\underline{k}) = 0$

$-6\omega_x + 9\omega_z = 0$ $\omega_x = 1.5\omega_z$ (4)

SUBSTITUTE INTO (2)

$-54 = 9(1.5\omega_z) + 6\omega_z$; $-54 = 19.5\omega_z$ $\omega_z = -2.769$ rad/s

EQ (1): $0 = -9\omega_y - 2(-2.769)$ $\omega_y = 0.615$ rad/s

EQ (4): $\omega_x = 1.5(-2.769)$ $\omega_x = -4.154$ rad/s

$\underline{\omega} = -(2.77\text{ rad/s})\,\underline{i} + (0.615\text{ rad/s})\,\underline{j} - (4.15\text{ rad/s})\,\underline{k}$ ◀

ALTERNATE SOLUTION: (SEE FOOTNOTE page 689)

$\underline{r}_{B/G} \times \underline{r}_{B/A}$ IS ⊥ TO PLANE OF AB AND BG (AND EB).

$\underline{r}_{E/G} \times (\underline{r}_{B/G} \times \underline{r}_{B/A})$ IS ⊥ TO BG AND IS CONTAINED IN PLANE OF AB AND BG (AND EB). THUS, IT IS IN DIRECTION OF EB. FOR CLEVIS, WE HAVE

$\underline{\omega} \cdot [\underline{r}_{B/G} \times (\underline{r}_{B/G} \times \underline{r}_{B/A})] = 0$

(CONTINUED)

15.174 CONTINUED

ALTERNATE SOLUTION

$\underline{r}_{B/A} = (6\text{ in.})\underline{i} + (2\text{ in.})\underline{j} - (9\text{ in.})\underline{k}$

$\underline{r}_{B/G} = (2\text{ in.})\underline{j}$

$\underline{\omega} \cdot [\underline{r}_{B/G} \times (\underline{r}_{B/G} \times \underline{r}_{B/A})] = 0$

$\underline{\omega} \cdot \{2\underline{j} \times [2\underline{j} \times (6\underline{i} + 2\underline{j} - 9\underline{k})]\} = 0$

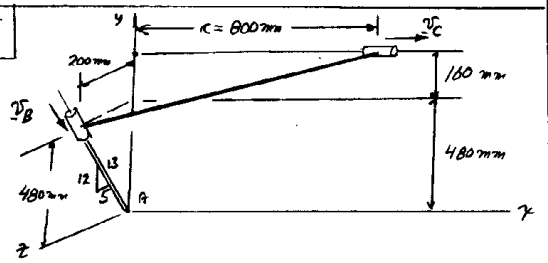
$\underline{\omega} \cdot [2\underline{j} \times (-12\underline{k} - 18\underline{i})] = 0$

$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot [-24\underline{i} + 36\underline{k}] = 0$

$-24\omega_x + 36\omega_z = 0 \qquad \omega_x = 1.5\omega_z$

WE AGAIN OBTAIN EQ (4). REMAINDER OF SOLUTION IS SAME AS DONE ON PRECEDING PAGE.

15.175



GIVEN: $v_B = 390\text{ mm/s}$ ↘

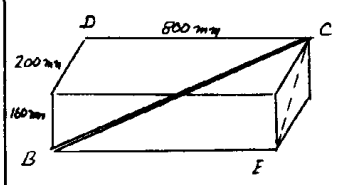
USING SAME SOLUTION AS IN PROB. 15.168, WE AGAIN HAVE

$v_C = -200\omega_y - 160\omega_z$ (1)

$360 = +200\omega_x + 800\omega_z$ (2)

$150 = +160\omega_x - 800\omega_y$ (3)

WE AGAIN FIND: $v_C = -34.5 \qquad \underline{v}_C = -(34.5\text{ mm/s})\underline{i}$ ◀



CLEVIS PREVENTS ROTATION OF BC ABOUT AXIS CE WHICH IS ⊥ TO THE HORIZONTAL ROD AT C AND LIES IN PLANE CONTAINING CD AND BC.

$\underline{r}_{E/C} = -(160\text{ mm})\underline{j} + (200\text{ mm})\underline{k}$

$\underline{\omega} \cdot \underline{r}_{E/C} = 0: \quad (\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot (-160\underline{j} + 200\underline{k}) = 0$

$-160\omega_y + 200\omega_z = 0 \qquad \omega_z = 0.8\omega_y$ (4)

SUBSTITUTE INTO (1):

$v_C = -34.5 = 200\omega_y - 160(0.8\omega_y) \qquad \omega_y = -0.4792\text{ rad/s}$

EQ(3) $\quad 150 = 160\omega_x - 800(-0.4792) \qquad \omega_x = -1.458\text{ rad/s}$

EQ(4) $\quad \omega_z = 0.8(-1.458) \qquad \omega_z = -1.167\text{ rad/s}$

$\underline{\omega} = -(1.458\text{ rad/s})\underline{i} - (0.479\text{ rad/s})\underline{j} - (1.167\text{ rad/s})\underline{k}$ ◀

ALTERNATE SOLUTION: (See footnote at Sample Prob 15.12)

$\underline{r}_{C/D} \times \underline{r}_{C/B}$ IS ⊥ TO PLANE OF CD AND CB.

$\underline{r}_{C/D} \times (\underline{r}_{C/D} \times \underline{r}_{C/B})$ IS ⊥ TO CD AND IS CONTAINED IN PLANE OF CD AND CB (AND CE). THUS, IT IS IN THE DIRECTION OF CE. FOR THE CLEVIS, WE HAVE

$\underline{\omega} \cdot [\underline{r}_{C/D} \times (\underline{r}_{C/D} \times \underline{r}_{C/B})] = 0$

(CONTINUED)

15.175 CONTINUED

$\underline{r}_{C/B} = (800\text{ mm})\underline{i} + (160\text{ mm})\underline{j} - (200\text{ mm})\underline{k}$

$\underline{r}_{C/D} = (800\underline{i})$

$\underline{\omega} \cdot [\underline{r}_{C/D} \times (\underline{r}_{C/D} \times \underline{r}_{C/B})] = 0$

$\underline{\omega} \cdot \{800\underline{i} \times [800\underline{i} \times (800\underline{i} + 160\underline{j} - 200\underline{k})]\} = 0$

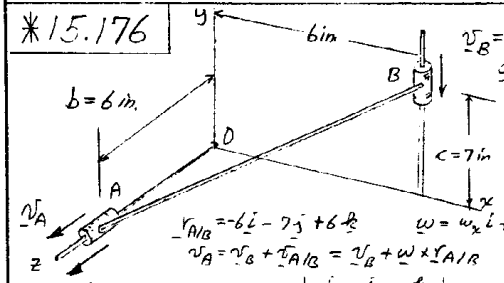
$\underline{\omega} \cdot [800\underline{i} \times (128 \times 10^3 \underline{k} + 160 \times 10^3 \underline{j})] = 0$

$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot [-128\underline{j} + 160\underline{k}]\, 800 \times 10^3 = 0$

$(-128\omega_y + 160\omega_z)\, 400 \times 10^3 = 0 \qquad \omega_z = 0.8\omega_y$

WE AGAIN OBTAIN EQ (4). REMAINDER OF SOLUTION IS SAME AS DONE IN PRECEDING COLUMN.

*15.176



$\underline{r}_{A/B} = -6\underline{i} - 7\underline{j} + 6\underline{k} \qquad \underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{r}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$v_A\underline{k} = -54\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -6 & -7 & 6 \end{vmatrix}$

EQUATE COEF. OF $\underline{i}, \underline{j}, \underline{k}$

$\underline{i}: \quad 0 = 6\omega_y + 7\omega_z$

$\underline{j}: \quad 54 = -6\omega_x - 6\omega_z$

$\underline{k}: \quad v_A = -7\omega_x + 6\omega_y$

} SINCE $\underline{\omega}$ IS INDETERMINATE, LET $\omega_y = 0$, THEN $\omega_z = 0$

THE "$\underline{j}$" EQUATION YIELDS, $54 = -6\omega_z \qquad \omega_x = -9\text{ rad/s}$

THUS, A POSSIBLE $\underline{\omega}$ IS: $\underline{\omega} = -(9\text{ rad/s})\underline{i}$ △

FROM "$\underline{k}$" EQUATION: $v_A = -7\omega_x + 6\omega_y = -7(-9) + 0 \qquad \underline{v}_A = (63\text{ in/s})\underline{k}$ △

$\underline{r}_{A/B} = -(6\text{ in.})\underline{i} - (7\text{ in.})\underline{j} + (6\text{ in.})\underline{k}$

$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = -9\underline{i} \times (-6\underline{i} - 7\underline{j} + 6\underline{k})$

$\underline{v}_{A/B} = (54\text{ in/s})\underline{j} + (63\text{ in/s})\underline{k}$ △

ACCELERATION OF A.

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A/B})$

$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$

$\underline{a}_A = +0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -6 & -7 & 6 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -9 & 0 & 0 \\ 0 & 54 & 63 \end{vmatrix}$

$a_A\underline{k} = (6\alpha_y + 7\alpha_z)\underline{i} + (-6\alpha_z - 6\alpha_x)\underline{j} + (-7\alpha_x + 6\alpha_y)\underline{k} + 567\underline{j} - 486\underline{k}$

EQUATE COEF. OF $\underline{i}, \underline{j}, \underline{k}$,

$0 = 6\alpha_y + 7\alpha_z$ (4)

$-567 = -6\alpha_x - 6\alpha_z$ (5)

$a_A + 486 = -7\alpha_x + 6\alpha_y$ (6)

$\alpha_x, \alpha_y, \alpha_z$ ARE INDETERMINATE. THUS, WE MAY ASSIGN ANY VALUE TO ONE COMPONENT.

FOR EXAMPLE, LET $\alpha_z = 0$, EQ(4) YIELDS, $\alpha_y = 0$

EQ(5): $\quad -567 = -6\alpha_x \qquad \alpha_x = 94.5\text{ rad/s}^2$

EQ(6): $\quad a_A + 486 = -7(94.5) \qquad a_A = -1147.5\text{ in/s}^2$

$\underline{a}_A = -(1148\text{ in/s}^2)\underline{k}$ ◀

(CONTINUED)

✱15.176 CONTINUED

ALTERNATE SOLUTION

SEE ALTERNATE SOLUTION OF PROB 15.167, EQ(6)

$$b\frac{db}{dt} + c\frac{dc}{dt} = 0$$

$$\frac{d}{dt}: \quad b\frac{d^2b}{dt^2} + \left(\frac{db}{dt}\right)^2 + c\frac{d^2c}{dt^2} + \left(\frac{dc}{dt}\right)^2 = 0$$

$\frac{d^2b}{dt^2} = a_A$; $\frac{d^2c}{dt^2} = a_B = 0$ (GIVEN)

$\frac{db}{dt} = v_A$; $\frac{dc}{dt} = v_B$

$$b\,a_A + v_A^2 + 0 + v_B^2 = 0$$

$$a_A = -\frac{v_A^2 + v_B^2}{b} \qquad (7)$$

WE HAVE: $v_A = 12$ in/s, $v_B = -54$ in/s, $b = 9$ in.

$$a_A = -\frac{(12)^2 + (54)^2}{9} = -1147.5 \text{ in/s}^2 \qquad \underline{a}_A = -(1148 \text{ in/s}^2)\,\underline{k}$$ ◀

✱15.177

FOLLOW STEPS OF SOLUTION OF PROB 15.176.

WE NOW HAVE: $c = 2$ in., $b = 9$ in.

FROM PROB 15.167, WE RECALL:

$$0 = 7\omega_y + 2\omega_z \qquad (1)$$

$$54 = -7\omega_x - 6\omega_z \qquad (2)$$

$$v_A = -2\omega_x + 6\omega_y \qquad (3)$$

WE HAVE SOLVED THESE FOR v_A: $\underline{v}_A = (12 \text{ in/s})\,\underline{k}$

A POSSIBLE SET OF COMPONENTS OF $\underline{\omega}$ IS:

ASSUME: $\omega_y = 0$, EQ(1) $\omega_z = 0$,

EQ(2) $54 = -7\omega_x$, $\omega_x = -6$ rad/s

WE HAVE: $\underline{\omega} = -(6 \text{ rad/s})\,\underline{i}$ ◁

$$\underline{r}_{A/B} = -(6 \text{ in.})\underline{i} - (12 \text{ in.})\underline{j} + (9 \text{ in.})\underline{k}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = -(6\underline{i}) \times (-6\underline{i} - 2\underline{j} + 9\underline{k})$$

$$\underline{v}_{A/B} = (54 \text{ in/s})\underline{j} + (12 \text{ in/s})\underline{k}$$ ◁

ACCELERATION OF A.

$$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A/B})$$

$$\underline{a}_A = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$\underline{a}_A = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -6 & -2 & 9 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 0 & 0 \\ 0 & 54 & 12 \end{vmatrix}$$

$$a_A\underline{k} = (9\alpha_y + 2\alpha_z)\underline{i} + (-6\alpha_z - 9\alpha_x)\underline{j} + (-2\alpha_x + 6\alpha_y)\underline{k} + 72\underline{j} - 324\underline{k}$$

EQUATE COEF. OF $\underline{i}, \underline{j}, \underline{k}$:

$$0 = 9\alpha_y + 2\alpha_z \qquad (4)$$

$$-72 = -9\alpha_x - 6\alpha_z \qquad (5)$$

$$a_A + 324 = -2\alpha_x + 6\alpha_y \qquad (6)$$

SINCE α IS INDETERMINATE, WE MAY ASSUME α_z = ANY VALUE. FOR EXAMPLE:

$\alpha_z = 0$ EQ(4) YIELDS $\alpha_y = 0$

EQ(5): $-72 = -9\alpha_x$ $\alpha_x = 8$ rad/s²

EQ(6): $a_A + 324 = -2(8)$

$a_A = -340$ in/s² $\underline{a}_A = -(340 \text{ in/s}^2)\,\underline{k}$ ◀

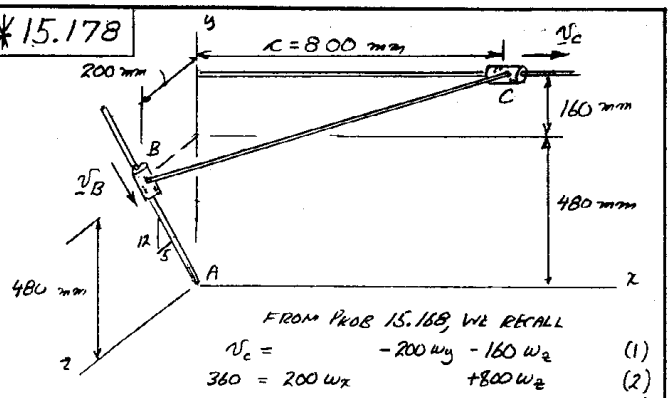
ALTERNATE SOLUTION

SEE TOP OF PAGE (PROB 15.176) FOR DERIVATION OF EQ(7) $a_A = -\frac{v_A^2 + v_B^2}{b}$

DATA: $v_A = 12$ in/s, $v_B = -54$ in/s, $b = 9$ in.

$$a_A = -\frac{(12)^2 + (54)^2}{9} = -340 \text{ in/s}^2 \qquad \underline{a}_A = -(340 \text{ in/s}^2)\,\underline{k}$$ ◀

✱15.178



FROM PROB 15.168, WE RECALL

$$v_C = -200\omega_y - 160\omega_z \qquad (1)$$

$$360 = 200\omega_x + 800\omega_z \qquad (2)$$

$$150 = 160\omega_x - 800\omega_y \qquad (3)$$

WE HAVE SOLVED THESE FOR v_C: $\underline{v}_C = (-34.5 \text{ mm/s})\,\underline{i}$

A POSSIBLE SET OF COMPONENTS OF $\underline{\omega}$ IS:

ASSUME $\omega_x = 0$:

EQ(2): $360 = 0 + 800\omega_z$ $\omega_z = 0.450$ rad/s

EQ(3): $150 = 0 - 800\omega_y$ $\omega_y = -0.1875$ rad/s

$$\underline{\omega} = -(0.1875 \text{ rad/s})\,\underline{j} + (0.450 \text{ rad/s})\,\underline{k}$$ ◁

$$\underline{r}_{C/B} = (800 \text{ mm})\,\underline{i} + (160 \text{ mm})\,\underline{j} - (200 \text{ mm})\,\underline{k}$$

$$\underline{v}_{C/B} = \underline{\omega} \times \underline{r}_{C/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -0.1875 & 0.450 \\ 800 & 160 & -200 \end{vmatrix} = (37.5 - 72)\underline{i} + (360)\underline{j} + (150)\underline{k}$$

$$\underline{v}_{C/B} = -(34.5 \text{ mm/s})\,\underline{i} + (360 \text{ mm/s})\,\underline{j} + (150 \text{ mm/s})\,\underline{k}$$ ◁

ACCELERATION OF C. $a_B = 0$ (GIVEN)

$$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = \underline{a}_B + \underline{\alpha} \times \underline{r}_{C/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C/B})$$

$$\underline{a}_C = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r}_{C/B} + \underline{\omega} \times \underline{v}_{C/B}$$

$$\underline{a}_C = \underline{a}_B + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 800 & 160 & -200 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -0.1875 & 0.450 \\ -34.5 & 360 & 150 \end{vmatrix}$$

$$a_C\underline{i} = 0 + (-200\alpha_y - 160\alpha_z)\underline{i} + (800\alpha_z + 200\alpha_x)\underline{j} + (160\alpha_x - 800\alpha_y)\underline{k} + (-28.125 - 162)\underline{i} - 15.525\underline{j} - 6.4688\underline{k}$$

EQUATE COEF OF $\underline{i}, \underline{j}, \underline{k}$:

$$a_C + 190.125 = -200\alpha_y - 160\alpha_z \qquad (4)$$

$$15.525 = 200\alpha_x + 800\alpha_z \qquad (5)$$

$$6.4688 = 160\alpha_x - 800\alpha_y \qquad (6)$$

SINCE α IS INDETERMINATE, WE MAY ASSUME ANY VALUE FOR ONE COMPONENT. FOR EXAMPLE, ASSUME $\alpha_x = 0$:

EQ(5): $15.525 = 0 + 800\alpha_z$ $\alpha_z = 19.41 \times 10^{-3}$ rad/s²

EQ(6): $6.4688 = 0 - 800\alpha_y$ $\alpha_y = -8.086 \times 10^{-3}$ rad/s²

EQ(4): $a_C + 190.125 = -(200)(-8.086 \times 10^{-3}) - (160)(19.41 \times 10^{-3})$

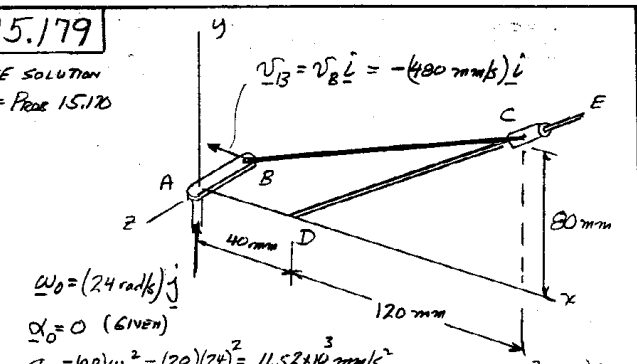
$a_C + 190.125 = +1.617 - 3.105$

$a_C = -191.6$ mm/s²

$$\underline{a}_C = -(191.6 \text{ mm/s}^2)\,\underline{i}$$ ◀

*15.179

SEE SOLUTION OF PROB 15.170



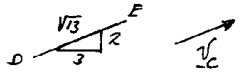
$\underline{v}_{/B} = v_B \underline{i} = -(480 \text{ mm/s})\underline{i}$

$\underline{\omega}_0 = (24 \text{ rad/s})\underline{j}$

$\underline{\alpha}_0 = 0$ (GIVEN)

$a_B = (AB)\omega_0^2 = (20)(24)^2 = 11.52\times10^3 \text{ mm/s}^2$ $\quad \underline{a}_B = +(11.52\times10^3 \text{ mm/s}^2)\underline{k}$ ◁

SINCE COLLAR C MOVES ALONG ROD DE



$\underline{v}_C$ $\quad (v_C)_y = \frac{2}{3}(v_C)_x$;

$\underline{a}_C$ $\quad (a_C)_y = \frac{2}{3}(a_C)_x$;

FROM PROB 15.170, WE HAVE

$$(v_C)_x + 480 = 20\omega_y - 80\omega_z \quad (1)$$
$$(v_C)_y = -20\omega_x + 160\omega_z \quad (2)$$
$$0 = 80\omega_x - 160\omega_y \quad (3)$$

WE HAVE ALREADY SOLVED THESE FOR $(v_C)_x$ AND $(v_C)_y$

$(v_C)_x = -360 \text{ mm/s}$ $\quad (v_C)_y = -240 \text{ mm/s}$

A POSSIBLE SET OF COMPONENTS OF $\underline{\omega}$ IS:

LET $\omega_x = 0$, EQ (3) YIELDS $\omega_y = 0$

EQ(2): $-240 \text{ mm/s} = +160\omega_z$ $\quad \omega_z = -1.5 \text{ rad/s}$

$\underline{\omega} = -(1.5 \text{ rad/s})\underline{k}$ ◁

SUBSTITUTE INTO EQ(1) TO CHECK VALUE

$-360 + 480 = 20(0) - 80(-1.5)$

$+120 = +120$ OK, CHECKS

VELOCITY OF C RELATIVE TO B:

$\underline{r}_{C/B} = (160 \text{ mm})\underline{i} + (80 \text{ mm})\underline{j} + (20 \text{ mm})\underline{k}$

$\underline{v}_{C/B} = \underline{\omega}\times\underline{r}_{C/B} = -(1.5 \text{ rad/s})\underline{k}\times[(160 \text{ mm})\underline{i} + (80 \text{ mm})\underline{j} + (20 \text{ mm})\underline{k}]$

$\underline{v}_{C/B} = (120 \text{ mm/s})\underline{i} - (240 \text{ mm/s})\underline{j}$ ◁

ACCELERATION OF C $\quad \underline{a}_C = (a_C)_x\underline{i} + (a_C)_y\underline{j}$

$\underline{\alpha} = \alpha_x\underline{i} + \alpha_y\underline{j} + \alpha_z\underline{k}$

$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = \underline{a}_B + \underline{\alpha}\times\underline{r}_{C/B} + \underline{\omega}\times\underline{v}_{C/B}$

$$(a_C)_x\underline{i} + (a_C)_y\underline{j} = (11.52\times10^3 \text{ mm/s}^2)\underline{k} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 160 & 80 & 20 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -1.5 \\ 120 & -240 & 0 \end{vmatrix}$$

$$(a_C)_x\underline{i} + (a_C)_y\underline{j} = 11.52\times10^3\underline{k} + (20\alpha_y - 80\alpha_z)\underline{i} + (160\alpha_z - 20\alpha_x)\underline{j} + (80\alpha_x - 160\alpha_y)\underline{k} - 360\underline{i} - 180\underline{j}$$

EQUATE COEFFICIENTS OF $\underline{i}$, $\underline{j}$, $\underline{k}$

$$\underline{i}: \quad (a_C)_x + 360 = 20\alpha_y - 80\alpha_z \quad (4)$$
$$\underline{j}: \quad (a_C)_y + 180 = -20\alpha_x + 160\alpha_z \quad (5)$$
$$\underline{k}: \quad -11.52\times10^3 = 80\alpha_x - 160\alpha_y \quad (6)$$

(CONTINUED)

*15.179 CONTINUED

ROD BC MAY SPIN FREELY ABOUT ITS AXIS, THUS $\underline{\alpha}$ IS INDETERMINATE AND WE MAY ASSUME ANY VALUE FOR ONE COMPONENT.

LET $\alpha_x = 0$, THEN

EQ(6): $-11.52\times10^3 = 80(0) - 160\alpha_y$ $\quad \alpha_y = +72 \text{ rad/s}^2$

SUBSTITUTING $\alpha_y = +72 \text{ rad/s}^2$ AND $(a_C)_y = \frac{2}{3}(a_C)_x$ WE WRITE

$$\text{EQ(4)} \quad (a_C)_x + 360 = 20(72) - 80\alpha_z \quad (4)$$
$$\text{EQ(5)} \quad \tfrac{2}{3}(a_C)_x + 180 = -2(0) + 160\alpha_z \quad (5)$$

MULTIPLY EQ.(4) BY 2.0

$$2(a_C)_x + 720 = +2880 - 160\alpha_z \quad (7)$$

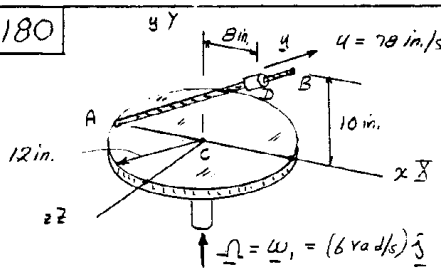
ADD EQ(5) AND EQ(7): $\frac{8}{3}(a_C)_x + 900 = +2880$

$\frac{8}{3}(a_C)_x = +1980$ $\quad (a_C)_x = +742.5 \text{ mm/s}^2$

$(a_C)_y = \frac{2}{3}(a_C)_x$ $\quad (a_C)_y = +495 \text{ mm/s}^2$

$\underline{a}_C = (742.5 \text{ mm/s}^2)\underline{i} + (495 \text{ mm/s}^2)\underline{j}$ ◀

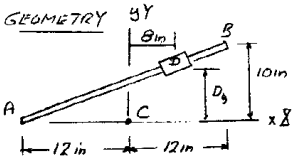
15.180



POINT C IS FIXED

$\underline{\Omega} = \underline{\omega}_1 = (6 \text{ rad/s})\underline{j}$

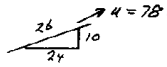
GEOMETRY



$\frac{D_y}{10 \text{ in.}} = \frac{(12+8)\text{in}}{(12+12)\text{in}}$ $\quad D_y = 8.33 \text{ in}$

$\underline{r}_{D/C} = (8 \text{ in.})\underline{i} + (8.33 \text{ in.})\underline{j}$

$AB^2 = 10^2 + 24^2$ $\quad AB = 26 \text{ in.}$ $\quad \underline{\lambda}_{AB} = \frac{24}{26}\underline{i} + \frac{10}{26}\underline{j}$



$\underline{u} = 78\underline{\lambda} = (72 \text{ in./s})\underline{i} + (30 \text{ in/s})\underline{j}$

(a) VELOCITY OF D $\quad \underline{v}_{D/Z} = \underline{u} = (72 \text{ in./s})\underline{i} + (30 \text{ in./s})\underline{j}$

$\underline{v}_{D'} = \underline{\Omega}\times\underline{r}_{D/C} = (6 \text{ rad/s})\underline{j}\times[(8 \text{ in.})\underline{i} + (8.33 \text{ in.})\underline{j}]$

$\underline{v}_{D'} = -(48 \text{ in/s})\underline{k}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/Z} = -48\underline{k} + 72\underline{i} + 30\underline{j}$

$\underline{v}_D = (72 \text{ in./s})\underline{i} + (30 \text{ in./s})\underline{j} - (48 \text{ in./s})\underline{k}$ ◀

(b) ACCELERATION OF D

$\underline{a}_{D/Z} = 0$ $\quad \dot{\underline{\Omega}} = 0$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/Z} + \underline{a}_C$

$\underline{a}_{D'} = \dot{\underline{\Omega}}\times\underline{r}_{D/C} + \underline{\Omega}\times\underline{\Omega}\times\underline{r}_{D/C} = \underline{\Omega}\times\underline{v}_{D'}$

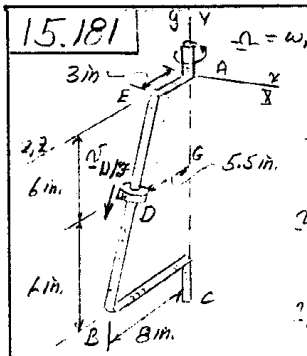
$= (6 \text{ rad/s})\underline{j}\times(-48 \text{ in/s})\underline{k} = -(288 \text{ in/s}^2)\underline{i}$

$\underline{a}_C = 2\underline{\Omega}\times\underline{v}_{D/Z} = 2(6 \text{ rad/s})\underline{j}\times[(72 \text{ in./s})\underline{i} + (30 \text{ in/s})\underline{j}]$

$= -(864 \text{ in/s}^2)\underline{k}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/Z} + \underline{a}_C$ $\quad \underline{a}_D = -(288 \text{ in/s}^2)\underline{i} - (864 \text{ in/s}^2)\underline{k}$ ◀

15.181



$\underline{\Omega} = \omega_1 = +(4\ \text{rad/s})\,\underline{j}$

$\underline{\partial}_{EB} = -\frac{12}{13}\underline{j} + \frac{5}{13}\underline{k}$

$v_{D/G} = u = 65\ \text{in./s}$

$\underline{v}_{D/G} = v_{D/G}\,\underline{\partial}_{EB} = (65)\left(-\frac{12}{13}\underline{j} + \frac{5}{13}\underline{k}\right)$

$\underline{v}_{D/G} = -(60\ \text{in./s})\underline{j} + (25\ \text{in./s})\underline{k}$

(a) VELOCITY OF D. $\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/G} = (4\underline{j}) \times (5.5\underline{k}) = (22\ \text{in./s})\,\underline{i}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G} = (22\ \text{in./s})\,\underline{i} - (60\ \text{in./s})\,\underline{j} + (25\ \text{in./s})\,\underline{k}$ ◀

(b) ACCELERATION OF D. $\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/G} + \underline{a}_C$

$v_{D/G}$ = CONSTANT $\quad \underline{a}_{D/G} = 0$

$\Omega = \omega_1$ = CONSTANT $\quad \dot{\underline{\Omega}} = 0$

$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/A} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/A})$

$= 0 + \underline{\Omega} \times \underline{v}_{D'}$

$= 0 + (4\ \text{rad/s})\underline{j} \times (22\ \text{in./s})\,\underline{i}$

$\underline{a}_{D'} = -(88\ \text{in./s}^2)\,\underline{k}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/G} = 2(4\ \text{rad/s})\underline{j} \times (-60\underline{j} + 25\underline{k})$

$= (200\ \text{in./s}^2)\,\underline{i}$

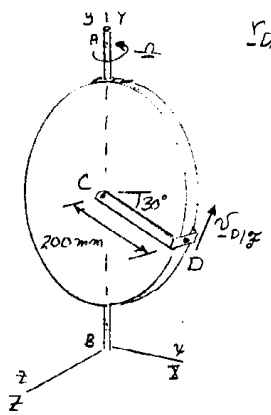
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/G} + \underline{a}_C$

$= -(88\ \text{in./s}^2)\,\underline{k} + 0 + (200\ \text{in./s}^2)\,\underline{i}$

$\underline{a}_D = (200\ \text{in./s}^2)\,\underline{i} - (88\ \text{in./s}^2)\,\underline{k}$ ◀

15.182

$\underline{\Omega} = \omega_1 = (10\ \text{rad/s})\,\underline{j} \qquad \dot{\underline{\Omega}} = 0$



$\underline{r}_{D/C} = (200\times\cos 30^\circ)\,\underline{i} - (200\times\sin 30^\circ)\,\underline{j}$

$= (173.21\ \text{mm})\,\underline{i} - (100\ \text{mm})\,\underline{j}$

$v_{D/G} = 1.5\ \text{m/s}$ ↗

$\underline{v}_{D/G} = (1.5\sin 30^\circ)\,\underline{i} + (1.5\cos 30^\circ)\,\underline{j}$

$= (0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j}$

$a_{D/G} = \frac{u^2}{r} = \frac{(1.5\ \text{m/s})^2}{0.2\ \text{m}} = 11.25\ \text{m/s}^2$ ◀

$\underline{a}_{D/G} = -(11.25\cos 30^\circ)\,\underline{i} + (11.25\sin 30^\circ)\,\underline{j}$

$= -(9.743\ \text{m/s}^2)\,\underline{i} + (5.625\ \text{m/s}^2)\,\underline{j}$

(a) VELOCITY OF D. $\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/C}$

$= (10\ \text{rad/s})\,\underline{j} \times [(0.17321\ \text{m})\,\underline{i} - (0.10\ \text{m})\,\underline{j}]$

$= -(1.7321\ \text{m/s})\,\underline{k}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G} = -(1.7321\ \text{m/s})\,\underline{k} + (0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j}$

$\underline{v}_D = (0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j} - (1.732\ \text{m/s})\,\underline{k}$ ◀

(CONTINUED)

15.182 CONTINUED

(b) ACCELERATION OF D.

WE HAVE ALREADY FOUND: $\dot{\underline{\Omega}} = 0$ AND

$\underline{a}_{D/G} = -(9.743\ \text{m/s}^2)\,\underline{i} + (5.625\ \text{m/s}^2)\,\underline{j}$

$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/G} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/G}) = 0 + \underline{\Omega} \times \underline{v}_{D'}$

$= (10\ \text{rad/s})\,\underline{j} \times (-1.7321\ \text{m/s})\,\underline{k} = -(17.32\ \text{m/s}^2)\,\underline{i}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/G} = 2(10\ \text{rad/s})\,\underline{j} \times [(0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j}]$

$= -(15.00\ \text{m/s}^2)\,\underline{k}$

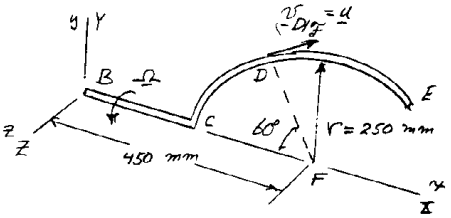
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/G} + \underline{a}_C$

$= -(17.32\ \text{m/s}^2)\,\underline{i} - (9.743\ \text{m/s}^2)\,\underline{i} + (5.625\ \text{m/s}^2)\,\underline{j} - (15.00\ \text{m/s}^2)\,\underline{k}$

$\underline{a}_D = -(27.1\ \text{m/s}^2)\,\underline{i} + (5.63\ \text{m/s}^2)\,\underline{j} - (15.00\ \text{m/s}^2)\,\underline{k}$ ◀

15.183

$\underline{\Omega} = \omega_1 = (0.4\ \text{rad/s})\,\underline{i} \qquad v_{D/G} = 150\ \text{mm/s}$



$\underline{v}_{D/G} = (150\ \text{mm/s})(\cos 30^\circ\,\underline{i} + \sin 30^\circ\,\underline{j})$

$= (129.9\ \text{mm/s})\,\underline{i} + (75\ \text{mm/s})\,\underline{j}$

$\underline{r}_{D/F} = -(125\ \text{mm})\,\underline{i} + (216.5\ \text{mm})\,\underline{j}$

(a) VELOCITY OF D. $\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/F} = (0.4\ \text{rad/s})\,\underline{i} \times (-125\,\underline{i} + 216.5\,\underline{j})$

$= (86.6\ \text{mm/s})\,\underline{k}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/G} = (86.6\ \text{mm/s})\,\underline{k} + (129.9\ \text{mm/s})\,\underline{i} + (75\ \text{mm/s})\,\underline{j}$

$\underline{v}_D = (129.9\ \text{mm/s})\,\underline{i} + (75\ \text{mm/s})\,\underline{j} + (86.6\ \text{mm/s})\,\underline{k}$ ◀

(b) ACCELERATION OF D. $\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/G} + \underline{a}_C$

$v_{D/G}$ = CONST., $\quad \Omega$ = CONSTANT, $\quad \dot{\Omega} = 0$

$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/F} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/F}) = 0 + \underline{\Omega} \times \underline{v}_{D'}$

$= (0.4\ \text{rad/s})\,\underline{i} \times (86.6\ \text{mm/s})\,\underline{k} = -(34.64\ \text{mm/s}^2)\,\underline{j}$

$\underline{a}_{D/F} = (u^2/r)\,\underline{\partial}_{DF} = (150^2/250)\,\underline{\partial}_{DF} = 90\,(\cos 60^\circ\,\underline{i} - \sin 60^\circ\,\underline{j})$

$= (45.0\ \text{mm/s}^2)\,\underline{i} - (77.94\ \text{mm/s}^2)\,\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/G} = 2(0.4\ \text{rad/s})\,\underline{i} \times [(129.9\ \text{mm/s})\,\underline{i} + (75\ \text{mm/s})\,\underline{j}]$

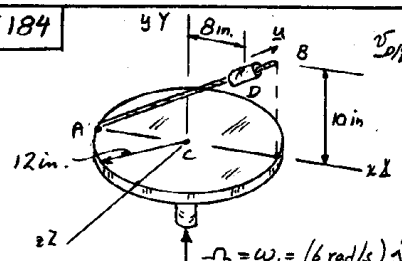
$= (60\ \text{mm/s}^2)\,\underline{k}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/G} + \underline{a}_C$

$= -(34.64\ \text{mm/s}^2)\,\underline{j} + (45.0\ \text{mm/s}^2)\,\underline{i} - (77.94\ \text{mm/s}^2)\,\underline{j} + (60\ \text{mm/s}^2)\,\underline{k}$

$\underline{a}_D = (45.0\ \text{mm/s}^2)\,\underline{i} - (112.6\ \text{mm/s}^2)\,\underline{j} + (60\ \text{mm/s}^2)\,\underline{k}$ ◀

15.184



$\underline{v}_{D/\mathcal{F}} = \underline{v} = 78\ \text{in/s} \rightarrow$

$\underline{a}_{D/\mathcal{F}} = 208\ \text{in/s}^2 \swarrow$

POINT C IS FIXED

$\underline{\Omega} = \underline{\omega}_1 = (6\ \text{rad/s})\,\underline{j}$

$\dot{\underline{\Omega}} = -(10\ \text{rad/s}^2)\,\underline{j}$

$\underline{\partial}_{AB} = \dfrac{\overline{AB}}{AB} = \dfrac{24\underline{i} + 10\underline{j}}{26}$; $\underline{v}_{D/\mathcal{F}} = 78\,\underline{\partial}_{AB} = (72\ \text{in/s})\,\underline{i} + (30\ \text{in/s})\,\underline{j}$

$\underline{a}_{D/\mathcal{F}} = -208\,\underline{\partial}_{AB} = -(192\ \text{in/s}^2)\,\underline{i} - (80\ \text{in/s}^2)\,\underline{j}$

$\underline{r}_{D/C} = (8\ \text{in.})\,\underline{i} + (r_{D/C})_y\,\underline{j}$

NOTE! $(r_{D/C})_y = 8.33$ in. BUT THIS VALUE DOES NOT ENTER SOLUTION

(a) VELOCITY OF D. SAME AS PROB 15.180

$\underline{v}_{D'} = -(48\ \text{in/s})\,\underline{k}$

$\underline{v}_D = (72\ \text{in/s})\,\underline{i} + (30\ \text{in/s})\,\underline{j} - (48\ \text{in/s})\,\underline{k}$ ◀

(b) ACCELERATION OF D.

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_C$

$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/C} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/C}) = \dot{\underline{\Omega}} \times \underline{r}_{D/C} + \underline{\Omega} \times \underline{v}_{D'}$

$= -(10\ \text{rad/s}^2)\,\underline{j} \times \left[(8\ \text{in.})\,\underline{i} + (r_{D/C})_y\,\underline{j}\right] + (6\ \text{rad/s})\,\underline{j} \times (-48\ \text{in/s})\,\underline{k}$

$= (80\ \text{in/s}^2)\,\underline{k} - (288\ \text{in/s}^2)\,\underline{i}$

$\underline{a}_{D/\mathcal{F}} = -(192\ \text{in/s}^2)\,\underline{i} - (80\ \text{in/s}^2)\,\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(6\ \text{rad/s})\,\underline{j} \times \left[(72\ \text{in/s})\,\underline{i} + (30\ \text{in/s})\,\underline{j}\right]$

$= -(864\ \text{in/s}^2)\,\underline{k}$

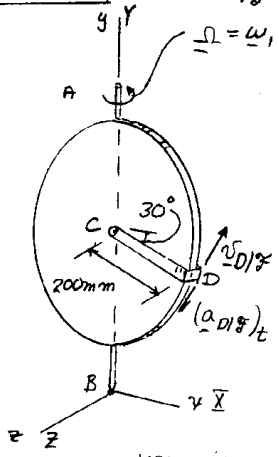
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_C$

$= (80\ \text{in/s}^2)\,\underline{k} - (288\ \text{in/s}^2)\,\underline{i} - (192\ \text{in/s}^2)\,\underline{i} - (80\ \text{in/s}^2)\,\underline{j} - (864\ \text{in/s}^2)\,\underline{k}$

$\underline{a}_D = -(480\ \text{in/s}^2)\,\underline{i} - (80\ \text{in/s}^2)\,\underline{j} - (784\ \text{in/s}^2)\,\underline{k}$ ◀

15.185

$\underline{v}_{D/\mathcal{F}} = 1.5\ \text{m/s} \nearrow$ $\underline{a}_{D/\mathcal{F}} = 3\ \text{m/s}^2 \swarrow$



$\underline{\Omega} = \underline{\omega}_1 = (10\ \text{rad/s})\,\underline{j}$; $\dot{\underline{\Omega}} = -(25\ \text{rad/s}^2)\,\underline{j}$

FROM PROB 15.182:

$\underline{r}_{D/C} = (173.21\ \text{mm})\,\underline{i} - (100\ \text{mm})\,\underline{j}$

$\underline{v}_{D/\mathcal{F}} = (0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j}$

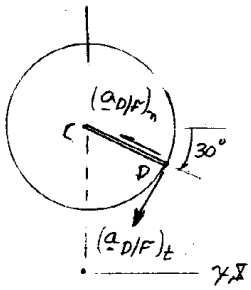
$\underline{v}_{D'} = -(1.7321\ \text{m/s})\,\underline{k}$

VELOCITY OF D: SAME AS PROB 15.182

$\underline{v}_D = (0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j} - (1.732\ \text{m/s})\,\underline{k}$ ◀

(CONTINUED)

15.185 CONTINUED



(b) ACCELERATION OF D.

$(\underline{a}_{D/\mathcal{F}})_t = 3\ \text{m/s}^2\ \angle 60°$

$= -(1.5\ \text{m/s}^2)\,\underline{i} - (2.598\ \text{m/s}^2)\,\underline{j}$

$(a_{D/\mathcal{F}})_n = \dfrac{v_{D/F}^2}{r} = \dfrac{(1.5\ \text{m/s})^2}{0.2} = 11.25\ \text{m/s}^2$

$(\underline{a}_{D/\mathcal{F}})_n = (11.25\ \text{m/s}^2) \measuredangle 30°$

$= -(9.743\ \text{m/s}^2)\,\underline{i} + (5.625\ \text{m/s}^2)\,\underline{j}$

$\underline{a}_{D/\mathcal{F}} = (\underline{a}_{D/\mathcal{F}})_t + (\underline{a}_{D/\mathcal{F}})_n$

$= (-1.5\ \text{m/s}^2)\,\underline{i} - (2.598\ \text{m/s}^2)\,\underline{j} - (9.743\ \text{m/s}^2)\,\underline{i} + (5.625\ \text{m/s}^2)\,\underline{j}$

$= -(11.242\ \text{m/s}^2)\,\underline{i} + (3.207\ \text{m/s}^2)\,\underline{j}$

$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/C} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/C}) = \dot{\underline{\Omega}} \times \underline{r}_{D/C} + \underline{\Omega} \times \underline{v}_{D'}$

$= -(25\ \text{rad/s}^2)\,\underline{j} \times \left[(0.17321\ \text{m})\,\underline{i} - (0.100\ \text{m})\,\underline{j}\right]$

$+ (10\ \text{rad/s})\,\underline{j} \times (-1.7321\ \text{m/s})\,\underline{k}$

$= +(4.3301\ \text{m/s}^2)\,\underline{k} - (17.321\ \text{m/s}^2)\,\underline{i}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(10\ \text{rad/s})\,\underline{j} \times \left[(0.75\ \text{m/s})\,\underline{i} + (1.299\ \text{m/s})\,\underline{j}\right]$

$= -(15.00\ \text{m/s}^2)\,\underline{k}$

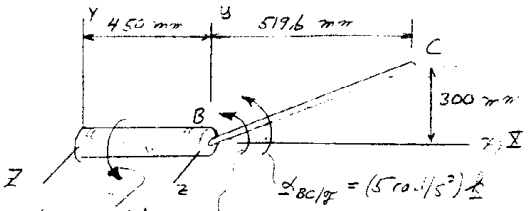
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_C$

$= (4.3301\ \text{m/s}^2)\,\underline{k} - (17.321\ \text{m/s}^2)\,\underline{i} - (11.242\ \text{m/s}^2)\,\underline{i}$

$+ (3.207\ \text{m/s}^2)\,\underline{j} - (15.00\ \text{m/s}^2)\,\underline{k}$

$\underline{a}_D = (28.6\ \text{m/s}^2)\,\underline{i} + (3.207\ \text{m/s}^2)\,\underline{j} - (10.67\ \text{m/s}^2)\,\underline{k}$ ◀

15.186



$\underline{\Omega} = \underline{\omega}_1 = (3\ \text{rad/s})\,\underline{i}$

$\dot{\underline{\Omega}} = 0$

$\underline{\omega}_{BC/\mathcal{F}} = (4\ \text{rad/s})\,\underline{k}$

$\underline{r}_{C/B} = (519.6\ \text{mm})\,\underline{i} + (300\ \text{mm})\,\underline{j}$

(a) VELOCITY OF C. $\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/B} = (3\ \text{rad/s})\,\underline{i} \times \left[(519.6\ \text{mm})\,\underline{i} + (300\ \text{mm})\,\underline{j}\right]$

$= (900\ \text{mm/s})\,\underline{k}$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_{BC/\mathcal{F}} \times \underline{r}_{C/B} = (4\ \text{rad/s})\,\underline{k} \times \left[(519.6\ \text{mm})\,\underline{i} + (300\ \text{mm})\,\underline{j}\right]$

$= (2078.5\ \text{mm/s})\,\underline{j} - (1200\ \text{mm/s})\,\underline{i}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = (900\ \text{mm/s})\,\underline{k} + (2078.5\ \text{mm/s})\,\underline{j} - (1200\ \text{mm/s})\,\underline{i}$

$\underline{v}_C = -(1200\ \text{mm/s})\,\underline{i} + (2078\ \text{mm/s})\,\underline{j} + (900\ \text{mm/s})\,\underline{k}$

$\underline{v}_C = -(1.200\ \text{m/s})\,\underline{i} + (2.078\ \text{m/s})\,\underline{j} + (0.900\ \text{m/s})\,\underline{k}$ ◀

(CONTINUED)

15.186 CONTINUED

(b) ACCELERATION OF C. $\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_R$

$\underline{a}_{C'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{C/B}) = \underline{\Omega} \times \underline{v}_{C'}$

$= (3 \text{ rad/s})\underline{i} \times (900 \text{ mm/s})\underline{k}$

$= -(2700 \text{ mm/s}^2)\underline{j}$

$\underline{a}_{C/\mathcal{F}} = \underline{\alpha}_{BC/\mathcal{F}} \times \underline{r}_{C/B} + \underline{\omega}_{BC/\mathcal{F}} \times (\underline{\omega}_{BC/\mathcal{F}} \times \underline{r}_{C/B})$

$= \underline{\alpha}_{BC/\mathcal{F}} \times \underline{r}_{C/B} + \underline{\omega}_{BC/\mathcal{F}} \times \underline{v}_{C/\mathcal{F}}$

$= (5 \text{ rad/s}^2)\underline{k} \times [(519.6 \text{ mm})\underline{i} + (300 \text{ mm})\underline{j}]$

$+ (4 \text{ rad/s})\underline{k} \times [(2078 \text{ mm/s})\underline{j} - (1200 \text{ mm/s})\underline{i}]$

$= +2598\,\underline{j} - 1500\,\underline{i} - 8312\,\underline{i} - 4800\,\underline{j}$

$= -(9812 \text{ mm/s}^2)\underline{i} - (2202 \text{ mm/s}^2)\underline{j}$

$\underline{a}_R = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}}$

$= 2(3 \text{ rad/s})\underline{i} \times [(2078 \text{ mm/s})\underline{j} - (1200 \text{ mm/s})\underline{i}]$

$(12\,468 \text{ mm/s}^2)\underline{k}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_R$

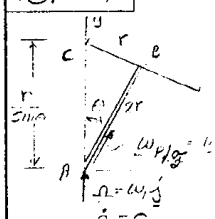
$= -(2700 \text{ mm/s}^2)\underline{j} - (9812 \text{ mm/s}^2)\underline{i} - (2202 \text{ mm/s}^2)\underline{j}$

$+ (12\,468 \text{ mm/s}^2)\underline{k}$

$\underline{a}_C = -(9812 \text{ mm/s}^2)\underline{i} - (4902 \text{ mm/s}^2)\underline{j} + (12\,468 \text{ mm/s}^2)\underline{k}$

$\underline{a}_C = -(9.81 \text{ m/s}^2)\underline{i} - (4.90 \text{ m/s}^2)\underline{j} + (12.47 \text{ m/s}^2)\underline{k}$ ◀

15.187



$\underline{\omega}_{PLATE/F} = \underline{\omega}_{P/F} = \omega_2 \sin\theta\,\underline{i} + \omega_2 \cos\theta\,\underline{j}$

(a) ACCELERATION OF DISK.

$\underline{\alpha} = \underline{\dot{\Omega}} \times \underline{\omega}_{P/F}$

$= \omega_1 \underline{j} \times (\omega_2 \sin\theta\,\underline{i} + \omega_2 \cos\theta\,\underline{j})$

$\underline{\alpha} = -\omega_1 \omega_2 \sin\theta\,\underline{k}$ ◀

(b) VELOCITY OF C. $\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/F}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = \omega_1 \underline{j} \times \frac{r}{\sin\theta}\underline{j} = 0$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_{P/\mathcal{F}} \times \underline{r}_{C/A} = (\omega_2 \sin\theta\,\underline{i} + \omega_2 \cos\theta\,\underline{j}) \times \frac{r}{\sin\theta}\underline{j}$

$= r\omega_2\,\underline{k}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = 0 + r\omega_2\,\underline{k}$ $\qquad \underline{v}_C = r\omega_2\,\underline{k}$ ◀

(c) ACCELERATION OF C. $\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/F} + \underline{a}_R$

$\underline{a}_{C'} = \underline{\dot{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{C/A}) = 0 + \underline{\Omega} \times \underline{v}_{C'} = 0 + 0$

$\underline{a}_{C/\mathcal{F}} = \underline{\omega}_{P/\mathcal{F}} \times (\underline{\omega}_{P/\mathcal{F}} \times \underline{r}_{C/A}) = \underline{\omega}_{P/F} \times \underline{v}_{C/F}$

$= (\omega_2 \sin\theta\,\underline{i} + \omega_2 \cos\theta\,\underline{j}) \times r\omega_2\,\underline{k}$

$= -r\omega_2^2 \sin\theta\,\underline{j} + r\omega_2^2 \cos\theta\,\underline{i}$

$\underline{a}_R = 2\underline{\Omega} \times \underline{v}_{C/F} = 2(\omega_1 \underline{j}) \times r\omega_2\,\underline{k} = 2r\omega_1\omega_2\,\underline{i}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_R = 0 - r\omega_2^2 \sin\theta\,\underline{j} + r\omega_2^2 \cos\theta\,\underline{i} + 2r\omega_1\omega_2\,\underline{i}$

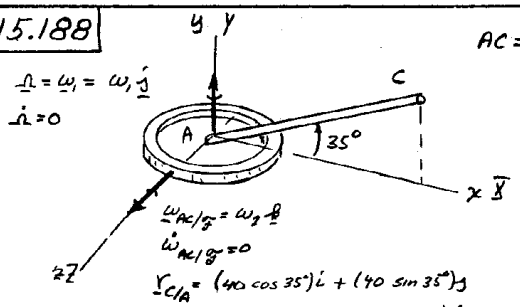
$\sin\theta = \frac{1}{\sqrt{5}} \quad \cos\theta = \frac{2}{\sqrt{5}}$

$\underline{a}_C = -r\omega_2^2 \frac{1}{\sqrt{5}}\underline{j} + r\omega_2^2 \frac{2}{\sqrt{5}}\underline{i} + 2r\omega_1\omega_2\,\underline{i}$

$= \left(2r\omega_1\omega_2 + \frac{2r}{\sqrt{5}}\omega_2^2\right)\underline{i} - \frac{1}{\sqrt{5}} r\omega_2^2\,\underline{j}$

$\underline{a}_C = 2r\left(\omega_1\omega_2 + \frac{\omega_2^2}{\sqrt{5}}\right)\underline{i} - r\frac{\omega_2^2}{\sqrt{5}}\underline{j}$ ◀

15.188



$\underline{r}_{C/A} = (40 \cos 35°)\underline{i} + (40 \sin 35°)\underline{j}$

$\underline{r}_{C/A} = (32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}$

GIVEN $\omega_1 = 0.15$ rad/s $\qquad \omega_2 = 0.25$ rad/s

(a) ANGULAR ACCELERATION: SAME AS IN PROB 15.186

$\underline{\alpha} = (0.0375 \text{ rad/s}^2)\underline{i}$ ◀

(b) VELOCITY OF C: $\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (0.15 \text{ rad/s})\underline{j} \times [(32.77 \text{ in})\underline{i} + (22.94 \text{ in})\underline{j}]$

$= -(4.915 \text{ in/s})\underline{k}$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_{AC/\mathcal{F}} \times \underline{r}_{C/A} = (0.25 \text{ rad/s})\underline{k} \times [(32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}]$

$= (8.192 \text{ in/s})\underline{j} - (5.736 \text{ in./s})\underline{i}$

$\underline{v}_C = -(5.74 \text{ in./s})\underline{i} + (8.19 \text{ in./s})\underline{j} - (4.91 \text{ in./s})\underline{k}$ ◀

(c) ACCELERATION OF C:

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_R$

$\underline{a}_{C'} = \underline{\dot{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{C/A}) = 0 + \underline{\Omega} \times \underline{v}_{C'}$

$= (0.15 \text{ rad/s})\underline{j} \times (-4.915 \text{ in/s})\underline{k} = -(0.7373 \text{ in/s}^2)\underline{i}$

$\underline{a}_{C/\mathcal{F}} = \underline{\omega}_{AC/\mathcal{F}} \times \underline{\omega}_{AC/\mathcal{F}} \times \underline{r}_{C/A} = \underline{\omega}_{AC/\mathcal{F}} \times \underline{v}_{C/\mathcal{F}}$

$= (0.25 \text{ rad/s})\underline{k} \times [(8.192 \text{ in./s})\underline{j} - (5.736 \text{ in/s})\underline{i}]$

$= -(2.048 \text{ in/s}^2)\underline{i} - (1.434 \text{ in./s}^2)\underline{j}$

$\underline{a}_R = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(0.15 \text{ rad/s})\underline{j} \times [8.192\underline{j} - 5.736\underline{i}]$

$= (1.721 \text{ in./s}^2)\underline{k}$

$\underline{a}_C = -(0.7373 \text{ in/s}^2)\underline{i} - (2.048 \text{ in/s}^2)\underline{i} - (1.434 \text{ in/s}^2)\underline{j} + (1.721 \text{ in/s}^2)\underline{k}$

$\underline{a}_C = -(2.79 \text{ in./s}^2)\underline{i} - (1.434 \text{ in./s}^2)\underline{j} + (1.721 \text{ in./s}^2)\underline{k}$ ◀

15.189

SEE SOLUTION OF PROB 15.188. WE NOW HAVE $\omega_1 = 0.25$ rad/s $\qquad \omega_2 = 0.15$ rad/s

(a) ANGULAR ACCELERATION SAME $\underline{\alpha} = (0.0375 \text{ rad/s}^2)\underline{i}$ ◀

(b) VELOCITY OF C: $\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (0.25 \text{ rad/s})\underline{j} \times [(32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}] = -(8.192 \text{ in/s})\underline{k}$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_{AC/\mathcal{F}} \times \underline{r}_{C/A} = (0.15 \text{ rad/s})\underline{k} \times [(32.77 \text{ in.})\underline{i} + (22.94 \text{ in.})\underline{j}]$

$= (4.915 \text{ in/s})\underline{j} - (3.442 \text{ in./s})\underline{i}$

$\underline{v}_C = -(3.44 \text{ in./s})\underline{i} + (4.91 \text{ in./s})\underline{j} - (8.19 \text{ in/s})\underline{k}$ ◀

(c) ACCELERATION OF C: $\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_R$

$\underline{a}_{C'} = -\underline{\Omega} \times \underline{v}_{C'} = (0.25 \text{ rad/s})\underline{j} \times (-8.192 \text{ in/s})\underline{k}$

$= -(2.048 \text{ in/s}^2)\underline{i}$

$\underline{a}_{C/\mathcal{F}} = \underline{\omega}_{AC/\mathcal{F}} \times \underline{v}_{C/\mathcal{F}}$

$= (0.15 \text{ rad/s})\underline{k} \times [(4.915 \text{ in/s})\underline{j} - (3.442 \text{ in/s})\underline{i}]$

$= -(0.7373 \text{ in/s}^2)\underline{i} - (0.5163 \text{ in/s}^2)\underline{j}$

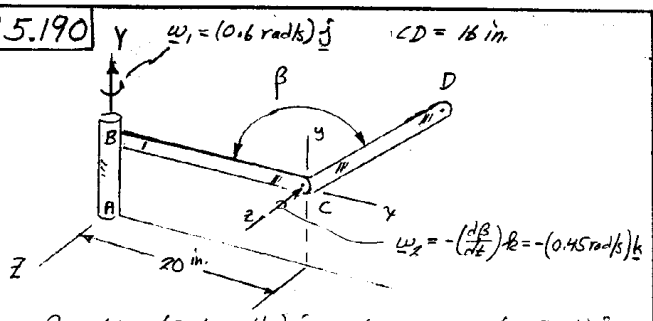
$\underline{a}_R = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(0.25 \text{ rad/s})\underline{j} \times [4.915\underline{j} - 3.442\underline{i}]$

$= +(1.721 \text{ in/s}^2)\underline{k}$

$\underline{a}_C = -(2.048 \text{ in/s}^2)\underline{i} - (0.7373 \text{ in/s}^2)\underline{i} - (0.5163 \text{ in/s}^2)\underline{j} + (1.721 \text{ in/s}^2)\underline{k}$

$\underline{a}_C = -(2.79 \text{ in./s}^2)\underline{i} - (0.516 \text{ in/s}^2)\underline{j} + (1.721 \text{ in/s}^2)\underline{k}$ ◀

15.190



$\underline{\Omega} = \omega_1 = (0.6 \text{ rad/s})\underline{j}$ $\quad \omega_{D/F} = \omega_2 = -(0.45 \text{ rad/s})\underline{k}$

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = (0.6 \text{ rad/s})\underline{j} - (0.45 \text{ rad/s})\underline{k}$

(a) ANGULAR ACCELERATION OF CD.

$$\underline{\alpha} = \underline{\Omega} \times \underline{\omega} = (0.6 \text{ rad/s})\underline{j} \times [(0.6 \text{ rad/s})\underline{j} - (0.45 \text{ rad/s})\underline{k}]$$

$$\underline{\alpha} = -(0.27 \text{ rad/s}^2)\underline{i} \quad \blacktriangleleft$$

FOR $\beta = 120°$

$$\underline{r}_{D/C} = (16 \text{ in.})\sin 30°\,\underline{i} + (16 \text{ in.})\cos 30°\,\underline{j} = (8 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}$$

$$\underline{r}_{D/B} = (20 \text{ in.})\underline{i} + \underline{r}_{D/C} = (28 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}$$

(b) VELOCITY OF D. $\quad \underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$

$$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/B} = (0.6 \text{ rad/s})\underline{j} \times [(28 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}] = -(16.8 \text{ in./s})\underline{k}$$

$$\underline{v}_{D/F} = \underline{\omega}_{D/F} \times \underline{r}_{D/C} = -(0.45 \text{ rad/s})\underline{k} \times [(8 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}]$$

$$= -(3.6 \text{ in./s})\underline{j} + (6.2354 \text{ in./s})\underline{i}$$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$:

$$\underline{v}_D = (6.24 \text{ in./s})\underline{i} - (3.6 \text{ in./s})\underline{j} - (16.8 \text{ in./s})\underline{k} \quad \blacktriangleleft$$

(c) ACCELERATION OF D. $\quad \underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$

$$\underline{a}_{D'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/B}) = \underline{\Omega} \times \underline{v}_{D'}$$

$$= (0.6 \text{ rad/s})\underline{j} \times (-16.8 \text{ in./s})\underline{k} = -(10.08 \text{ in./s}^2)\underline{i}$$

$$\underline{a}_{D/F} = \underline{\omega}_{D/F} \times (\underline{\omega}_{D/F} \times \underline{r}_{D/C}) = \underline{\omega}_{D/F} \times \underline{v}_{D/F}$$

$$= -(0.45 \text{ rad/s})\underline{k} \times [-(3.6 \text{ in./s})\underline{j} + (6.2354 \text{ in./s})\underline{i}]$$

$$= -(1.620 \text{ in./s}^2)\underline{i} - (2.806 \text{ in./s}^2)\underline{j}$$

$$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/F}$$

$$= 2(0.6 \text{ rad/s})\underline{j} \times [-(3.6 \text{ in./s})\underline{j} + (6.2354 \text{ in./s})\underline{i}]$$

$$= -(7.482 \text{ in./s}^2)\underline{k}$$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$$

$$= -(10.08 \text{ in./s}^2)\underline{i} - (1.620 \text{ in./s}^2)\underline{i} - (2.806 \text{ in./s}^2)\underline{j} - (7.482 \text{ in./s}^2)\underline{k}$$

$$\underline{a}_D = -(11.70 \text{ in./s}^2)\underline{i} - (2.81 \text{ in./s}^2)\underline{j} - (7.48 \text{ in./s}^2)\underline{k} \quad \blacktriangleleft$$

15.191

SAME AS PROB. 15.190, EXCEPT $\beta = 60°$.

(a) ACCELERATION OF CD.

α IS INDEPENDENT OF β. SEE PROB. 15.190(a)

$$\underline{\alpha} = -(0.27 \text{ rad/s}^2)\underline{i} \quad \blacktriangleleft$$

$$\underline{r}_{D/C} = -(8 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}; \quad \underline{r}_{D/B} = (20 \text{ in.})\underline{i} + \underline{r}_{D/C} = (12 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}$$

(b) VELOCITY OF D: $\quad \underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$

$$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/B} = (0.6 \text{ rad/s})\underline{j} \times [(12 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}] = -(7.2 \text{ in./s})\underline{k}$$

$$\underline{v}_{D/F} = \underline{\omega}_{D/F} \times \underline{r}_{D/C} = -(0.45 \text{ rad/s})\underline{k} \times [-(8 \text{ in.})\underline{i} + (13.856 \text{ in.})\underline{j}]$$

$$= (3.6 \text{ in/s})\underline{j} + (6.2354 \text{ in.})\underline{i}$$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$:

$$\underline{v}_D = (6.24 \text{ in/s})\underline{i} + (3.6 \text{ in./s})\underline{j} - (7.2 \text{ in./s})\underline{k} \quad \blacktriangleleft$$

(CONTINUED)

15.191 CONTINUED

(c) ACCELERATION OF D

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$$

$$\underline{a}_{D'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/B}) = \underline{\Omega} \times \underline{v}_{D'}$$

$$= (0.6 \text{ rad/s})\underline{j} \times -(7.2 \text{ in./s})\underline{k} = -(4.32 \text{ in./s}^2)\underline{i}$$

$$\underline{a}_{D/F} = \underline{\omega}_{D/F} \times (\underline{\omega}_{D/F} \times \underline{r}_{D/C}) = \underline{\omega}_{D/F} \times \underline{v}_{D/F}$$

$$= -(0.45 \text{ rad/s})\underline{k} \times [(3.6 \text{ in./s})\underline{j} + (6.2354 \text{ in./s})\underline{i}]$$

$$= (1.620 \text{ in./s}^2)\underline{i} - (2.806 \text{ in./s}^2)\underline{j}$$

$$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/F} = 2(0.6 \text{ rad/s})\underline{j} \times [(3.6 \text{ in./s})\underline{j} + (6.2354 \text{ in./s})\underline{i}]$$

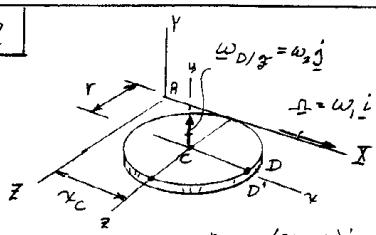
$$= -(7.482 \text{ in./s}^2)\underline{k}$$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$$

$$= -(4.32 \text{ in./s}^2)\underline{i} + (1.620 \text{ in./s}^2)\underline{i} - (2.806 \text{ in./s}^2)\underline{j} - (7.482 \text{ in./s}^2)\underline{k}$$

$$\underline{a}_D = -(2.70 \text{ in/s}^2)\underline{i} - (2.81 \text{ in./s}^2)\underline{j} - (7.48 \text{ in./s}^2)\underline{k} \quad \blacktriangleleft$$

15.192



$$\underline{r}_{D/A} = (r_C + r)\underline{i} + r\underline{k}$$

VELOCITIES:

$$\underline{\omega}_{D/F} = \omega_2\underline{j}$$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$:

$$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = \omega_1\underline{i} \times [(r_C + r)\underline{i} + r\underline{k}] = -r\omega_1\underline{j}$$

$$\underline{v}_{D/F} = \underline{\omega}_{D/F} \times \underline{r}_{D/C} = \omega_2\underline{j} \times r\underline{i} = -r\omega_2\underline{k}$$

$$\underline{v}_D = -r\omega_1\underline{j} - r\omega_2\underline{k} \quad \triangleleft$$

ACCELERATIONS:

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$$

$$\underline{a}_{D'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{DA} = \underline{\Omega} \times \underline{v}_{D'} = \omega_1\underline{i} \times (-r\omega_1\underline{j}) = -r\omega_1^2\underline{k}$$

$$\underline{a}_{D/F} = \underline{\omega}_{D/F} \times \underline{\omega}_{D/F} \times \underline{r}_{D/C} = \underline{\omega}_{D/F} \times \underline{v}_{D/F} = \omega_2\underline{j} \times (-r\omega_2\underline{k}) = -r\omega_2^2\underline{i}$$

$$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/F} = 2(\omega_1\underline{i}) \times (-r\omega_2\underline{k}) = 2r\omega_1\omega_2\underline{j}$$

$$\underline{a}_D = -r\omega_2^2\underline{i} + 2r\omega_1\omega_2\underline{j} - r\omega_1^2\underline{k} \quad \triangleleft$$

GIVEN DATA:

$r = 0.18 \text{ m}$; $\quad \omega_1 = 6 \text{ rad/s}$; $\quad \omega_2 = 8 \text{ rad/s}$

$$\underline{v}_D = -(0.18 \text{ m})(6 \text{ rad/s})\underline{j} - (0.18 \text{ m})(8 \text{ rad/s})\underline{k}$$

$$\underline{v}_D = -(1.08 \text{ m/s})\underline{j} - (1.44 \text{ m/s})\underline{k} \quad \blacktriangleleft$$

$$\underline{a}_D = -(0.18 \text{ m})(8 \text{ rad/s})^2\underline{i} + 2(0.18 \text{ m})(6 \text{ rad/s})(8 \text{ rad/s})\underline{j} - (0.18 \text{ m})(6 \text{ rad/s})^2\underline{k}$$

$$\underline{a}_D = -(11.52 \text{ m/s}^2)\underline{i} + (17.28 \text{ m/s}^2)\underline{j} - (6.48 \text{ m/s}^2)\underline{k} \quad \blacktriangleleft$$

15.193

SEE SOLUTION OF PROB. 15.192:

GIVEN DATA

$r = 0.18 \text{ m}$; $\quad \omega_1 = 8 \text{ rad/s}$; $\quad \omega_2 = 6 \text{ rad/s}$

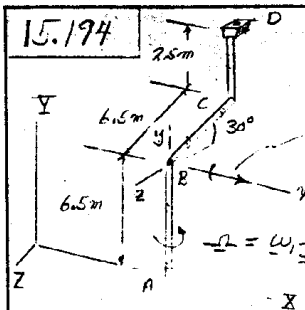
$$\underline{v}_D = -(0.18 \text{ m})(8 \text{ rad/s})\underline{j} - (0.18 \text{ m})(6 \text{ rad/s})\underline{k}$$

$$\underline{v}_D = -(1.44 \text{ m/s})\underline{j} - (1.08 \text{ m/s})\underline{k} \quad \blacktriangleleft$$

$$\underline{a}_D = -(0.18 \text{ m})(6 \text{ rad/s})^2\underline{i} + 2(0.18 \text{ m})(8 \text{ rad/s})(6 \text{ rad/s})\underline{j} - (0.18 \text{ m})(8 \text{ rad/s})^2\underline{k}$$

$$\underline{a}_D = -(6.48 \text{ m/s}^2)\underline{i} + (17.28 \text{ m/s}^2)\underline{j} - (11.52 \text{ m/s}^2)\underline{k} \quad \blacktriangleleft$$

15.194



$\underline{\omega}_{D/F} = \omega_D \underline{i} = \frac{d\beta}{dt}\underline{i} = (0.04\ \text{rad/s})\underline{i}$

$\underline{\Omega} = \omega_1 \underline{j} = (0.03\ \text{rad/s})\underline{j}$

$\underline{r}_{D/B} = +(6.5\sin 30°)\underline{j} + 2.5\underline{j} - (6.5\cos 30°)\underline{k}$

$= +(5.75\ \text{m})\underline{j} - (5.629\ \text{m})\underline{k}$

$\underline{r}_{D/A} = (6.5\ \text{m})\underline{j} + \underline{r}_{D/B} = (12.25\ \text{m})\underline{j} - (5.629\ \text{m})\underline{k}$

(a) INITIAL ACCELERATION OF ECU.

$\underline{v}_{D/F} = (0.04\ \text{rad/s})\underline{i} + (0.03\ \text{rad/s})\underline{j}$

$\underline{\alpha} = \underline{\Omega} \times \underline{\omega} = (0.03\ \text{rad/s})\underline{j} \times [(0.04\ \text{rad/s})\underline{i} + (0.03\ \text{rad/s})\underline{j}]$

$\underline{\alpha} = -(1.2 \times 10^{-3}\ \text{rad/s}^2)\underline{k}$ ◀

(b) VELOCITY OF D. $\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = (0.03\ \text{rad/s})\underline{j} \times [(12.25\ \text{m})\underline{j} - (5.629\ \text{m})\underline{k}]$

$= -(168.9\ \text{mm/s})\underline{i}$

$\underline{v}_{D/F} = \underline{\omega}_{D/F} \times \underline{r}_{D/B} = (0.04\ \text{rad/s})\underline{i} \times [(5.75\ \text{m})\underline{j} - (5.629\ \text{m})\underline{k}]$

$= (230\ \text{mm/s})\underline{k} + (225.16\ \text{mm/s})\underline{j}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}$

$\underline{v}_D = -(168.9\ \text{mm/s})\underline{i} + (225\ \text{mm/s})\underline{j} + (230\ \text{mm/s})\underline{k}$ ◀

(c) ACCELERATION OF D. $\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$

$\underline{a}_{D'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/A}) = \underline{\Omega} \times \underline{v}_{D'}$

$= (0.03\ \text{rad/s})\underline{j} \times (-168.9\ \text{mm/s})\underline{i}$

$= (5.067\ \text{mm/s}^2)\underline{k}$

$\underline{a}_{D/F} = \underline{\omega}_{D/F} \times (\underline{\omega}_{D/F} \times \underline{r}_{D/B}) = \underline{\omega}_{D/F} \times \underline{v}_{D/F}$

$= (0.04\ \text{rad/s})\underline{i} \times [(230\ \text{mm/s})\underline{k} + (225.16\ \text{mm/s})\underline{j}]$

$= -(9.20\ \text{mm/s}^2)\underline{j} + (9.006\ \text{mm/s}^2)\underline{k}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/F}$

$= 2(0.03\ \text{rad/s})\underline{j} \times [(230\ \text{mm/s})\underline{k} + (225.16\ \text{mm/s})\underline{j}]$

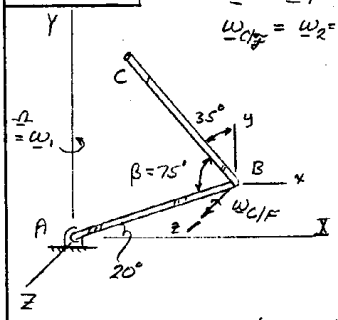
$= (13.80\ \text{mm/s}^2)\underline{i}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$

$= (5.067\ \text{mm/s}^2)\underline{k} - (9.20\ \text{mm/s}^2)\underline{j} + (9.006\ \text{mm/s}^2)\underline{k} + (13.80\ \text{mm/s}^2)\underline{i}$

$\underline{a}_D = (13.80\ \text{mm/s}^2)\underline{i} - (9.20\ \text{mm/s}^2)\underline{j} + (14.07\ \text{mm/s}^2)\underline{k}$ ◀

15.195



$\underline{\Omega} = \underline{\omega}_1 = (0.15\ \text{rad/s})\underline{j}$

$\underline{\omega}_{C/F} = \underline{\omega}_2 = -\frac{d\beta}{dt}\underline{k} = +(0.20\ \text{rad/s})\underline{k}$

$AB = BC = 4.5\ \text{m}$

$\underline{r}_{C/B} = -4.5\sin 35°\,\underline{i} + 4.5\cos 35°\,\underline{j}$

$\underline{r}_{C/B} = -(2.581\ \text{m})\underline{i} + (3.686\ \text{m})\underline{j}$

$\underline{r}_{B/A} = 4.5\cos 20°\,\underline{i} + 4.5\sin 20°\,\underline{j}$

$= +(4.229\ \text{m})\underline{i} + (1.539\ \text{m})\underline{j}$

$\underline{r}_{C/A} = \underline{r}_{B/A} + \underline{r}_{C/B} = (1.648\ \text{m})\underline{i} + (5.225\ \text{m})\underline{j}$

$\underline{v}_{C/F} = \underline{\omega}_{C/F} \times \underline{r}_{C/B} = +(0.20\ \text{rad/s})\underline{k} \times [-(2.581\ \text{m})\underline{i} + (3.686\ \text{m})\underline{j}]$

$= -(0.5162\ \text{m/s})\underline{j} - (0.7372\ \text{m/s})\underline{i}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (0.15\ \text{rad/s})\underline{j} \times [(1.648\ \text{m})\underline{i} + (5.225\ \text{m})\underline{j}]$

$= -(0.2472\ \text{m/s})\underline{k}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/F} = -(0.737\ \text{m/s})\underline{i} - (0.516\ \text{m/s})\underline{j} - (0.247\ \text{m/s})\underline{k}$

ACCELERATION OF C. $\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/F} + \underline{a}_C$

$\underline{a}_{C'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{C/A}) - \underline{\Omega} \times \underline{v}_{C'}$

$= (0.15\ \text{rad/s})\underline{j} \times [-0.2472\ \text{m/s})\underline{k}] = -(0.0371\ \text{m/s}^2)\underline{i}$

$\underline{a}_{C/F} = \underline{\omega}_{C/F} \times (\underline{\omega}_{C/F} \times \underline{r}_{C/B}) = \underline{\omega}_{C/F} \times \underline{v}_{C/F}$

$= +(0.20\ \text{rad/s})\underline{k} \times [-(0.5162\ \text{m/s})\underline{j} - (0.7372\ \text{m/s})\underline{i}]$

$= +(0.1032\ \text{m/s}^2)\underline{i} - (0.1474\ \text{m/s}^2)\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{C/F} = 2(0.15\ \text{rad/s})\underline{j} \times [-(0.5762\ \text{m/s})\underline{j} - (0.2372\ \text{m/s})\underline{i}]$

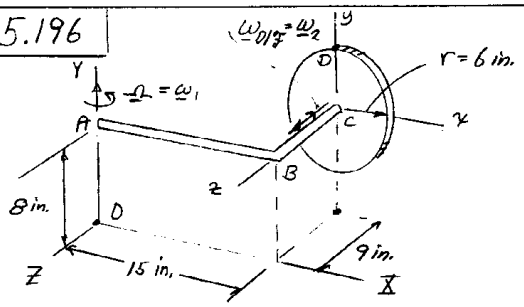
$= +(0.2212\ \text{m/s}^2)\underline{k}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/F} + \underline{a}_C$

$= -(0.0371\ \text{m/s}^2)\underline{i} + (0.1032\ \text{m/s}^2)\underline{i} - (0.1474\ \text{m/s}^2)\underline{j} + (0.2212\ \text{m/s}^2)\underline{k}$

$\underline{a}_C = (0.0661\ \text{m/s}^2)\underline{i} - (0.1474\ \text{m/s}^2)\underline{j} + (0.2212\ \text{m/s}^2)\underline{k}$ ◀

15.196



$\underline{\Omega} = \underline{\omega}_1 = (3\ \text{rad/s})\underline{j}$ $\underline{\omega}_{D/F} = \underline{\omega}_2 = (4\ \text{rad/s})\underline{k}$

$\underline{\omega} = \underline{\Omega} + \underline{\omega}_{D/F} = (3\ \text{rad/s})\underline{j} + (4\ \text{rad/s})\underline{k}$

$\underline{\alpha} = \underline{\Omega} \times \underline{\omega}$

$= (3\ \text{rad/s})\underline{j} \times [(3\ \text{rad/s})\underline{j} + (4\ \text{rad/s})\underline{k}]$

$\underline{\alpha} = (12\ \text{rad/s}^2)\underline{i}$ ◀

(CONTINUED)

MAKRON Books
COPYDESK
Marketing Universitário
(0xx11) 3845-2105
(0xx11) 3845-6729
fax (0xx11) 3845-5095

15.196 CONTINUED

$\underline{r}_{D/C} = (6\text{ in.})\underline{j}$

$\underline{r}_{D/A} = (15\text{ in.})\underline{i} + (6\text{ in.})\underline{j} - (9\text{ in.})\underline{k}$

VELOCITY OF D.

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = (3\text{ rad/s})\underline{j} \times [(15\text{ in.})\underline{i} + (6\text{ in.})\underline{j} - (9\text{ in.})\underline{k}]$

$= -(45\text{ in./s})\underline{k} - (27\text{ in./s})\underline{i}$

$\underline{v}_{D/F} = \underline{\omega}_{D/F} \times \underline{r}_{D/C} = (4\text{ rad/s})\underline{k} \times (6\text{ in.})\underline{j}$

$= -(24\text{ in./s})\underline{i}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F} = -(51\text{ in./s})\underline{i} - (45\text{ in./s})\underline{k}$

(b) ACCELERATION OF D. $\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$

$\underline{a}_{D'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{D/A}) = \underline{\Omega} \times \underline{v}_{D'}$

$= (3\text{ rad/s})\underline{j} \times [-(45\text{ in./s})\underline{k} - (27\text{ in./s})\underline{i}]$

$= -(135\text{ in./s}^2)\underline{i} + (81\text{ in./s}^2)\underline{k}$

$\underline{a}_{D/F} = \underline{\omega}_{D/F} \times (\underline{\omega}_{D/F} \times \underline{r}_{D/C}) = \underline{\omega}_{D/F} \times \underline{v}_{D/F}$

$= (4\text{ rad/s})\underline{k} \times (-24\text{ in./s})\underline{i} = -(96\text{ in./s}^2)\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/F} = 2(3\text{ rad/s})\underline{j} \times (-24\text{ in./s})\underline{i}$

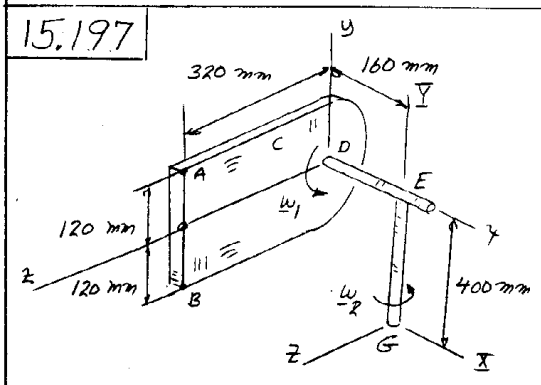
$= (144\text{ in./s}^2)\underline{k}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_C$

$= -(135\text{ in./s}^2)\underline{i} + (81\text{ in./s}^2)\underline{k} - (96\text{ in./s}^2)\underline{j} + (144\text{ in./s}^2)\underline{k}$

$\underline{a}_D = -(135\text{ in./s}^2)\underline{i} - (96\text{ in./s}^2)\underline{j} + (225\text{ in./s}^2)\underline{k}$ ◀

15.197



$\underline{\Omega} = \underline{\omega}_2 = (2\text{ rad/s})\underline{j}$

$\underline{\omega}_{ABC/F} = \underline{\omega}_1 = (2.4\text{ rad/s})\underline{i}$

$\underline{r}_{A/D} = (120\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$

$\underline{r}_{A/G} = -(160\text{ mm})\underline{i} + (520\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$

(a) VELOCITY OF A. $\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/F}$

$\underline{v}_{A'} = \underline{\Omega} \times \underline{r}_{A/G}$

$= (2\text{ rad/s})\underline{j} \times [-(160\text{ mm})\underline{i} + (520\text{ mm})\underline{j} + (320\text{ mm/s})\underline{k}]$

$= +(320\text{ mm/s})\underline{k} + (640\text{ mm/s})\underline{i}$

$\underline{v}_{A/F} = \underline{\omega}_{ABC/F} \times \underline{r}_{A/D} = (2.4\text{ rad/s})\underline{i} \times [(120\text{ mm})\underline{j} + (320\text{ mm})\underline{k}]$

$= (288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}$

(CONTINUED)

15.197 CONTINUED

$\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/F}$

$\underline{v}_A = (320\text{ mm/s})\underline{k} + (640\text{ mm/s})\underline{i} + (288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}$

$\underline{v}_A = (0.64\text{ m/s})\underline{i} - (0.768\text{ m/s})\underline{j} + (0.608\text{ m/s})\underline{k}$ ◀

(b) ACCELERATION OF A. $\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C$

$\underline{a}_{A'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/G}) = \underline{\Omega} \times \underline{v}_{A'}$

$= (2\text{ rad/s})\underline{j} \times [(320\text{ mm})\underline{k} + (640\text{ mm})\underline{i}]$

$= (640\text{ mm/s}^2)\underline{i} - (1280\text{ mm/s}^2)\underline{k}$

$\underline{a}_{A/F} = \underline{\omega}_{ABC/F} \times (\underline{\omega}_{ABC/F} \times \underline{r}_{A/D}) = \underline{\omega}_{ABC/F} \times \underline{v}_{A/F}$

$= (2.4\text{ rad/s})\underline{i} \times [(288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}]$

$= -(691\text{ mm/s}^2)\underline{j} - (1843\text{ mm/s}^2)\underline{k}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{A/F} = 2(2\text{ rad/s})\underline{j} \times [(288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}]$

$= (1152\text{ mm/s}^2)\underline{i}$

$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C$

$= (640\text{ mm/s}^2)\underline{i} - (1280\text{ mm/s}^2)\underline{k} - (691\text{ mm/s}^2)\underline{j}$

$- (1843\text{ mm/s}^2)\underline{k} + (1152\text{ mm/s}^2)\underline{i}$

$\underline{a}_A = (1.792\text{ m/s}^2)\underline{i} - (0.691\text{ m/s}^2)\underline{j} - (3.12\text{ m/s}^2)\underline{k}$ ◀

15.198

SEE SOLUTION & SKETCH OF PROB. 15.197.

$\underline{\Omega} = (2\text{ rad/s})\underline{j}$ $\underline{\omega}_{ABC/F} = (2.4\text{ rad/s})\underline{i}$

$\underline{r}_{B/D} = -(120\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$

$\underline{r}_{B/G} = -(160\text{ mm})\underline{i} + (280\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$

(a) VELOCITY OF B. $\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/F}$

$\underline{v}_{B'} = \underline{\Omega} \times \underline{r}_{B/G}$

$= (2\text{ rad/s})\underline{j} \times [-(160\text{ mm})\underline{i} + (280\text{ mm})\underline{j} + (320\text{ mm})\underline{k}]$

$= +(320\text{ mm/s})\underline{k} + (640\text{ mm/s})\underline{i}$

$\underline{v}_{B/F} = \underline{\omega}_{ABC/F} \times \underline{r}_{B/D} = (2.4\text{ rad/s})\underline{i} \times [-(120\text{ mm})\underline{j} + (320\text{ mm})\underline{k}]$

$= -(288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}$

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/F} = (640\text{ mm/s})\underline{i} - (768\text{ mm/s})\underline{j} + (32\text{ mm/s})\underline{k}$ ◀

(b) ACCELERATION OF B. $\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/F} + \underline{a}_C$

$\underline{a}_{B'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{B/G}) = \underline{\Omega} \times \underline{v}_{B'}$

$= (2\text{ rad/s})\underline{j} \times [(320\text{ mm/s})\underline{k} + (640\text{ mm/s})\underline{i}]$

$= (640\text{ mm/s}^2)\underline{i} - (1280\text{ mm/s}^2)\underline{k}$

$\underline{a}_{B/F} = \underline{\omega}_{ABC/F} \times (\underline{\omega}_{ABC/F} \times \underline{r}_{B/D}) = \underline{\omega}_{ABC/F} \times \underline{v}_{B/F}$

$= (2.4\text{ rad/s})\underline{i} \times [(-288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}]$

$= (691\text{ mm/s}^2)\underline{j} - (1843\text{ mm/s}^2)\underline{k}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{B/F} = 2(2\text{ rad/s})\underline{j} \times [(-288\text{ mm/s})\underline{k} - (768\text{ mm/s})\underline{j}]$

$= -(1152\text{ mm/s}^2)\underline{i}$

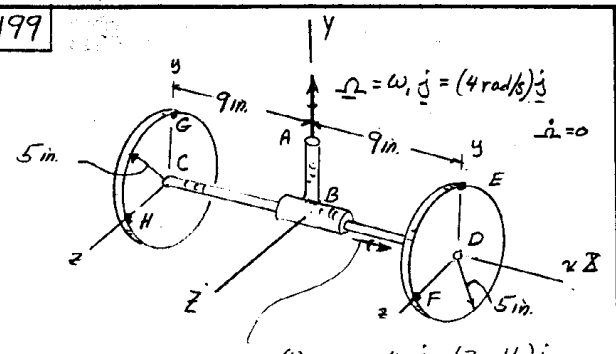
$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/F} + \underline{a}_C$

$= (640\text{ mm/s}^2)\underline{i} - (1280\text{ mm/s}^2)\underline{k} + (691\text{ mm/s}^2)\underline{j}$

$- (1843\text{ mm/s}^2)\underline{k} - (1152\text{ mm/s}^2)\underline{i}$

$\underline{a}_B = -(0.512\text{ m/s}^2)\underline{i} + (0.691\text{ m/s}^2)\underline{j} - (3.12\text{ m/s}^2)\underline{k}$ ◀

15.199



$\underline{\Omega} = \omega_1 \underline{j} = (4\ \text{rad/s})\underline{j}$ $\quad \dot{\underline{\Omega}} = 0$

$\underline{\omega}_{DISKS/\mathcal{F}} = \omega_2 \underline{i} = (3\ \text{rad/s})\underline{i}$ $\quad \dot{\underline{\omega}}_{D/\mathcal{F}} = 0$

(a) POINT E

VELOCITY $\quad \underline{v}_E = \underline{v}_{E'} + \underline{v}_{E/\mathcal{F}}$

$\underline{r}_{E/B} = (9\text{in.})\underline{i} + (5\text{in.})\underline{j} \qquad \underline{r}_{E/D} = (5\text{in.})\underline{j}$

$\underline{v}_{E'} = \underline{\Omega} \times \underline{r}_{E/B} = (4\ \text{rad/s})\underline{j} \times [(9\text{in.})\underline{i} + (5\text{in.})\underline{j}]$

$= -(36\ \text{in./s})\underline{k}$

$\underline{v}_{E/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{E/D} = (3\ \text{rad/s})\underline{i} \times (5\text{in.})\underline{j} = (15\ \text{in./s})\underline{k}$

$\underline{v}_E = -(36\ \text{in./s})\underline{k} + (15\ \text{in./s})\underline{k} \qquad \underline{v}_E = -(21\ \text{in./s})\underline{k}$ ◀

ACCELERATION

$\underline{a}_E = \underline{a}_{E'} + \underline{a}_{E/\mathcal{F}} + \underline{a}_C$

$\underline{a}_{E'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{E/B} = \underline{\Omega} \times \underline{v}_{E'} = (4\ \text{rad/s})\underline{j} \times (-36\ \text{in/s})\underline{k} = -(144\ \text{in/s}^2)\underline{i}$

$\underline{a}_{E/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{E/D} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{v}_{E/\mathcal{F}}$

$= (3\ \text{rad/s})\underline{i} \times (15\ \text{in./s})\underline{k} = -(45\ \text{in./s}^2)\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{E/\mathcal{F}} = 2(4\ \text{rad/s})\underline{j} \times (15\ \text{in./s})\underline{k} = (120\ \text{in/s}^2)\underline{i}$

$\underline{a}_E = -(144\ \text{in./s})\underline{i} - (45\ \text{in./s}^2)\underline{j} + (120\ \text{in/s}^2)\underline{i}$

$\underline{a}_E = -(24\ \text{in./s}^2)\underline{i} - (45\ \text{in./s}^2)\underline{j}$ ◀

(b) POINT F

VELOCITY $\quad \underline{v}_F = \underline{v}_{F'} + \underline{v}_{F/\mathcal{F}}$

$\underline{r}_{F/B} = (9\text{in.})\underline{i} + (5\text{in.})\underline{k} \qquad \underline{r}_{F/D} = (5\text{in.})\underline{k}$

$\underline{v}_{F'} = \underline{\Omega} \times \underline{r}_{F/B} = (4\ \text{rad/s})\underline{j} \times [(9\text{in.})\underline{i} + (5\text{in.})\underline{k}]$

$= -(36\ \text{in./s}^2)\underline{k} + (20\ \text{in./s})\underline{i}$

$\underline{v}_{F/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{F/D} = (3\ \text{rad/s})\underline{i} \times (5\text{in.})\underline{k} = -(15\text{in./s})\underline{j}$

$\underline{v}_F = (20\ \text{in./s})\underline{i} - (15\ \text{in./s})\underline{j} - (36\ \text{in/s})\underline{k}$ ◀

ACCELERATION $\quad \underline{a}_F = \underline{a}_{F'} + \underline{a}_{F/\mathcal{F}} + \underline{a}_C$

$\underline{a}_{F'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{F/B} = \underline{\Omega} \times \underline{v}_{F'} = (4\ \text{rad/s})\underline{j} \times [-(36\text{in./s})\underline{k} + (20\text{in./s})\underline{i}]$

$= -(144\ \text{in./s}^2)\underline{i} - (80\ \text{in/s}^2)\underline{k}$

$\underline{a}_{F/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{F/D} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{v}_{F/\mathcal{F}}$

$= (3\ \text{rad/s})\underline{i} \times (-15\text{in./s})\underline{j} = -(45\ \text{in/s}^2)\underline{k}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{F/\mathcal{F}} = 2(4\ \text{rad/s})\underline{j} \times (-15\text{in/s})\underline{j} = 0$

$\underline{a}_F = -(144\ \text{in./s}^2)\underline{i} - (80\ \text{in./s}^2)\underline{k} - (45\ \text{in./s}^2)\underline{k} + 0$

$\underline{a}_F = -(144\ \text{in/s}^2)\underline{i} - (125\ \text{in/s}^2)\underline{k}$ ◀

15.200

SEE SKETCH OF PROB 15.199

NOTE THAT DISKS ARE WELDED TO ROD CD. THUS FOR DISK BCH WE ALSO HAVE

$\underline{\omega}_{DISK/\mathcal{F}} = \omega_2 \underline{i} = (3\ \text{rad/s})\underline{i}$

$\dot{\underline{\omega}}_{DISK/\mathcal{F}} = 0$

ALSO $\underline{\Omega} = \omega_1 \underline{j} = (4\ \text{rad/s})\underline{j}$ $\qquad \dot{\underline{\Omega}} = 0$

(a) POINT G

VELOCITY $\quad \underline{v}_G = \underline{v}_{G'} + \underline{v}_{G/\mathcal{F}}$

$\underline{r}_{G/B} = -(9\text{in.})\underline{i} + (5\text{in.})\underline{j} \qquad \underline{r}_{G/C} = (5\text{in.})\underline{j}$

$\underline{v}_{G'} = \underline{\Omega} \times \underline{r}_{G/B} = (4\ \text{rad/s})\underline{j} \times [-(9\text{in.})\underline{i} + (5\text{in.})\underline{j}]$

$= (36\text{in./s})\underline{k}$

$\underline{v}_{G/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{G/C} = (3\ \text{rad/s})\underline{i} \times (5\text{in.})\underline{j} = (15\text{in./s})\underline{k}$

$\underline{v}_G = (36\ \text{in./s})\underline{k} + (15\ \text{in./s})\underline{k} \qquad \underline{v}_G = (51\ \text{in./s})\underline{k}$ ◀

ACCELERATION $\quad \underline{a}_G = \underline{a}_{G'} + \underline{a}_{G/\mathcal{F}} + \underline{a}_C$

$\underline{a}_{G'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{G/B} = \underline{\Omega} \times \underline{v}_{G'} = (4\ \text{rad/s})\underline{j} \times (36\ \text{in./s})\underline{k} = (144\ \text{in/s}^2)\underline{i}$

$\underline{a}_{G/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{G/C} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{v}_{G/\mathcal{F}}$

$= (3\ \text{rad/s})\underline{i} \times (15\ \text{in./s})\underline{k} = -(45\text{in./s}^2)\underline{j}$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{G/\mathcal{F}} = 2(4\ \text{rad/s})\underline{j} \times (15\text{in./s})\underline{k} = (120\ \text{in/s}^2)\underline{i}$

$\underline{a}_G = (144\text{in./s}^2)\underline{i} - (45\text{in./s}^2)\underline{j} + (120\text{in./s}^2)\underline{i}$

$\underline{a}_G = (264\text{in./s}^2)\underline{i} - (45\text{in/s}^2)\underline{j}$ ◀

(b) POINT H

VELOCITY $\quad \underline{v}_H = \underline{v}_{H'} + \underline{v}_{H/\mathcal{F}}$

$\underline{r}_{H/B} = -(9\text{in.})\underline{i} + (5\text{in.})\underline{k} \qquad \underline{r}_{H/C} = (5\text{in.})\underline{k}$

$\underline{v}_{H'} = \underline{\Omega} \times \underline{r}_{H/B} = (4\ \text{rad/s})\underline{j} \times [-(9\text{in.})\underline{i} + (5\text{in})\underline{k}]$

$= (36\ \text{in./s})\underline{k} + (20\ \text{in/s})\underline{i}$

$\underline{v}_{H/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{H/C} = (3\ \text{rad/s})\underline{i} \times (5\text{in.})\underline{k} = -(15\text{in./s})\underline{j}$

$\underline{v}_H = (20\text{in/s})\underline{i} - (15\text{in./s})\underline{j} + (36\text{in./s})\underline{k}$ ◀

ACCELERATION $\quad \underline{a}_H = \underline{a}_{H'} + \underline{a}_{H/\mathcal{F}} + \underline{a}_C$

$\underline{a}_{H'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{H/B} = \underline{\Omega} \times \underline{v}_{H'} = (4\ \text{rad/s})\underline{j} \times [(36\text{in/s})\underline{k} + (20\text{in/s})\underline{i}]$

$= (144\ \text{in./s}^2)\underline{i} - (80\text{in./s}^2)\underline{k}$

$\underline{a}_{H/\mathcal{F}} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{r}_{H/C} = \underline{\omega}_{DISKS/\mathcal{F}} \times \underline{v}_{H/\mathcal{F}}$

$= (3\ \text{rad/s})\underline{i} \times (-15\text{in./s})\underline{j} = -(45\text{in./s}^2)\underline{k}$

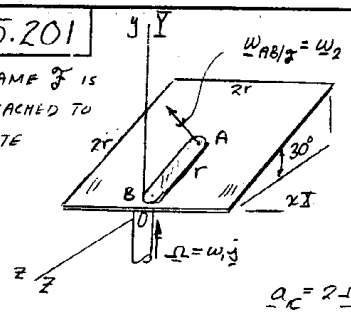
$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{H/\mathcal{F}} = 2(4\ \text{rad/s})\underline{j} \times (-15\text{in/s})\underline{j} = 0$

$\underline{a}_H = (144\text{in/s}^2)\underline{i} - (80\text{in./s}^2)\underline{k} - (45\text{in/s}^2)\underline{k} + 0$

$\underline{a}_H = (144\ \text{in/s}^2)\underline{i} - (125\text{in./s}^2)\underline{k}$ ◀

15.201

FRAME $\mathcal{F}$ IS ATTACHED TO PLATE



$\dot{\omega}_1 = \dot{\omega}_2 = 0$

(a) $\Theta = 0$

$\underline{v}_{B/\mathcal{F}} = r\omega_2 \underline{i}$

$\underline{a}_{B/\mathcal{F}} = -\omega_2^2 \underline{r}_{B/A} = r\omega_2^2(\sin 30° \underline{j} - \cos 30° \underline{k})$

$\underline{a}_{B'} = 0$

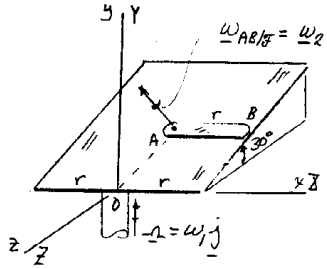
$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{B/\mathcal{F}} = 2(\omega_1 \underline{j}) \times (r\omega_2 \underline{i})$

$= -2r\omega_1\omega_2 \underline{k}$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$

$= 0 + r\omega_2^2(\sin 30° \underline{j} - \cos 30° \underline{k}) - 2r\omega_1\omega_2 \underline{k}$

$\underline{a}_B = r\omega_2^2 \sin 30° \underline{j} - (r\omega_2^2 \cos 30° + 2r\omega_1\omega_2)\underline{k}$ ◀



(b) $\Theta = 90°$

$\underline{v}_{B/\mathcal{F}} = r\omega_2(\sin 30° \underline{j} - \cos 30° \underline{k})$ $\quad \underline{a}_{B/\mathcal{F}} = -r\omega_2^2 \underline{i}$

$\underline{r}_{B/O} = r\underline{i} + r\sin 30° \underline{j} - r\cos 30° \underline{k}$

$\underline{a}_{B'} = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{OB})$

$= \omega_1 \underline{j} \times [\omega_1 \underline{j} \times (r\underline{i} + r\sin 30° \underline{j} - r\cos 30° \underline{k})]$

$= -r\omega_1^2 \underline{i} + r\omega_1^2 \cos 30° \underline{k}$

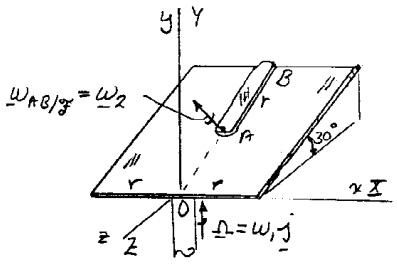
$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{B/\mathcal{F}} = 2(\omega_1 \underline{j}) \times (r\omega_2(\sin 30° \underline{j} - \cos 30° \underline{k})$

$= -2r\omega_1\omega_2 \cos 30° \underline{i}$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$

$= [-r\omega_1^2 \underline{i} + r\omega_1^2 \cos 30° \underline{k}] - r\omega_2^2 \underline{i} - 2r\omega_1\omega_2 \cos 30° \underline{i}$

$\underline{a}_B = -r(\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2 \cos 30°)\underline{i} + r\omega_1^2 \cos 30° \underline{k}$ ◀



(c) $\Theta = 180°$

$\underline{v}_{B/\mathcal{F}} = -r\omega_2 \underline{i}$

$\underline{a}_{B'} = +(2r\cos 30°)\omega_1^2 \underline{k} = +2r\omega_1^2 \cos 30° \underline{k}$

$\underline{a}_{B/\mathcal{F}} = r\omega_2^2(-\sin 30° \underline{j} + \cos 30° \underline{k})$

$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{B/\mathcal{F}} = 2(\omega_1 \underline{j}) \times (-r\omega_2 \underline{i}) = 2r\omega_1\omega_2 \underline{k}$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_C$

$= 2r\omega_1^2 \cos 30° \underline{k} + r\omega_2^2(-\sin 30° \underline{j} + \cos 30° \underline{k}) + 2r\omega_1\omega_2 \underline{k}$

$\underline{a}_B = -r\omega_2^2 \sin 30° \underline{j} + r(2\omega_1^2 \cos 30° + \omega_2^2 \cos 30° + 2\omega_1\omega_2)\underline{k}$ ◀

15.202

SEE SOLUTION OF PROB. 15.201.

GIVEN DATA: $\omega_1 = 3$ rad/s, $\omega_2 = 2$ rad/s, $r = 0.12$ m

(a) $\Theta = 0$: $\underline{a}_B = (0.12\text{m})(2\text{rad/s})^2 \sin 30° \underline{j}$

$-[(0.12\text{m})(2\text{rad/s})^2 \cos 30° + 2(0.12\text{m})(3\text{rad/s})(2\text{rad/s})]\underline{k}$

$\underline{a}_B = (0.24\text{m/s}^2)\underline{j} - (1.856\text{m/s}^2)\underline{k}$ ◀

(b) $\Theta = 90°$: $\underline{a}_B = -(0.12\text{m})[(3\text{rad/s})^2 + (2\text{rad/s})^2 + 2(3\text{rad/s})(2\text{rad/s})\cos 90°]\underline{i}$

$+ (0.12\text{m})(3\text{rad/s})^2 \cos 30° \underline{k}$

$\underline{a}_B = -(2.81\text{m/s}^2)\underline{i} + (0.935\text{m/s}^2)\underline{k}$ ◀

(c) $\Theta = 180°$: $\underline{a}_B = -(0.12\text{m})(2\text{rad/s})^2 \sin 30° \underline{j}$

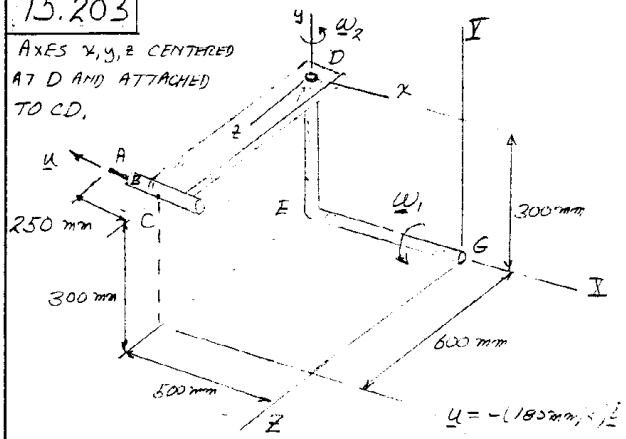
$+ (0.12\text{m})[2(3\text{rad/s})^2 \cos 30° + (2\text{rad/s})^2 \cos 30°$

$+ 2(3\text{rad/s})(2\text{rad/s})]\underline{k}$

$\underline{a}_B = -(0.24\text{m/s}^2)\underline{j} + (3.73\text{m/s}^2)\underline{k}$ ◀

15.203

AXES x, y, z CENTERED AT D AND ATTACHED TO CD.



$\underline{u} = -(180\text{mm/s})\underline{i}$

$\underline{\omega}_1 = (1.2\text{ rad/s})\underline{i}$ $\quad \underline{\omega}_2 = (1.6\text{ rad/s})\underline{j}$

$\underline{r}_{D/G} = -(500\text{ mm})\underline{i} + (300\text{ mm})\underline{j}$

$\underline{r}_{A/D} = -(250\text{ mm})\underline{i} + (600\text{ mm})\underline{k}$

(a) VELOCITY OF A: $\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/\mathcal{F}}$

$\underline{v}_{A'} = \underline{v}_D + \underline{v}_{A/D}$ WHERE $\underline{v}_{A/D}$ IS VELOCITY OF A RELATIVE TO THE FRAME AXES ATTACHED AT D.

$\underline{v}_{A'} = \underline{\omega}_1 \times \underline{r}_{D/G} + (\underline{\omega}_1 + \underline{\omega}_2) \times \underline{r}_{A/D}$

$= (1.2\text{ rad/s})\underline{i} \times [-(500\text{ mm})\underline{i} + (300\text{ mm})\underline{j}]$

$+ [(1.2\text{ rad/s})\underline{i} + (1.6\text{ rad/s})\underline{j}] \times [-(250\text{ mm})\underline{i} + (600\text{ mm})\underline{k}]$

$= (360\text{ mm/s})\underline{k} - (720\text{ mm/s})\underline{j} + (400\text{ mm/s})\underline{i} + (960\text{ mm/s})\underline{j}$

$\underline{v}_{A'} = (960\text{ mm/s})\underline{i} - (720\text{ mm/s})\underline{j} + (360\text{ mm/s})\underline{k}$

$\underline{v}_{A/\mathcal{F}} = \underline{u} = -(180\text{ mm/s})\underline{i}$

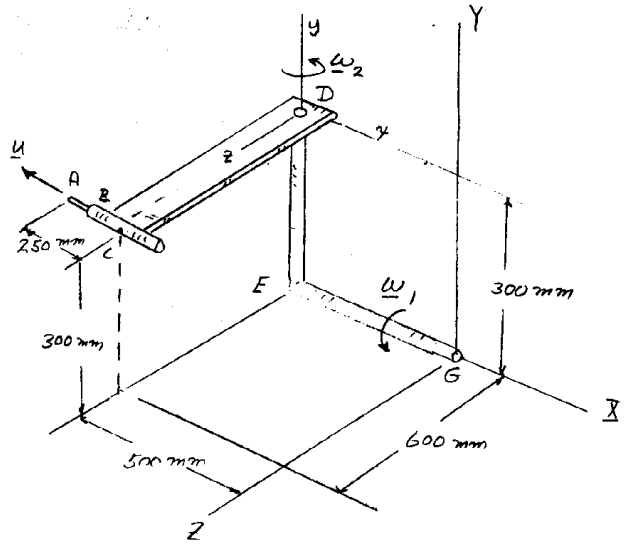
$\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/\mathcal{F}} = (960\text{ mm/s})\underline{i} - (720\text{ mm/s})\underline{j} + (360\text{ mm/s})\underline{k} - (180\text{ mm/s})\underline{i}$

$\underline{v}_A = (0.78\text{ m/s})\underline{i} - (0.72\text{ m/s})\underline{j} + (0.36\text{ m/s})\underline{k}$ ◀

(CONTINUED)

15.203 CONTINUED

WE REPEAT THE FIGURE:



[AB]: $\omega_1 = (1.2\ \text{rad/s})\underline{i}$ $\quad \underline{u} = -(180\ \text{mm/s})\underline{i}$

$\underline{\omega}_2 = (1.6\ \text{rad/s})\underline{k}$

WE HAVE CALCULATED:

$\underline{r}_{D/G} = -(500\ \text{mm})\underline{i} + (300\ \text{mm})\underline{j}$

$\underline{r}_{A/D} = -(250\ \text{mm})\underline{i} + (600\ \text{mm})\underline{k}$

$\underline{v}_A = (0.78\ \text{m/s})\underline{i} - (0.72\ \text{m/s})\underline{j} + (0.76\ \text{m/s})\underline{k}$

NOTE: AXES x, y, z ARE CENTERED AT D AND ARE ATTACHED TO CD.

b) ACCELERATION OF A. $\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C$

$\underline{a}_{A'} = \underline{a}_D + \underline{a}_{A'/D}$ WHERE $\underline{a}_{A'/D}$ IS ACCELERATION OF A RELATIVE TO TRANSLATING AXES ATTACHED AT D.

FOR ARM CD, WE COMPUTE:

$\underline{\omega}_{CD} = \underline{\omega}_1 + \underline{\omega}_2 = (1.2\ \text{rad/s})\underline{i} + (1.6\ \text{rad/s})\underline{j}$

$\underline{\alpha}_{CD} = \underline{\omega}_1 \times \underline{\omega}_{CD} = (1.2\ \text{rad/s})\underline{i} \times [(1.2\ \text{rad/s})\underline{i} + (1.6\ \text{rad/s})\underline{j}]$

$= (1.92\ \text{rad/s}^2)\underline{k}$

$\underline{a}_D = \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{D/G})$

$= (1.2\ \text{rad/s})\underline{i} \times \{(1.2\ \text{rad/s})\underline{i} \times [-(500\ \text{mm})\underline{i} + (300\ \text{mm})\underline{j}]\}$

$= -(432\ \text{mm/s}^2)\underline{j}$

$\underline{a}_{A'/D} = \underline{\alpha}_{CD} \times \underline{r}_{A/D} + \underline{\omega}_{CD} \times (\underline{\omega}_{CD} \times \underline{r}_{A/D})$

$= (1.92\ \text{rad/s}^2)\underline{k} \times [-(250\ \text{mm})\underline{i} + (600\ \text{mm})\underline{k}]$

$+ \underline{\omega}_{CD} \times \{[(1.2\ \text{rad/s})\underline{i} + (1.6\ \text{rad/s})\underline{j}] \times [-250\underline{i} + 600\underline{k}]\}$

(CONTINUED)

15.203 CONTINUED

$\underline{a}_{A'/D} = -(480\ \text{mm/s}^2)\underline{j} + \underline{\omega}_{CD} \times [-(720\ \text{mm/s})\underline{j} + (400\ \text{mm/s})\underline{k} + (960\ \text{mm/s})\underline{i}]$

$= -480\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.2 & 1.6 & 0 \\ 960 & -720 & 400 \end{vmatrix}$

$= -480\underline{j} + 640\underline{i} - 480\underline{j} - 864\underline{k} - 1536\underline{k}$

$\underline{a}_{A'/D} = (640\ \text{mm/s}^2)\underline{i} - (960\ \text{mm/s}^2)\underline{j} - (2400\ \text{mm/s}^2)\underline{k}$

$\underline{a}_{A'} = \underline{a}_D + \underline{a}_{A'/D} = -(432\ \text{mm/s}^2)\underline{j} + [\quad]$

$\underline{a}_{A'} = (640\ \text{mm/s}^2)\underline{i} - (1392\ \text{mm/s}^2)\underline{j} - (2400\ \text{mm/s}^2)\underline{k}$

$\underline{a}_{A/F} = 0$ SINCE $\underline{v}_{A/F} = \underline{u} = -(180\ \text{mm/s})\underline{i}$ = CONSTANT

$\underline{a}_C = 2\underline{\omega}_{CD} \times \underline{v}_{A/F}$

$= 2[(1.2\ \text{rad/s})\underline{i} + (1.6\ \text{rad/s})\underline{j}] \times (-180\ \text{mm/s})\underline{i}$

$= +(576\ \text{mm/s}^2)\underline{k}$

$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C$

$= (640\ \text{mm/s}^2)\underline{i} - (1392\ \text{mm/s}^2)\underline{j} - (2400\ \text{mm/s}^2)\underline{k}$

$+ (576\ \text{mm/s}^2)\underline{k}$

$\underline{a}_A = (0.64\ \text{m/s}^2)\underline{i} - (1.392\ \text{m/s}^2)\underline{j} - (1.824\ \text{m/s}^2)\underline{k}$ ◀

ALTERNATE SOLUTION (INVOLVING TWO CORIOLIS TERMS)

WE USE A REFERENCE FRAME WITH ORIGIN AT D AND ROTATING WITH THE COMPONENT DEG.

VELOCITY OF A. $\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/F}$

$\underline{v}_{A'} = \underline{\omega}_1 \times \underline{r}_{A/G} = (1.2\ \text{rad/s})\underline{i} \times [-(500\ \text{mm})\underline{i} + (300\ \text{mm})\underline{j} + 600\underline{k}]$

$= (360\ \text{mm/s})\underline{k} - (720\ \text{mm/s})\underline{j}$

$\underline{v}_{A/F} = \underline{\omega}_2 \times \underline{r}_{A/D} + \underline{u}$

$= (1.6\ \text{rad/s})\underline{j} \times [-(250\ \text{mm})\underline{i} + (600\ \text{mm})\underline{k}] - (180\ \text{mm/s})\underline{i}$

$= (400\ \text{mm/s})\underline{k} + (960\ \text{mm/s})\underline{i} - (180\ \text{mm/s})\underline{i}$

$\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/F} = 360\underline{k} - 720\underline{j} + 400\underline{k} + 960\underline{i} - 180\underline{i}$

$\underline{v}_A = (0.78\ \text{m/s})\underline{i} - (0.72\ \text{m/s})\underline{j} + (0.76\ \text{m/s})\underline{k}$ ◀

ACCELERATION OF A. $\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C$

$\underline{a}_{A'} = \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{A/G}) = \underline{\omega}_1 \times \underline{v}_{A'}$

$= (1.2\ \text{rad/s})\underline{i} \times [(360\ \text{mm/s})\underline{k} - (720\ \text{mm/s})\underline{j}]$

$= -(432\ \text{mm/s}^2)\underline{j} - (864\ \text{mm/s}^2)\underline{k}$

$\underline{a}_{A/F}$ (SINCE A MOVES ON CD WHICH ROTATES AT RATE ω_2, WE HAVE A CORIOLIS TERM HERE)

$\underline{a}_{A/F} = \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{A/D}) + 2\underline{\omega}_2 \times \underline{u}$

$= \underline{\omega}_2 \times \{(1.6\ \text{rad/s})\underline{j} \times [-(250\ \text{mm})\underline{i} + (600\ \text{mm})\underline{k}]\} + 2\underline{\omega}_2 \times \underline{u}$

$= (1.6\ \text{rad/s})\underline{j} \times [(400\ \text{mm/s})\underline{k} + (960\ \text{mm/s})\underline{i}]$

$+ 2(1.6\ \text{rad/s})\underline{j} \times (-180\ \text{mm})\underline{i}$

$= (640\ \text{mm/s}^2)\underline{i} - (1536\ \text{mm/s}^2)\underline{k} + (576\ \text{mm/s}^2)\underline{k}$

$= (640\ \text{mm/s}^2)\underline{i} - (960\ \text{mm/s}^2)\underline{k}$

$\underline{a}_C = 2\underline{\omega}_1 \times \underline{v}_{A/F}$

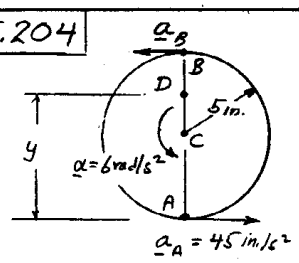
$= 2(1.2\ \text{rad/s})\underline{i} \times [(400\ \text{mm})\underline{k} + (180\ \text{mm/s})\underline{i}]$

$= -(960\ \text{mm/s}^2)\underline{j}$

$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/F} + \underline{a}_C = -432\underline{j} - 864\underline{k} + 640\underline{i} - 960\underline{k} - 960\underline{j}$

$\underline{a}_A = (0.64\ \text{m/s}^2)\underline{i} - (1.392\ \text{m/s}^2)\underline{j} - (1.824\ \text{m/s}^2)\underline{k}$ ◀

15.204



(a) ACCELERATION OF B

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$= \underline{a}_A + (\overrightarrow{AB})\underline{\alpha}$

$= [45\text{ in./s}^2 \rightarrow] + [(10\text{ in.})(6\text{ rad/s}^2) \leftarrow]$

$= [45\text{ in./s}^2 \rightarrow] + [60\text{ in./s}^2 \leftarrow]$

$\underline{a}_B = 15\text{ in./s}^2 \leftarrow$ ◀

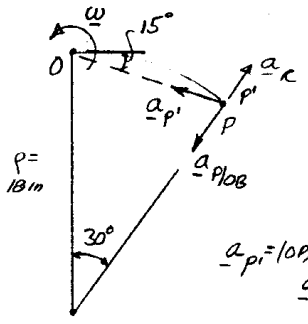
(b) LET D BE POINT ON DIAMETER AB LOCATED AT DISTANCE y FROM A.

$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A} = \underline{a}_A + (y)\underline{\alpha}$

$a_D = [45\text{ in./s}^2 \rightarrow] + [y(6\text{ rad/s}^2) \leftarrow]$

For $a_D = 0$: $\pm\ 45 - 6y = 0$ $\quad y = 7.5$ in. FROM A ◀

15.205



$\underline{\omega} = 150$ rpm $= 15.71$ rad/s

$\underline{v}_{P/O} = 45$ ft/s ∢ 30°

$a_{P/O} = \dfrac{(v_{P/O})^2}{\rho} = \dfrac{(45\text{ ft/s})^2}{(18/12\text{ ft})}$

$a_{P/O} = 1350\text{ ft/s}^2$ ∢ 60°

$OP = 2\rho \sin 15° = 2(18\text{ in.})\sin 15°$

$OP = 9.317\text{ in.} = 0.7765\text{ ft}$

$a_{P'} = (OP)\omega^2 = (0.7765\text{ ft})(15.71\text{ rad/s})^2$

$a_{P'} = 191.6\text{ ft/s}^2$ ∢ 15°

$a_n = 2\omega v_{P/O} = 2(15.71\text{ rad/s})(45\text{ ft/s})$

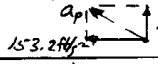
$a_n = 1414\text{ ft/s}^2$ ∢ 60°

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/O} + \underline{a}_n$

$= 191.6\text{ ft/s}^2$ ∢ 15° $+ 1350\text{ ft/s}^2$ ∢ 60° $+ 1413.7\text{ ft/s}^2$ ∢ 60°

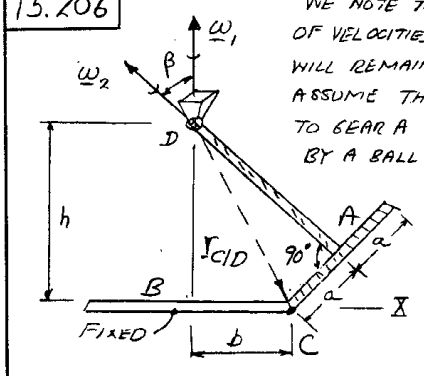
$\pm (a_P)_x = -185.1 - 675 + 706.9 = -153.2\text{ ft/s}^2$

$+\uparrow (a_P)_y = +49.6 - 1169.1 + 1224.3 = +104.8\text{ ft/s}^2$



$\underline{a}_P = 185.6\text{ ft/s}^2$ ∢ 34.4° ◀

15.206



WE NOTE THAT THE DISTRIBUTION OF VELOCITIES AND ACCELERATIONS WILL REMAIN UNCHANGED IF WE ASSUME THAT AXLE AD IS WELDED TO GEAR A AND SUPPORTED AT D BY A BALL AND SOCKET. IN THIS WAY WE OBTAIN A RIGID BODY WITH A FIXED POINT D AND ANGULAR VELOCITY $\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2$

(a) RATE OF SPIN. WE NOTE THAT $\underline{v}_C = 0$

BUT $\underline{v}_C = \underline{\omega} \times \underline{r}_{C/D} = (\underline{\omega}_1 + \underline{\omega}_2) \times \underline{r}_{C/D}$

$= \underline{\omega}_1 \times \underline{r}_{C/D} + \underline{\omega}_2 \times \underline{r}_{C/D}$

$= \omega_1 \underline{j} \times (b\underline{i} - h\underline{j}) + \omega_2 \underline{\partial}_{AD} \times (\underline{r}_{A/D} + \underline{r}_{C/A})$

$= -\omega_1 b \underline{k} + \omega_2 \underline{\partial}_{AD} \times a\underline{\partial}_{AC}$

(CONTINUED

15.206 CONTINUED

WE NOTE THAT $\underline{\partial}_{AD} \times \underline{\partial}_{AC} = \underline{k}$

$\underline{v}_C = -\omega_1 b \underline{k} + \omega_2 a \underline{k} = (-\omega_1 b + \omega_2 a)\underline{k}$

SINCE $v_C = 0$, $\quad -\omega_1 b + \omega_2 a = 0$ $\quad \omega_2 = \dfrac{b}{a}\omega_1$ ◀

(b) ANGULAR ACCELERATION OF GEAR A. GEAR AND AXLE MOVE ABOUT VERTICAL WITH CONSTANT ANGULAR VELOCITY $\underline{\Omega} = \underline{\omega}_1$

$\underline{\alpha} = \dfrac{d\underline{\omega}}{dt} = \underline{\Omega} \times \underline{\omega} = \underline{\omega}_1 \times (\underline{\omega}_1 + \underline{\omega}_2) = \underline{\omega}_1 \times \underline{\omega}_2$

$\underline{\alpha} = \omega_1 \underline{j} \times (-\omega_2 \sin\beta\, \underline{i} + \omega_2 \cos\beta\, \underline{j}) = \omega_1 \omega_2 \sin\beta\, \underline{k}$

OR SINCE $\omega_2 = \dfrac{b}{a}\omega_1$, $\quad \underline{\alpha} = \dfrac{b}{a}\omega_1^2 \sin\beta\, \underline{k}$ ◀

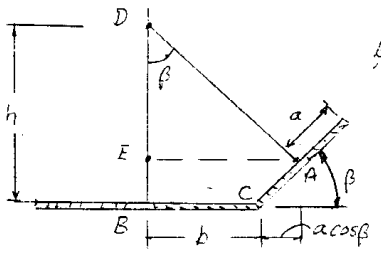
(c) ACCELERATION OF TOOTH C OF GEAR A.

$\underline{a}_C = \underline{\alpha} \times \underline{r}_{C/D} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C/D}) = \underline{\alpha} \times \underline{r}_{C/D} + \underline{\omega} \times \underline{v}_C$

BUT $\underline{v}_C = 0$,

$\underline{a}_C = \underline{\alpha} \times \underline{r}_{C/D} = (\omega_1 \omega_2 \sin\beta\, \underline{k}) \times (b\underline{i} - h\underline{j})$

$\underline{a}_C = \omega_1 \omega_2 \sin\beta\, [h\underline{j} + b\underline{i}]$ (2)



WE NOTE THAT:

$DE = (AC)\sin\beta = a\sin\beta$

$AE = BC + (AC)\cos\beta$

$= b + a\cos\beta$

$DE = \dfrac{AE}{\tan\beta} = \dfrac{b + a\cos\beta}{\tan\beta}$

$h = BD = BE + DE = a\sin\beta + (b + a\cos\beta)\dfrac{\cos\beta}{\sin\beta}$

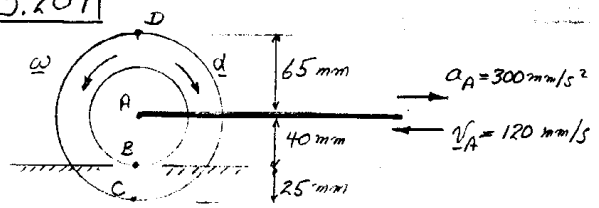
$= \dfrac{a\sin^2\beta + b\cos\beta + a\cos^2\beta}{\sin\beta}$ $\quad h = \dfrac{a + b\cos\beta}{\sin\beta}$

EQ. (2): $\underline{a}_C = \omega_1 \omega_2 \sin\beta \left[b\underline{j} + \dfrac{a + b\cos\beta}{\sin\beta}\underline{i} \right]$

$= \omega_1 \omega_2 \left[b\sin\beta\, \underline{j} + (a + b\cos\beta)\underline{i} \right]$

$\underline{a}_C = \dfrac{b}{a}\omega_1^2 \left[(a + b\cos\beta)\underline{i} + (b\sin\beta)\underline{j} \right]$ ◀

15.207



ANGULAR VELOCITY OF WHEEL. (INSTANTANEOUS CENTER AT B)

$$v_A = r\omega \curvearrowleft$$

$$[120\ mm/s \leftarrow] = [40\ mm]\omega \qquad \omega = 3\ rad/s \curvearrowright$$

ACCELERATION. SINCE A MOVES IN A HORIZONTAL STRAIGHT LINE, ITS ACCELERATION IS $a_A \rightarrow$. ALSO SINCE WHEEL ROLLS ON RAIL $a_B \uparrow$.

$$a_B = a_A + a_{B/A}$$

$$[a_B \uparrow] = [a_A \rightarrow] + [r_{B/A}\alpha \leftarrow] + [r_{B/A}\omega^2 \uparrow]$$

$\pm$ X COMPONENTS: $\quad 0 = +300\ mm/s^2 - (40\ mm)\alpha$

$$\alpha = 7.5\ rad/s^2 \qquad \alpha = 7.5\ rad/s^2 \curvearrowright$$

$+\uparrow$ Y COMPONENTS: $\quad a_B = (40\ mm)(3\ rad/s)^2$

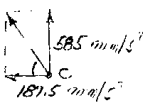
$$a_B = 360\ mm/s^2 \qquad \underline{a}_B = 360\ mm/s^2 \uparrow$$

(b)

$$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A}$$

$$= a_A \rightarrow + r_{C/A}\alpha \leftarrow + r_{C/A}\omega^2 \downarrow$$

$$= (300\ mm/s^2)\underline{i} - (65\ mm)(7.5\ rad/s^2)\underline{i} + (65\ mm)(3\ rad/s)^2\underline{j}$$



$$\underline{a}_C = -(187.5\ mm/s^2)\underline{i} + (585\ mm/s^2)\underline{j}$$

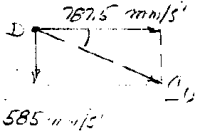
$$a_C = 614\ mm/s^2 \quad 72.2^\circ$$

(c)

$$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A}$$

$$= a_A \rightarrow + r_{D/A}\alpha \rightarrow + r_{D/A}\omega^2 \downarrow$$

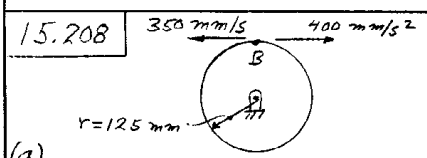
$$= (300\ mm/s^2)\underline{i} + (65\ mm)(7.5\ rad/s^2)\underline{i} - (65\ mm)(3\ rad/s)^2\underline{j}$$



$$\underline{a}_D = (787.5\ mm/s^2)\underline{i} - (585\ mm/s^2)\underline{j}$$

$$a_D = 981\ mm/s^2 \quad 36.6^\circ$$

15.208



(a)

$$v_B = r\omega; \qquad 350\ mm/s = (125\ mm)\omega$$

$$\omega = 2.8\ rad/s \qquad \underline{\omega} = 2.8\ rad/s \curvearrowright$$

$$(a_B)_t = 400\ mm/s^2 \rightarrow$$

$$(a_B)_t = r\alpha: \qquad 400\ mm/s^2 = (125\ mm)\alpha$$

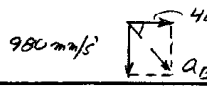
(b)

$$\alpha = 3.2\ rad/s^2 \qquad \underline{\alpha} = 3.2\ rad/s^2 \curvearrowright$$

ACCELERATION OF B.

$$(\underline{a}_B)_t = 400\ mm/s^2 \rightarrow$$

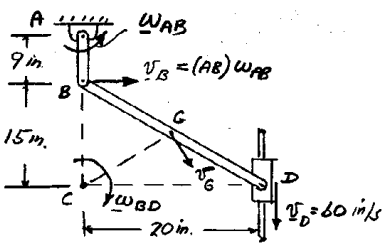
$$(\underline{a}_B)_n = r\omega^2 = (125\ mm)(2.8\ rad/s)^2 = 980\ mm/s^2 \downarrow$$



$$a_B = 1058\ mm/s^2 \quad 67.8^\circ$$

$$\underline{a}_B = 1.058\ m/s^2 \quad 67.8^\circ$$

15.209



ROD BD

(a) DRAW LINES $\perp$ TO $\underline{v}_B$ AND $\underline{v}_D$ TO LOCATE INSTANTANEOUS CENTER OF BD AT POINT C.

(b)

$$v_D = (CD)\omega_{BD}$$

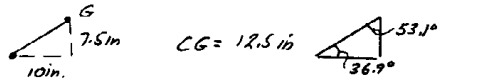
$$60\ in/s = (20\ in.)\omega_{BD} \qquad \omega_{BD} = 3\ rad/s \curvearrowright$$

$$v_B = (BC)\omega_{BD} = (15\ in.)(3\ rad/s) = 45\ in/s \rightarrow$$

CRANK AB: $\quad v_B = (AB)\omega_{AB}$

$$45\ in/s = (9\ in.)\omega_{AB} \qquad \underline{\omega}_{AB} = 5\ rad/s \curvearrowright$$

(c) ROD BD



$$CG = 12.5\ in$$

$$v_G = (CG)\omega_{BD} = (12.5\ in.)(3\ rad/s) = 37.5\ in/s$$

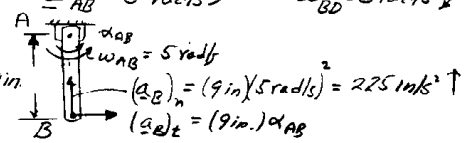
$$\underline{v}_G = 37.5\ in/s \quad 53.1^\circ$$

15.210

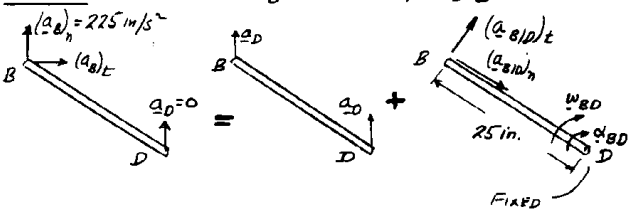
VELOCITY SEE PROB 15.209

$$\underline{\omega}_{AB} = 5\ rad/s \curvearrowright \qquad \omega_{BD} = 3\ rad/s \curvearrowright$$

CRANK AB



LINK AB NOTE v_D = CONSTANT, $\therefore\ a_D = 0$



PLANE MOTION = TRANSLATION WITH D + ROTATION ABOUT D

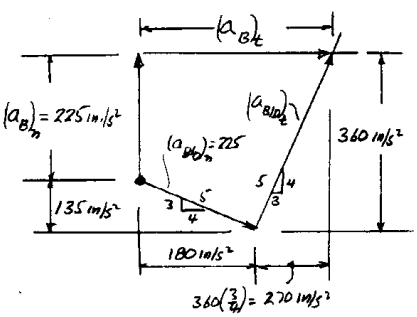
$$(\underline{a}_{B/D})_t = (25\ in.)\alpha_{BD} \quad \tfrac{4}{3}$$

$$(\underline{a}_{B/D})_n = (BD)\omega_{BD}^2 = (25\ in.)(3\ rad/s)^2 = 225\ in/s^2 \quad \tfrac{4}{5}\ \tfrac{3}{5}$$

$$\underline{a}_B = \underline{a}_D + \underline{a}_{B/D}$$

$$(\underline{a}_B)_n + (\underline{a}_B)_t = \underline{a}_D + (\underline{a}_{B/D})_n + (\underline{a}_{B/D})_t$$

$$[225\ in/s^2 \uparrow] + [(a_B)_t \rightarrow] = 0 + [225\ in/s^2\ \tfrac{4}{5}\ \tfrac{3}{5}] + [(a_{B/D})_t\ \tfrac{4}{3}\ \tfrac{3}{5}]$$



$$(a_{B/D})_t = (360\ in/s^2)\tfrac{5}{4}$$

$$(25in)\alpha_{BD} = 450\ in/s^2$$

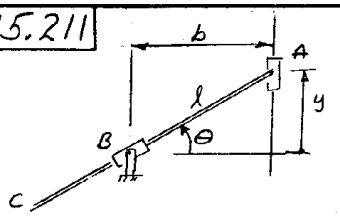
$$\underline{\alpha}_{BD} = 18\ rad/s^2 \curvearrowright$$

$$(a_B)_t = 180 + 270 = 450\ in/s^2$$

$$(9in.)\alpha_{AB} = 450\ in/s^2$$

$$\underline{\alpha}_{AB} = 50\ rad/s^2 \curvearrowright$$

15.211



(a) $y = b\tan\theta$

$\dot{y} = b\sec^2\theta\,\dot{\theta}$

$v_A = b\sec^2\theta\,\omega$

$\omega = \frac{v_A}{b}\cos^2\theta$ ◀

(b) $v_B = \frac{dl}{dt} = \frac{d}{dt}(b\sec\theta)$

$= b\sec\theta\tan\theta\,\dot{\theta} = b\sec\theta\tan\theta\,\omega$

SUBSTITUTE FOR ω:

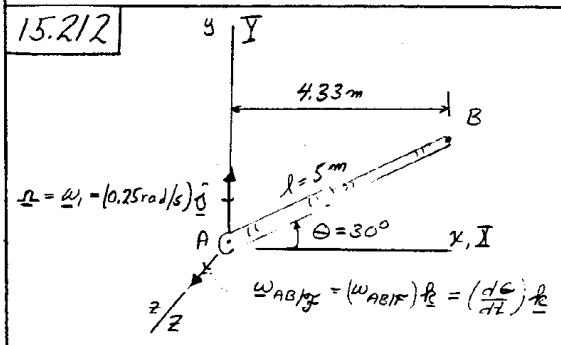
$v_B = b\sec\theta\tan\theta\left(\frac{v_A}{b}\cos^2\theta\right)$ $\quad v_B = v_A\sin\theta$ ◀

(c) $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v_A}{b}\cos^2\theta\right) = -2\frac{v_A}{b}\cos\theta\sin\theta\,\dot{\theta}$

$= -2\frac{v_A}{b}\cos\theta\sin\theta\left(\frac{v_A}{b}\cos^2\theta\right)$

$\alpha = -2\left(\frac{v_A}{b}\right)^2\sin\theta\cos^3\theta$ ◀

15.212



DATA: $\underline{\omega}_{AB/F} = 0$, $\quad \frac{dl}{dt} = 0.20\ \text{m/s} = \text{CONSTANT}$

$\underline{v}_{B/F} = \frac{dl}{dt} \rightarrow = 0.20\ \text{m/s}(\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$

$\underline{v}_{B/F} = (0.1732\ \text{m/s})\underline{i} + (0.100\ \text{m/s})\underline{j}$

$\underline{r}_{B/A} = (4.33\ \text{m})\underline{i} + (2.5\ \text{m})\underline{j}$

ACCELERATION OF B. $\quad \underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/F} + \underline{a}_C$

$\underline{a}_{B'} = -(4.33\ \text{m})\omega_1^2\,\underline{i} = -(4.33)(0.25)^2\,\underline{i} = -(0.2706\ \text{m/s}^2)\underline{i}$

$\underline{a}_{B/F} = 0$

$\underline{a}_C = 2\underline{\Omega}\times\underline{v}_{B/F} = 2(0.25\ \text{rad/s})\underline{j}\times\left[(0.1732\ \text{m/s})\underline{i} + (0.100\ \text{m/s})\underline{j}\right]$

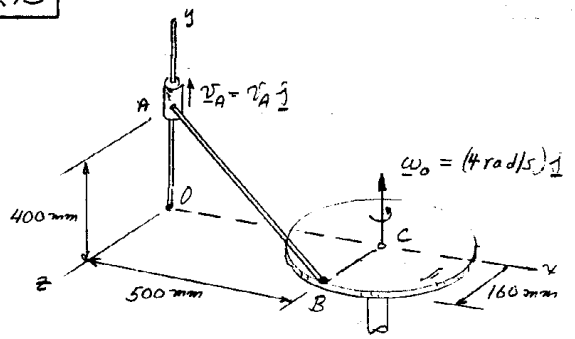
$= -(0.0866\ \text{m/s}^2)\underline{k}$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/F} + \underline{a}_C$

$= -(0.2706\ \text{m/s}^2)\underline{i} + 0 - (0.0866\ \text{m/s}^2)\underline{k}$

$\underline{a}_B = -(271\ \text{mm/s}^2)\underline{i} - (86.6\ \text{mm/s}^2)\underline{k}$ ◀

15.213



DISK: $\underline{v}_B = \underline{\omega}_0\times\underline{r}_{B/C} = (4\ \text{rad/s})\underline{j}\times(160\ \text{mm})\underline{k} = (640\ \text{mm/s})\underline{i}$

ROD AB

$\underline{r}_{A/B} = -(500\ \text{mm})\underline{i} + (400\ \text{mm})\underline{j} - (160\ \text{mm})\underline{k}$

$\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega}\times\underline{r}_{B/A}$

$v_A\underline{j} = (640\ \text{mm/s})\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -500 & 400 & -160 \end{vmatrix}$

$v_A\underline{j} = 640\underline{i} + (-160\omega_y - 400\omega_z)\underline{i} + (-500\omega_z + 160\omega_x)\underline{j}$

$+ (400\omega_x + 500\omega_y)\underline{k}$

$\underline{i}$: $\quad -640 = -160\omega_y - 400\omega_z$ $\quad (1)$

$\underline{j}$: $\quad v_A = 160\omega_x - 500\omega_z$ $\quad (2)$

$\underline{k}$: $\quad 0 = 400\omega_x + 500\omega_y$ $\quad (3)$

$\underline{\omega}$ IS INDETERMINATE. WE MAY FREELY CHOOSE ONE COMPONENT OF $\underline{\omega}$. FOR EXAMPLE:

ASSUME $\omega_y = 0$

EQ(3): $\quad 0 = 400\omega_x + 0$ $\quad \omega_x = 0$

EQ(1): $\quad -640 = 160(0) - 400\,\omega_z$ $\quad \omega_z = 1.6$ rad/s

EQ(2): $\quad v_A = 160(0) - 500(1.6)$

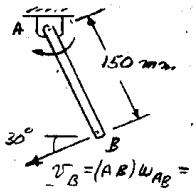
$v_A = -800\ \text{mm/s}$

$\underline{v}_A = -(800\ \text{mm/s})\underline{j}$ ◀

15.214

LINK AB

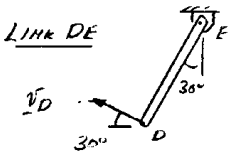
$\omega_{AB} = 4$ rad/s



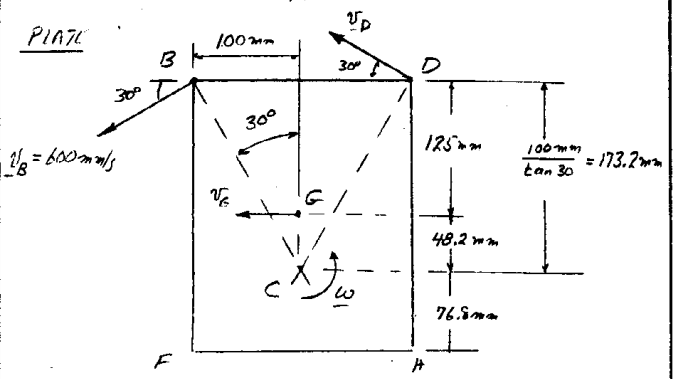
$v_B = (AB)\omega_{AB} = (150\text{ mm})(4\text{ rad/s})$

$v_B = 600$ mm/s ∠ 30°

LINK DE



PLATE



DRAW LINES ⊥ TO v_B AND v_D TO LOCATE INSTANTANEOUS CENTER C.

$BC = (100\text{ mm})/\sin 30° = 200$ mm

(a)

$v_B = (BC)\omega$

$600\text{ mm/s} = (200\text{ mm})\,\omega$ $\qquad \omega = 3$ rad/s ↻ ◀

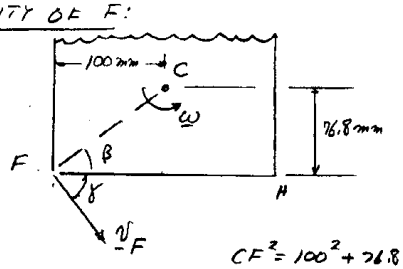
(b) VELOCITY OF G:

$CG = 173.2\text{ mm} - 125\text{ mm} = 48.2$ mm

$v_G = (CG)\omega = (48.2\text{ mm})(3\text{ rad/s}) = 144.6$ mm/s

$v_G = 144.6$ mm/s ← ◀

(c) VELOCITY OF F:



$CF^2 = 100^2 + 76.8^2$

$CF = 126.1$ mm

$\beta = 37.5°$

$\gamma = 90° - 37.5° = 52.5°$

$v_F = (CF)\omega = (126.1\text{ mm})(3\text{ rad/s}) = 378.3$ mm/s

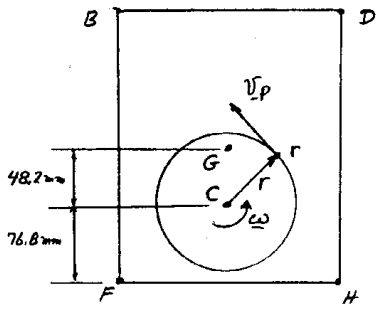
$v_F = 378$ mm/s ↘ 52.5° ◀

15.215

SEE PROB. 15.214 FOR LOCATION OF THE INSTANTANEOUS CENTER C.

WE ALSO RECALL THE VALUE OF ω.

$\omega = 3$ rad/s ↻



FOR AN ARBITRARY POINT P LOCATED AT A DISTANCE r FROM C,

$v_P = r\omega$

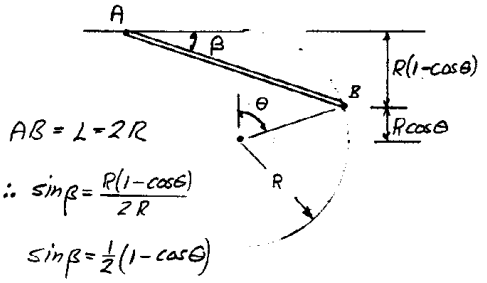
IF v_P IS TO BE EQUAL TO 150 mm/s,

$150\text{ mm/s} = r(3\text{ rad/s})$

$r = 50$ mm

THUS, $v_P \le 150$ mm/s ON OR INSIDE A CIRCLE OF $r = 50$ mm CENTERED ON THE INSTANTANEOUS CENTER C. ◀

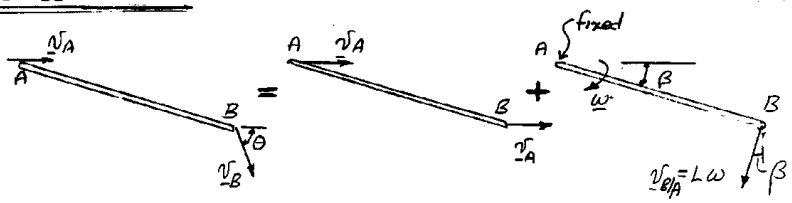
PROBLEM 15.C1



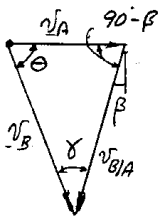
$AB = L = 2R$

$\therefore \sin\beta = \frac{R(1-\cos\theta)}{2R}$

$\sin\beta = \frac{1}{2}(1-\cos\theta)$



PLANE MOTION = TRANSLATION + ROTATION



$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\gamma = \beta + (90° - \theta) = 90° + \beta - \theta$

$$\frac{v_A}{\sin\gamma} = \frac{v_B}{\sin(90°-\beta)} = \frac{v_{B/A}}{\sin\theta}$$

$$v_{B/A} = v_B \frac{\sin\theta}{\sin(90°-\beta)} = v_B \frac{\sin\theta}{\cos\beta}$$

$$v_A = v_B \frac{\sin\gamma}{\sin(90°-\beta)} = v_B \frac{\sin(90°+\beta-\theta)}{\sin(90°-\beta)} = v_B \frac{\cos(\beta-\theta)}{\cos\beta}$$

$$\omega = \frac{v_{B/A}}{L}$$

```
10 '*****************************************************************
20 '*                        PROBLEM 15.C1                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '             L = length of rod AB    (meters)
90 '             R = radius of circular slot    (meters)
100 '           VA = speed of A  (m/s)
110 '           VB = speed of B  (m/s)
120 '          VBA = velocity of B relative to A  (m/s)
130 '        OMEGA = angular velocity of rod  (rad/s)
140 '        THETA = angle theta    (degrees)
150 '         BETA = angle between rod and horizontal  (degrees)
160 '
170 '   Data
180     LET L = .4      'meters
190     LET R = .2      'meters
200     LET VB = 1.2    'm/s
210     K = 4*ATN(1)/180
220 '
230     PRINT "  THETA          OMEGA            VA
240     PRINT " degrees         rad/s            m/s";PRINT
250 '
260     FOR THETA = 0 TO 150 STEP 10
270        SB = (R/L)*(1 - COS(K*THETA))       'SB = sine of angle beta
280        BETA = (ATN(SB/SQR(1-SB^2)))/K
290        VBA = VB*SIN(K*THETA)/COS(K*BETA)
300        OMEGA = VBA/L
310        VA = VB*COS(K*(BETA - THETA))/COS(K*BETA)
320        PRINT USING "  ###      ";THETA,
330        PRINT USING "     ##.###     ";OMEGA,VA
340     NEXT:PRINT
350     PRINT "   OMEGA is positive clockwise.
360     PRINT "   VA is positive to the right.
```

THETA degrees	OMEGA rad/s	VA m/s
0	0.000	1.200
10	0.521	1.183
20	1.027	1.140
30	1.503	1.080
40	1.942	1.010
50	2.336	0.938
60	2.683	0.868
70	2.985	0.803
80	3.244	0.745
90	3.464	0.693
100	3.649	0.648
110	3.802	0.610
120	3.928	0.578
130	4.029	0.552
140	4.109	0.532
150	4.168	0.516

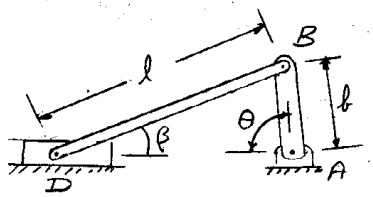
OMEGA is positive clockwise.
VA is positive to the right.

```
C*****************************************************************
C*                        PROBLEM 15.C1                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C            Definition of Symbols
C
C                 L = length of rod AB    (meters)
C                 R = radius of circular slot    (meters)
C               VA = speed of A   (m/s)
C               VB = speed of B   (m/s)
C              VBA = velocity of B relative to A   (m/s)
C            OMEGA = angular velocity of rod   (rad/s)
C            THETA = angle theta    (degrees)
C             BETA = angle between rod and horizontal   (degrees)
C
      REAL OMEGA, L, K
C           DATA :
      L = 0.4
      R = 0.2
      VB = 1.2
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,'(/,2X,A\)') ' THETA           OMEGA           VA'
      WRITE (*,'(/,2X,A\)') 'degrees          rad/s           m/s'
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,150,10
        THETA = I
        BETA = ASIN((R/L)*(1.- COS(K*THETA)))/K
        VBA = VB*SIN(K*THETA)/COS(K*BETA)
      OMEGA = VBA/L
         VA = VB*COS(K*(BETA - THETA))/COS(K*BETA)
         WRITE (*,20) THETA,OMEGA,VA
   20    FORMAT (4X,F4.0,10X,F6.3,10X,F6.3)
   10 CONTINUE
      WRITE (*,'(/,5X,A\)') 'OMEGA is positive clockwise.'
      WRITE (*,'(/,5X,A\)') 'VA is positive to the right.'
      END
```

THETA degrees	OMEGA rad/s	VA m/s
0.	.000	1.200
10.	.521	1.183
20.	1.027	1.140
30.	1.503	1.080
40.	1.942	1.010
50.	2.336	.938
60.	2.683	.868
70.	2.985	.803
80.	3.244	.745
90.	3.464	.693
100.	3.649	.648
110.	3.802	.610
120.	3.928	.578
130.	4.029	.552
140.	4.109	.532
150.	4.168	.516

OMEGA is positive clockwise.
VA is positive to the right.

PROBLEM 15.C2

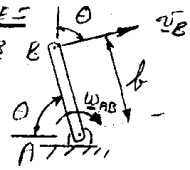


GEOMETRY LAW OF SINES:

$$\frac{\sin\beta}{b} = \frac{\sin\theta}{l} \qquad \sin\beta = \frac{b}{l}\sin\theta$$ ◁

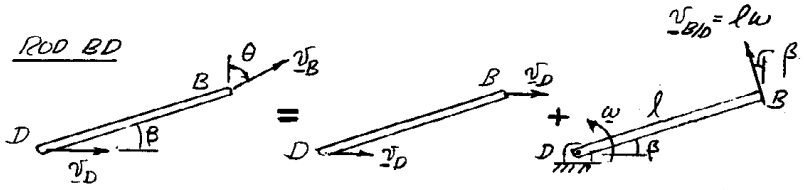
VELOCITIES

CRANK AB



$$v_B = b\,\omega_{AB}$$ ◁

ROD BD

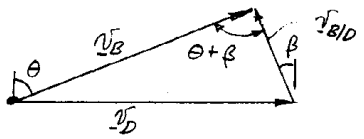


PLANE MOTION = TRANSLATION + ROTATION

$$\underline{v}_B = \underline{v}_D + \underline{v}_{B/D}$$

$$[\underline{v}_B \ \theta] = [\underline{v}_D \rightarrow] + [\underline{v}_{B/D} \perp \beta]$$

VECTOR POLYGON



LAW OF SINES

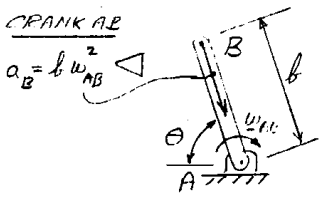
$$\frac{v_D}{\sin(\theta+\beta)} = \frac{v_B}{\sin(90°-\beta)} = \frac{v_{B/D}}{\sin(90°-\theta)}$$

$$v_D = v_B\frac{\sin(\theta+\beta)}{\sin(90°-\beta)} \qquad v_D = v_B\frac{\sin(\theta+\beta)}{\cos\beta}$$ ◁

$$v_{B/D} = v_B\frac{\sin(90°-\theta)}{\sin(90°-\beta)} \qquad v_{B/D} = v_B\frac{\cos\theta}{\cos\beta}$$ ◁

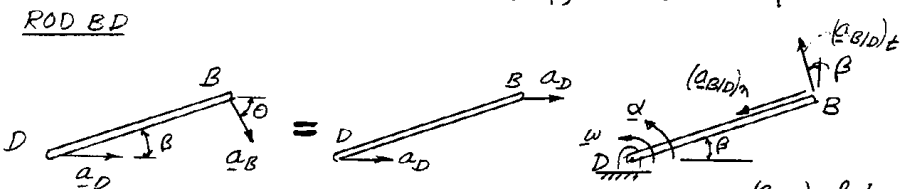
ACCELERATIONS

CRANK AB



$$a_B = b\,\omega_{AB}^2$$ ◁

ROD BD



PLANE MOTION = TRANSLATION + ROTATION

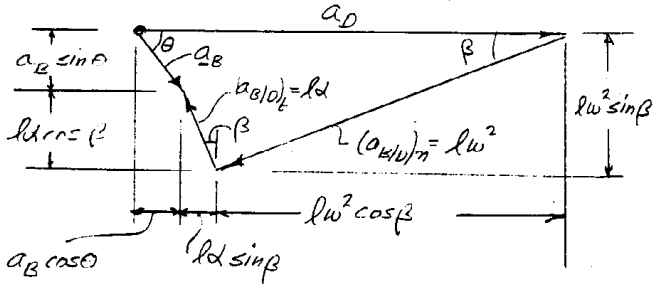
$$(a_{B/D})_t = l\alpha$$

$$(a_{B/D})_n = l\omega^2$$

$$\underline{a}_B = \underline{a}_D + \underline{a}_{B/D}$$

$$[a_B \ \theta] = [\underline{a}_D \rightarrow] + [l\alpha \perp \beta] + [l\omega^2 \rightarrow \beta]$$

VECTOR POLYGON



$+\downarrow$ comps: $a_B\sin\theta = -l\alpha\cos\beta + l\omega^2\sin\beta$

$$\alpha = \frac{l\omega^2\sin\beta - a_B\sin\theta}{l\cos\beta}$$ ◁

$+\rightarrow$ comps:

$$a_B\cos\theta = a_D - l\alpha\sin\beta - l\omega^2\cos\beta$$

$$a_D = l\alpha\sin\beta + l\omega^2\cos\beta + a_B\cos\theta$$ ◁

CONTINUED

PROBLEM 15.C2 CONTINUED

```
10 '*****************************************************************
20 '*                        PROBLEM 15.C2                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '
70 '            Definition of Symbols
80 '
90 '          B = length of crank AB    (ft)
100 '         L = length of connecting rod BD   (ft)
110 '    ANGVEL = angular velocity of crank AB  (rpm)
120 '        TH = angle theta   (degrees)
130 '      BETA = angle between rod BD and horizontal   (degrees)
140 '
150 '     OMEGA = angular velocity of rod BD  (rad/s)
160 '        VB = velocity of pin at B   (ft/s)
170 '        VD = velocity of piston P   (ft/s)
180 '       VBD = velocity of B relative to D   (ft/s)
190 '
200 '     ALPHA = angular acceleration of rod BD, [+ ccwise]  (rad/s^2)
210 '        AB = acceleration of pin at B   (ft/s^2)
220 '        AD = acceleration of piston P, [+ to the right]  (ft/s^2)
230 '       ABD = acceleration of B relative to D   (ft/s^2)
240 '
250   PI = 4*ATN(1):     K = PI/180
260 '
270   B = 3     'inches
280   L = 8     'inches
290   B = B/12:      L = L/12        'express B and L in feet
300 '
310   ANGVEL = 2000     'rpm
320   ANGVEL = ANGVEL*(2*PI)/60     'express in rad/s
330   PRINT"THETA       OMEGA          ALPHA           VD            AD
340   PRINT"                rad/s        rad/s^2         ft/s        ft/s^2
350   PRINT
360   FOR TH = 0 TO 180 STEP 10
370 '
380         'Calculation of velocities
390     SB = (B/L)*SIN(K*TH)          'SB = sine of angle BETA
400     BETA = (ATN(SB/SQR(1-SB^2)))/K
410     VB = B * ANGVEL
420     VD = VB * SIN(K*TH + K*BETA)/COS(K*BETA)
430     VBD =VB * COS(K*TH)/COS(K*BETA)
440     OMEGA = VBD/L
450 '
460         'Calculation of accelerations
470     AB = B*(ANGVEL)^2
480     ALPHA = (L*(OMEGA)^2*SIN(K*BETA) - AB*SIN(K*TH))/(L*COS(K*BETA))
490     AD = ALPHA*L*SIN(K*BETA) + L*(OMEGA)^2*COS(K*BETA) +AB*COS(K*TH)
500 '
510     PRINT USING " ###     "; TH,
520     PRINT USING "  ###.##         ######      ";OMEGA,ALPHA,VD,AD
530   NEXT:PRINT
540   PRINT "NOTE: Positive directions are"
550   PRINT  "            counterclockwise and to the right.
```

THETA	OMEGA rad/s	ALPHA rad/s^2	VD ft/s	AD ft/s^2
0	78.54	0	0.00	15079
10	77.51	-2470	12.46	14689
20	74.42	-4957	24.27	13543
30	69.25	-7458	34.84	11705
40	61.99	-9940	43.62	9290
50	52.71	-12322	50.20	6463
60	41.52	-14470	54.33	3437
70	28.70	-16207	55.95	457
80	14.68	-17346	55.18	-2233
90	-0.00	-17744	52.36	-4436
100	-14.68	-17346	47.95	-6041
110	-28.70	-16207	42.46	-7044
120	-41.52	-14470	36.36	-7529
130	-52.71	-12322	30.02	-7635
140	-61.99	-9940	23.69	-7511
150	-69.25	-7458	17.52	-7289
160	-74.42	-4957	11.55	-7067
170	-77.51	-2470	5.73	-6910
180	-78.54	0	-0.00	-6854

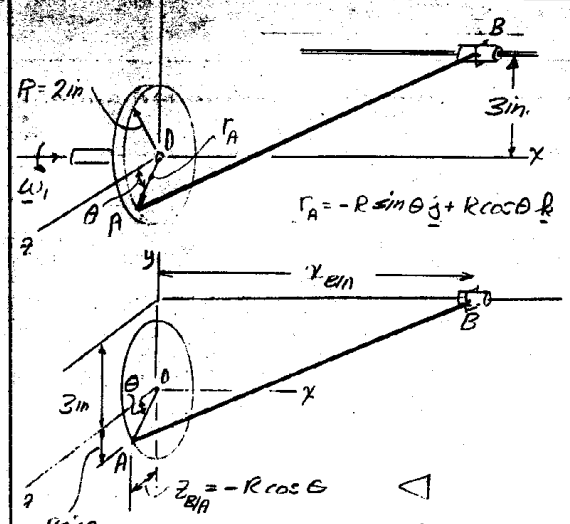
NOTE: Positive directions are
counterclockwise and to the right.

```
C*****************************************************************
C*                        PROBLEM 15.C2                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C             Definition of Symbols
C
C          B = length of crank AB   (ft)
C          L = length of connecting rod BD  (ft)
C     ANBVEL = angular velocity of crank AB  (rpm)
C         TH = angle theta  (degrees)
C       BETA = angle between rod BD and horizontal  (degrees)
C
C      OMEGA = angular velocity of rod BD  (rad/s)
C         VB = velocity of pin at B  (ft/s)
C         VD = velocity of piston P  (ft/s)
C        VBD = velocity of B relative to D  (ft/s)
C
C      ALPHA = angular acceleration of rod BD, [+ ccwise]  (rad/s**2)
C         AB = acceleration of pin at B  (ft/s**2)
C         AD = acceleration of piston P, [+ to the right]  (ft/s**2)
C        ABD = acceleration of B relative to D  (ft/s**2)
C
      REAL OMEGA, L, K, TH
C
      PI = 4.*ATAN(1.)
      K = PI/180.
C
C       Length of AB and BD in inches.
      B = 3.
      L = 8.
C       Express length of AB and BD in feet.
      B = B/12.
      L = L/12.
C       Angular velocity of AB in RPM.
      ANGVEL = 2000.
C       Express ANGVEL in rad/s.
      ANGVEL = ANGVEL*(2.*PI)/60.
C
      WRITE (*,'(/,2X,A\)') 'THETA          OMEGA          ALPHA
     + VD              AD '
      WRITE (*,'(/,2X,A\)') '                rad/s        rad/s**2
     +m/s            ft/s**2'
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,180,10
            TH = I
C
C           Calculation of velocites
C
         BETA = ASIN((B/L)*SIN(K*TH))/K
           VB = B * ANGVEL
           VD = VB * SIN(K*TH + K*BETA)/COS(K*BETA)
          VBD = VB * COS(K*TH)/COS(K*BETA)
        OMEGA = VBD/L
C
C           Calculation of accelerations
C
           AB = B*(ANGVEL)**2
        ALPHA = (L*OMEGA**2*SIN(K*BETA) - AB*SIN(K*TH))/(L*COS(K*BETA))
           AD = ALPHA*L*SIN(K*BETA)+L*OMEGA**2*COS(K*BETA)+AB*COS(K*TH)
         WRITE (*,20) TH,OMEGA,ALPHA,VD,AD
   20    FORMAT (3X,F4.0,8X,F6.2,7X,F7.0,9X,F5.2,5X,F6.0)
   10 CONTINUE
      WRITE (*,'(/,3X,A\)') 'NOTE: Positive directions are'
      WRITE (*,'(/,3X,A\)') '      counterclockwise and to the right.'
C
      WRITE (*,'(/,A\)')
      END
```

THETA	OMEGA rad/s	ALPHA rad/s**2	VD m/s	AD ft/s**2
0.	78.54	0.	.00	15079.
10.	77.51	-2470.	12.46	14689.
20.	74.42	-4957.	24.27	13543.
30.	69.25	-7458.	34.84	11705.
40.	61.99	-9940.	43.62	9290.
50.	52.71	-12322.	50.20	6463.
60.	41.52	-14470.	54.33	3437.
70.	28.70	-16207.	55.95	457.
80.	14.68	-17346.	55.18	-2233.
90.	.00	-17744.	52.36	-4436.
100.	-14.68	-17346.	47.95	-6041.
110.	-28.70	-16207.	42.46	-7044.
120.	-41.52	-14470.	36.36	-7529.
130.	-52.71	-12322.	30.02	-7635.
140.	-61.99	-9940.	23.69	-7511.
150.	-69.25	-7458.	17.52	-7289.
160.	-74.42	-4957.	11.55	-7067.
170.	-77.51	-2470.	5.73	-6910.
180.	-78.54	0.	.00	-6854.

NOTE: Positive directions are
counterclockwise and to the right.

PROBLEM 15.C3



$\underline{r}_A = -R\sin\theta\,\underline{j} + R\cos\theta\,\underline{k}$

$z_{B/A} = -R\cos\theta$ ◁

$y_{B/A} = 2 + R\sin\theta$ ◁

$x_{B/A} = \sqrt{L^2 - y_{B/A}^2 - z_{B/A}^2}$ ◁

KNOWN: $\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$L = AB = 7$ in.

NOTE: See Sample Problem 15.12 for discussion of Cases A, B, C.

$\underline{\omega}_1 = \omega_1\underline{i}$ $\qquad v_A = R\omega_1$ ◁

$\underline{v}_A = \underline{\omega}_1 \times \underline{r}_A = \omega_1\underline{i} \times (-R\sin\theta\,\underline{j} + R\cos\theta\,\underline{k})$

$= -R\omega_1\sin\theta\,\underline{k} - R\omega_1\cos\theta\,\underline{j}$

$(v_A)_y = -v_A\cos\theta$; $(v_A)_z = -v_A\sin\theta$ ◁

VELOCITY OF B

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}$

$$v_B\underline{i} = (v_A)_y\underline{j} + (v_A)_z\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ x_{B/A} & y_{B/A} & z_{B/A} \end{vmatrix}$$

$$\begin{aligned} v_B &= &&+ z_{B/A}\omega_y &&- y_{B/A}\omega_z && (1)\\ -(v_A)_y &= -z_{B/A}\omega_x &&&&+ x_{B/A}\omega_z && (2)\\ -(v_A)_z &= y_{B/A}\omega_x &&- x_{B/A}\omega_y && && (3) \end{aligned}$$

MULTIPLY (1) BY $x_{B/A}$, (2) BY $y_{B/A}$, (3) BY $z_{B/A}$, AND ADD:

$v_B x_{B/A} - (v_A)_y y_{B/A} - (v_A)_z z_{B/A} = 0$

$v_B = \left[(v_A)_y y_{B/A} + (v_A)_z z_{B/A}\right] / x_{B/A}$ ◁ (4)

ANGULAR VELOCITY $\underline{r}_{B/E} = y_{B/A}\underline{j} + z_{B/A}\underline{k}$

$\underline{r}_{B/E} \cdot \underline{\omega} = 0$: $(y_{B/A}\underline{j} + z_{B/A}\underline{k}) \cdot (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) = 0$

$y_{B/A}\omega_y + z_{B/A}\omega_z = 0$ (5)

MULTIPLY (1) BY $z_{B/A}$, (5) BY $y_{B/A}$, AND ADD:

$v_B z_{B/A} = \omega_y\left(z_{B/A}^2 + y_{B/A}^2\right)$: $\quad \omega_y = v_B / \left[z_{B/A} + (y_{B/A}^2 / z_{B/A})\right]$ ◁ (6)

FROM (5): $\quad \omega_z = -(y_{B/A}/z_{B/A})\,\omega_y$ ◁ (7)

FROM (3): $\quad \omega_x = \left[-(v_A)_z + x_{B/A}\omega_y\right] / y_{B/A}$ ◁ (8)

```
10 '*************************************************************
20 '*                      PROBLEM 15.C3                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '          Definition of symbols
70 '
80 '            L = length of rod AB   (in.)
90 '       OMEGA1 = angular velocity of disk   (rad/s)
100 '           R = radius of disk   (in.)
110 '          TH = angle theta   (degrees)
120 '          VA = velocity of point A on the disk   (in./s)
130 '          VB = velocity of collar B   (in./s)
140 '       OMEGAX = x component of angular velocity of rod  (rad/s)
150 '       OMEGAY = y component of angular velocity of rod  (rad/s)
160 '       OMEGAZ = z component of angular velocity of rod  (rad/s)
170 '
180 '     NOTE: Follow solution of Sample Prob. 15.12, page 689.
190 '
200 '       Data
210       LET L = 7           'in.
220       LET OMEGA1 = 12     'rad/s
230       LET R = 2           'in.
240       VA = R*OMEGA1       'rad/s
250       K = 4*ATN(1)/180
```

SEE NEXT PAGE FOR FULL PROGRAM

```
C*************************************************************
C*                      PROBLEM 15.C3                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C          Definition of Symbols
C
C              L = length of rod AB   (in.)
C         OMEGA1 = angular velocity of disk  (rad/s)
C              R = radius of disk  (in.)
C             TH = angle theta  (degrees)
C             VA = velocity of point A on the disk (in./s)
C             VB = velocity of collar B (in./s)
C         OMEGAX = x component of angular velocity of rod  (rad/s)
C         OMEGAZ = y component of angular velocity of rod  (rad/s)
C         OMEGAZ = z component of angular velocity of rod  (rad/s)
C
C          NOTE : Follow solution of Sample Prob. 15.12 page 689.
C
      REAL OMEGA1, OMEGAX, OMEGAY, OMEGAZ, L, K
C
C       Data
      L = 7.
      R = 2.
      OMEGA1 = 12.
      VA = R*OMEGA1
      K = 4.*ATAN(1.)/180.
```

SEE NEXT PAGE FOR FULL PROGRAM

PROBLEM 15.C3 CONTINUED

```
10 '*****************************************************************
20 '*                      PROBLEM 15.C3                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '          Definition of symbols
70 '
80 '            L = length of rod AB   (in.)
90 '       OMEGA1 = angular velocity of disk   (rad/s)
100 '           R = radius of disk   (in.)
110 '          TH = angle theta   (degrees)
120 '          VA = velocity of point A on the disk   (in./s)
130 '          VB = velocity of collar B   (in./s)
140 '      OMEGAX = x component of angular velocity of rod  (rad/s)
150 '      OMEGAY = y component of angular velocity of rod  (rad/s)
160 '      OMEGAZ = z component of angular velocity of rod  (rad/s)
170 '
180 '     NOTE: Follow solution of Sample Prob. 15.12, page 689.
190 '
200 '       Data
210      LET L = 7           'in.
220      LET OMEGA1 = 12     'rad/s
230      LET R = 2           'in.
240      VA = R*OMEGA1       'rad/s
250      K = 4*ATN(1)/180
260 '
270      PRINT"  theta","  V sub B","  omega x","  omega y","  omega z"
280      PRINT" degrees","  in./s","   rad/s","   rad/s","   rad/s"
290      PRINT
300 '
310      FOR TH = 0 TO 360 STEP 30
320         VAY = -VA*COS(K*TH)
330         VAZ = -VA*SIN(K*TH)
340         YBA = 3 + R*SIN(K*TH)
350         ZBA = -R*COS(K*TH)
360         XBA = SQR(L^2 - YBA^2 - ZBA^2)
370         VB = (VAY*YBA + VAZ*ZBA)/XBA      'from Eq.(4)
380 '
390 '        'Calculation of components of angular velocity of rod
400         OMEGAY = VB/(ZBA +YBA^2/ZBA)          'from Eq.(6)
410         OMEGAZ = -(YBA/ZBA)*OMEGAY            'from Eq.(7)
420         OMEGAX = (1/YBA)*(-VAZ +XBA*OMEGAY)   'from Eq.(8)
430 '       PRINT TH, VB,OMEGAX, OMEGAY, OMEGAZ
440         PRINT USING "  ###           ";TH,
450         PRINT USING "###.###       ";VB,OMEGAX,OMEGAY,OMEGAZ
460      NEXT
```

theta degrees	V sub B in./s	omega x rad/s	omega y rad/s	omega z rad/s
0	-12.000	3.692	1.846	2.769
30	-11.384	4.421	1.038	2.397
60	-7.114	4.718	0.304	1.439
90	0.000	4.800	0.000	-0.000
120	7.114	4.718	0.304	-1.439
150	11.384	4.421	1.038	-2.397
180	12.000	3.692	1.846	-2.769
210	9.621	1.714	2.381	-2.749
240	5.285	-5.504	2.027	-2.570
270	-0.000	-24.000	0.000	0.000
300	-5.285	-5.504	2.027	2.570
330	-9.621	1.714	2.381	2.749
360	-12.000	3.692	1.846	2.769

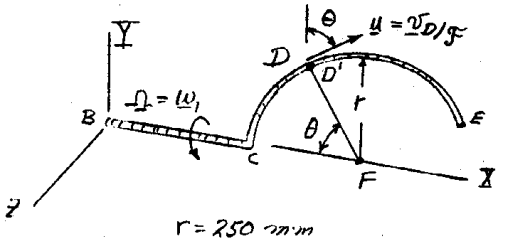
```
C*****************************************************************
C*                      PROBLEM 15.C3                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C          Definition of Symbols
C
C               L = length of rod AB   (in.)
C          OMEGA1 = angular velocity of disk  (rad/s)
C               R = radius of disk  (in.)
C              TH = angle theta  (degrees)
C              VA = velocity of point A on the disk (in./s)
C              VB = velocity of collar B (in./s)
C          OMEGAX = x component of angular velocity of rod  (rad/s)
C          OMEGAZ = y component of angular velocity of rod  (rad/s)
C          OMEGAZ = z component of angular velocity of rod  (rad/s)
C
C          NOTE : Follow solution of sample Prob. 15.12 page 689.
C
      REAL OMEGA1, OMEGAX, OMEGAY, OMEGAZ, L, K
C
C       Data
      L = 7.
      R = 2.
      OMEGA1 = 12.
      VA = R*OMEGA1
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,100)
  100 FORMAT (T5,'theta',T17,'V sub B'T33,'omega x',T48,'omega y',T63,
     +'omega z')
      WRITE (*,101)
  101 FORMAT (T4,'degrees',T18,'in./s',T34,'rad/s',T49,'rad/s',T64,
     +'rad/s')
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,360,30
         TH = I
         VAY = -VA*COS(K*TH)
         VAZ = -VA*SIN(K*TH)
         YBA = 3.+ R*SIN(K*TH)
         ZBA = -R*COS(K*TH)
         XBA = SQRT(L**2 - YBA**2 - ZBA**2)
C           Using Eq.(4), we write
         VB = (VAY*YBA + VAZ*ZBA)/XBA
C
C          Calculation of components of angular velocity of rod
C             Using Eq.(6), we write
         OMEGAY = VB/(ZBA + YBA**2/ZBA)
C             Using Eq.(7), we write
         OMEGAZ = -(YBA/ZBA)*OMEGAY
C             Using Eq.(8), we write
         OMEGAX = (1./YBA)*(-VAZ + XBA*OMEGAY)
C
         WRITE (*,20) TH,VB,OMEGAX,OMEGAY,OMEGAZ
   20    FORMAT (4X,F4.0,8X,F7.3,8X,F7.3,8X,F7.3,8X,F7.3)
   10 CONTINUE
      END
```

theta degrees	V sub B in./s	omega x rad/s	omega y rad/s	omega z rad/s
0.	-12.000	3.692	1.846	2.769
30.	-11.384	4.421	1.038	2.397
60.	-7.114	4.718	.304	1.439
90.	.000	4.800	.000	.000
120.	7.114	4.718	.304	-1.439
150.	11.384	4.421	1.038	-2.397
180.	12.000	3.692	1.846	-2.769
210.	9.621	1.714	2.381	-2.749
240.	5.285	-5.504	2.027	-2.570
270.	.000	-24.000	.000	.000
300.	-5.285	-5.504	2.027	2.570
330.	-9.621	1.714	2.381	2.749
360.	-12.000	3.692	1.846	2.769

PROBLEM 15.C4

$\dot{u} = 0$

$\dot{\Omega} = 0$



$r = 250$ mm

$\underline{\Omega} = \underline{\omega}_1 = (0.4 \text{ rad/s})\,\underline{i}$

THE ROTATING FRAME CONSISTS OF THE XYZ FRAME WHICH ROTATES WITH THE ARM

$$\underline{v}_{D/F} = \underline{u} = u\sin\theta\,\underline{i} + u\cos\theta\,\underline{j}$$

FOR $u = 150$ mm/s,

$$\underline{v}_{D/F} = 150\sin\theta\,\underline{i} + 150\cos\theta\,\underline{j}$$

$$\underline{r}_{D/F} = -r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}$$

FOR $r = 250$ mm,

$$\underline{r}_{D/F} = -250\cos\theta\,\underline{i} + 250\sin\theta\,\underline{j}$$

FOR COINCIDING POINT D',

$$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/F} = (0.4\underline{i}) \times (-250\cos\theta\,\underline{i} + 250\sin\theta\,\underline{j})$$

$$\underline{v}_{D'} = 100\sin\theta\,\underline{k}$$

VELOCITY OF D

$$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/F}; \quad \underline{v}_D = 100\sin\theta\,\underline{k} + 150\sin\theta\,\underline{i} + 150\cos\theta\,\underline{j}$$

$$\underline{v}_D = \begin{cases} v_x = 150\sin\theta \\ v_y = 150\cos\theta \\ v_z = 100\sin\theta \end{cases} \quad \triangle\triangle\triangle$$

ACCELERATION OF D

$$\underline{a}_{D'} = \underline{\Omega} \times \underline{v}_{D'} = (0.4\underline{i}) \times (100\sin\theta\,\underline{k})$$

$$\underline{a}_{D'} = -40\sin\theta\,\underline{j}$$

$$\underline{a}_{D/F} = \frac{u^2}{r} = \frac{(150 \text{ mm/s})^2}{250 \text{ mm}} = 90 \text{ mm/s}^2 \text{ DIRECTED ALONG DF TOWARD F}$$

$$\underline{a}_{D/F} = 90\cos\theta\,\underline{i} - 90\sin\theta\,\underline{j}$$

CORIOLIS ACCEL: $\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{D/F} = 2(0.4\underline{i}) \times (150\sin\theta\,\underline{i} + 150\cos\theta\,\underline{j}) = 120\cos\theta\,\underline{k}$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/F} + \underline{a}_c = -40\sin\theta\,\underline{j} + (90\cos\theta\,\underline{i} - 90\sin\theta\,\underline{j}) + 120\cos\theta\,\underline{k}$$

$$\underline{a}_D = \begin{cases} a_x = 90\cos\theta \\ a_y = -130\sin\theta \\ a_z = 120\cos\theta \end{cases} \quad \triangle\triangle\triangle$$

```
10 '*****************************************************************
20 '*                      PROBLEM 15.C4                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '           Definition of Symbols
70 '
80 '             V = magnitude of velocity of point D   (mm/s)
90 '      VX,VY,VZ = rectangular components of V         (mm/s)
100 '            A = magnitude of acceleration of point D  (mm/s^2)
110 '     AX,AY,AZ = rectangular components of A          (mm/s^2)
120 '           TH = angle theta  (degrees)
130 '
140 '       NOTE: For specified values of theta, calculate the
150 '             components of V and A.
160 '
170     K = 4*ATN(1)/180
180 '
190           'calculate velocities
200     PRINT " theta          VX             VY             VZ"
210     PRINT "degrees        mm/s           mm/s           mm/s"
220     PRINT
230     FOR TH = 0 TO 150 STEP 10
240        VX = 150*SIN(K*TH)
250        VY = 150*COS(K*TH)
260        VZ = 100*SIN(K*TH)
270        PRINT USING " ###          ";TH,
280        PRINT USING "####.##         ";VX,VY,VZ
290     NEXT
```

CONTINUED

```
C*****************************************************************
C*                      PROBLEM 15.C4                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C                V = magnitude of velocity of point D  (mm/s)
C         VX,VY,VZ = rectangular components of V  (mm/s)
C                A = magnitude of acceleration of point D  (mm/s**2)
C         AX,AY,AZ = rectangular components of A  (mm/s)
C               TH = angle theta  (degrees)
C
C            NOTE : For specified values of theta, calculate the
C                   components of V and A.
C
      REAL K
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,51)
   51 FORMAT (T5,'theta',T20,'VX',T34,'VY',T47,'VZ')
      WRITE (*,52)
   52 FORMAT (T4'degrees',T19,'mm/s',T33,'mm/s',T46,'mm/s')
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,150,10
           TH = I
           VX = 150.*SIN(K*TH)
           VY = 150.*COS(K*TH)
           VZ = 100.*SIN(K*TH)
           WRITE (*,20) TH,VX,VY,VZ
   20      FORMAT (4X,F4.0,8X,F7.2,7X,F7.2,7X,F7.2)
   10 CONTINUE
```

CONTINUED

PROBLEM 15.C4 CONTINUED

BASIC VERSION (CONTINUED)

```
300     PRINT:PRINT
310 '
320           'calculate accelerations
330     PRINT " theta          AX              AY              AZ"
340     PRINT "degrees       mm/s^2          mm/s^2          mm/s^2"
350     PRINT
360     FOR TH = 0 TO 150 STEP 10
370        AX =  90*COS(K*TH)
380        AY = -130*SIN(K*TH)
390        AZ =  120*COS(K*TH)
400        PRINT USING " ###          ";TH,
410        PRINT USING "####.##        ";AX,AY,AZ
420     NEXT
```

theta degrees	VX mm/s	VY mm/s	VZ mm/s
0	0.00	150.00	0.00
10	26.05	147.72	17.36
20	51.30	140.95	34.20
30	75.00	129.90	50.00
40	96.42	114.91	64.28
50	114.91	96.42	76.60
60	129.90	75.00	86.60
70	140.95	51.30	93.97
80	147.72	26.05	98.48
90	150.00	-0.00	100.00
100	147.72	-26.05	98.48
110	140.95	-51.30	93.97
120	129.90	-75.00	86.60
130	114.91	-96.42	76.60
140	96.42	-114.91	64.28
150	75.00	-129.90	50.00

theta degrees	AX mm/s^2	AY mm/s^2	AZ mm/s^2
0	90.00	0.00	120.00
10	88.63	-22.57	118.18
20	84.57	-44.46	112.76
30	77.94	-65.00	103.92
40	68.94	-83.56	91.93
50	57.85	-99.59	77.13
60	45.00	-112.58	60.00
70	30.78	-122.16	41.04
80	15.63	-128.03	20.84
90	-0.00	-130.00	-0.00
100	-15.63	-128.03	-20.84
110	-30.78	-122.16	-41.04
120	-45.00	-112.58	-60.00
130	-57.85	-99.59	-77.13
140	-68.94	-83.56	-91.93
150	-77.94	-65.00	-103.92

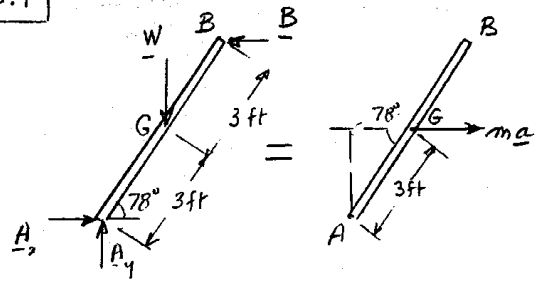
FORTRAN VERSION (CONTINUED)

```
C
      WRITE (*,53)
   53 FORMAT (//,T5,'theta',T20,'AX',T34,'AY',T47,'AZ')
      WRITE (*,54)
   54 FORMAT (T4'degrees',T17,'mm/s**2',T31,'mm/s**2',T44,'mm/s**2')
      WRITE (*,'(/,A\)')
C
      DO 30 I = 0,150,10
           TH = I
           AX = 90.*COS(K*TH)
           AY = -130.*SIN(K*TH)
           AZ = 120.*COS(K*TH)
C
           WRITE (*,40) TH,AX,AY,AZ
   40      FORMAT (4X,F4.0,8X,F7.2,7X,F7.2,7X,F7.2)
   30 CONTINUE
      END
```

theta degrees	VX mm/s	VY mm/s	VZ mm/s
0.	.00	150.00	.00
10.	26.05	147.72	17.36
20.	51.30	140.95	34.20
30.	75.00	129.90	50.00
40.	96.42	114.91	64.28
50.	114.91	96.42	76.60
60.	129.90	75.00	86.60
70.	140.95	51.30	93.97
80.	147.72	26.05	98.48
90.	150.00	.00	100.00
100.	147.72	-26.05	98.48
110.	140.95	-51.30	93.97
120.	129.90	-75.00	86.60
130.	114.91	-96.42	76.60
140.	96.42	-114.91	64.28
150.	75.00	-129.90	50.00

theta degrees	AX mm/s**2	AY mm/s**2	AZ mm/s**2
0.	90.00	.00	120.00
10.	88.63	-22.57	118.18
20.	84.57	-44.46	112.76
30.	77.94	-65.00	103.92
40.	68.94	-83.56	91.93
50.	57.85	-99.59	77.13
60.	45.00	-112.58	60.00
70.	30.78	-122.16	41.04
80.	15.63	-128.03	20.84
90.	.00	-130.00	.00
100.	-15.63	-128.03	-20.84
110.	-30.78	-122.16	-41.04
120.	-45.00	-112.58	-60.00
130.	-57.85	-99.59	-77.13
140.	-68.94	-83.56	-91.93
150.	-77.94	-65.00	-103.92

16.1



For the board to remain in position shown, we need $B \geq 0$. When board is just about to start rotating about A, we have $B = 0$.

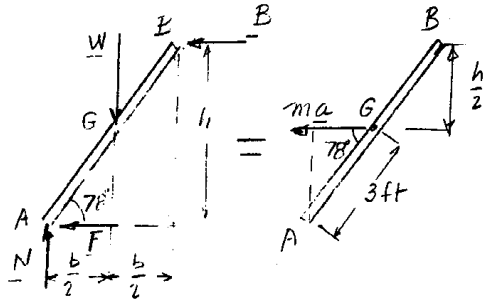
$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$$W(3\text{ ft})\cos 78^\circ = \frac{W}{g} a\,(3\text{ ft})\sin 78^\circ$$

$$a = g\cot 78^\circ = (32.2\text{ ft/s}^2)\cot 78^\circ$$

$$a = 6.84\text{ ft/s}^2 \blacktriangleleft$$

16.2



We have $b = (6\text{ ft})\cos 78^\circ = 1.2475\text{ ft}$

$h = (6\text{ ft})\sin 78^\circ = 5.869\text{ ft}$

When board is about to slide:

$$F = F_m = \mu_s N = 0.3N$$

$+\uparrow \Sigma F_y = 0$: $N - W = 0$ $\quad N = W$

$$F = 0.3N = 0.3W$$

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$:

$$W\left(\frac{b}{2}\right) - Fh - Nb = -ma\left(\frac{h}{2}\right)$$

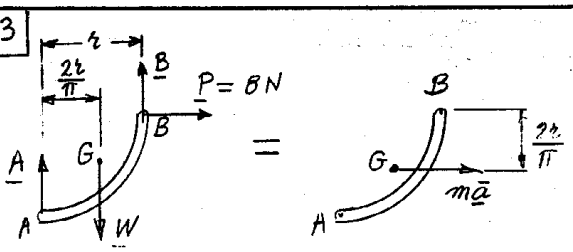
$$Wb - (0.3W)(2h) - W(2b) = -\frac{W}{g} ah$$

$$ah = g(0.6h + b)$$

$$5.869\,a = 32.2\,[0.6(5.869) + 1.2475]$$

$$a = 26.2\text{ ft/s}^2 \blacktriangleleft$$

16.3



(a) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad P = m\bar{a}$

$$\bar{a} = \frac{P}{m} = \frac{8N}{2.5\text{kg}} = 3.20\text{ m/s}^2$$

$$\underline{\bar{a}} = 3.20\text{ m/s}^2 \rightarrow \blacktriangleleft$$

$(L) +\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$:

$$W\left(r - \frac{2r}{\pi}\right) - Ar = m\bar{a}\left(\frac{2r}{\pi}\right)$$

$$A = W\left(1 - \frac{2}{\pi}\right) - m\bar{a}\left(\frac{2}{\pi}\right) = mg\left(1 - \frac{2}{\pi}\right) - P\left(\frac{2}{\pi}\right)$$

$$= (2.5\text{kg})(9.81\text{m/s}^2)\left(1 - \frac{2}{\pi}\right) - (8N)\left(\frac{2}{\pi}\right)$$

$$= 8.912\text{ N} - 5.093\text{ N} = 3.819\text{ N} \quad \underline{A} = 3.82\text{ N}\uparrow \blacktriangleleft$$

$+\uparrow \Sigma F_y = 0$: $A + B - W = 0$

$B = W - A = (2.5)(9.81) - 3.819$, $\quad \underline{B} = 20.71\text{ N}\uparrow \blacktriangleleft$

16.4

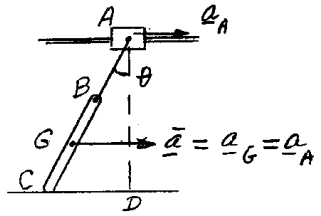
Geometry and Kinematics:

Distance between collar and floor

$$= AD = 0.25\text{ m} + 0.35\text{ m} = 0.60\text{ m}$$

When cord and rod lie in a straight line:

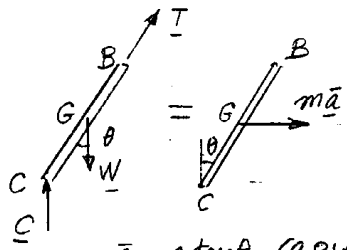
$$AC = AB + BC = 0.25\text{ m} + 0.4\text{ m} = 0.65\text{ m}$$



$$\cos\theta = \frac{AD}{AC} = \frac{0.60\text{ m}}{0.65\text{ m}}$$

$$\theta = 22.62^\circ$$

Kinetics



(a) $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:

$$W(CG)\sin\theta = m\bar{a}(CG)\cos\theta$$

$$m\bar{a} = mg\tan\theta$$

$$\bar{a} = g\tan\theta = (9.81\text{m/s}^2)\tan 22.62^\circ$$

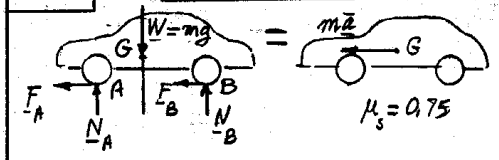
$$\underline{a_A} = \underline{\bar{a}} = 4.09\text{ m/s}^2 \rightarrow \blacktriangleleft$$

(b) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad T\sin\theta = m\bar{a} = mg\tan\theta$

$$T = \frac{mg}{\cos\theta} = \frac{(4\text{kg})(9.81\text{m/s}^2)}{0.60/0.65}$$

$$T = 42.5\text{ N} \blacktriangleleft$$

16.5 (a) Four-wheel drive



$F_A = \mu_s N_A$

$F_B = \mu_s N_B$

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - W = 0$ $\quad N_A + N_B = W = mg$

Thus: $F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s (N_A + N_B) = \mu_s mg$

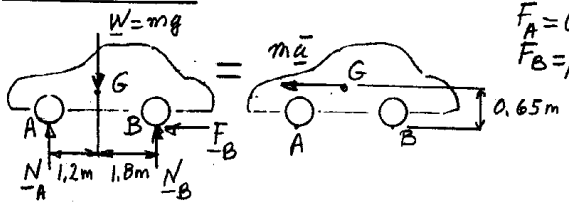
$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F_A + F_B = m\bar{a}$

$\mu_s mg = m\bar{a}$

$\bar{a} = \mu_s g = 0.75\,(9.81\ m/s^2)$

$\bar{a} = 7.36\ m/s^2 \leftarrow$ ◀

(b) Rear-wheel drive



$F_A = 0$

$F_B = \mu_s N_B$

$+\circlearrowleft \Sigma M_A = 0$: $-mg(1.2) + N_B(3) = m\bar{a}(0.65)$

$N_B = \frac{m}{3}(0.65\bar{a} + 1.2g)$

Thus: $F_B = \mu_s N_B = 0.75 N_B = m(0.1625\bar{a} + 0.3g)$

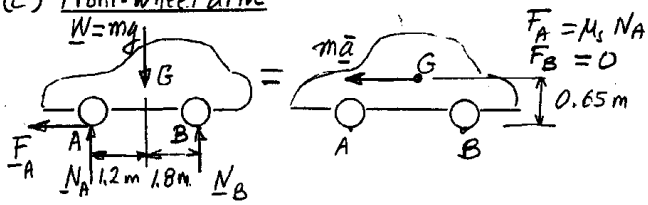
$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F_B = m\bar{a}$

$m(0.1625\bar{a} + 0.3g) = m\bar{a}$

$\bar{a}(1 - 0.1625) = 0.3\,(9.81\ m/s^2)$

$\bar{a} = 0.3582\,(9.81\ m/s^2)$ $\quad \bar{a} = 3.51\ m/s^2 \leftarrow$ ◀

(c) Front-wheel drive



$F_A = \mu_s N_A$

$F_B = 0$

$+\circlearrowleft \Sigma M_B = 0$: $mg(1.8) - N_A(3) = m\bar{a}(0.65)$

$N_A = \frac{m}{3}(1.8g - 0.65\bar{a})$

Thus: $F_A = \mu_s N_A = 0.75 N_A = m(0.45g - 0.1625\bar{a})$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F_A = m\bar{a}$

$m(0.45g - 0.1625\bar{a}) = m\bar{a}$

$\bar{a}(1 + 0.1625) = 0.45(9.81\ m/s^2)$

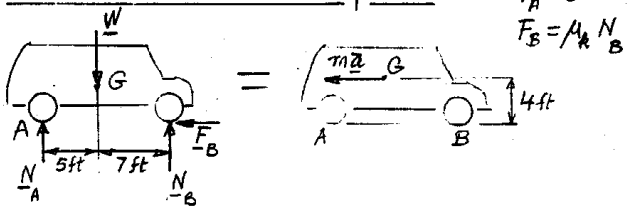
$\bar{a} = 0.3871(9.81\ m/s^2)$

$\bar{a} = 3.80\ m/s^2 \leftarrow$ ◀

16.6 From solution of Sample Prob. 16.1:

$\mu_k = 0.699$

(a) if rear-wheel brakes fail to operate:



$F_A = 0$

$F_B = \mu_k N_B$

$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $N_B(12\ ft) - W(5\ ft) = m\bar{a}(4\ ft)$

$N_B = \frac{5}{12} W + \frac{1}{3} \frac{W}{g} \bar{a}$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F_B = m\bar{a}$ $\quad \mu_k N_B = \frac{W}{g} \bar{a}$

$0.699\left(\frac{5}{12} W + \frac{1}{3} \frac{W}{g} \bar{a}\right) = \frac{W}{g} \bar{a}$

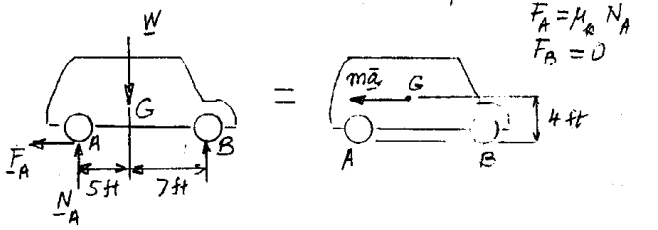
$\bar{a} = \frac{0.699(5/12)(32.2\ ft/s^2)}{1 - 0.233}$ $\quad \bar{a} = 12.227\ ft/s^2 \leftarrow$

Uniformly accelerated motion:

$v^2 = v_0^2 + 2ax$ $\quad 0 = (30\ ft/s)^2 + 2(-12.227\ ft/s^2)x$

$x = 36.8\ ft$ ◀

(b) if front-wheel brakes fail to operate:



$F_A = \mu_k N_A$

$F_B = 0$

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $W(7\ ft) - N_A(12\ ft) = m\bar{a}(4\ ft)$

$N_A = \frac{7}{12} W - \frac{1}{3} \frac{W}{g} \bar{a}$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F_A = m\bar{a}$ $\quad \mu_k N_A = \frac{W}{g} \bar{a}$

$0.699\left(\frac{7}{12} W - \frac{1}{3} \frac{W}{g} \bar{a}\right) = \frac{W}{g} \bar{a}$

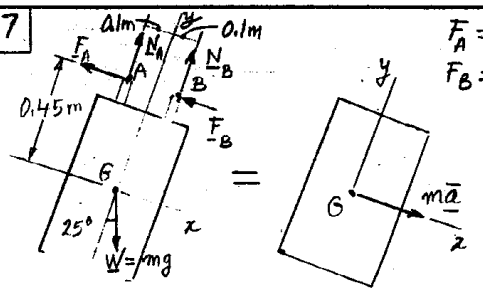
$\bar{a} = \frac{0.699(7/12)(32.2\ ft/s^2)}{1 + 0.233}$ $\quad \bar{a} = 10.649\ ft/s^2 \leftarrow$

Uniformly accelerated motion:

$v^2 = v_0^2 + 2ax$ $\quad 0 = (30\ ft/s)^2 + 2(-10.648\ ft/s^2)x$

$x = 42.3\ ft$ ◀

16.7



$F_A = \mu_k N_A$

$F_B = \mu_k N_B$

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - W\cos 25° = 0$

$N_A + N_B = mg\cos 25°$ (1)

$+\circlearrowleft \Sigma M_G = 0$: $N_B(0.1\text{m}) - N_A(0.1\text{m}) + (F_A + F_B)(0.45\text{m}) = 0$

But $F_A + F_B = \mu_k(N_A + N_B) = \mu_k mg\cos 25°$ (2)

Substituting for $F_A + F_B$ and solving for $N_B - N_A$:

$N_B - N_A = -4.5\mu_k mg\cos 25°$ (3)

Adding (1) and (3): $2N_B = mg\cos 25°(1 - 4.5\mu_k)$ (4)

Given data: $m = 25$ kg, $g = 9.81$ m/s², $\mu_k = 0.20$

Substituting in (4):

$2N_B = (25\text{ kg})(9.81\text{ m/s}^2)\cos 25°(1 - 0.90) = 22.227$ N

$N_B = 11.11$ N ◀

16.8 See solution of Prob. 16.7 for diagram and derivation of Eq. (4):

$2N_B = mg\cos 25°(1 - 4.5\mu_k)$ (4)

(a) Making $N_B = 0$ in (4):

$1 - 4.5\mu_k = 0$ $\quad \mu_k = 0.222$ ◀

(b) Referring to diagram:

$\pm \Sigma F_x = \Sigma (F_x)_{\text{eff}}$:

$mg\sin 25° - (F_A + F_B) = m\bar{a}$

But, from Eq. (2) of solution of Prob. 16.7:

$F_A + F_B = \mu_k mg\cos 25°$

Substituting for $F_A + F_B$:

$mg\sin 25° - \mu_k mg\cos 25° = m\bar{a}$

$\bar{a} = g(\sin 25° - \mu_k \cos 25°)$

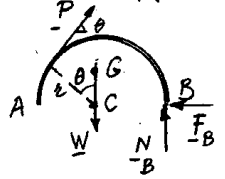
Substituting the value obtained for μ_k in part (a):

$\bar{a} = (9.81\text{ m/s}^2)(\sin 25° - 0.222\cos 25°)$

$\bar{a} = 2.17\text{ m/s}^2 \searrow 25°$ ◀

16.9 (a) Sliding and tipping are both impending.

Since pipe is in equilibrium, we draw FB diagram



Since tipping is impending, there is no reaction at A.

Since sliding is impending:

$F_B = \mu_s N_B$ (1)

$+\circlearrowleft \Sigma M_C = 0$: $N_B r - Pr = 0$ $\quad N_B = P$ (2)

$\pm \Sigma F_x = 0$: $P\cos\theta - F_B = 0$

Recalling (1) and (2): $P\cos\theta = \mu_s P$ $\quad \theta = \cos^{-1}\mu_s$ (3) ◀

$+\uparrow \Sigma F_y = 0$: $P\sin\theta - W + N_B = 0$

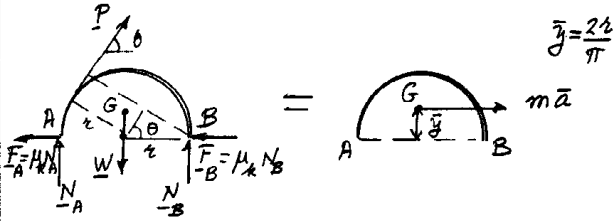
Recalling (2): $P\sin\theta - W + P = 0$

$P(1 + \sin\theta) = W$ (4)

$P = \dfrac{W}{1+\sin\theta}$ (4') ◀

(b) Acceleration of pipe if P is slightly increased

We assume that P has the value found in part (a), but that pipe starts sliding, causing value of coefficient of friction to drop from μ_s to μ_k



$+\uparrow \Sigma F_y = 0$: $N_A + N_B - W + P\sin\theta = 0$, $\quad N_A + N_B = W - P\sin\theta$ (5)

$\pm \Sigma F_x = m\bar{a}$: $P\cos\theta - \mu_k(N_A + N_B) = m\bar{a}$

Substitute for $N_A + N_B$ from (5):

$P\cos\theta - \mu_k W + \mu_k P\sin\theta = m\bar{a}$

Substitute for P from (4'):

$\dfrac{W\cos\theta}{1+\sin\theta} - \mu_k W + \dfrac{\mu_k W\sin\theta}{1+\sin\theta} = m\bar{a}$

$\dfrac{W}{1+\sin\theta}(\cos\theta - \mu_k - \cancel{\mu_k \sin\theta} + \cancel{\mu_k \sin\theta}) = m\bar{a}$

Recalling from (3) that $\cos\theta = \mu_s$: $\bar{a} = g\dfrac{\mu_s - \mu_k}{1+\sin\theta}$ (6) ◀

(c) Value of N_A

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}$: $P(r + r\sin\theta) - Wr + N_A(2r) = m\bar{a}\bar{y}$

From (4), we note that $P(r + r\sin\theta) - Wr = 0$. Thus:

$N_A(2r) = m\bar{a}\bar{y} = m\bar{a}\dfrac{2r}{\pi}$ $\quad N_A = \dfrac{m\bar{a}}{\pi}$ (7)

From (6): $N_A = \dfrac{m}{\pi} g \dfrac{\mu_s - \mu_k}{1+\sin\theta}$ $\quad N_A = \dfrac{W}{\pi}\dfrac{\mu_s - \mu_k}{1+\sin\theta}$ ◀

16.10 See solution of Prob. 16.9 for derivation of Eqs. (3), (4'), (6), and (7).

Given data: $W = 200$ lb, $\mu_s = 0.50$, $\mu_k = 0.40$

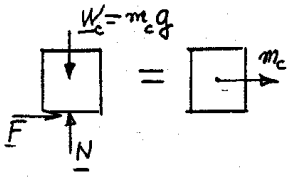
(a) Eq. (3): $\theta = \cos^{-1}\mu_s = \cos^{-1} 0.50$ $\quad \theta = 60.0°$ ◀

Eq. (4'): $P = \dfrac{W}{1+\sin\theta} = \dfrac{200\text{ lb}}{1+\sin 60°}$ $\quad P = 107.2$ lb ◀

(b) Eq. (6): $\bar{a} = g\dfrac{\mu_s - \mu_k}{1+\sin\theta} = 32.2\dfrac{0.50 - 0.40}{1+\sin 60°}$, $\quad \bar{a} = 1.726\text{ ft/s}^2$ ◀

(c) Eq. (7): $N_A = \dfrac{m\bar{a}}{\pi} = \dfrac{200(1.726)}{32.2\,\pi}$ $\quad N_A = 3.41$ lb ◀

16.11 We first determine the required friction force if the crate is not to slide. The deceleration of the crate is then the same as that of the truck.



$\pm \Sigma F_x = \Sigma (F_x)_{eff}$:

$F = m_c a$ (1)

$= (1500\ kg)(3\ m/s^2)$

$= 4500\ N = 4.5\ kN$

On the other hand:

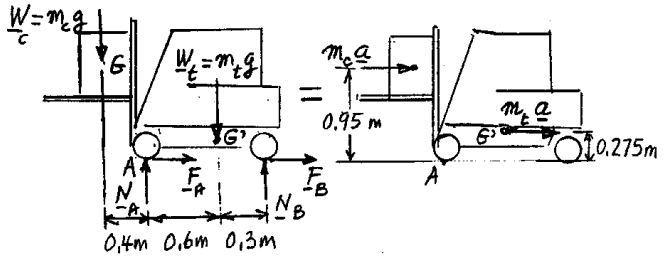
$+\uparrow \Sigma F_y = 0$: $N - m_c g = 0$

$N = m_c g = (1500\ kg)(9.81\ m/s^2)$

$= 14.715\ kN$

$F_m = \mu_s N = 0.50(14.715\ kN) = 7.3575\ kN$ (2)

Since $F < F_m$, the crate will not slide



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$N_B(0.9m) + m_c g(0.4m) - m_t g(0.6m) = -m_c a(0.95m) - m_t a(0.275m)$

$0.9 N_B + (1500 \times 0.4 - 2800 \times 0.6)(9.81) + (1500 \times 0.95 + 2800 \times 0.275)a = 0$

$0.9 N_B - 10\,594.8 + 2195\, a = 0$ (3)

For $a = 3\ m/s^2$, Eq. (3) yields

$0.9 N_B - 10\,594.8 + 2195(3) = 0$ $N_B = 4455.3\ N$

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - (1500 + 2800)(9.81) = 0$

$N_A = 42183 - N_B = 42183 - 4455.3 = 37\,728\ N$

At each front wheel: $\underline{A} = \frac{1}{2}\underline{N}_A = 18.86\ kN \uparrow$ ◀

At each rear wheel: $\underline{B} = \frac{1}{2}\underline{N}_B = 2.23\ kN \uparrow$ ◀

16.12 For no sliding: See solution of Prob. 16.11 for first diagram and Eqs. (1) and (2).

Making $F = F_m = 7357.5\ N$ and $m = 1500\ kg$ in (1):

$7357.5\ N = (1500\ kg)a$ $\underline{a} = 4.91\ m/s^2 \rightarrow$ ◁

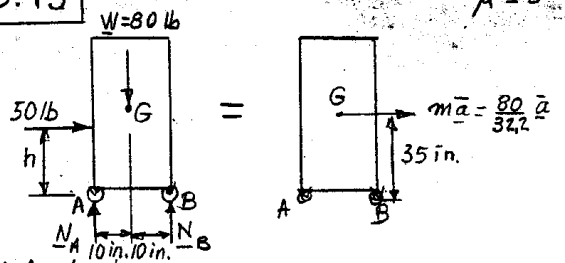
For no tipping: See solution of Prob. 16.11 for second diagram and derivation of Eq. (3).

Making $N_B = 0$ in Eq. (3):

$-10\,594.8 + 2195\, a = 0$ $\underline{a} = 4.83\ m/s^2 \rightarrow$ ◁

The maximum allowable deceleration of the truck if the crate is not to slip and the truck not to tip is thus $\underline{a} = 4.83\ m/s^2 \rightarrow$ ◀

16.13 $\mu = 0$



(a) Acceleration

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $50 = \frac{80}{32.2}\bar{a}$ $\bar{a} = 20.1\ ft/s^2 \rightarrow$ ◀

(b) Range of values of h for no tipping

When tipping to the right is impending: $N_A = 0$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $(50\ lb)h - (80\ lb)(10\ in.) = \left(\frac{80}{32.2}\bar{a}\right)(35\ in.)$

$50h - 80(10) = 50(35)$

$h = 16 + 35$ $h = 51.0$ ◁

When tipping to the left is impending: $N_B = 0$

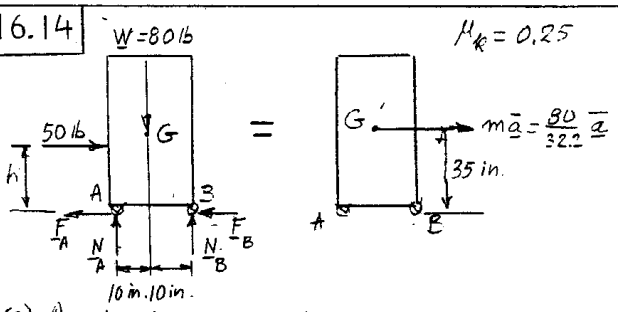
$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $(50\ lb)h + (80\ lb)(10\ in.) = \left(\frac{80}{32.2}\bar{a}\right)(35\ in.)$

$50h + 80(10) = 50(35)$

$h = -16 + 35$ $h = 19.00\ in$ ◁

For no tipping: $19.00\ in. \le h \le 51.0\ in.$ ◀

16.14



(a) Acceleration

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - 80\ lb = 0$ $N_A + N_B = 80\ lb$

Thus: $F_A + F_B = \mu_k (N_A + N_B) = 0.25(80\ lb) = 20\ lb$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $50 - (F_A + F_B) = \frac{80}{32.2}\bar{a}$

$\frac{80}{32.2}\bar{a} = 50 - 20 = 30$ $\bar{a} = 12.08\ ft/s^2 \rightarrow$ ◀

(b) Range of values of h for no tipping.

When tipping to the right is impending: $N_A = F_A = 0$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $(50\ lb)h - (80\ lb)(10\ in.) = \left(\frac{80}{32.2}\bar{a}\right)(35\ in.)$

$50h - 80(10) = 30(35)$

$h = 16 + 21$ $h = 37.0\ in.$ ◁

When tipping to the left is impending: $N_B = F_B = 0$

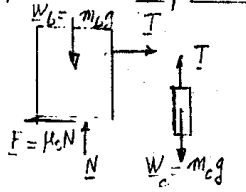
$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $(50\ lb)h + (80\ lb)(10\ in.) = \left(\frac{80}{32.2}\bar{a}\right)(35\ in.)$

$50h + 80(10) = 30(35)$

$h = -16 + 21$ $h = 5.00\ in.$ ◁

Thus, for no tipping: $5.00\ in. \le h \le 37.0\ in.$ ◀

16.15 We first check that barrel actually slides by determining the mass m_c of the cylinder required for impending motion.



Barrel:

$\Sigma F_y = 0$: $N = m_b g$

$\Sigma F_x = 0$: $T = F = \mu_s N = \mu_s m_b g$

Cylinder: $W_c = T$

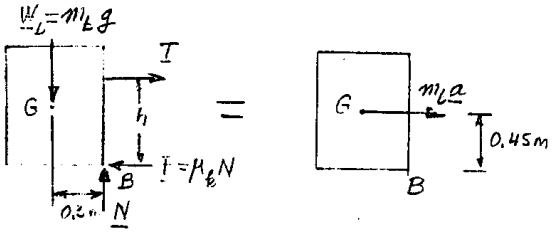
$m_c g = \mu_s m_b g$

$m_c = \mu_s m_b$ (1)

Substituting data in (1): $(m_c)_{req} = 0.35(125\ kg) = 43.75\ kg$

Since $m_c = 60\ kg$, $m_c > (m_c)_{req}$; barrel actually slides.

Equations of motion for barrel:



$+\uparrow \Sigma F_y = 0$: $N - W_b = 0$ $\quad N = W_b = m_b g$

Thus: $F = \mu_k N = \mu_k m_b g$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $T - F = m_b a$

$T = \mu_k m_b g + m_b a$

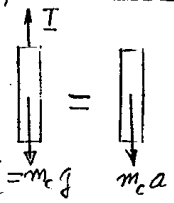
$T = (\mu_k g + a) m_b$ (2)

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$ (when barrel about to tip):

$Th - m_b g (0.3m) = m_b a (0.45m)$

$Th = (0.3g + 0.45a) m_b$ (3)

Equations of motion for cylinder:



$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$m_c g - T = m_c a$

$T = (g - a) m_c$ (4)

(a) Equating the right-hand members in (2) and (4):

$(\mu_k g + a) m_b = (g - a) m_c$

$a = \dfrac{m_c - \mu_k m_b}{m_c + m_b} g$ (5)

Given data: $m_b = 125\ kg$, $m_c = 60\ kg$, $\mu_k = 0.30$

$a = \dfrac{60 - 0.30(125)}{60 + 125} g = 0.12162\, g$

$a = 0.12162(9.81\ m/s^2)$ $\quad \underline{a = 1.193\ m/s^2 \rightarrow}$ ◀

(b) Substituting for a and given data in (2) and (3):

$T = (0.30g + 0.12162g)(125) = 52.70\, g$

$Th = (0.3g + 0.45 \times 0.12162g)(125) = 44.34\, g$

Thus: $h = \dfrac{Th}{T} = \dfrac{44.34g}{52.70g} = 0.841\ m = 841\ mm$

The barrel will not tip for $h \le 841\ mm$ ◀

16.16 See solution of Prob. 16.15 for diagrams and derivation of Eqs. (1) through (5).

Dividing (3) by (2) member by member, we have

$$h = \frac{0.3g + 0.45a}{\mu_k g + a}$$

Substituting the given data, $h = 0.6m$, $\mu_k = 0.40$:

$0.6(0.40g + a) = 0.3g + 0.45a$

$0.15a = 0.06g$ $\quad a = 0.4g$

Substituting into (5):

$$0.4g = \frac{m_c - 0.40 m_b}{m_c + m_b} g$$

$0.4 m_c + 0.4 m_b = m_c - 0.40 m_b$

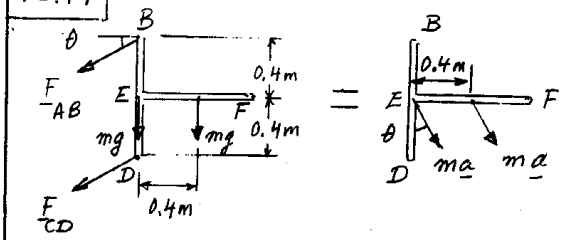
$m_c = \dfrac{0.4 + 0.40}{1 - 0.4} m_b = \dfrac{0.8}{0.6}(160\ kg) = 213.3\ kg$

The barrel will not tip for $m_c \le 213\ kg$ ◀

Note: The above analysis is valid only if the barrel slides. From Eq. (1) of Prob. 16.15 we note that it will not slide for $m_c \le \mu_s m_b = 0.48(160\ kg) = 76.8\ kg$. However, we may check from statics that barrel will not tip for $m_c = 76.8\ kg$: We have $T = m_c g = 76.8g$ and

$+\circlearrowleft \Sigma M_B = (160g)(0.3) - (76.8g)(0.6) = +1.92g > 0$ (O.K.)

16.17



$\theta \nearrow \Sigma F = \Sigma F_{eff}$: $2mg\cos\theta = 2ma$

$a = g\cos\theta$ (1)

$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{eff}$:

$F_{AB}\cos\theta(0.8m) - mg(0.4m) = -2ma\sin\theta(0.4m) - ma\cos\theta(0.4m)$

$F_{AB} = \frac{1}{2}\dfrac{mg}{\cos\theta} - \frac{1}{2}ma(2\tan\theta + 1)$

Substitute from (1): $F_{AB} = \frac{1}{2}mg\left(\dfrac{1}{\cos\theta} - 2\sin\theta - \cos\theta\right)$ (2)

$\theta \searrow \Sigma F = \Sigma F_{eff}$: $F_{AB} + F_{CD} + 2mg\sin\theta = 0$

$F_{CD} = -F_{AB} - 2mg\sin\theta$ (3)

Given data: $m = 2\ kg$, $g = 9.81\ m/s^2$, $\theta = 30°$

Eq. (2): $F_{AB} = \frac{1}{2}(2)(9.81)\left(\dfrac{1}{\cos 30°} - 2\sin 30° - \cos 30°\right)$

$= 9.81(1.1547 - 1 - 0.8660) = -6.978\ N$

$F_{AB} = 6.98\ N\ C$ ◀

Eq. (3): $F_{CD} = -(-6.978) - 2(2)(9.81)\sin 30°$

$= 6.978 - 19.62 = -12.64\ N$ $\quad F_{CD} = 12.64\ N\ C$ ◀

16.18 Use portion of solution of Prob. 16.17 above dashed line.

Given data: $m = 2\ kg$, $g = 9.81\ m/s^2$, $\theta = 60°$

Eq. (2): $F_{AB} = \frac{1}{2}(2)(9.81)\left(\dfrac{1}{\cos 60°} - 2\sin 60° - \cos 60°\right)$

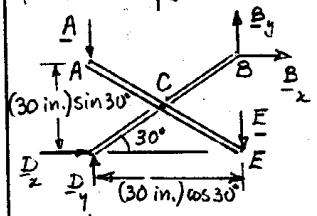
$= 9.81(2 - 1.732 - 0.5) = -2.276\ N$, $F_{AB} = 2.28\ N\ C$ ◀

Eq. (3): $F_{CD} = -(-2.276) - 2(2)(9.81)\sin 60°$

$= 2.276 - 33.983 = -31.71\ N$ $\quad F_{CD} = 31.7\ N\ C$ ◀

16.19 **Analysis of linkage**

Since members ACE and DCB are of negligible mass, their effective forces may also be neglected and the methods of statics may be applied to their analysis.

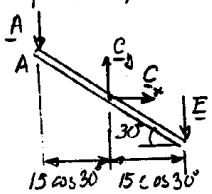


Free Body: Entire Linkage:

$+\circlearrowleft \Sigma M_D = 0$:

$(B_y - E)(30\cos 30°) - B_x(30\sin 30°) = 0$

$(B_y - E)\cos 30° - B_x \sin 30° = 0$ (1)



Free Body: Member ACE

$+\circlearrowleft \Sigma M_C = 0$:

$A(15\cos 30°) - E(15\cos 30°) = 0$

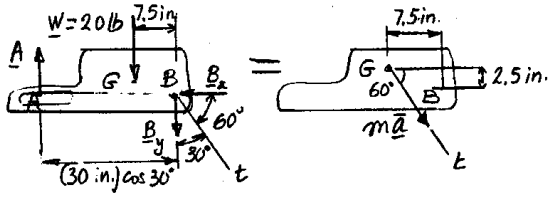
$E = A$

Carrying into Eq. (1):

$(B_y - A)\cos 30° - B_x \sin 30° = 0$ (2)

Equations of motion for member AB

Since member AB is in curvilinear translation, its mass center G has the same acceleration as B. It is thus perpendicular to BD, or $\measuredangle 60°$



(a) $+\measuredangle 60° \Sigma F_t = \Sigma(F_t)_{eff}$:

$(B_y - A)\cos 30° - B_x \sin 30° + W\cos 30° = m\bar{a}$

Recalling Eq. (2), we have

$W\cos 30° = m\bar{a}$ $\quad \bar{a} = \frac{W}{m}\cos 30° = g\cos 30°$

$\bar{a} = (32.2 \text{ ft/s}^2)\cos 30°$ $\quad \underline{\bar{a}} = 27.9 \text{ ft/s}^2 \measuredangle 60°$ ◀

(b) $+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$W(7.5\text{in.}) - A(30\text{in.})\cos 30° = (m\bar{a}\sin 60°)(7.5\text{in.}) - (m\bar{a}\cos 60°)(2.5\text{in.})$

But $m\bar{a} = W\cos 30°$ as found above. Thus:

$7.5W - 30A\cos 30° = W\cos 30°(7.5\sin 60° - 2.5\cos 60°)$

$A = W\left(\frac{1}{4\cos 30°} - \frac{1}{4}\sin 60° + \frac{1}{12}\cos 60°\right)$

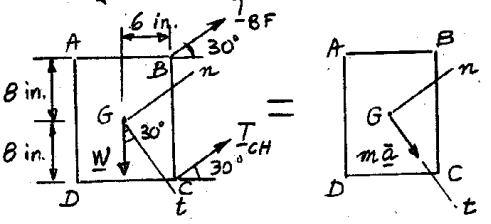
$= 0.11384\,W$

Recalling that $W = 20$ lb,

$A = 0.11384(20\text{ lb}) = 2.2768 \text{ lb}$

$\underline{A} = 2.28 \text{ lb} \uparrow$ ◀

16.20 The plate is in curvilinear translation. Thus, its mass center G has the same acceleration as B, that is $\measuredangle 60°$.



(a) $+\measuredangle \Sigma F_t = \Sigma(F_t)_{eff}$: $\quad W\cos 30° = m\bar{a}$

$\bar{a} = \frac{W}{m}\cos 30° = g\cos 30°$

$\bar{a} = (32.2 \text{ ft/s}^2)\cos 30°$ $\quad \underline{\bar{a}} = 27.9 \text{ ft/s}^2 \measuredangle 60°$ ◀

(b) $+\nearrow \Sigma F_n = \Sigma(F_n)_{eff}$: $\quad T_{BF} + T_{CH} - W\sin 30° = 0$

$T_{BF} + T_{CH} = 0.5\,W$ (1)

$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$:

$(T_{BF}\sin 30°)(6) - (T_{BF}\cos 30°)(8) + (T_{CH}\sin 30°)(6) + (T_{CH}\cos 30°)(8) = 0$

$-3.9282\,T_{BF} + 9.9282\,T_{CH} = 0$

$T_{CH} = 0.3957\,T_{BF}$ (2)

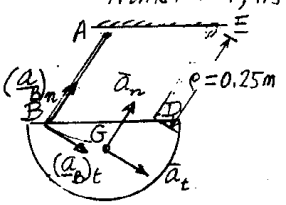
Substitute for T_{CH} from (2) into (1):

$1.3957\,T_{BF} = 0.5\,W$ $\quad T_{BF} = 0.3582\,W$

From (2): $T_{CH} = 0.3957(0.3582\,W) = 0.14174\,W$

With $W = 16$ lb: $\quad T_{BF} = 5.73$ lb; $T_{CH} = 2.27$ lb ◀

16.21 **Kinematics.** Since the plate is in curvilinear translation, its mass center G has the same acceleration as B. We note that B describes a circle of radius $r = 0.25$m centered at A.



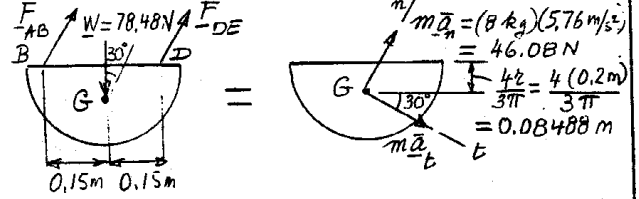
$\underline{\bar{a}} = \underline{a}_B = (\underline{a}_B)_t + (\underline{a}_B)_n$

$= \bar{a}_t \measuredangle 30° + \frac{v^2}{\rho} \measuredangle 60°$

$= \bar{a}_t \measuredangle 30° + \frac{(1.2)^2}{0.25} \measuredangle 60°$

$\underline{\bar{a}} = \bar{a}_t \measuredangle 30° + (5.76 \text{ m/s}^2) \measuredangle 60°$

Equations of motion $\quad W = (8\text{kg})(9.81\text{m/s}^2) = 78.48\text{ N}$



$+\nearrow \Sigma F_n = \Sigma(F_n)_{eff}$: $\quad F_{AB} + F_{DE} - 78.48\cos 30° = 46.08$ (1)

$+\circlearrowleft \Sigma M_G = 0$: $(F_{AB}\cos 30°)(0.15) + (F_{AB}\sin 30°)(0.08488) - (F_{DE}\cos 30°)(0.15) + (F_{DE}\sin 30°)(0.08488) = 0$

$0.17234\,F_{AB} - 0.08746\,F_{DE} = 0$ (2)

(CONTINUED)

16.21 CONTINUED

We repeat Eqs. (1) and (2):

$$F_{AE} + F_{DE} - 78.48\cos 30^\circ = 46.08 \quad (1)$$

$$0.17234\,F_{AB} - 0.08746\,F_{DE} = 0 \quad (2)$$

Solving (2) for F_{AB} and substituting into (1):

$$F_{AB} = 0.5075\,F_{DE} \quad (3)$$

$$1.5075\,F_{DE} - 67.766 = 46.08$$

$$F_{DE} = 75.652 \qquad F_{DE} = 75.7\ N\ T \blacktriangleleft$$

From (3):

$$F_{AB} = 0.5075(75.652) \qquad F_{AB} = 38.4\ N\ T \blacktriangleleft$$

Note: The component a_t of the acceleration of G may be obtained by writing

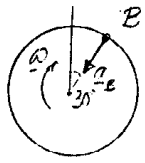
$+\searrow \Sigma F_t = \Sigma (F_t)_{eff}$:

$$W\sin 30^\circ = m\bar{a}_t$$

$$a_t = \frac{W}{m}\sin 30^\circ = 0.5g = 4.905\ m/s^2$$

16.22

We first determine the acceleration of point B of disk:



$\omega = 180\ rpm = 18.85\ rad/s$

Since ω = constant,

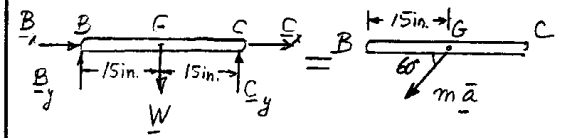
$$a_B = (a_B)_n = r\omega^2$$

$$= \left(\frac{8}{12}\ ft\right)(18.85\ rad/s)^2$$

$$\underline{a}_B = 236.9\ ft/s^2\ \measuredangle 60^\circ$$

Since rod BC is in translation,

$$\underline{\bar{a}} = \underline{a}_B = 236.9\ ft/s^2\ \measuredangle 60^\circ$$



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$:

$$C_y(30\text{ in.}) - W(15\text{ in.}) = -m\bar{a}\sin 60^\circ (15\text{ in.})$$

Since $W = 15$ lb and $m\bar{a} = \frac{15\text{ lb}}{32.2}(236.9) = 110.36$ lb:

$$30C_y - (15)(15) = -110.36\sin 60^\circ (15) = -95.57(15)$$

$$2C_y = -75.57 + 15 = -80.57, \quad C_y = -40.285\ lb$$

$$\underline{C}_y = 40.3\ lb \downarrow \blacktriangleleft$$

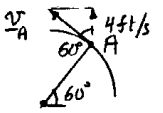
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $B_y - W + C_y = -m\bar{a}\sin 60^\circ$

$$B_y = W - C_y - \frac{15}{32.2}(236.9)\sin 60^\circ = 15 + 40.285 - 95.57$$

$$B_y = -40.285\ lb \qquad \underline{B}_y = 40.3\ lb \downarrow \blacktriangleleft$$

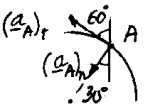
16.23

Kinematics. Since the T-shaped rod is in curvilinear translation, all points have the same velocity and same acceleration at a given instant. Thus the vertical component of $\underline{v}_A$ is also 4 ft/s ↑, and the vertical component of $\underline{a}_A$ is also 15 ft/s² ↑.



Therefore $v_A = \frac{4\ ft/s}{\cos 60^\circ} = 8\ ft/s$

Accelerations: $(a_A)_n = \frac{v_A^2}{\rho} = \frac{8^2}{7.5/12} = 102.4\ ft/s^2$



Since $(a_A)_y = 15\ ft/s^2$, we write

$$(a_A)_t\cos 60^\circ - (a_A)_n\cos 30^\circ = 15\ ft/s^2$$

$$(a_A)_t\cos 60^\circ - 102.4\cos 30^\circ = 15$$

$$0.5(a_A)_t = 15 + 102.4\cos 30^\circ = 103.68$$

$$(a_A)_t = 207.36\ ft/s^2$$

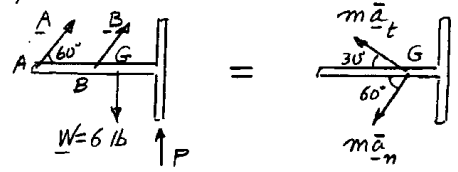
Therefore: $\underline{\bar{a}}_n = (\underline{a}_A)_n = 102.4\ ft/s^2\ \measuredangle 60^\circ$

$\underline{\bar{a}}_t = (\underline{a}_A)_t = 207.36\ ft/s^2\ \measuredangle 30^\circ$

$$m\bar{a}_n = \frac{6}{32.2}(102.4) = 19.081\ lb\ \measuredangle 60^\circ$$

$$m\bar{a}_t = \frac{6}{32.2}(207.36) = 38.639\ lb\ \measuredangle 30^\circ$$

Equations of motion.



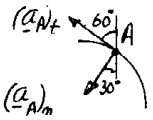
$30^\circ\ \measuredangle +\ \Sigma F_t = \Sigma (F_t)_{eff}$: $P\cos 60^\circ - W\cos 60^\circ = m\bar{a}_t$

$$0.5P - 0.5(6\ lb) = 38.639\ lb$$

$$P = 83.3\ lb \blacktriangleleft$$

16.24

See solution of Prob. 16.23 for determination of v_A and $(a_A)_n$: $(a_A)_n = 102.4\ ft/s^2$



Since $(a_A)_y = 0$, we must have

$$(a_A)_t\cos 60^\circ - (a_A)_n\cos 30^\circ = 0$$

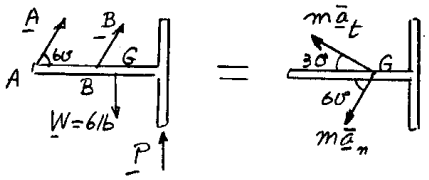
$$(a_A)_t = 1.732(102.4\ ft/s^2) = 177.36\ ft/s^2$$

Therefore:

$$m\bar{a}_n = \frac{6}{32.2}(102.4) = 19.081\ lb\ \measuredangle 60^\circ$$

$$m\bar{a}_t = \frac{6}{32.2}(177.36) = 33.049\ lb\ \measuredangle 30^\circ$$

Equations of motion.



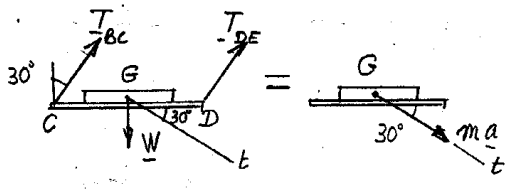
$30^\circ\ \measuredangle +\ \Sigma F_t = \Sigma (F_t)_{eff}$: $P\cos 60^\circ - W\cos 60^\circ = m\bar{a}_t$

$$0.5P - 0.5(6\ lb) = 33.049\ lb$$

$$P = 72.1\ lb \blacktriangleleft$$

***16.25** (a) Block attached to platform

Block & Platform



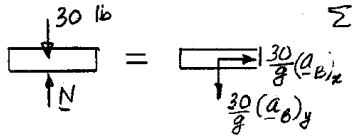
$+\searrow \Sigma F_t = \Sigma (F_t)_{eff}$: $W\sin 30° = ma$

$a = \frac{W}{m}\sin 30° = \frac{1}{2}g = 16.1\ ft/s^2$

$\underline{a} = 16.1\ ft/s^2 \measuredangle 30°$ ◀

(b) Block can slide with no friction

Equations of motion for block:

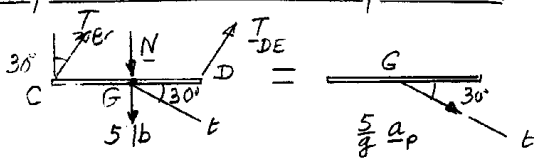


$\Sigma F_x = \Sigma (F_x)_{eff}$: $(a_B)_x = 0$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $30 - N = \frac{30}{g}(a_B)_y$

$a_B = (a_B)_y = g\left(1 - \frac{N}{30}\right)$ (1)

Equations of motion for platform:



$+\searrow \Sigma F_t = \Sigma (F_t)_{eff}$: $(N+5)\sin 30° = \frac{5}{g}a_P$

$a_P = \frac{1}{2}g\left(1 + \frac{N}{5}\right)$ (2)

But vertical components of accelerations of block and platform must be equal if contact is maintained:

$(a_B)_y = a_P \sin 30°$ (3)

Substituting from (1) and (2) into (3):

$g\left(1 - \frac{N}{30}\right) = \frac{1}{2}g\left(1 + \frac{N}{5}\right)\sin 30°$

$30 - N = 7.5 + 1.5N$

$N = 9\ lb$

Substituting into (1) and (2):

$a_B = (32.2\ ft/s^2)\left(1 - \frac{9}{30}\right) = 22.54\ ft/s^2$

$\underline{a}_B = 22.5\ ft/s^2 \downarrow$ ◀

$a_P = \frac{1}{2}(32.2\ ft/s^2)\left(1 + \frac{9}{5}\right) = 45.08\ ft/s^2$

$\underline{a}_P = 45.1\ ft/s^2 \measuredangle 30°$ ◀

Note: Since $N > 0$, we check that contact between block and platform is maintained

***16.26** We first assume that block does not slide

From the solution of Prob. 16.25a, we find that the common acceleration of the block and platform is $\underline{a} = \frac{1}{2}g \measuredangle 30°$. Considering the block alone:

$\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $F = (15\ lb)\cos 30° = 12.99\ lb$

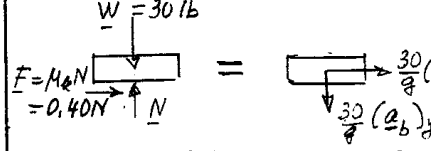
$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $30\ lb - N = (15\ lb)\sin 30°$

$N = 30 - 7.5 = 22.5\ lb$

Thus $F_m = \mu_s N = 0.50(22.5\ lb) = 11.25\ lb$

Since $F > F_m$, the block will slide

Equations of motion for block: (assuming sliding)



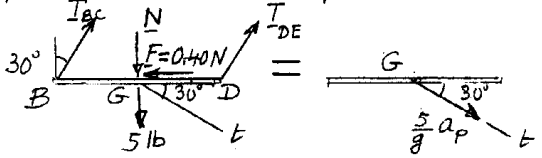
$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $30 - N = \frac{30}{g}(a_b)_y$

$(a_b)_y = g\left(1 - \frac{N}{30}\right)$ (1)

$\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $0.40N = \frac{30}{g}(a_b)_x$

$(a_b)_x = g\left(\frac{N}{75}\right)$ (2)

Equations of motion for platform



$+\searrow \Sigma F_t = \Sigma (F_t)_{eff}$: $(N+5)\sin 30° - (0.40N)\cos 30° = \frac{5}{g}a_P$

$a_P = g(0.5 + 0.030718N)$ (3)

If contact is maintained between block and platform, we must have

$(a_b)_y = (a_P)_y = a_P \sin 30°$ (4)

Substituting from (1) and (3) into (4):

$g\left(1 - \frac{N}{30}\right) = g(0.5 + 0.030718N)\sin 30°$

$(0.015359 + 0.033333)N = 0.75$

$N = 15.403\ lb$

Substituting for N in (2) and (1):

$(a_b)_x = (32.2\ ft/s^2)\frac{15.403}{75}$, $(\underline{a}_b)_x = 6.61\ ft/s^2 \rightarrow$ ◀

$(a_b)_y = (32.2\ ft/s^2)\left(1 - \frac{15.403}{30}\right)$, $(\underline{a}_b)_y = 15.67\ ft/s^2 \downarrow$ ◀

Substituting for N in (3):

$a_P = (32.2\ ft/s^2)(0.5 + 0.030718 \times 15.403) = 31.335\ ft/s^2$

$\underline{a}_P = 31.3\ ft/s^2 \measuredangle 30°$ ◀

Note: Since $N > 0$, we check that contact between block and platform is maintained.

***16.27** From solution of Prob. 16.17:

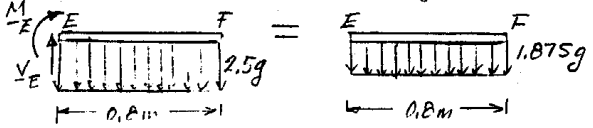
$a = g\cos\theta = g\cos 30^\circ = 0.8660g \nwarrow 60^\circ$

$a_y = a\sin 60^\circ = 0.8660g \sin 60^\circ \quad \underline{a}_y = 0.75g \downarrow$

Distributed mass per unit length $= m' = \frac{2\text{ kg}}{0.8\text{m}} = 2.5\text{ kg/m}$

Vert. comp. of effective forces $= m'a_y = 2.5(0.75g) = 1.875g$

Distributed weight $= w = m'g = 2.5g$



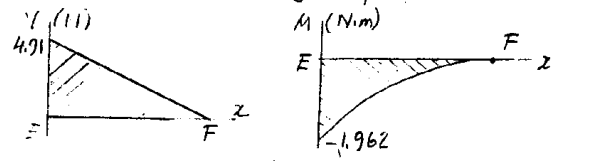
$+\sum F_y = \sum (F_y)_{eff}: \quad (2.5g)(0.8\text{m}) - V_E = (1.875g)(0.8\text{m})$

$V_E = 0.5g = 0.5(9.81) \qquad V_E = 4.91\text{ N}$ ◀

$+\circlearrowleft \sum M_E = \sum (M_E)_{eff}: \quad M_E + (2.5g)(0.8\text{m})(0.4\text{m}) = (1.875g)(0.8\text{m})(0.4\text{m})$

$M_E = -0.2g = -0.2(9.81) \qquad M_E = -1.962\text{ N·m}$ ◀

We observe that $V_E = M_E = 0$ and that V-diagram will be linear and M-diagram parabolic.



***16.28** From solution of Prob. 16.22:

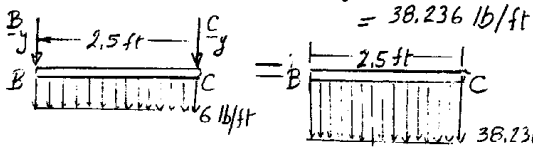
$\underline{a} = 236.7\text{ ft/s}^2 \searrow 60^\circ$

$a_y = 236.7\sin 60^\circ \qquad \underline{a}_y = 205.2\text{ ft/s}^2 \downarrow$

Distributed weight per unit length $= w = \frac{15\text{ lb}}{(30/12)\text{ ft}} = 6\text{ lb/ft}$

Distributed mass per unit length $= \frac{w}{g} = \frac{6}{g} = \frac{6}{32.2}$

Vert. comp. of effective forces $= \frac{w}{g}a_y = \frac{6}{32.2}(205.2) = 38.236\text{ lb/ft}$

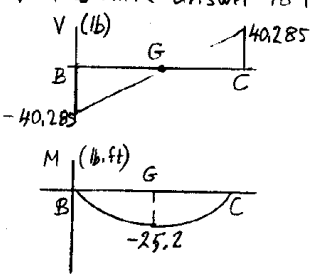


$+\downarrow \sum F_y = \sum (F_y)_{eff}: \quad B_y + C_y + (2.5\text{ ft})(6\text{ lb/ft}) = (2.5\text{ ft})(38.236\text{ lb/ft})$

$B_y + C_y = 80.59\text{ lb}$

From symmetry: $B_y = C_y = 40.285\text{ lb} \qquad \underline{B}_y = \underline{C}_y = 40.3\text{ lb} \downarrow$

We thus check answer to Prob. 16.22



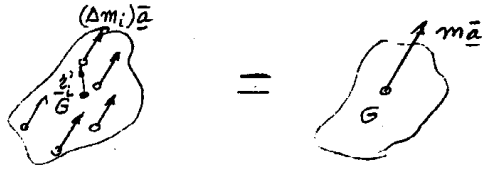
Max. value of BM occurs at G, where $V = 0$:

$|M|_{max} =$ Area under V-diagram from B to G $= \frac{1}{2}(40.285\text{ lb})(1.25\text{ ft})$

$|M|_{max} = 25.2\text{ lb·ft}$ ◀

$V_B = -40.3\text{ lb}$ ◀

16.29 Since slab is in translation, each particle has the same acceleration as G, namely, $\underline{a}$. The effective forces, therefore, consist of vectors $(\Delta m_i)\underline{a}$



The sum of these vectors is

$$\sum(\Delta m_i)\underline{a} = (\sum \Delta m_i)\underline{a}$$

or, since $\sum \Delta m_i = m$,

$$\sum(\Delta m_i)\underline{a} = m\underline{a}$$

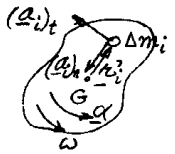
The sum of the moments about G of these vectors is

$$\sum \underline{r}_i' \times (\Delta m_i)\underline{a} = \left(\sum \Delta m_i \underline{r}_i'\right) \times \underline{a} \qquad (1)$$

But $\sum \Delta m_i \underline{r}_i' = m\bar{\underline{r}}' = 0$, because G is the mass center. It follows that the right-hand member of Eq. (1) is zero. Thus, the moment about G of the vector $m\underline{a}$ must also be zero, which means that its line of action passes through G and that it may be attached at G.

16.30 Since slab is in centroidal rotation, we have for any given particle

$\underline{a}_i = (\underline{a}_i)_t + (\underline{a}_i)_n = \underline{\alpha} \times \underline{r}_i' - \omega^2 \underline{r}_i'$



The effective forces, therefore, are

$(\Delta m_i)\underline{a}_i = (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - (\Delta m_i)\omega^2 \underline{r}_i'$

The sum of these vectors is

$\sum(\Delta m_i)\underline{a}_i = \sum(\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - \sum(\Delta m_i)\omega^2 \underline{r}_i'$

$= \underline{\alpha} \times \sum(\Delta m_i)\underline{r}_i' - \omega^2 \sum(\Delta m_i)\underline{r}_i'$

But $\sum(\Delta m_i)\underline{r}_i' = m\bar{\underline{r}}' = 0$, since G is the mass center.

Thus: $\sum(\Delta m_i)\underline{a}_i = 0$

which shows that the effective forces reduce to a couple. The moment of this couple is equal to the sum of the moments about G of the effective forces:

$\sum(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \sum[\underline{r}_i' \times (\Delta m_i)(\underline{\alpha} \times \underline{r}_i')] - \sum \underline{r}_i' \times (\Delta m_i)\omega^2 \underline{r}_i'$

But $\underline{r}_i' \times (\Delta m_i)\omega^2 \underline{r}_i' = \omega^2(\Delta m_i)(\underline{r}_i' \times \underline{r}_i') = 0$

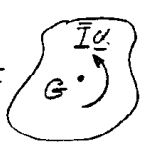
and $\underline{r}_i' \times (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') = (\Delta m_i) r_i'^2 \underline{\alpha}$

Thus: $\sum(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \sum(\Delta m_i) r_i'^2 \underline{\alpha}$

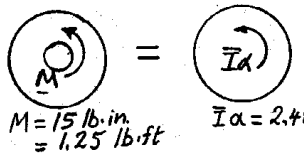
$= \left[\sum(\Delta m_i) r_i'^2\right] \underline{\alpha}$

Since $\sum(\Delta m_i) r_i'^2 = \bar{I}$

the moment of the couple is $\bar{I}\underline{\alpha}$



16.31 $\bar{I} = m\bar{k}^2 = \frac{180}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2 = 2.484\ \text{lb·ft·s}^2$



$M = 15\ \text{lb·in.} = 1.25\ \text{lb·ft}$ $\quad \bar{I}\alpha = 2.484\alpha$

$\Sigma M = \Sigma(M_{eff})$: $\quad 1.25 = 2.484\alpha$

$\alpha = 0.5032\ \text{rad/s}^2$

Uniformly accelerated rotation: $\omega_0 = 1200\text{ rpm} = 125.7\ \text{rad/s}$

$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad 0 = (125.7)^2 + 2(-0.5032)\theta$

$\theta = 15{,}700\ \text{rad}$

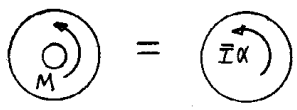
$\theta = 2500\ \text{rev}$ ◀

16.32 $\omega_0 = 3600\text{ rpm} = 376.99\ \text{rad/s}$

$t = 7.10\text{ min} = 7.10\ (60\text{s}) = 426\ \text{s}$

$\omega = \omega_0 + \alpha t \qquad 0 = 376.99 + \alpha(426)$

$\alpha = -0.88495\ \text{rad/s}^2$



$\bar{I} = m\bar{k}^2 = (1850\ \text{kg})(0.234\text{ m})^2 = 101.30\ \text{kg·m}^2$

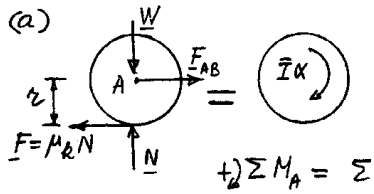
$\Sigma M = \Sigma(M)_{eff}$: $\quad M = \bar{I}\alpha = (101.30\ \text{kg·m}^2)(0.88495\ \text{rad/s}^2)$

$M = 89.6\ \text{N·m}$ ◀

16.33 $W = mg \qquad \bar{I} = \frac{1}{2}mr^2$

We note that link AB is a two-force member.

(a)



$+\uparrow \Sigma F_y = \Sigma(F)_{eff}$: $\quad N - W = 0$

$N = W$

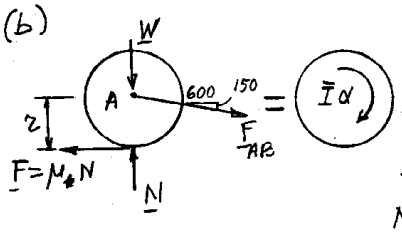
Thus: $F = \mu_k N = \mu_k W$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad Fr = \bar{I}\alpha$

$\alpha = \frac{Fr}{\bar{I}} = \frac{\mu_k W r}{\frac{1}{2}mr^2} = \frac{2\mu_k g}{r}$

$\alpha = \frac{2(0.30)(9.81)}{0.150\text{ m}} = 39.24 \qquad \underline{\alpha} = 39.2\ \text{rad/s}^2\ \circlearrowleft$ ◀

(b)



$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $\quad (F_{AB})_x - \mu_k N = 0$

$(F_{AB})_x = \mu_k N \qquad (1)$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad N - W - (F_{AB})_y = 0$

$(F_{AB})_y = N - W \qquad (2)$

Dividing (1) by (2):

$\frac{\mu_k N}{N-W} = \frac{(F_{AB})_x}{(F_{AB})_y} = \frac{600}{150} = 4$

$\mu_k N = 4N - 4W \qquad N = \frac{4W}{4-\mu_k} = \frac{4W}{4-0.30} = 1.081W$

$F = \mu_k N = 0.30(1.081W) = 0.3243W$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad Fr = \bar{I}\alpha$

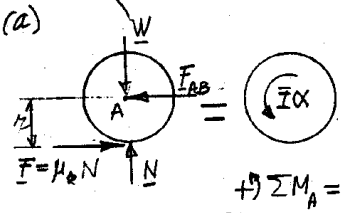
$\alpha = \frac{Fr}{\bar{I}} = \frac{0.3243Wr}{\frac{1}{2}mr^2} = \frac{0.6486\,g}{r} = \frac{0.6486(9.81\text{ m/s}^2)}{0.150\text{ m}}$

$\underline{\alpha} = 42.4\ \text{rad/s}^2\ \circlearrowleft$ ◀

16.34 $W = mg \qquad \bar{I} = \frac{1}{2}mr^2$

We note that link AB is a two-force member.

(a)



$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad N - W = 0$

$N = W$

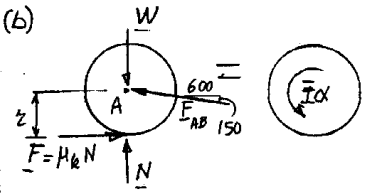
Thus: $F = \mu_k N = \mu_k W$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad Fr = \bar{I}\alpha$

$\alpha = \frac{Fr}{\bar{I}} = \frac{\mu_k W r}{\frac{1}{2}mr^2} = \frac{2\mu_k g}{r}$

$\alpha = \frac{2(0.30)(9.81)}{0.150\text{ m}} = 39.24 \qquad \underline{\alpha} = 39.2\ \text{rad/s}^2\ \circlearrowleft$ ◀

(b)



$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $\quad \mu_k N - (F_{AB})_x = 0$

$\mu_k N = (F_{AB})_x \qquad (1)$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad N - W + (F_{AB})_y = 0$

$W - N = (F_{AB})_y \qquad (2)$

Dividing (1) by (2): $\quad \frac{\mu_k N}{W-N} = \frac{(F_{AB})_x}{(F_{AB})_y} = \frac{600}{150} = 4$

$\mu_k N = 4W - 4N \qquad N = \frac{4}{4+\mu_k}W = \frac{4}{4+0.30}W = 0.9302\,W$

$F = \mu_k N = 0.30(0.9302\,W) = 0.2791\,W$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad Fr = \bar{I}\alpha$

$\alpha = \frac{Fr}{\bar{I}} = \frac{0.2791Wr}{\frac{1}{2}mr^2} = \frac{0.5582\,g}{r} = \frac{0.5582(9.81)}{0.150}$

$\underline{\alpha} = 36.5\ \text{rad/s}^2\ \circlearrowleft$ ◀

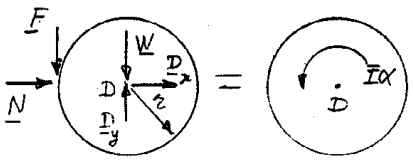
16.35 Kinematics: $\omega_0 = 240\text{ rpm}\circlearrowleft = 25.133\ \text{rad/s}\circlearrowleft$

$\theta = 60\text{ rev} = 60(2\pi) = 376.99\ \text{rad}$

$\omega^2 = \omega_0^2 + 2\alpha\theta$: $\quad 0 = (25.133)^2 + 2\alpha(376.99)$

$\alpha = -0.8378 \qquad \underline{\alpha} = 0.8378\ \text{rad/s}^2\ \circlearrowright$

Equation of motion for flywheel and drum



$r = \frac{10}{12}\text{ ft}$

$\bar{I} = 15\ \text{lb·ft·s}^2$

$\bar{I}\alpha = (15)(0.8378) = 12.567\ \text{lb·ft}$

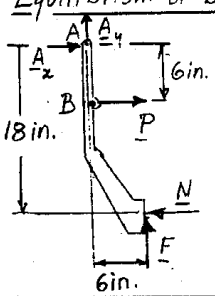
$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$: $\quad Fr = \bar{I}\alpha$

$F\left(\frac{10}{12}\text{ ft}\right) = 12.567\ \text{lb·ft}$

$F = 15.08\ \text{lb}$

But $F = \mu_k N \qquad N = \frac{F}{\mu_k} = \frac{15.08\text{ lb}}{0.40} = 37.70\ \text{lb}$

Equilibrium of brake



$+\circlearrowleft \Sigma M_A = 0$: $\quad 6P + 6F - 18N = 0$

$P = 3N - F$

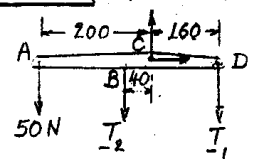
$= 3(37.70) - 15.08$

$= 98.02\ \text{lb}$

$P = 98.0\ \text{lb}$ ◀

16.36 Equilibrium of lever AD

(Dimensions in mm)

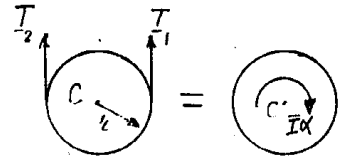


$+\circlearrowleft \Sigma M_C = 0$:

$(50\text{ N})(200) + T_2(40) - T_1(160) = 0$

$4T_1 - T_2 = 250\text{ N}$ (1)

Equations of motion for flywheel and drum



$\bar{I} = m\bar{k}^2$

$= (300\text{ kg})(0.600\text{ m})^2$

$= 108\text{ kg}\cdot\text{m}^2$

$r = 0.100\text{ m}$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $T_2 r - T_1 r = \bar{I}\alpha$

$(T_2 - T_1)(0.100\text{ m}) = 108\alpha$

$\alpha = (925.93\times10^{-6})(T_2 - T_1)$ (2)

Belt Friction

We recall Eq. (8.14), page 351 of Statics. Using μ_k instead of μ_s since brake band is slipping:

$\frac{T_2}{T_1} = e^{\mu_k\beta}$ or $T_2 = T_1 e^{\mu_k\beta}$ (3)

Making $\mu_k = 0.30$ and $\beta = 180^\circ = \pi$ rad in (3):

$T_2 = T_1 e^{0.30\pi}$ $T_2 = 2.5663\,T_1$ (4)

Substituting for T_2 from (4) into (1):

$4T_1 - 2.5663\,T_1 = 250\text{ N}$ $T_1 = 174.38\text{ N}$

From (1): $T_2 = 4(174.38) - 250$ $T_2 = 447.51\text{ N}$

Substituting for T_1 and T_2 into (2):

$\alpha = (925.93\times10^{-6})(447.51 - 174.38)$, $\underline{\alpha = 0.2529\text{ rad/s}^2}$

Kinematics

$\underline{\omega_0 = 180\text{ rpm}}$ $\omega_0 = +18.850\text{ rad/s}$

$\underline{\alpha = 0.2529\text{ rad/s}^2}$ $\alpha = -0.2529\text{ rad/s}^2$

$\omega = \omega_0 + \alpha t$: $0 = 18.850 - 0.2529\,t$

$t = 74.5\text{ s}$ ◀

16.37 Use solution of Prob. 16.36 up to dashed line. We recall the following equations:

$4T_1 - T_2 = 250\text{ N}$ (1)

$\alpha = (925.93\times10^{-6})(T_2 - T_1)$ (2)

$T_2 = T_1 e^{\mu_k\beta}$ (3)

Making $\mu_k = 0.40$ and $\beta = 180^\circ = \pi$ rad in Eq. (3):

$T_2 = T_1 e^{0.40\pi}$ $T_2 = 3.5136\,T_1$ (4)

Substituting for T_2 from (4) into (1):

$4T_1 - 3.5136\,T_1 = 250\text{ N}$ $T_1 = 514.0\text{ N}$

From (1): $T_2 = 4(514.0) - 250$ $T_2 = 1806\text{ N}$

Substituting for T_1 and T_2 into (2):

$\alpha = (925.93\times10^{-6})(1806 - 514)$ $\underline{\alpha = 1.1963\text{ rad/s}^2}$

(CONTINUED)

16.37 CONTINUED Kinematics:

$\underline{\omega_0 = 180\text{ rpm}}$, $\omega_0 = +18.850\text{ rad/s}$

$\underline{\alpha = 1.1963\text{ rad/s}^2}$, $\alpha = -1.1963\text{ rad/s}^2$

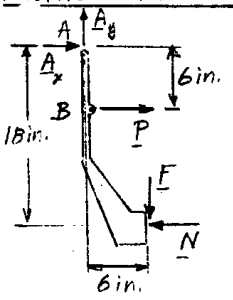
$\omega = \omega_0 + \alpha t$: $0 = 18.850 - 1.1963\,t$ $t = 15.76\text{ s}$ ◀

16.38 Kinematics. Same as for Prob. 16.35, except for senses of $\underline{\omega_0}$ and $\underline{\alpha}$. We now have

$\underline{\alpha = 0.8378\text{ rad/s}^2}$

Motion of flywheel and drum. Same as for Prob. 16.35, except that senses of $\bar{I}\alpha$ and F are reversed.

Equilibrium of brake



$+\circlearrowleft \Sigma M_A = 0$: $6P - 6F - 18N = 0$

$P = 3N + F$

$= 3(37.70) + 15.08$

$= 128.18\text{ lb}$

$P = 128.2\text{ lb}$ ◀

16.39 Kinematics: $\omega_0 = 360\text{ rpm} = 37.699\text{ rad/s}$

$\theta = 25\text{ rev} = 157.08\text{ rad}$

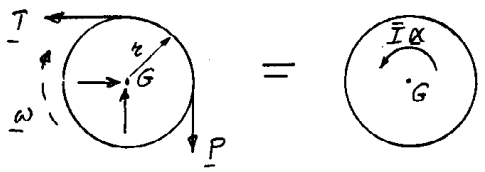
$\omega^2 = \omega_0^2 + 2\alpha\theta$ $0 = (37.699)^2 + 2\alpha(157.08)$

$\alpha = -4.524\text{ rad/s}^2$ (sense opposite to θ)

$\underline{\alpha = 4.524\text{ rad/s}^2}\ \circlearrowleft$

Equation of motion for flywheel

$r = 15\text{ in.} = 1.25\text{ ft}$



$\bar{I} = \frac{1}{2}mr^2$

$= \frac{1}{2}\frac{240}{32.2}(1.25)^2$

$= 5.823\text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $Tr - Pr = \bar{I}\alpha$

$T - P = \frac{\bar{I}\alpha}{r} = \frac{5.823(4.524)}{1.25}$

$T - P = 21.075$ (1)

Belt Friction: We recall Eq. (8.14), page 351 of Statics. Using μ_k instead of μ_s, since band brake is slipping, and noting that $T_1 = P$ and $T_2 = T$:

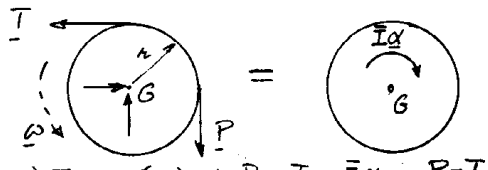
$\frac{T_2}{T_1} = e^{\mu_k\beta}$; $\frac{T}{P} = e^{0.35(\pi/2)}$ $T = 1.7329\,P$

Substituting into (1):

$1.7329\,P - P = 21.075$ $P = 28.8\text{ lb}$ ◀

16.40 Kinematics: Same as in Prob. 16.39, except that senses are reversed: $\underline{\alpha = 4.524\text{ rad/s}^2}\ \circlearrowright$

Equation of motion for flywheel.



See solution of Prob. 16.39 for numerical data

$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $Pr - Tr = \bar{I}\alpha$, $P - T = 21.075$ (1')

(CONTINUED)

16.40 CONTINUED

Eq. of motion yielded $P - T = 21.075$ (1')

Belt Friction: We use Eq. (8.14), page 351 of Statics. We note that, since ω is ↻, P pulls and T resists, that is, $T_1 = T$ and $T_2 = P$:

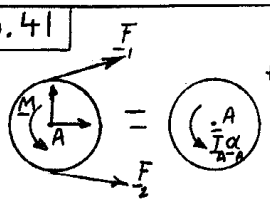
$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{P}{T} = e^{0.3\pi(\pi/2)} = 1.7329$$

$$T = \frac{P}{1.7329} = 0.5771\,P$$

Substituting for T into (1'):

$$P - 0.5771P = 21.075 \qquad P = 49.8\ \text{lb}$$ ◀

16.41

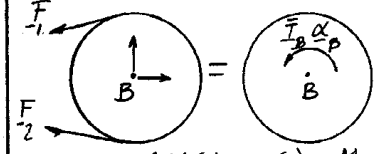


Disk A $\quad \bar{I}_A = \frac{1}{2} m_A r_A^2$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}:$$

$$M + F_2 r_A - F_1 r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\frac{M}{r_A} + F_2 - F_1 = \frac{1}{2} m_A r_A \alpha_A \quad (1)$$



Disk B $\quad \bar{I}_B = \frac{1}{2} m_B r_B^2$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}:$$

$$F_1 r_B - F_2 r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$F_1 - F_2 = \frac{1}{2} m_B r_B \alpha_B \quad (2)$$

Add (1) and (2): $\quad \frac{M}{r_A} = \frac{1}{2} m_A r_A \alpha_A + \frac{1}{2} m_B r_B \alpha_B$

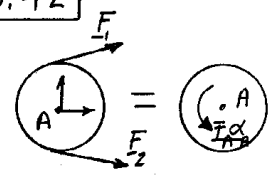
But $a_{belt} = r_A \alpha_A = r_B \alpha_B$. Thus: $\quad \frac{M}{r_A} = \frac{1}{2}(m_A r_A \alpha_A + m_B r_A \alpha_A)$

$$\frac{M}{r_A} = \frac{m_A + m_B}{2} r_A \alpha_A, \qquad \alpha_A = \frac{2M}{(m_A + m_B) r_A^2}$$

With given data: $\alpha_A = \dfrac{2(2.70\ \text{N·m})}{(2\ \text{kg} + 4\ \text{kg})(0.2\ \text{m})^2}$, $\underline{\alpha_A = 22.5\ \text{rad/s}^2}$ ◀

From $r_A \alpha_A = r_B \alpha_B$: $\alpha_B = \frac{r_A}{r_B}\alpha_A = \frac{0.2\ \text{m}}{0.5\ \text{m}}(22.5)$, $\underline{\alpha_B = 15.00\ \text{rad/s}^2}$ ↻ ◀

16.42

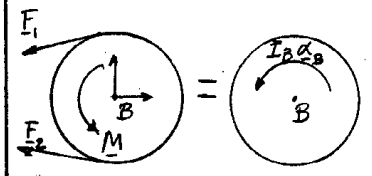


Disk A $\quad \bar{I}_A = \frac{1}{2} m_A r_A^2$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}:$$

$$F_2 r_A - F_1 r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$F_2 - F_1 = \frac{1}{2} m_A r_A \alpha_A \quad (1)$$



Disk B $\quad \bar{I}_B = \frac{1}{2} m_B r_B^2$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}:$$

$$M + F_1 r_B - F_2 r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\frac{M}{r_B} + F_1 - F_2 = \frac{1}{2} m_B r_B \alpha_B \quad (2)$$

Add (1) and (2): $\quad \frac{M}{r_B} = \frac{1}{2} m_A r_A \alpha_A + \frac{1}{2} m_B r_B \alpha_B$

But $a_{belt} = r_A \alpha_A = r_B \alpha_B$. Thus: $\quad \frac{M}{r_B} = \frac{1}{2}(m_A r_A \alpha_A + m_B r_A \alpha_A)$

$$\frac{M}{r_B} = \frac{m_A + m_B}{2} r_A \alpha_A, \qquad \alpha_A = \frac{2M}{(m_A + m_B) r_A r_B}$$

With given data: $\alpha_A = \dfrac{2(2.70\ \text{N·m})}{(2\ \text{kg} + 4\ \text{kg})(0.2\ \text{m})(0.3\ \text{m})} = 15.00\ \text{rad/s}^2$

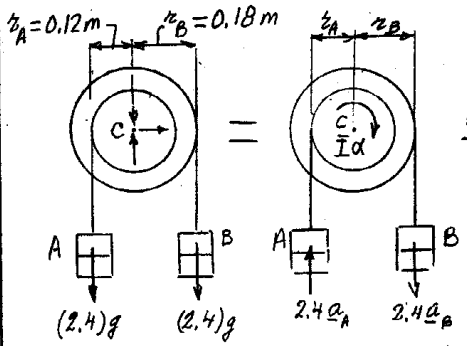
$\underline{\alpha_A = 15.00\ \text{rad/s}^2}$ ↻ ◀

From $r_A \alpha_A = r_B \alpha_B$: $\alpha_B = \frac{r_B}{r_B}\alpha_A = \frac{0.2\ \text{m}}{0.3\ \text{m}}(15.00\ \text{rad/s}^2)$

$\underline{\alpha_B = 10.00\ \text{rad/s}^2}$ ↻ ◀

16.43

Consider pulley and collars as a single system.



$\underline{a_A = r_A \alpha} \uparrow$

$\underline{a_B = r_B \alpha} \downarrow$

$\bar{I} = (6\ \text{kg})(0.135\ \text{m})^2 = 0.10935\ \text{kg·m}^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:

$$(2.4g) r_B - (2.4g) r_A = \bar{I}\alpha + (2.4 a_A) r_A + (2.4 a_B) r_B$$

$$(2.4g)(r_B - r_A) = 0.10935\alpha + 2.4 r_A^2 \alpha + 2.4 r_B^2 \alpha$$

$$(2.4)(9.81)(0.06) = [0.10935 + 2.4(0.12)^2 + 2.4(0.18)^2]\alpha$$

$$1.41264 = 0.22167\,\alpha$$

$$\alpha = 6.3727\ \text{rad/s}^2 \qquad \underline{\alpha = 6.37\ \text{rad/s}^2} \circlearrowright$$ ◀

(b) Kinematics

$$a_A = r_A \alpha = (0.12\ \text{m})(6.3727\ \text{rad/s}^2)$$

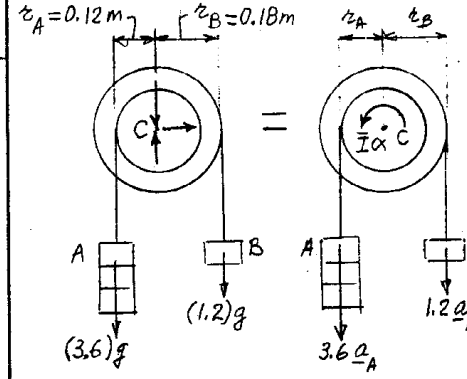
$$\underline{a_A = 0.76472\ \text{m/s}^2} \uparrow$$

$$v_A = v_0 + a_A t = 0 + (0.76472\ \text{m/s}^2)(2.5\ \text{s}) = 1.9118\ \text{m/s}$$

$\underline{v_A = 1.912\ \text{m/s}} \uparrow$ ◀

16.44

Consider pulley and collars as a single system.



$\underline{a_A = r_A \alpha} \downarrow$

$\underline{a_B = r_B \alpha} \uparrow$

$\bar{I} = (6\ \text{kg})(0.135\ \text{m})^2 = 0.10935\ \text{kg·m}^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:

$$(3.6g) r_A - (1.2g) r_B = \bar{I}\alpha + (3.6 a_A) r_A + (1.2 a_B) r_B$$

$$(3.6 r_A - 1.2 r_B) g = 0.10935\alpha + 3.6 r_A^2 \alpha + 1.2 r_B^2 \alpha$$

$$(3.6 \times 0.12 - 1.2 \times 0.18)(9.81) = [0.10935 + 3.6(0.12)^2 + 1.2(0.18)^2]\alpha$$

$$2.1190 = 0.20007\,\alpha$$

$$\alpha = 10.591\ \text{rad/s}^2 \qquad \underline{\alpha = 10.59\ \text{rad/s}^2} \circlearrowright$$ ◀

(b) Kinematics

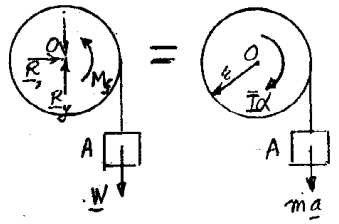
$$a_A = r_A \alpha = (0.12\ \text{m})(10.591\ \text{rad/s}^2)$$

$$\underline{a_A = 1.2709\ \text{m/s}^2} \downarrow$$

$$v_A = v_0 + at = 0 + (1.2709\ \text{m/s}^2)(2.5\ \text{s}) = 3.177\ \text{m/s}$$

$\underline{v_A = 3.18\ \text{m/s}} \downarrow$ ◀

16.45 Consider flywheel and block A as a single system.



$+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$:

$Wr - M_f = \bar{I}\alpha + (ma)r$

But $\alpha = \frac{a}{r}$ and $m = \frac{W}{g}$

Thus: $Wr - M_f = \bar{I}\frac{a}{r} + \frac{W}{g}ar$ (1)

With first block:

$y = \frac{1}{2}at^2$: $12\text{ ft} = \frac{1}{2}a_1(4.50\text{ s})^2$ $a_1 = 1.18519\text{ ft/s}^2$

Substituting for a_1 and making $W = 20$ lb, $r = 1.5$ ft in (1):

$(20)(1.5) - M_f = \bar{I}\frac{1.18519}{1.5} + \frac{20}{32.2}(1.18519)(1.5)$ (2)

With second block:

$y = \frac{1}{2}a_2t^2$: $12\text{ ft} = \frac{1}{2}a_2(2.80\text{ s})^2$ $a_2 = 3.06122\text{ ft/s}^2$

Substituting for a_2 and making $W = 40$ lb, $r = 1.5$ ft in (1):

$(40)(1.5) - M_f = \bar{I}\frac{3.06122}{1.5} + \frac{40}{32.2}(3.06122)(1.5)$ (3)

Subtracting (2) from (3):

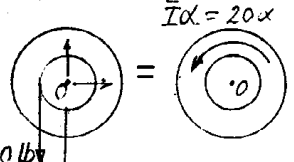
$20 - 30 = \bar{I}\frac{3.06122 - 1.18519}{1.5} + \frac{122.449 - 23.704}{32.2}(1.5)$

$1.2507\,\bar{I} = 30 - 4.5799$

$\bar{I} = 20.3\text{ lb·ft·s}^2$ ◀

16.46 For each of the double pulleys: $\bar{I}\alpha = 20\alpha$

Case 1



(a) $+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$:

$(200\text{ lb})\left(\frac{9}{12}\text{ ft}\right) = 20\alpha$

$\alpha = 7.5\text{ rad/s}^2\,\circlearrowleft$ ◀

(b) $\omega = \alpha t = 7.5(3\text{ s})$

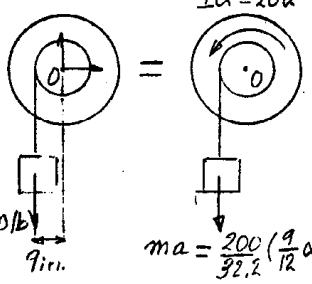
$\omega = 22.5\text{ rad/s}\,\circlearrowleft$ ◀

(c) $\theta = \frac{10\text{ ft}}{(9/12)\text{ ft}} = 13.333\text{ rad}$

$\omega^2 = 2\alpha\theta = 2(7.5)(13.333) = 200$

$\omega = 14.14\text{ rad/s}\,\circlearrowleft$ ◀

Case 2



(a) $+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$:

$(200)\left(\frac{9}{12}\right) = 20\alpha + ma\left(\frac{9}{12}\right)$

$= 20\alpha + \frac{200}{32.2}\left(\frac{9}{12}\right)^2\alpha$

$150 = (20 + 3.494)\alpha$

$\alpha = 6.3846\text{ rad/s}^2$

$\alpha = 6.38\text{ rad/s}^2\,\circlearrowleft$ ◀

(b) $\omega = \alpha t = (6.3846\text{ rad/s}^2)(3\text{ s})$ $\omega = 19.15\text{ rad/s}\,\circlearrowleft$ ◀

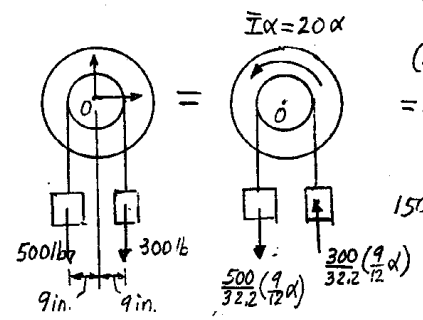
(c) As above: $\theta = 13.333$ rad

$\omega^2 = 2\alpha\theta = 2(6.3846)(13.333) = 170.26$

$\omega = 13.05\text{ rad/s}\,\circlearrowleft$ ◀

(CONTINUED)

16.46 CONTINUED Case 3



(a) $+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$:

$(500)\left(\frac{9}{12}\right) - (300)\left(\frac{9}{12}\right) = 20\alpha + \frac{500}{32.2}\left(\frac{9}{12}\right)^2\alpha + \frac{300}{32.2}\left(\frac{9}{12}\right)^2\alpha$

$150 = (20 + 8.734 + 5.241)\alpha$

$\alpha = 4.415\text{ rad/s}^2$

$\alpha = 4.41\text{ rad/s}^2\,\circlearrowleft$ ◀

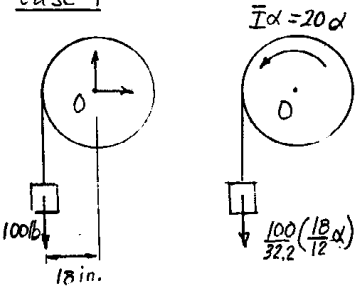
(b) $\omega = \alpha t = (4.415\text{ rad/s}^2)(3\text{ s})$ $\omega = 13.24\text{ rad/s}\,\circlearrowleft$ ◀

(c) As above: $\theta = 13.333$ rad

$\omega^2 = 2\alpha\theta = 2(4.415)(13.333) = 117.73$

$\omega = 10.85\text{ rad/s}\,\circlearrowleft$ ◀

Case 4



(a) $+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$:

$(100)\left(\frac{18}{12}\right) = 20\alpha + \frac{100}{32.2}\left(\frac{18}{12}\right)^2\alpha$

$150 = (20 + 6.988)\alpha$

$\alpha = 5.558\text{ rad/s}^2$

$\alpha = 5.56\text{ rad/s}^2\,\circlearrowleft$ ◀

(b) $\omega = \alpha t = (5.558)(3\text{ s})$

$\omega = 16.67\text{ rad/s}\,\circlearrowleft$ ◀

(c) $\theta = \frac{10\text{ ft}}{(18/12)\text{ ft}} = 6.6667$ rad

$\omega^2 = 2\alpha\theta = 2(5.558)(6.6667) = 74.11$

$\omega = 8.61\text{ rad/s}\,\circlearrowleft$ ◀

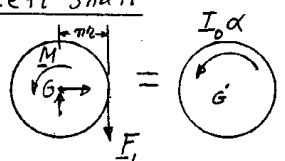
16.47 For a disk of thickness t, radius r, and density ρ, we have

$m = \rho(\pi r^2)t$ $I = \frac{1}{2}mr^2 = \frac{1}{2}\rho\pi r^4 t$

Thus, if $I = I_0$ for disk of radius nr, then $I = I_0/n^4$ for disk of radius r

Kinematics: If for left shaft, we have $\alpha\,\circlearrowleft$, then for middle shaft, we have $n\alpha\,\circlearrowright$, and for right shaft and coder, we have $n^2\alpha\,\circlearrowleft$.

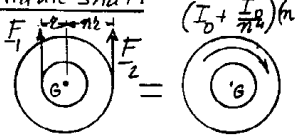
Left shaft



$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:

$M - F_1(nr) = I_0\alpha$ (1)

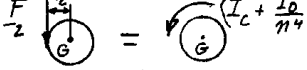
Middle shaft



$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:

$F_1 r - F_2(nr) = \left(I_0 + \frac{I_0}{n^4}\right)(n\alpha)$ (2)

Right-hand shaft and coder



$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:

$F_2 r = \left(I_c + \frac{I_0}{n^4}\right)n^2\alpha$ (3)

(CONTINUED)

16.47 CONTINUED We repeat Eqs. (1), (2), and (3), multiplying (2) by n and (3) by n^2:

$$M - nF_1 r = I_0\alpha \quad (1)$$

$$nF_1 r - n^2F_2 r = \left(I_0 + \frac{I_0}{n^4}\right)n^2\alpha \quad (2')$$

$$n^2F_2 r = \left(I_C + \frac{I_0}{n^4}\right)n^4\alpha \quad (3')$$

Adding (1), (2'), and (3'):

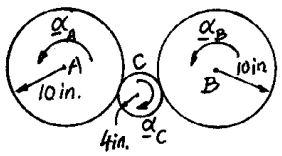
$$M = \left(I_0 + n^2I_0 + \frac{I_0}{n^2} + n^4I_C + I_0\right)\alpha$$

$$= \left[\left(n^2 + 2 + \frac{1}{n^2}\right)I_0 + n^4I_C\right]\alpha$$

$$M = \left[\left(n + \frac{1}{n}\right)^2 I_0 + n^4I_C\right]\alpha$$

$$I_R = \frac{M}{\alpha} = \left(n + \frac{1}{n}\right)^2 I_0 + n^4I_C$$

16.48 Kinematics

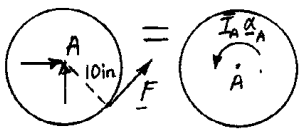


We express that the tangential components of the accelerations of the gear teeth are equal:

$$a_t = 10\alpha_A = 10\alpha_B = 4\alpha_C$$

$$\alpha_B = \alpha_A \qquad \alpha_C = 2.5\alpha_A \quad (1)$$

Kinetics



Gear A:

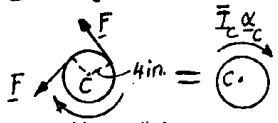
$$\bar{I}_A = m_A\bar{k}_A^2 = \frac{20}{32.2}\left(\frac{7.5}{12}\right)^2 = 0.24262$$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$$F\left(\frac{10}{12}\right) = 0.24262\,\alpha_A$$

$$F = 0.29115\,\alpha_A \quad (2)$$

Because of symmetry, gear C exerts an equal force F on gear B.



Gear C:

(a) $\bar{I}_C = m_C\bar{k}_C^2 = \frac{5}{32.2}\left(\frac{3}{12}\right)^2 = 0.009705$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$$M - 2Fr_C = \bar{I}_C\alpha_C$$

$$\frac{50}{12}\text{ lb·ft} - 2F\left(\frac{4}{12}\text{ ft}\right) = 0.009705\,\alpha_C$$

Substituting for α_C from (1) and for F from (2):

$$4.1667 - 2(0.29115\alpha_A)\left(\frac{1}{3}\right) = 0.009705(2.50\alpha_A)$$

$$4.1667 - 0.19410\,\alpha_A = 0.02426\,\alpha_A$$

$$4.1667 = 0.21836\,\alpha_A$$

$$\alpha_A = 19.081\text{ rad/s}^2$$

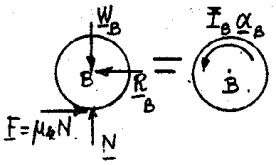
$$\underline{\alpha_A = 19.08\text{ rad/s}^2}\ \circlearrowleft$$

(b) Substituting for α_A into (2):

$$F = 0.29115(19.081)$$

$$\underline{F = 5.56\text{ lb}}\ \nearrow$$

16.49 (a) Disk B



$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$$N - W_B = 0$$

$$N = W_B = m_Bg$$

Thus: $F = \mu_k N = \mu_k m_B g$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$$Fr_B = \bar{I}_B\alpha_B$$

$$\mu_k m_B g r_B = \frac{1}{2}m_B r_B^2\alpha_B$$

$$\alpha_B = \frac{2\mu_k g}{r_B}\ \circlearrowleft \quad (1)$$

For given data:

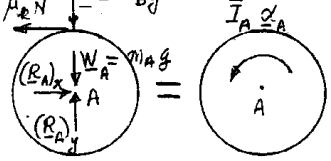
$$\alpha_B = \frac{2(0.35)(9.81\text{ m/s}^2)}{0.180\text{ m}} = 38.15\text{ rad/s}^2, \quad \underline{\alpha_B = 38.2\text{ rad/s}^2}\ \circlearrowleft$$

$$W_B = m_Bg = (1.6\text{ kg})(9.81\text{ m/s}^2) = 15.696\text{ N}$$

$$F = \mu_k m_B g = 0.35(15.696) = 5.4936\text{ N}$$

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $F - R_B = 0$ $\quad \underline{R_B = 5.4936\text{ N}}\ \leftarrow$

Disk A



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$$Fr_A = \bar{I}_A\alpha_A$$

$$\mu_k m_B g r_A = \frac{1}{2}m_A r_A^2\alpha_A$$

$$\alpha_A = \frac{2\mu_k r_B}{r_A}\frac{m_B}{m_A}\ \circlearrowleft \quad (2)$$

For given data: $\alpha_A = \frac{2(0.35)(9.81\text{ m/s}^2)}{0.300\text{ m}}\frac{1.6\text{ kg}}{4\text{ kg}} = 9.156\text{ rad/s}^2$

$$\underline{\alpha_A = 9.16\text{ rad/s}^2}\ \circlearrowleft$$

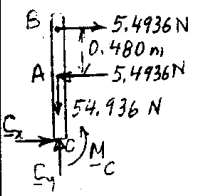
$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $(R_A)_x - F = 0$ $\quad (R_A)_x = 5.4936\text{ N}\ \rightarrow$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $(R_A)_y - m_Ag - m_Bg = 0$

$$(R_A)_y = (m_A + m_B)g = (4\text{ kg} + 1.6\text{ kg})(9.81\text{ m/s}^2)$$

$$= 54.936\text{ N} \qquad \underline{(R_A)_y = 54.936\text{ N}}\ \uparrow$$

(b) Reaction at C



$\pm\rightarrow \Sigma F_x = 0$: $C_x = 0$

$+\uparrow \Sigma F_y = 0$: $C_y - 54.936\text{ N} = 0$

$C_y = 54.936\text{ N}$ $\quad \underline{C = 54.9\text{ N}}\ \uparrow$

$+\circlearrowleft \Sigma M_C = 0$: $M_C - (5.4936\text{ N})(0.480\text{ m}) = 0$

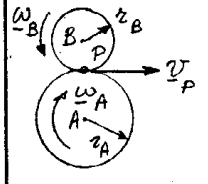
$$\underline{M_C = 2.64\text{ N·m}}\ \circlearrowleft$$

16.50 See solution of Prob. 16.49 for derivation of Eqs. (1) and (2) defining α_B and α_A.

Disk A: $\omega_A = \omega_0 - \alpha_A t = \omega_0 - \frac{2\mu_k g}{r_A}\frac{m_B}{m_A}t\ \circlearrowleft$ (3)

Disk B: $\omega_B = \alpha_B t = \frac{2\mu_k g}{r_B}t\ \circlearrowleft$ (4)

When disks have stopped slipping:



$$v_P = \omega_A r_A = \omega_B r_B$$

$$\omega_0 r_A - (2\mu_k g)\frac{m_B}{m_A}t = 2\mu_k g t$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{1}{1 + \frac{m_B}{m_A}}$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A + m_B} \quad (5)$$

(CONTINUED)

16.50 CONTINUED Substituting for t from (5) into (3) and (4):

$$\omega_A = \omega_0 - \frac{2\mu_k g}{r_A}\frac{m_B}{m_A}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A+m_B} = \omega_0 - \frac{\omega_0 m_B}{m_A+m_B}$$

$$\omega_A = \omega_0\frac{m_A+m_B-m_B}{m_A+m_B} \qquad \underline{\omega_A = \omega_0\frac{m_A}{m_A+m_B}} \quad (6)$$

$$\omega_B = \frac{2\mu_k g}{r_B}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A+m_B} \qquad \underline{\omega_B = \omega_0\frac{r_A}{r_B}\frac{m_A}{m_A+m_B}} \quad (7)$$

(a) From (6) and (7), it is apparent that ω_A and ω_B are independent of μ_k. However if $\mu_k = 0$, we have from (3) and (4) $\underline{\omega_A = \omega_0}$ and $\underline{\omega_B = 0}$.

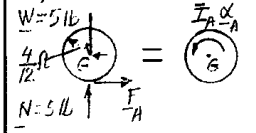
(b) We can write (6) in the form $\underline{\omega_A = \omega_0/(1+m_B/m_A)}$ ◀

which shows that ω_A depends only upon ω_0 and m_B/m_A

16.51 Assume that no slipping occurs.

Then: $a_{belt} = (4\text{in.})\alpha_A = (8\text{in.})\alpha_B \qquad \alpha_B = \frac{1}{2}\alpha_A$ (1)

Cylinder A

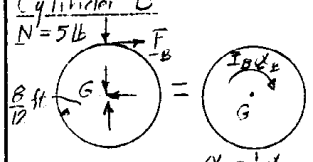


$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}: \quad F_A\left(\tfrac{4}{12}\text{ ft}\right) = \bar{I}_A\alpha_A$$

$$F_A\left(\tfrac{4}{12}\right) = \tfrac{1}{2}\tfrac{5}{g}\left(\tfrac{4}{12}\right)^2\alpha_A$$

$$\underline{F_A = \frac{5}{6}\frac{\alpha_A}{g}} \quad (2)$$

Cylinder B



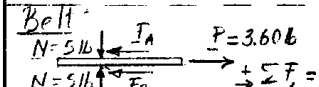
$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}:$$

$$F_B\left(\tfrac{8}{12}\text{ ft}\right) = \bar{I}_B\alpha_B$$

$$F_B\left(\tfrac{8}{12}\right) = \tfrac{1}{2}\tfrac{20}{g}\left(\tfrac{8}{12}\right)^2\left(\tfrac{1}{2}\alpha_A\right)$$

$$\underline{F_B = \frac{20}{6}\frac{\alpha_A}{g}} \quad (3)$$

Belt



$$\rightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0 \quad (4)$$

$$3.60 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$

$$\alpha_A = \frac{(3.60)6}{25}g = 0.864g \qquad \underline{\alpha_A = 27.82\text{ rad/s}^2\circlearrowright}$$

Check that belt does not slip:

From (2): $F_A = \frac{5}{6}(0.864) = 0.720$ lb

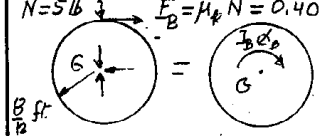
From (4): $F_B = P - F_A = 3.60 - 0.720 = 2.88$ lb

But $F_m = \mu_s N = 0.50(5\text{ lb}) = 2.50$ lb

Since $F_B > F_m$, assumption is wrong;

Slipping occurs between disk B and belt ◀

We redo analysis of B, assuming slipping: ($\alpha_B \neq \frac{1}{2}\alpha_A$)



$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}:$$

$$(2\text{lb})\left(\tfrac{8}{12}\text{ ft}\right) = \tfrac{1}{2}\tfrac{20}{g}\left(\tfrac{8}{12}\right)^2\alpha_B$$

$$\alpha_B = 0.3g, \qquad \underline{\alpha_B = 9.66\text{ rad/s}^2\circlearrowright} \quad ◀$$

Belt Eq (4): $P - F_A - F_B = 0 \qquad F_A = P - F_B = 3.60 - 2 = 1.60$ lb

Since $F_A < F_m$, there is no slipping between A and belt ◀

Our analysis of Disk A, therefore is valid. Using Eq. (2), we have

$$1.60\text{ lb} = \frac{5}{6}\frac{\alpha_A}{g}, \qquad \alpha_A = 1.92g \qquad \underline{\alpha_A = 61.8\text{ rad/s}^2\circlearrowright} \quad ◀$$

16.52 Assume that no slipping occurs

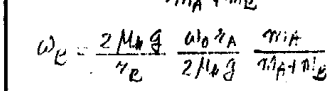
See solution of Prob. 16.51 up to the dashed line. We obtain

$$\alpha_B = \tfrac{1}{2}\alpha_A \quad (1)$$

$$F_A = \frac{5}{6}\frac{\alpha_A}{g} \quad (2)$$

$$F_B = \frac{20}{6}\frac{\alpha_A}{g} \quad (3)$$

Belt



$$\rightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$2.00 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$

$$\alpha_A = \frac{(2.00)6}{25}g = 0.480g \qquad \alpha_A = 15.46\text{ rad/s}^2\circlearrowright$$

From (2): $F_A = \frac{5}{6}(0.480) = 0.400$ lb

From (3): $F_B = \frac{20}{6}(0.480) = 1.600$ lb

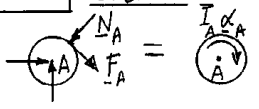
But $F_m = \mu_s N = 0.50(5\text{ lb}) = 2.50$ lb

Thus, F_A and F_B are both $< F_m$. Our assumption was right:

There is no slipping between disks and belt ◀

$\underline{\alpha_A = 15.46\text{ rad/s}^2\circlearrowright}$ From (1): $\underline{\alpha_B = 7.73\text{ rad/s}^2\circlearrowright}$ ◀

16.53 Caster A

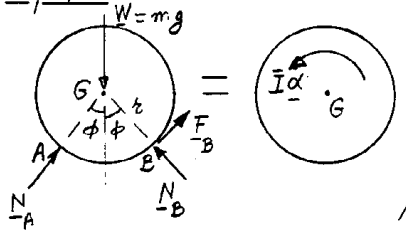


$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}:$$

$$F_A r_A = \bar{I}_A\alpha_A$$

Since $\bar{I}_A \approx 0$: $\quad F_A \approx 0$

Cylinder



Since roller B slips on disk:

$$F_B = \mu_k N_B$$

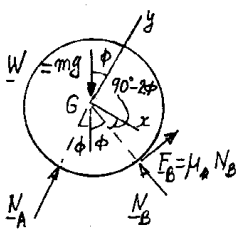
$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}:$$

$$F_B r = \bar{I}\alpha$$

$$\mu_k N_B r = \tfrac{1}{2}mr^2\alpha$$

$$\underline{\alpha = \frac{2\mu_k N_B}{mr}} \quad (1)$$

Selecting y axis along AG ∀ φ and x axis ◁ φ:



$$\nearrow \Sigma F_x = \Sigma(F_x)_{eff}:$$

$$W\sin\phi + F_B\sin(90°-2\phi) - N_B\cos(90°-2\phi) = 0$$

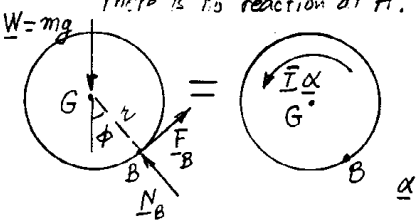
$$mg\sin\phi + \mu_k N_B\cos 2\phi - N_B\sin 2\phi = 0$$

$$\underline{N_B = \frac{mg\sin\phi}{\sin 2\phi - \mu_k\cos 2\phi}} \quad (2)$$

Substituting for N_B from (2) into (1):

$$\alpha = \frac{2\mu_k mg\sin\phi}{mr(\sin 2\phi - \mu_k\cos 2\phi)} \qquad \underline{\alpha = \frac{2\mu_k g\sin\phi}{r(\sin 2\phi - \mu_k\cos 2\phi)}\circlearrowright} \quad ◀$$

16.54 When contact is to be lost with caster at B, there is no reaction at B.



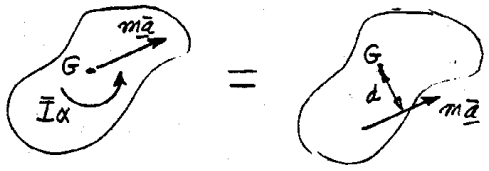
$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}:$$

$$Wr\sin\phi = \bar{I}\alpha$$

$$mgr\sin\phi = \tfrac{1}{2}mr^2\alpha$$

$$\underline{\alpha = \frac{2g}{r}\sin\phi\circlearrowright} \quad ◀$$

16.55 We know that the system of the effective forces may be reduced to the vector $m\bar{a}$ at G and the couple $\bar{I}\alpha$. We further know from Chap. 3 of Statics that a force-couple system in a plane may be further reduced to a single force:



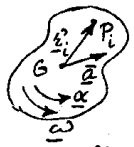
The perpendicular distance d from G to the line of action of the single vector $m\bar{a}$ is obtained by writing

$$+\circlearrowleft \Sigma M_G = \Sigma M_G: \quad \bar{I}\alpha = (m\bar{a})d$$

$$d = \frac{\bar{I}\alpha}{m\bar{a}} = \frac{m\bar{k}^2\alpha}{m\bar{a}} \qquad d = \frac{\bar{k}^2\alpha}{\bar{a}}$$

16.56 Kinematics. The acceleration of point P_i is

$$\underline{a}_i = \underline{\bar{a}} + \underline{a}_{P_i/G}$$

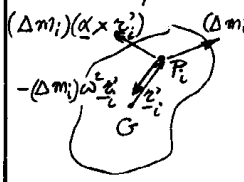


$$\underline{a}_i = \underline{\bar{a}} + \underline{\alpha} \times \underline{r}_i' + \underline{\omega} \times (\underline{\omega} \times \underline{r}_i')$$

$$= \underline{\bar{a}} + \underline{\alpha} \times \underline{r}_i' - \omega^2 \underline{r}_i'$$

Note that $\underline{\alpha} \times \underline{r}_i'$ is perpendicular to $\underline{r}_i'$.

Thus, the effective forces are as shown in Fig. P16.56 and as repeated here:



$$(\Delta m_i)\underline{a}_i = (\Delta m_i)\underline{\bar{a}} + (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - (\Delta m_i)\omega^2 \underline{r}_i'$$

The sum of the effective forces is

$$\Sigma(\Delta m_i)\underline{a}_i = \Sigma(\Delta m_i)\underline{\bar{a}} + \Sigma(\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - \Sigma(\Delta m_i)\omega^2 \underline{r}_i'$$

$$= \underline{\bar{a}} \Sigma(\Delta m_i) + \underline{\alpha} \times \Sigma(\Delta m_i)\underline{r}_i' - \omega^2 \Sigma(\Delta m_i)\underline{r}_i'$$

But $\Sigma(\Delta m_i) = m$ = total mass and, since G is the mass center, we have $\Sigma(\Delta m_i)\underline{r}_i' = m\underline{\bar{r}}' = 0$. Thus

$$\Sigma(\Delta m_i)\underline{a}_i = m\underline{\bar{a}} \qquad (1)$$

The sum of the moments about G of the effective forces is

$$\Sigma(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \Sigma \underline{r}_i' \times \Delta m_i \underline{\bar{a}} + \Sigma \underline{r}_i' \times (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - \Sigma \underline{r}_i' \times (\Delta m_i)\omega^2 \underline{r}_i'$$

$$= \left(\Sigma \underline{r}_i' \Delta m_i\right) \times \underline{\bar{a}} + \Sigma\left[\underline{r}_i' \times (\underline{\alpha} \times \underline{r}_i')\Delta m_i\right] - \omega^2 \Sigma(\underline{r}_i' \times \underline{r}_i')\Delta m_i$$

Again, since G is the mass center, $\Sigma \underline{r}_i' \Delta m_i = 0$. On the other hand, for each particle, $\underline{r}_i' \times \underline{r}_i' = 0$. The expression obtained reduces to the middle term. But, since $\underline{\alpha} \perp \underline{r}_i'$, we have $\underline{r}_i' \times (\underline{\alpha} \times \underline{r}_i') = r_i'^2 \underline{\alpha}$ and

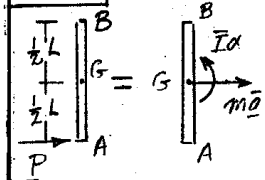
$$\Sigma(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \Sigma r_i'^2 (\Delta m_i)\underline{\alpha} = (\Sigma r_i'^2 \Delta m_i)\underline{\alpha}$$

Since $\Sigma r_i'^2 \Delta m_i$ represents the moment of inertia $\bar{I}$ about G,

$$\Sigma(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \bar{I}\underline{\alpha} \qquad (2)$$

From (1) and (2) we conclude that the system of the effective forces reduces to a vector $m\bar{a}$ attached at G and a couple $\bar{I}\underline{\alpha}$.

16.57



(a) Angular acceleration

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}: \quad P\frac{L}{2} = \bar{I}\alpha$$

$$P\frac{L}{2} = \frac{1}{12} m L^2 \alpha$$

$$\alpha = \frac{6P}{mL} = \frac{6(2.70\ \text{N})}{(1.50\ \text{kg})(0.9\text{m})}$$

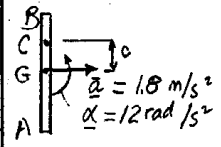
$$\underline{\alpha = 12.00\ \text{rad/s}^2\ \circlearrowleft}$$

(b) Acceleration of G

$$\pm \Sigma F_x = \Sigma(F_x)_{eff}: \quad P = m\bar{a} \qquad \bar{a} = \frac{P}{m} = \frac{2.70\ \text{N}}{1.50\ \text{kg}}$$

$$\underline{\bar{a} = 1.800\ \text{m/s}^2 \rightarrow}$$

(c) Point C with no acceleration.



$$a_C = \bar{a} - c\alpha$$

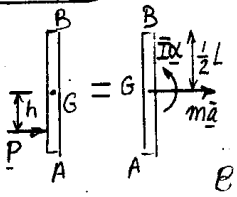
$$0 = 1.8 - c(12)$$

$$c = 0.15\text{m} = 150\text{mm}$$

above G

or: 300 mm from B

16.58



$$(a) \pm \Sigma F_x = \Sigma(F_x)_{eff}: \quad P = m\bar{a}$$

$$\bar{a} = P/m \qquad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}: \quad Ph = \bar{I}\alpha$$

$$Ph = \frac{1}{12} m L^2 \alpha$$

$$\alpha = \frac{12Ph}{mL^2} \qquad (2)$$

But $a_B = \bar{a} - \frac{1}{2}L\alpha = 0 \qquad (3)$

Substituting from (1) and (2) into (3):

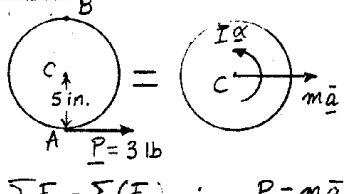
$$\frac{P}{m} - \frac{1}{2}\frac{12\,PhL}{mL^2} = 0, \quad h = \frac{1}{6}L = \frac{1}{6}(900\text{mm}) = 150\text{ mm}$$

or 300 mm from A

(b) From (2): $\alpha = \frac{12(2.70\text{N})(0.150\text{m})}{(1.50\text{kg})(0.9\text{m})^2}$, $\underline{\alpha = 4.00\ \text{rad/s}^2\ \circlearrowleft}$

From (1): $\bar{a} = 2.70\text{N}/1.50\text{kg}$, $\underline{\bar{a} = 1.800\ \text{m/s}^2 \rightarrow}$

16.59 Point C is mass center. $m = \frac{15}{g}$



$$\bar{I} = \frac{1}{2} m r^2 = \frac{1}{2}\frac{15}{g}\left(\frac{5}{12}\right)^2$$

$$\pm \Sigma F_x = \Sigma(F_x)_{eff}: \quad P = m\bar{a}$$

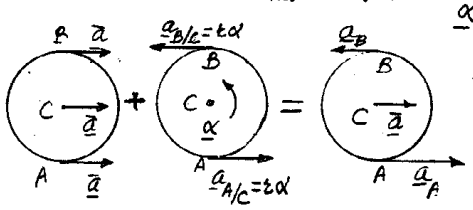
$$3 = \frac{15}{g}\bar{a} \qquad \underline{\bar{a} = 0.2g \rightarrow}$$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}: \quad Pr = \bar{I}\alpha$$

$$3\left(\frac{5}{12}\right) = \frac{1}{2}\frac{15}{g}\left(\frac{5}{12}\right)^2 \alpha$$

$$\underline{\alpha = 0.96g\ \circlearrowleft}$$

Kinematics:



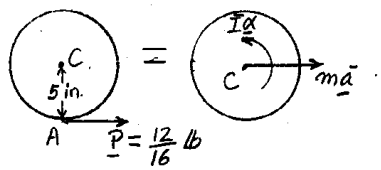
(a) $a_A = \bar{a} + r\alpha = 0.2g + \frac{5}{12}(0.96g) = (0.2 + 0.4)g = 0.6g$

$$\underline{a_A = 19.32\ \text{ft/s}^2 \rightarrow}$$

(b) $a_B = \bar{a} - r\alpha = 0.2g - 0.4g = -0.2g$ $\underline{a_B = 6.44\ \text{ft/s}^2 \leftarrow}$

16.60

Point C is mass center. $m = \frac{15}{g}$



$\bar{I} = \frac{1}{2} m r^2 = \frac{1}{2}\frac{15}{g}\left(\frac{5}{12}\right)^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $Pr = \bar{I}\alpha$

$\frac{12}{16}\left(\frac{5}{12}\right) = \frac{1}{2}\frac{15}{g}\left(\frac{5}{12}\right)^2 \alpha$

$\alpha = \frac{12}{16}\frac{12}{5}\frac{2}{15} g = 0.24g$, $\underline{\alpha} = 7.73 \text{ rad/s}^2 \circlearrowleft$ ◀

(b) $\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $P = m\bar{a}$ $\quad \frac{12}{16} = \frac{15}{g}\bar{a}$

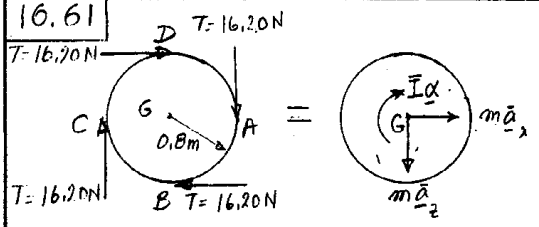
$\bar{a} = \frac{12}{16(15)} g = 0.05g$ $\quad \underline{\bar{a}} = 1.610 \text{ ft/s}^2 \rightarrow$ ◀

(c) Accel. of tape relative to C $= a_{T/C} = r\alpha$

Length of tape unwound $= \frac{1}{2} a_{T/C} t^2 = \frac{1}{2} r\alpha t^2$

$= \frac{1}{2}(5 \text{ in.})(0.24g \text{ rad/s}^2)(2\text{s})^2 = 2.4g = 77.3 \text{ in.}$ ◀

16.61



$m = 120 \text{ kg}$, $\bar{I} = m k_y^2 = (120 \text{ kg})(0.6 \text{ m})^2 = 43.2 \text{ kg·m}^2$

(a) With all 4 rockets fired:

$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $0 = m\underline{\bar{a}}$ $\quad \underline{\bar{a}} = 0$ ◀

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $4Tr = \bar{I}\alpha$

$4(16.20 \text{ N})(0.8 \text{ m}) = 43.2\alpha$ $\quad \underline{\alpha} = 1.200 \text{ rad/s}^2 \circlearrowleft$

$\underline{\alpha} = -(1.200 \text{ rad/s}^2)\underline{j}$ ◀

(b) With all rockets except D:

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $-16.20 \text{ N} = 120\,\bar{a}_x$ $\quad \bar{a}_x = -0.1350 \text{ m/s}^2$

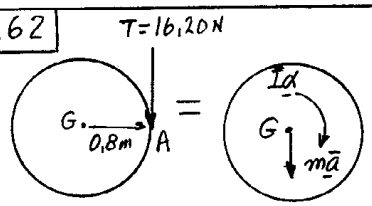
$+\downarrow \Sigma F_z = \Sigma (F_z)_{eff}$: $0 = 120\,\bar{a}_z$ $\quad \bar{a}_z = 0$

$\underline{\bar{a}} = -(0.1350 \text{ m/s}^2)\underline{i}$ ◀

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $3Tr = \bar{I}\alpha$

$3(16.20 \text{ N})(0.8 \text{ m}) = 43.2\alpha$ $\quad \underline{\alpha} = -(0.900 \text{ rad/s}^2)\underline{j}$ ◀

16.62



$m = 120 \text{ kg}$

$\bar{I} = m k_y^2 = (120 \text{ kg})(0.6 \text{ m})^2 = 43.2 \text{ kg·m}^2$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $Tr = \bar{I}\alpha$

$(16.20 \text{ N})(0.8 \text{ m}) = 43.2\alpha$ $\quad \underline{\alpha} = 0.300 \text{ rad/s}^2 \circlearrowleft$

$\underline{\alpha} = -(0.300 \text{ rad/s}^2)\underline{j}$ ◀

$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $(16.20 \text{ N})\underline{k} = 120\,\underline{\bar{a}}$

$\underline{\bar{a}} = (0.1350 \text{ m/s}^2)\underline{k}$ ◀

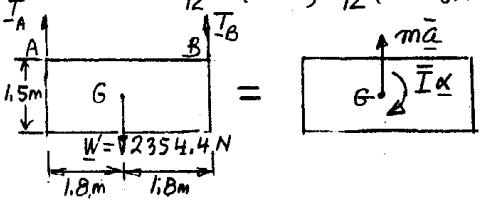
16.63

Kinetics: $m = 240 \text{ kg}$

$W = mg = 240(9.81) = 2354.4 \text{ N}$

$\bar{I} = \frac{1}{12} m(b^2 + c^2) = \frac{1}{12}(240 \text{ kg})(3.6^2 + 1.5^2)$

$= 304.2 \text{ kg·m}^2$



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $T_A + T_B - W = m\bar{a}$

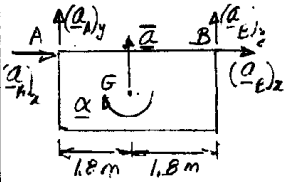
$T_A + T_B = 2354.4 + 240\,\bar{a}$ $\quad (1)$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $T_A(1.8\text{m}) - T_B(1.8\text{m}) = \bar{I}\alpha$

$T_A(1.8) - T_B(1.8) = 304.2\,\alpha$

$T_A - T_B = 169.0\,\alpha$ $\quad (2)$

Kinematics:



$(a_A)_y = \bar{a} + 1.8\alpha$ $\quad (3)$

$(a_B)_y = \bar{a} - 1.8\alpha$ $\quad (4)$

Given: $(a_A)_y = 7 \text{ m/s}^2$

$(a_B)_y = 1 \text{ m/s}^2$

Substitute in (3) and (4): $\bar{a} + 1.8\alpha = 7$ $\quad (3')$

$\bar{a} - 1.8\alpha = 1$ $\quad (4')$

Adding (3') and (4'): $2\bar{a} = 8$ $\quad \underline{\bar{a}} = 4.00 \text{ m/s}^2 \uparrow$

Subtracting (4') from (3'): $3.6\alpha = 6$, $\underline{\alpha} = 1.667 \text{ rad/s}^2 \circlearrowleft$

Carrying the values found for $\bar{a}$ and α into (1) and (2):

$T_A + T_B = 2354.4 + 240(4) = 3314.4$ $\quad (1')$

$T_A - T_B = 169.0(1.667) = 281.67$ $\quad (2')$

Adding (1') and (2'): $2T_A = 3596.1$ $\quad T_A = 1798 \text{ N}$ ◀

Subtracting (2') from (1'): $2T_B = 3032.7$ $\quad T_B = 1516 \text{ N}$ ◀

16.64

See solution of Prob. 16.63 for derivation of Eqs. (1), (2), (3), and (4).

Given: $T_A = 1900 \text{ N}$, $T_B = 1600 \text{ N}$

Substitute in (1) and (2):

$1900 \text{ N} + 1600 \text{ N} = 2354.4 \text{ N} + 240\,\bar{a}$

$\underline{\bar{a}} = 4.773 \text{ m/s}^2 \uparrow$

$1900 \text{ N} - 1600 \text{ N} = 169.0\,\alpha$

$\underline{\alpha} = 1.7751 \text{ rad/s}^2 \circlearrowleft$

Carrying the values found for $\bar{a}$ and α into (3) and (4):

$(a_A)_y = 4.773 + 1.8(1.7751) = 7.97 \text{ m/s}^2$

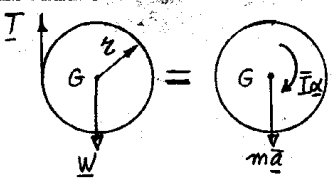
$(a_B)_y = 4.773 - 1.8(1.7751) = 1.578 \text{ m/s}^2$

For cables:

$\underline{a}_A = 7.97 \text{ m/s}^2 \uparrow$ ◀

$\underline{a}_B = 1.578 \text{ m/s}^2 \uparrow$ ◀

16.65 (a) $m = 15$ kg; $r = 0.5$ m, $\bar{a} = 0.8$ m/s² ↓



$W = (15)(9.81) = 147.15$ N

$m\bar{a} = (15)(0.8) = 12.00$ N

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$W - T = m\bar{a}$

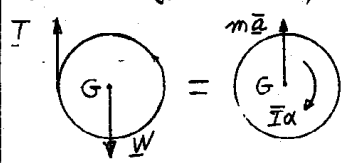
$147.15 - T = 12.00$

$T = 135.15$ N $T = 135.2$ N ◀

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $Tr = \bar{I}\alpha$ $Tr = \frac{1}{2} mr^2 \alpha$

$\alpha = \frac{2T}{mr} = \frac{2(135.15)}{15(0.5)} = 36.04$ $\alpha = 36.0$ rad/s² ↻ ◀

(b) $m = 15$ kg, $r = 0.5$ m, $\bar{a} = 0.8$ m/s² ↑



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$T - W = m\bar{a}$

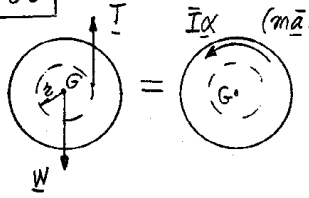
$T - 147.15 = 12.00$

$T = 159.15$ N, $T = 159.2$ N ◀

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $Tr = \bar{I}\alpha$ $Tr = \frac{1}{2} mr^2 \alpha$

$\alpha = \frac{2T}{mr} = \frac{2(159.15)}{15(0.5)} = 42.44$ $\alpha = 42.4$ rad/s² ↻ ◀

16.66



$(m\bar{a} = 0)$

(a) $+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$T - W = 0$

$T = W$ ◀

(b) $+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:

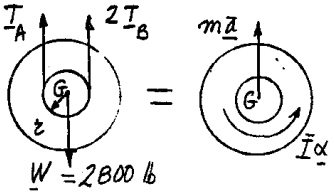
$Tr = \bar{I}\alpha$

$Wr = \frac{W}{g} \bar{k}^2 \alpha$

$\alpha = rg/\bar{k}^2$ ↻ ◀

16.67 $m = \frac{2800 \text{ lb}}{32.2 \text{ ft/s}^2} = 86.957$ $r = 2.25$ in.

$\bar{I} = m\bar{k}^2 = 86.957 \left(\frac{6}{12} \text{ ft}\right)^2 = 21.739$



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$2T_A + 2T_B - W = m\bar{a}$

$2(T_A + T_B) - 2800 = 86.957\bar{a}$ (1)

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $(2T_B - 2T_A)r = \bar{I}\alpha$

$2(T_B - T_A)\left(\frac{2.25}{12} \text{ ft}\right) = 21.739\alpha$ (2)

Given data: $T_A = 690$ lb, $T_B = 730$ lb.

(a) Substitute in (2):

$2(730 - 690)\frac{2.25}{12} = 21.739\alpha$

$15 = 21.739\alpha$ $\alpha = 0.690$ rad/s² ↻ ◀

(b) Substitute in (1):

$2(690 + 730) - 2800 = 86.957\bar{a}$

$40 = 86.957\bar{a}$ $\bar{a} = 0.460$ ft/s² ↑ ◀

16.68 See solution of Prob. 16.67 for diagram and derivation of Eqs. (1) and (2):

$2(T_A + T_B) - 2800 = 86.957\bar{a}$ (1)

$2(T_B - T_A)\left(\frac{2.25}{12}\right) = 21.739\alpha$ (2)

Given data: $T_A = 680$ lb, $\bar{a} = -6$ in./s² $= -0.5$ ft/s²

(a) Substitute in (1):

$2(680 \text{ lb}) + 2T_B - 2800 = 86.957(-0.5)$

$2T_B = 2800 - 1360 - 43.48 = 1396.48$ lb

$T_B = 698.26$ lb $T_B = 698$ lb ◀

(b) Substitute for T_A and T_B into (2):

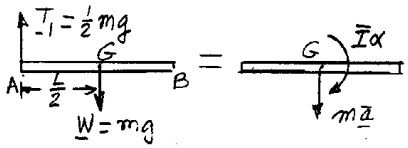
$2(698.26 - 680)\frac{2.25}{12} = 21.739\alpha$ $\alpha = 0.315$ rad/s² ↻ ◀

16.69 Before spring 2 breaks, tension in spring 1 is

$T_1 = \frac{1}{2} W = \frac{1}{2} mg$

Immediately after spring 2 breaks, elongation of spring 1 is unchanged. Thus, we still have

$T_1 = \frac{1}{2} mg$



(a) $+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $T_1\left(\frac{L}{2}\right) = \bar{I}\alpha$

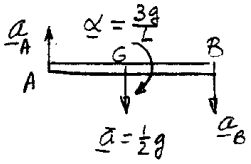
$\frac{1}{2} mg\left(\frac{L}{2}\right) = \frac{1}{12} mL^2 \alpha$

$\alpha = \frac{3g}{L}$ ↻ ◀

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $W - T_1 = m\bar{a}$

$mg - \frac{1}{2} mg = m\bar{a}$ $\bar{a} = \frac{1}{2} g \downarrow$

(b) Acceleration of A



$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$+\downarrow a_A = \bar{a} - \frac{L}{2}\alpha$

$= \frac{1}{2} g - \frac{L}{2}\frac{3g}{L} = -g$

$a_A = g \uparrow$ ◀

(c) Acceleration of B

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

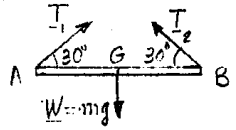
$+\downarrow a_B = \bar{a} + \frac{L}{2}\alpha = \frac{1}{2} g + \frac{L}{2}\frac{3g}{L} = +2g$

$a_B = 2g \downarrow$ ◀

Note. Compare the solutions of Probs. 16.69-72, where remaining support is a spring, with the solutions of Probs. 16.90-93, where remaining support is a pin and bracket.

16.70

Before spring 2 breaks:

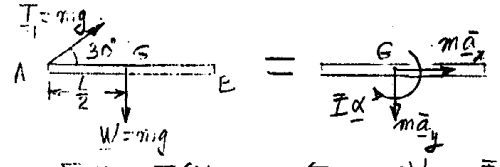


$+\uparrow \Sigma F_y = 0: T_1 \sin 30° + T_2 \sin 30° - W = 0$

$2T_1 \sin 30° = W = mg$

$T_1 = mg \measuredangle 30°$

Immediately after spring 2 breaks, elongation of spring 1 is unchanged. Thus, we still have $T_1 = mg \measuredangle 30°$.



(a) $+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}: \quad (T_1 \sin 30°)\frac{L}{2} = \bar{I}\alpha$

$(mg \sin 30°)\frac{L}{2} = \frac{1}{12} m L^2 \alpha$

$\alpha = \frac{3g}{L}$ ↻ ◀

$\pm \rightarrow \Sigma F_x = \Sigma (F_x)_{eff}: \quad T_1 \cos 30° = m\bar{a}_x$

$mg \cos 30° = m\bar{a}_x$

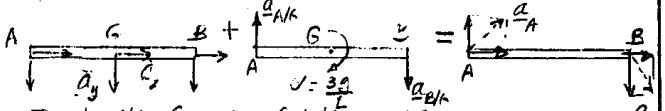
$\bar{a}_x = 0.866g \rightarrow$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad W - T_1 \sin 30° = m\bar{a}_y$

$mg - 0.5mg = m\bar{a}_y$

$\bar{a}_y = \frac{1}{2} g \downarrow$

Accelerations of A and B



(b) Acceleration of A

$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = [0.866g \rightarrow + 0.5g \downarrow] + \left(\frac{3g}{L}\right)\left(\frac{L}{2}\right)\uparrow$

$= (0.866g \rightarrow) + (g \uparrow)$

$\underline{a}_A = 1.323g \measuredangle 49.1°$ ◀

(c) Acceleration of B

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = [0.866g \rightarrow + 0.5g \downarrow] + \left(\frac{3g}{L}\right)\left(\frac{L}{2}\right)\downarrow$

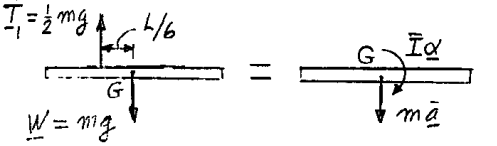
$= (0.866g \rightarrow) + (2g \downarrow)$

$\underline{a}_B = 2.18g \searrow 66.6°$ ◀

16.71

Before spring 2 breaks, tension in spring 1 is $T_1 = \frac{1}{2} W = \frac{1}{2} mg$.

Immediately after spring 2 breaks, elongation of spring 1 is unchanged. Thus, we still have $T_1 = \frac{1}{2} mg$



$(+\circlearrowleft) \Sigma M_G = \Sigma (M_G)_{eff}: \quad T_1 \left(\frac{L}{6}\right) = \bar{I}\alpha$

$\frac{1}{2} mg \left(\frac{L}{6}\right) = \frac{1}{12} m L^2 \alpha$

$\alpha = \frac{g}{L}$ ↻ ◀

(CONTINUED)

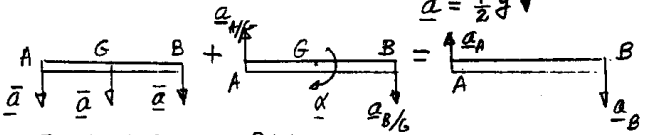
16.71 CONTINUED

Accelerations of A and B

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad W - T_1 = m\bar{a}$

$mg - \frac{1}{2} mg = m\bar{a}$

$\bar{a} = \frac{1}{2} g \downarrow$



(b) Acceleration of A

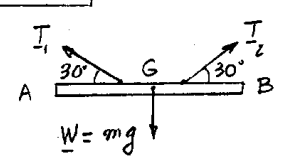
$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \frac{1}{2} g \downarrow + \left(\frac{3g}{L}\right)\left(\frac{L}{2}\right)\uparrow = 0 \qquad \underline{a}_A = 0$ ◀

(c) Acceleration of B

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \frac{1}{2} g \downarrow + \left(\frac{3g}{L}\right)\left(\frac{L}{2}\right)\downarrow = g \downarrow \qquad \underline{a}_B = g \downarrow$ ◀

16.72

Before spring 2 breaks:



$+\uparrow \Sigma F_y = 0:$

$T_1 \sin 30° + T_2 \sin 30° - W = 0$

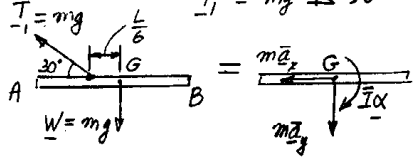
Since $T_1 = T_2$ by symmetry,

$2T_1 \sin 30° = W = mg$

$T_1 = mg \measuredangle 30°$

Immediately after spring 2 breaks, elongation of spring 1 is unchanged, Thus, we still have

$T_1 = mg \measuredangle 30°$



$(+\circlearrowleft) \Sigma M_G = \Sigma (M_G)_{eff}: \quad (T_1 \sin 30°)\frac{L}{6} = \bar{I}\alpha$

$(mg \sin 30°)\frac{L}{6} = \frac{1}{12} m L^2 \alpha \qquad \alpha = \frac{g}{L}$ ↻ ◀

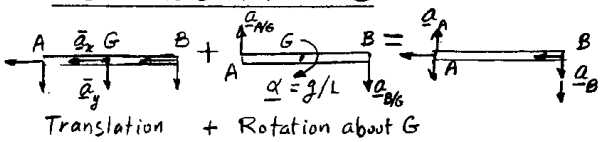
$\pm \Sigma F_x = \Sigma (F_x)_{eff}: \quad T_1 \cos 30° = m\bar{a}_x$

$mg \cos 30° = m\bar{a}_x \qquad \bar{a}_x = 0.866g \leftarrow$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad W - T_1 \sin 30° = m\bar{a}_y$

$mg - mg \sin 30° = m\bar{a}_y \qquad \bar{a}_y = 0.5g \downarrow$

Accelerations of A and B



(b) Acceleration of A

$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = [0.866g \leftarrow + 0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\uparrow$

$= 0.866g \leftarrow + 0 \qquad \underline{a}_A = 0.866g \leftarrow$ ◀

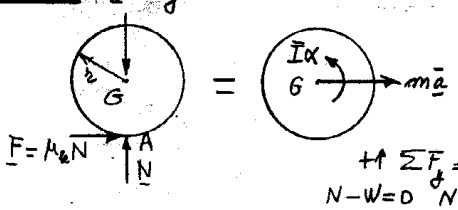
(c) Acceleration of B

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = [0.866g \leftarrow + 0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\downarrow$

$= 0.866g \leftarrow + g \downarrow \qquad \underline{a}_B = 1.323g \searrow 49.1°$ ◀

Note: Compare the solutions of Probs. 16.69-72, where remaining support is a spring, with the solutions of Probs. 16.90-93, where remaining support is a pin and bracket.

16.73



$+\uparrow \Sigma F_y = 0$: $N - W = 0$ $N = W = mg$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}$

$\mu_k mg = m\bar{a}$ $\quad \bar{a} = \mu_k g \rightarrow$ (1)

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $Fr = \bar{I}\alpha$

$\mu_k mgr = \bar{I}\alpha$ (2)

For sphere: $\bar{I} = \frac{2}{5} mr^2$ $\quad \mu_k mgr = \frac{2}{5} mr^2 \alpha$

$\alpha = \frac{5}{2} \frac{\mu_k g}{r}$ ↻ (3)

Kinematics: $+\circlearrowleft \omega = \omega_0 - \alpha t = \omega_0 - \frac{5}{2} \frac{\mu_k g}{r} t$ (4)

$\pm \bar{v} = \bar{a} t = \mu_k g t$ (5)

$\pm v_A = \bar{v} - \omega r = \mu_k g t - (\omega_0 - \frac{5}{2} \frac{\mu_k g}{r} t) r$

$v_A = \frac{7}{2} \mu_k g t - \omega_0 r$

(a) When sphere starts rolling ($t = t_1$), we have $v_A = 0$:

$\frac{7}{2} \mu_k g t_1 - \omega_0 r = 0$ $\quad t_1 = \frac{2}{7} \frac{\omega_0 r}{\mu_k g}$ ◀

(b) Substituting for t in (5) and (4) the value found for t_1:

$\bar{v}_1 = \mu_k g \left(\frac{2}{7} \frac{\omega_0 r}{\mu_k g} \right)$ $\quad \bar{v}_1 = \frac{2}{7} \omega_0 r \rightarrow$ ◀

$\omega_1 = \omega_0 - \frac{5}{2} \frac{\mu_k g}{r} \left(\frac{2}{7} \frac{\omega_0 r}{\mu_k g} \right)$ $\quad \omega_1 = \frac{2}{7} \omega_0 \circlearrowleft$ ◀

16.74 See solution of Prob. 16.73 for diagram and derivation of Eqs. (1) and (2):

$\bar{a} = \mu_k g \rightarrow$ (1)

$\mu_k mgr = \bar{I}\alpha$ (2)

For disk: $\bar{I} = \frac{1}{2} mr^2$ $\quad \mu_k mgr = \frac{1}{2} mr^2 \alpha$

$\alpha = \frac{2\mu_k g}{r}$ ↻ (3)

Kinematics: $+\circlearrowleft \omega = \omega_0 - \alpha t = \omega_0 - \frac{2\mu_k g}{r} t$ (4)

$\pm \bar{v} = \bar{a} t = \mu_k g t$ (5)

$\pm v_A = \bar{v} - \omega r = \mu_k g t - (\omega_0 - \frac{2\mu_k g}{r} t) r$

$v_A = 3\mu_k g t - \omega_0 r$

(a) When disk starts rolling ($t = t_1$), we have $v_A = 0$:

$3\mu_k g t_1 - \omega_0 r = 0$ $\quad t_1 = \frac{1}{3} \frac{\omega_0 r}{\mu_k g}$ ◀

(b) Substituting for t in (5) and (4) the value found for t_1:

$v_1 = \mu_k g \left(\frac{1}{3} \frac{\omega_0 r}{\mu_k g} \right)$ $\quad v_1 = \frac{1}{3} \omega_0 r \rightarrow$ ◀

$\omega_1 = \omega_0 - \frac{2\mu_k g}{r} \left(\frac{1}{3} \frac{\omega_0 r}{\mu_k g} \right)$ $\quad \omega_1 = \frac{1}{3} \omega_0 \circlearrowleft$ ◀

Note: We check that $v_1 = \omega_1 r$ in both Prob. 16.73 and 16.74.

16.75 See solution of Prob. 16.73 for diagram and derivation of Eqs. (1) and (3):

$\bar{a} = \mu_k g \rightarrow$ (1)

$\alpha = \frac{5}{2} \frac{\mu_k g}{r}$ ↻ (3)

Kinematics: $+\circlearrowleft \omega = \alpha t = \frac{5}{2} \frac{\mu_k g}{r} t$ (4)

$\pm \bar{v} = \bar{a} t = \mu_k g t$ (5)

$\pm v_A = \bar{v} + \omega r = \mu_k g t + (\frac{5}{2} \frac{\mu_k g}{r} t) r$

$v_A = \frac{7}{2} \mu_k g t$

(a) When disk starts rolling ($t = t_1$), we have

$v_A = v_1$ $\quad \frac{7}{2} \mu_k g t_1 = v_1$ $\quad t_1 = \frac{2}{7} \frac{v_1}{\mu_k g}$ ◀

(b) Substituting for t in (5) and (4) the value found for t_1:

$\bar{v} = \mu_k g \left(\frac{2}{7} \frac{v_1}{\mu_k g} \right)$ $\quad \bar{v} = \frac{2}{7} v_1 \rightarrow$ ◀

$\omega = \frac{5}{2} \frac{\mu_k g}{r} \left(\frac{2}{7} \frac{v_1}{\mu_k g} \right)$ $\quad \omega = \frac{5}{7} \frac{v_1}{r} \circlearrowleft$ ◀

Note. We check that $\bar{v} = v_1 - \omega r$.

16.76 See solution of Prob. 16.73 for diagram and derivation of Eqs. (1) and (2):

$\bar{a} = \mu_k g \rightarrow$ (1)

$\mu_k mgr = \bar{I}\alpha$ (2)

Making $\bar{I} = m\bar{k}^2$ in Eq. (2):

$\mu_k mgr = m\bar{k}^2 \alpha$

$\alpha = \frac{\mu_k g r}{\bar{k}^2}$ ↻ (3)

Kinematics: $+\circlearrowleft \omega = \alpha t = \frac{\mu_k g r}{\bar{k}^2} t$ (4)

$\pm \bar{v} = \bar{a} t = \mu_k g t$ (5)

$\pm v_A = \bar{v} + \omega r = \mu_k g t + (\frac{\mu_k g r}{\bar{k}^2} t) r$

$v_A = \mu_k g \left(1 + \frac{r^2}{\bar{k}^2} \right) t$

(a) When wheel starts rolling ($t = t_1$), we have

$v_A = v_1$ $\quad \mu_k g \left(1 + \frac{r^2}{\bar{k}^2} \right) t_1 = v_1$

$\mu_k g \frac{\bar{k}^2 + r^2}{\bar{k}^2} t_1 = v_1$

$t_1 = \frac{v_1}{\mu_k g} \frac{\bar{k}^2}{r^2 + \bar{k}^2}$ ◀

(b) Substituting for t in (5) and (4) the value found for t_1:

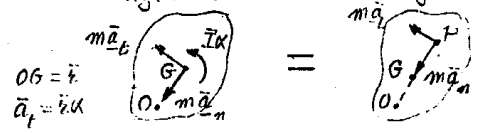
$\bar{v} = \mu_k g \left(\frac{v_1}{\mu_k g} \frac{\bar{k}^2}{r^2 + \bar{k}^2} \right)$, $\quad \bar{v} = v_1 \frac{\bar{k}^2}{r^2 + \bar{k}^2} \rightarrow$ ◀

$\omega = \frac{\mu_k g r}{\bar{k}^2} \left(\frac{v_1}{\mu_k g} \frac{\bar{k}^2}{r^2 + \bar{k}^2} \right)$, $\quad \omega = \frac{v_1 r}{r^2 + \bar{k}^2} \circlearrowleft$ ◀

Note. We check that $\bar{v} = v_1 - \omega r$.

16.77 Noncentroidal Rotation

Fig. 16.15b: Fig. P16.78:



We first check that the sum of the vectors is the same in both figures. To have the same sum of the moments about G, we must have

$$+\circlearrowleft \Sigma M_G = \Sigma M_G: \quad \bar{I}\alpha = (m\bar{a}_t)(GP)$$

$$m\bar{k}^2\alpha = m\bar{r}\alpha\,(GP)$$

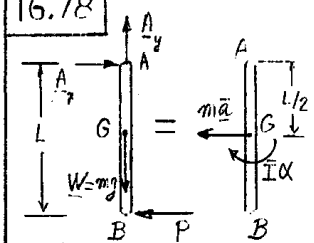
$$GP = \frac{\bar{k}^2}{\bar{r}} \qquad \text{(Q.E.D.)}$$

Note: The center of rotation and the center of percussion are interchangeable. Indeed, the relation obtained above may be expressed as

$$GP = \frac{\bar{k}^2}{GO} \quad \text{or as} \quad GO = \frac{\bar{k}^2}{GP}$$

Thus, if P is selected as the center of rotation, then O will be the center of percussion.

16.78



$$\bar{a} = \frac{L}{2}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$$PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= (m\tfrac{L}{2}\alpha)\frac{L}{2} + \tfrac{1}{12}mL^2\alpha$$

$$= \tfrac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3P}{mL} = \frac{3(12\text{N})}{(2\text{ kg})(0.75\text{ m})} = 24\text{ rad/s}^2 \qquad \underline{\alpha = 24.0\text{ rad/s}^2}\ \circlearrowleft$$

$$\bar{a} = \frac{L}{2}\alpha = (0.375\text{ m})(24.0\text{ rad/s}^2) \qquad \underline{\bar{a} = 9.00\text{ m/s}^2}\ \leftarrow$$

(b) $\pm \rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x - P = -m\bar{a}$

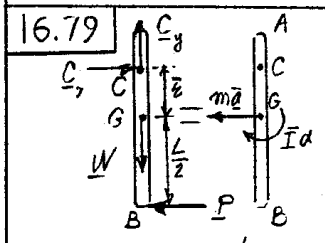
$$A_x = P - m\bar{a} = 12\text{ N} - (2\text{ kg})(9\text{ m/s}^2) = 12 - 18 = -6\text{ N}$$

$$\underline{A_x = 6.00\text{ N}}\ \leftarrow$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A_y - W = 0$

$$A_y = W = mg = (2\text{ kg})(9.81\text{ m/s}^2) \qquad \underline{A_y = 19.62\text{ N}}\ \uparrow$$

16.79



$$\bar{a} = \bar{r}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$$P\left(\bar{r} + \frac{L}{2}\right) = (m\bar{a})\bar{r} + \bar{I}\alpha$$

$$= (m\bar{r}\alpha)\bar{r} + \frac{1}{12}mL^2\alpha$$

$$= \frac{W}{g}\left(\bar{r}^2 + \frac{L^2}{12}\right)\alpha$$

$$\alpha = \frac{Pg}{W}\,\frac{\bar{r} + \frac{L}{2}}{\bar{r}^2 + \frac{L^2}{12}} \qquad \underline{\alpha = \frac{Pg}{WL}\,\frac{(\bar{r}/L) + (1/2)}{(\bar{r}/L)^2 + (1/12)}}\ \circlearrowleft \quad (1)$$

For $P = 2.5$ lb, $W = 6$ lb, $L = 30$ in. $= 2.5$ ft, $\bar{r}/L = 1/4$:

$$\alpha = \frac{(2.5)g}{6(2.5)}\,\frac{\frac{1}{4} + \frac{1}{2}}{\frac{1}{16} + \frac{1}{2}} = \frac{g}{6}\,\frac{36}{7} = \frac{6}{7}g$$

$$\alpha = \frac{6}{7}(32.2) \qquad \underline{\alpha = 27.6\text{ rad/s}^2}\ \circlearrowleft$$

(CONTINUED)

16.79 CONTINUED

$$\bar{a} = \bar{r}\alpha = \left(\frac{7.5}{12}\text{ ft}\right)\left(\frac{6}{7}g\right) = \frac{15}{28}g$$

$\pm \rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $C_x - P = -m\bar{a}$

$$C_x = P - m\bar{a} = P - m\left(\frac{15}{28}g\right) = P - \frac{15}{28}W = 2.5\text{ lb} - \frac{15}{28}(6\text{ lb})$$

$$C_x = -0.714\text{ lb} \qquad \underline{C_x = 0.714\text{ lb}}\ \leftarrow$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $C_y - W = 0$

$$C_y = W = 6\text{ lb} \qquad \underline{C_y = 6.00\text{ lb}}\ \uparrow$$

16.80 See solution of Prob. 16.79 for diagram and derivation of Eq. (1).

$\pm \rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $C_x - P = -m\bar{a}$

For $C_x = 0$, we have $m\bar{a} = P$

$$\bar{a} = \frac{P}{m} = \frac{P}{W}g = \frac{2.5\text{ lb}}{6\text{ lb}}g = \frac{5}{12}g$$

Since $\bar{a} = \bar{r}\alpha$: $\alpha = \dfrac{\bar{a}}{\bar{r}} = \dfrac{5}{12}\dfrac{g}{\bar{r}}$ (2)

Substituting for α from (2) into (1) and noting that $\frac{P}{W} = \frac{2.5\text{ lb}}{6\text{ lb}} = \frac{5}{12}$:

$$\frac{5}{12}\frac{g}{\bar{r}} = \frac{5}{12}\frac{g}{L}\,\frac{(\bar{r}/L) + (1/2)}{(\bar{r}/L)^2 + (1/12)}$$

$$\left(\frac{\bar{r}}{L}\right)^2 + \frac{1}{12} = \frac{\bar{r}}{L}\left(\frac{\bar{r}}{L} + \frac{1}{2}\right)$$

$$\frac{1}{12} = \frac{1}{2}\frac{\bar{r}}{L} \qquad \frac{\bar{r}}{L} = \frac{1}{6}$$

$$\bar{r} = \frac{1}{6}(30\text{ in.}) \qquad \bar{r} = 5.00\text{ in.}$$

Substituting for $\bar{r}$ into Eq. (2):

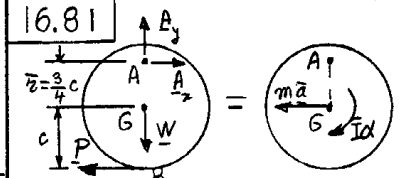
$$\alpha = \frac{5}{12}\,\frac{32.2\text{ ft/s}^2}{(5/12)\text{ ft}} \qquad \underline{\alpha = 32.2\text{ rad/s}^2}\ \circlearrowleft$$

Alternate solution. Noting that to have no horizontal reaction at C, point B must be the center of percussion, we have from Prob. 16.77:

$$GB = \frac{\bar{k}^2}{\bar{r}} \qquad \bar{r} = \frac{\bar{k}^2}{GB} = \frac{\frac{1}{12}L^2}{L/2} = \frac{1}{6}L = \frac{30}{6} = 5\text{ in.}$$

We then substitute for $\bar{r}$ in Eq (2) to find $\underline{\alpha}$

16.81



$$W = mg$$

$$\bar{a} = \bar{r}\alpha = \tfrac{3}{4}c\alpha \qquad \bar{I} = \tfrac{1}{2}mc^2$$

(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $P\left(\frac{7}{4}c\right) = (m\bar{a})\bar{r} + \bar{I}\alpha$

$$\frac{7}{4}Pc = \left(m\tfrac{3}{4}c\alpha\right)\left(\tfrac{3}{4}c\right) + \tfrac{1}{2}mc^2\alpha$$

$$\frac{7}{4}Pc = \left(\frac{9}{16} + \frac{1}{2}\right)mc^2\alpha$$

$$\alpha = \frac{28}{17}\frac{P}{mc} = \frac{28}{17}\,\frac{20\text{N}}{(6\text{kg})(0.16\text{m})} = 34.314\text{ rad/s}^2$$

$$\underline{\alpha = 34.3\text{ rad/s}^2}\ \circlearrowleft$$

$$\bar{a} = \bar{r}\alpha = \tfrac{3}{4}c\alpha = \tfrac{3}{4}(0.16\text{m})(34.314\text{ rad/s}^2) = 4.118\text{ m/s}^2$$

(b) $\pm \rightarrow \Sigma F_x = (\Sigma F_x)_{eff}$: $A_x - P = -m\bar{a}$

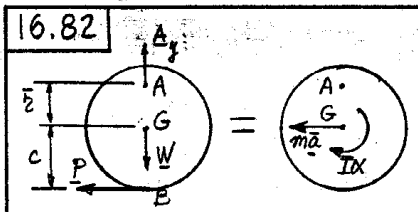
$$A_x = P - m\bar{a} = 20\text{ N} - (6\text{ kg})(4.118\text{ m/s}^2) = -4.71\text{ N}$$

$$\underline{A_x = 4.71\text{ N}}\ \leftarrow$$

$+\uparrow \Sigma F_y = (\Sigma F_y)_{eff}$: $A_y - W = 0$

$$A_y = W = mg = (6\text{ kg})(9.81\text{ m/s}^2) \qquad \underline{A_y = 58.9\text{ N}}\ \uparrow$$

16.82



We have $\underline{A}_x = 0$

$W = mg$

$\bar{a} = \bar{r}\alpha$

$\bar{I} = \frac{1}{2}mc^2$

(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $P(\bar{r}+c) = (m\bar{a})\bar{r} + \bar{I}\alpha$

$= (m\bar{r}\alpha)\bar{r} + \frac{1}{2}mc^2\alpha$

$P(\bar{r}+c) = m\left(\bar{r}^2 + \frac{1}{2}c^2\right)\alpha$ (1)

$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $P = m\bar{a}$ $P = m\bar{r}\alpha$ (2)

Substituting for P from (2) into (1):

$m\bar{r}\alpha(\bar{r}+c) = m\left(\bar{r}^2 + \frac{1}{2}c^2\right)\alpha$

$\bar{r}^2 + \bar{r}c = \bar{r}^2 + \frac{1}{2}c^2$

$\bar{r}c = \frac{1}{2}c^2$ $\bar{r} = \frac{1}{2}c = \frac{1}{2}(160\text{ mm})$

$\bar{r} = 80\text{ mm}$ ◀

(b) Substitute for $\bar{r}$ into Eq. (2):

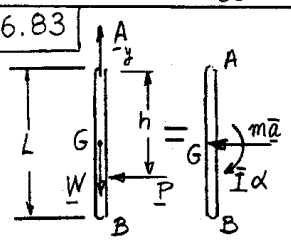
$P = m\left(\frac{1}{2}c\right)\alpha$ $\alpha = \frac{2P}{mc} = \frac{2(20\text{ N})}{(6\text{ kg})(0.16\text{ m})} = 41.67\text{ rad/s}^2$

$\underline{\alpha = 41.7\text{ rad/s}^2}$ ◀

Alternate solution for part (a): Noting that B must be the center of percussion if the horizontal component of the reaction at A is to be zero, we have from Prob. 16.77

$GB = \frac{\bar{k}^2}{\bar{r}}$ $\bar{r} = \frac{\bar{k}^2}{GB} = \frac{\frac{1}{2}c^2}{c} = \frac{1}{2}c = \frac{1}{2}(160\text{ mm}) = 80\text{ mm}$

16.83



We have $\underline{A}_x = 0$

$W = mg$

$\bar{a} = \frac{L}{2}\alpha$

$\bar{I} = \frac{1}{12}mL^2$

(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $Ph = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$

$= \left(m\frac{L}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$

$Ph = \frac{1}{3}mL^2\alpha$ (1)

$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $P = m\bar{a}$ $P = m\left(\frac{L}{2}\alpha\right)$ (2)

Substituting for P from (2) into (1):

$\left(m\frac{L}{2}\alpha\right)h = \frac{1}{3}mL^2\alpha$

$h = \frac{2}{3}L = \frac{2}{3}(750\text{ mm})$

$h = 500\text{ mm}$ ◀

(b) Solving Eq. (2) for α:

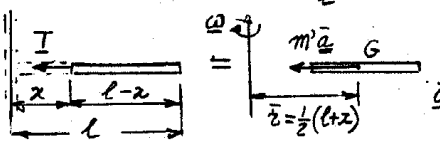
$\alpha = \frac{2P}{mL} = \frac{2(12\text{ N})}{(2\text{ kg})(0.75\text{ m})} = 16.00\text{ rad/s}^2$

$\underline{\alpha = 16.00\text{ rad/s}^2}$ ◀

Alternate Solution for part (a): Noting that the force $\underline{P}$ must be applied at the center of percussion P if the horizontal component of the reaction at A is to be zero, we have from Prob. 16.77:

$GP = \frac{\bar{k}^2}{\bar{r}} = \frac{\frac{1}{12}L^2}{\frac{1}{2}L} = \frac{1}{6}L$, $h = \frac{L}{2} + \frac{L}{6} = \frac{2L}{3} = \frac{2}{3}(750) = 500\text{ mm}$

16.84 Consider the portion of rod of length $\ell - x$ and mass $m' = m\frac{\ell - x}{\ell}$:



We have $\underline{\alpha} = 0$

$\bar{a} = \bar{a}_n = \omega^2\bar{r}$ ←

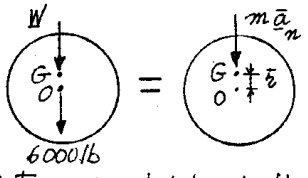
$\pm \Sigma F_n = \Sigma(F_n)_{eff}$: $T = m'\bar{a} = m'\omega^2\bar{r}$

$= \left(m\frac{\ell - x}{\ell}\right)\omega^2\frac{\ell + x}{2}$

$T = \frac{1}{2}\frac{m}{\ell}\omega^2(\ell^2 - x^2)$ ◀

16.85 $\omega = 1200\text{ rpm} = 125.66\text{ rad/s}$ $\alpha = 0$

(1) Force exerted by shaft on flywheel = 6000 lb↓:

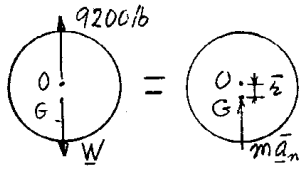


$+\downarrow \Sigma F_n = \Sigma(F_n)_{eff}$:

$W + 6000 = m\bar{a}_n$

$W + 6000 = \frac{W}{g}\omega^2\bar{r}$ (1)

(2) Force exerted by shaft on flywheel = 9200 lb↑:



$+\uparrow \Sigma F_n = \Sigma(F_n)_{eff}$:

$9200 - W = m\bar{a}_n$

$9200 - W = \frac{W}{g}\omega^2\bar{r}$ (2)

Subtract (2) from (1): $W + 6000 - 9200 + W = 0$

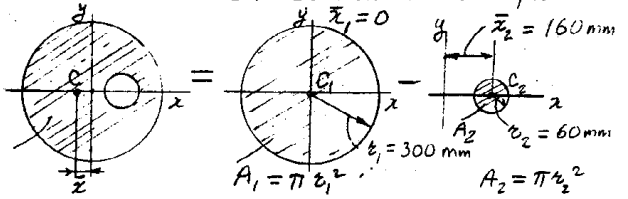
$W = 1600\text{ lb}$ ◀

Add (1) and (2): $6000 + 9200 = 2\frac{W}{g}\omega^2\bar{r}$

$15,200 = 2\frac{1600}{32.2}(125.66)^2\bar{r}$, $\bar{r} = 0.009686 = 0.1162\text{ in.}$ ◀

16.86 Determination of mass center of disk

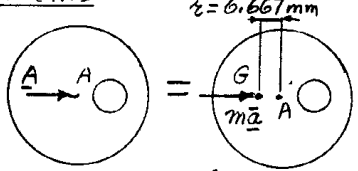
We determine the centroid of the composite area



$\bar{x}A = \bar{x}_1A_1 - \bar{x}_2A_2$ or $\bar{x}(A_1 - A_2) = \bar{x}_1A_1 - \bar{x}_2A_2$

$\bar{x} = \frac{\bar{x}_1A_1 - \bar{x}_2A_2}{A_1 - A_2} = \frac{0 - (160)\pi(60)^2}{\pi(300)^2 - \pi(60)^2} = -\frac{160(60)^2}{(300)^2 - (60)^2} = -6.667\text{ mm}$

Kinetics



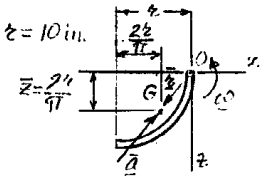
$\omega = 480\text{ rpm} = 50.265\text{ rad/s}$

$\bar{a} = \bar{r}\omega^2 = (6.667\times10^{-3}\text{ m})(50.265\text{ rad/s})^2 = 16.844\text{ m/s}^2$

$\Sigma F = \Sigma(F)_{eff}$: $A = m\bar{a} = (30\text{ kg})(16.844\text{ m/s}^2)$

$\underline{A = 505\text{ N}} \rightarrow$ ◀

16.87 Determination of $m\bar{a}$



Since $\alpha = 0$, $\bar{a} = \bar{a}_n = -\omega^2 \bar{r}$

$\bar{a} = -\omega^2(\bar{x}\,i + \bar{z}\,k)$

But $\bar{x} = -\left(r - \frac{2r}{\pi}\right) = -r\left(1 - \frac{2}{\pi}\right)$

$= -\left(\frac{10}{12}\text{ft}\right)(0.36338)$

$= -0.30282$ ft

and $\bar{z} = \frac{2r}{\pi} = \frac{2}{\pi}\left(\frac{10}{12}\text{ft}\right) = 0.53052$ ft

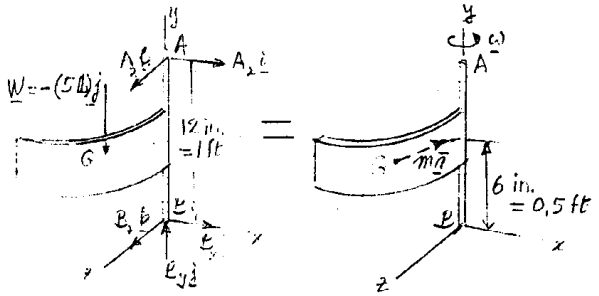
$\omega = 180$ rpm $= 18.85$ rad/s.

Thus: $\bar{a} = -(18.85)^2(-0.30282\,i + 0.53052\,k)$

$= (+107.60\,i - 188.51\,k)$ ft/s^2

and $m\bar{a} = \frac{5}{32.2}\bar{a} = (16.708\,i - 29.271\,k)$ lb (1)

Equivalence of external and effective forces:



External forces:

$\underline{W} = -(5\text{lb})\underline{j}$ at $\underline{r}_G = -(0.30282\text{ ft})\underline{i} + (0.5\text{ ft})\underline{j} + (0.53052\text{ ft})\underline{k}$

$\underline{A} = A_x\underline{i} + A_z\underline{k}$ at $\underline{r}_A = (1\text{ ft})\underline{j}$

$\underline{B} = B_x\underline{i} + B_y\underline{j} + B_z\underline{k}$ at $\underline{r}_B = 0$

Effective forces:

From (1): $m\bar{a} = (16.708\text{ lb})\underline{i} - (29.271\text{ lb})\underline{k}$ at $\underline{r}_G$

$\Sigma \underline{H}_B = \Sigma(\underline{H}_B)_{eff}$: $\underline{r}_G \times \underline{W} + \underline{r}_A \times \underline{A} = \underline{r}_G \times m\bar{a}$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.30282 & 0.5 & 0.53052 \\ 0 & -5 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & 0 \\ A_x & 0 & A_z \end{vmatrix} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.30282 & 0.5 & 0.53052 \\ 16.708 & 0 & -29.271 \end{vmatrix}$$

Equating the coefficients of $\underline{k}$:

$(0.30282)(5) - (1)A_x = -(16.708)(0.5)$

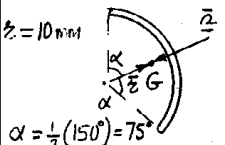
$A_x = +9.868$ lb $\quad \underline{A}_x = (9.87\text{ lb})\underline{i}$ ◀

Equating the coefficients of $\underline{i}$:

$(0.53052)(5) + (1)A_z = (0.5)(-29.271)$

$A_z = -17.288$ lb $\quad \underline{A}_z = -(17.29\text{ lb})\underline{k}$ ◀

16.88 Determination of $m\bar{a}$ for member AB



$\alpha = \frac{1}{2}(150°) = 75°$

$= \frac{75}{180}\pi = 1.3090$ rad

Since ω = constant:

$\bar{a} = \bar{a}_n = -\omega^2 \bar{r}$

But $\bar{r} = r\frac{\sin\alpha}{\alpha} = 0.01\text{m}\,\frac{\sin 75°}{1.3090}$

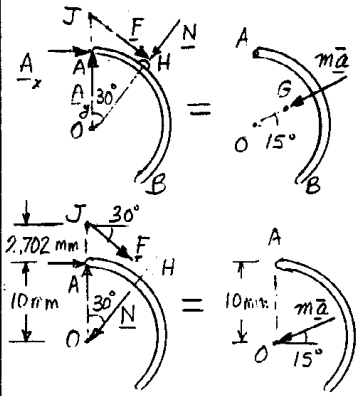
$\bar{r} = 7.3791 \times 10^{-3}$ m

$\bar{a} = -\omega^2(7.3791\times10^{-3})\angle 15°$

$m = (4.5\times10^{-3}\text{ kg})$

$m\bar{a} = -(33.206\times10^{-6})\omega^2\angle 15°$ (1)

Member AB:



We note from triangle OHJ:

$OH = 11$ mm

$OJ = \frac{OH}{\cos 30°} = \frac{11\text{mm}}{\cos 30°}$

$OJ = 12.702$ mm

$AJ = 12.702 - 10 = 2.702$ mm

To compute moments more easily, we slide the vectors $\underline{F}$, $\underline{N}$, and $m\bar{a}$ along their lines of action as shown

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$(F\cos 30°)(2.702\text{ mm}) + (N\sin 30°)(10\text{ mm}) = (m\bar{a}\cos 15°)(10\text{ mm})$

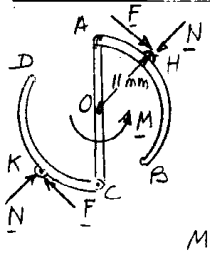
But $F = \mu_k N = 0.35N$ $\quad N = \frac{F}{0.35}$

Substituting for N:

$F\left(2.702\cos 30° + \frac{10\sin 30°}{0.35}\right) = 10\,m\bar{a}\cos 15°$

$F = 0.58098\,m\bar{a}$

Arm and both members. Since $\alpha = 0$ and $\bar{a} = 0$ for this system, it is in equilibrium



$+\circlearrowleft \Sigma M_O = 0$: $M - 2F(0.011\text{m}) = 0$

$M = 2(0.58098\,m\bar{a})(0.011)$

$= (12.782\times10^{-3})\,m\bar{a}$

or, recalling Eq. (1):

$M = (12.782\times10^{-3})(33.206\times10^{-6})\,\omega^2$

$M = (424.4\times10^{-9})\,\omega^2$ (2)

For $\omega = 3000$ rpm $= 314.16$ rad/s, we have

$M = (424.4\times10^{-3})(314.16)^2 = 41.89\times10^{-3}$ N·m

$\underline{M} = 41.9$ N·mm ↻ ◀

16.89

See solution of Prob. 16.88 for derivation of Eq. (2):

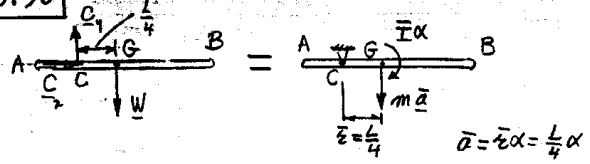
$M = (424.4\times10^{-9})\,\omega^2$ (2)

For $M = 30$ N·mm $= 30\times10^{-3}$ N·m:

$30\times10^{-3} = (424.4\times10^{-9})\,\omega^2$

$\omega = 265.87$ rad/s $\quad \omega = 2540$ rpm ◀

16.90



$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $W(\frac{L}{4}) = (m\bar{a})\frac{L}{4} + \bar{I}\alpha$

$= (m\frac{L}{4}\alpha)\frac{L}{4} + \frac{1}{12}mL^2\alpha$

$\frac{1}{4}WL = (\frac{1}{16}+\frac{1}{12})mL^2\alpha$

$\alpha = \frac{12}{7}\frac{W}{mL} = \frac{12}{7}\frac{g}{L}$

(a) $a_B = (CB)\alpha = \frac{3L}{4}(\frac{12}{7}\frac{g}{L})$ $\quad \underline{a_B} = \frac{9}{7}g \downarrow$ ◀

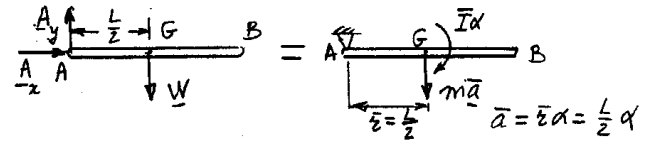
(b) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $C_x = 0$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $C_y - W = -m\bar{a}$

$C_y = W - m\bar{a} = W - \frac{W}{g}(\frac{L}{4}\alpha) = W - \frac{W}{g}\frac{L}{4}(\frac{12}{7}\frac{g}{L}) = \frac{4}{7}W$

$\underline{C} = \frac{4}{7}W \uparrow$ ◀

16.91



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $W(\frac{L}{2}) = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$

$= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12}mL^2\alpha$

$\frac{1}{2}WL = \frac{1}{3}mL^2\alpha$

$\alpha = \frac{3}{2}\frac{W}{mL} = \frac{3}{2}\frac{g}{L}$

(a) $a_B = (AB)\alpha = L(\frac{3}{2}\frac{g}{L})$ $\quad \underline{a_B} = \frac{3}{2}g \downarrow$ ◀

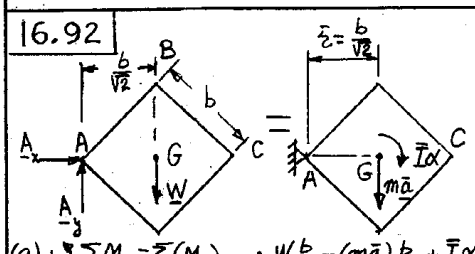
(b) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $A_x = 0$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - W = -m\bar{a}$

$A_y = W - m\bar{a} = W - \frac{W}{g}(\frac{L}{2}\alpha) = W - \frac{W}{g}\frac{L}{2}(\frac{3}{2}\frac{g}{L}) = \frac{1}{4}W$

$\underline{A} = \frac{1}{4}W \uparrow$ ◀

16.92



$\bar{a} = \bar{r}\alpha = \frac{b}{\sqrt{2}}\alpha$

$\bar{I} = \frac{1}{12}m(b^2+b^2) = \frac{1}{6}mb^2$

(a) $+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $W\frac{b}{\sqrt{2}} = (m\bar{a})\frac{b}{\sqrt{2}} + \bar{I}\alpha$

$W\frac{b}{\sqrt{2}} = (m\frac{b}{\sqrt{2}}\alpha)\frac{b}{\sqrt{2}} + \frac{1}{6}mb^2\alpha = \frac{2}{3}mb^2\alpha$

$\alpha = \frac{3}{2\sqrt{2}}\frac{W}{mb} = \frac{3\sqrt{2}}{4}\frac{g}{b}$ $\quad \underline{\alpha} = 1.061\frac{g}{b}$ ◀

(b) $\underline{a_C} = (AC)\alpha = (2\frac{b}{\sqrt{2}})(\frac{3\sqrt{2}}{4}\frac{g}{b})$ $\quad \underline{a_C} = \frac{3}{2}g \downarrow$ ◀

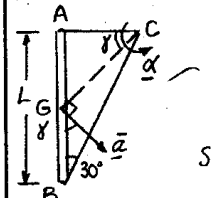
(c) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $A_x = 0$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - W = -m\bar{a}$

$A_y = W - m\bar{a} = W - \frac{W}{g}(\frac{b}{\sqrt{2}}\alpha) = W - \frac{W}{g}\frac{b}{\sqrt{2}}(\frac{3\sqrt{2}}{4}\frac{g}{b}) = \frac{1}{4}W$

$\underline{A} = \frac{1}{4}W \uparrow$ ◀

16.93 Geometry and Kinematics



$AC = L\tan 30° = \frac{L}{\sqrt{3}} = 0.57735L$

$GC = \sqrt{(AC)^2 + (AG)^2} = \sqrt{(\frac{L}{\sqrt{3}})^2 + (\frac{L}{2})^2} = L\sqrt{\frac{1}{3}+\frac{1}{4}}$

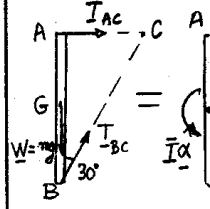
$GC = \sqrt{\frac{7}{12}}L = 0.76376L$

$\tan\gamma = \frac{AG}{AC} = \frac{L/2}{L/\sqrt{3}} = \frac{\sqrt{3}}{2}$, $\gamma = 40.89°$

Since rod rotates about C:

$\bar{a} = (GC)\alpha = 0.76376L\alpha$

Kinetics



(a) $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:

$W(AC) = \bar{I}\alpha + m\bar{a}(GC)$

$mg(0.57735L) = \frac{1}{12}mL^2\alpha + m(0.76376L\alpha)(0.76376L)$

$0.57735gL = 0.66667L^2\alpha$

$\alpha = 0.86603\frac{g}{L}$, $\underline{\alpha} = 0.866\frac{g}{L}$ ◀

(b) We first determine $m\bar{a}$:

$m\bar{a} = m(0.76376L\alpha) = m(0.76376L)(0.86603\frac{g}{L})$

$m\bar{a} = 0.66144mg = 0.66144W$

$+\nwarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $T_{BC}\cos 30° - W = -m\bar{a}\cos\gamma$

$T_{BC}\cos 30° = W - 0.66144W\cos 40.89° = 0.500W$

$T_{BC} = 0.57735W$ $\quad T_{BC} = 0.577W$ ◀

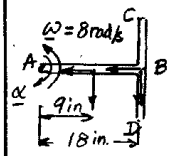
$\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $T_{AC} + T_{BC}\sin 30° = m\bar{a}\sin\gamma$

$T_{AC} = (0.577W)\sin 30° + (0.66144W)\sin 40.89° = 0.14431W$

$T_{AC} = 0.1443W$ ◀

16.94

We shall consider separately the effective forces of rods AB and CD.



$\bar{a}_{AB} = \bar{r}\alpha \downarrow + \bar{r}\omega^2 \leftarrow = \frac{9}{12}\alpha \downarrow + \frac{9}{12}(8)^2 \leftarrow$

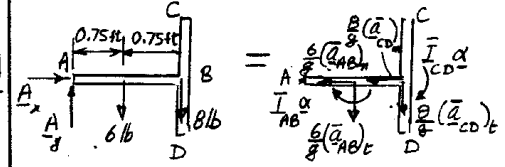
$= 0.75\alpha \downarrow + 48\ ft/s^2 \leftarrow$

$\bar{a}_{CD} = \bar{r}\alpha \downarrow + \bar{r}\omega^2 \leftarrow = \frac{18}{12}\alpha \downarrow + \frac{18}{12}(8)^2 \leftarrow$

$= 1.50\alpha \downarrow + 96\ ft/s^2 \leftarrow$

We also note that $\bar{I}_{AB} = \frac{1}{12}m\ell^2 = \frac{1}{12}\frac{6}{g}(\frac{18}{12})^2 = \frac{1.125}{g}$

$\bar{I}_{CD} = \frac{1}{12}m\ell^2 = \frac{1}{12}\frac{8}{g}(\frac{24}{12})^2 = \frac{2.6667}{g}$



(a) $+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$(6\ lb)(0.75\ ft) + (8\ lb)(1.5\ ft) = \frac{6}{g}(\bar{a}_{AB})_t(0.75ft) + \bar{I}_{AB}\alpha + \frac{8}{g}(\bar{a}_{CD})_t(1.5ft) + \bar{I}_{CD}\alpha$

$16.5 = \frac{6}{g}(0.750\alpha)(0.75) + \frac{1.125}{g}\alpha + \frac{8}{g}(1.50\alpha)(1.5) + \frac{2.6667}{g}\alpha$

$16.5 = 25.1667\frac{\alpha}{g}$ $\quad \alpha = 0.65563g$ $\quad \underline{\alpha} = 21.1\ rad/s^2$ ◀

(b) $\pm\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $A_x = -\frac{6}{g}(\bar{a}_{AB})_n - \frac{8}{g}(\bar{a}_{CD})_n$

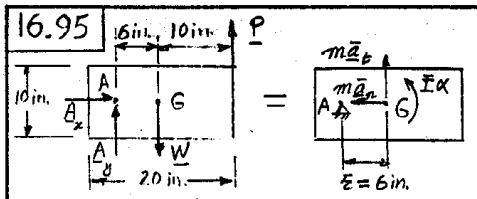
$A_x = -\frac{6}{g}(48\ ft/s^2) - \frac{8}{g}(96\ ft/s^2) = -\frac{1056}{32.2}$, $\underline{A_x} = 32.8\ lb \leftarrow$ ◀

$+\nwarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - 6\ lb - 8\ lb = -\frac{6}{g}(\bar{a}_{AB})_t - \frac{8}{g}(\bar{a}_{CD})_t$

$A_y = 14 - \frac{6}{g}(0.75\alpha) - \frac{8}{g}(1.50\alpha) = 14 - 16.5\frac{\alpha}{g} = 14 - 16.5(0.65563)$

$A_y = 14 - 10.818 = 3.182\ lb$ $\quad \underline{A_y} = 3.18\ lb \uparrow$ ◀

16.95



$\bar{a}_t = \bar{r}\alpha = (\frac{6}{12}\text{ ft})(30\text{ rad/s}^2) = 15\text{ ft/s}^2$

$\bar{a}_n = \bar{r}\omega^2 = (\frac{6}{12}\text{ ft})(20\text{ rad/s})^2 = 200\text{ ft/s}^2$.

$\bar{I} = \frac{m}{12}\left[(\frac{10}{12})^2 + (\frac{20}{12})^2\right] = (0.28935)m$

(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $P(\frac{16}{12}\text{ ft}) - W(\frac{6}{12}\text{ ft}) = m\bar{a}_t(\frac{6}{12}\text{ ft}) + \bar{I}\alpha$

$\frac{4}{3}P = \frac{1}{2}W + \frac{W}{g}(15)(\frac{1}{2}) + 0.28935\frac{W}{g}(30)$

$P = \frac{3}{8}W + \frac{3}{4}\frac{W}{32.2}(7.5 + 8.6805) = (0.375 + 0.37688)W$

$= 0.75188\,(12\text{ lb}) = 9.0225\text{ lb}$

$\underline{P = 9.02\text{ lb}\uparrow}$ ◀

(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x = -m\bar{a}_n = -\frac{12}{32.2}(200)$

$A_x = -74.53\text{ lb}$ $\underline{A_x = 74.5\text{ lb}\leftarrow}$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A_y + P - W = m\bar{a}_t$

$A_y = W + m\bar{a}_t - P = 12 + \frac{12}{32.2}(15) - 9.02 = 8.57\text{ lb}$

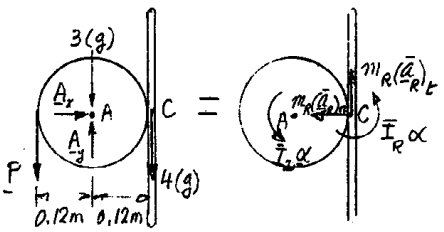
$\underline{A_y = 8.57\text{ lb}\uparrow}$ ◀

16.96

We represent the effective forces of the disk by a couple $\bar{I}_D\underline{\alpha}$ and the effective forces of the rod by vectors $m_R(\bar{a}_R)_t$ and $m_R(\bar{a}_R)_n$ attached at C and a couple $\bar{I}_R\underline{\alpha}$. We have

$\bar{I}_D = \frac{1}{2}(3\text{ kg})(0.120\text{ m})^2 = 21.6\times10^{-3}\text{ kg}\cdot\text{m}^2$

$m_R = 4\text{ kg}$, $\bar{I}_R = \frac{1}{12}(4\text{ kg})(0.48\text{ m})^2 = 76.8\times10^{-3}\text{ kg}\cdot\text{m}^2$



$(\bar{a}_R)_t = 0.12\alpha$

$(\bar{a}_R)_n = 0.12\omega^2$

$+\circlearrowleft \Sigma H_A = \Sigma(H_A)_{eff}$:

$P(0.12) - 4g(0.17) = \bar{I}_D\alpha + \bar{I}_R\alpha + m_R(a_R)_t(0.12)$

$0.12(P - 4g) = 21.6\times10^{-3}\alpha + 76.8\times10^{-3}\alpha + 4(0.12\alpha)(0.12)$

$P - 4g = 1.300\alpha$ (1)

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x = -m_R(a_R)_n$

$A_x = -4(0.12\omega^2)$ $A_x = -0.48\omega^2$ (2)

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A_y - P - 4g - 3g = m_R(\bar{a}_R)_t$

$A_y = P + 7g + 4(0.12\alpha)$ $A_y = P + 7g + 0.48\alpha$ (3)

(a) Making $\alpha = 36\text{ rad/s}^2$ in (1):

$P = 4(9.81) + 1.3(36) = 86.04\text{ N}$ $\underline{P = 86.0\text{ N}\downarrow}$ ◀

(b) Making $\omega = 12\text{ rad/s}$ in (2) and $P = 86.04$, $\alpha = 36$ in (3):

$A_x = -0.48(12)^2 = -69.12\text{ N}$ $\underline{A_x = 69.1\text{ N}\leftarrow}$ ◀

$A_y = 86.04 + 7(9.81) + 0.48(36)$ $\underline{A_y = 172.0\text{ N}\uparrow}$ ◀

16.97

See solution of Prob. 16.96 for derivation of Eqs. (1), (2), and (3).

(a) Making $P = 0$ in Eq. (1), we have

$\alpha = -\frac{4g}{1.300} = -\frac{4(9.81)}{1.300} = -30.18$ $\underline{\alpha = 30.2\text{ rad/s}^2\circlearrowleft}$ ◀

(b) Making $\omega = 15\text{ rad/s}$ in (2) and $P = 0$, $\alpha = -30.18$ in (3):

$A_x = -0.48\omega^2 = -0.48(15)^2 = -108.0\text{ N}$

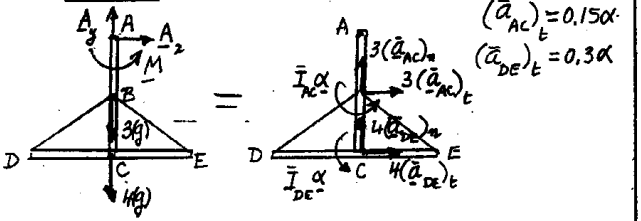
$\underline{A_x = 108.0\text{ N}\leftarrow}$ ◀

$A_y = P + 7g + 0.48\alpha = 0 + 7(9.81) + 0.48(-30.18)$

$= +54.18\text{ N}$ $\underline{A_y = 54.2\text{ N}\uparrow}$ ◀

16.98

We first consider the entire system and express that the external forces are equivalent to the effective forces of both rods.



$(\bar{a}_{AC})_t = 0.15\alpha$

$(\bar{a}_{DE})_t = 0.3\alpha$

$\bar{I}_{AC} = \frac{1}{12}(3\text{ kg})(0.3\text{ m})^2 = 22.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$\bar{I}_{DE} = \frac{1}{12}(4\text{ kg})(0.4\text{ m})^2 = 53.333\times10^{-3}\text{ kg}\cdot\text{m}^2$

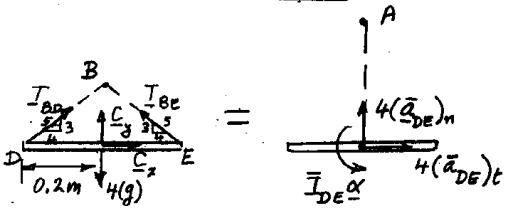
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$M = \bar{I}_{AC}\alpha + 3(\bar{a}_{AC})_t(0.15) + \bar{I}_{DE}\alpha + 4(\bar{a}_{DE})_t(0.3)$

$M = 22.5\times10^{-3}\alpha + 3(0.15\alpha)(0.15) + 53.333\times10^{-3}\alpha + 4(0.3\alpha)(0.3)$

$M = 0.50333\alpha$ (1)

We now consider rod DE alone:



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$(\frac{3}{5}T_{BE})(0.2\text{ m}) - (\frac{3}{5}T_{BD})(0.2\text{ m}) = \bar{I}_{DE}\alpha$

$0.12(T_{BE} - T_{BD}) = 53.333\times10^{-3}\alpha$

$T_{BE} - T_{BD} = 0.44444\alpha$ (2)

Given data: $T_{BD} = 2\text{ N}$ $T_{BE} = 8\text{ N}$

(a) Substitute in (2):

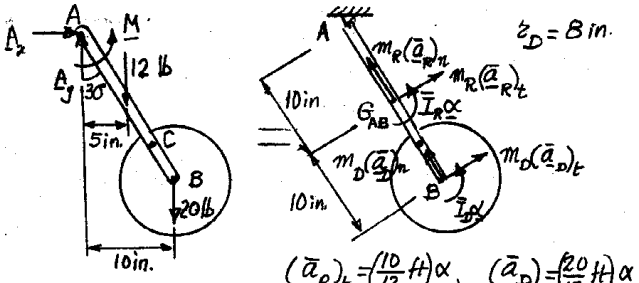
$8 - 2 = 0.44444\alpha$ $\underline{\alpha = 13.50\text{ rad/s}^2\circlearrowleft}$ ◀

(b) Carry value of α into (1):

$M = 0.50333(13.50)$

$\underline{M = 6.79\text{ N}\cdot\text{m}\circlearrowleft}$ ◀

16.99 We first consider the entire system and express that the external forces are equivalent to the effective forces of the disk and the rod.



$(\bar{a}_R)_t = (\frac{10}{12}\text{ ft})\alpha$, $(\bar{a}_D)_t = (\frac{20}{12}\text{ ft})\alpha$

(a) $\Sigma M_A = \Sigma(M_A)_{eff}$:

$$M - (12\text{ lb})(\tfrac{5}{12}\text{ ft}) - (20\text{ lb})(\tfrac{10}{12}\text{ ft}) = \bar{I}_R\alpha + m_R(\bar{a}_R)_t(\tfrac{10}{12}) + \bar{I}_D\alpha + m_D(\bar{a}_D)_t(\tfrac{20}{12})$$

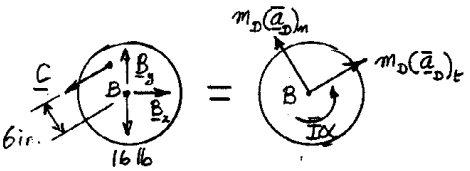
$$M - 5 - \frac{200}{12} = \frac{1}{12}\frac{12}{g}\left(\frac{20}{12}\right)^2\alpha + \frac{12}{g}\left(\frac{10}{12}\alpha\right)\left(\frac{10}{12}\right) + \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2\alpha + \frac{20}{g}\left(\frac{20}{12}\alpha\right)\left(\frac{20}{12}\right)$$

$$M - 5 - 16.667 = \frac{1}{g}(2.7778 + 8.3333 + 4.4444 + 55.5555)\alpha$$

$$M - 21.667 = \frac{71.111}{32.2}(25\text{ rad/s}^2)$$

$$M = 76.877\text{ lb·ft}, \quad \underline{M = 76.9\text{ lb·ft}}\ \text{↻}$$ ◀

(b) Consider now the disk alone:

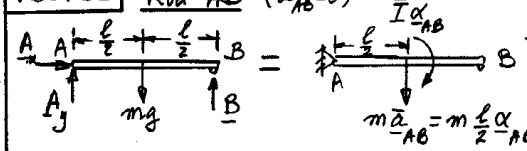


$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $C(\frac{8}{12}\text{ ft}) = \bar{I}_D\alpha$

$$C = 2\bar{I}_D\alpha = 2\left[\frac{1}{2}\frac{20}{32.2}\left(\frac{8}{12}\right)^2(25\text{ rad/s}^2)\right] = 6.901\text{ lb}$$

Force exerted by disk on rod: $\underline{C = 6.90\text{ lb} \angle 30°}$ ◀

16.100 Rod AB ($\omega_{AB} = 0$)



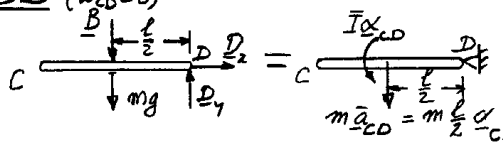
$\bar{I} = \frac{1}{12}m\ell^2$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $mg(\frac{\ell}{2}) - B\ell = \bar{I}\alpha_{AB} + m\bar{a}_{AB}(\frac{\ell}{2})$

$$\frac{1}{2}mg\ell - B\ell = \frac{1}{12}m\ell^2\alpha_{AB} + (m\frac{\ell}{2}\alpha_{AB})\frac{\ell}{2}$$

$$\frac{1}{2}mg\ell - B\ell = \frac{1}{3}m\ell^2\alpha_{AB} \qquad (1)$$

Rod CD ($\omega_{CD} = 0$)



$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$: $mg(\frac{\ell}{2}) + B(\frac{\ell}{2}) = \bar{I}\alpha_{CD} + m\bar{a}_{CD}(\frac{\ell}{2})$

$$\frac{1}{2}mg\ell + \frac{1}{2}B\ell = \frac{1}{12}m\ell^2\alpha_{CD} + (m\frac{\ell}{2}\alpha_{CD})\frac{\ell}{2}$$

Multiplying by 2:

$$mg\ell + B\ell = \frac{2}{3}m\ell^2\alpha_{CD} \qquad (2)$$

Adding (1) and (2):

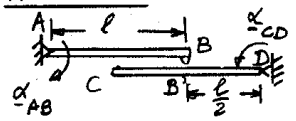
$$\frac{3}{2}mg\ell = m\ell^2(\frac{1}{3}\alpha_{AB} + \frac{2}{3}\alpha_{CD}) \qquad (3)$$

(CONTINUED)

16.100 CONTINUED We rewrite Eq. (3) as follows:

$$\alpha_{AB} + 2\alpha_{CD} = \frac{g}{\ell}\frac{g}{\ell} \qquad (3')$$

Kinematics



We must have

$$(\underline{a}_B\downarrow) = (\underline{a}_{B'}\downarrow)$$

$$\ell\alpha_{AB} = \frac{\ell}{2}\alpha_{CD}$$

$$\alpha_{AB} = \frac{1}{2}\alpha_{CD} \qquad (4)$$

Substituting for α_{AB} from (4) into (3'):

$$\frac{1}{2}\alpha_{CD} + 2\alpha_{CD} = \frac{g}{\ell}\frac{g}{\ell}$$

$$\frac{5}{2}\alpha_{CD} = \frac{g}{\ell}\frac{g}{\ell} \qquad \alpha_{CD} = 1.8\frac{g}{\ell} \qquad (5)$$

(a) Acceleration of C:

$$a_C = \ell\alpha_{CD} = \ell(1.8\frac{g}{\ell}) \qquad \underline{a_C = 1.8g}\downarrow$$ ◀

(b) Force on knob B:

Substituting for α_{CD} from (5) into (2):

$$mg\ell + B\ell = \frac{2}{3}m\ell^2(1.8\frac{g}{\ell})$$

$$B = 1.2mg - mg \qquad \underline{B = 0.2mg}\uparrow$$ ◀

6.101 See portion of solution of Prob. 6.98 above dashed line for derivation of Eqs. (1) and (2):

$$M = 0.50333\alpha \qquad (1)$$

$$T_{BE} - T_{BD} = 0.44444\alpha \qquad (2)$$

Given data:

$$M = 7.55\text{ N·m}$$

$$T_{BD} = 0$$

(a) Substitute $M = 7.55$ N·m into Eq. (1):

$$7.55 = 0.50333\alpha$$

$$\alpha = 15.00\text{ rad/s}^2$$

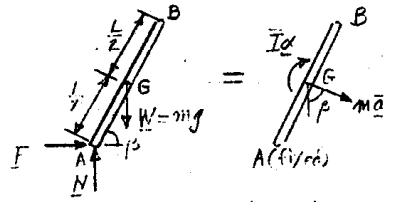
$$\underline{\alpha = 15.00\text{ rad/s}^2}\ \text{↻}$$ ◀

(b) Substitute $\alpha = 15.00$ rad/s² and $T_{BD} = 0$ into Eq. (2):

$$T_{BE} - 0 = 0.44444(15.00)$$

$$T_{BE} = 6.67\text{ N}$$ ◀

16.102 We note that rod rotates about A and that, just after release, $\omega = 0$.



$\bar{I} = \frac{1}{12} m L^2$

$\bar{a} = \frac{L}{2}\alpha$

+) $\Sigma M_A = \Sigma (M_A)_{eff}$: $W\left(\frac{L}{2}\cos\beta\right) = m\bar{a}\left(\frac{L}{2}\right) + \bar{I}\alpha$

$\frac{1}{2} mgL\cos\beta = m\left(\frac{L}{2}\alpha\right)\frac{L}{2} + \frac{1}{12} mL^2\alpha$

$= \frac{1}{3} mL^2\alpha$

$\alpha = \frac{3}{2}\frac{g}{L}\cos\beta$ (1)

+→ $\Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}\sin\beta$

$= m\left(\frac{L}{2}\alpha\right)\sin\beta$

Substitute from (1): $= m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\beta\right)\sin\beta$

$F = \frac{3}{4} mg\sin\beta\cos\beta$ (2)

+↑ $\Sigma F_y = \Sigma (F_y)_{eff}$: $N - W = -m\bar{a}\cos\beta$

$= -m\left(\frac{L}{2}\alpha\right)\cos\beta$

$N - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\beta\right)\cos\beta$

$N = mg\left(1 - \frac{3}{4}\cos^2\beta\right)$ (3)

(a) Make $\beta = 65°$ in Eq. (1):

$\alpha = \frac{3}{2}\frac{g}{L}\cos 65° = 0.63393\frac{g}{L}$ $\quad \underline{\alpha = 0.634\frac{g}{L}}$ ◀

(b) Make $\beta = 65°$ in (2) and (3):

$F = \frac{3}{4} mg\sin 65°\cos 65° = 0.28727 mg$, $\underline{F = 0.287 mg}$ ◀

$N = mg\left(1 - \frac{3}{4}\cos^2 65°\right) = 0.86605 mg$, $\underline{N = 0.866 mg\uparrow}$ ◀

(c) Minimum $\mu_s = \frac{F}{N} = \frac{0.28727}{0.86605} = 0.3317$, $(\mu_s)_{min} = 0.332$ ◀

***16.103** See solution of Prob. 16.102 for derivation of Eqs. (1), (2), and (3). Slipping is impending when

$F = \mu_s N$ or $F = 0.35 N$

Substituting for F and N from (2) and (3):

$\frac{3}{4} mg\sin\beta\cos\beta = 0.35 mg\left(1 - \frac{3}{4}\cos^2\beta\right)$

$\sin\beta\cos\beta = 0.46667 - 0.35\cos^2\beta$ (4)

Squaring: $\sin^2\beta\cos^2\beta = 0.217778 - 0.326667\cos^2\beta + 0.1225\cos^4\beta$

$(1 - \cos^2\beta)\cos^2\beta =$

$\cos^2\beta - \cos^4\beta = 0.217778 - 0.326667\cos^2\beta + 0.1225\cos^4\beta$

$1.1225\cos^4\beta - 1.326667\cos^2\beta + 0.217778 = 0$

Solving for $\cos^2\beta$:

$\cos^2\beta = 0.98490$ and $\cos^2\beta = 0.19699$

$\cos\beta = 0.99242$ and $\cos\beta = 0.44383$

$\beta = 7.06°$ and $\beta = 63.65°$

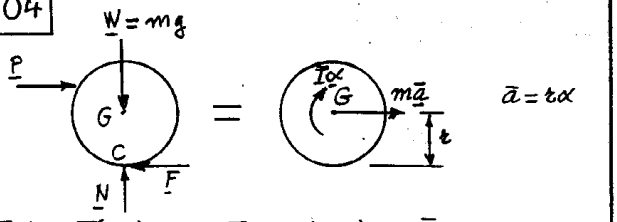
We check that both values obtained for β satisfy the original equation (4). Thus slipping is impending for $\beta = 7.06°$ and $\beta = 63.65°$

Rod will not slip for very small values of β (less than 7.06° or for large values of β (greater than 63.65° Thus,

Rod will slip if $7.1° < \beta < 63.6°$ ◀

Note: We check from Prob. 16.102 that rod will not slip for $\beta = 65°$, since $\mu_s = 0.35$ is larger than $(\mu_s)_{min} = 0.332$.

16.104



$\bar{a} = r\alpha$

+) $\Sigma M_C = \Sigma (M_C)_{eff}$: $\Sigma M_C = (m\bar{a})r + \bar{I}\alpha$

$= (mr\alpha)r + \bar{I}\alpha$

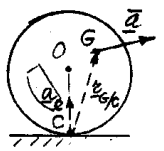
$= (mr^2 + \bar{I})\alpha$

But, by parallel-axis theorem, $I_C = \bar{I} + mr^2$

Thus: $\Sigma M_C = I_C\alpha$ (Q.E.D.) ◀

Note: See also Prob. 16.105.

16.105 Kinematics. For rolling motion, instantaneous center is at C, and $\underline{a}_C$ is directed toward the geometric center O. The acceleration $\underline{\bar{a}}$ of the mass center G may be expressed as

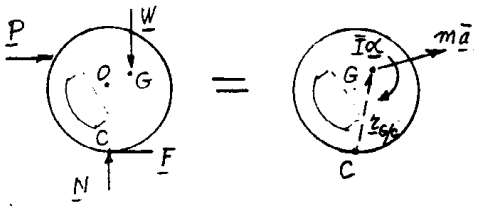


$\underline{\bar{a}} = \underline{a}_G = \underline{a}_C + \underline{a}_{G/C}$

$= \underline{a}_C + \underline{\alpha} \times \underline{r}_{G/C} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{G/C})$

or, since $\underline{\omega} \perp \underline{r}_{G/C}$: $\underline{\bar{a}} = \underline{a}_C + \underline{\alpha} \times \underline{r}_{G/C} - \omega^2 \underline{r}_{G/C}$ (1)

Kinetics:



$\Sigma \underline{M}_C = \Sigma (\underline{M}_C)_{eff}$: $\Sigma \underline{M}_C = \bar{I}\underline{\alpha} + \underline{r}_{G/C} \times m\underline{\bar{a}}$

or, recalling Eq. (1):

$\Sigma \underline{M}_C = \bar{I}\underline{\alpha} + \underline{r}_{G/C} \times m(\underline{a}_C + \underline{\alpha} \times \underline{r}_{G/C} - \omega^2 \underline{r}_{G/C})$

$= \bar{I}\underline{\alpha} + \underline{r}_{G/C} \times m\underline{a}_C + m\underline{r}_{G/C} \times (\underline{\alpha} \times \underline{r}_{G/C}) - m\omega^2 \underline{r}_{G/C} \times \underline{r}_{G/C}$

But $\underline{r}_{G/C} \times \underline{r}_{G/C} = 0$ and, since $\underline{\alpha} \perp \underline{r}_{G/C}$:

$\underline{r}_{G/C} \times m(\underline{\alpha} \times \underline{r}_{G/C}) = mr^2_{G/C}\underline{\alpha}$

Thus: $\Sigma \underline{M}_C = \left(\bar{I} + mr^2_{G/C}\right)\underline{\alpha} + \underline{r}_{G/C} \times m\underline{a}_C$

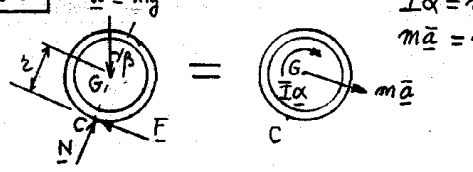
By parallel axis theorem, $\bar{I} + mr^2_{G/C} = I_C$

We write

$\Sigma \underline{M}_C = \bar{I}_C\underline{\alpha} + \underline{r}_{G/C} \times m\underline{a}_C$ (2)

Eq. (2) reduces to $\Sigma M_C = \bar{I}_C\alpha$ when $\underline{r}_{G/C} \times m\underline{a}_C = 0$, that is, when $\underline{r}_{G/C}$ and $\underline{a}_C$ are collinear. Referring to the first diagram, we note that this will occur only when the points G, O, and C lie in a straight line. (Q.E.D.) ◀

16.106



$\bar{I}\alpha = m\bar{k}^2\alpha$

$m\bar{a} = mr\alpha$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(W\sin\beta)r = (m\bar{a})r + \bar{I}\alpha$

$mg\sin\beta\, r = (mr\alpha)r + m\bar{k}^2\alpha$

$rg\sin\beta = (r^2 + \bar{k}^2)\alpha$

$\alpha = \dfrac{rg\sin\beta}{r^2 + \bar{k}^2}$

$\bar{a} = r\alpha = r\dfrac{rg\sin\beta}{r^2+\bar{k}^2}$ $\qquad \bar{a} = \dfrac{r^2}{r^2+\bar{k}^2} g\sin\beta$ •

16.107

Kinematics: Motion of mass center (unif. acc.)

$\bar{x} = \bar{x}_0 + \bar{v}_0 t + \frac{1}{2}\bar{a}t^2$

where $\bar{x} = 2.25\text{ m}$, $\bar{x}_0 = \bar{v}_0 = 0$, $t = 15\text{ s}$

$2.25\text{ m} = \frac{1}{2}\bar{a}(15\text{ s})^2$ $\qquad \bar{a} = 20\times10^{-3}\text{ m/s}^2$

Kinetics. See solution of Prob. 16.106 for derivation of

$$\bar{a} = \frac{r^2}{r^2+\bar{k}^2} g\sin\beta$$

Substitute $\bar{a} = 20\times10^{-3}\text{ m/s}^2$, $r = 0.06\text{ m}$, $\beta = 18°$:

$20\times10^{-3} = \dfrac{(0.06)^2}{(0.06)^2+\bar{k}^2}(9.81)\sin 18°$

$(0.06)^2 + \bar{k}^2 = \dfrac{(0.06)^2}{20\times10^{-3}}(9.81)\sin 18° = 0.54566$

$\bar{k}^2 = 0.54566 - 0.00360 = 0.54206$

$\bar{k} = 0.7362\text{ m}$ $\qquad \bar{k} = 736\text{ mm}$

16.108

Kinetics. See solution of Prob. 16.106 for derivation of

$$\bar{a} = \frac{r^2}{r^2+\bar{k}^2} g\sin\beta$$

Substitute $r = 0.04\text{ m}$ $\bar{k} = 0.5\text{ m}$, $\beta = 18°$:

$\bar{a} = \dfrac{(0.04)^2}{(0.04)^2+(0.5)^2}(9.81)\sin 18° = 19.278\times10^{-3}\text{ m/s}^2$

Kinematics. Motion of mass center is unif. accel.:

$\bar{x} = \bar{x}_0 + \bar{v}_0 t + \frac{1}{2}\bar{a}t^2 = 0 + 0 + \frac{1}{2}(19.278\times10^{-3}\text{ m/s}^2)(20\text{ s})^2$

$\bar{x} = 3.856\text{ m}$ $\qquad \bar{x} = 3.86\text{ m}$

16.109

See solution of Prob. 16.106 for derivation of

$$\bar{a} = \frac{r^2}{r^2+\bar{k}^2} g\sin\beta$$

For cylinder C: $I_C = \frac{1}{2}m_C r^2$, $\bar{k}_C^2 = \dfrac{I_C}{m_C} = \frac{1}{2}r^2$

For pipe P: $I_P = m_P r^2$, $\bar{k}_P^2 = \dfrac{I_P}{m_P} = r^2$

Acceleration of C relative to P:

$a_{C/P} = a_C - a_P = \dfrac{r^2}{r^2+\frac{1}{2}r^2} g\sin\beta - \dfrac{r^2}{r^2+r^2} g\sin\beta$

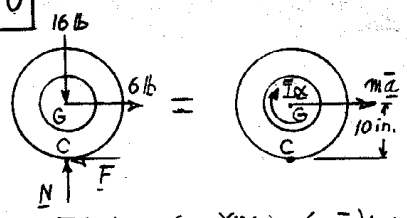
$= \left(\dfrac{1}{1+\frac{1}{2}} - \dfrac{1}{1+1}\right) g\sin\beta = \left(\dfrac{2}{3} - \dfrac{1}{2}\right) g\sin\beta = \dfrac{1}{6} g\sin\beta$

Distance between C and P: $x_{C/P} = \frac{1}{2}a_{C/P}t^2 = \left(\frac{1}{12}g\sin\beta\right)t^2$

$x_{C/P} = \frac{1}{12}(9.81\text{ m/s}^2)\sin 15°(2.5\text{ s})^2$ $\qquad x_{C/P} = 1.322\text{ m}$

or $x_{C/P} = \frac{1}{12}(32.2\text{ ft/s}^2)\sin 15°(2.5\text{ s})^2$ $\qquad x_{C/P} = 4.34\text{ ft}$

16.110



$r = \frac{10}{12}\text{ ft}$

$\bar{a} = r\alpha$

$\bar{a} = \frac{10}{12}\alpha$

$\bar{I} = m\bar{k}^2 = \frac{16}{g}\left(\frac{7.5}{12}\right)^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(6\text{ lb})\left(\frac{10}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$= \frac{16}{g}\left(\frac{10}{12}\alpha\right)\frac{10}{12} + \frac{16}{g}\left(\frac{7.5}{12}\right)^2\alpha$

$5 = \frac{16}{g}\frac{156.25}{144}\alpha$ $\qquad \alpha = 0.288\,g$ $\qquad \alpha = 9.27\text{ rad/s}^2$ ◀

$\bar{a} = r\alpha = \frac{10}{12}(0.288\,g) = 0.240\,g$ $\qquad \bar{a} = 7.73\text{ ft/s}^2 \rightarrow$ ◀

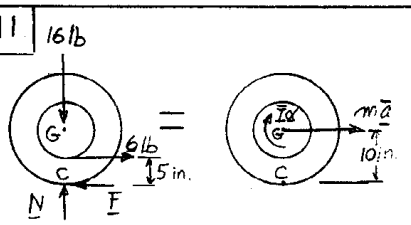
(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $6\text{ lb} - F = m\bar{a}$

$F = 6 - m\bar{a} = 6 - \frac{16}{g}(0.240\,g) = +2.16\text{ lb}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 16\text{ lb} = 0$ $\qquad N = 16\text{ lb}$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{2.16\text{ lb}}{16\text{ lb}}$ $\qquad (\mu_s)_{min} = 0.135$ ◀

16.111



$r = \frac{10}{12}\text{ ft}$

$\bar{a} = r\alpha$

$\bar{a} = \frac{10}{12}\alpha$

$\bar{I} = m\bar{k}^2 = \frac{16}{g}\left(\frac{7.5}{12}\right)^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(6\text{ lb})\left(\frac{5}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$= \frac{16}{g}\left(\frac{10}{12}\alpha\right)\frac{10}{12} + \frac{16}{g}\left(\frac{7.5}{12}\right)^2\alpha$

$2.5 = \frac{16}{g}\frac{156.25}{144}\alpha$ $\qquad \alpha = 0.144\,g$ $\qquad \alpha = 4.64\text{ rad/s}^2$ ◀

$\bar{a} = r\alpha = \frac{10}{12}(0.144\,g) = 0.120\,g$ $\qquad \bar{a} = 3.86\text{ ft/s}^2 \rightarrow$ ◀

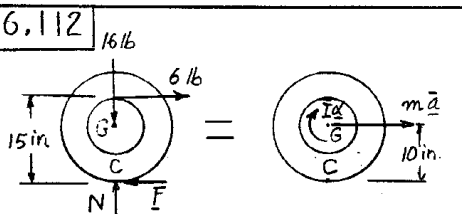
(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $6\text{ lb} - F = m\bar{a}$

$F = 6 - m\bar{a} = 6 - \frac{16}{g}(0.120\,g) = +4.08\text{ lb}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 16\text{ lb} = 0$ $\qquad N = 16\text{ lb}$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{4.08\text{ lb}}{16\text{ lb}}$ $\qquad (\mu_s)_{min} = 0.255$ ◀

16.112



$r = \frac{10}{12}\text{ ft}$

$\bar{a} = r\alpha$

$\bar{a} = \frac{10}{12}\alpha$

$\bar{I} = m\bar{k}^2 = \frac{16}{g}\left(\frac{7.5}{12}\right)^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(6\text{ lb})\left(\frac{15}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$= \frac{16}{g}\left(\frac{10}{12}\alpha\right)\left(\frac{10}{12}\right) + \frac{16}{g}\left(\frac{7.5}{12}\right)^2\alpha$

$7.5 = \frac{16}{g}\frac{156.25}{144}\alpha$ $\qquad \alpha = 0.432\,g$ $\qquad \alpha = 13.91\text{ rad/s}^2$ ◀

$\bar{a} = r\alpha = \frac{10}{12}(0.432\,g) = 0.360\,g$ $\qquad \bar{a} = 11.59\text{ ft/s}^2 \rightarrow$ ◀

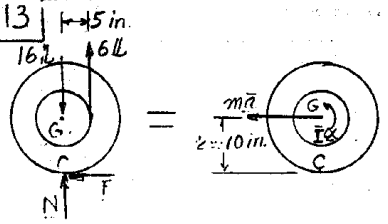
(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $6\text{ lb} - F = m\bar{a}$

$F = 6 - m\bar{a} = 6 - \frac{16}{g}(0.360\,g) = +0.240\text{ lb}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 16\text{ lb} = 0$ $\qquad N = 16\text{ lb}$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{0.240\text{ lb}}{16\text{ lb}}$ $\qquad (\mu_s)_{min} = 0.015$ ◀

16.113



$r = \frac{10}{12}$ ft

$\bar{a} = r\alpha$

$\bar{a} = \frac{10}{12}\alpha$

$\bar{I} = m\bar{k}^2 = \frac{16}{g}\left(\frac{7.5}{12}\right)^2$

(a) $+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(6\text{ lb})\left(\frac{5}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$2.5 = \frac{16}{g}\left(\frac{10}{12}\alpha\right)\frac{10}{12} + \frac{16}{g}\left(\frac{7.5}{12}\right)^2\alpha$

$2.5 = \frac{16}{g}\frac{156.25}{144}\alpha \quad \alpha = 0.144\,g \quad \underline{\alpha = 4.64\text{ rad/s}^2\,\circlearrowleft}$

$\bar{a} = r\alpha = \frac{10}{12}(0.144\,g) = 0.120\,g \quad \underline{\bar{a} = 3.86\text{ ft/s}^2 \leftarrow}$

(b) $+\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $F = m\bar{a}$

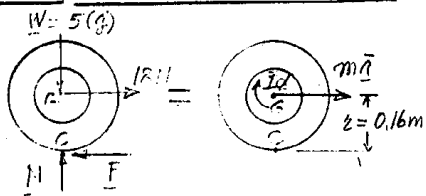
$F = \frac{16}{g}(0.120\,g) = +1.920\text{ lb} \quad \underline{F = 1.920\text{ lb} \leftarrow}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N + 6\text{ lb} - 16\text{ lb} = 0 \quad N = 10\text{ lb}\uparrow$

$(\mu_s)_{min} = \frac{F}{N} = \frac{1.920\text{ lb}}{10\text{ lb}} \quad (\mu_s)_{min} = 0.192$

16.114

Assume that the disk rolls



$\bar{a} = r\alpha$

$\bar{a} = 0.16\alpha$

$\bar{I} = m\bar{k}^2 = (5\text{ kg})(0.12\text{ m})^2 = 0.072\text{ kg}\cdot\text{m}^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(18\text{ N})(0.16\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$2.88 = 5(0.16\alpha)(0.16) + 0.072\alpha$

$2.88 = 0.200\alpha \quad \underline{\alpha = 14.4\text{ rad/s}^2\,\circlearrowleft}$

$a = r\alpha = (0.16)(14.4) = 2.304\text{ m/s}^2 \quad \underline{\bar{a} = 2.30\text{ m/s}^2 \rightarrow}$

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $18\text{ N} - F = m\bar{a}$

$F = 18 - m\bar{a} = 18 - 5(2.304) = 6.48\text{ N}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 5(g) = 0 \quad N = 5(9.81) = 49.05\text{ N}$

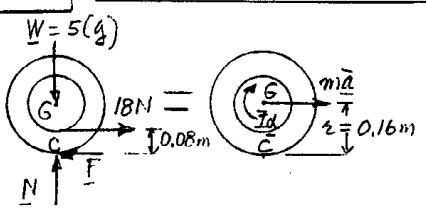
$F_m = \mu_s N = 0.20(49.05\text{ N}) = 9.81\text{ N}$

Since $F = 6.48\text{ N} < F_m$, disk actually rolls. Thus:

(a) Disk does not slide. (b) $\alpha = 14.4\text{ rad/s}^2\,\circlearrowleft$; $\underline{\bar{a} = 2.30\text{ m/s}^2 \rightarrow}$

16.115

Assume that the disk rolls



$\bar{a} = r\alpha$

$\bar{a} = 0.16\alpha$

$I = m\bar{k}^2 = (5\text{ kg})(0.12\text{ m})^2 = 0.072\text{ kg}\cdot\text{m}^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(18\text{ N})(0.08\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$1.44 = 5(0.16\alpha)(0.16) + 0.072\alpha$

$1.44 = 0.200\alpha \quad \underline{\alpha = 7.20\text{ rad/s}^2\,\circlearrowleft}$

$\bar{a} = r\alpha = (0.16)(7.20) = 1.152\text{ m/s}^2 \quad \underline{\bar{a} = 1.152\text{ m/s}^2 \rightarrow}$

(CONTINUED)

16.115 CONTINUED

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $18\text{ N} - F = m\bar{a}$

$F = 18 - m\bar{a} = 18 - 5(1.152) = 12.24\text{ N}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 5(g) = 0 \quad N = 5(9.81) = 49.05\text{ N}$

$F_m = \mu_s N = 0.20(49.05\text{ N}) = 9.81\text{ N}$

(a) Since $F = 12.24\text{ N} > F_m$, our assumption is wrong

Disk rotates and slides

(b) Knowing that disk slides, we write

$F = \mu_k N = 0.15(49.05\text{ N}) = 7.3575\text{ N}$

$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $F(0.16\text{ m}) - (18\text{ N})(0.08\text{ m}) = \bar{I}\alpha$

$(7.3575)(0.16) - (18)(0.08) = 0.072\alpha$

$\alpha = -3.65\text{ rad/s}^2 \quad \underline{\alpha = 3.65\text{ rad/s}^2\,\circlearrowright}$

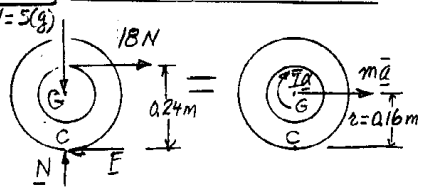
$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $18\text{ N} - F = m\bar{a}$

$18 - 7.3575 = 5\bar{a}$

$\bar{a} = +2.1285\text{ m/s}^2 \quad \underline{a = 2.13\text{ m/s}^2 \rightarrow}$

16.116

Assume that the disk rolls.



$\bar{a} = r\alpha$

$\bar{a} = 0.16\alpha$

$\bar{I} = m\bar{k}^2 = (5\text{ kg})(0.12\text{ m})^2 = 0.072\text{ kg}\cdot\text{m}^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(18\text{ N})(0.24\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$4.32 = 5(0.16\alpha)(0.16) + 0.072\alpha$

$4.32 = 0.200\alpha \quad \underline{\alpha = 21.6\text{ rad/s}^2\,\circlearrowleft}$

$\bar{a} = r\alpha = (0.16)(21.6) = 3.456\text{ m/s}^2 \quad \underline{\bar{a} = 3.46\text{ m/s}^2 \rightarrow}$

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $18\text{ N} - F = m\bar{a}$

$F = 18 - m\bar{a} = 18 - 5(3.456) = 0.720\text{ N}$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - 5(g) = 0 \quad N = 5(9.81) = 49.05\text{ N}$

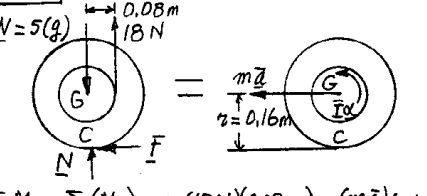
$F_m = \mu_s N = 0.20(49.05\text{ N}) = 9.81\text{ N}$

Since $F = 0.720\text{ N} < F_m$, disk actually rolls. Thus:

(a) Disk does not slide. (b) $\alpha = 21.6\text{ rad/s}^2\,\circlearrowleft$; $\underline{\bar{a} = 3.46\text{ m/s}^2 \rightarrow}$

16.117

Assume that disk rolls



$\bar{a} = r\alpha$

$\bar{a} = 0.16\alpha$

$\bar{I} = m\bar{k}^2 = (5\text{ kg})(0.12\text{ m})^2 = 0.072\text{ kg}\cdot\text{m}^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(18\text{ N})(0.08\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$1.44 = 5(0.16\alpha)(0.16) + 0.072\alpha$

$1.44 = 0.200\alpha \quad \underline{\alpha = 7.20\text{ rad/s}^2\,\circlearrowleft}$

$\bar{a} = r\alpha = (0.16)(7.20) = 1.152\text{ m/s}^2 \quad \underline{\bar{a} = 1.152\text{ m/s}^2 \leftarrow}$

$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $F = m\bar{a} = 5(1.152) = 5.76 \quad \underline{F = 5.76\text{ N} \leftarrow}$

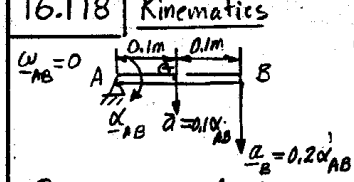
$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N + 18 - 5(9.81) = 0 \quad \underline{N = 31.05\text{ N}\uparrow}$

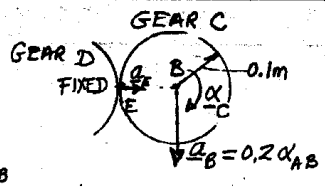
$F_m = \mu_s N = 0.20(31.05\text{ N}) = 6.21\text{ N}$

Since $F = 5.76\text{ N} < F_m$, disk actually rolls. Thus:

(a) Disk does not slide. (b) $\underline{\alpha = 7.20\text{ rad/s}^2\,\circlearrowleft}$; $\underline{\bar{a} = 1.152\text{ m/s}^2 \leftarrow}$

16.118 Kinematics





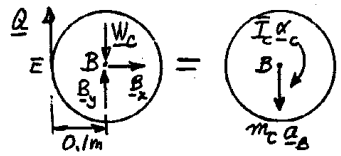
Since gear D is fixed, we have for point E of gear C:

$\downarrow (a_E)_t = 0$ But $\underline{a}_E = \underline{a}_B + \underline{a}_{E/B}$

$+\downarrow (a_E)_t = (a_B)_t + (a_{E/B})_t$

$0 = 0.2\alpha_{AB} - 0.1\alpha_C$ $\alpha_{AB} = \frac{1}{2}\alpha_C$ (1)

Kinetics - Gear C



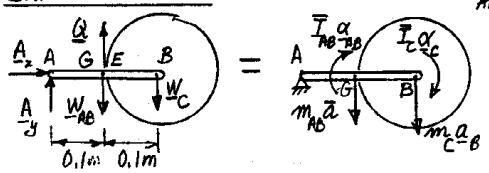
$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$Q(0.1\text{m}) = \bar{I}_C\alpha_C$

$= (5\text{kg})(0.075\text{m})^2\alpha_C$

$Q = 0.28125\alpha_C$ (2)

Bar AB and Gear C



$\bar{I}_{AB} = \frac{1}{12} m_{AB} L^2 = \frac{1}{12}(3\text{kg})(0.2\text{m})^2$

(a) $+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$W_{AB}(0.1) + W_C(0.2) - Q(0.3) = (m_{AB}\bar{a})0.1 + \bar{I}_{AB}\alpha_{AB} + (m_C a_B)0.2 - \bar{I}_C\alpha_C$

$(3)g(0.1) + (5)g(0.2) - Q(0.3) = (3)(0.1\alpha_{AB})0.1 + \frac{1}{12}(3)(0.2)^2\alpha_{AB} + 5(0.2\alpha_{AB})0.2 - 5(0.075)^2\alpha_C$

$(1.3)g - 0.1Q = 0.24\alpha_{AB} + 0.028125\alpha_C$

Substituting for α_{AB} and Q from (2) and (1):

$1.3g - 0.1(0.28125\alpha_C) = 0.24(\frac{1}{2}\alpha_C) + 0.028125\alpha_C$

$1.3g = 0.17625\alpha_C$ $\alpha_C = 7.3759(9.81)$

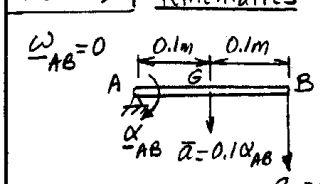
$\alpha_C = 72.36$ $\underline{\alpha}_C = 72.4\ \text{rad/s}^2\ \circlearrowright$ ◀

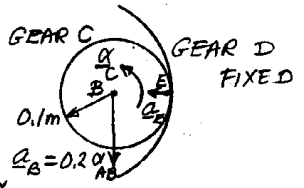
(b) $a_B = 0.2\alpha_{AB} = 0.2(\frac{1}{2}\alpha_C) = 0.1\alpha_C = 0.1(72.36)$

$\underline{a}_B = 7.24\ \text{m/s}^2 \downarrow$ ◀

Note: The same numerical values are obtained for α_C and a_B in Prob. 16.119.

16.119 Kinematics





Since gear D is fixed, we have for point E of gear C:

$\downarrow (a_E)_t = 0$ But $\underline{a}_E = \underline{a}_B + \underline{a}_{E/B}$

$+\downarrow (a_E)_t = (a_B)_t + (a_{E/B})_t$

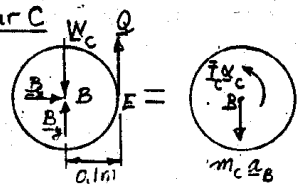
$0 = 0.2\alpha_{AB} - 0.1\alpha_C$

$\alpha_{AB} = \frac{1}{2}\alpha_C$ (1)

(CONTINUED)

16.119 CONTINUED Kinetics

Gear C



$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

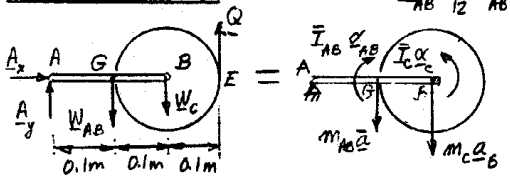
$Q(0.1\text{m}) = \bar{I}_C\alpha_C$

$= (5\text{kg})(0.075\text{m})^2\alpha_C$

$Q = 0.28125\alpha_C$ (2)

Bar AB and Gear C

$\bar{I}_{AB} = \frac{1}{12} m_{AB} L^2 = \frac{1}{12}(3\text{kg})(0.2\text{m})^2$



(a) $+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$W_{AB}(0.1) + W_C(0.2) - Q(0.3) = (m_{AB}\bar{a})0.1 + \bar{I}_{AB}\alpha_{AB} + (m_C a_B)0.2 - \bar{I}_C\alpha_C$

$(3)g(0.1) + (5)g(0.2) - Q(0.3) = (3)(0.1\alpha_{AB})0.1 + \frac{1}{12}(3)(0.2)^2\alpha_{AB} + 5(0.2\alpha_{AB})0.2 - 5(0.075)^2\alpha_C$

$(1.3)g - 0.3Q = 0.24\alpha_{AB} - 0.028125\alpha_C$

Substituting for α_{AB} and Q from (2) and (1):

$1.3g - 0.3(0.28125\alpha_C) = 0.24(\frac{1}{2}\alpha_C) - 0.028125\alpha_C$

$1.3g = 0.17625\alpha_C$ $\alpha_C = 7.3759(9.81)$

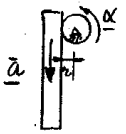
$\alpha_C = 72.36$ $\underline{\alpha}_C = 72.4\ \text{rad/s}^2\ \circlearrowright$ ◀

(b) $a_B = 0.2\alpha_{AB} = 0.2(\frac{1}{2}\alpha_C) = 0.1\alpha_C = 0.1(72.36)$

$\underline{a}_B = 7.24\ \text{m/s}^2 \downarrow$ ◀

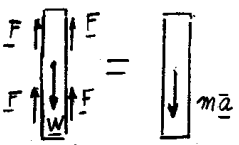
Note: The same numerical values were obtained for α_C and a_B in Prob. 16.118.

16.120 Kinematics:



$\bar{a} = r\alpha$ $\alpha = \frac{\bar{a}}{r}$

Kinetics of bar

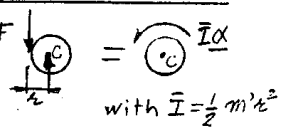


with $W = mg$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$W - 4F = m\bar{a}$ (1)

Kinetics of one disk



with $\bar{I} = \frac{1}{2} m' r^2$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$:

$Fr = \bar{I}\alpha$

$Fr = \frac{1}{2} m' r^2 (\frac{\bar{a}}{r})$

$F = \frac{1}{2} m' \bar{a}$ (2)

Substitute for F from (2) into (1):

$mg - 4(\frac{1}{2} m'\bar{a}) = m\bar{a}$

$mg = (m + 2m')\bar{a}$ $\bar{a} = \frac{mg}{m+2m'}$

(a) $m = 5\text{kg}$, $m' = 2\text{kg}$:

$\bar{a} = \frac{5}{5+2(2)} g$ $\underline{\bar{a}} = \frac{5}{9} g \downarrow$ ◀

(b) $m' = 0$: $\bar{a} = g$ $\underline{\bar{a}} = g \downarrow$ ◀

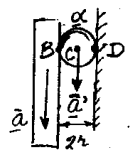
(c) $m = 0$ $\bar{a} = 0$ $\underline{\bar{a}} = 0$ ◀

16.121 Kinematics:

Disk is rolling on vertical wall.

$\bar{a}' = a_C = r\alpha$

$\bar{a} = a_B = 2r\alpha$

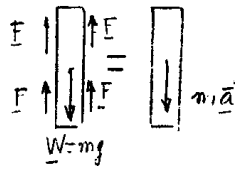


Therefore:

$\alpha = \frac{\bar{a}}{2r}$

$\bar{a}' = r\alpha = \frac{1}{2}\bar{a}$

Kinetics of bar

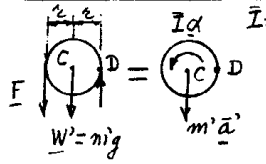


$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$W - 4F = m\bar{a}$

$mg - 2(2F) = m\bar{a}$ (1)

Kinetics of one disk



$\bar{I} = \frac{1}{2}m'r^2$

$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$:

$F(2r) + m'g\,r = \bar{I}\alpha + m'\bar{a}'r$

$2Fr + m'gr = \frac{1}{2}m'r^2\left(\frac{\bar{a}}{r}\right) + m'\frac{\bar{a}}{2}r$

$2F = m'\left(\frac{1}{4}\bar{a} + \frac{1}{2}\bar{a}\right) - m'g$ (2)

Substitute for 2F from (2) into (1):

$mg - 2m'\left(\frac{3}{4}\bar{a}\right) + 2m'g = m\bar{a}$

$\left(m + \frac{3}{2}m'\right)\bar{a} = (m + 2m')g$

$\bar{a} = \frac{m+2m'}{m+\frac{3}{2}m'}g$

(a) $m = 5$ kg, $m' = 2$ kg: $\bar{a} = \frac{5+4}{5+3}g$ $\quad \underline{\bar{a}} = \frac{9}{8}g \downarrow$ ◀

(b) $m' = 0$: $\bar{a} = g$ $\quad \underline{\bar{a}} = g \downarrow$ ◀

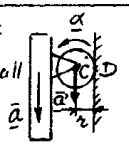
(c) $m = 0$: $\bar{a} = \frac{2}{3/2}g$ $\quad \underline{\bar{a}} = \frac{4}{3}g \downarrow$ ◀

Note: In parts (a) and (c), $\bar{a} > g$. From Eq. (1) it follows that $F < 0$. The disks push the bar down.

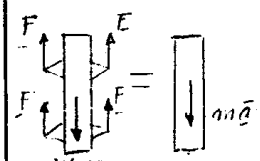
16.122 Kinematics:

Disk is rolling on vertical wall

$\bar{a} = \bar{a}' = r\alpha$



Kinetics of bar

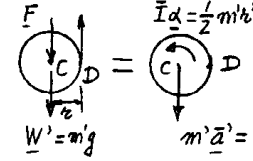


$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$W - 4F = m\bar{a}$

$mg - 4F = m\bar{a}$ (1)

Kinetics of one disk



$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$:

$(F + W')r = \bar{I}\alpha + m'\bar{a}'r$

$(F + m'g)r = \frac{1}{2}m'r^2\left(\frac{\bar{a}}{r}\right) + m'\bar{a}r$

$F = \frac{3}{2}m'\bar{a} - m'g$ (2)

Substitute for F from (2) into (1):

$mg - 6m'\bar{a} + 4m'g = m\bar{a}$

$(m + 6m')\bar{a} = (m + 4m')g$

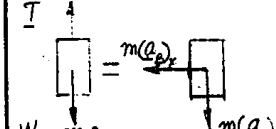
$\bar{a} = \frac{m+4m'}{m+6m'}g$

(a) $m = 5$ kg, $m' = 2$ kg: $\bar{a} = \frac{5+8}{5+12}$ $\quad \underline{\bar{a}} = \frac{13}{17}g \downarrow$ ◀

(b) $m' = 0$: $\bar{a} = g$ $\quad \underline{\bar{a}} = g \downarrow$ ◀

(c) $m = 0$: $\bar{a} = \frac{4}{6}g$ $\quad \underline{\bar{a}} = \frac{2}{3}g \downarrow$ ◀

16.123 (a) Kinetics of Block B (immediately after release)



$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $(a_B)_x = 0$ (1)

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$:

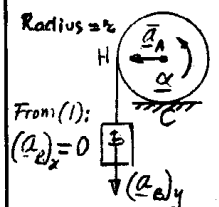
$mg - T = m(a_B)_y$ or $mg - T = ma_B$ (2)

(CONTINUED)

16.123 CONTINUED

Kinematics of block and cylinder

Since cylinder rolls: $\underline{a}_A = r\alpha \leftarrow$ (3)



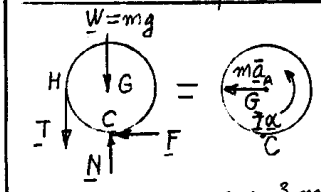
$\underline{a}_H = \underline{a}_A + \underline{a}_{H/A} = r\alpha \leftarrow + r\alpha \downarrow$

But, assuming cord inextensible:

$(\underline{a}_B)_y = (\underline{a}_H)_y = r\alpha \downarrow$

and thus: $\underline{a}_B = r\alpha \downarrow$ (4)

Kinetics of Cylinder A



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$Tr = (m\bar{a}_A)r + \bar{I}\alpha$

$= (mr\alpha)r + \frac{1}{2}mr^2\alpha$

$T = \frac{3}{2}mr\alpha$ (5)

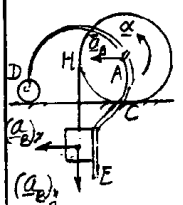
Substituting from (4) and (5) into (2):

$mg - \frac{3}{2}mr\alpha = mr\alpha \qquad \alpha = \frac{2}{5}\frac{g}{r}$

Substituting for α into (3) and (4):

$\bar{a}_A = a_B = r\left(\frac{2}{5}\frac{g}{r}\right) = \frac{2}{5}g \qquad \underline{\bar{a}}_A = \frac{2}{5}g \leftarrow ; \underline{a}_B = \frac{2}{5}g \downarrow$ ◀

(b) Kinematics of block and cylinder



Since cylinder rolls: $\underline{a}_A = r\alpha \leftarrow$ (6)

$\underline{a}_H = \underline{a}_A + \underline{a}_{H/A} = r\alpha \leftarrow + r\alpha \downarrow$

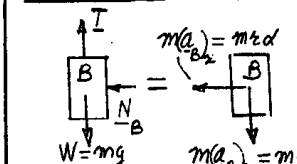
But, assuming cord inextensible:

$(\underline{a}_B)_y = (\underline{a}_H)_y = r\alpha \downarrow$

Also, constraint requires that $(\underline{a}_B)_x = \underline{a}_A = r\alpha \leftarrow$

Thus: $\underline{a}_B = r\alpha \leftarrow + r\alpha \downarrow$ (7)

Kinetics of Block B



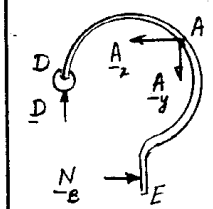
$\pm \Sigma F_x = \Sigma(F_x)_{eff}$:

$N_B = mr\alpha$ (8)

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$mg - T = mr\alpha$ (9)

Equilibrium of Member DAE



Since its mass is negligible, its effective forces are also negligible and DAE is in equilibrium

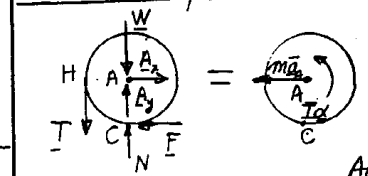
$\pm \Sigma F_x = 0$: $N_B - A_x = 0$

$A_x = N_B$

Recalling (8):

$A_x = mr\alpha$ (10)

Kinetics of Cylinder A



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$Tr - A_x r = (m\bar{a}_A)r + \bar{I}\alpha$

$= (mr\alpha)r + \frac{1}{2}mr^2\alpha$

$T - A_x = \frac{3}{2}mr\alpha$ (11)

Adding (9), (10), and (11):

$mg - T + A_x + T - A_x = mr\alpha + mr\alpha + \frac{3}{2}mr\alpha$

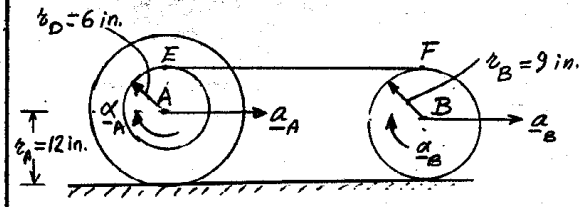
$mg = \frac{7}{2}mr\alpha \qquad \alpha = \frac{2}{7}\frac{g}{r}$

Substituting for α into (6) and (7):

$\bar{a}_A = (a_B)_x = (a_B)_y = r\left(\frac{2}{7}\frac{g}{r}\right) = \frac{2}{7}g \qquad \underline{\bar{a}}_A = \frac{2}{7}g \leftarrow$ ◀

$(\underline{a}_B)_x = \frac{2}{7}g \leftarrow, \quad (\underline{a}_B)_y = \frac{2}{7}g \downarrow$ ◀

16.124 Kinematics of system



$\underline{a}_A = r_A \alpha_A = 12\alpha_A \rightarrow$

$\underline{a}_E = \underline{a}_A + \underline{a}_{E/A} = 12\alpha_A \rightarrow + 6\alpha_A \rightarrow = 18\alpha_A \rightarrow$

$\underline{a}_B = r_B \alpha_B = 9\alpha_B \rightarrow$

$\underline{a}_F = \underline{a}_B + \underline{a}_{F/B} = 9\alpha_B \rightarrow + 9\alpha_B \rightarrow = 18\alpha_B \rightarrow$

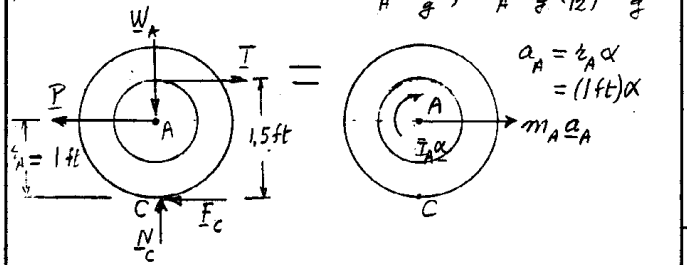
Assuming cord inextensible: $\underline{a}_E = \underline{a}_F$, and thus:

$18\alpha_A = 18\alpha_B$ or $\alpha_A = \alpha_B = \alpha$. Thus:

$$\underline{a}_A = r_A \alpha \rightarrow, \quad \underline{a}_B = r_B \alpha \rightarrow \qquad (1)$$

Disk-and-drum A

$m_A = \frac{24}{g}$, $\bar{I}_A = \frac{24}{g}\left(\frac{8}{12}\right)^2 = \frac{10.667}{g}$



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$: $T(1.5) - P(1) = (m_A a_A) r_A + \bar{I}_A \alpha$

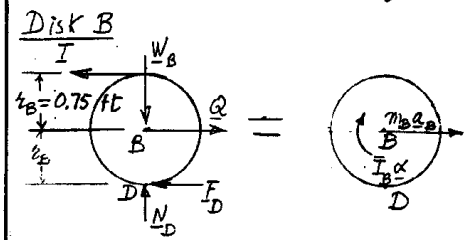
$T(1.5) - P(1) = \frac{24}{g}(\alpha)(1) + \frac{10.667}{g}\alpha$

$1.5T - P = 34.667\frac{\alpha}{g}$ (2)

Disk B

$m_B = \frac{16}{g}$

$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}\frac{16}{g}(0.75)^2 = \frac{4.5}{g}$



$a_B = r_B \alpha = 0.75\alpha$

$+\circlearrowright \Sigma M_D = \Sigma (M_D)_{eff}$: $Q(0.75) - T(1.5) = (m_B a_B) r_B + I_B \alpha$

$Q(0.75) - T(1.5) = \frac{16}{g}(0.75\alpha)(0.75) + \frac{4.5}{g}\alpha$

$0.75Q - 1.5T = \frac{13.5}{g}\alpha$ (3)

Adding (2) and (3): $0.75Q - P = 48.167\frac{\alpha}{g}$ (4)

Substituting $P = 0$ and $Q = 6$ lb into (4):

$0.75(6) - 0 = 48.167\frac{\alpha}{g}$ $\quad \alpha = 0.093426g$

From (1):

$a_A = r_A \alpha = (1\text{ ft})(0.093426)(32.2) = 3.008\text{ ft/s}^2$

$a_B = r_B \alpha = (0.75\text{ ft})(0.093426)(32.2) = 2.256\text{ ft/s}^2$

$\underline{a}_A = 3.01\text{ ft/s}^2 \rightarrow$; $\underline{a}_B = 2.26\text{ ft/s}^2 \rightarrow$ ◀

16.125

See solution of Prob. 16.124 up to dashed line for derivation of Eqs. (1) and (4):

$$a_A = r_A \alpha \qquad a_B = r_B \alpha \qquad (1)$$

$$0.75Q - P = 48.167\frac{\alpha}{g} \qquad (4)$$

Substituting $P = 6$ lb and $Q = 0$ in Eq. (4):

$-6\text{ lb} = 48.167\frac{\alpha}{g}$

$\alpha = -0.12457g$

$= -4.011\text{ rad/s}^2$

The minus sign indicates that $\underline{\alpha}$ has a sense opposite to that assumed in the sketches in the solution of Prob. 16.124. Thus:

$\underline{\alpha} = 4.011\text{ rad/s}^2$ ↻

From Eqs. (1):

$a_A = r_A \alpha = (1\text{ ft})(-4.011\text{ rad/s}^2) = -4.011\text{ ft/s}^2$

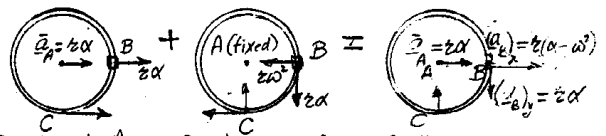
$a_B = r_B \alpha = (0.75\text{ ft})(-4.011\text{ rad/s}^2) = -3.008\text{ ft/s}^2$

The minus signs obtained indicate that $\underline{a}_A$ and $\underline{a}_B$ have senses opposite to those assumed. Therefore:

$\underline{a}_A = 4.01\text{ ft/s}^2 \leftarrow$; $\underline{a}_B = 3.01\text{ ft/s}^2 \leftarrow$ ◀

16.126

Kinematics: Rolling motion with $\omega = \sqrt{\frac{g}{2r}}$

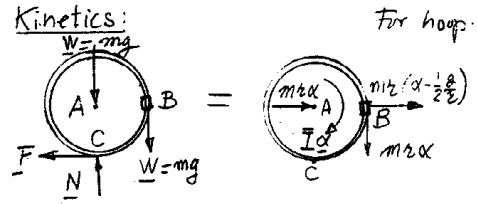


Transl. with A + Rotation about A = Rolling motion

$\bar{a}_A = r\alpha \rightarrow$ $\quad \underline{a}_B = r(\alpha - \omega^2) \rightarrow + r\alpha \downarrow$

$= r\left(\alpha - \frac{1}{2}\frac{g}{r}\right) \rightarrow + r\alpha \downarrow$ (1)

Kinetics: For hoop: $\bar{I} = mr^2$



(a) $+\circlearrowright \Sigma M_C = \Sigma (M_C)_{kin}$:

$mgr = (mr\alpha)r + \bar{I}\alpha + (mr\alpha)r + m\frac{r}{2}\left(\alpha - \frac{1}{2}\frac{g}{r}\right)r$

$= mr^2\alpha + mr^2\alpha + mr^2\alpha + mr^2\alpha - \frac{1}{2}mgr$

$\frac{3}{2}mgr = 4mr^2\alpha$ $\quad \underline{\alpha} = \frac{3}{8}\frac{g}{r}$ ↻ ◀

(b) From (1):

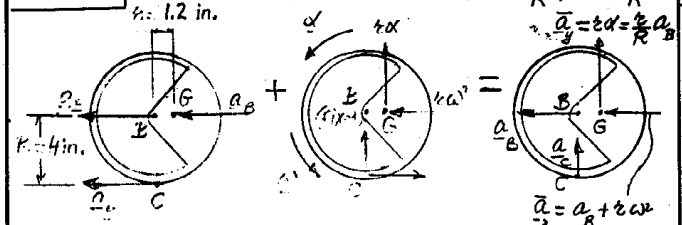
$\pm\rightarrow (a_B)_x = r\left(\frac{3}{8}\frac{g}{r} - \frac{1}{2}\frac{g}{r}\right) = -\frac{1}{8}g$ $\quad (\underline{a}_B)_x = \frac{1}{8}g \leftarrow$

$+\uparrow (a_B)_y = -r\left(\frac{3}{8}\frac{g}{r}\right) = -\frac{3}{8}g$ $\quad (\underline{a}_B)_y = \frac{3}{8}g \downarrow$

$\underline{a}_B = 0.395g \swarrow 71.6°$ ◀

Note: We could not use the equation $\Sigma M_C = \bar{I}_C \alpha$ for the hoop-block combination, since C, A, and G are not aligned (cf. Prob. 16.105).

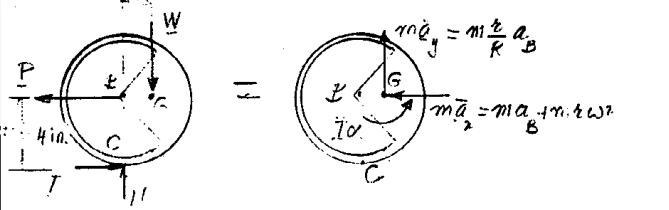
16.127 **Kinematics:** (Rolling motion) $\omega = \frac{v_B}{R}$, $\alpha = \frac{a_B}{R}$



Translation with B + Rotation about B = Rolling motion

$\bar{a} = (a_B + r\omega^2) \leftarrow + \frac{r}{R} a_B \uparrow$

Kinetics: $r = 1.2$ in. $\quad \bar{I} = m\bar{k}^2$



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $Pr = (m\bar{a}_y)r + (m\bar{a}_x)R + \bar{I}\alpha$

$Pr = (m\frac{r}{R}a_B)r + ma_B R + mr\omega^2 R + mk^2 \frac{r}{R}a_B$

$= ma_B\left(\frac{r^2}{R} + R + \frac{k^2}{R}\right) + mr\left(\frac{v_B}{R}\right)^2 R$

$P = W\left(\frac{r}{R}\right) + \frac{W}{g}a_B\left(1 + \frac{r^2+k^2}{Rr}\right) + \frac{W}{g}\frac{r}{R^2}v_B^2$

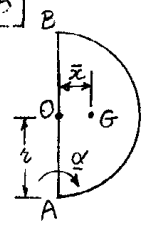
Data: $W = 4$ lb, $g = 32.2$ ft/s^2, $r = 1.2$ in. $= 0.1$ ft, $k = 3$ in. $= 0.25$ ft, $R = 4$ in. $= 1/3$ ft, $v_B = 2$ ft/s, $a_B = 5$ ft/s^2:

$P = 4\left(\frac{0.1}{1/3}\right) + \frac{4}{32.2}(5)\left[1 + \frac{(0.1)^2+(0.25)^2}{(1/3)^2}\right] + \frac{4}{32.2}\frac{0.1}{(1/3)^2}(2)^2$

$= 1.2 + 1.0764 + 0.4472 = 2.6736$ $\quad \underline{P = 2.67 \text{ lb}} \leftarrow$ ◀

Note: We could not use the equation $\Sigma M_C = I_C\alpha$ in this problem, since points C, B, and G are not aligned (cf. Prob. 16.105).

16.128 B



Kinematics $\omega = 0$

$\underline{a}_O = \underline{a}_A + \underline{a}_{O/A} = 0 + [r\alpha \rightarrow]$

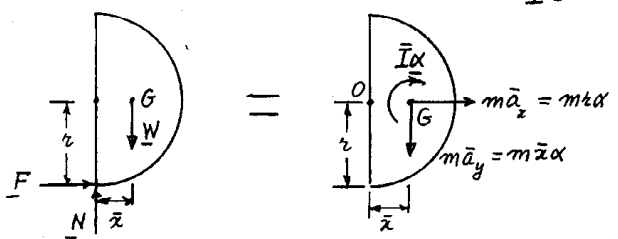
$\underline{\bar{a}} = \underline{a}_G = \underline{a}_O + \underline{a}_{G/O}$

$= [r\alpha \rightarrow] + [\bar{x}\alpha \downarrow]$

Thus: $\underline{\bar{a}}_x = r\alpha \rightarrow$, $\underline{\bar{a}}_y = \bar{x}\alpha \downarrow$ (1)

Kinetics

$\bar{I} = m\bar{k}^2$



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $W\bar{x} = (m\bar{a}_x)r + (m\bar{a}_y)\bar{x} + \bar{I}\alpha$

$mg\bar{x} = (mr\alpha)r + (m\bar{x}\alpha)\bar{x} + m\bar{k}^2\alpha$

$\alpha = \frac{g\bar{x}}{r^2 + \bar{x}^2 + \bar{k}^2}$ (2)

(CONTINUED)

16.128 CONTINUED

$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $F = m\bar{a}_x$

$F = mr\alpha$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - W = -m\bar{a}_y$

$N = mg - m\bar{x}\alpha$

$\mu_{min} = \frac{F}{N} = \frac{mr\alpha}{mg - m\bar{x}\alpha}$ $\quad \mu_{min} = \frac{r\alpha}{g - \bar{x}\alpha}$ (3)

For a hemisphere:

$\bar{x} = OG = \frac{3}{8}r$ $\quad \bar{I} = I_O - m\bar{x}^2 = \frac{2}{5}mr^2 - m\left(\frac{3}{8}r\right)^2$

$\bar{I} = \frac{2}{5}mr^2 - \frac{9}{64}mr^2$ $\quad \bar{k}^2 = \frac{\bar{I}}{m} = \left(\frac{2}{5} - \frac{9}{64}\right)r^2$

Substituting into (2):

$\alpha = \frac{g\left(\frac{3}{8}r\right)}{r^2 + \frac{9}{64}r^2 + \left(\frac{2}{5} - \frac{9}{64}\right)r^2} = \frac{3/8}{7/5}\frac{g}{r}$ $\quad \alpha = \frac{15}{56}\frac{g}{r}$

(a) Substituting into (3):

$\mu_{min} = \frac{15/56}{1 - (3/8)(15/56)} = \frac{0.26786}{0.89955}$ $\quad \mu_{min} = 0.298$ ◀

(b) $a_B = (2r)\alpha = (2r)(15g/56r) = 30g/56$, $\quad \underline{a_B = 0.536g} \rightarrow$ ◀

Note: In this problem and the next, we cannot use the equation $\Sigma M_A = I_A\alpha$, since points A, O, and G are not aligned (cf. Prob. 16.106).

16.129 See solution of Prob. 16.128 up to dashed line.

For a half cylinder:

$\bar{x} = OG = \frac{4r}{3\pi}$ $\quad \bar{I} = I_O - m\bar{x}^2 = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$

$\bar{k}^2 = \frac{\bar{I}}{m} = \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2$

Substituting into (2):

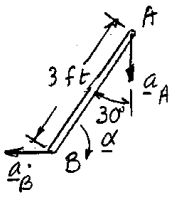
$\alpha = \frac{g(4r/3\pi)}{r^2 + \left(\frac{4r}{3\pi}\right)^2 + \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2} = \frac{4/3\pi}{3/2}\frac{g}{r}$ $\quad \alpha = \frac{8}{9\pi}\frac{g}{r}$

(a) Substituting into (3):

$\mu_{min} = \frac{8/9\pi}{1 - (4/3\pi)(8/9\pi)} = \frac{0.28294}{0.87992}$ $\quad \mu = 0.322$ ◀

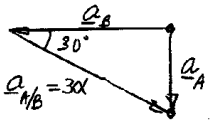
(b) $a_B = (2r)\alpha = (2r)(8g/9\pi r) = 16g/9\pi$, $\quad \underline{a_B = 0.566g}$ ◀

16.130 **Kinematics** $\omega = 0$. Assume $\underline{\alpha}$ ↺

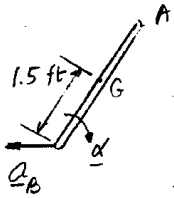


$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_A \updownarrow = \overleftrightarrow{a_B} + (3\text{ ft})\alpha \quad \measuredangle 30°$



$a_B = (3\alpha)\cos 30° = 2.598\alpha \leftarrow$



$\underline{\bar{a}} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B}$

$= 2.598\alpha \leftarrow + 1.5\alpha \measuredangle 30°$

$\pm a_x = -2.598\alpha + (1.5\cos 30°)\alpha = -1.299\alpha$

$\underline{a}_x = 1.299\alpha \leftarrow$

$+\uparrow a_y = -(1.5\sin 30°)\alpha = -0.75\alpha$

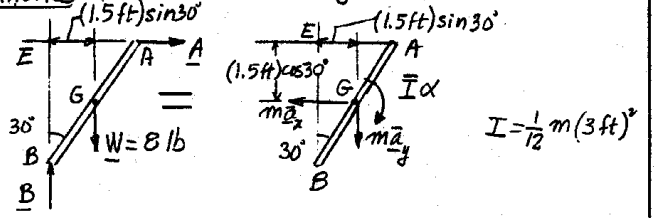
$\underline{a}_y = 0.75\alpha \downarrow$

(CONTINUED)

16.130 CONTINUED We recall the results obtained on the preceding page:

Assuming α), we have $\bar{a}_x = 1.299\alpha \leftarrow$, $\bar{a}_y = 0.75\alpha \downarrow$ (1)

Kinetics



$I = \frac{1}{12} m(3\text{ ft})^2$

(a) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$:

$W(1.5\sin30°) = \bar{I}\alpha + (m\bar{a}_x)(1.5\cos30°) + (m\bar{a}_y)(1.5\sin30°)$

$0.75W = \frac{1}{12}m(3)^2\alpha + m(1.299\alpha)(1.299) + m(0.75\alpha)(0.75)$

$0.75W = m(0.75 + 1.6875 + 0.5625)\alpha = 3m\alpha$

$\alpha = \frac{0.75W}{3m} = 0.25g = 0.25(32.2\text{ ft/s}^2)$

$\underline{\alpha = 8.05\text{ rad/s}^2\circlearrowleft}$ ◀

Recalling (1):

$\bar{a}_x = 1.299(8.05)$ $\quad \underline{\bar{a}_x = 10.457\text{ ft/s}^2 \leftarrow}$

$\bar{a}_y = 0.75(8.05)$ $\quad \underline{\bar{a}_y = 6.0375\text{ ft/s}^2 \leftarrow}$

(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $A = -m\bar{a}_x$

$A = -\frac{8}{32.2}(10.457) = -2.598\text{ lb}$ $\quad \underline{A = 2.60\text{ lb} \leftarrow}$ ◀

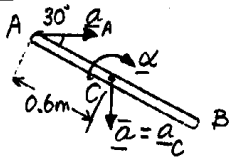
(c) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $B - W = -m\bar{a}_y$

$B = W - m\bar{a}_y = 8 - \frac{8}{32.2}(6.0375) = +6.50\text{ lb}$

$\underline{B = 6.50\text{ lb}\uparrow}$ ◀

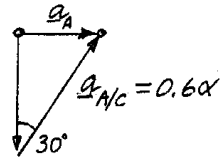
16.131 Kinematics

Assume α)



$\underline{a}_A = \underline{a}_C + \underline{a}_{A/C}$

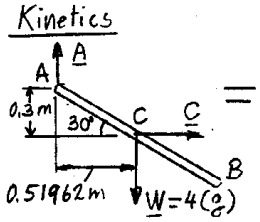
$\overleftrightarrow{a}_A = \bar{a}\updownarrow + 0.6\alpha \measuredangle 60°$



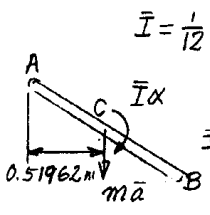
$\bar{a} = 0.6\alpha\cos30° = 0.51962\alpha$

$\underline{\bar{a} = 0.51962\alpha\downarrow}$

Kinetics



$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(4\text{ kg})(1.2\text{ m})^2 = 0.48\text{ kg}\cdot\text{m}^2$



$\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $C = 0$

(a) $\Sigma(M_A) = \Sigma(M_A)_{eff}$: $W(0.51962) - C(0.3) = (m\bar{a})(0.51962) + \bar{I}\alpha$

$(4)(9.81)(0.51962) - (0)(0.3) = (4)(0.51962)^2\alpha + 0.48\alpha$

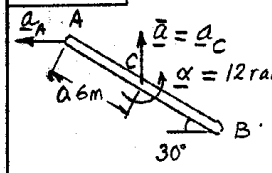
$20.390 = 1.5600\alpha$ $\quad \underline{\alpha = 13.07\text{ rad/s}^2\circlearrowleft}$ ◀

(b) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A - W = -m\bar{a}$

$A = W - m\bar{a} = (4)(9.81) - (4)(0.51962 \times 13.07)$

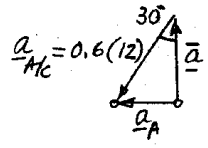
$A = +12.074\text{ N}$ $\quad \underline{A = 12.07\uparrow}$ ◀

16.132 Kinematics $\quad \underline{\alpha = 12\text{ rad/s}^2\circlearrowleft}$



$\underline{a}_A = \underline{a}_C + \underline{a}_{A/C}$

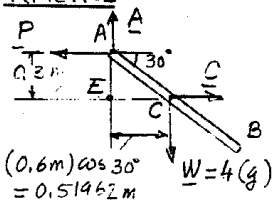
$\overleftrightarrow{a}_A = \bar{a}\updownarrow + 0.6(12)\measuredangle 60°$

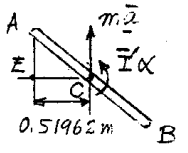


$\bar{a} = 0.6(12)\cos30°$

$\underline{\bar{a} = 6.2354\text{ m/s}^2\uparrow}$

Kinetics





$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(4\text{ kg})(1.2\text{ m})^2 = 0.48\text{ kg}\cdot\text{m}^2$

(a) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$: $P(0.3\text{m}) - W(0.51962\text{m}) = m\bar{a}(0.51962\text{m}) + \bar{I}\alpha$

$P(0.3) - 4(9.81)(0.51962) = 4(6.2354)(0.51962) + 0.48(12)$

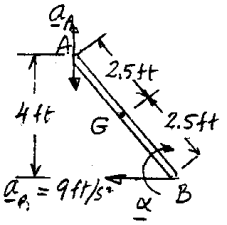
$P = +130.37$ $\quad \underline{P = 130.4\text{ N} \leftarrow}$ ◀

(b) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A - W = m\bar{a}$

$A = W + m\bar{a} = 4(9.81) + 4(6.2354)$

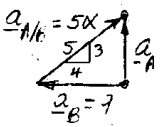
$= +64.18\text{ N}$ $\quad \underline{A = 64.2\text{ N}\uparrow}$ ◀

16.133 Kinematics. Initially, end A is constrained to move $\updownarrow$ and end B with $\underline{a}_B = 9\text{ ft/s}^2 \leftarrow$. Thus:



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_A\updownarrow = 9\leftarrow + 5\alpha\ \frac{5}{4}\ \frac{3}{}$



Similar triangles: $\frac{5\alpha}{5} = \frac{9}{4}$

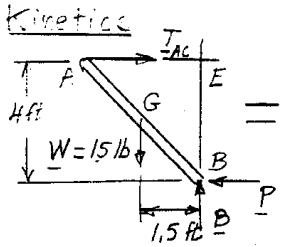
$\underline{\alpha = 2.25\text{ rad/s}^2\circlearrowleft}$

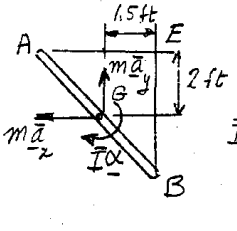
$\bar{a} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = 9\leftarrow + 2.5\alpha\ \frac{5}{4}\ \frac{3}{}$

$\bar{a}_x = 9\leftarrow + \frac{4}{5}(2.5)(2.25)\rightarrow = 4.5\text{ ft/s}^2\leftarrow$

$\bar{a}_y = \frac{3}{5}(2.5)(2.25)\uparrow = 3.375\text{ ft/s}^2\uparrow$

Kinetics





$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}\frac{15}{g}(5)^2 = \frac{31.25}{g}$

(a) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$: $P(4) - W(1.5) = \bar{I}\alpha + m\bar{a}_x(2) + m\bar{a}_y(1.5)$

$4P - (15)(1.5) = \frac{31.25}{g}(2.25) + \frac{15}{g}(4.5)(2) + \frac{15}{g}(3.375)(1.5)$

$4P - 22.5 = \frac{281.25}{32.2} = 8.7345$ $\quad P = +7.3086$

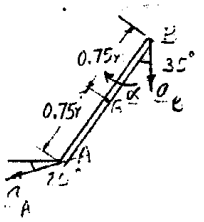
$\underline{P = 7.31\text{ lb} \leftarrow}$ ◀

(b) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $T_{AC} - P = -m\bar{a}_x$

$T_{AC} = P - m\bar{a}_x = 7.8086 - \frac{15}{32.2}(4.5) = 5.7123\text{ lb}$

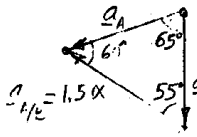
$\underline{T = 5.71\text{ lb}}$ ◀

16.134 Kinematics Assume α ↻ ($\omega=0$)



$\underline{a}_A = \underline{a}_E + \underline{a}_{A/E}$

$a_A \measuredangle 25° = a_E \uparrow + 1.5\alpha \measuredangle 35°$



Law of sines:

$\frac{a_E}{\sin 65°} = \frac{1.5\alpha}{\sin 65°}$

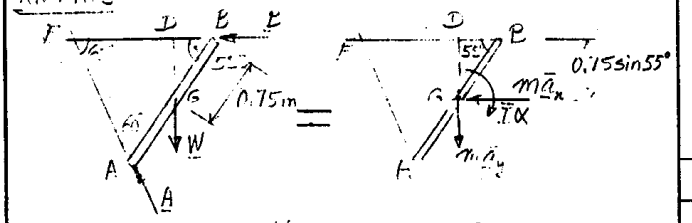
$a_E = 1.4333\alpha$

$\underline{a} = \underline{a}_G = \underline{a}_E + \underline{a}_{G/E} = 1.4333\alpha \downarrow + 0.75\alpha \measuredangle 35°$

$\bar{a}_x = 0.75\alpha\cos 35° \leftarrow = 0.61436\alpha \leftarrow$

$\bar{a}_y = 1.4333\alpha \downarrow + 0.75\alpha \sin 35° \downarrow = 1.0031\alpha \downarrow$

Kinetics



Law of sines: $\frac{EB}{\sin 60°} = \frac{AB}{\sin 45°}$, $EB = \frac{\sin 60°}{\sin 45°}(1.5\text{m}) = 1.4333\text{m}$

$ED = EB - DB = 1.4333 - 0.75\cos 55° = 1.0031\text{ m}$

$DG = 0.75\sin 55° = 0.61436\text{ m}$

(a) $+\circlearrowleft \sum M_E = \sum (M_E)_{eff}$: $W(FD) = \bar{I}\alpha + m\bar{a}_x(DG) + m\bar{a}_y(ED)$

$(6)(9.81)(1.0031) = \frac{1}{12}(6)(1.5)^2\alpha + (6)(0.61436\alpha)(0.61436) + (6)(1.0031\alpha)(1.0031)$

$59.04 = (1.125 + 2.7646 + 6.3778)\alpha$

$59.04 = 9.427\alpha \qquad \alpha = +6.263, \quad \underline{\alpha = 6.26\text{ rad/s}^2}$ ↻ ◀

$\bar{a}_x = 0.61436(6.263) = 3.848\text{ m/s}^2$

$\bar{a}_y = 1.0031(6.263) = 6.282\text{ m/s}^2$

(b) $+\circlearrowleft \sum M_A = \sum (M_A)_{eff}$:

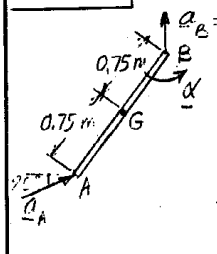
$W(7.5\cos 55°) - B(1.5\sin 55°) = \bar{I}\alpha + m\bar{a}_y(0.75\cos 55°) - m\bar{a}_x(0.75\sin 55°)$

$6(9.81)(0.75\cos 55°) - B(1.5\sin 55°) = \frac{1}{12}(6)(1.5)^2(6.263) + 6(6.282)(0.75\cos 55°) - 6(3.848)(0.75\sin 55°)$

$25.321 - 1.2287B = 2.046 + 16.214 - 14.184$

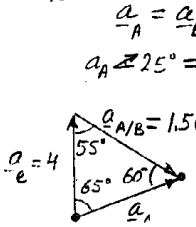
$16.245 = 1.2287B \qquad \underline{B = 13.22\text{ N}}$ ◀

16.135 Kinematics $\underline{a}_B = 4\text{ m/s}^2 \uparrow$ ($\omega=0$)



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_A \measuredangle 25° = 4\uparrow + 1.5\alpha \measuredangle 35°$



Law of sines:

$\frac{1.5\alpha}{\sin 65°} = \frac{4}{\sin 60°}$

$\alpha = 2.7907\text{ rad/s}^2$ ↻

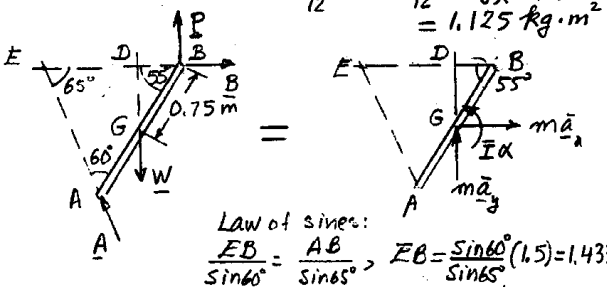
$\underline{a} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = 4\uparrow + 0.75(2.7907) \measuredangle 35°$

$\bar{a}_x = 0.75(2.7907)\cos 35° \rightarrow = 1.7145\text{ m/s}^2 \rightarrow$

$\bar{a}_y = 4\uparrow + 0.75(2.7907)\sin 35° \downarrow = 2.7995\text{ m/s}^2 \uparrow$

16.135 CONTINUED Kinetics

$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(6\text{kg})(1.5\text{m})^2 = 1.125\text{ kg}\cdot\text{m}^2$



Law of sines: $\frac{EB}{\sin 60°} = \frac{AB}{\sin 65°}$, $EB = \frac{\sin 60°}{\sin 65°}(1.5) = 1.4333\text{m}$

$ED = EB - DB = 1.4333 - 0.75\cos 55° = 1.0031\text{ m}$

$DG = 0.75\sin 55° = 0.61436\text{m}$

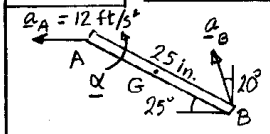
$+\circlearrowleft \sum M_E = \sum (M_E)_{eff}$: $P(EB) - W(ED) = \bar{I}\alpha + m\bar{a}_x(DG) + m\bar{a}_y(ED)$

$P(1.4333) - 6(9.81)(1.0031) = 1.125(2.7907) + 6(1.7145)(0.61436) + 6(2.7995)(1.0031)$

$1.4333P - 59.0425 = 3.1395 + 6.3199 + 16.8491$

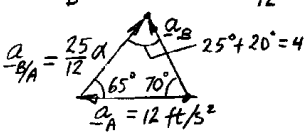
$1.4333P = 85.351 \qquad \underline{P = 59.5\text{ N}} \uparrow$ ◀

16.136 Kinematics $\underline{a}_A = 12\text{ ft/s}^2 \leftarrow$ ($\omega=0$)



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$a_B \measuredangle 20° = 12 \leftarrow + (\frac{25}{12}\text{ft})\alpha \measuredangle 25°$



Law of sines: $\frac{(25/12)\alpha}{\sin 70°} = \frac{12}{\sin 45°}$

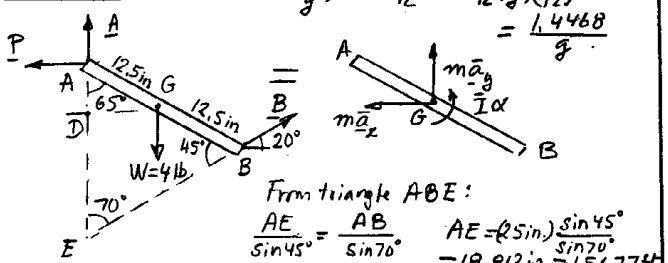
$\underline{\alpha = 7.6546\text{ rad/s}^2}$ ↻

$\underline{a} = \underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 12 \leftarrow + \frac{12.5}{12}(7.6546) \measuredangle 65°$

$\pm \rightarrow \bar{a}_x = -12 + \frac{12.5}{12}(7.6546)\cos 65° = -8.6302, \quad \underline{\bar{a}_x = 8.6302} \leftarrow$

$+\uparrow \bar{a}_y = +\frac{12.5}{12}(7.6546)\sin 65° = +7.2265, \quad \underline{\bar{a}_y = 7.2265} \uparrow$

Kinetics $\quad m = \frac{4}{g}, \quad \bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(\frac{4}{g})(\frac{25}{12})^2 = \frac{1.4468}{g}$



From triangle ABE: $\frac{AE}{\sin 45°} = \frac{AB}{\sin 70°}$ $\quad AE = (25\sin)\frac{\sin 45°}{\sin 70°} = 18.812\text{ in.} = 1.5677\text{ ft}$

$GD = (12.5\text{ in.})\sin 65° = 11.329\text{ in.} = 0.9441\text{ ft}$

$DE = AE - AD = 18.812 - (12.5)\cos 65° = 13.579\text{ in.} = 1.1275\text{ ft}$

(a) $+\circlearrowleft \sum M_E = \sum (M_E)_{eff}$: $P(AE) - W(GD) = \bar{I}\alpha + m\bar{a}_x(DE) + m\bar{a}_y(GD)$

$P(1.5677) - 4(0.9441) = \frac{1.4468}{g}(7.6546) + \frac{4}{g}(8.6302)(1.1275) + \frac{4}{g}(7.2265)(0.9441)$

$1.5677P - 3.776 = \frac{1}{32.2}(11.075 + 38.922 + 27.289) = 2.400$

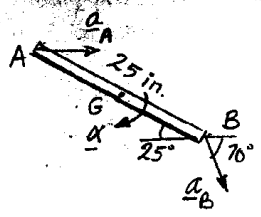
$P = 3.940\text{ lb} \qquad \underline{P = 3.94\text{ lb}} \leftarrow$ ◀

(b) $\pm \rightarrow \sum F_x = \sum (F_x)_{eff}$: $B\cos 20° - P = -m\bar{a}_x$

$B\cos 20° = 3.940 - \frac{4}{32.2}(8.6302) = 2.868$

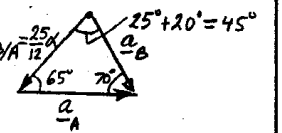
$B = 3.052\text{ lb} \qquad \underline{B = 3.05\text{ lb} \measuredangle 20°}$ ◀

16.137 Kinematics Assume α ↻ ($\omega = 0$)



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$a_B \nabla 70° = a_A \rightarrow + \left(\frac{25}{12}\text{ft}\right)\alpha \nabla 65°$



Law of sines: $\frac{a_A}{\sin 45°} = \frac{a_B}{\sin 65°} = \frac{(25/12)\alpha}{\sin 70°}$

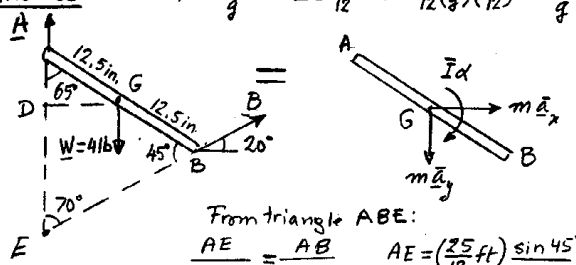
$\underline{a}_A = 1.5677\alpha \rightarrow$ $\underline{a}_B = 2.0093\alpha \nabla 70°$

$\underline{\bar{a}} = \underline{a}_G = \frac{1}{2}(\underline{a}_A + \underline{a}_B) = 0.7838\alpha \rightarrow + 1.0047\alpha \nabla 70°$

$\bar{a}_x = 0.7838\alpha + 1.0047\alpha \cos 70° = 1.1275\alpha \rightarrow$

$\bar{a}_y = 1.0047\alpha \sin 70° = 0.9441\alpha \downarrow$

Kinetics $m = \frac{4}{g}$ $\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}\left(\frac{4}{g}\right)\left(\frac{25}{12}\right)^2 = \frac{1.4468}{g}$



From triangle ABE:

$\frac{AE}{\sin 45°} = \frac{AB}{\sin 70°}$ $AE = \left(\frac{25}{12}\text{ft}\right)\frac{\sin 45°}{\sin 70°}$

$AE = 1.5677\text{ ft}$

$GD = \left(\frac{12.5}{12}\text{ft}\right)\sin 65° = 0.9441\text{ ft}$

$DE = AE - AD = 1.5677\text{ ft} - \left(\frac{12.5}{12}\text{ft}\right)\cos 65° = 1.1275\text{ ft}$

(a) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$: $W(GD) = \bar{I}\alpha + m\bar{a}_x(DE) + m\bar{a}_y(GD)$

$4(0.9441) = \frac{1.4468}{g}\alpha + \frac{4}{g}(1.1275\alpha)(1.1275) + \frac{4}{g}(0.9441\alpha)(0.9441)$

$3.7764 = \frac{\alpha}{g}(1.4468 + 5.0850 + 3.5653)$

$3.7764 = 10.097\frac{\alpha}{g}$, $\alpha = 0.3740g$, $\underline{\alpha} = 12.04\text{ rad/s}^2$ ↻ ◀

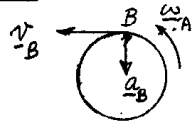
(b) $\pm\Sigma F_x = \Sigma(F_x)_{eff}$: $B\cos 20° = m\bar{a}_x = m(1.1275\alpha)$

$B\cos 20° = \frac{4}{g}(1.1275)(0.3740g) = 1.6867\text{ lb}$

$\underline{B} = 1.795\text{ lb} \angle 20°$ ◀

16.138 Kinematics

For disk A: ($\alpha_A = 0$)

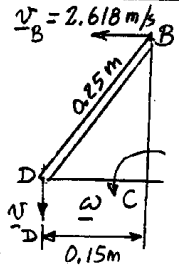


$\omega_A = 500\text{ rpm} = 52.36\text{ rad/s}$

$v_B = r\omega_A = (0.05\text{ m})(52.36\text{ rad/s}) = 2.618\text{ m/s}$

$a_B = r\omega_A^2 = (0.05\text{ m})(52.36\text{ rad/s})^2 = 137.08\text{ m/s}^2$

For rod (velocities)



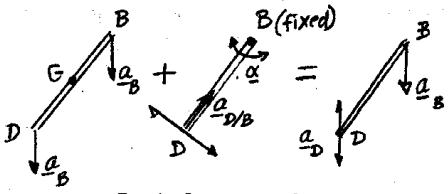
$BC = \sqrt{(0.25)^2 - (0.15)^2} = 0.20\text{ m}$

$\omega = \frac{v_B}{BC} = \frac{2.618\text{ m/s}}{0.20\text{ m}}$

$\underline{\omega} = 13.09\text{ rad/s}$ ↺

(CONTINUED)

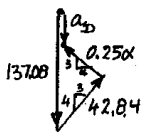
16.138 CONTINUED Kinematics of rod (accelerations)



$\underline{a}_B + \underline{a}_{D/B} = \underline{a}_D$

$a_B \downarrow + (a_{D/B})_n \frac{3}{5}\frac{4}{5} + (a_{D/B})_t \frac{4}{5}\frac{3}{5} = a_D \downarrow$

$137.08 \downarrow + 0.25(13.09)^2 \frac{3}{5}\frac{4}{5} + 0.25\alpha \frac{4}{5}\frac{3}{5} = a_D \downarrow$



$\pm$ components:

$\frac{3}{5}(42.84) - \frac{4}{5}(0.25\alpha) = 0$

$25.704 - 0.2\alpha = 0$

$\underline{\alpha} = 128.52\text{ rad/s}^2$ ↻

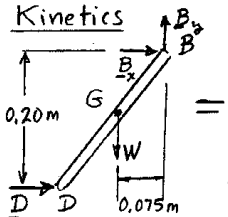
$\underline{\bar{a}} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = 137.08 \downarrow + 0.125(13.09)^2 \rightarrow + 0.125(128.52) \nwarrow$

$= 137.08 \downarrow + 21.42 \frac{3}{5}\frac{4}{5} + 16.065 \frac{4}{5}\frac{3}{5}$

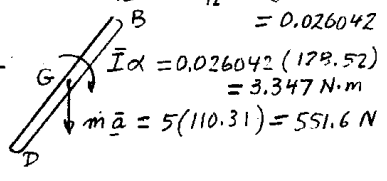
$\pm \bar{a}_x = \frac{3}{5}(21.42) - \frac{4}{5}(16.065) = 0$

$+\downarrow \bar{a}_y = 137.08 - \frac{4}{5}(21.42) - \frac{3}{5}(16.065) = 110.31\text{ m/s}^2$

Kinetics



$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(5\text{ kg})(0.25\text{ m})^2 = 0.026042$



$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $D(0.20\text{m}) + W(0.075\text{m}) = m\bar{a}(0.075) - \bar{I}\alpha$

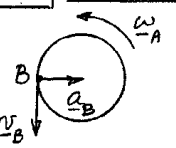
$0.2D + 5(9.81)(0.075) = 551.6(0.075) - 3.347$

$0.2D = 41.370 - 3.347 - 3.679 = 34.344$

$D = 171.7\text{ N}$ $\underline{D} = 171.7\text{N} \rightarrow$ ◀

16.139 Kinematics

For disk A: ($\alpha_A = 0$)

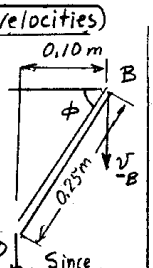


$\omega_A = 500\text{ rpm} = 52.36\text{ rad/s}$

$v_B = r\omega_A = (0.05\text{m})(52.36\text{ rad/s}) = 2.618\text{ m/s}$

$a_B = r\omega_A^2 = (0.05\text{m})(52.36\text{ rad/s})^2 = 137.08\text{ m/s}^2$

For rod (velocities)

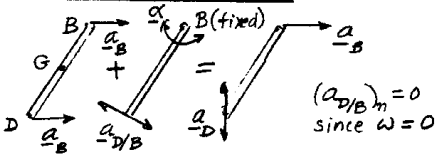


Since $\underline{v}_D \parallel \underline{v}_B$, we have $\underline{\omega} = 0$.

We also note that $\cos\phi = \frac{0.10}{0.25}$

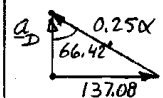
$\phi = 66.42°$

For rod (accelerations)



$\underline{a}_B + \underline{a}_{D/B} = \underline{a}_D$

$137.08 \rightarrow + 0.25\alpha \nwarrow = a_D \downarrow$



$(0.25\alpha)\sin 66.42° = 137.08$

$\underline{\alpha} = 598.3\text{ rad/s}^2$ ↻

$\underline{\bar{a}} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B}$

$\pm \bar{a}_x = 137.08 - 0.125(598.3)\sin 66.42° = +68.54\text{ m/s}^2$

$+\uparrow \bar{a}_y = 0.125(598.3)\cos 66.42° = +29.92\text{ m/s}^2$

(CONTINUED)

16.139 CONTINUED

We recall from previous page:

$\alpha = 598.3$ rad/s^2, $\bar{a}_x = 68.54$ m/s^2 →, $\bar{a}_y = 29.92$ m/s^2 ↑

Kinetics

$\bar{I} = \frac{1}{12} m\ell^2 = \frac{1}{12}(5\text{ kg})(0.25\text{ m})^2 = 0.026042$

$\bar{I}\alpha = 0.026042(598.3) = 15.581$ N·m

$m\bar{a}_x = 5(68.54) = 342.7$ N

$m\bar{a}_y = 5(29.92) = 149.6$ N

We recall that $\phi = 66.42°$, Thus: $h = (0.25\text{ m})\sin\phi = 0.2291$ m

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $Dh + W(0.05) = m\bar{a}_x(\frac{h}{2}) - m\bar{a}_y(0.05) - \bar{I}\alpha$

$D(0.2291) + 5(9.81)(0.05) = (342.7)\frac{0.2291}{2} - (149.6)(0.05) - 15.581$

$0.2291D = 39.256 - 7.480 - 15.581 - 2.453 = 13.742$, $\underline{D = 60.0\text{ N}}$ ◀

16.140

Kinematics

Crank AB:

$\omega_{AB} = 12$ rad/s ↻, $\alpha_{AB} = 80$ rad/s^2 ↻

$(a_B)_n = r\omega^2 = (\frac{3}{12}\text{ ft})(12\text{ rad/s})^2 = 36.0$ ft/s^2

$(a_B)_t = r\alpha = (\frac{3}{12}\text{ ft})(80\text{ rad/s}^2) = 20.0$ ft/s^2

Rod BD

Velocities:

$v_B = (AB)\omega_{AB} = \frac{3}{12}(12) = 3$ rad/s

$\omega_{BD} = \frac{v_B}{BC} = \frac{3\text{ ft/s}}{0.5\text{ ft}}$

$\omega_{BD} = 6$ rad/s ↻

Accelerations:

Transl. with B + Rotation about B = Plane motion

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$

$a_D \downarrow = 20 \leftarrow + 36 \uparrow + (a_{D/B})_n \frac{3}{5} + (a_{D/B})_t \frac{4}{5}$ (1)

where $(a_{D/B})_n = (BD)\omega_{BD}^2 = (\frac{10}{12})(6)^2 = 30.0$ ft/s^2

and $(a_{D/B})_t = (BD)\alpha_{BD} = \frac{10}{12}\alpha_{BD}$

$+\rightarrow$ x components:

$-20 - \frac{4}{5}(30) + \frac{3}{5}(\frac{10}{12}\alpha_{BD}) = 0$

$\frac{1}{2}\alpha_{BD} = 20 + 24 = 44.0$

$\alpha_{BD} = 88.0$ rad/s^2

Since $(\underline{a}_{D/B})_t$ is directed ↗:

$\underline{\alpha}_{BD} = 88.0$ rad/s^2 ↻

$\bar{\underline{a}} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + \frac{1}{2}\underline{a}_{D/B}$

$\bar{\underline{a}} = 20 \leftarrow + 36 \uparrow + \frac{1}{2}(30)\frac{3}{5} + \frac{1}{2}(\frac{10}{12})(88)\frac{4}{5}$

$+\rightarrow \bar{a}_x = -20 - \frac{4}{5}(15) + \frac{3}{5}(36.667) = -10.00$, $\bar{a}_x = 10.00$ ft/s^2 ←

$+\uparrow \bar{a}_y = 36 + \frac{3}{5}(15) + \frac{4}{5}(36.667) = 74.33$ $\bar{a}_y = 74.33$ ft/s^2 ↑

(CONTINUED)

16.140 CONTINUED

We recall from previous column:

$\alpha_{BD} = 88.0$ rad/s^2 ↻; $\bar{a}_x = 10.00$ ft/s^2 ←, $\bar{a}_y = 74.33$ ft/s^2 ↑

Kinetics

$\bar{I} = \frac{1}{12}\frac{3}{g}(\frac{10}{12})^2 = \frac{0.17361}{g}$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $D(\frac{6}{12}\text{ ft}) - W(\frac{4}{12}) = \bar{I}\alpha + m\bar{a}_y(\frac{4}{12}) - m\bar{a}_x(\frac{3}{12})$

$0.5D - 3(\frac{1}{3}) = \frac{0.17361}{g}(88) + \frac{3}{g}(74.33)(\frac{1}{3}) - \frac{3}{g}(10)(\frac{1}{4})$

$0.5D - 1 = \frac{1}{32.2}(82.108)$, $D = 7.100$ lb $\underline{D = 7.10\text{ lb}}$ ◀

16.141

Kinematics $\omega_{AB} = 12$ rad/s ↻, $\alpha_{AB} = 80$ rad/s^2 ↻

Same analysis as for Prob. 16.140, except that $\underline{\alpha}_{AB}$ ↻ resulting in $(\underline{a}_B)_t$ →. Eq. (1) must be replaced by

$a_D \downarrow = 20 \rightarrow + 36 \uparrow + (\underline{a}_{D/B})_n \frac{3}{5} + (\underline{a}_{D/B})_t \frac{4}{5}$ (1')

where we have again $(a_{D/B})_n = 30.0$ ft/s^2

and $(a_{D/B})_t = \frac{10}{12}\alpha_{BD}$

$+\rightarrow$ x components:

$20 - \frac{4}{5}(30) + \frac{3}{5}(\frac{10}{12}\alpha_{BD}) = 0$

$\frac{1}{2}\alpha_{BD} = -20 + 24 = 4.00$

$\alpha_{BD} = 8.00$ rad/s^2

Since $(\underline{a}_{D/B})_t$ is directed ↗:

$\underline{\alpha}_{BD} = 8.00$ rad/s^2 ↻

$\bar{\underline{a}} = \underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + \frac{1}{2}\underline{a}_{D/B}$

$\bar{\underline{a}} = 20 \rightarrow + 36 \uparrow + \frac{1}{2}(30)\frac{3}{5} + \frac{1}{2}(\frac{10}{12})(8.00)\frac{4}{5}$

$+\rightarrow \bar{a}_x = 20 - \frac{4}{5}(15) + \frac{3}{5}(3.333) = +10.00$ ft/s^2

$+\uparrow \bar{a}_y = 36 + \frac{3}{5}(15) + \frac{4}{5}(3.333) = +47.67$ ft/s^2

Thus: $\bar{a}_x = 10.00$ ft/s^2 →, $\bar{a}_y = 47.67$ ft/s^2 ↑

Kinetics

$\bar{I} = \frac{1}{12}\frac{3}{g}(\frac{10}{12})^2 = \frac{0.17361}{g}$

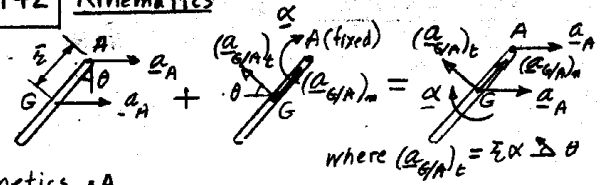
$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $D(\frac{6}{12}\text{ ft}) - W(\frac{4}{12}) = \bar{I}\alpha + m\bar{a}_x(\frac{3}{12}) + m\bar{a}_y(\frac{4}{12})$

$0.5D - 3(\frac{1}{3}) = \frac{0.17361}{g}(8.00) + \frac{3}{g}(10)(\frac{1}{4}) + \frac{3}{g}(47.67)(\frac{1}{3})$

$0.5D - 1 = \frac{1}{32.2}(56.556)$ $D = 5.513$ lb

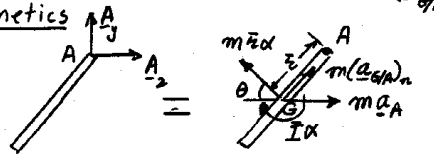
$\underline{D = 5.51\text{ lb}}$ ◀

16.142 Kinematics



where $(a_{G/A})_t = \bar{r}\alpha$

Kinetics



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{kin}: \quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - ma_A(\bar{r}\cos\theta)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2+\bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\frac{d\omega}{d\theta}$ and noting that $\bar{k}=0.3$ m and $\bar{r}=0.525$ m:

$$\omega\frac{d\omega}{d\theta} = \frac{0.525\,a_A}{(0.3)^2+(0.525)^2}\cos\theta$$

$$\omega\,d\omega = 1.4359\,a_A\cos\theta\,d\theta$$

Integrating in θ from 0 to $\frac{\pi}{2}$ and in ω from 0 to ω_f:

$$\int_0^{\omega_f}\omega\,d\omega = 1.4359\,a_A\int_0^{\pi/2}\cos\theta\,d\theta$$

$$\left[\tfrac{1}{2}\omega^2\right]_0^{\omega_f} = 1.4359\,a_A\left[\sin\theta\right]_0^{\pi/2}$$

$$\omega_f^2 = 2.8718\,a_A \qquad (1)$$

Given data: $a_A = a = 1.95$ m/s^2

$\omega_f^2 = 2.8718(1.95) = 5.600$ $\qquad \omega_f = 2.37$ rad/s ◀

16.143

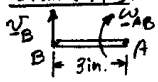
See solution of Prob. 16.142 for derivation of Eq. (1).

Given data: $\omega_f = 1.8$ rad/s. Substitute into (1):

$(1.8)^2 = 2.8718\,a_A$ $\qquad a = a_A = 1.128$ m/s^2 ◀

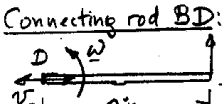
16.144 Kinematics - Velocities

Crank AB: $\omega_{AB} = 2000$ rpm $= 209.44$ rad/s



$$v_B = r\omega_{AB} = \left(\tfrac{3}{12}\text{ ft}\right)(209.44\text{ rad/s}) = 52{,}360\text{ ft/s}$$

Connecting rod BD:



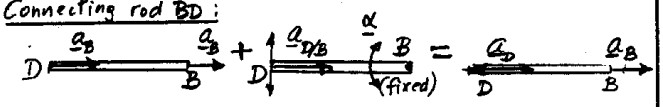
Instant. center is at intersection of perpendiculars to v_B and v_D. Thus, it is at D.

$$\omega = \frac{v_B}{BD} = \frac{52{,}360}{8/12} = 78{,}540\text{ rad/s}$$

Accelerations

Crank AB: $a_B = r\omega_{AB}^2 = \left(\frac{3}{12}\text{ ft}\right)(209.44)^2$, $a_B = 10{,}966 \rightarrow$

Connecting rod BD:



$$a_B + a_{D/B} = a_D$$

$$a_B \rightarrow + (a_{D/B})_n \rightarrow + (a_{D/B})_t \uparrow = a_D \leftrightarrow$$

It follows that $(a_{D/B})_t = 0$ and $\alpha = 0$

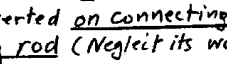
Also $\quad a_B \rightarrow + (BD)\omega^2 \rightarrow = a_D$

$$a_D = 10{,}966 \rightarrow + \left(\tfrac{8}{12}\right)(78{,}540)^2 = 15{,}079\text{ ft/s}^2 \rightarrow$$

$$\bar{a} = \tfrac{1}{2}(a_B + a_D) = \tfrac{1}{2}(10{,}966 + 15{,}079) = 13{,}022\text{ ft/s}^2 \rightarrow$$ (CONTINUED)

16.144 CONTINUED Kinetics

Piston

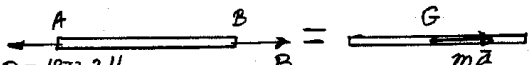


$$ma_D = \frac{4}{32.2}(15{,}079) = 1873.2\text{ lb}$$

$D = 1873.2$ lb →

Force exerted on connecting rod at D is 1873.2 lb ←

Connecting rod (Neglect its weight)



$$+\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}:$$

$$B - D = m\bar{a}$$

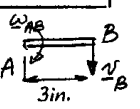
$$B - 1873.2 = \frac{5}{32.2}(13{,}022)$$

$$B = 1873.2 + 2022.0 = 3895\text{ lb}$$

The forces exerted on the connecting rod at B and D are

$B = 3895$ lb →; $\quad D = 1873$ lb ← ◀

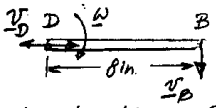
16.145 Kinematics - Velocities



$\omega_{AB} = 2000$ rpm $= 209.44$ rad/s

$$v_B = r\omega_{AB} = \left(\tfrac{3}{12}\text{ ft}\right)(209.44\text{ rad/s}) = 52{,}360\text{ ft/s}$$

Connecting rod BD: Instant. center is at D. Thus

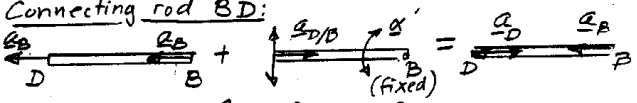


$$\omega = \frac{v_B}{BD} = \frac{52{,}360}{8/12} = 78{,}540\text{ rad/s}$$

Accelerations - Crank AB: $a_B = r\omega_{AB}^2 = \left(\frac{3}{12}\right)(209.44)^2$

$a_B = 10{,}966$ ft/s^2 ←

Connecting rod BD:



$$a_B + a_{D/B} = a_D$$

$$a_B \leftarrow + (a_{D/B})_n \rightarrow + (a_{D/B})_t \downarrow = a_D \leftrightarrow$$

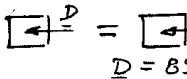
y comp.: $\quad (a_{D/B})_t = 0$ and $\alpha = 0$

x comp.: $\quad 10{,}966 \leftarrow + \left(\frac{8}{12}\right)(78{,}540)^2 \rightarrow = a_D$

$$a_D = 6854\text{ ft/s}^2 \leftarrow$$

$$\bar{a} = a_G = \tfrac{1}{2}(a_B + a_D) = \tfrac{1}{2}(10{,}966 \leftarrow + 6{,}854 \leftarrow) = 8{,}910\text{ ft/s}^2 \leftarrow$$

Kinetics of Piston

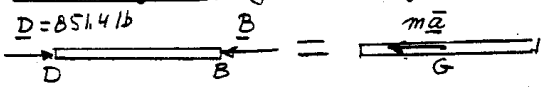


$$ma_D = \frac{4}{32.2}(6854) = 851.4\text{ lb}$$

$D = 851.4$ lb ←

Force exerted on connecting rod at D is 851.4 lb →

Connecting rod (Neglect its weight)



$$+\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}: \quad B - 851.4 = \frac{5}{32.2}(8910)$$

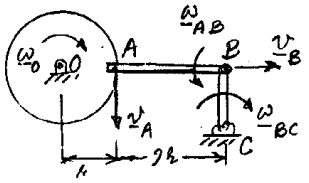
$$B = 851.4 + 1383.5 = 2235\text{ lb}$$

The forces exerted on the connecting rod at B and D are

$B = 2235$ lb ←; $\quad D = 851$ lb → ◀

16.146 Kinematics

Velocities



$\underline{v}_A = r\omega_0 \downarrow$

Instant. center of AB is at B. Therefore:

$\omega_{AB} = \frac{v_A}{2r} = \frac{r\omega_0}{2r}$

$\underline{\omega}_{AB} = \frac{1}{2}\omega_0 \circlearrowleft$

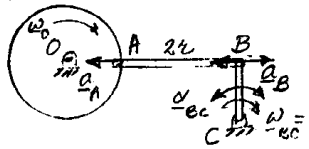
Also: $\underline{v}_B = 0$ and $\underline{\omega}_{BC} = 0$

Accelerations

Disk: Since ω_0 = constant:

$\alpha = 0$

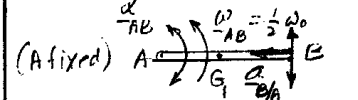
$\underline{a}_A = (\underline{a}_A)_n = r\omega_0^2 \leftarrow$



Rod BC: Since $\omega_{BC} = 0$:

$\underline{a}_B = (\underline{a}_B)_t = r\underline{\alpha}_{BC} \leftrightarrow$

Rod AB:



Relative motion about A:

$\underline{a}_{B/A} = (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\underline{a}_{B/A} = -[2r(\frac{1}{2}\omega_0)^2 \rightarrow] + [2r\alpha_{AB} \uparrow]$

We now write: $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$[r\alpha_{BC} \leftrightarrow] = [r\omega_0^2 \leftarrow] + [\frac{1}{2}r\omega_0^2 \rightarrow] + [2r\alpha_{AB} \uparrow]$

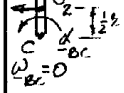
Vertical components: $0 = 2r\alpha_{AB}$ $\quad \alpha_{AB} = 0$

Horiz. components: $r\alpha_{BC} = \frac{3}{2}r\omega_0^2$, $\alpha_{BC} = \frac{3}{2}\omega_0^2 \circlearrowleft$

$\underline{a}_{AB} = \underline{a}_A + \underline{a}_{G_1/A} = [r\omega_0^2 \leftarrow] + [r\omega_{AB}^2 \rightarrow] = [r\omega_0^2 + r(\frac{1}{2}\omega_0)^2]$

$\underline{a}_{AB} = \frac{5}{4}r\omega_0^2 \leftarrow$

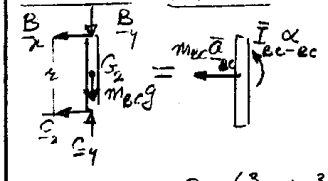
Rod BC:



$\underline{a}_{BC} = \underline{a}_{G_2} = \frac{1}{2}r\alpha_{BC} = \frac{1}{2}r(\frac{3}{2}\omega_0^2) = \frac{3}{4}r\omega_0^2$

$\underline{a}_{BC} = \frac{3}{4}r\omega_0^2 \leftarrow$

Kinetics - Rod BC



$m_{BC} = m'r$

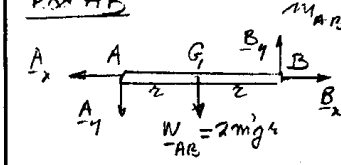
$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$B_x r = \bar{I}_{BC}\alpha_{BC} + (m_{BC}\bar{a}_{BC})\frac{r}{2}$

$B_x r = \frac{1}{12}(m'r)r^2(\frac{3}{2}\omega_0^2) + m'r(\frac{3}{4}r\omega_0^2)\frac{r}{2}$

$B_x = (\frac{3}{24} + \frac{3}{8})m'r^2\omega_0^2 \qquad B_x = \frac{1}{2}m'r^2\omega_0^2$

Rod AB



$m_{AB} = m'(2r) = 2m'r$

$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x - B_x = m_{AB}\bar{a}_{AB}$

$A_x - \frac{1}{2}m'r^2\omega_0^2 = (2m'r)(\frac{5}{4}r\omega_0^2)$

$\underline{A}_x = 3m'r^2\omega_0^2 \leftarrow$, $\underline{B}_x = \frac{1}{2}m'r^2\omega_0^2 \rightarrow$ ◀

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $A_y(2r) + (2m'gr)r = 0$

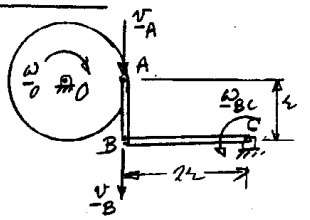
$A_y = -m'gr \qquad \underline{A}_y = m'gr \uparrow$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $-A_y - W_{AB} + B_y = 0$

$B_y = A_y + W_{AB} = -m'gr + 2m'gr \qquad \underline{B}_y = m'gr \uparrow$ ◀

16.147 Kinematics

Velocities



$\underline{v}_A = r\omega_0 \downarrow$

Since $\underline{v}_A$ and $\underline{v}_B$ are both vertical, instant. center of AB is at an infinite distance. Thus:

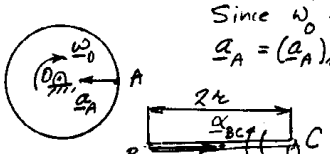
$\underline{\omega}_{AB} = 0$

Also: $\underline{v}_B = \underline{v}_A = r\omega_0 \downarrow$

For rod BC: $\omega_{BC} = \frac{v_B}{2r} = \frac{r\omega_0}{2r} \qquad \underline{\omega}_{BC} = \frac{1}{2}\omega_0 \circlearrowleft$

Accelerations

Disk: Since ω_0 = constant:



$\underline{a}_A = (\underline{a}_A)_n = r\omega_0^2 \leftarrow$

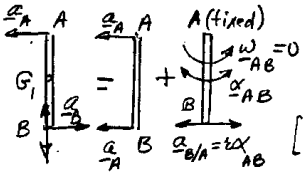
Rod BC:

$\underline{a}_B = (\underline{a}_B)_n + (\underline{a}_B)_t$

$= [2r\omega_{BC}^2 \rightarrow] + [2r\alpha_{BC} \downarrow]$

$\underline{a}_B = [2r(\frac{1}{2}\omega_0)^2 \rightarrow] + [2r\alpha_{BC} \downarrow]$

$= [\frac{1}{2}r\omega_0^2 \rightarrow] + [2r\alpha_{BC} \downarrow]$



Rod AB:

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$[\frac{1}{2}r\omega_0^2 \rightarrow] + [2r\alpha_{BC} \downarrow] = [r\omega_0^2 \leftarrow] + [\underline{a}_{B/A}]$

$\pm \rightarrow x$ components: $\frac{1}{2}r\omega_0^2 = -r\omega_0^2 + a_{B/A}$ $\qquad \underline{a}_{B/A} = \frac{3}{2}r\omega_0^2 \rightarrow$

$a_{B/A} = r\alpha_{AB} = \frac{3}{2}r\omega_0^2 \qquad \underline{\alpha}_{AB} = \frac{3}{2}\omega_0^2 \circlearrowleft$

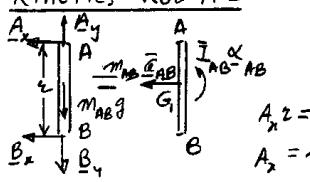
$+\uparrow$ y components: $2r\alpha_{BC} = 0 \qquad \underline{\alpha}_{BC} = 0$

$\underline{\bar{a}}_{AB} = \underline{a}_{G_1} = \underline{a}_A + \frac{r}{2}\underline{\alpha}_{AB} \rightarrow = \overleftarrow{r\omega_0^2} + \frac{r}{2}(\overrightarrow{\frac{3}{2}\omega_0^2})$, $\qquad \underline{\bar{a}}_{AB} = \frac{1}{4}r\omega_0^2 \leftarrow$

Rod BC: $\underline{\bar{a}}_{BC} = \underline{a}_{G_2} = [r\omega_{BC}^2 \rightarrow] + [r\underline{\alpha}_{BC} \downarrow] = r(\frac{\omega_0}{2})^2 \rightarrow + 0$

$\underline{\bar{a}}_{BC} = \frac{1}{4}r\omega_0^2 \rightarrow$

Kinetics - Rod AB



$m_{AB} = m'r$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$A_x r = (m_{AB}\bar{a}_{AB})\frac{r}{2} + \bar{I}_{AB}\alpha_{AB}$

$A_x r = m'r(\frac{1}{4}r\omega_0^2)\frac{r}{2} + \frac{1}{12}m'r\,r^2(\frac{3}{2}\omega_0^2)$

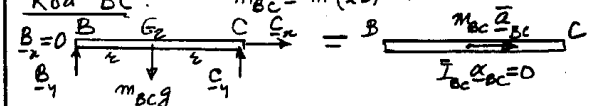
$A_x = m'r^2\omega_0^2(\frac{1}{8} + \frac{1}{8})$

$\underline{A}_x = \frac{1}{4}m'r^2\omega_0^2 \leftarrow$ ◀

$\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x + B_x = m_{AB}\bar{a}_{AB}$

$\frac{1}{4}m'r^2\omega_0^2 + B_x = m'r(\frac{1}{4}r\omega_0^2)$ $\qquad \underline{B}_x = 0$ ◀

Rod BC:



$m_{BC} = m'(2r) = 2m'r$

$+\circlearrowleft \Sigma(M_C) = \Sigma(M_C)_{eff}$: $-B_y(2r) + m_{BC}g\,r = 0$

$B_y = \frac{1}{2}m_{BC}g = \frac{1}{2}(2m'r)g \qquad \underline{B}_y = m'gr \uparrow$ (on BC)

Rod AB:

On AB: $\qquad B_y = m'gr \downarrow$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A_y - B_y - m_{AB}g = 0$

$A_y = B_y + m_{AB}g = m'gr + (m'r)g = 2m'gr$

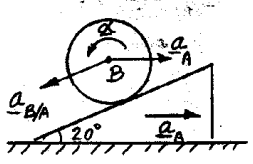
$\underline{A}_y = 2m'gr \uparrow$ ◀

*16.148 Kinematics

We resolve $\underline{a}_B$ into $\underline{a}_A$ and a component $\underline{a}_{B/A}$ parallel to the incline:

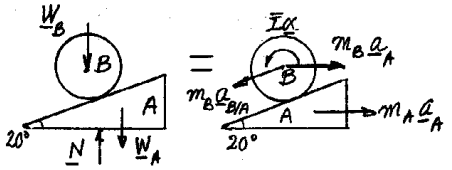
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

where $a_{B/A} = r\alpha$, since cylinder B rolls on wedge A.



Kinetics. Cylinder and Wedge

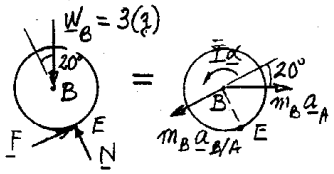
$r = 0.1\ m$
$m_A = 2\ kg$
$m_B = 3\ kg$
$\bar{I} = \frac{1}{2} m_B r^2 = 0.015\ kg\cdot m^2$
$a_{B/A} = (0.1)\alpha$



$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $0 = m_A a_A + m_B a_A - m_B a_{B/A}\cos 20°$

$0 = (2+3)a_A - (3)(0.1\alpha)\cos 20°$, $a_A = (0.06\cos 20°)\alpha$ (1)

Cylinder



$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}$:

$(W_B \sin 20°)r = \bar{I}\alpha + (m_B a_{B/A})r - (m_B a_A \cos 20°)r$

$3(g)\sin 20°(0.1) = (0.015)\alpha + 3(0.1\alpha)(0.1) - 3(0.1)a_A \cos 20°$

Substitute for a_A from (1):

$0.3(9.81)\sin 20° = (0.015 + 0.03 - 0.018\cos^2 20°)\alpha$

$1.0066 = 0.029106\alpha$, $\alpha = 34.583$, $\underline{\alpha} = 34.6\ rad/s^2 \circlearrowleft$ ◀

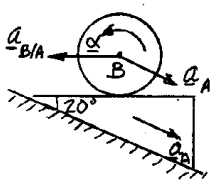
From (1): $a_A = (0.06\cos 20°)(34.583)$ $\underline{a}_A = 1.950\ m/s^2 \rightarrow$ ◀

*16.149 Kinematics

We resolve $\underline{a}_B$ into $\underline{a}_A$ and the horizontal component $\underline{a}_{B/A}$:

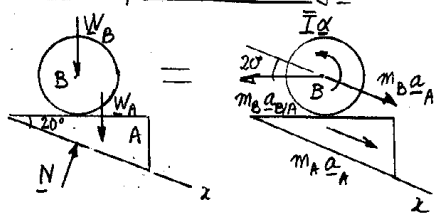
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

where $a_{B/A} = r\alpha$, since cylinder B rolls on wedge A.



Kinetics. Cylinder and Wedge

$r = 0.1\ m$
$m_A = 2\ kg$
$m_B = 3\ kg$
$\bar{I} = \frac{1}{2} m_B r^2 = 0.015\ kg\cdot m^2$
$a_{B/A} = (0.1)\alpha$



$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $(W_A + W_B)\sin 20° = (m_A + m_B)a_A - m_B a_{B/A}\cos 20°$

$(2+3)g\sin 20° = (2+3)a_A - 3(0.1)\alpha\cos 20°$

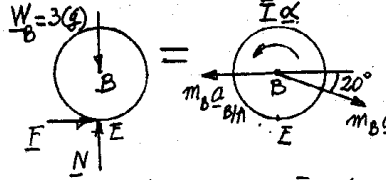
$a_A = (9.81)\sin 20° + (0.06\cos 20°)\alpha$ (1)

(CONTINUED)

*16.149 CONTINUED Cylinder

We recall:

$m_B = 3\ kg$
$r = 0.1\ m$
$\bar{I} = 0.015\ kg\cdot m^2$
$a_{B/A} = (0.1\alpha)$



$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}$: $0 = \bar{I}\alpha + (m_B a_{B/A})r - (m_B a_A \cos 20°)r$

$0.015\alpha + 3(0.1\alpha)(0.1) - 3a_A\cos 20°(0.1) = 0$

$\alpha = \left(\frac{20}{3}\cos 20°\right)a_A$ (2)

(a) Substitute for α from (2) into (1):

$a_A = 9.81\sin 20° + (0.4\cos^2 20°)a_A$

$a_A = 5.1875\ m/s^2$ $\underline{a}_A = 5.19\ m/s^2 \nwarrow 20°$ ◀

(b) From (2): $\alpha = \left(\frac{20}{3}\cos 20°\right)(5.1875)$ $\underline{\alpha} = 32.5\ rad/s^2 \circlearrowleft$ ◀

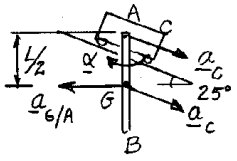
*16.150 Kinematics

We resolve the acceleration of G into the accel. of the cart and the accel. of G relative to A:

$$\underline{a}_R = \underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$$

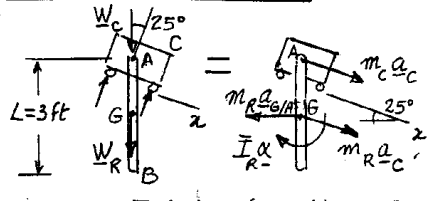
$$\underline{a}_R = \underline{a}_C + \underline{a}_{G/A}$$

where $a_{G/A} = \frac{1}{2} L\alpha$



Kinetics. Cart and Rod

$W_R = 30\ lb$
$W_C = 40\ lb$
$L = 3\ ft$
$\bar{I}_R = \frac{1}{12} m_R L^2$
$a_{G/A} = \frac{1}{2}(3)\alpha = 1.5\alpha$



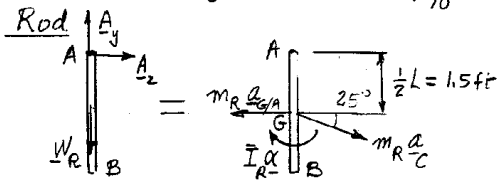
$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $(W_C + W_R)\sin 25° = (m_C + m_R)a_C - m_R a_{G/A}\cos 25°$

$(40+30)\sin 25° = \frac{1}{32.2}\left[(40+30)a_C - 30(1.5\alpha)\cos 25°\right]$

$70a_C = 70(32.2)\sin 25° + (45\cos 25°)\alpha$

$a_C = (32.2)\sin 25° + \left(\frac{45}{70}\cos 25°\right)\alpha$ (1)

Rod



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $0 = \bar{I}\alpha + (m_R a_{G/A})\frac{L}{2} - (m_R a_C \cos 25°)\frac{L}{2}$

$\frac{1}{12}\frac{30}{g}(3)^2\alpha + \frac{30}{g}(1.5\alpha)(1.5) - \left(\frac{30}{g}a_C\cos 25°\right)(1.5) = 0$

$22.5\alpha + 67.5\alpha - (45\cos 25°)a_C = 0$

$\alpha = (0.5\cos 25°)a_C$ (2)

(a) Substitute for α from (2) into (1):

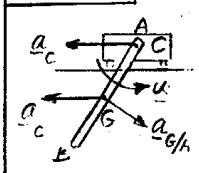
$a_C = (32.2)\sin 25° + \left(\frac{45}{70}\cos 25°\right)(0.5\cos 25°)a_C$

$a_C = \frac{32.2\sin 25°}{1 - \frac{22.5}{70}\cos^2 25°} = 18.490\ ft/s^2$ $\underline{a}_C = 18.49\ ft/s^2 \searrow 25°$ ◀

(b) From (2): $\alpha = (0.5\cos 25°)(18.490) = 8.379\ rad/s^2$

$\underline{\alpha} = 8.38\ rad/s^2 \circlearrowleft$ ◀

***16.151** **Kinematics.** We resolve the acceleration of G into the accel. of the cart and the accel. of G relative to A:



$$\bar{a}_P = \bar{a}_G = \bar{a}_A + \bar{a}_{G/A}$$

$$\bar{a}_R = \bar{a}_C + \bar{a}_{G/A}$$

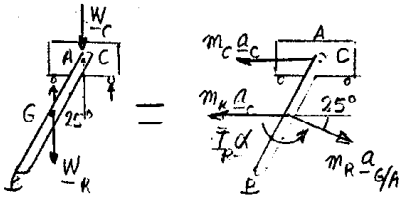
where $a_{G/A} = \frac{1}{2}L\alpha$

Kinetics. Cart and Rod

$W_R = 30$ lb
$W_C = 40$ lb
$L = 3$ ft

$\bar{I}_R = \frac{1}{12} m_R L^2$

$a_{G/A} = \frac{1}{2}(3\text{ ft})\alpha = 1.5\alpha$

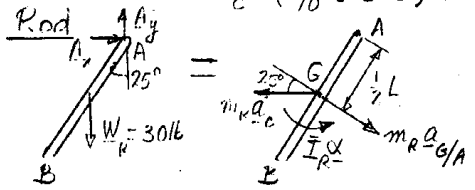


$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $0 = m_R a_{G/A}\cos 25° - (m_C + m_R)a_C$

$$\frac{30}{g}(1.5\alpha)\cos 25° - \frac{40+30}{g}a_C = 0$$

$$a_C = \left(\frac{45}{70}\cos 25°\right)\alpha \qquad (1)$$

Rod



$+\circlearrowleft \Sigma (M_A) = \Sigma (M_A)_{eff}$:

$$W_R\left(\frac{1}{2}\sin 25°\right) = \bar{I}_R\alpha + (m_R a_{G/A})\frac{L}{2} - (m_R a_C \cos 25°)\frac{L}{2}$$

$$30(1.5\sin 25°) = \frac{1}{12}\frac{30}{g}(3)^2\alpha + \frac{30}{g}(1.5\alpha)(1.5) - \left(\frac{30}{g}a_C\cos 25°\right)(1.5)$$

$$45\sin 25° = \frac{30}{g}(0.75 + 2.25)\alpha - \left(\frac{45}{g}\cos 25°\right)a_C$$

$$45(32.2)\sin 25° = 30(3)\alpha - (45\cos 25°)a_C$$

$$\alpha = 16.1\sin 25° + (0.5\cos 25°)a_C \qquad (2)$$

(a) Substitute for α from (2) into (1):

$$a_C = \left(\frac{45}{70}\cos 25°\right)(16.1\sin 25°) + \left(\frac{45}{70}\cos 25°\right)(0.5\cos 25°)a_C$$

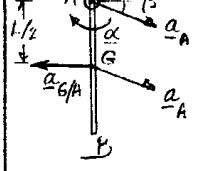
$$a_C = \frac{10.350\sin 25°\cos 25°}{1 - \frac{22.5}{70}\cos^2 25°} = 5.3864\text{ ft/s}^2$$

$$\underline{a_C = 5.39\text{ ft/s}^2} \blacktriangleleft$$

(b) Substitute into (1): $5.3864 = \left(\frac{45}{70}\cos 25°\right)\alpha$

$$\alpha = 9.24502\text{ rad/s}^2 \qquad \underline{\alpha = 9.25\text{ rad/s}^2} \blacktriangleleft$$

***16.152** **Kinematics.** We resolve the acceleration of G into the acceleration of A and the accel. of G relative to A:



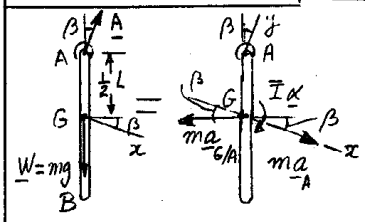
$$\bar{a} = \bar{a}_G = \bar{a}_A + \bar{a}_{G/A}$$

$$\bar{a} = \bar{a}_A + \bar{a}_{G/A}$$

where $a_{G/A} = \frac{1}{2}L\alpha \qquad (1)$

(CONTINUED)

***16.152 CONTINUED** **Kinetics**



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$$0 = \bar{I}\alpha + (m a_{G/A})\left(\frac{L}{2}\right) - (m a_A \cos\beta)\left(\frac{L}{2}\right)$$

But, from (1):

$$a_{G/A} = \frac{1}{2}L\alpha$$

and $\bar{I} = \frac{1}{12}mL^2$

Thus: $\frac{1}{12}mL^2\alpha + m\left(\frac{1}{2}L\alpha\right)\frac{L}{2} - (m a_A\cos\beta)\frac{L}{2} = 0$

$$a_A\cos\beta = \frac{2}{3}L\alpha \qquad (2)$$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$:

$$W\sin\beta = m a_A - (m a_{G/A})\cos\beta$$

$$mg\sin\beta = m a_A - m\left(\frac{1}{2}L\alpha\right)\cos\beta$$

$$a_A = g\sin\beta + \frac{1}{2}L\alpha\cos\beta \qquad (3)$$

Substitute for a_A from (3) into (2):

$$g\sin\beta\cos\beta + \frac{1}{2}L\alpha\cos^2\beta = \frac{2}{3}L\alpha$$

$$g\sin\beta\cos\beta = \frac{1}{6}L\alpha(4 - 3\cos^2\beta)$$

$$\alpha = \frac{6g}{L}\frac{\sin\beta\cos\beta}{4 - 3\cos^2\beta} \qquad (4)$$

Substitute for α into (2) and solving for a_A:

$$a_A = \frac{2L}{3\cos\beta}\frac{6g}{L}\frac{\sin\beta\cos\beta}{4 - 3\cos^2\beta} \qquad \underline{a_A = \frac{4g\sin\beta}{4 - 3\cos^2\beta}} \measuredangle\beta \qquad (5)$$

We note that the senses of α and a_A are always as indicated, since $4 - 3\cos^2\beta > 0$.

$+\nearrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A - W\cos\beta = -m a_{G/A}\sin\beta$

$$A = mg\cos\beta - m\left(\frac{1}{2}L\alpha\right)\sin\beta$$

Substitute for α the expression obtained:

$$A = mg\cos\beta - m\frac{L\sin\beta}{2}\frac{6g}{L}\frac{\sin\beta\cos\beta}{4 - 3\cos^2\beta}$$

$$= mg\cos\beta\left(1 - \frac{3\sin^2\beta}{4 - 3\cos^2\beta}\right) = mg\cos\beta\left(\frac{4 - 3\cos^2\beta - 3\sin^2\beta}{4 - 3\cos^2\beta}\right)$$

$$\underline{A = mg\frac{\cos\beta}{4 - 3\cos^2\beta}} \nearrow\beta \qquad (6)$$

Given data: $m = 4$ kg, $L = 1.5$ m, $\beta = 20°$, $g = 9.81\text{ m/s}^2$

(a) Substitute into Eq. (4):

$$\alpha = \frac{6(9.81)}{1.5}\frac{\sin 20°\cos 20°}{4 - 3\cos^2 20°} = 39.24\frac{0.32139}{1.35093} = 9.3354$$

$$\underline{\alpha = 9.34\text{ rad/s}^2} \circlearrowleft \blacktriangleleft$$

(b) Substitute data into Eq. (5):

$$a_A = \frac{4(9.81)\sin 20°}{4 - 3\cos^2 20°} = \frac{13.4209}{1.35093} = 9.9345$$

$$\underline{a_A = 9.93\text{ m/s}^2} \measuredangle 20° \blacktriangleleft$$

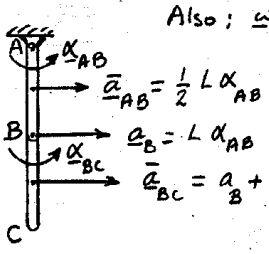
(c) Substitute data into Eq. (6):

$$A = (4)(9.81)\frac{\cos 20°}{4 - 3\cos^2 20°} = \frac{36.874}{1.35093} = 27.295$$

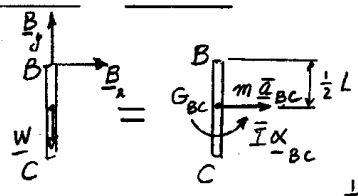
$$\underline{A = 27.3\text{ N}} \measuredangle 70° \blacktriangleleft$$

16.153 Kinematics. Assume α_{AB} ↻, α_{BC} ↻

Also: $\omega_{AB} = \omega_{BC} = 0$



Kinetics. Bar BC



+↻ $\Sigma M_B = \Sigma (M_B)_{eff}$:

$0 = \bar{I}\alpha_{BC} + (m a_{BC})(\frac{1}{2} L)$

$= \frac{1}{12} m L^2 \alpha_{BC} + m(L\alpha_{AB} + \frac{1}{2} L \alpha_{BC})(\frac{1}{2} L)$

$\frac{1}{2} \alpha_{AB} + \frac{1}{4} \alpha_{BC} + \frac{1}{12} \alpha_{BC} = 0$

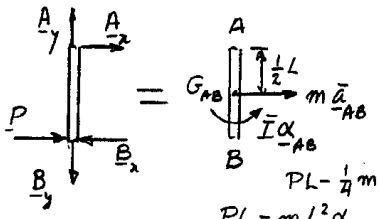
$\frac{1}{3} \alpha_{AB} + \frac{1}{3} \alpha_{BC} = 0 \qquad \alpha_{BC} = -3 \alpha_{AB}$ (1)

$\pm \rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $B_x = m \bar{a}_{BC} = m(L\alpha_{AB} + \frac{1}{2} L \alpha_{BC})$

Recalling (1): $B_x = mL(\alpha_{AB} - \frac{3}{4} \alpha_{AB})$

$B_x = \frac{1}{4} m L \alpha_{AB}$ (2)

Bar AB



+↻ $\Sigma M_A = \Sigma (M_A)_{eff}$:

$PL - B_x L = \bar{I}\alpha_{AB} + (m \bar{a}_{AB})(\frac{1}{2} L)$

Substituting from (2):

$PL - \frac{1}{4} m L^2 \alpha_{AB} = \frac{1}{12} m L^2 \alpha_{AB} + m(\frac{1}{2} L \alpha_{AB})\frac{L}{2}$

$PL = m L^2 \alpha_{AB} (\frac{1}{12} + \frac{1}{4} + \frac{1}{4}) = \frac{7}{12} m L^2 \alpha_{AB}$

Thus: $\alpha_{AB} = \frac{12}{7} \frac{P}{mL}$ (3)

Substituting into (1):

$\alpha_{BC} = -\frac{3}{2}(\frac{12}{7} \frac{P}{mL}) = -\frac{18}{7} \frac{P}{mL}$ (4)

Given Data:

$L = 15$ in. $= 1.25$ ft, $W = 4$ lb, $P = 3.5$ lb

Substitute into (3):

$\alpha_{AB} = \frac{12}{7} \frac{3.5}{(4/32.2)(1.25)} = 38.64$ rad/s²

$\alpha_{AB} = 38.6$ rad/s² ↻ ◀

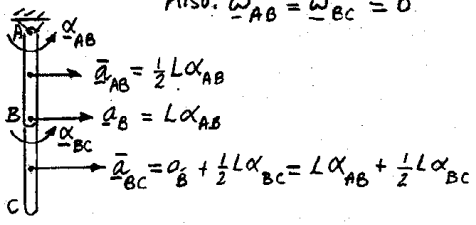
Substitute data into (4):

$\alpha_{BC} = -\frac{18}{7} \frac{3.5}{(4/32.2)(1.25)} = -57.96$ rad/s²

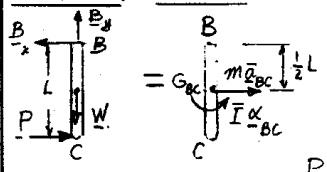
$\alpha_{BC} = 58.0$ rad/s² ↺ ◀

***16.154** Kinematics. Assume α_{AB} ↻, α_{BC} ↻

Also: $\omega_{AB} = \omega_{BC} = 0$



Kinetics. Bar BC



+↻ $\Sigma M_B = \Sigma (M_B)_{eff}$:

$PL = \bar{I}\alpha_{BC} + (m \bar{a}_{BC})(\frac{1}{2} L)$

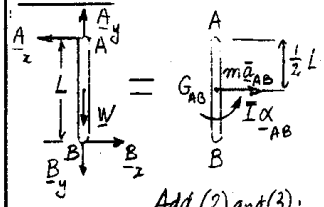
$= \frac{1}{12} m L^2 \alpha_{BC} + m(L\alpha_{AB} + \frac{1}{2} L \alpha_{BC})(\frac{1}{2} L)$

$P = \frac{1}{2} m L \alpha_{AB} + \frac{1}{3} m L \alpha_{BC}$ (1)

$\pm \rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $P - B_x = m \bar{a}_{BC}$

$P - B_x = m(L\alpha_{AB} + \frac{1}{2} L \alpha_{BC})$ (2)

Bar AB



+↻ $\Sigma M_A = \Sigma (M_A)_{eff}$:

$B_x L = \bar{I}\alpha_{AB} + (m \bar{a}_{AB})(\frac{1}{2} L)$

$= \frac{1}{12} m L^2 \alpha_{AB} + m(\frac{1}{2} L \alpha_{AB})(\frac{1}{2} L)$

$B_x = \frac{1}{3} m L \alpha_{AB}$ (3)

Add (2) and (3): $P = \frac{4}{3} m L \alpha_{AB} + \frac{1}{2} m L \alpha_{BC}$ (4)

Subtract (1) from (4): $0 = \frac{5}{6} m L \alpha_{AB} + \frac{1}{6} m L \alpha_{BC}$

$\alpha_{BC} = -5 \alpha_{AB}$ (5)

Substitute for α_{BC} in (1):

$P = \frac{1}{2} m L \alpha_{AB} + \frac{1}{3} m L (-5 \alpha_{AB}) = -\frac{7}{6} m L \alpha_{AB}$

$\alpha_{AB} = -\frac{6}{7} \frac{P}{mL}$ (6)

Substitute into (5):

$\alpha_{BC} = -5(-\frac{6}{7} \frac{P}{mL}) \qquad \alpha_{BC} = \frac{30}{7} \frac{P}{mL}$ (7)

Given Data:

$L = 15$ in. $= 1.25$ ft, $W = 4$ lb, $P = 3.5$ lb

Substitute into (6):

$\alpha_{AB} = -\frac{6}{7} \frac{3.5}{(4/32.2)(1.25)} = -19.32$ rad/s²

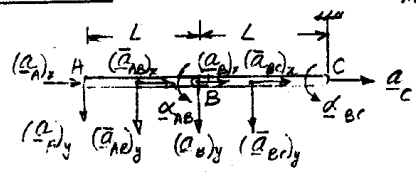
$\alpha_{AB} = 19.32$ rad/s² ↺ ◀

Substitute data into (7):

$\alpha_{BC} = \frac{30}{7} \frac{3.5}{(4/32.2)(1.25)} = 96.60$ rad/s²

$\alpha_{BC} = 96.6$ rad/s² ↻ ◀

*16.155 Kinematics. Assume α_{AB}, α_{BC}, a_C →



Also, we have $\omega_{AB} = \omega_{BC} = 0$

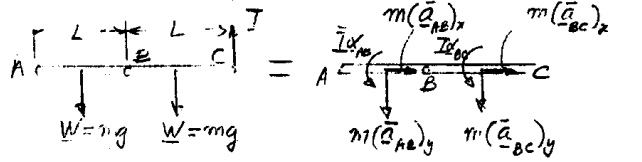
Since $\omega_{AB} = \omega_{BC} = 0$: $(a_A)_x = (\bar a_{AB})_x = (a_B)_x = (\bar a_{BC})_x = a_C$

$(\bar a_{BC})_y = \frac{1}{2} L \alpha_{BC}$ ↓ $\qquad (a_B)_y = L\alpha_{BC}$ ↓

$(\bar a_{AB})_y = (a_B)_y + \frac{1}{2} L \alpha_{AB} = (L\alpha_{BC} + \frac{1}{2} L \alpha_{AB})$ ↓

$(a_A)_y = (a_B)_y + L\alpha_{AB} = (L\alpha_{BC} + L\alpha_{AB})$ ↓

Kinetics. Entire system

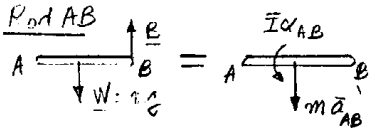


$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad 0 = m(\bar a_{AB})_x + m(\bar a_{BC})_x$

$0 = 2ma_C$

Thus: $(a_A)_x = (\bar a_{AB})_x = (a_B)_x = (\bar a_{BC})_x = a_C = 0$

Rod AB



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $\quad mg(\frac{1}{2}L) = \bar I\alpha_{AB} + (m\bar a_{AB})(\frac{1}{2}L)$

$\frac{1}{2} mgL = \frac{1}{12} mL^2 \alpha_{AB} + m(L\alpha_{BC} + \frac{1}{2} L\alpha_{AB})(\frac{1}{2}L)$

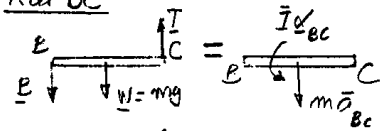
$\frac{1}{2} g = \frac{1}{3} L\alpha_{AB} + \frac{1}{2} L\alpha_{BC}$

$\frac{2}{3} L\alpha_{AB} + L\alpha_{BC} = g \qquad (1)$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad B - W = -m\bar a_{AB}$

$B = mg - m(L\alpha_{BC} + \frac{1}{2} L\alpha_{AB}) \qquad (2)$

Rod BC



$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $\quad BL + W(\frac{1}{2}L) = \bar I\alpha_{BC} + (m\bar a_{BC})(\frac{1}{2}L)$

$BL + \frac{1}{2} mgL = \frac{1}{12} mL^2 \alpha_{BC} + m(\frac{1}{2} L\alpha_{BC})(\frac{1}{2}L)$

$B = -\frac{1}{2} mg + \frac{1}{3} mL\alpha_{BC} \qquad (3)$

Subtract (3) from (2):

$0 = \frac{3}{2} mg - m(\frac{4}{3} L\alpha_{BC} + \frac{1}{2} L\alpha_{AB})$

$\frac{3}{8} L\alpha_{AB} + L\alpha_{BC} = \frac{9}{8} g \qquad (4)$

Subtract (1) from (4): $\quad (\frac{3}{8} - \frac{2}{3}) L\alpha_{AB} = \frac{1}{8} g$

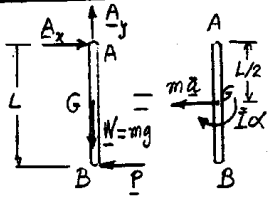
$-\frac{7}{24} L\alpha_{AB} = \frac{1}{8} g \qquad \alpha_{AB} = -\frac{3}{7} \frac{g}{L}$

Substitute in (1): $\quad \frac{2}{3} L(-\frac{3}{7}\frac{g}{L}) + L\alpha_{BC} = g \qquad \alpha_{BC} = \frac{9}{7}\frac{g}{L}$

(a) $a_A = L\alpha_{BC} + L\alpha_{AB} = \frac{9}{7} g - \frac{3}{7} g = \frac{6}{7} g \qquad \underline{a_A = \frac{6}{7} g}$ ↓ ◀

(b) $a_B = L\alpha_{BC} = \frac{9}{7} g \qquad \underline{a_B = \frac{9}{7} g}$ ↓ ◀

*16.156 Entire Rod AB: $\quad \bar a = \frac{L}{2}\alpha$



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$.

$PL = (m\bar a)\frac{L}{2} + \bar I\alpha$

$= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12} mL^2\alpha$

$= \frac{1}{3} mL^2\alpha$

$\alpha = \frac{3P}{mL}$ ↻ $\qquad (1)$

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad A_x - P = -m\bar a$

$A_x = P - m\frac{L}{2}\alpha = P - m\frac{L}{2}\frac{3P}{mL} = -\frac{1}{2} P \qquad \underline{A_x = \frac{1}{2} P}$ ←

Portion AJ of rod.

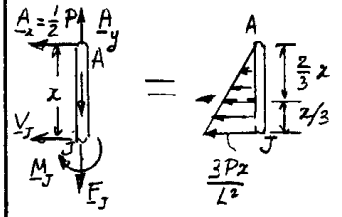
External forces: They include the reaction at A, the weight of AJ, and the axial force F_J, shear V_J, and bending-moment couple M_J at point J.

Effective forces: Since the accel. of any given point is proportional to its distance from A, the effective forces are linearly distributed. Noting that the mass per unit length is m/L, we find that the effective force per unit length at point J is

$(\frac{m}{L}) a_J = \frac{m}{L}(x\alpha)$

or, recalling (1):

$= \frac{m}{L}(x\frac{3P}{mL}) = \frac{3Px}{L^2}$



$+\circlearrowleft \Sigma M_J = \Sigma (M_J)_{eff}$: $\quad M_J - A_x x = -\frac{1}{2}(\frac{3Px}{L^2}) x (\frac{2}{3})$

$M_J = \frac{1}{2} Px - \frac{1}{2}\frac{P}{L^2} x^3 \qquad (2)$

For maximum value of M_J:

$\frac{dM_J}{dx} = \frac{1}{2} P - \frac{3}{2}\frac{P}{L^2} x^2 = 0 \qquad x = \frac{L}{\sqrt{3}} \qquad (3)$

Substituting in (2):

$(M_J)_{max} = \frac{1}{2} P\frac{L}{\sqrt{3}} - \frac{1}{2}\frac{P}{L^2}(\frac{L}{\sqrt{3}})^3 = \frac{1}{2}\frac{PL}{\sqrt{3}}(1 - \frac{1}{3})$

$(M_J)_{max} = \frac{PL}{3\sqrt{3}} \qquad (4)$

(a) Substituting the given data in (3) and (4):

$x = \frac{750\text{ mm}}{\sqrt{3}} \qquad x = 433.0\text{ mm}$

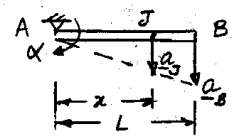
$(M_J)_{max} = \frac{(12\text{ N})(0.75\text{ m})}{3\sqrt{3}} = 1.732\text{ N·m}$

Thus: $M_{max} = 1.732$ N·m at 433 mm below A. ◀

(b) From (2) and (3) we verify that x and $(M_J)_{max}$ are independent of m. ◀

***16.157** From the answer to Prob. 16.91, we have

$$\underline{a}_B = \tfrac{3}{2} g \downarrow \qquad \underline{A} = \tfrac{1}{4} W \uparrow$$



It follows that

$$\underline{\alpha} = \frac{a_B}{L} = \frac{3g}{2L} \circlearrowleft$$

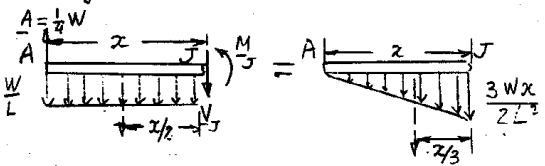
$$\underline{a}_J = x\alpha = \frac{3g}{2L} x$$

Portion AJ of beam

External forces: They include the reaction $\underline{A}$, the distributed load per unit length W/L, the shear $\underline{V}_J$, and the bending-moment couple $\underline{M}_J$ at J.

Effective forces: Since the accel. of any given point is proportional to its distance from A, the effective forces are linearly distributed. Noting that the mass per unit length is W/gL, we find that the effective force per unit length at J is

$$\frac{W}{gL} a_J = \frac{W}{gL} \frac{3g}{2L} x = \frac{3Wx}{2L^2}$$



$$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad \frac{W}{L} x - \frac{1}{4} W + V_J = \frac{1}{2} \left(\frac{3Wx}{2L^2} \right) x$$

$$V_J = \frac{1}{4} W - \frac{W}{L} x + \frac{3}{4} \frac{W}{L^2} x^2$$

$$+\circlearrowleft \Sigma M_J = \Sigma (M_J)_{eff}: \quad \left(\frac{W}{L} x \right) \frac{x}{2} - \left(\frac{1}{4} W \right) x + M_J = \frac{1}{2} \left(\frac{3Wx}{2L^2} \right) x \left(\frac{x}{3} \right)$$

$$M_J = \frac{1}{4} Wx - \frac{1}{2} \frac{W}{L} x^2 + \frac{1}{4} \frac{W}{L^2} x^3$$

To help draw V- and M-diagrams, we note that V_J is minimum when

$$\frac{dV_J}{dx} = -\frac{W}{L} + \frac{3}{2} \frac{W}{L^2} x = 0 \qquad x = \frac{2}{3} L$$

$$(V_J)_{min} = \frac{1}{4} W - \frac{W}{L} \frac{2L}{3} + \frac{3}{4} \frac{W}{L^2} \left(\frac{2}{3} L \right)^2 = -\frac{1}{12} W$$

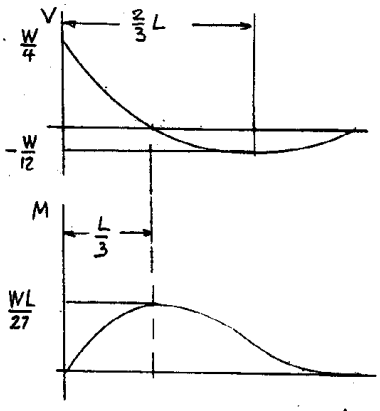
M_J is maximum or minimum when $dM_J/dx = V_J = 0$:

$$\frac{1}{4} W - \frac{W}{L} x + \frac{3}{4} \frac{W}{L^2} x^2 = 0$$

$$3x^2 - 4Lx + L^2 = 0 \qquad (3x - L)(x - L) = 0, \quad x = \frac{L}{3} \text{ and } x = L$$

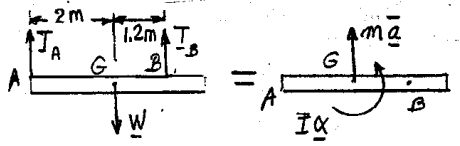
$$M_{max} = \frac{1}{4} W \frac{L}{3} - \frac{1}{2} \frac{W}{L} \left(\frac{L}{3} \right)^2 + \frac{1}{4} \frac{W}{L^2} \left(\frac{L}{3} \right)^3 = \frac{1}{27} WL$$

$$M_{min} = \frac{1}{4} WL - \frac{1}{2} \frac{W}{L} L^2 + \frac{1}{4} \frac{W}{L^2} L^3 = 0$$



$M_{max} = \frac{WL}{27}$ at $\frac{1}{3} L$ from A. ◀

16.158



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad T_A + T_B - W = m\bar{a}$$

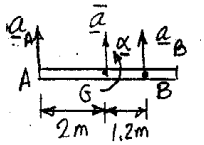
$$1000\,N + 1800\,N - (200\,kg)(9.81\,m/s^2) = (200\,kg)\bar{a}$$

$$\bar{a} = 4.190\ m/s^2 \uparrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}: \quad T_B (1.2m) - T_A (2m) = \bar{I}\alpha$$

$$(1800\,N)(1.2m) - (1000\,N)(2m) = \frac{1}{12} (200\,kg)(4m)^2 \alpha$$

$$\underline{\alpha} = 0.600\ rad/s^2$$



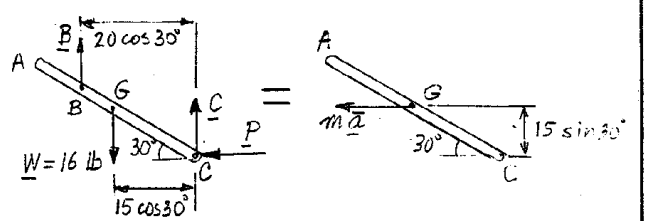
$$a_A = \bar{a} - (2m)\alpha = 4.190 - 2(0.600) = 2.990\ m/s^2$$

$$\underline{a}_A = 2.99\ m/s^2 \uparrow \blacktriangleleft$$

$$a_B = \bar{a} + (1.2m)\alpha = 4.190 + 1.2(0.600) = 4.910\ m/s^2$$

$$\underline{a}_B = 4.91\ m/s^2 \uparrow \blacktriangleleft$$

16.159



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}:$$

$$(16\,lb)(15\cos 30°) - B(20\cos 30°) = m\bar{a}(15\sin 30°) \qquad (1)$$

$$\pm \Sigma F_x = \Sigma (F_x)_{eff}: \quad P = m\bar{a} \qquad (2)$$

(a) Making $P = 5\,lb$ and $m = 16/32.2$ in Eq. (2):

$$5\,lb = m\bar{a} \qquad 5 = \frac{16}{32.2} a \qquad \underline{a} = 10.06\ ft/s^2 \leftarrow \blacktriangleleft$$

(b) Substituting $m\bar{a} = 5\,lb$ into Eq. (1):

$$16(15\cos 30°) - B(20\cos 30°) = 5(15\sin 30°)$$

$$B = \frac{16(15)}{20} - \frac{5(15)}{20} \tan 30° = 12 - 2.17 = 9.83$$

$$\underline{B} = 9.83\ lb \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + C - W = 0$$

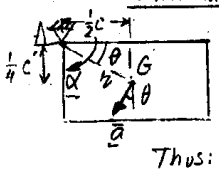
$$C = W - B = 16 - 9.83 = 6.17\ lb$$

$$\underline{C} = 6.17\ lb \uparrow \blacktriangleleft$$

16.160

(1) Plate attached to pins

Kinematics Assume α ↻ $(\omega = 0)$



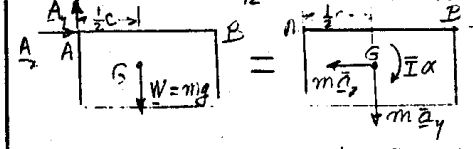
$\bar{a} = r\alpha \angle \theta$

$\pm$ x comp: $\bar{a}_x = r\alpha \sin\theta = (r\sin\theta)\alpha$

$+\downarrow$ y comp: $\bar{a}_y = r\alpha\cos\theta = (r\cos\theta)\alpha$

Thus: $\bar{a}_x = \frac{1}{4}c\alpha \leftarrow$; $\bar{a}_y = \frac{1}{2}c\alpha \downarrow$ (1)

Kinetics $\bar{I} = \frac{1}{12}m[c^2 + (\frac{c}{2})^2] = \frac{5}{48}mc^2$



(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $W(\frac{c}{2}) = \bar{I}\alpha + (m\bar{a}_x)(\frac{c}{4}) + (m\bar{a}_y)(\frac{c}{2})$

$\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + m(\frac{1}{4}c\alpha)(\frac{c}{4}) + m(\frac{1}{2}c\alpha)(\frac{c}{2})$

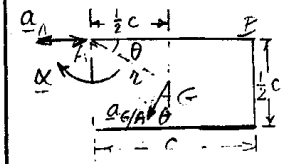
$\frac{1}{2}mgc = \frac{20}{48}mc^2\alpha$ $\alpha = 1.2\frac{g}{c}$ $\underline{\alpha = 1.2\frac{g}{c}}$ ↻ ◀

(b) From (1): $\bar{a}_x = \frac{1}{4}c\alpha = \frac{1}{4}(1.2g)$ $\underline{\bar{a}_x = 0.3g} \leftarrow$ ◀

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}(1.2g)$ $\underline{\bar{a}_y = 0.6g} \downarrow$ ◀

(2) Plate suspended from wires.

Kinematics Assume α ↻ $(\omega = 0)$



$\bar{a} = \bar{a}_G = \bar{a}_A + \bar{a}_{G/A}$

$= a_A \leftarrow + r\alpha \angle \theta$

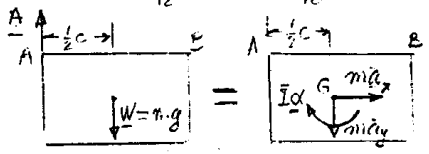
$+\downarrow$ y comp.

$\bar{a}_y = 0 - r\alpha\cos\theta$

$= -(r\cos\theta)\alpha$

$r\cos\theta = \frac{1}{2}c$ Thus: $\bar{a}_y = -\frac{1}{2}c\alpha$ $\underline{\bar{a}_y = \frac{1}{2}c\alpha} \downarrow$ (2)

Kinetics $\bar{I} = \frac{1}{12}m[c^2 + (\frac{c}{2})^2] = \frac{5}{48}mc^2$



$\pm \Sigma F_x = \Sigma(F_x)_{eff}$

$0 = m\bar{a}_x$

$\bar{a}_x = 0$

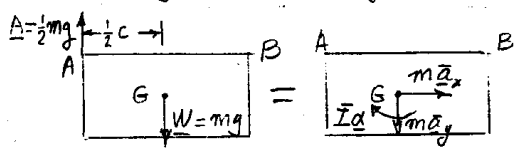
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $W(\frac{1}{2}c) = \bar{I}\alpha + (m\bar{a}_y)(\frac{1}{2}c)$

Peculiar: (1): $\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + (\frac{1}{2}mc\alpha)(\frac{1}{2}c)$

$\frac{1}{2}mgc = \frac{17}{48}mc^2\alpha$ $\alpha = \frac{24}{17}\frac{g}{c}$ $\underline{\alpha = \frac{24}{17}\frac{g}{c}}$ ↻ ◀

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}c(\frac{24}{17}\frac{g}{c}) = \frac{12}{17}g$ $\underline{\bar{a} = \frac{12}{17}g} \downarrow$ ◀

(3) Plate suspended from springs. Immediately after spring B is released, the tension in spring A is still $\frac{1}{2}mg$ since its elongation is unchanged



(a) $+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $(\frac{1}{2}mg)(\frac{1}{2}c) = \bar{I}\alpha$

$\frac{1}{4}mgc = \frac{5}{48}mc^2\alpha$ $\underline{\alpha = 2.4\frac{g}{c}}$ ↻ ◀

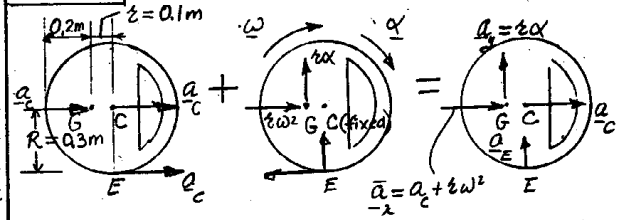
(b) $\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $0 = m\bar{a}_x$ $\bar{a}_x = 0$

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $mg - \frac{1}{2}mg = m\bar{a}_y$

$\bar{a}_y = \frac{1}{2}g$ $\underline{\bar{a} = 0.5g} \downarrow$ ◀

16.161

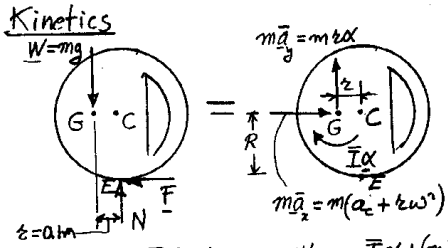
Kinematics: (Rolling motion) $a_C = R\alpha$



Transl. with C + Rotation about C = Rolling motion

$\bar{a} = (a_C + r\omega^2) \rightarrow + r\alpha \uparrow$

Kinetics



$\bar{I} = m\bar{k}^2$

$m = 5\,kg$

$\bar{k} = 0.15\,m$

$R = 0.3\,m$

$r = 0.1\,m$

$a_C = R\alpha$

$\omega = 8\,rad/s$

$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$: $-Wr = \bar{I}\alpha + (m\bar{a}_x)R + (m\bar{a}_y)r$

$-mgr = m\bar{k}^2\alpha + m(a_C + r\omega^2)R + mr\alpha r$

$= m\bar{k}^2\alpha + mR^2\alpha + mr R\omega^2 + mr^2\alpha$

$-gr = (\bar{k}^2 + R^2 + r^2)\alpha + rR\omega^2$

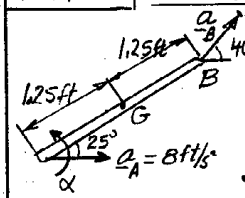
$-9.81(0.1) = [(0.15)^2 + (0.3)^2 + (0.1)^2]\alpha + (0.1)(0.3)(8)^2$

$\alpha = -23.68\ rad/s^2$ $\underline{\alpha = 23.7\ rad/s^2}$ ↻ ◀

Note. We could not apply the equation $\Sigma M_E = I_E\alpha$ to this problem, since points E, C, and G are not aligned (cf. P16.105)

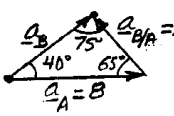
16.162

Kinematics $\underline{a}_A = 8\,ft/s^2 \leftarrow$ $(\omega = 0)$



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$a_B \angle 40° = 8 \rightarrow + 2.5\alpha \angle 65°$



Law of sines:

$\frac{2.5\alpha}{\sin 40°} = \frac{8}{\sin 75°}$

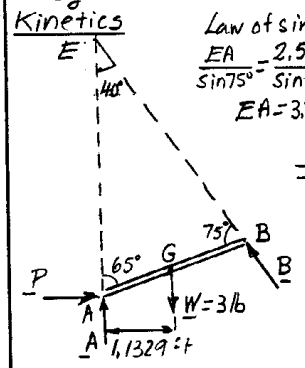
$\alpha = 2.1295\,rad/s^2$ ↻

$\bar{a} = \underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 8 \rightarrow + 1.25(2.1295) \angle 65°$

$\bar{a}_x = 8 \rightarrow + 1.25(2.1295)\cos 65° \leftarrow = 6.8751\,ft/s^2 \rightarrow$

$\bar{a}_y = 1.25(2.1295)\sin 65° \uparrow = 2.4125\,ft/s^2 \uparrow$

Kinetics



Law of sines:

$\frac{EA}{\sin 75°} = \frac{2.5\,ft}{\sin 40°}$

$EA = 3.7568\,ft$

$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}\frac{3}{g}(2.5)^2 = \frac{1.5625}{g}$

$ED = EA - DA$

$= 3.7568 - 1.25\sin 25° = 3.2285\,ft$

(a) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$: $P(EA) - W(1.1329) = \bar{I}\alpha + m\bar{a}_x(ED) + m\bar{a}_y(1.1329)$

$P(3.7568) - 3(1.1329) = \frac{1.5625}{g}(2.1295) + \frac{3}{g}(6.8751)(3.2285) + \frac{3}{g}(2.4125)(1.1329)$

$P(3.7568) = 3.3987 + \frac{78.115}{32.2}$ $P = 1.5504\,lb$ $\underline{P = 1.550\,lb} \rightarrow$ ◀

(b) $\pm \Sigma F_x = \Sigma(F_x)_{eff}$: $1.5504 - B\sin 40° = \frac{3}{32.2}(6.8751)$, $\underline{B = 1.416\,lb \angle 50°}$ ◀

16.163

(a) Force in rim (no radial force)

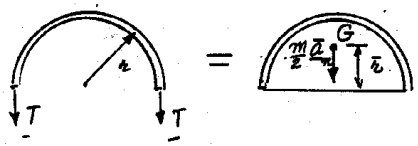
Consider half of rim $\therefore \frac{m}{2} = \frac{1}{2}(1800\text{ kg}) = 800\text{ kg}$

$r = 0.6\text{ m}$, $\bar{r} = \frac{2r}{\pi} = \frac{1.2}{\pi} = 0.3820\text{ m}$

$\alpha = 0$

$\omega = 360\text{ rpm} = 12\pi\text{ rad/s}$

$a_n = \omega^2 r = (12\pi)^2(0.3820) = 542.9\text{ m/s}^2$



$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $2T = \frac{m}{2}\bar{a}_n$

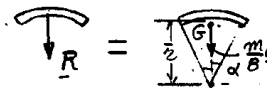
$2T = (800\text{ kg})(542.9\text{ m/s}^2)$ $\quad T = 217\text{ kN}$ ◀

(b) Force in each spoke (no tangential force)

Consider 1/8 of rim: $\frac{m}{8} = \frac{1}{8}(1800\text{ kg}) = 225\text{ kg}$

$\alpha = \frac{\pi}{8} = 22.5°$

$\bar{r} = \frac{r\sin\alpha}{\alpha} = \frac{0.6\sin 22.5°}{\pi/8} = 0.5847\text{ m}$



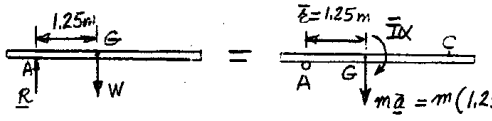
$\bar{a}_n = \omega^2\bar{r} = (12\pi)^2(0.5847) = 831.0\text{ m/s}^2$

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $R = \frac{m}{8}\bar{a}_n = (200\text{ kg})(831.0\text{ m/s}^2)$

$R = 166.2\text{ kN}$ ◀

16.164

First consider plank alone, with no worker



$L = 3.5\text{ m}$

$\omega = 0$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $W(1.25) = m\bar{a}(1.25) + \bar{I}\alpha$

$mg(1.25) = m(1.25\alpha)(1.25) + \frac{1}{12}m(3.5)^2\alpha$

$1.25g = 2.5833\alpha$ $\quad \alpha = 0.4839\,g$

Accel. of midpoint G: $a_G = 1.25\alpha = 1.25(0.4839\,g)$

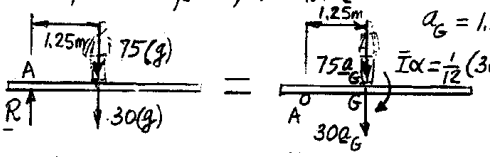
$= 0.605\,g$

Accel. of point above pipe CD:

$a_C = 2.5\alpha = 2.5(0.4839\,g) = 1.210\,g$

(a) Accel. of worker standing in middle G of plank

Since the accel. of G (without worker) is less than g, and since accel. of worker alone would be g, contact between worker and plank is maintained. Considering the worker-plank system, we have



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $(30+75)(g)(1.25) = (30+75)a_G(1.25) + \frac{1}{12}(30)(3.5)^2\alpha$

$131.25g = 131.25(1.25\alpha) + 30.625\alpha$

$= 194.6875\alpha$ $\quad \alpha = 0.67416\,g$

$a_{worker} = a_G = 1.25\alpha = 1.25(0.67416\,g) = 0.84270\,g$

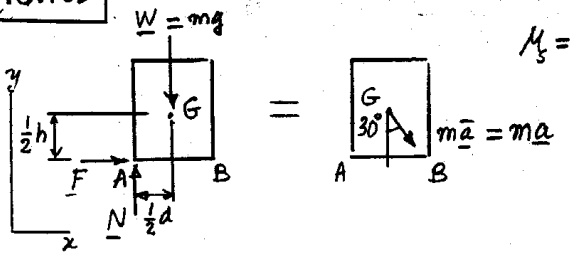
$= 0.84270(9.81\text{ m/s}^2)$ $\quad a_{worker} = 8.27\text{ m/s}^2\downarrow$ ◀

(b) Accel. of worker standing above pipe CD

Since the accel. of point above pipe CD was found to be $1.210\,g$, that is, larger than g, worker will lose contact with plank and $a_{worker} = g\downarrow = 9.81\text{ m/s}^2\downarrow$ ◀

16.165

$\mu_s = 0.25$



(a) $\pm\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $F = ma\sin 30°$ (1)

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $mg - N = ma\cos 30°$

$N = mg - ma\cos 30°$ (2)

Sliding impends when $F = F_m = \mu_s N$. Substituting from (1) and (2):

$ma\sin 30° = \mu_s(mg - ma\cos 30°)$

$a = \frac{\mu_s g}{\sin 30° + \mu_s\cos 30°} = \frac{0.25g}{0.71651}$ $\quad a = 0.349g$ ◀

(b) Tipping about A will impend when reactions F and N are applied as shown.

$+\circlearrowleft \Sigma M_G = 0$: $F(\frac{1}{2}h) - N(\frac{1}{2}d) = 0$

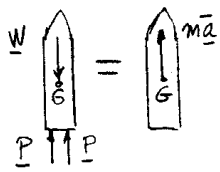
$\frac{h}{d} = \frac{N}{F}$ But $F = F_m = 0.25N$ Thus

$\frac{h}{d} = \frac{N}{0.25N}$ $\quad \frac{h}{d} = 4$ ◀

16.166

Let P = Thrust of one engine.

With both engines operating:



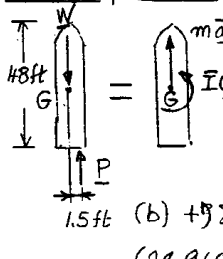
$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$2P - W = m\bar{a}$

$2P - 25{,}000 = \frac{25{,}000}{32.2}(45)$

$P = 29{,}969\text{ lb}$

With only one engine operating:



(a) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$P - W = m\bar{a}$

$29{,}969 - 25{,}000 = \frac{25{,}000}{32.2}\bar{a}$

$\bar{a} = 6.40\text{ ft/s}^2\uparrow$ ◀

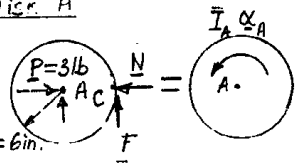
(b) $+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $P(1.5\text{ft}) = \bar{I}\alpha$

$(29{,}969\text{ lb})(1.5\text{ft}) = \frac{1}{12}\frac{25{,}000\text{ lb}}{32.2\text{ft/s}^2}(48\text{ft})^2\alpha$

$\alpha = 0.302\text{ rad/s}^2\circlearrowleft$ ◀

16.167 (a) While the disks are slipping, a friction force $F\uparrow$ is applied to disk A and $F\downarrow$ to disk B.

Disk A



$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2}\frac{5}{32.2}\left(\frac{6}{12}\right)^2 = 0.019410$

$+\uparrow \Sigma F_x = 0$: $P - N = 0$, $N = P = 3$ lb

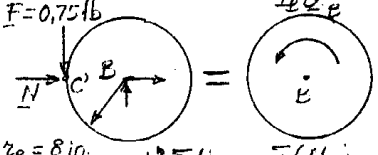
Thus: $F = \mu_k N = 0.25(3\text{ lb}) = 0.75$ lb

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $Fr_A = \bar{I}_A\alpha_A$

$(0.75\text{ lb})\left(\frac{6}{12}\text{ ft}\right) = 0.019410\,\alpha_A$

$\alpha_A = 19.320$ rad/s² $\quad \underline{\alpha}_A = 19.32$ rad/s² $\circlearrowleft$ ◀

Disk B



$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}\frac{10}{32.2}\left(\frac{8}{12}\right)^2 = 0.069013$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $Fr_B = \bar{I}_B\alpha_B$

$(0.75\text{ lb})\left(\frac{8}{12}\text{ ft}\right) = 0.069013\,\alpha_B$

$\alpha_B = 7.245$ rad/s² $\quad \underline{\alpha}_B = 7.25$ rad/s² $\circlearrowleft$ ◀

(b) Kinematics: $(\omega_A)_0 = 0$, $(\omega_B)_0 = 900\text{ rpm} = 94.248$ rad/s $\circlearrowleft$

Disks will stop sliding when $v_C = v_{C'}$, that is, when

$\omega_A r_A = \omega_B r_B$

$(\alpha_A t) r_A = [(\omega_B)_0 - \alpha_B t] r_B$

$(19.320t)6 = [94.248 - 7.245t]8$

$173.880t = 753.984 \qquad t = 4.336$ s

At that time, the angular velocities of the disks are

$\omega_A = 19.320(4.336) = 83.772$ rad/s $= 800$ rpm

$\underline{\omega}_A = 800\text{ rpm}\ \circlearrowleft$ ◀

$\omega_B = 94.248 - 7.245(4.336) = 62.834$ rad/s $= 600$ rpm

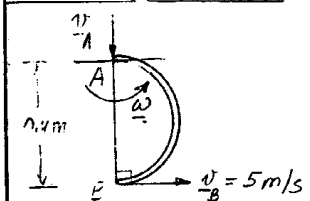
$\underline{\omega}_B = 600\text{ rpm}\ \circlearrowleft$ ◀

16.168 Kinematics

Velocities

Inst. center at A

$\omega = \frac{v_B}{AB} = \frac{5\text{ m/s}}{0.4\text{ m}} = 12.5$ rad/s



Accelerations. Since $\underline{a}_B = 0$,

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = 0 + \underline{a}_{A/B}$

$a_A\downarrow = (AB)\omega^2\downarrow + (AB)\alpha\leftarrow$

It follows that $\alpha = 0$

$\bar{a} = a_G = a_{G/B} = -\omega^2 r_{G/B}$

But $r_{G/B} = \frac{2r}{\pi}i + rj$

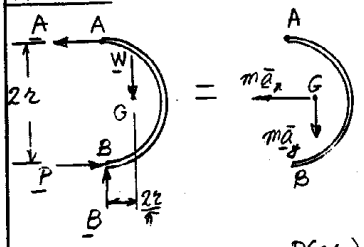
Thus: $\bar{a} = -\omega^2 r\left(\frac{2}{\pi}i + j\right)$ $\quad \bar{a}_x = \frac{2}{\pi}\omega^2 r\leftarrow$, $\bar{a}_y = \omega^2 r\downarrow$

(CONTINUED)

16.168 CONTINUED We recall from previous column:

$\alpha = 0$; $\bar{a}_x = \frac{2}{\pi}\omega^2 r\leftarrow$, $\bar{a}_y = \omega^2 r\downarrow$

Kinetics



(a) $\bar{I}\alpha = 0$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$W\left(\frac{2r}{\pi}\right) - P(2r) = (m\bar{a}_x)r + (m\bar{a}_y)\frac{2r}{\pi}$

$= m\left(\frac{2}{\pi}\omega^2 r\right)r + m(\omega^2 r)\frac{2r}{\pi} = \frac{4}{\pi}m\omega^2 r^2$

$P(2r) = \frac{mg}{\pi}(2r) - \frac{4}{\pi}m\omega^2 r^2$

$P = \frac{m}{\pi}(g - 2\omega^2 r)$

Making $m = 1.5$ kg, $\omega = 12.5$ rad/s, $r = 0.2$ m, $g = 9.81$ m/s²:

$P = \frac{1.5}{\pi}[9.81 - 2(12.5)^2(0.2)] = -25.158$ N

$P = 25.2$ N ← ◀

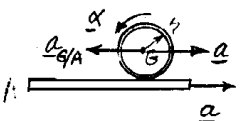
(b) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $B - mg = -m\bar{a}_y$

$B = mg - m\omega^2 r = m(g - \omega^2 r) = 1.5[9.81 - (12.5)^2(0.2)]$

$B = -32.16$ N $\qquad \underline{B} = 32.2$ N $\downarrow$ ◀

16.169 Kinematics.

Consider the frame of reference attached to point A of the plate and in translation with the plate. Since the pipe rolls on the plate, we have



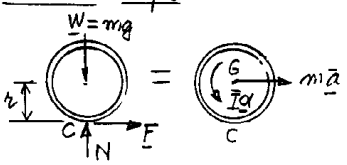
$\underline{a}_{G/A} = r\alpha\leftarrow$

and, therefore, with respect to a Newtonian frame,

$\bar{a} = \underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$

$+\rightarrow \quad \bar{a} = a - r\alpha \qquad (1)$

Kinetics. Pipe



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$0 = \bar{I}\alpha - (m\bar{a})r \qquad (2)$

Solving (1) for α: $\alpha = \frac{a - \bar{a}}{r}$

Substitute in (2): $\bar{I}\left(\frac{a-\bar{a}}{r}\right) - m\bar{a}r = 0$

or, setting $\bar{I} = m\bar{k}^2$, $\frac{m\bar{k}^2}{r}(a - \bar{a}) - m\bar{a}r = 0$

$\bar{k}^2 a - \bar{k}^2\bar{a} - r^2\bar{a} = 0$

$\bar{a} = \frac{\bar{k}^2}{\bar{k}^2 + r^2}a \qquad (3)$

(a) For pipe: $\bar{k} = r$ $\quad \bar{a} = \frac{r^2}{r^2 + r^2}a$, $\bar{a} = \frac{1}{2}a$

$\underline{a} = \frac{1}{2}a\rightarrow$ ◀

(b) $\underline{a}_{G/A} = \bar{a} - \underline{a} = \frac{1}{2}a\leftarrow$

Thus $a = 2a_{G/A}$ $\quad a_{plate} = 2(a_{pipe/plate})$

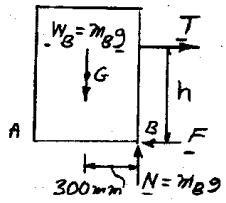
and $x_{plate} = 2(x_{pipe/plate}) = 2d$ $\quad x_{plate} = 2d\rightarrow$ ◀

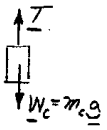
PROBLEM 16.C1

WE MUST INVESTIGATE SEVERAL CASES

SYSTEM DOES NOT ACCELERATE
TIPPING IMPENDS ABOUT E.

$F_m = \mu_s N = \mu_s m_B g$





$\Sigma F_y = 0$: $T = m_C g$

$\pm \Sigma F_x = 0$: $T - F = 0$ $\quad F = T = m_C g$

NO ACCELERATION IF $F \le F_m$: $m_C g \le \mu_s m_B g$ $\quad m_C \le \mu_s m_B$ ◁

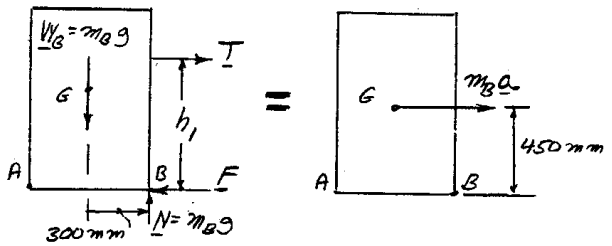
$+\circlearrowleft \Sigma M_B = 0$: $T h - m_B g (300\text{ mm}) = 0$

$m_C g h - m_B g (300\text{ mm}) = 0$ $\quad h = 300 \dfrac{m_B}{m_C}$ ◁

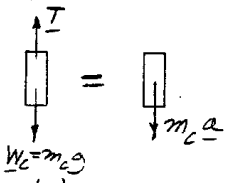
SYSTEM ACCELERATES
TIPPING IMPENDS ABOUT E

SINCE BARREL SLIDES, $F = \mu_k N = \mu_k m_B g$

BARREL



CYLINDER



$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: $T - F = m_B a$

$T - \mu_k m_B g = m_B a$ $\quad T = (\mu_k g + a) m_B$ ②

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $m_C g - T = m_C a$

$T = m_C g - m_C a$ ①

FROM ① AND ②:

$T = T$: $m_C g - m_C a = \mu_k g m_B + a m_B$

$a = \dfrac{m_C - \mu_k m_B}{m_C + m_B} g$ ◁

$+\circlearrowleft \Sigma M_B = \Sigma M_{B\,eff}$: $T h_1 - m_B g (300) = m_B a (450)$

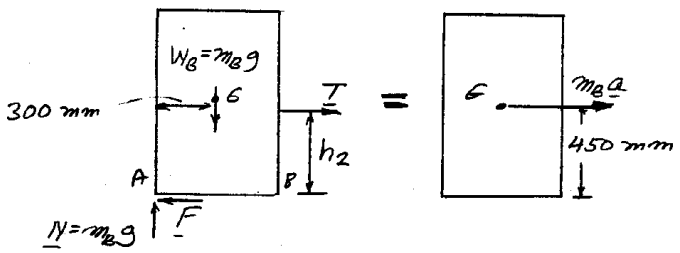
$T h_1 = (300 g + 450 a) m_B$ ③

DIVIDE ③ BY ② MEMBER BY MEMBER:

$h_1 = \dfrac{300 g + 450 a}{\mu_k g + a}$ ◁

BARREL WILL NOT TIP ABOUT E FOR $h \le h_1$ ◁

SYSTEM ACCELERATES
TIPPING IMPENDS ABOUT A



CYLINDER
SAME AS ABOVE

$\pm \Sigma F_x = \Sigma (F_x)_{eff}$: SAME AS ABOVE: $T = (\mu_k g + a) m_B$ ②

$+\circlearrowleft \Sigma M_A = \Sigma M_{A\,eff}$: $T h_2 + m_B g (300) = m_B a (450)$; $T h_2 = (-300 g + 450 a) m_B$ ④

DIVIDE ④ BY ② MEMBER BY MEMBER: $h_2 = \dfrac{-300 g + 450 a}{\mu_k g + a}$ ◁

BARREL WILL NOT TIP ABOUT A FOR $h \ge h_2$ ◁
(WE ALSO OBSERVE THAT
h IS ALWAYS POSITIVE)

CONTINUED

PROBLEM 16.C1 CONTINUED

```
10 '*****************************************************************
20 '*                        PROBLEM 16.C1                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '              A = acceleration of barrel and cylinder   (m/s^2)
90 '             MB = mass of the barrel      (kg)
100 '             MC = mass of the cylinder    (kg)
110 '              H = height of cable         (mm)
120 '             US = coefficient of static friction
130 '             UK = coefficient of kinetic friction
140 '
150         'Data
160     LET MB = 125
170     LET US = .35
180     LET UK = .3
190     LET G = 9.810001
200 '
210     PRINT " Mass of       Acceleration of          Barrel
220     PRINT " cylinder         cylinder          does not tip for
230     PRINT
240 '
250     FOR MC = 25 TO 75 STEP 5
260        GOSUB 370
270     NEXT
280 '
290     FOR MC = 100 TO 400 STEP 25
300        GOSUB 370
310     NEXT
320     END
330 '
340             'SUBROUTINE
350      'System does not accelerate.
360      'But, the system tips for one range of values of H
370     IF MC > US*MB GOTO 450
380     H = 300*MB/MC
390     PRINT USING " ### kg "¡MC,
400     PRINT "       Ø",
410     PRINT USING "       H <= #### mm"¡H
420     GOTO 580
430 '
440      'System accelerates and tips for one range of values of H
450     A = ((MC - UK*MB)/(MC + MB))*G
460     H1 = ( 300*G + 450*A)/(UK*G + A)
470     H2 = (-300*G + 450*A)/(UK*G + A)
480     IF H2 > Ø GOTO 550
490     PRINT USING " ### kg "¡MC,
500     PRINT USING "       #.### m/s^2 "¡A,
510     PRINT USING "         H <= #### mm"¡H1
520     GOTO 580
530 '
540      'System accelerates and tips for two ranges of values of H
550     PRINT USING " ### kg "¡MC,
560     PRINT USING"       #.### m/s^2 "¡A,
570     PRINT USING " ###.## mm <= H <= #### mm"¡H2,H1
580     RETURN
```

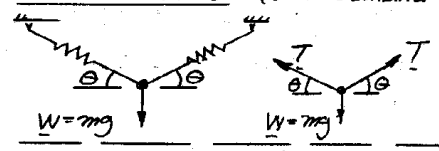
Mass of cylinder	Acceleration of cylinder	Barrel does not tip for
25 kg	Ø	H <= 1500 mm
30 kg	Ø	H <= 1250 mm
35 kg	Ø	H <= 1071 mm
40 kg	Ø	H <= 938 mm
45 kg	Ø.433 m/s^2	H <= 929 mm
50 kg	Ø.701 m/s^2	H <= 894 mm
55 kg	Ø.954 m/s^2	H <= 865 mm
60 kg	1.193 m/s^2	H <= 841 mm
65 kg	1.420 m/s^2	H <= 821 mm
70 kg	1.635 m/s^2	H <= 804 mm
75 kg	1.839 m/s^2	H <= 788 mm
100 kg	2.725 m/s^2	H <= 736 mm
125 kg	3.434 m/s^2	H <= 704 mm
150 kg	4.013 m/s^2	H <= 683 mm
175 kg	4.496 m/s^2	H <= 668 mm
200 kg	4.905 m/s^2	H <= 656 mm
225 kg	5.255 m/s^2	H <= 647 mm
250 kg	5.559 m/s^2	H <= 640 mm
275 kg	5.825 m/s^2	H <= 635 mm
300 kg	6.059 m/s^2	H <= 630 mm
325 kg	6.268 m/s^2	H <= 626 mm
350 kg	6.454 m/s^2	H <= 622 mm
375 kg	6.622 m/s^2	3.85 mm <= H <= 619 mm
400 kg	6.774 m/s^2	10.82 mm <= H <= 617 mm

```
C*****************************************************************
C*                        PROBLEM 16.C1                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C           Definition of Symbols
C
C               MB = mass of the barrel  (kg)
C               MC = mass of the cylinder  (kg)
C                H = height of cable  (mm)
C               US = coefficient of static friction
C               UK = coefficient of kinetic friction
C
      REAL MB, MC
C
      MB = 125.
      US = Ø.35
      UK = Ø.3
      G = 9.81
C
      WRITE (*,11)
   11 FORMAT (T5,'Mass of',T20,'Acceleration of',T50,'Barrel')
      WRITE (*,12)
   12 FORMAT (T4,'cylinder',T23,'cylinder',T45,'does not tip for')
      WRITE (*,'(/,A\)')
C
      DO 10 I = 25,75,5
           MC = I
           CALL ACCEL (MC,MB,US,UK,G)
   10 CONTINUE
C
      DO 20 I = 100,400,25
           MC = I
           CALL ACCEL (MC,MB,US,UK,G)
   20 CONTINUE
C
      WRITE (*,'(/,A\)')
      END
C
                SUBROUTINE ACCEL (MC,MB,US,UK,G)
C        System does not accelerate.
C        but, the system tips for one range of values of H
      REAL MC,MB
      IF (MC.GT.(US*MB)) GOTO 440
      H = 300.*MB/MC
      WRITE (*,10) MC,H
   10 FORMAT (4X,F4.Ø,' kg',9X,'Ø.ØØØ m/s**2',12X,' H <=  ',F6.1,' mm')
      GOTO 570
C
C        System accelerates and tips for one range of values of H
  440 A = ((MC - UK*MB)/(MC + MB))*G
      H1 = (300.*G + 450.*A)/(UK*G + A)
      H2 = (-300.*G + 450.*A)/(UK*G + A)
      IF (H2.GT.Ø.) GOTO 540
      WRITE (*,20) MC,A,H1
   20 FORMAT (4X,F4.Ø,' kg',9X,F5.3,' m/s**2',12X,' H <=  ',F6.1,' mm')
      GOTO 570
C
C        System accelerates and tips for two ranges of values of H
  540 WRITE (*,30) MC,A,H2,H1
   30 FORMAT (4X,F4.Ø,' kg',9X,F5.3,' m/s**2',5X,F5.2,' mm  <= H <= ',
     +F5.1,' mm')
  570 RETURN
      END
```

Mass of cylinder	Acceleration of cylinder	Barrel does not tip for
25. kg	Ø.ØØØ m/s**2	H <= 1500.Ø mm
30. kg	Ø.ØØØ m/s**2	H <= 1250.Ø mm
35. kg	Ø.ØØØ m/s**2	H <= 1071.4 mm
40. kg	Ø.ØØØ m/s**2	H <= 937.5 mm
45. kg	.433 m/s**2	H <= 929.5 mm
50. kg	.701 m/s**2	H <= 894.2 mm
55. kg	.954 m/s**2	H <= 865.4 mm
60. kg	1.193 m/s**2	H <= 841.3 mm
65. kg	1.420 m/s**2	H <= 821.Ø mm
70. kg	1.635 m/s**2	H <= 803.6 mm
75. kg	1.839 m/s**2	H <= 788.5 mm
100. kg	2.725 m/s**2	H <= 735.6 mm
125. kg	3.434 m/s**2	H <= 703.8 mm
150. kg	4.013 m/s**2	H <= 682.7 mm
175. kg	4.496 m/s**2	H <= 667.6 mm
200. kg	4.905 m/s**2	H <= 656.3 mm
225. kg	5.255 m/s**2	H <= 647.4 mm
250. kg	5.559 m/s**2	H <= 640.4 mm
275. kg	5.825 m/s**2	H <= 634.6 mm
300. kg	6.059 m/s**2	H <= 629.8 mm
325. kg	6.268 m/s**2	H <= 625.7 mm
350. kg	6.454 m/s**2	H <= 622.3 mm
375. kg	6.622 m/s**2	3.85 mm <= H <= 619.2 mm
400. kg	6.774 m/s**2	10.82 mm <= H <= 616.6 mm

PROBLEM 16.C2

TENSION IN SPRINGS (STATIC LOADING)



$\Sigma F_y = 0$

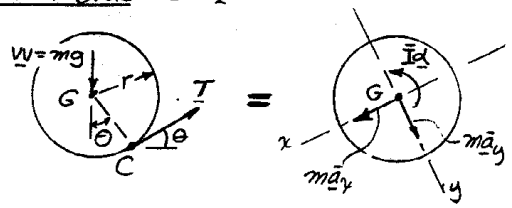
$2T\sin\theta - mg = 0$

$T = \dfrac{mg}{2\sin\theta}$ ◁

$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $Tr = \bar{I}\alpha$

$\alpha = T\dfrac{r}{\bar{I}} = \left(\dfrac{mg}{2\sin\theta}\right)\dfrac{r}{\frac{1}{2}mr^2} = \dfrac{g}{r\sin\theta}$ ◁

FREE BODY DISK $\bar{I} = \frac{1}{2}mr^2$



$\rightarrow \Sigma F_x = \Sigma (F_x)_{eff}$: $mg\sin\theta - T = m\bar{a}_x$

$\bar{a}_x = (mg\sin\theta - T)/m$ ◁

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $mg\cos\theta = m\bar{a}_y$ $\bar{a}_y = g\cos\theta$ ◁

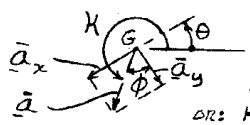
ACCELERATION OF C: $\underline{a}_C = \underline{\bar{a}} + \underline{a}_{C/G} = \underline{\bar{a}} + r\alpha\angle\theta$

$(a_C)_x = \bar{a}_x - r\alpha$ ◁

$(a_C)_y = \bar{a}_y$ ◁

$\bar{a} = \sqrt{(\bar{a}_x)^2 + (\bar{a}_y)^2}$ ◁ $a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2}$ ◁

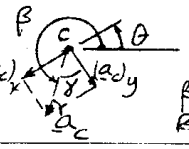
DIRECTIONS OF ACCELERATIONS:



$\tan\phi = \dfrac{\bar{a}_x}{\bar{a}_y}$ ◁

$K = \theta + 270° - \phi$

OR: $K = \theta - \phi - 90°$ ◁



$\tan\gamma = \dfrac{(a_C)_x}{(a_C)_y}$ ◁

$\beta = \theta + 270° - \gamma$

$\beta = \theta - \gamma - 90°$ ◁

```
10 '*****************************************************************
20 '*'                      PROBLEM 16.C2                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         TH = angle theta   (degrees)
90 '          T = tension in spring
100 '     ALPHA = angular acceleration of disk
110 '     AX,AY = components of acceleration of center of disk
120 '        AG = magnitude of acceleration of center of disk
130 '     KAPPA = angle between horizontal and acceleration AG
140 '               (+ KAPPA is up to the right)
150 '  ACX,ACY = components of acceleration of point C
160 '        AC = magnitude of acceleration of point C
170 '      BETA = angle between horizontal and acceleration AC
180 '               (+ BETA is up to the right)
185 '
190 '  NOTE: +x axis is down to the left along spring CD
200 '        +y axis is down to right perpendicular to spring CD
210 '
220     K = 3.14159/180
230     PRINT "THETA          [AG]/G        KAPPA",
240       PRINT "    [AC]/G              BETA
250     PRINT"degrees    dimensionless    degrees",
260       PRINT "dimensionless     degrees":PRINT
270     G = 1:   M = 1:   R = 1
280 '
290     FOR TH = 30 TO 90 STEP 5
300          'Tension in spring
310        T = M*G/(2*SIN(K*TH))
320          'Acceleration of mass center of disk
330        AX = (M*G*SIN(K*TH) - T)/M:      AY = G*COS(K*TH)
340        AG = SQR(AX^2 +AY^2)
350        PHI = (ATN(AX/AY))/K
360        KAPPA = TH - PHI - 90
370         'Acceleration of point C
380        ALPHA = G/(R*SIN(K*TH))
390        ACX = AX - R*ALPHA:      ACY = AY
400        AC = SQR(ACX^2 + ACY^2)
410        GAMMA = (ATN(ACX/ACY))/K
420        BETA = TH - GAMMA - 90
430        PRINT USING " ##          ";TH,
440        PRINT USING " ###.###          ###.##      ";AG,KAPPA,
450        PRINT USING "   ###.###          ###.##        ";AC,BETA
460     NEXT
```

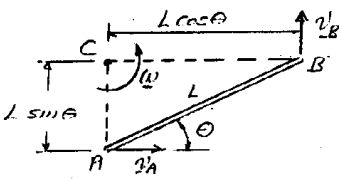
THETA degrees	[AG]/G dimensionless	KAPPA degrees	[AC]/G dimensionless	BETA degrees
30	1.000	-30.00	2.646	10.89
35	0.872	-35.00	2.200	13.14
40	0.778	-40.00	1.856	15.63
45	0.707	-45.00	1.581	18.44
50	0.653	-50.00	1.354	21.67
55	0.610	-55.00	1.163	25.46
60	0.577	-60.00	1.000	30.00
65	0.552	-65.00	0.860	35.56
70	0.532	-70.00	0.740	42.48
75	0.518	-75.00	0.642	51.21
80	0.508	-80.00	0.566	62.12
85	0.502	-85.00	0.517	75.29
90	0.500	-90.00	0.500	90.00

```
C*****************************************************************
C*                      PROBLEM 16.C2                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C            Definition of Symbols
C
C
      REAL  K, M, KAPPA
C
      WRITE (*,11)
   11 FORMAT (T4,'THETA',T18,'[AG]/G',T32,'KAPPA',T47,'[AC]/G',
     +T62,'BETA')
      WRITE (*,12)
   12 FORMAT (T3,,'degrees',T14,'dimensionless',T31,'degrees',
     +T43,'dimensionless',T61'degrees')
      WRITE (*,'(/,A\)')
C
      K = 3.14159/180
      G = 1.
      M = 1.
      R = 1.
C
      DO 10 I = 30,90,5
           TH = I
C
C        Tension in spring
         T = M*G/(2.*SIN(K*TH))
C           Acceleration of mass center of disk
         AX = (M*G*SIN(K*TH) - T)/M
         AY = G*COS(K*TH)
         AG = SQRT(AX**2 + AY**2)
         PHI = (ATAN(AX/AY))/K
         KAPPA = TH - PHI - 90
C           Acceleration of point C
         ALPHA = G/(R*SIN(K*TH))
         ACX = AX - R*ALPHA
         ACY = AY
         AC = SQRT(ACX**2 + ACY**2)
         GAMMA = (ATAN(ACX/ACY))/K
         BETA = TH - GAMMA - 90.
C
      WRITE (*,20) TH,AG,KAPPA,AC,BETA
   20 FORMAT (4X,F3.0,8X,F7.3,8X,F6.2,9X,F7.3,8X,F6.2)
   10 CONTINUE
      END
```

THETA degrees	[AG]/G dimensionless	KAPPA degrees	[AC]/G dimensionless	BETA degrees
30.	1.000	-30.00	2.646	10.89
35.	.872	-35.00	2.200	13.14
40.	.778	-40.00	1.856	15.63
45.	.707	-45.00	1.581	18.44
50.	.653	-50.00	1.354	21.67
55.	.610	-55.00	1.163	25.46
60.	.577	-60.00	1.000	30.00
65.	.552	-65.00	.860	35.56
70,	.532	-70.00	.740	42.48
75.	.518	-75.00	.642	51.21
80.	.508	-80.00	.566	62.12
85.	.502	-85.00	.517	75.29
90.	.500	-90.00	.500	90.00

PROBLEM 16.C3

KINEMATICS VELOCITIES

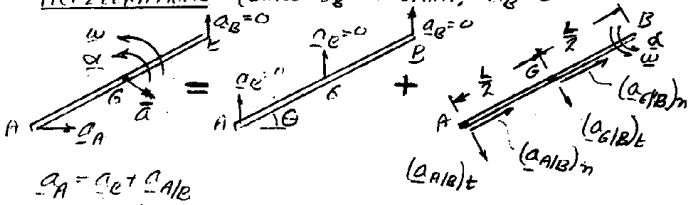


INST. CENTER AT C,

$v_B = (L\cos\theta)\omega$

$\omega = \dfrac{v_B}{L\cos\theta}$ ◁

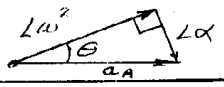
ACCELERATIONS (SINCE v_B = CONSTANT, $a_B = 0$)



$\underline{a}_A = \underline{a}_e + \underline{a}_{A/B}$

$\underline{a}_A = 0 + (a_{A/B})_t \nabla\theta + (a_{A/B})_n \angle\theta$

$a_A = L\alpha \nabla\theta + L\omega^2 \angle\theta$



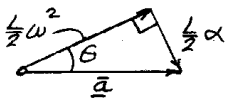
$L\alpha = L\omega^2 \tan\theta$

$\alpha = \omega^2 \tan\theta$ ◁

KINEMATICS ACCELERATIONS (CONTINUED)

$\bar{a} = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + (\underline{a}_{G/B})_t \nabla\theta + (a_{G/B}) \angle\theta$

$\bar{a} = 0 + \frac{1}{2}\alpha \nabla\theta + \frac{1}{2}\omega^2 \angle\theta$

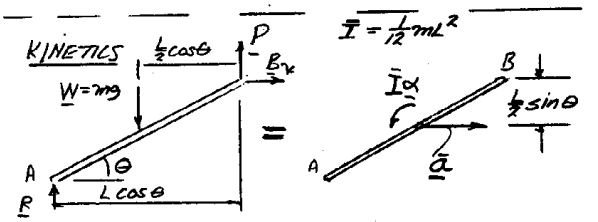


FROM THE RIGHT TRIANGLE:

$\bar{a} = \frac{1}{2}\omega^2/\cos\theta$ ◁

$\bar{I} = \frac{1}{12}mL^2$

KINETICS



$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $mg\frac{1}{2}\cos\theta - RL\cos\theta = \bar{I}\alpha + m\bar{a}\frac{1}{2}\sin\theta$

$R = \frac{1}{2}mg - \dfrac{\bar{I}\alpha}{L\cos\theta} - \dfrac{m\bar{a}L\sin\theta}{L\cos\theta}$

$R = \frac{1}{2}mg - \dfrac{\bar{I}\alpha}{L\cos\theta} - \frac{1}{2}m\bar{a}\tan\theta$ ◁

```
10 '*****************************************************************
20 '*                        PROBLEM 16.C3                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '             L = length of rod     (meters)
90 '            VB = constant upward velocity of end B  (m/s)
100 '           TH = angle theta between rod and horizontal  (degrees)
110 '        OMEGA = angular velocity of rod       (rad/s)
120 '        ALPHA = angular acceleration of rod  (rad/s^2)
130 '         ABAR = acceleration of mass center of the rod  (m/s^2)
140 '            M = mass of rod    (kg)
150 '            R = reaction at end A     (newtons)
160 '
170 '       Data
180     L = 1.2    '(meters)
190     VB = .5    '(m/s)
200     M = 6      '(kg)
210     G = 9.810001   '(m/s^2)
220     I = (M*L^2)/12
230
240     K = 4*ATN(1)/180
250     PRINT " theta     Reaction at A"
260     PRINT "degrees       newtons"
270     PRINT " "
280     FOR TH = 0 TO 75 STEP 5
290        OMEGA = VB/(L*COS(K*TH))
300        ALPHA = OMEGA^2*TAN(K*TH)
310        ABAR = (.5*L*OMEGA^2)/COS(K*TH)
320        R = .5*M*G - (I*ALPHA)/(L*COS(K*TH)) - .5*M*ABAR*TAN(K*TH)
340        IF R < 0  GOTO 380
350        PRINT USING "  ##           ##.###        ";TH,R
370     NEXT
380     PRINT USING "  ##";TH,
390     PRINT "       End A leaves surface, reaction at A is zero.
400     END
```

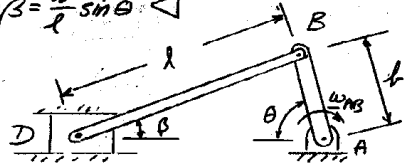
```
 theta      Reaction at A
degrees       newtons

   0           29.430
   5           29.393
  10           29.353
  15           29.306
  20           29.247
  25           29.169
  30           29.060
  35           28.899
  40           28.652
  45           28.251
  50           27.560
  55           26.277
  60           23.656
  65           17.592
  70            0.817
  75       End A leaves surface, reaction at A is zero.
```

```
C*****************************************************************
C*                        PROBLEM 16.C3                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C            Definition of Symbols
C
C                 L = length of rod    (meters)
C                VB = constant upward velocity of end B  (m/s)
C                TH = angle between rod and horizontal  (degrees)
C             OMEGA = angular velocity of rod     (rad/s)
C             ALPHA = angular acceleration of rod     (rad/s**2)
C              ABAR = acceleration of mass center of the rod    (m/s**2)
C                 M = mass of rod    (kg)
C                 R = Reaction at end A  (newtons)
C
      REAL I, L, M, K
      L = 1.2
      VB = 0.5
      M = 6.
      G = 9.81
      I = (M*L**2)/12.
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,'(/,2X,A\)') ' theta      Reaction at A'
      WRITE (*,'(/,2X,A\)') 'degrees        newtons      '
      WRITE (*,'(/,A\)')
C
      DO 10 J = 0,75,5
           TH = J
        OMEGA = VB/(L*COS(K*TH))
        ALPHA = OMEGA**2*TAN(K*TH)
        ABAR = (0.5*L*OMEGA**2)/COS(K*TH)
        R = 0.5*M*G - (I*ALPHA)/(L*COS(K*TH)) - 0.5*M*ABAR*TAN(K*TH)
        IF (R.LT.0.0) GOTO 350
            WRITE (*,20) TH,R
 20         FORMAT (4X,F3.0,10X,F6.3)
            GOTO 10
350  WRITE (*,30) TH
 30  FORMAT(T5,F3.0,T13,'End A leaves surface, reaction at A is zero.')
 10  CONTINUE
     END
```

```
       theta      Reaction at A
      degrees        newtons

         0.           29.430
         5.           29.393
        10.           29.353
        15.           29.306
        20.           29.247
        25.           29.169
        30.           29.060
        35.           28.899
        40.           28.652
        45.           28.251
        50.           27.560
        55.           26.277
        60.           23.656
        65.           17.592
        70.             .817
        75.       End A leaves surface, reaction at A is zero.
```

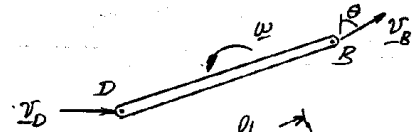
PROBLEM 16.C4

$\sin\beta = \frac{b}{l}\sin\theta$ ◁



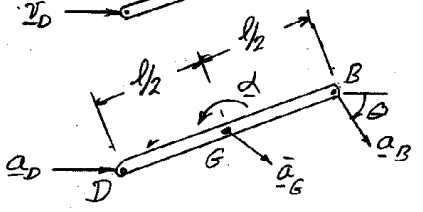
KINEMATICS REFER TO SOLUTION OF PROB 15.C2 FOR DETERMINATION OF:

$\underline{a}_B$ = ACCELERATION OF B
$\underline{a}_D$ = ACCELERATION OF PISTON D
α = ANGULAR ACCELERATION OF ROD BD



VELOCITIES

$v_B = b\omega_{AB}$ $\quad v_D = v_B\frac{\sin(\theta+\beta)}{\cos\beta}$ $\quad v_{B/D} = v_B\frac{\cos\theta}{\cos\beta}$ $\quad \omega = \frac{v_{B/D}}{l}$



ACCELERATIONS

$a_B = b\omega_{AB}^2$ ↘ $\quad \alpha = \dfrac{l\omega^2\sin\beta - a_B\sin\theta}{l\cos\beta}$ ◁

FOR POSITIVE DIRECTIONS UP AND TO THE RIGHT

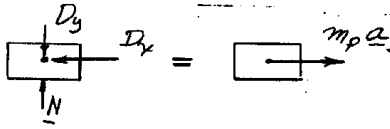
$$\begin{cases} a_D = l\alpha\sin\beta + l\omega^2\cos\beta + a_B\cos\theta & ◁ \\ (a_B)_x = a_B\cos\theta & ◁ \\ (a_B)_y = -a_B\sin\theta & ◁ \end{cases}$$

WE NOTE THAT G IS AT MIDPOINT OF UNIFORM ROD BD.

$$\underline{\bar{a}}_G = \tfrac{1}{2}(\underline{a}_D + \underline{a}_B) \quad \begin{cases} (\bar{a}_G)_x = \frac{1}{2}\left[a_D + (a_B)_x\right] & ◁ \\ (\bar{a}_G)_y = \frac{1}{2}(a_B)_y & ◁ \end{cases}$$

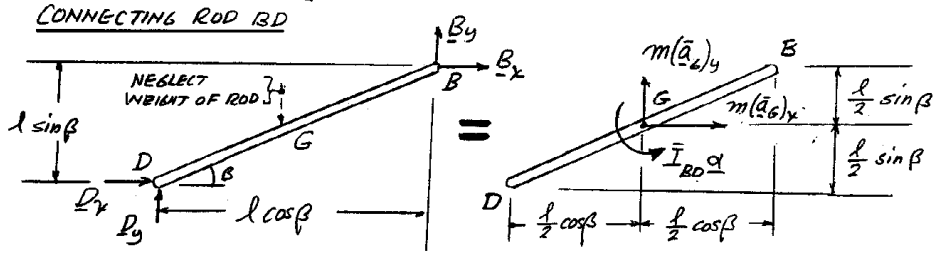
KINETICS

PISTON D



$\overset{+}{\leftarrow}\Sigma F_x = \Sigma(F_x)_{eff}$: $\quad D_x = -m_p a_D$ ◁

CONNECTING ROD BD



$\bar{I}_{BD} = \frac{1}{12}ml^2$ ◁

$+\circlearrowleft\Sigma M_B = \Sigma(M_B)_{eff}$:

$$D_x l\sin\beta - D_y l\cos\beta = \bar{I}_{BD}\alpha - m(\bar{a}_G)_y\frac{l}{2}\cos\beta + m(\bar{a}_G)_x\frac{l}{2}\sin\beta$$

$$D_y = \left[D_x l\sin\beta - \bar{I}_{BD}\alpha + m(\bar{a}_G)_y\frac{l}{2}\cos\beta - m(\bar{a}_G)_x\frac{l}{2}\sin\beta\right]/l\cos\beta$$

$$D_y = D_x\tan\beta - \frac{\bar{I}_{BD}\alpha}{l\cos\beta} + \frac{1}{2}m(\bar{a}_G)_y - \frac{1}{2}m(\bar{a}_G)_x\tan\beta$$ ◁

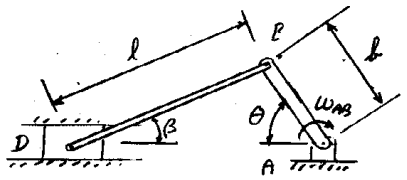
$+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}$: $\quad B_y + D_y = m(\bar{a}_G)_y$ $\qquad B_y = m(\bar{a}_G)_y - D_y$ ◁

$\overset{+}{\rightarrow}\Sigma F_x = \Sigma(F_x)_{eff}$: $\quad B_x + D_x = m(\bar{a}_G)_x$ $\qquad B_x = m(a_G)_x - D_x$ ◁

NOTE: FOR CONNECTING ROD BD, POSITIVE DIRECTIONS OF COMPONENTS ARE UP AND TO THE RIGHT, POSITIVE SENSE OF ALPHA IS COUNTERCLOCKWISE.

(CONTINUED)

PROBLEM 16.C4 CONTINUED



ω_{AB} = 2000 rpm

b = 3 in.

l = 8 in.

WEIGHT OF PISTON = 4 lb

WEIGHT OF ROD BD = 5 lb

```
10 '*****************************************************************
20 '*'                      PROBLEM 16.C4                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '           B = length of crank AB    (ft)
90 '           L = length of connecting rod BD   (ft)
100 '          M = mass of connecting rod BD   (lb.s^2/ft)
110 '         MP = mass of piston  (lb.s^2/ft)
120 '        IBD = mass moment of inertia of connecting rod (lb.ft.s^2)
130 '     ANGVEL = angular velocity of crank AB  (rpm)
140 '         TH = angle theta   (degrees)
150 '       BETA = angle between rod BD and horizontal   (degrees)
160 '
170 '      OMEGA = angular velocity of rod BD  (rad/s)
180 '         VB = velocity of pin at B   (ft/s)
190 '         VD = velocity of piston P   (ft/s)
200 '        VBD = velocity of B relative to D   (ft/s)
210 '
220 '      ALPHA = angular acceleration of rod BD  (rad/s^2)
230 '         AB = acceleration of pin at B   (ft/s^2)
240 '        ABX = horizontal component of AB
250 '        ABY = vertical component of AB
260 '         AD = acceleration of piston P   (ft/s^2)
270 '        ABD = acceleration of B relative to D   (ft/s^2)
280 '
290 '         AG = acceleration of mass center of rod BD   (ft/s^2)
300 '        AGX = horizontal component of AG
310 '        AGY = vertical component of AG
320 '      BX,BY = components of reaction at B   (lb)
330 '      DX,DY = components of reaction at D   (lb)
340 '
350  PI = 4*ATN(1):     K = PI/180
360 '
370 '    Data
380  LET B = 3     'inches
390  LET L = 8     'inches
400       B = B/12:      L = L/12        'express B and L in feet
410  LET ANGVEL = 2000     'rpm
420       ANGVEL = ANGVEL*(2*PI)/60     'express in rad/s
430  LET MP = 4/32.2
440  LET M = 5/32.2       'lb*ft*s^2
450  IBD = (1/12)*M*L^2
460 '
470  PRINT "           NOTE: Positive directions of components are ",
480  PRINT "                 up and to the right":PRINT
490  PRINT" theta       Bx            By            Dx            Dy"
500  PRINT"degrees      lb            lb            lb            lb
510  PRINT
520  FOR TH = 0 TO 180 STEP 10
530     GOSUB 670
540     DX = -MP*AD
550     ABX = AB*COS(K*TH):       ABY = -AB*SIN(K*TH)
560     AGX = .5*(AD + ABX)
570     AGY = .5*ABY
580     DY =  DX*TAN(K*BETA) - (IBD*ALPHA)/(L*COS(K*BETA))
                + .5*M*AGY - .5*M*AGX*TAN(K*BETA)
590     BY = M*AGY - DY
600     BX = M*ABX - DX
610     PRINT USING " ###       ";TH,
620     PRINT USING "#####.##       ";BX,BY,DX,DY
630  NEXT
640  END
650 '      SUBROUTINE to calculate acceleration of rod (cf.Prob 15.C2)
660 '       Calculation of velocities
670       SB = (B/L)*SIN(K*TH)          'SB = sine of angle BETA
680       BETA = (ATN(SB/SQR(1-SB^2)))/K
690       VB = B * ANGVEL
700       VD = VB * SIN(K*TH + K*BETA)/COS(K*BETA)
710       VBD = VB * COS(K*TH)/COS(K*BETA)
720       OMEGA = VBD/L
730 '
740 '       Calculation of accelerations
750       AB = B*(ANGVEL)^2
760       ALPHA = (L*(OMEGA)^2*SIN(K*BETA)-AB*SIN(K*TH))/(L*COS(K*BETA))
770       AD = L*ALPHA*SIN(K*BETA)+L*OMEGA^2*COS(K*BETA) + AB*COS(K*TH)
780  RETURN
```

CONTINUED

```
C*****************************************************************
C*                      PROBLEM 16.C4                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C            B = length of crank AB   (ft)
C            L = length of connection of rod BD   (ft)
C            M = mass of connecting rod BD   (lb.s**2/ft)
C           MP = mass of piston P   (lb.s**2/ft)
C          IBD = mass moment of inertia of connecting rod  (lb.ft.s**2)
C       ANGVEL = angular velocity of crank AB   (rpm)
C           TH = angle theta   (degrees)
C         BETA = angle between rod BD and horizontal   (degrees)
C
C        OMEGA = angular velocity of rod BD    (rad/s)
C           VB = velocity of pin at B   (ft/s)
C           VD = velocity of piston P   (ft/s)
C          VBD = velocity of B relative to D   (ft/s)
C
C        ALPHA = angular acceleration of rod BD   (rad/s**2)
C           AB = acceleration of pin at B   (ft/s**2)
C          ABX = horizontal component of AB
C          ABY = vertical component of AB
C           AD = acceleration of piston P   (ft/s**2)
C          ABD = acceleration of B relative to D   (ft/s**2)
C
C           AG = acceleration of mass center of rod BD  (ft/s**2)
C          AGX = horizontal component of AG
C          AGY = vertical component of AG
C        BX,BY = components of reaction at B   (lb)
C        DX,DY = components of reaction at D   (lb)
C
      REAL K, MP, L, M, IBD
C
      PI = 4.*ATAN(1.)
      K = PI/180.
C
C        Length of AB and BD in inches.
      B = 3.
      L = 8.
C        Express length of AB and BD in feet.
      B = B/12.
      L = L/12.
C        Angular velocity of crank in rpm.
      ANGVEL = 2000.
C        Express ANGVEL in rad/s
      ANGVEL = ANGVEL*(2.*PI)/60.
      MP = 4./32.2
      M = 5./32.2
      IBD = (1./12.)*M*L**2
C
      WRITE (*,'(/,2X,A\)') ' NOTE : Positive directions of components'
      WRITE (*,'(/,2X,A\)') '        are up and to the right'
C
      WRITE (*,11)
   11 FORMAT (//,T5,'theta',T19,'Bx',T33,'By',T48,'Dx',T62,'Dy')
      WRITE (*,12)
   12 FORMAT (T4,'degrees',T19,'lb',T33,'lb',T48,'lb',T62,'lb',/)
C
      DO 10 I = 0,180,10
         TH = I
            CALL ACCEL (L,B,ANGVEL,MP,M,IBD,TH,K,AD,AB,ALPHA,BETA)
         DX = -MP*AD
        ABX = AB*COS(K*TH)
        ABY = -AB*SIN(K*TH)
        AGX = 0.5*(AD + ABX)
        AGY = 0.5*ABY
         DY = DX*TAN(K*BETA) - (IBD*ALPHA)/(L*COS(K*BETA)) + 0.5*M*AGY
     $                - 0.5*M*AGX*TAN(K*BETA)
         BY = M*AGY - DY
         BX = M*AGX - DX
```

CONTINUED

PROBLEM 16.C4 CONTINUED

BASIC VERSION CONTINUED

NOTE: Positive directions of components are up and to the right

theta degrees	Bx lb	By lb	Dx lb	Dy lb
0	3895.22	0.00	-1873.11	0.00
10	3803.69	88.37	-1824.74	-236.21
20	3533.82	148.57	-1682.31	-439.78
30	3100.10	156.30	-1454.00	-582.01
40	2527.49	95.19	-1154.01	-642.47
50	1851.92	-39.00	-802.86	-613.23
60	1119.57	-235.13	-426.99	-502.21
70	383.53	-466.52	-56.82	-333.55
80	-302.90	-695.56	277.38	-142.92
90	-895.48	-883.41	551.06	31.99
100	-1367.40	-1001.08	750.49	162.60
110	-1713.12	-1036.69	875.02	236.62
120	-1945.53	-995.25	935.27	257.90
130	-2088.49	-892.49	948.44	240.27
140	-2168.51	-747.17	933.10	199.89
150	-2208.80	-575.59	905.51	149.88
160	-2226.67	-389.47	877.91	98.27
170	-2233.37	-196.16	858.39	48.31
180	-2234.97	0.00	851.42	-0.00

FORTRAN VERSION CONTINUED

```
      WRITE (*,20) TH,BX,BY,DX,DY
   20 FORMAT (4X,F4.0,7X,F8.2,6X,F8.2,7X,F8.2,7X,F7.2)
   10 CONTINUE
C
      WRITE (*,'(/,A\)')
      END
C
      SUBROUTINE ACCEL (L,B,ANGVEL,MP,M,IBD,TH,K,AD,AB,ALPHA,BETA)
      REAL L, MP, M, IBD, K
C     Calculation of velocities
      BETA = (ASIN((B/L)*SIN(K*TH)))/K
      RADICAL = SQRT(1.- ((B/L)*SIN(K*TH))**2)
      VB = B * ANGVEL
      VD = VB * SIN(K*TH + K*BETA)/COS(K*BETA)
      VBD = VB * COS(K*TH)/COS(K*BETA)
      OMEGA = VBD/L
C
C     Calculation of accelerations
      AB = B*(ANGVEL)**2
      ALPHA = (L*(OMEGA)**2*SIN(K*BETA)-AB*SIN(K*TH))/(L*COS(K*BETA))
      AD = L*ALPHA*SIN(K*BETA) +L*OMEGA**2*COS(K*BETA) +AB*COS(K*TH)
      RETURN
      END
```

NOTE : Positive directions of components are up and to the right

theta degrees	Bx lb	By lb	Dx lb	Dy lb
0.	3895.22	.00	-1873.11	.00
10.	3803.69	88.37	-1824.74	-236.21
20.	3533.82	148.57	-1682.31	-439.77
30.	3100.10	156.30	-1454.00	-582.01
40.	2527.49	95.19	-1154.01	-642.47
50.	1851.92	-39.00	-802.86	-613.23
60.	1119.57	-235.13	-426.99	-502.21
70.	383.53	-466.52	-56.82	-333.55
80.	-302.90	-695.56	277.38	-142.92
90.	-895.48	-883.41	551.06	31.99
100.	-1367.40	-1001.08	750.49	162.60
110.	-1713.12	-1036.69	875.02	236.62
120.	-1945.53	-995.25	935.27	257.90
130.	-2088.49	-892.49	948.44	240.27
140.	-2168.51	-747.17	933.10	199.89
150.	-2208.80	-575.59	905.51	149.88
160.	-2226.67	-389.47	877.91	98.27
170.	-2233.37	-196.16	858.39	48.31
180.	-2234.96	.00	851.42	.00

17.1

$\bar{I} = m\bar{k}^2 = \frac{300\,lb}{32.2\,ft/s^2}\left(\frac{9}{12}\,ft\right)^2 = 5.241\ lb\cdot ft\cdot s^2$

$\omega_1 = 3600\ rpm = 377\ rad/s \qquad \omega_2 = 0$

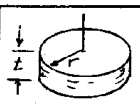
$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(5.241)(377)^2 = 372{,}400\ ft\cdot lb$

$T_2 = 0 \qquad U_{1-2} = -M\theta = -\left(\frac{10}{12}\,lb\cdot ft\right)\theta$

$T_1 + U_{1-2} = T_2: \quad 372{,}400 - \frac{10}{12}\theta = 0$

$\theta = 446{,}960\ rad \qquad \theta = 71{,}100\ rev$ ◀

17.2



FOR ANY DISK: $m = \rho(\pi r^2 t)$

$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}\pi\rho t r^4$

DISK A: $I_A = \frac{1}{2}\pi\rho b r^4$

DISK B: $I_B = \frac{1}{2}\pi\rho(3b)(nr)^4 = 3n^4\left[\frac{1}{2}\pi\rho b r^4\right] = 3n^4 I_A$

$I_{TOTAL} = I_A + I_B = (1+3n^4)I_A$

WORK-ENERGY $T_1 = 0 \qquad U_{1-2} = M\theta = M(4\pi\ rad)$

$T_2 = \frac{1}{2}I_{TOTAL}\omega_2^2$

$T_1 + U_{1-2} = T_2: \quad 0 + M(4\pi) = \frac{1}{2}(1+3n^4)I_A\omega_2^2$

$\omega_2^2 = \frac{8\pi M}{(1+3n^4)I_A}$

FOR POINT D ON RIM OF DISK B

$v_D = (nr)\omega_2 \quad$ OR $\quad v_D^2 = n^2 r^2 \omega_2^2 = \frac{8\pi M r^2}{I_A}\cdot\frac{n^2}{1+3n^4}$

FOR MAXIMUM v_D: $\frac{d}{dn}\left(\frac{n^2}{1+3n^4}\right) = 0$

$\frac{1}{(1+3n^4)^2}\left[n^2(12n^3) - (1+3n^4)(2n)\right] = 0$

$12n^5 - 2n - 6n^5 = 0$

$2n(3n^4 - 1) = 0$

$n = 0$ AND $n = (1/3)^{0.25} = 0.7598 \qquad n = 0.760$ ◀

17.3

SEE SOLUTION OF PROB 17.2 FOR:

$I_A = \frac{1}{2}mr^2 \qquad I_{TOTAL} = (1+3n^4)I_A$

$T_1 = 0 \qquad T_2 = \frac{1}{2}I_{TOTAL}\omega_2^2 \qquad U_{1-2} = M\theta \qquad M = 20\ N\cdot m$

$\omega_2 = 600\ rpm = 62.83\ rad/s \qquad \theta = 4\ rev = 4(2\pi) = 8\pi\ rad$

$T_1 + U_{1-2} = T_2: \quad 0 + (20\ N\cdot m)(8\pi\ rad) = \frac{1}{2}I_{TOTAL}(62.83\ rad/s)^2$

$I_{TOTAL} = 0.2547\ lb\cdot ft\cdot s^2 \qquad (1)$

$I_A = \frac{1}{2}mr^2 = \frac{1}{2}(15\ kg)(0.125\ m)^2 = 0.11719\ lb\cdot ft\cdot s^2$

$I_{TOTAL} = (1+3n^4)I_A: \quad 0.2547 = (1+3n^4)(0.11719)$

$1 + 3n^4 = 2.173 \qquad n^4 = \frac{1}{3}(1.173) \qquad n = 0.7908$

RADIUS OF B: $r_B = nr = (0.7908)(125\ mm) \qquad r_B = 98.8\ mm$ ◀

17.4

$\bar{I} = m\bar{k}^2 = \frac{900\,lb}{32.2\,ft/s^2}\left(\frac{30}{12}\,ft\right)^2 = 174.69\ lb\cdot ft\cdot s^2$

(a) $\omega_1 = 200\ rpm = 20.944\ rad/s$

$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(174.69)(20.944)^2 = 38{,}314\ ft\cdot lb$

$U_{1-2} = -2000\ ft\cdot lb$

$T_1 + U_{1-2} = T_2: \quad 38314 - 2000 = T_2$

$T_2 = 36{,}314\ ft\cdot lb$

$T_2 = \frac{1}{2}\bar{I}\omega_2^2: \quad 36314 = \frac{1}{2}(174.69)\omega_2^2$

$\omega_2 = 20.39\ rad/s = 194.71\ rpm \qquad \omega_2 = 194.7\ rpm$ ◀

(b) $U_{2-1} = M\theta: \quad 2000\ ft\cdot lb = (251\ lb\cdot ft)\theta$

$\theta = 80\ rad \qquad \theta = 12.73\ rev$ ◀

17.5

$\omega_1 = 300\ rpm = 31.416\ rad/s; \quad \omega_2 = 0.90\,\omega_1$

(a) $U_{1-2} = -500\ J$

$T_1 + U_{1-2} = T_2: \quad \frac{1}{2}\bar{I}\omega_1^2 - 500\ J = \frac{1}{2}\bar{I}\omega_2^2$

$\frac{1}{2}\bar{I}(\omega_1^2 - \omega_2^2) = 500$

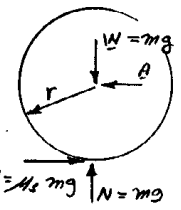
$\bar{I} = \frac{2(500)}{\omega_1^2 - \omega_2^2} = \frac{1000}{\omega_1^2(1-0.9^2)} = \frac{1000}{(31.416)^2(1-0.81)} = 5.333\ kg\cdot m^2$

$I = 5.33\ kg\cdot m^2$ ◀

(b) $U_{2-1} = M\theta: \quad 500\ J = (25\ N\cdot m)\theta$

$\theta = 20\ rad \qquad \theta = 3.18\ rev$ ◀

17.6



ONLY FORCE DOING WORK IS FRICTION FORCE F.

MOMENT OF F ABOUT CENTER A IS

$M = Fr$

$M = \mu_s mg\, r$

WORK: $U_{1-2} = M\theta = \mu W r\theta$

INITIAL VELOCITY = 0 $\qquad T_1 = 0$

FINAL VELOCITY = $\omega_2 = \frac{v}{r} \qquad \bar{I} = \frac{1}{2}mr^2$

$T_2 = \frac{1}{2}\bar{I}\omega_2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$

WORK-ENERGY

$T_1 + U_{1-2} = T_2$

$0 + \mu_k mg r\theta = \frac{1}{4}mv^2$

$\theta = \frac{v^2}{4r\mu_k g}\ rad \qquad (1)$

DATA: $v = 18\ m/s$

$m = 4\ kg$

$r = 75\ mm = 0.075\ m$

$\mu_k = 0.25$

EQ(1) $\quad \theta = \frac{(18\ m/s)^2}{4(0.075\ m)(0.25)(9.81\ m/s^2)} = 440.4\ rad$

$\theta = (440.4\ rad)/2\pi = 70.09\ rev$

$\theta = 70.1\ rev$ ◀

ANSWER IS INDEPENDENT OF MASS OF DISK

17.7

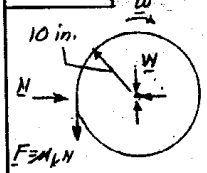
SEE PROB 17.6 FOR DERIVATION OF EQ (1)

$\theta = \frac{v^2}{4r\mu_k g}\ rad$

$\theta = \frac{1}{2\pi}\left[\frac{v^2}{4r\mu_k g}\right]\ rev$

$\theta = \frac{v^2}{8\pi r\mu_k g}\ rev$ ◀

17.8



$\omega_1 = 240 \text{ rpm} = 8\pi \text{ rad/s}$ $\quad \mu_k = 0.40$

$\bar{I} = 16 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053 \text{ ft}\cdot\text{lb}$

$\omega_2 = 0 \quad T_2 = 0$

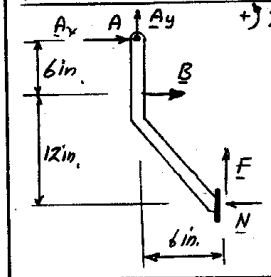
$\theta = 75 \text{ rev} = 75(2\pi) = 150\pi \text{ rad}$

$U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ ft}\right)\right](150\pi \text{ rad}) = -392.7F$

$T_1 + U_{1-2} = T_2$: $\quad 5053 - 392.7F = 0 \quad F = 12.868 \text{ lb}$

$F = \mu_k N$: $\quad 12.868 = (0.40)N \quad N = 32.17 \text{ lb}$

FREE BODY BRAKE ARM:



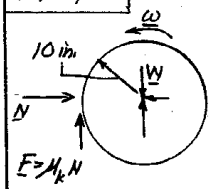
$+\circlearrowleft \Sigma M_A = 0$: $\quad B(6\text{in.}) + F(6\text{in.}) - N(18\text{in.}) = 0$

$B(6\text{in.}) + (12.868\text{lb})(6\text{in.}) - (32.17\text{lb})(18\text{in.}) = 0$

$B = 83.64 \text{ lb}$

$\underline{B} = 83.6 \text{ lb} \rightarrow$ ◀

17.9



$\omega_1 = 240 \text{ rpm} = 8\pi \text{ rad/s}$ $\quad \mu_k = 0.40$

$\bar{I} = 16 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053 \text{ ft}\cdot\text{lb}$

$\omega_2 = 0 \quad T_2 = 0$

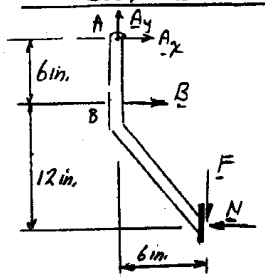
$\theta = 75 \text{ rev} = 75(2\pi) = 150\pi \text{ rad}$

$U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ ft}\right)\right](150\pi \text{ rad}) = -392.7F$

$T_1 + U_{1-2} = T_2$: $\quad 5053 - 392.7F = 0 \quad F = 12.868 \text{ lb}$

$F = \mu_k N$: $\quad 12.868 = 0.40N \quad N = 32.17 \text{ lb}$

FREE BODY BRAKE ARM:



$+\circlearrowleft \Sigma M_A = 0$

$B(6\text{in.}) - F(6\text{in.}) - N(18\text{in.}) = 0$

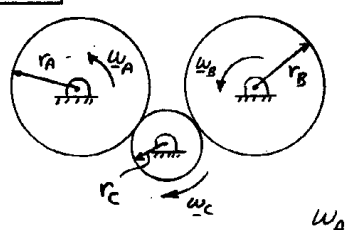
$B(6\text{in.}) - (12.868\text{lb})(6\text{in.}) - (32.17\text{lb})(18\text{in.}) = 0$

$B = 109.37 \text{ lb}$

$\underline{B} = 109.4 \text{ lb} \rightarrow$ ◀

17.10

KINEMATICS



$r_A\omega_A = r_C\omega_C = r_B\omega_B$

SINCE $r_A = r_B$

$\omega_A = \omega_B = \frac{r_C}{r_A}\omega_C$

FOR $r_A = r_B = 10\text{in.}$

$r_C = 4\text{in}$

$\omega_A = \omega_B = \frac{4\text{in.}}{10\text{in.}}\omega_C$

$\omega_A = \omega_B = 0.4\,\omega_C$

(CONTINUED)

17.10 CONTINUED

$W_B = W_B = 20 \text{ lb} \quad W_C = 5 \text{ lb}$

$\bar{r}_A = \bar{r}_B = 7.5 \text{ in} \quad \bar{r}_C = 3 \text{ in.}$

GEARS A AND B: $\bar{I}_A = \bar{I}_B = m\bar{r}^2 = \frac{20}{32.2}\left(\frac{7.5}{12}\right)^2 = 0.24262 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

GEAR C: $\bar{I}_C = m\bar{r}^2 = \frac{5}{32.2}\left(\frac{3}{12}\right)^2 = 0.00970 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

POSITION 1: $\omega_C = 450 \text{ rpm} = 47.12 \text{ rad/s}$; $\omega_A = \omega_B = 0.4\omega_C = 18.85 \text{ rad/s}$

$T_1 = 2\left[\frac{1}{2}I_A\omega_A^2\right] + \frac{1}{2}I_C\omega_C^2$

$= 2\left[\frac{1}{2}(0.24262)(18.85)^2\right] + \frac{1}{2}(0.00970)(47.12)^2 = 96.98 \text{ ft}\cdot\text{lb}$

POSITION 2: $\omega_C = 1800 \text{ rpm} = 188.50 \text{ rad/s}$; $\omega_A = \omega_B = 75.4 \text{ rad/s}$

$T_2 = 2\left[\frac{1}{2}(0.24262)(75.4)^2\right] + \frac{1}{2}(0.00970)(188.5)^2 = 1551.6 \text{ ft}\cdot\text{lb}$

NOTE. SINCE $T \sim \omega^2$ AND $\frac{\omega_2}{\omega_1} = 4$ WE CHECK THAT $\frac{T_2}{T_1} = (4)^2 = 16$

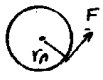
$U_{1-2} = M\theta_C = (50\text{lb}\cdot\text{in})\theta_C = \left(\frac{50}{12}\text{ lb}\cdot\text{ft}\right)\theta_C$

WORK-ENERGY (1)

$T_1 + U_{1-2} = T_2$ $\quad 96.98 \text{ ft}\cdot\text{lb} + \left(\frac{50}{12}\text{ lb}\cdot\text{ft}\right)\theta_C = 1551.6 \text{ ft}\cdot\text{lb}$

$\theta_C = 349.1 \text{ rad} \quad \theta_C = 55.6 \text{ rev}$ ◀

(b) GEAR A $\quad \theta_A = 0.4\theta_C = 139.64 \text{ rad} \quad r_A = \frac{10}{12}\text{ ft}$



$T_1 + U_{1-2} = T_2$: $\quad \frac{1}{2}I_A\omega_{A_1}^2 + Fr_A\theta_A = \frac{1}{2}I_A\omega_{A_2}^2$

$\frac{1}{2}(0.24262)(18.85)^2 + F\left(\frac{10}{12}\right)(139.64) = \frac{1}{2}(0.24262)(75.4)^2$

$F = 5.56 \text{ lb}$ ◀

17.11

SEE SOLUTION OF PROB. 17.10. THE 50 lb·in COUPLE IS NOW APPLIED TO GEAR B.

(a) EQ.(1) IS NOW: $96.98 \text{ ft}\cdot\text{lb} + \left(\frac{50}{12}\text{ lb}\cdot\text{ft}\right)\theta_B = 1551.6 \text{ ft}\cdot\text{lb}$

$\theta_B = 349.1 \text{ rad}$

THE CORRESPONDING ROTATION OF GEAR C IS

$\theta_C = \theta_B/0.4 = (349.1 \text{ rad})/0.4$

$\theta_C = 872.75 \text{ rad} \quad \theta_C = 138.9 \text{ rev}$ ◀

(b) GEAR A $\quad \theta_A = \theta_B = 349.1 \text{ rad} \quad r_A = \frac{10}{12}\text{ ft}$

$T_1 + U_{1-2} = T_2$: $\quad \frac{1}{2}I_A\omega_{A_1}^2 + Fr_A\theta_A = \frac{1}{2}I_A\omega_{A_2}^2$

$\frac{1}{2}(0.24262)(18.85)^2 + F\left(\frac{10}{12}\right)(349.1) = \frac{1}{2}(0.24262)(75.4)^2$

$F = 2.22 \text{ lb}$ ◀

17.12

FOR A DISK OF RADIUS r AND THICKNESS t:

$m = \rho(\pi r^2)t = \rho\pi t r^2$

$\bar{I}_0 = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi t r^2)r^2 = \frac{1}{2}\rho\pi t r^4$

FOR A DISK OF RADIUS nr AND THICKNESS t,

$\bar{I} = \frac{1}{2}\rho\pi t(nr)^4 \qquad \bar{I} = n^4\bar{I}_0$

KINEMATICS: IF FOR SHAFT A WE HAVE ω_A

THEN, FOR SHAFT B WE HAVE $\omega_B = \omega_A/n$

AND, FOR SHAFT C WE HAVE $\omega_C = \omega_A/n^2$.

PRINCIPLE OF WORK-ENERGY:

COUPLE M_0 APPLIED TO SHAFT C FOR ONE REVOLUTION, $\theta = 2\pi$ radians

$T_1 = 0$, $\quad U_{1-2} = M_0\theta = M_0(2\pi \text{ radians}) = 2\pi M_0$

$T_2 = \frac{1}{2}\left(\bar{I}_{\text{SHAFT A}}\right)\omega_A^2 + \frac{1}{2}\left(\bar{I}_{\text{SHAFT B}}\right)\omega_B^2 + \frac{1}{2}\left(\bar{I}_{\text{SHAFT C}}\right)\omega_C^2$

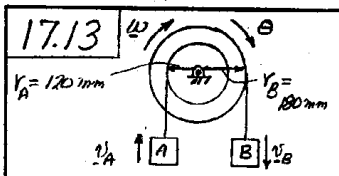
$= \frac{1}{2}\bar{I}_0\omega_A^2 + \frac{1}{2}\left(\bar{I}_0 + n^4\bar{I}_0\right)\left(\frac{\omega_A}{n}\right)^2 + \frac{1}{2}\left(n^4\bar{I}_0\right)\left(\frac{\omega_A}{n^2}\right)^2$

$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n^2 + 2 + \frac{1}{n^2}\right) = \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$

$T_1 + U_{1-2} = T_2$: $\quad 0 + 2\pi M_0 = \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$

$\omega_A^2 = \frac{4\pi M_0}{\bar{I}_0}\frac{1}{\left(n + \frac{1}{n}\right)^2} \qquad \omega_A = \frac{2n}{n^2+1}\sqrt{\frac{\pi M_0}{\bar{I}_0}}$ ◀

17.13



$m_A = 2(1.5) = 3.0$ kg
$m_B = 2(1.5) = 3.0$ kg

KINEMATICS ASSUME ω)

$\underline{v}_A = (0.12\,m)\omega\ \uparrow$

$v_B = (0.18\,m)\omega\ \downarrow$

FOR $y_A = 900$ mm $\uparrow = 0.9$ m $\uparrow$

$\theta = \frac{y_A}{r_A} = \frac{900\text{ mm}}{120\text{ mm}} = 7.5$ rad)

$y_B = r_B\theta = (0.18)(7.5) = 1.35$ m $\downarrow$

KINETICS: $I = m_i \bar{k}_i^2 = (6\text{ kg})(0.140\text{ m})^2 = 0.1176$ kg·m²

$T_1 = 0$

$T_2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$T_2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m_A (0.12\,\omega)^2 + \frac{1}{2} m_B (0.18\,\omega)^2$

$= \frac{1}{2}\left[I + 14.4\times10^{-3} m_A + 32.4\times10^{-3} m_B\right]\omega^2$ (1)

$= \frac{1}{2}\left[0.1176 + 14.4\times10^{-3}(3.0) + 32.4\times10^{-3}(3.0)\right]\omega^2$

$T_2 = 0.1290\,\omega^2$

$U_{1-2} = -W_A y_A + W_B y_B - M\theta$ $\quad M = 0.5$ N·m

$= -(3.0)(9.81)(0.9) + (3.0)(9.81)(1.35) - (0.5)(7.5)$

$= 9.4935$ J

PRINCIPLE OF WORK-ENERGY

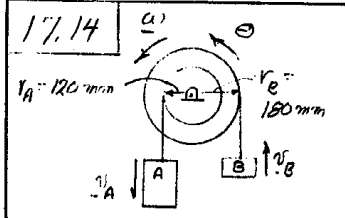
$T_1 + U_{1-2} = T_2$: $\quad 0 + 9.4935 = 0.1290\,\omega^2$

$\omega^2 = 73.593$ $\quad \omega = 8.5786$ rad/s

$v_A = (0.12\,m)\omega = (0.12)(8.5786) = 1.0294$ m/s

$\underline{v}_A = 1.029$ m/s ↑ ◀

17.14



$m_A = 2(1.5) = 4.5$ kg
$m_B = 1.5$ kg

KINEMATICS ASSUME ω)

$\underline{v}_A = (0.12\,m)\omega\ \downarrow$

$v_B = (0.18\,m)\omega\ \uparrow$

$y_A = 0.9$ m $\downarrow$

$\theta = \frac{y_A}{r_A} = \frac{0.9\text{ m}}{0.12\text{ m}} = 7.5$ rad)

$y_B = r_B\theta = (0.18)(7.5) = 1.35$ m ↑

KINETICS $\quad T_1 = 0$

SEE PROB 17.13 WE AGAIN HAVE

EQ(1): $T_2 = \frac{1}{2}\left[I + 14.4\times10^{-3} m_A + 32.4\times10^{-3} m_B\right]\omega^2$

$= \frac{1}{2}\left[0.1176 + 14.4\times10^{-3}(4.5) + 32.4\times10^{-3}(1.5)\right]\omega^2$

$T_2 = 0.1155\,\omega^2$

$U_{1-2} = +W_A y_A - W_B y_B - M\theta$ $\quad M = 0.5$ N·m

$= (4.5)(9.81)(0.9) - (1.5)(9.81)(1.35) - 0.5(7.5)$

$= 16.115$

PRINCIPLE OF WORK-ENERGY

$T_1 + U_{1-2} = T_2$

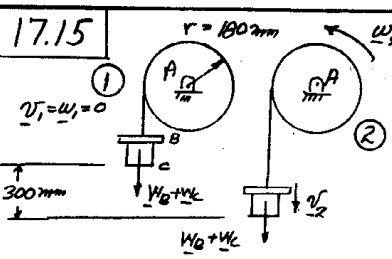
$0 + 16.115 = 0.1155\,\omega^2$

$\omega^2 = 139.526$ $\quad \omega = 11.812$ rad/s

$v_A = r_A\omega = (0.12\,m)(11.812\text{ rad/s}) = 1.4174$

$\underline{v}_A = 1.417$ m/s ↓ ◀

17.15



$\omega_2 = \frac{v_2}{r} = \frac{v_2}{0.18\,m}$

$\bar{I} = m\bar{k}^2 = (5\text{ kg})(0.15\text{ m})^2$

$= 0.1125$ kg·m²

$W_B + W_C = (1.8\text{ kg} + 3\text{ kg})g$

$= 47.088$ N

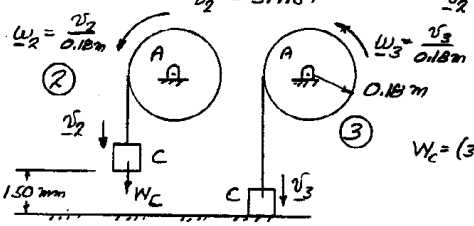
COLLAR WILL BE REMOVED AT POSITION 2.

$T_1 + U_{1-2} = T_2$: $\quad 0 + (W_B + W_C)(0.3\text{ m}) = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}(m_B + m_C)v_2^2$

$0 + (47.088)(0.3) = \frac{1}{2}(0.1125)\left(\frac{v_2}{0.18}\right)^2 + \frac{1}{2}(1.8+3)v_2^2$

$14.1264 = (1.736 + 2.4)v_2^2$

$v_2^2 = 3.4154$ $\quad \underline{v}_2 = 1.848$ m/s ↓



$W_C = (3\text{ kg})g = 29.43$ N

MOTION AFTER COLLAR HAS BEEN REMOVED

$T_2 = \frac{1}{2}m_C v_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m v_2^2 + \frac{1}{2}\bar{I}\left(\frac{v_2}{0.18}\right)^2$

$= \frac{1}{2}(3\text{ kg})v_2^2 + \frac{1}{2}(0.1125\text{ kg·m}^2)\frac{1}{(0.18\text{m})^2}v_2^2$

$= (1.5 + 1.736)v_2^2 = 3.236\,(3.4154) = 11.0522$ J

$U_{2-3} = W_C(0.15\text{ m}) = (29.43\text{ N})(0.15\text{ m}) = 4.4145$ J

$T_3 = \frac{1}{2}m_C v_3^2 + \frac{1}{2}\bar{I}\omega_3^2$

$= \frac{1}{2}(3\text{ kg})v_3^2 + \frac{1}{2}(0.1125\text{ kg·m}^2)\left(\frac{v_3}{0.18}\right)^2 = (1.5 + 1.736)v_3^2$

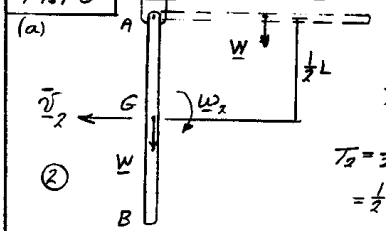
$T_2 + U_{2-3} = T_3$: $\quad 11.0522\text{ J} + 4.4145\text{ J} = (1.5 + 1.736)v_3^2$

$15.4667 = 3.236\,v_3^2$

$v_3^2 = 4.7796$ $\quad \underline{v}_3 = 2.19$ m/s ↓ ◀

17.16

(a)



$\omega_1 = v_1 = 0$

$v_2 = \frac{L}{2}\omega_2$

$T_1 = 0$

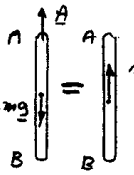
$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$

$U_{1-2} = W\frac{L}{2} = mg\frac{L}{2}$ $\quad T_2 = \frac{1}{6}mL^2\omega_2^2$

$T_1 + U_{1-2} = T_2$: $\quad 0 + mg\frac{L}{2} = \frac{1}{6}mL^2\omega_2^2$

$\omega_2^2 = \frac{3g}{L}$ $\quad \underline{\omega}_2 = \sqrt{3g/L}$) ◀



$m\bar{a} = m r\omega_2^2 = m\frac{L}{2}\left(\frac{3g}{L}\right) = \frac{3}{2}mg$

$+\uparrow \Sigma F_y = \Sigma F_{eff}$: $\quad A - mg = \frac{3}{2}mg$

$A = 2.5\,mg$ $\quad A = 2.5\,W$ ↑ ◀

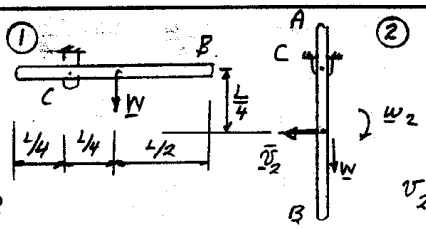
(b) DATA: $\quad W = mg = 6$ lb $\quad L = 2.5$ ft

$\omega_2^2 = 3g/L = (3)(32.2\text{ ft/s}^2)/2.5\text{ ft}$ $\quad \underline{\omega}_2 = 6.22$ rad/s) ◀

$A = 2.5\,mg = 2.5(6\text{ lb})$ $\quad \underline{A} = 15$ lb ↑ ◀

17.17

(a)



$\omega_1 = v_1 = 0$

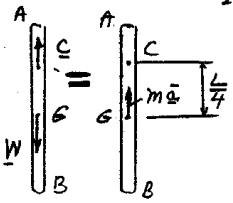
$T_1 = 0$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2} m \left(\frac{L}{4}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12} m L^2\right)\omega_2^2 = \frac{7}{96} m L^2 \omega_2^2$

$V_{1-2} = W \frac{L}{4} = mg \frac{L}{4}$

$T_1 + V_{1-2} = T_2$: $\quad 0 + mg\frac{L}{4} = \frac{7}{96} m L^2 \omega_2^2$

$\omega_2^2 = \frac{24}{7}\frac{g}{L}$ $\qquad \underline{\omega_2 = \sqrt{\frac{24g}{7L}}}$ ◀



$m\bar{a} = m\frac{L}{4}\omega_2^2 = m\frac{L}{4}\left(\frac{24}{7}\frac{g}{L}\right) = \frac{6}{7}mg$

$+\uparrow \Sigma F_y = \Sigma F_{eff}$

$C - W = \frac{6}{7} mg$

$C - W = \frac{6}{7} W$ $\qquad \underline{C = \frac{13}{7} W \uparrow}$ ◀

(b) DATA: $W = 6$ lb, $L = 2.5$ ft

$\omega_2^2 = \frac{24g}{7L} = \frac{24(32.2\ \text{ft/s}^2)}{7(2.5\ \text{ft})} = 44.16$ $\qquad \underline{\omega_2 = 6.65\ \text{rad/s}}$ ◀

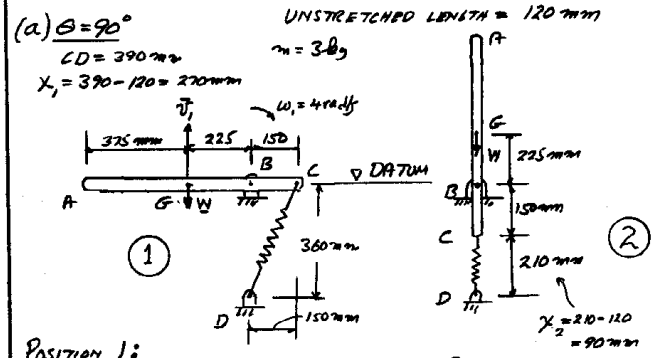
$C = \frac{13}{7} W = \frac{13}{7}(6\ \text{lb})$ $\qquad \underline{C = 11.14\ \text{lb} \uparrow}$ ◀

17.18

SPRING: $k = 300$ N/m

UNSTRETCHED LENGTH = 120 mm

(a) $\theta = 90°$



POSITION 1:

SPRING: $V_e = \frac{1}{2} k x_1^2 = \frac{1}{2}(300\ \text{N/m})(0.270\ \text{m})^2 = 10.935$ J

GRAVITY: $V_g = 0$

KINETIC ENERGY $\quad \bar{v}_1 = (BG)\omega_1 = (0.225\ \text{m})(4\ \text{rad/s}) = 0.9$ m/s

$T_1 = \frac{1}{2} m \bar{v}_1^2 + \frac{1}{2} \bar{I} \omega_1^2 = \frac{1}{2}(3\ \text{kg})(0.9\ \text{m/s})^2 + \frac{1}{2}\left[\frac{1}{12}(3\ \text{kg})(0.75\ \text{m})^2\right](4\ \text{rad/s})^2$

$T_1 = 1.215 + 1.125 = 2.34$ J

POSITION 2:

SPRING: $V_e = \frac{1}{2} k x_2^2 = \frac{1}{2}(300\ \text{N/m})(0.09\ \text{m})^2 = 1.215$ J

GRAVITY: $V_g = Wh = mgh = (3\ \text{kg})(9.81\ \text{m/s}^2)(0.225\ \text{m}) = 6.6218$ J

KINETIC ENERGY: $\bar{v}_2 = (BG)\omega_2 = (0.225\ \text{m})\omega_2$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2 = \frac{1}{2}(3\ \text{kg})(0.225\omega_2)^2 + \frac{1}{2}\left[\frac{1}{12}(3\ \text{kg})(0.75\ \text{m})^2\right]\omega_2^2$

$T_2 = 0.075938\omega_2^2 + 0.070313\omega_2^2 = 0.14625\omega_2^2$

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$: $\quad 2.34\ \text{J} + 10.935\ \text{J} + 0 = 0.14625\omega_2^2 + 1.215\ \text{J} + 6.6218\ \text{J}$

$5.4383 = 0.14625\omega_2^2$

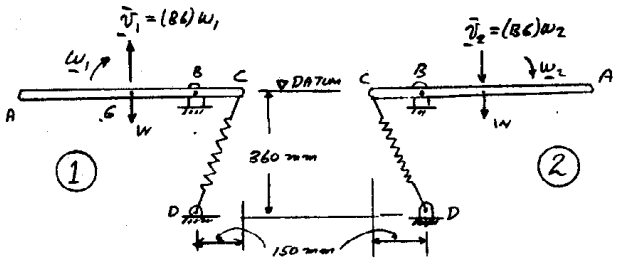
$\omega_2^2 = 37.184$ $\qquad \omega_2 = 6.098$ rad/s

$\underline{\omega_2 = 6.10\ \text{rad/s}}$ ◀

(CONTINUED)

17.18 CONTINUED

(b) $\theta = 180°$



WE NOTE THAT IN POSITIONS 1 AND 2 THE LENGTH OF SPRING IS THE SAME. ALSO THE POTENTIAL WITH RESPECT TO GRAVITY IS THE SAME.

THUS $V_1 = V_2$

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$

SINCE $V_1 = V_2$, $\quad T_1 = T_2$ AND $\omega_1 = \omega_2$

RECALL THAT $\omega_1 = 4$ rad/s, $\quad \therefore$ $\underline{\omega_2 = 4\ \text{rad/s}}$ ◀

17.19

THE ROD NOW ROTATES IN A HORIZONTAL PLANE.

WE MAY FOLLOW SOLUTION OF PROB. 17.18 AND REMOVE TERMS CORRESPONDING TO POTENTIAL ENERGY WITH RESPECT TO GRAVITY.

(a) $\theta = 90°$

POSITION 1:

SPRING: $V_e = 10.935$ J $\qquad V_1 = 10.935$ J

KINETIC ENERGY: $T_1 = 2.34$ J

POSITION 2:

SPRING: $V_e = 1.215$ J $\qquad V_2 = 1.215$ J

KINETIC ENERGY:

$T_2 = 0.14625\omega_2^2$

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$

$2.34\ \text{J} + 10.935\ \text{J} = 0.14625\omega_2^2 + 1.215\ \text{J}$

$12.06 = 0.14625\omega_2^2$

$\omega_2^2 = 82.46$ $\qquad \omega_2 = 9.081$ rad/s

$\underline{\omega_2 = 9.08\ \text{rad/s}}$ ◀

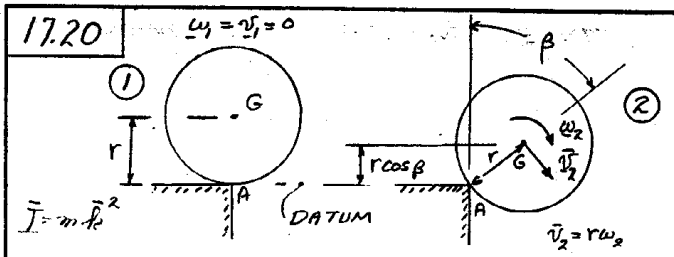
(b) $\theta = 180°$

SEE PART (b) OF PROB 17.18, WE AGAIN NOTE THAT $V_1 = V_2$ AND THUS $T_1 = T_2$ AND $\omega_1 = \omega_2$

SINCE $\omega_1 = 4$ rad/s,

$\underline{\omega_2 = 4\ \text{rad/s}}$ ◀

17.20



$T_1 = 0$ $\quad V_1 = mgr$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$ $\quad V_2 = mg(r\cos\beta)$

$= \frac{1}{2}m(r\omega_2)^2 + \frac{1}{2}m\bar{k}^2\omega_2^2 = \frac{1}{2}m(r^2+\bar{k}^2)\omega_2^2$

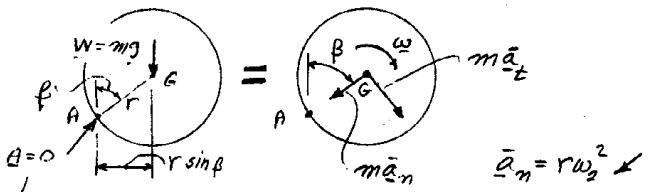
CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + mgr = \frac{1}{2}m(r^2+\bar{k}^2)\omega_2^2 + mgr\cos\beta$

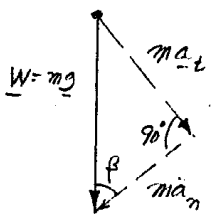
$$\omega_2^2 = \frac{2g(1-\cos\beta)r}{r^2+\bar{k}^2}$$

NEWTON'S SECOND LAW

SINCE μ_s IS LARGE, SPHERE WILL NOT SLIP AT A AND WILL LOSE CONTACT WHEN N IS ZERO.



SINCE $N=0$, $F = \mu N = 0$



$m\bar{a}_n = mg\cos\beta$

$r\omega_2^2 = g\cos\beta$ $\quad (1)$

$$\frac{2g(1-\cos\beta)r^2}{r^2+\bar{k}^2} = g\cos\beta$$

$$\frac{\cos\beta}{1-\cos\beta} = \frac{2}{1+\frac{\bar{k}^2}{r^2}} \quad (2)$$

EQ(1): $\omega_2 = \sqrt{\frac{g}{r}\cos\beta}$ $\quad \bar{v}_2 = r\omega_2 = \sqrt{gr\cos\beta}$ $\quad (3)$

FOR A SPHERE: $m\bar{k}^2 = \frac{2}{5}mr^2$ $\quad \frac{\bar{k}^2}{r^2} = \frac{2}{5}$

(a) EQ(2): $\frac{\cos\beta}{1-\cos\beta} = \frac{2}{1+\frac{2}{5}} = \frac{10}{7}$

$7\cos\beta = 10 - 10\cos\beta$

$\cos\beta = \frac{10}{17}$ $\quad \beta = 53.97°$ $\quad \beta = 54.0°$ ◀

(b) EQ(3): $\bar{v}_2 = \sqrt{gr\left(\frac{10}{17}\right)} = 0.7670\sqrt{gr}$

$\bar{v}_2 = 0.767\sqrt{gr}$ ↖ 54.0° ◀

17.21

SEE SOLUTION OF PROB 17.20:

FOR A CYLINDER: $m\bar{k}^2 = \frac{1}{2}mr^2$ $\quad \frac{\bar{k}^2}{r^2} = \frac{1}{2}$

(a) EQ(2): $\frac{\cos\beta}{1-\cos\beta} = \frac{2}{1+\frac{1}{2}} = \frac{4}{3}$

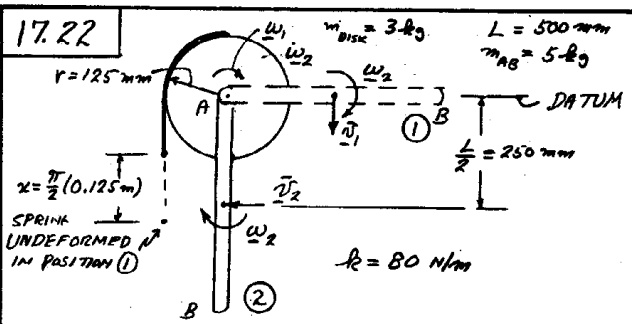
$3\cos\beta = 4 - 4\cos\beta$ $\quad \cos\beta = \frac{4}{7}$ $\quad \beta = 55.15°$

$\beta = 55.2°$ ◀

(b) EQ(3): $\bar{v}_2 = \sqrt{gr\cos\beta} = \sqrt{gr\left(\frac{4}{7}\right)} = 0.7559\sqrt{gr}$

$\bar{v}_2 = 0.756\sqrt{gr}$ ↖ 55.2° ◀

17.22



KINETIC ENERGY FOR DISK-ROD ASSEMBLY

$\bar{v} = \frac{L}{2}\omega = (0.25\text{ m})\omega$

$T = \frac{1}{2}I_{DISK}\omega^2 + \left[\frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2\right]$

$= \frac{1}{2}\left[\frac{1}{2}(3\text{ kg})(0.125\text{ m})^2\right]\omega^2 + \frac{1}{2}(5\text{ kg})(0.25\text{ m})^2 + \frac{1}{2}\left[\frac{1}{12}(5\text{ kg})(0.5\text{ m})^2\right]\omega^2$

$= (0.011719 + 0.15625 + 0.05208)\omega^2$

$T = 0.22005\omega^2$ $\quad (1)$

POSITION 1: $\omega_1 = \bar{v}_1 = 0$, $T_1 = 0$ $\quad V_1 = 0$

POSITION 2: $T_2 = 0.22005\omega_2^2$

$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(80\text{ N/m})\left(\frac{\pi}{2}0.125\text{ m}\right)^2 = 1.5421\text{ J}$

$V_g = -Wh = -(5\text{ kg})(9.81\text{ m/s}^2)(0.25\text{ m}) = -12.263\text{ J}$

$V_2 = 1.5421 - 12.263 = -10.721\text{ J}$

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0.22005\omega_2^2 - 10.721\text{ J}$

$\omega_2^2 = 48.717$

$\omega_2 = 6.979\text{ rad/s}$ $\quad \underline{\omega_2 = 6.98\text{ rad/s}}$ ◀

17.23

SEE SOLUTION OF PROB. 17.22 WE NOW HAVE $\omega_1 \neq 0$ AND WE MUST HAVE $\omega_2 = 9$ rad/s

POSITION 1:

EQ(1): $T_1 = 0.22005\omega_1^2$

$V_1 = 0$

POSITION 2: $\omega_2 = 9$ rad/s

EQ(1) $T_2 = 0.22005(9)^2 = 17.824\text{ J}$

$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(80\text{ N/m})\left(\frac{\pi}{2}0.125\text{ m}\right)^2 = 1.5421\text{ J}$

$V_g = -Wh = -(5\text{ kg})(9.81\text{ m/s}^2)(0.125\text{ m}) = -12.263\text{ J}$

$V_2 = 1.542\text{ J} - 12.263\text{ J} = -10.721\text{ J}$

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$

$0.22005\omega_1^2 + 0 = 17.824 - 10.721$

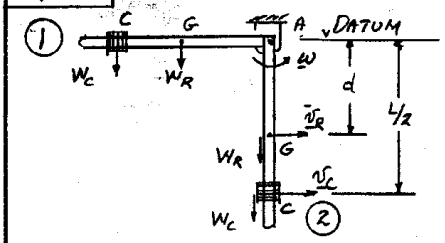
$0.22005\omega_1^2 = 7.103$

$\omega_1^2 = 32.279$

$\omega_1 = 5.681\text{ rad/s}$

$\underline{\omega_1 = 5.68\text{ rad/s}}$ ◀

17.24



KINEMATICS

ROD $\bar{v}_R = \frac{L}{2}\omega$

COLLAR $v_C = d\omega$

POSITION 1 $\omega = 0$

$T_1 = 0 \quad V_1 = 0$

POSITION 2 $T_2 = \frac{1}{2} m_R \bar{v}_R^2 + \frac{1}{2}\bar{I}_R \omega^2 + \frac{1}{2} m_C v_C^2$

$= \frac{1}{2} m_R \left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12} m_R L^2\right)\omega^2 + \frac{1}{2} m_C d^2 \omega^2$

$= \frac{1}{6} m_R L^2 \omega^2 + \frac{1}{2} m_C d^2 \omega^2$

$V_2 = -W_C d - W_R \frac{L}{2}$

$T_1 + V_1 = T_2 + V_2$: $0 + 0 = \left(\frac{1}{6}\frac{W_R}{g}L^2 + \frac{1}{2}\frac{W_C}{g}d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$

$0 = \frac{1}{6g}\left(W_R L^2 + 3W_C d^2\right)\omega^2 - \frac{1}{2}\left(2W_C d + W_R L\right)$

$$\omega^2 = 3g\,\frac{2W_C d + W_R L}{3W_C d^2 + W_R L^2} \qquad (1)$$

DATA: $W_C = 2$ lb; $W_R = 6$ lb

$d = 1$ ft; $L = 2$ ft $\quad \omega^2 = 3g\,\frac{2(2)(1)+(6)(2)}{3(2)(1)^2+(6)(2)^2} = 3g\left(\frac{16}{30}\right)$

$\omega^2 = 3(32.2)\left(\frac{16}{30}\right) = 51.52 \qquad \omega = 7.18$ rad/s ◀

17.25

TO FIND ω_{MAX} WE DIFFERENTIATE EQ(1) OF PROB 17.24 WITH RESPECT TO d

$$2\omega\left(\frac{\partial\omega}{\partial d}\right) = \frac{2W_C(3W_C d^2 + W_R L^2) - 6W_C d(2W_C d + W_R L)}{(3W_C d^2 + W_R L^2)^2}$$

SETTING $\frac{\partial\omega}{\partial d} = 0$: $2W_C(3W_C d^2 + W_R L^2) - 6W_C d(2W_C d + W_R L) = 0$

$3W_C d^2 + W_R L^2 - 6W_C d^2 - 3W_R L d = 0$

$3W_C d^2 + 3W_R L d - W_R L^2 = 0$

DATA: $W_C = 2$ lb; $W_R = 6$ lb

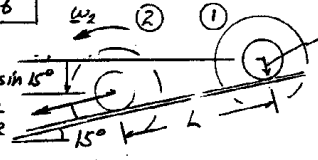
$6d^2 + 18Ld - 6L^2 = 0$

$d^2 + 3Ld - L^2 = 0$

$$d = \frac{-3 + \sqrt{3^2+4}}{2} L = 0.3028L$$

For $L = 24$ in.: $L = (0.3028)(24\text{ in})$ $\qquad L = 7.27$ in. ◀

17.26



$\bar{v}_2 = r\omega_2 \quad \omega_2 = \frac{\bar{v}_2}{r}$

$T_1 = 0$

$U_{1-2} = Wh = mgh$

$T_2 = \frac{1}{2} m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2} m\bar{v}_2^2 + \frac{1}{2} m\bar{k}^2\left(\frac{\bar{v}_2}{r}\right)^2 = \frac{1}{2} m\left(1 + \frac{\bar{k}^2}{r^2}\right)\bar{v}_2^2$

$T_1 + U_{1-2} = T_2$: $0 + mgh = \frac{1}{2} m\left(1 + \frac{\bar{k}^2}{r^2}\right)\bar{v}_2^2$

$$1 + \frac{\bar{k}^2}{r^2} = \frac{2gh}{\bar{v}_2^2} \qquad (1)$$

DATA: $\bar{k} = 24$ in., $r = 1.25$ in., $L = 8$ ft, THUS $h = (8\text{ ft})\sin 15°$

$1 + \frac{(24\text{ in.})^2}{(1.25\text{ in})^2} = \frac{2(32.2\text{ ft/s}^2)(8\text{ ft})\sin 15°}{\bar{v}_2^2}$

$(1 + 368.6)\bar{v}_2^2 = 133.34 \qquad \bar{v}_2^2 = 0.3608\ \text{ft}^2/\text{s}^2$

$\bar{v}_2 = 0.6007\text{ ft/s} = 7.208\text{ in./s} \qquad \bar{v}_2 = 7.21\text{ in./s} \angle 15°$ ◀

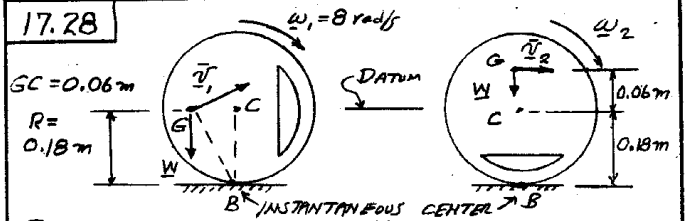
17.27

SEE PROB 17.26 FOR DERIVATION OF EQ(1)

DATA: $r = 40$ mm, $\bar{v}_2 = 0.16$ m/s, $L = 1.5$ m, THUS $h = (1.5\text{ m})\sin 15°$

EQ(1): $1 + \frac{\bar{k}^2}{(40\text{ mm})^2} = \frac{2(9.81\text{ m/s}^2)(1.5\text{ m})\sin 15°}{(0.16\text{ m/s})^2}$ $\qquad \bar{k} = 689$ mm ◀

17.28



$\bar{v}_1 = (BG)\omega_1 = \sqrt{(0.18)^2 + (0.06)^2}(8) = 8\sqrt{0.036}$ m/s ; $\bar{v}_2 = 0.24\omega_2$

$m = 3$ kg $\qquad \bar{k} = 0.09$ m

POSITION 1: $V_1 = 0$

$T_1 = \frac{1}{2} m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(3)\left(8\sqrt{0.036}\right)^2 + \frac{1}{2}(3)(0.09)^2(8)^2 = 4.2336$ J

POSITION 2: $V_2 = Wh = mgh = (3)(9.81)(0.06) = 1.7658$ J

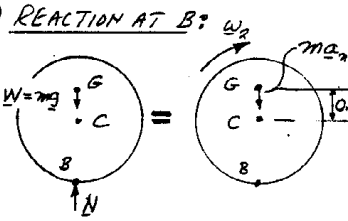
$T_2 = \frac{1}{2} m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}(3)(0.24\omega_2)^2 + \frac{1}{2}(3)(0.09)^2\omega_2^2 = 0.09855\omega_2^2$

a) CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

$4.2336\text{ J} + 0 = 0.09855\omega_2^2 + 1.7658\text{ J}$

$\omega_2^2 = 25.04 \qquad \omega_2 = 5.004$ rad/s $\qquad \omega_2 = 5.00$ rad/s ◀

(b) REACTION AT B:



$m\bar{a}_n = m(CG)\omega_2^2 = (3\text{ kg})(0.06\text{ m})(5\text{ rad/s})^2 = 4.50$ N

$+\uparrow \Sigma F = ma_n$

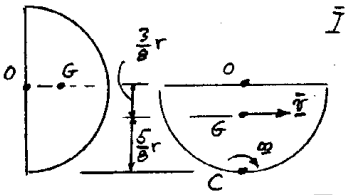
$N - (3\text{ kg})(9.81\text{ m/s}^2) = -4.50$ N

$N = 24.9$ N ↑ ◀

17.29

KINEMATICS $\bar{v} = \frac{5}{8} r\omega$

$\bar{I} = \frac{2}{5} mr^2 - m\left(\frac{3}{8}r\right)^2 = \left(\frac{2}{5} - \frac{9}{64}\right)mr^2$



(a) WORK–ENERGY

$T_1 = 0$

$T_2 = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$T_2 = \frac{1}{2} m\left(\frac{5}{8}r\right)^2\omega^2 + \frac{1}{2}\left(\frac{2}{5} - \frac{9}{64}\right)mr^2\omega^2$

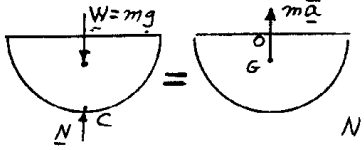
$T_2 = \frac{13}{40} mr^2\omega^2$

$U_{1-2} = mg\left(\frac{3}{8}r\right)$

$T_1 + U_{1-2} = T_2$: $0 + mg\left(\frac{3}{8}r\right) = \frac{13}{40} mr^2\omega^2$

$\omega^2 = \frac{15}{13}\frac{g}{r} \qquad \omega = 1.074\sqrt{\frac{g}{r}}$ ◀

(b) REACTION AT C $\qquad \alpha = 0 \quad a_O = 0$



$\bar{a} = a_O + a_{G/O} = \left(\frac{3}{8}r\right)\omega^2$

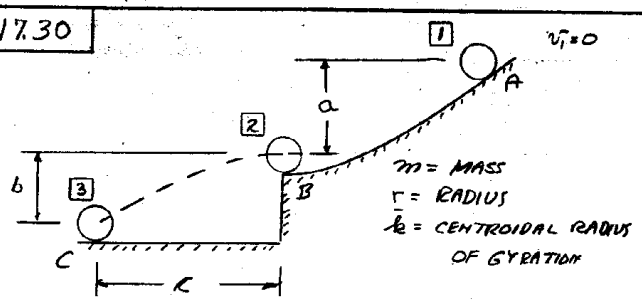
$+\uparrow \Sigma F = \Sigma F_{eff}$

$N - mg = m\left(\frac{3}{8}r\right)\omega^2$

$N = mg + m\left(\frac{3}{8}r\right)\left(\frac{15}{13}\frac{g}{r}\right) = mg\left(1 + \frac{45}{104}\right)$

$N = \frac{149}{104} mg \qquad N = 1.433\,mg$ ↑ ◀

17.30



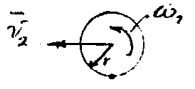
$v_1 = 0$

m = MASS
r = RADIUS
$\bar{k}$ = CENTROIDAL RADIUS OF GYRATION

WORK-ENERGY

POSITION 1: $v = 0 \quad T_1 = 0$

POSITION 2: ROLLING MOTION $\bar{v} = r\omega \quad \omega = \frac{\bar{v}}{r}$



$$T_2 = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$$
$$= \frac{1}{2} m \bar{v}^2 + \frac{1}{2} m \bar{k}^2 \left(\frac{\bar{v}}{r}\right)^2$$
$$T_2 = \frac{1}{2} m \left(1 + \frac{\bar{k}^2}{r^2}\right) \bar{v}^2 = \frac{1}{2} m \frac{r^2 + \bar{k}^2}{r^2} \bar{v}^2$$

$U_{1-2} = mga$

$T_1 + U_{1-2} = T_2$: $\quad 0 + mga = \frac{1}{2} m \frac{r^2 + \bar{k}^2}{r^2} \bar{v}^2$

$$\bar{v}^2 = 2ga \frac{r^2}{r^2 + \bar{k}^2} \quad (1)$$

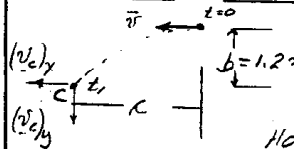
DATA SPHERE $\bar{k}^2 = \frac{2}{5} r^2$,

$$\bar{v}^2 = 2ga \frac{r^2}{r^2 + \frac{2}{5} r^2} = 2ga\left(\frac{5}{7}\right)$$

FOR $a = 1.5$ m: $\bar{v}^2 = 2(9.81)(1.5)\left(\frac{5}{7}\right) = 21.021$

$\bar{v} = 4.585$ m/s

FROM B TO C PROJECTILE MOTION



VERTICAL MOTION $(v_c)_y = \sqrt{2gb} = \sqrt{2(9.81)(1.2)}$

$(v_c)_y = 4.852$ m/s

HORIZ. MOTION $(v_c)_x = \bar{v} = 4.585$ m/s

$v_c = \sqrt{(v_c)_x^2 + (v_c)_y^2} = \sqrt{4.585^2 + 4.852^2}$ $\quad v_c = 6.68$ m/s ◀

$(v_c)_y = g t_1$; $4.852 = 9.81 t_1$; $\quad t_1 = 0.4946$ s

DISTANCE R: $R = \bar{v} t_1 = (4.585)(0.4946)$; $\quad R = 2.27$ m ◀

17.31

SEE SOLUTION OF PROB 17.30
WE NOW HAVE A PIPE FOR WHICH $\bar{k} = r$

EQ(1) $\bar{v}^2 = 2ga \frac{r^2}{r^2 + r^2} = 2ga\left(\frac{1}{2}\right) = ga$

FOR $a = 1.5$ m: $\bar{v}^2 = (9.81)(1.5) \quad \bar{v} = 3.836$ m/s

FROM B TO C $b = 1.2$ m, WE AGAIN HAVE:

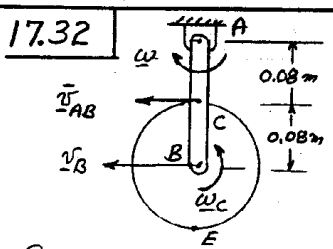
$(v_c)_y = 4.852$ m/s AND $(v_c)_x = \bar{v} = 3.836$ m/s

$v_c = \sqrt{(v_c)_x^2 + (v_c)_y^2} = \sqrt{3.836^2 + 4.852^2}$ $\quad v_c = 6.19$ m/s ◀

$(v_c)_y = g t_1$; $4.852 = 9.81 t_1$; $\quad t_1 = 0.4946$ s (SAME AS PROB 17.30)

DISTANCE R: $R = \bar{v} t_1 = (3.836)(0.4946)$; $\quad R = 1.897$ m ◀

17.32



KINEMATICS INSTANTANEOUS CENTER OF GEAR C IS AT E.

$v_B = (AB)\omega = (BE)\omega_C$

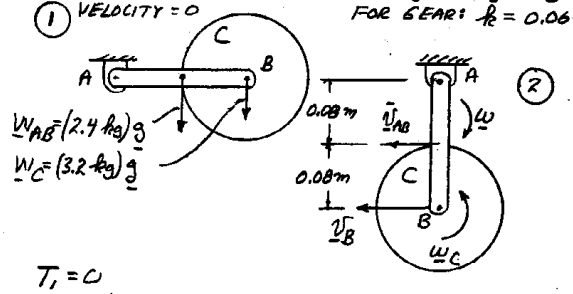
$v_B = (0.16\text{ m})\omega = (0.08)\omega_C$

$\omega_C = 2\omega$

$\bar{v}_{AB} = (0.08\text{ m})\omega$

PRINCIPLE OF WORK-ENERGY $\quad m_{AB} = 2.4$ kg, $m_C = 3.2$ kg

FOR GEAR: $\bar{k} = 0.06$ m



$T_1 = 0$

$U_{1-2} = (2.4\text{ kg})g(0.08\text{ m}) + (3.2\text{ kg})g(0.16\text{ m}) = 0.704(9.81) = 6.9062$ J

$$T_2 = \frac{1}{2} m_{AB} \bar{v}_{AB}^2 + \frac{1}{2} \bar{I}_{AB} \omega^2 + \frac{1}{2} m_C v_B^2 + \frac{1}{2} \bar{I}_C \omega_C^2$$
$$= \frac{1}{2}(2.4)(0.08\omega)^2 + \frac{1}{2}\left[\frac{1}{12}(2.4)(0.16)^2\right]\omega^2 + \frac{1}{2}(3.2)(0.16\omega)^2 + \frac{1}{2}\left[(3.2)(0.06)^2\right](2\omega)^2$$
$$= 0.07424\,\omega^2$$

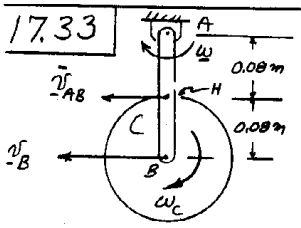
$T_1 + U_{1-2} = T_2$: $\quad 0 + 6.9062\text{ J} = 0.07424\,\omega^2$

$\omega^2 = 93.025 \quad \omega = 9.645$ rad/s

VELOCITY OF B. $v_B = (0.16\text{ m})\omega = (0.16\text{ m})(9.645\text{ rad/s})$

$v_B = 1.543$ m/s ◀

17.33



KINEMATICS SINCE GEAR C ROLLS ON FIXED GEAR D (NOT SHOWN HERE), THE INSTANTANEOUS CENTER OF GEAR C IS AT H.

$v_B = (AB)\omega = (HB)\omega_C$

$v_B = (0.16\text{ m})\omega = (0.08\text{ m})\omega_C$

$\omega_C = 2\omega$

$\bar{v}_{AB} = (0.08\text{ m})\omega$

WE NOTE THAT THIS KINEMATIC ANALYIS IS IDENTICAL TO THAT OF PROB 17.32 EXCEPT FOR THE DIRECTION OF ω_C. SINCE KINETIC ENERGY IS A SCALAR, THE EXPRESSION FOR T_2 WILL AGAIN BE

$$T_2 = 0.07424\,\omega^2$$

SINCE THE WORK DONE BY W_{AB} AND W_C IS ALSO THE SAME AS IN PROB 17.32, WE AGAIN HAVE

$$U_{1-2} = 6.9062\text{ J}$$

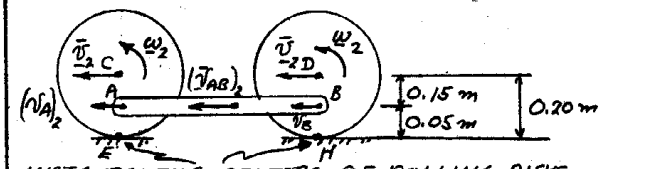
THE PRINCIPLE OF WORK-ENERGY AGAIN YIELDS:

$$T_1 + U_{1-2} = T_2$$
$$0 + 6.9062\text{ J} = 0.07424\omega^2$$
$$\omega = 9.645\text{ rad/s}$$

VELOCITY OF B: $v_B = (0.16\text{ m})\omega^2 = (0.16\text{ m})(9.645\text{ rad/s})$

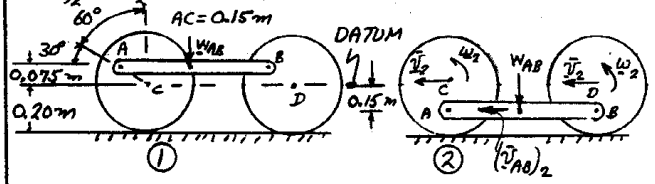
$v_B = 1.543$ m/s ◀

17.34 KINEMATICS WHEN $\theta = 180°$



$\bar{v}_2 = (CE)\omega_2 = (0.20\text{ m})\omega_2$

$(\bar{v}_{AB})_2 = (v_A)_2 = (0.05\text{ m})\omega_2$



ROD AB: $m_{AB} = 9$ kg

EACH DISK: $m_{DISK} = m = 6$ kg

(a) POSITION 1: VELOCITY = 0 $\quad T_1 = 0$

$V_1 = W_{AB}(0.075\text{ m}) = (9\text{ kg})(9.81\text{ m/s}^2)(0.075\text{ m}) = 6.6218\text{ J}$

POSITION 2:

$$T_2 = \tfrac{1}{2} m_{AB}(\bar{v}_{AB})_2^2 + 2\left[\tfrac{1}{2} m \bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2\right]$$

$$= \tfrac{1}{2}(9\text{ kg})(0.05\omega_2)^2 + 2\left[\tfrac{1}{2}(6\text{ kg})(0.20\omega_2)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}\right)(6\text{ kg})(0.20)^2\omega_2^2\right]$$

$$= 0.37125\,\omega_2^2$$

$V_2 = W_{AB}(-0.15\text{ m}) = -(9\text{ kg})(9.81\text{ m/s}^2)(0.15\text{ m}) = -13.2435\text{ J}$

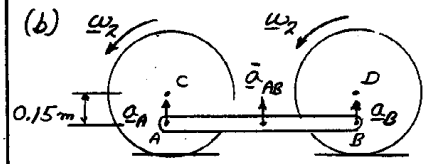
CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 6.6218 = 0.37125\,\omega_2^2 - 13.2435$

$\omega_2^2 = 53.509\ \text{rad}^2/\text{s}^2 \qquad \omega_2 = 7.315$ rad/s

$\underline{\omega_2 = 7.32\text{ rad/s}}$ ◀

(b)



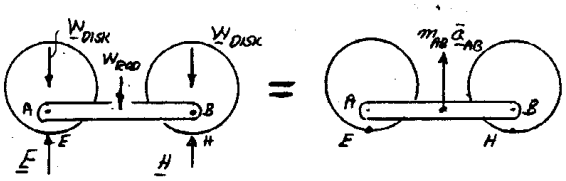
SINCE C AND D MOVE HORIZONTALLY $(a_C)_y = (a_D)_y = 0$

$+\uparrow \quad \underline{a}_A = (\underline{a}_C)_y + (CA)\omega_2^2\uparrow = 0 + (0.15\text{ m})(53.509\ \text{rad}^2/\text{s}^2) = 8.026\text{ m/s}^2\uparrow$

$\underline{a}_B = \underline{a}_A = 8.026\text{ m/s}^2\uparrow$

SINCE ROD AB IS IN CURVILINEAR TRANSLATION

$\bar{\underline{a}}_{AB} = \underline{a}_A = \underline{a}_B = 8.026\text{ m/s}^2\uparrow$



BY SYMMETRY: $E = H$

$+\uparrow \Sigma F = \Sigma(F)_{eff}$: $\quad 2E - 2W_{DISK} - W_{ROD} = m_{AB}\bar{a}_{AB}$

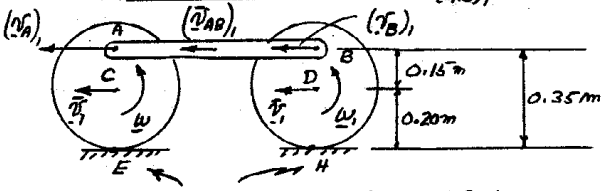
$2E - 2(6\text{ kg})(9.81\text{ m/s}^2) - (9\text{ kg})(9.81\text{ m/s}^2) = (9\text{ kg})(8.026\text{ m/s}^2)$

$2E - 117.72\text{ N} - 88.29\text{ N} = 72.23\text{ N}$

$E = 139.12\text{ N}$

EACH REACTION = 139.1 N ↑ ◀

17.35 KINEMATICS WHEN $\theta = 0$ AND $(\bar{v}_{AB})_1 = 0.84$ m/s ←



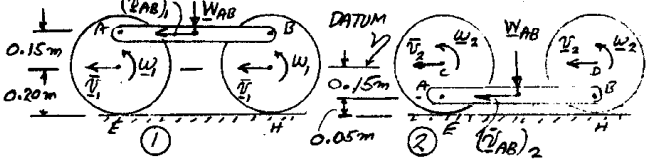
$(v_A)_1 = (v_B)_1 = (\bar{v}_{AB})_1 = 0.84\text{ m/s}$ ←

$\omega = \dfrac{(v_A)_1}{AE} = \dfrac{0.84\text{ m/s}}{0.35\text{ m}} = 2.4\text{ rad/s} \qquad \underline{\omega = 2.4\text{ rad/s}}$ ↻

$\bar{v}_1 = (CE)\omega = (0.20\text{ m})(2.4\text{ rad/s}) \qquad \underline{\bar{v}_1 = 0.48\text{ m/s}}$ ←

KINEMATICS WHEN $\theta = 180°$ SAME AS IN PROB 17.34

KINETICS $\quad m_{AB} = 9$ kg $\quad m_{DISK} = m = 6$ kg



(a) POSITION 1: $V_1 = W_{AB}(0.15\text{ m}) = (9\text{ kg})(9.81\text{ m/s}^2)(0.15\text{ m}) = 13.2435\text{ J}$

$$T_1 = \tfrac{1}{2} m_{AB}(\bar{v}_{AB})_1^2 + 2\left[\tfrac{1}{2} m \bar{v}_1^2 + \tfrac{1}{2}\bar{I}\omega_1^2\right]$$

$$= \tfrac{1}{2}(9\text{ kg})(0.84\text{ m/s})^2 + 2\left[\tfrac{1}{2}(6\text{ kg})(0.48\text{ m/s})^2 + \tfrac{1}{2}\left(\tfrac{1}{2}\right)(6\text{ kg})(0.20\text{ m})^2(2.4\text{ rad/s})^2\right]$$

$= 5.249\text{ J}$

POSITION 2: SAME AS POSITION 2 OF PROB 17.34

$V_2 = -13.2435\text{ J}$

$T_2 = 0.37125\,\omega_2^2 \qquad (\bar{v}_{AB})_2 = (0.05\text{ m})\omega_2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 5.249\text{ J} + 13.2435\text{ J} = 0.37125\,\omega_2^2 - 13.2435\text{ J}$

$\omega_2^2 = 85.484\ \text{rad}^2/\text{s}^2 \qquad \omega_2 = 9.245$ rad/s

VELOCITY OF ROD WHEN $\theta = 180°$:

$(\bar{v}_{AB})_2 = (0.05\text{ m})\omega_2 = (0.05\text{ m})(9.245\text{ rad/s}) = 0.4623\text{ m/s}$

$\underline{(\bar{v}_{AB})_2 = 462\text{ mm/s}}$ ← ◀

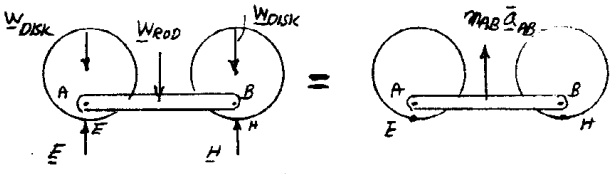
(b) SEE SKETCH OF PROB 17.34 (b)

$+\uparrow \quad \underline{a}_A = (\underline{a}_C)_y + (CA)\omega_2^2\uparrow = 0 + (0.15\text{ m})(85.484\ \text{rad}^2/\text{s}^2)$

$= 12.823\text{ m/s}^2\uparrow$

SINCE ROD AB IS IN CURVILINEAR TRANSLATION

$\bar{\underline{a}}_{AB} = \underline{a}_A = \underline{a}_B = 12.823\text{ m/s}^2\uparrow$



BY SYMMETRY: $E = H$

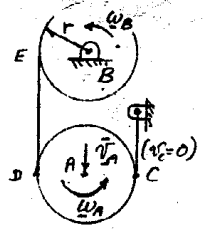
$+\uparrow \Sigma F = \Sigma(F)_{eff}$: $\quad 2E - 2W_{DISK} - W_{ROD} = m_{AB}\bar{a}_{AB}$

$2E - 2(6\text{ kg})(9.81\text{ m/s}^2) - (9\text{ kg})(9.81\text{ m/s}^2) = (9\text{ kg})(12.823\text{ m/s}^2)$

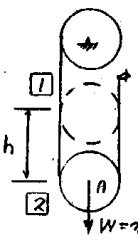
$2E - 117.72\text{ N} - 88.29 = 115.40\text{ N}$

$E = 160.71\text{ N}$ $\quad$ EACH REACTION = 160.7 N ↑ ◀

17.36



$r = 5$ in.



KINEMATICS $v_D = r\omega_B$

$\omega_A = \frac{v_D}{CD} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$

$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$ ALSO: $v_D = 2\bar{v}_A$

$T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega_A^2 + \frac{1}{2}\bar{I}\omega_B^2$

$T = \frac{1}{2}m\left(\frac{r}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2$

$T = \frac{7}{16}mr^2\omega_B^2$

POSITION 1: $\omega = 0$ $T_1 = 0$

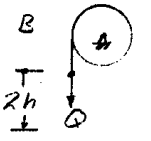
POSITION 2: $T_2 = \frac{7}{16}mr^2\omega_B^2$

WORK: $U_{1-2} = mgh$

$T_1 + U_{1-2} = T_2$: $0 + mgh = \frac{7}{16}mr^2\omega_B^2$

$\omega_B^2 = \frac{16}{7}\frac{gh}{r^2}$ (1)

CONSIDER CYLINDER B SEPARATELY: Q = TENSION IN CORD BETWEEN CYLINDERS. WHEN A MOVES DOWN h, CORD MOVES DOWN $2h$



$T_1 = 0$ $T_2 = \frac{1}{2}\bar{I}_B\omega_B^2$

$U_{1-2} = Q(2h)$

$T_1 + U_{1-2} = T_2$: $0 + Q(2h) = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2$

SUBSTITUTE FOR ω_B^2: $Q(2h) = \frac{1}{4}mr^2\left(\frac{16}{7}\frac{gh}{r^2}\right)$

$Q = \frac{2}{7}mg = \frac{2}{7}W$ (2)

DATA: $W = 21$ lb, $h = 4$ ft $r = 6$ in.

(a) $v_A = r\omega_B = \frac{1}{2}r\sqrt{\frac{16}{7}\frac{gh}{r^2}} = \sqrt{\frac{4}{7}gh}$

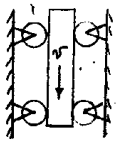
$v_A = \sqrt{\frac{4}{7}(32.2\ \text{ft/s}^2)(4\ \text{ft})} = 8.579$ ft/s $\quad \underline{v}_A = 8.58$ ft/s ↓

(b) EQ(2) $Q = \frac{2}{7}W = \frac{2}{7}(21\text{ lb})$ $\quad Q = 6$ lb

17.37

FOR EACH CASE: $T_1 = 0$ AND $W_{BAR} = 10$ lb, $W_{DISK} = 4$ lb

(a)



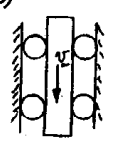
$T_2 = \frac{1}{2}\frac{10}{g}v^2 + 4\left[\frac{1}{2}\left(\frac{1}{2}\frac{4}{g}r^2\right)\left(\frac{v}{r}\right)^2\right]$

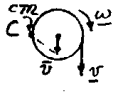
$U_{1-2} = (10\text{ lb})h$; $T_2 = \frac{1}{2}\frac{v^2}{g}(10+4) = \frac{v^2}{2g}(14)$

$T_1 + U_{1-2} = T_2$: $0 + 10h = \frac{v^2}{2g}(14)$

$v^2 = \frac{5}{7}(2gh)$ $\quad \underline{v} = 0.845\sqrt{2gh}$ ↓

(b)





$\bar{v} = \frac{1}{2}v$ $\quad \omega = \frac{v}{2r}$

$T_2 = \frac{1}{2}\frac{10}{g}\bar{v}^2 + 4\left[\frac{1}{2}\left(\frac{1}{2}\frac{4}{g}r^2\right)\left(\frac{v}{2r}\right)^2 + \frac{1}{2}\frac{4}{g}\left(\frac{v}{2}\right)^2\right]$

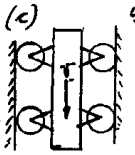
$= \frac{v^2}{2g}(10+2+4) = \frac{v^2}{2g}(16)$

DISKS MOVE DOWN $\frac{1}{2}h$. $U_{1-2} = (10\text{lb})h + 4[(4\text{lb})\frac{h}{2}] = 18h$

$T_1 + U_{1-2} = T_2$: $0 + 18h = \frac{v^2}{2g}(16)$

$v^2 = \frac{9}{8}\sqrt{2gh}$ $\quad \underline{v} = 1.061\sqrt{2gh}$ ↓

(c)



$T_2 = \frac{1}{2}\frac{10}{g}v^2 + 4\left[\frac{1}{2}\left(\frac{1}{2}\frac{4}{g}r^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}\frac{4}{g}v^2\right]$

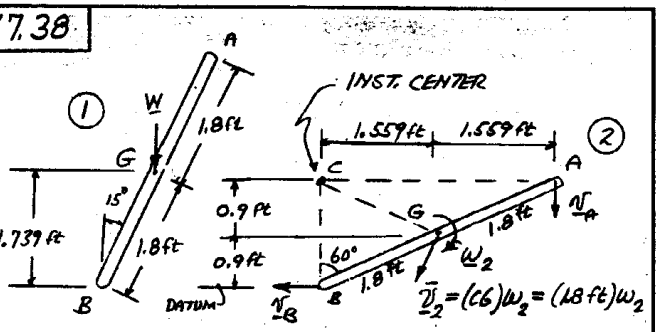
$= \frac{v^2}{2g}(10+8+16) = \frac{v^2}{2g}(34)$

$U_{1-2} = (10\text{lb})h + 4[(4\text{lb})h] = 26h$

$T_1 + U_{1-2} = T_2$: $26h = \frac{v^2}{2g}(34)$; $v^2 = \frac{13}{17}\sqrt{2gh}$

$v = 0.874\sqrt{2gh}$ ↓

17.38



POSITION 1: VELOCITY = 0 $T_1 = 0$

$V_1 = W(1.739\text{ ft}) = (6\text{ lb})(1.739\text{ ft}) = 10.43$ ft·lb

POSITION 2: $T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2}\frac{W}{g}(1.8\omega_2)^2 + \frac{1}{2}\left[\frac{1}{12}\frac{W}{g}(3.6\text{ ft})^2\right]\omega_2^2$

$= 2.16\frac{W}{g}\omega_2^2 = 2.16\frac{6\text{ lb}}{32.2}\omega_2^2 = 0.4025\omega_2^2$

$V_2 = W(0.9\text{ ft}) = (6\text{ lb})(0.9\text{ ft}) = 5.4$ ft·lb

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + 10.43 = 0.4025\omega_2^2 + 5.4$

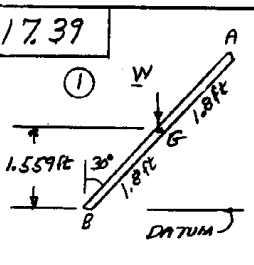
$\omega_2^2 = 12.50$ $\quad \underline{\omega}_2 = 3.535$ rad/s ↻

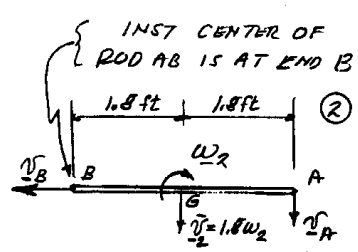
VELOCITY OF COLLARS WHEN $\theta = 60°$

$v_A = (CA)\omega_2 = 2(1.559\text{ ft})(3.535\text{ rad/s})$ $\quad \underline{v}_A = 11.02$ ft/s ↓ ◀

$v_B = (CB)\omega_2 = (1.8\text{ ft})(3.535\text{ rad/s})$ $\quad \underline{v}_B = 6.36$ ft/s ← ◀

17.39





POSITION 1: VELOCITY = 0 $T_1 = 0$

$V_1 = W(1.557\text{ ft}) = 1.557W$

POSITION 2: $V_2 = 0$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\frac{W}{g}(1.8\omega_2)^2 + \frac{1}{2}\left[\frac{1}{12}\frac{W}{g}(3.6)^2\right]\omega_2^2$

$= 2.16\frac{W}{g}\omega_2^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + 1.557W = 2.16\frac{W}{g}\omega_2^2 + 0$

$\omega_2^2 = 0.721g = 0.721(32.2) = 23.21$

$\omega_2 = 4.818$ rad/s

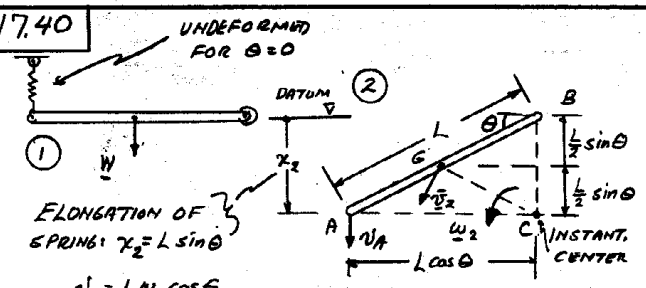
VELOCITY OF COLLAR WHEN $\theta = 180°$

$v_A = (AB)\omega_2 = (3.6\text{ ft})(4.818\text{ rad/s}) = 17.35$ ft/s

$\underline{v}_A = 17.35$ ft/s ↓ ◀

VELOCITY OF COLLAR B IS ZERO WHEN $\theta = 180°$

17.40



$v_A = L\omega_2\cos\theta$

$CG = \frac{L}{2}$; $\bar{v} = \frac{L}{2}\omega$

POSITION 1: $v=0$ $T_1=0$; $V_1=0$

POSITION 2: $T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$

$$T_2 = \frac{1}{6}mL^2\omega_2^2 \quad (1)$$

$$V_2 = -mg\frac{L}{2}\sin\theta + \frac{1}{2}k\chi_2^2$$

$$V_2 = -\frac{1}{2}mgL\sin\theta + \frac{1}{2}kL^2\sin^2\theta \quad (2)$$

CONSERVATION OF ENERGY

$T_1+V_1 = T_2+V_2$: $0+0 = \frac{1}{6}mL^2\omega_2^2 - \frac{1}{2}mgL\sin\theta + \frac{1}{2}kL^2\sin^2\theta$

$$\omega_2^2 = 3\left(\frac{g}{L}\sin\theta - \frac{k}{m}\sin^2\theta\right)$$

DATA: $m = 12$ kg, $L = 0.75$ m, $k = 120$ N/m

$$\omega_2^2 = 3\left(\frac{9.81}{0.75}\sin\theta - \frac{120}{12}\sin^2\theta\right)$$

$$\omega_2^2 = 3\left(13.08\sin\theta - 10\sin^2\theta\right)$$

(a) FOR $\theta = 30°$: $\omega_2^2 = 3(13.08\sin 30° - 10\sin^2 30°)$

$\omega_2^2 = 3(6.54 - 2.5) = 12.12$ $\omega_2 = 3.481$ rad/s

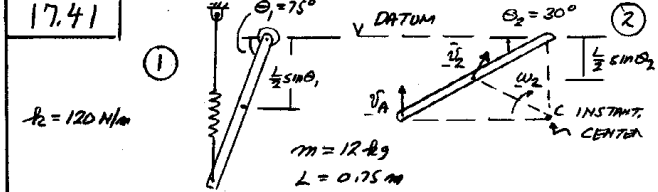
$v_A = L\omega_2\cos 30° = (0.75)(3.481)\cos 30°$ $\underline{v}_A = 2.26$ m/s ↓ ◀

(b) FOR $\theta = 60°$: $\omega_2^2 = 3(13.08\sin 60° - 10\sin^2 60°)$

$\omega_2^2 = 3(11.328 - 7.5) = 11.482$ $\omega_2 = 3.389$ rad/s

$v_A = L\omega_2\cos 60° = (0.75)(3.389)\cos 60°$ $\underline{v}_A = 1.271$ m/s ↓ ◀

17.41



POSITION 1: $\theta_1 = 75°$ $v=0$; $T_1=0$

USE EQ(2) OF PROB 17.40:

$$V_1 = -\frac{1}{2}(12\text{ kg})(9.81\text{ m/s}^2)(0.75\text{ m})\sin 75° + \frac{1}{2}(120\text{ N/m})(0.75\text{ m})^2\sin^2 75°$$

$V_1 = -42.641 + 31.489 = -11.152$ J

POSITION 2: $\theta = 30°$

USE EQ(1) OF PROB 17.40: $T_2 = \frac{1}{6}mL^2\omega_2^2$

$T_2 = \frac{1}{6}(12\text{ kg})(0.75\text{ m})^2\omega_2^2 = 1.125\omega_2^2$

USE EQ(2) OF PROB 17.40:

$$V_2 = -\frac{1}{2}(12\text{ kg})(9.81\text{ m/s}^2)(0.75\text{ m})\sin 30° + \frac{1}{2}(120\text{ N/m})(0.75\text{ m})^2\sin^2 30°$$

$= -22.073 + 8.438 = -13.635$ J

$T_1+V_1 = T_2+V_2$: $0 - 11.152\text{ J} = 1.125\omega_2^2 - 13.635\text{ J}$ ◀

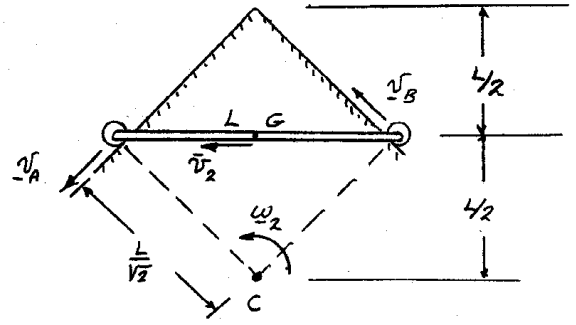
$\omega_2^2 = 2.207$ $\omega_2 = 1.486$ rad/s ◀

$v_A = L\omega_2\cos 30° = (0.75)(1.486)(\cos 30°)$ $\underline{v}_A = 0.965$ m/s ↑ ◀

17.42

$T_1+V_1 = T_2+V_2$

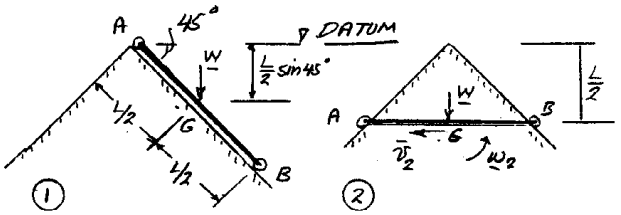
MAXIMUM VELOCITY WILL OCCUR WHEN KINETIC ENERGY IS MAXIMUM, THAT IS, WHE POTENTIAL ENERGY IS MINIMUM.



WE DRAW LINES ⊥ TO v_A AND v_B TO LOCATE INSTANTANEOUS CENTER C

$\bar{v}_2 = \frac{L}{2}\omega_2$

$v_A = \frac{L}{\sqrt{2}}\omega_2$



CONSERVATION OF ENERGY

POSITION 1: $T_1 = 0$; $V_1 = -W\frac{L}{2}\sin 45° = -mg\frac{L}{2\sqrt{2}}$

POSITION 2: $T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$T_2 = \frac{1}{6}mL^2\omega_2^2$$

$$V_2 = -W\frac{L}{2} = -mg\frac{L}{2}$$

$T_1+V_1 = T_2+V_2$:

$$0 - mg\frac{L}{2\sqrt{2}} = \frac{1}{6}mL^2\omega_2^2 - mg\frac{L}{2}$$

$$\omega_2^2 = \frac{3g}{L}\left(1 - \frac{1}{\sqrt{2}}\right) = 0.8787\frac{g}{L}$$

$$\omega_2 = 0.9374\sqrt{g/L} \quad (1)$$

CORRESPONDING v_A:

$$v_A = \frac{L}{\sqrt{2}}\omega_2 = 0.6628\sqrt{gL} \quad (2)$$

FOR $L = 0.75$ m

EQ(1): $\omega_2 = 0.9374\sqrt{\frac{9.81\text{ m/s}^2}{0.75\text{ m}}} = 3.390$ rad/s

$\omega_2 = 3.39$ rad/s ↻ ◀

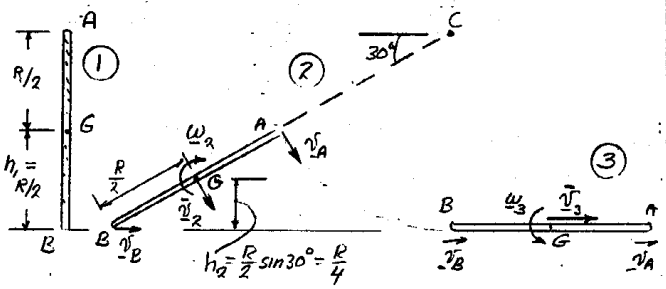
EQ(2): $v_A = \frac{0.75\text{ m}}{\sqrt{2}}(3.39\text{ rad/s}) = 1.7978$ m/s

$\underline{v}_A = 1.798$ m/s ↙ 45° ◀

17.43

NOTE: END B IS MOVED SLIGHTLY TO THE LEFT AND RELEASED

ROD MOVES FROM POSITION 1 TO POSITION 2 WHERE B, A, AND C LIE IN STRAIGHT LINE AND INSTANTANEOUS CENTER IS AT B. ROD THEN MOVES TO POSITION 3.



POSITION 1: $T_1 = 0 \quad V_1 = Wh_1 = mg\frac{R}{2}$

POSITION 2: SINCE INST. CENTER IS AT B

$\bar{v}_2 = \frac{1}{2}R\omega_2$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m\left(\frac{1}{2}R\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mk^2\right)\omega_2^2 = \frac{1}{6}mk^2\omega_2^2$

$V_2 = Wh_2 = mg\frac{R}{4}$

POSITION 3: $V_3 = 0$

SINCE BOTH $\bar{v}_A$ AND $\bar{v}_B$ ARE HORIZONTAL, $\omega_3 = 0$ (1)

$T_3 = \frac{1}{2}m\bar{v}_3^2$

(a) FROM 1 TO 2. CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{2}mgR = \frac{1}{6}mR^2\omega_2^2 + \frac{1}{4}mgR$

$\omega_2^2 = \frac{3}{2}\frac{g}{R} \qquad \omega_2 = \sqrt{\frac{3}{2}\frac{g}{R}} \qquad \underline{\omega}_2 = 1.225\sqrt{\frac{g}{R}}\ \curvearrowright$ ◀

$\bar{v}_2 = \frac{1}{2}R\omega_2 = \frac{1}{2}\sqrt{\frac{3}{2}gR} = \sqrt{\frac{3}{8}gR} \qquad \underline{\bar{v}}_2 = 0.612\sqrt{gR}\ \angle 60°$ ◀

(b) FROM 1 TO 2: CONSERVATION OF ENERGY

FROM EQ(1), WE HAVE $\quad \underline{\omega}_3 = 0$ ◀

$T_1 + V_1 = T_3 + V_3$: $\quad 0 + \frac{1}{2}mgR = \frac{1}{2}m\bar{v}_3^2$

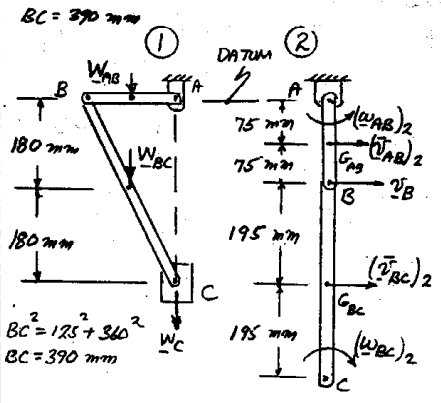
$\bar{v}_3 = gR \qquad \underline{\bar{v}}_3 = \sqrt{gR}\ \rightarrow$ ◀

17.44

IN POSITION 2 THE INSTANTANEOUS CENTER OF ROD BC IS LOCATED AT END C.

$BC^2 = 150^2 + 360^2$

$BC = 390$ mm



$BC^2 = 125^2 + 360^2$

$BC = 390$ mm

$m_{AB} = 3$ kg

$m_{BC} = 8$ kg

$m_C = 4$ kg

ROD AB

$(\bar{v}_{AB})_2 = (0.075\text{ m})(\omega_{AB})_2$

$\bar{v}_B = (0.150\text{ m})(\omega_{AB})_2$

ROD BC

INST. CENTER AT C

THUS $v_C = 0$

$\bar{v}_B = (0.390\text{ m})(\omega_{BC})_2$

$\bar{v}_B = \bar{v}_B$

$(0.150)(\omega_{AB})_2 = (0.390)(\omega_{BC})_2$

$(\omega_{BC})_2 = \frac{5}{13}(\omega_{AB})_2$

$(\bar{v}_{BC})_2 = (0.195\text{ m})(\omega_{BC})_2$

$= (0.195)\left(\frac{5}{13}\right)(\omega_{AB})_2$

$= 0.075\ (\omega_{AB})_2$

POSITION 1: $T_1 = 0$

$V_1 = -W_{BC}(0.180\text{ m}) - W_C(0.360\text{ m})$

$= -(8)(9.81)(0.180) - (4)(9.81)(0.360) = -28.253$ J (1)

POSITION 2:

$T_2 = \frac{1}{2}m_{AB}(\bar{v}_{AB})_2^2 + \frac{1}{2}\bar{I}_{AB}(\omega_{AB})_2^2 + \frac{1}{2}m_{BC}(\bar{v}_{BC})_2^2 + \frac{1}{2}I_{BC}(\omega_{BC})_2^2$

$= \frac{1}{2}(3\text{ kg})(0.075)^2(\omega_{AB})_2^2 + \frac{1}{2}\left[\frac{1}{12}(3\text{ kg})(0.150\text{ m})^2\right](\omega_{AB})_2^2$

$+ \frac{1}{2}(8\text{ kg})(0.075)^2(\omega_{AB})_2^2 + \frac{1}{2}\left[\frac{1}{12}(8\text{ kg})(0.390\text{ m})^2\right]\left(\frac{5}{13}\right)^2(\omega_{AB})_2^2$

$= (8.4375 + 2.8125 + 22.5 + 7.5)10^{-3}(\omega_{AB})_2^2$

$= 41.25 \times 10^{-3}(\omega_{AB})_2^2$ (2)

$V_2 = -W_{AB}(0.075\text{ m}) - W_{BC}(0.150\text{ m} + 0.195\text{ m}) - W_C(0.150\text{ m} + 0.390\text{ m})$

$= -(3)(9.81)(0.075) - (8)(9.81)(0.345) - (4)(9.81)(0.54)$

$= -50.4725$ J (3)

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$

$0 - 28.253 = 41.25 \times 10^{-3}(\omega_{AB})_2^2 - 50.4725$

$22.2195 = 41.25 \times 10^{-3}(\omega_{AB})_2^2$

$(\omega_{AB})_2^2 = 538.65 \qquad (\underline{\omega}_{AB})_2 = 23.209$ rad/s ↻

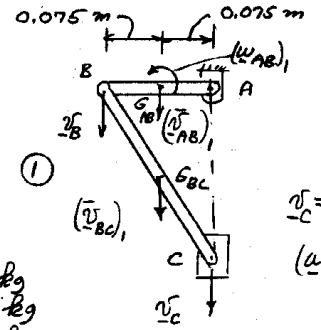
VELOCITY OF B.

$v_B = (0.150\text{ m})(\omega_{AB})_2 = (0.150\text{ m})(23.209\text{ rad/s})$

$v_B = 3.481$ m/s $\qquad \underline{v}_B = 3.48$ m/s $\rightarrow$ ◀

17.45

SEE SOLUTION OF PROB. 17.44. WE NOW HAVE THE INITIAL VELOCITY OF COLLAR C GIVEN: $\underline{v}_C = 0.9\ \text{m/s} \downarrow$



$m_{AB} = 3\ \text{kg}$
$m_{BC} = 8\ \text{kg}$
$m_C = 4\ \text{kg}$

KINEMATICS

SINCE $\underline{v}_C$ AND $\underline{v}_B$ ARE VERTICAL, INST. CENTER OF BC IS AT ∞, AND $(\omega_{BC})_1 = 0$

$\underline{v}_C = \underline{v}_B = (\underline{v}_{BC})_1 = 0.9\ \text{m/s} \downarrow$

$(\omega_{AB})_1 = \dfrac{v_B}{AB} = \dfrac{0.9\ \text{m/s}}{0.15\ \text{m}} = 6\ \text{rad/s} \circlearrowleft$

$(\bar{v}_{AB})_1 = (0.075\ \text{m})(6\ \text{rad/s}) = 0.45\ \text{m/s} \downarrow$

POSITION 1.

$$T_1 = \tfrac{1}{2} m_{AB}(\bar{v}_{AB})_1^2 + \tfrac{1}{2}\bar{I}(\omega_{AB})_1^2 + \tfrac{1}{2} m_{BC}(\bar{v}_{BC})_1^2 + \tfrac{1}{2} m_C v_C^2$$

$$= \tfrac{1}{2}(3\ \text{kg})(0.45\ \text{m/s})^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(3\ \text{kg})(0.150\ \text{m})^2\right](6\ \text{rad/s})^2 + \tfrac{1}{2}(8\ \text{kg})(0.9\ \text{m/s})^2 + \tfrac{1}{2}(4\ \text{kg})(0.9\ \text{m/s})^2$$

$$= 5.265\ \text{J}$$

SEE EQ(1) OF SOLUTION OF PROB 17.44 FOR:

$V_1 = -28.253\ \text{J}$

POSITION 2. SEE EQS(2) AND(3) OF PROB 17.44 FOR:

$T_2 = 41.25 \times 10^{-3}(\omega_{AB})_2^2$

$V_2 = -50.4725\ \text{J}$

CONSERVATION OF ENERGY.

$T_1 + V_1 = T_2 + V_2$:

$5.265\ \text{J} - 28.253\ \text{J} = 41.25 \times 10^{-3}(\omega_{AB})_2^2 - 50.4725\ \text{J}$

$27.4845 = 41.25 \times 10^{-3}(\omega_{AB})_2^2$

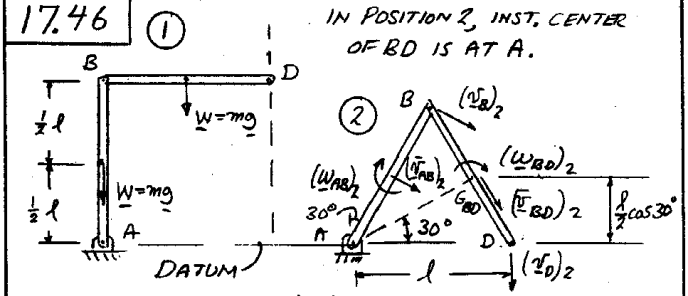
$(\omega_{AB})_2^2 = 666.29 \qquad (\underline{\omega}_{AB})_2 = 25.813\ \text{rad/s} \circlearrowleft$

VELOCITY OF B, IN POSITION 2.

$v_B = (AB)(\omega_{AB})_2 = (0.150\ \text{m})(25.813\ \text{rad/s}) = 3.872\ \text{m/s}$

$\underline{v}_B = 3.87\ \text{m/s} \rightarrow$ ◀

17.46



IN POSITION 2, INST. CENTER OF BD IS AT A.

(a)

$(v_B)_2 = l(\omega_{AB})_2 = l(\omega_{BD})_2$

$(\omega_{AB})_2 = (\omega_{BD})_2$

ROD AB

$(\bar{v}_{AB})_2 = \tfrac{1}{2} l(\omega_{AB})_2$

ROD BD $(\bar{v}_{BD})_2 = (AG_{BD})(\omega_{AB})_2 = l\cos 30°\,(\omega_{AB})_2$

(CONTINUED)

17.46 CONTINUED

POSITION 1: $T_1 = 0 \qquad V_1 = mg\tfrac{l}{2} + mgl = \tfrac{3}{2}mgl$

POSITION 2:

$$T_2 = \tfrac{1}{2} m(\bar{v}_{AB})_2^2 + \tfrac{1}{2}\bar{I}(\omega_{AB})_2^2 + \tfrac{1}{2} m(\bar{v}_{BD})_2^2 + \tfrac{1}{2}\bar{I}(\omega_{BD})_2^2$$

$$= \tfrac{1}{2} m\left[\tfrac{l}{2}(\omega_{AB})_2\right]^2 + \tfrac{1}{2}\left[\quad ml^2\right](\omega_{AB})_2^2 + \tfrac{1}{2} m\left[l\cos 30°(\omega_{AB})_2\right]^2 + \tfrac{1}{2}\left[\tfrac{1}{12}ml^2\right](\omega_{AB})_2^2$$

$$= ml^2(\omega_{AB})_2^2\left[\tfrac{1}{8} + \tfrac{1}{24} + \tfrac{1}{2}\cos^2 30° + \tfrac{1}{24}\right] = \tfrac{7}{12}ml^2(\omega_{AB})_2^2$$

$V_2 = 2mg\left(\tfrac{l}{2}\cos 30°\right) = \tfrac{\sqrt{3}}{2}mgl$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \tfrac{3}{2}mgl = \tfrac{7}{12}ml^2(\omega_{AB})_2^2 + \tfrac{\sqrt{3}}{2}mgl$

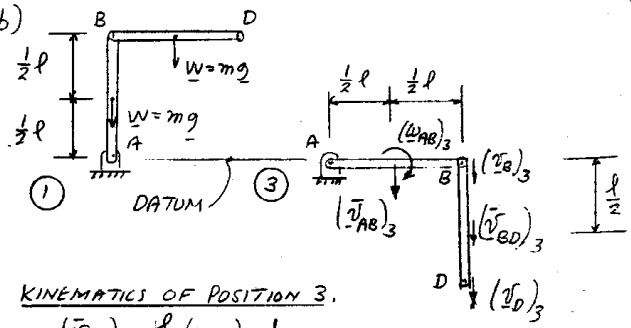
$(\omega_{AB})_2^2 = \dfrac{12}{7}\left(\tfrac{3}{2} - \tfrac{\sqrt{3}}{2}\right)\dfrac{g}{l} = 1.0868\,\dfrac{g}{l}$

$(\omega_{AB})_2 = 1.0425\sqrt{\dfrac{g}{l}}$

VELOCITY OF D.

$(v_D)_2 = (AD)(\omega_{AB})_2 = l\left(1.0425\sqrt{\dfrac{g}{l}}\right) \qquad \underline{v}_D = 1.043\sqrt{gl} \downarrow$ ◀

(b)



KINEMATICS OF POSITION 3.

$(\bar{v}_{AB})_3 = \tfrac{l}{2}(\omega_{AB})_3 \downarrow$

$(\underline{v}_B)_3 = l(\omega_{AB})_3 \downarrow$

SINCE $(\underline{v}_B)_3$ AND $(\underline{v}_D)_3$ ARE VERTICAL THE INSTANTANEOUS CENTER OF BD IS AT ∞

$(\omega_{BD})_3 = 0$

$(\bar{v}_{BD})_3 = (v_B)_3 = (v_D)_3 = l(\omega_{AB})_3$

POSITION 1: SEE PART (a):

$T_1 = 0 \qquad V_1 = \tfrac{3}{2}mgl$

POSITION 3:

$$T_3 = \tfrac{1}{2} m(\bar{v}_{AB})_3^2 + \tfrac{1}{2}\bar{I}(\omega_{AB})_3^2 + \tfrac{1}{2} m(\bar{v}_{BD})_3^2$$

$$= \tfrac{1}{2} m\left[\tfrac{l}{2}(\omega_{AB})_3\right]^2 + \tfrac{1}{2}\left[\tfrac{1}{12}ml^2\right](\omega_{AB})_3^2 + \tfrac{1}{2} m\left[l(\omega_{AB})_3\right]^2$$

$$= ml^2(\omega_{AB})_3^2\left[\tfrac{1}{8} + \tfrac{1}{24} + \tfrac{1}{2}\right] = \tfrac{2}{3}ml^2(\omega_{AB})_3^2$$

$V_3 = -mg\tfrac{l}{2}$

CONSERVATION OF ENERGY

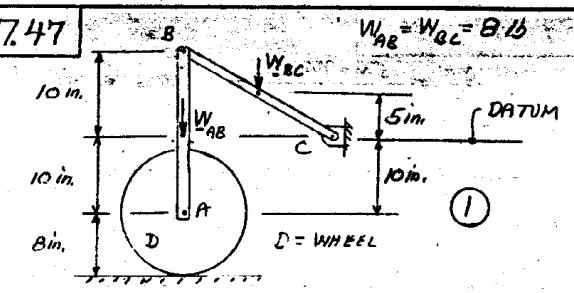
$T_1 + V_1 = T_3 + V_3$: $\quad 0 + \tfrac{3}{2}mgl = \tfrac{2}{3}ml^2(\omega_{AB})_3^2 - \tfrac{1}{2}mgl$

$(\omega_{AB})_3^2 = 3\,\dfrac{g}{l} \qquad (\omega_{AB})_3 = \sqrt{\dfrac{3g}{l}}$

VELOCITY OF D.

$(v_D)_3 = l(\omega_{AB})_3 = \sqrt{3gl} \qquad \underline{v}_D = 1.732\sqrt{gl} \downarrow$ ◀

17.47



$W_{AB} = W_{BC} = 8$ lb

WHEEL D: $k = 6$ in. $W_{WHEEL} = 15$ lb

$\bar{I}_D = m\bar{k}^2 = \frac{15\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{6}{12}\text{ ft}\right)^2 = 0.11646\text{ lb·ft·s}^2$

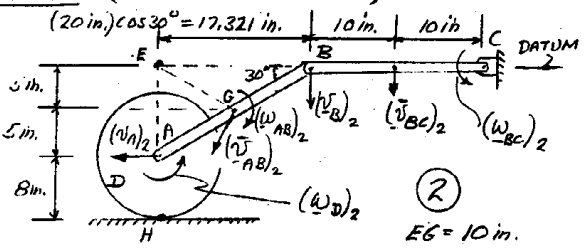
RODS AB AND BC: $W = 8$ lb

$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{8\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(\frac{20}{12}\text{ ft}\right)^2 = 0.05751\text{ lb·ft·s}^2$

POSITION 1: VELOCITY = 0, $T_1 = 0$

$V_1 = W_{BC}\left(\frac{5}{12}\text{ ft}\right) = (8\text{ lb})\left(\frac{5}{12}\text{ ft}\right) = 3.333\text{ ft·lb}$

POSITION 2: (ROD BC IS HORIZONTAL)



KINEMATICS FOR POSITION 2:

ROD BC. $(\vec{v}_{BC})_2 = \left(\frac{10}{12}\text{ ft}\right)(\omega_{BC})_2 = 0.8333(\omega_{BC})_2$

$(v_B)_2 = \left(\frac{20}{12}\text{ ft}\right)(\omega_{BC})_2$

ROD AB. INST. CENTER AT E.

$(v_B)_2 = \left(\frac{17.321}{12}\text{ ft}\right)(\omega_{AB})_2 = \left(\frac{20}{12}\text{ ft}\right)(\omega_{BC})_2$

$(\omega_{AB})_2 = 1.1547(\omega_{BC})_2$

$(\vec{v}_{AB})_2 = (EG)(\omega_{AB})_2 = \left(\frac{10}{12}\text{ ft}\right)(1.1547)(\omega_{BC})_2 = 0.96225(\omega_{BC})_2$

$(v_A)_2 = (EA)(\omega_{AB})_2 = \left(\frac{10}{12}\text{ ft}\right)(1.1547)(\omega_{BC})_2 = 0.96225(\omega_{BC})_2$

WHEEL D. INST. CENTER AT H.

$(\omega_D)_2 = \frac{(v_A)_2}{(AH)} = \frac{0.96225(\omega_{BC})_2}{(8/12)\text{ ft}} = 1.4434(\omega_{BC})_2$

KINETICS IN POSITION 2.

$$T_2 = \left[\frac{1}{2}m_D(v_A)_2^2 + \frac{1}{2}\bar{I}_D(\omega_D)_2^2\right] + \left[\frac{1}{2}m_{AB}(\vec{v}_{AB})_2^2 + \frac{1}{2}\bar{I}(\omega_{AB})_2^2\right] + \left[\frac{1}{2}m_{BC}(\vec{v}_{BC})_2^2 + \frac{1}{2}\bar{I}(\omega_{BC})_2^2\right]$$

$$= \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.96225)^2(\omega_{BC})_2^2 + \frac{1}{2}(0.11646)(1.4434)^2(\omega_{BC})_2^2\right] + \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.96225)^2(\omega_{BC})_2^2 + \frac{1}{2}(0.05751)(1.1547)^2(\omega_{BC})_2^2\right] + \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.8333)^2(\omega_{BC})_2^2 + \frac{1}{2}(0.05751)(\omega_{BC})_2^2\right]$$

(CONTINUED)

17.47 CONTINUED

$$T_2 = \left\{[0.21567 + 0.12132] + [0.11502 + 0.03839] + [0.08627 + 0.02876]\right\}(\omega_{BC})_2^2$$

$T_2 = 0.60536(\omega_{BC})_2^2$

$V_2 = -W_{AB}\left(\frac{5}{12}\text{ ft}\right) = -(8\text{ lb})\left(\frac{5}{12}\text{ ft}\right) = -3.333\text{ ft·lb}$

CONSERVATION OF ENERGY.

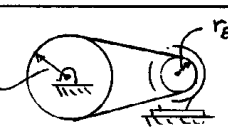
$T_1 + V_1 = T_2 + V_2$: $0 + 3.333 = 0.60536(\omega_{BC})_2^2 - 3.333$

$(\omega_{BC})_2^2 = 11.012$ $\quad (\omega_{BC})_2 = 3.318$ rad/s

VELOCITY OF A.

$(v_A)_2 = 0.96225(\omega_{BC})_2 = 0.96225(3.318)$ $\quad v_A = 3.19$ ft/s ◀

17.48



$r_A\omega_A = r_B\omega_B$

$(6\text{ in.})\omega_A = (2.5\text{ in.})\omega_B$

$\omega_B = 1200$ rpm $= 125.66$ rad/s $\quad \omega_A = 0.4167\,\omega_B$

$\omega_A = 0.4167\,\omega_B = 52.36$ rad/s

(a) $\text{HORSEPOWER} = \frac{M_A\omega_A}{550}$

$7.5\text{ hp} = \frac{M_A(52.36\text{ rad/s})}{550}$ $\quad M_A = 78.8\text{ lb·ft}$ ◀

(b) $\text{HORSEPOWER} = \frac{M_B\omega_B}{550}$

$7.5\text{ hp} = \frac{M_B(125.66\text{ rad/s})}{550}$ $\quad M_B = 32.8\text{ lb·ft}$ ◀

17.49

KINEMATICS. $\omega_{AB} = 30\text{ Hz} = 30(2\pi)\text{ rad/s} = 60\pi\text{ rad/s}$

GEARS B AND C.

$r_B = 75$ mm $\quad r_C = 180$ mm

$r_B\omega_{AB} = r_C\omega_{CD}$: $(75\text{ mm})(60\pi\text{ rad/s}) = (180\text{ mm})(\omega_{CD})$

$\omega_{CD} = 25\pi$ rad/s

GEARS D AND E.

$r_D = 75$ mm $\quad r_E = 180$ mm

$r_D\omega_{CD} = r_E\omega_{EF}$: $(75\text{ mm})(25\pi\text{ rad/s}) = (180\text{ mm})(\omega_{EF})$

$\omega_{EF} = 10.4167\pi$ rad/s

Power = 7.5 kW

(a) SHAFT AB.

$\text{Power} = M_{AB}\omega_{AB}$: $\quad 7500\text{ W} = M_{AB}(60\pi\text{ rad/s})$

$M_{AB} = 39.8\text{ N·m}$ ◀

(b) SHAFT CD.

$\text{Power} = M_{CD}\omega_{CD}$: $\quad 7500\text{ W} = M_{CD}(25\pi\text{ rad/s})$

$M_{CD} = 95.5\text{ N·m}$ ◀

(c) SHAFT EF.

$\text{Power} = M_{EF}\omega_{EF}$: $\quad 7500\text{ W} = M_{EF}(10.4167\pi\text{ rad/s})$

$M_{EF} = 229\text{ N·m}$ ◀

17.50

Power = 400 W

(a) $\omega = 3600 \text{ rpm} = 376.99 \text{ rad/s}$

Power $= M\omega$

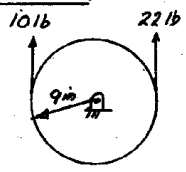
$400 \text{ W} = M(376.99 \text{ rad/s})$ $\quad M = 1.061 \text{ N·m}$ ◀

(b) $\omega = 1200 \text{ rpm} = 125.66 \text{ rad/s}$

Power $= M\omega$

$400 \text{ W} = M(125.66 \text{ rad/s})$ $\quad M = 3.18 \text{ N·m}$ ◀

17.51



$\omega = 200 \text{ rpm} = 20.944 \text{ rad/s}$

$M = (22 \text{ lb} - 10 \text{ lb})\left(\frac{9}{12} \text{ ft}\right) = 9 \text{ lb-ft}$

Power $= M\omega = (9 \text{ lb·ft})(20.944 \text{ rad/s})$

$= 188.5 \text{ ft·lb/s}$

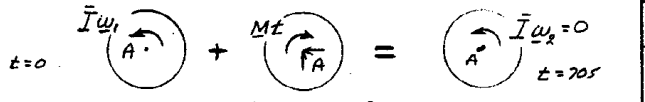
$= (188.5 \frac{\text{ft·lb}}{\text{s}})/(550 \frac{\text{ft·lb}}{\text{s·hp}})$

POWER = 0.343 horsepower ◀

17.52

$\omega_1 = 3600 \text{ rpm} = 120\pi \text{ rad/s}$ $\quad \omega_2 = 0$

$\bar{I} = m\bar{k}^2 = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{2}{12} \text{ ft}\right)^2 = 0.00518 \text{ lb·ft·s}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $\bar{I}\omega_1 - Mt = 0$ (1)

$(0.00518)(120\pi) - M(70 \text{ s}) = 0$

$M = 0.02788 \text{ lb·ft}$

$M = 0.33451 \text{ lb·in.}$

$M = 0.335 \text{ lb·in.}$ ◀

17.53

$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$

$\bar{I} = m\bar{k}^2 = (2750 \text{ kg})(1.15 \text{ m})^2 = 3636.9 \text{ kg·m}^2$

$t = 12.8 \text{ min} = 768 \text{ s}$

SEE SKETCH OF SOLUTION OF PROB 17.52 AND EQ(1)

$\bar{I}\omega_1 - Mt = 0$

$M = \frac{\bar{I}\omega_1}{t} = \frac{(3636.9 \text{ kg·m}^2)(6\pi \text{ rad/s})}{768 \text{ s}} = 89.26 \text{ N·m}$

$M = 89.3 \text{ N·m}$ ◀

17.54

DENOTE BY I_o THE MASS MOMENT OF INERTIA OF DISK A AND OF DISK B. DENOTE BY t THE THICKNESS OF DISKS.

$I_o = \frac{1}{2}mr^2 = \frac{1}{2}(\pi\rho t r^2)r^2 = \frac{1}{2}\pi\rho t r^4$

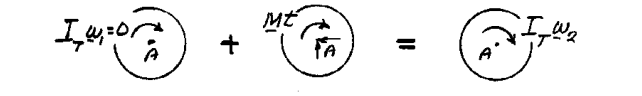
$I_C = \frac{1}{2}m_C(nr)^2 = \frac{1}{2}(\pi\rho t n^2 r^2)n^2 r^2 = \left(\frac{1}{2}\pi\rho t r^4\right)n^4 = n^4 I_o$

$I_T = I_{TOTAL} = 2I_o + n^4 I_o = I_o(2 + n^4)$ (1)

$m_A = m_B = 6 \text{ kg}$ $\quad r = 0.175 \text{ m}$ $\quad M = 15 \text{ N·m}$

$I_o = \frac{1}{2}m_A r^2 = \frac{1}{2}(6 \text{ kg})(0.175 \text{ m})^2 = 0.091875 \text{ kg·m}^2$

$\omega_1 = 0$ $\quad \omega_2 = 1200 \text{ rpm} = 40\pi \text{ rad/s}$ $\quad t = 4 \text{ s}$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $0 + Mt = I_T\omega_2$

$I_T = \frac{Mt}{\omega_2}$

(CONTINUED)

17.54 CONTINUED

$I_T = \frac{Mt}{\omega_2} = \frac{(15 \text{ N·m})(4 \text{ s})}{(40\pi \text{ rad/s})} = 0.47746 \text{ kg·m}^2$

EQ(1) $\quad I_T = I_o(2 + n^4)$

$0.47746 \text{ kg·m}^2 = (0.091875 \text{ kg·m}^2)(2 + n^4)$

$2 + n^4 = 5.1969$ $\quad n^4 = 3.1969$ $\quad n = 1.3372$

$r_C = nr = (1.3372)(175 \text{ mm})$ $\quad r = 234 \text{ mm}$ ◀

17.55

$\bar{I} = m\bar{k}^2 = \frac{42 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{10.8 \text{ in.}}{12 \text{ in}}\right)^2 = 1.0565 \text{ ft·lb·s}^2$

$t = 0.1 \text{ s}$ $\quad \omega_1 = 0$

$M = (25 \text{ lb})\left(\frac{18}{12} \text{ ft}\right) = 37.5 \text{ lb·ft}$

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

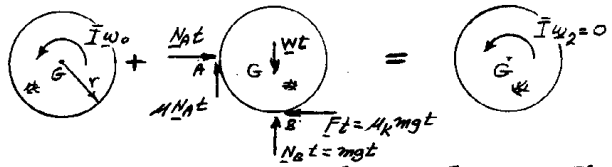
+↻ MOMENTS ABOUT A: $0 + Mt = \bar{I}\omega_2$

$\omega_2 = \frac{Mt}{\bar{I}} = \frac{(37.5 \text{ lb·ft})(0.1 \text{ s})}{1.0565 \text{ lb·ft·s}^2} = 3.549 \text{ rad/s}$

$\omega = 3.55 \text{ rad/s}$ ◀

17.56

SPHERE $\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}\frac{W}{g}r^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↑ y COMPONENTS: $0 + N_B t + \mu_k N_A t - Wt = 0$ (1)

+→ x COMPONENTS: $0 + N_A t - \mu_k N_B t = 0$ (2)

(3)

FROM EQ(2) $\quad N_A = \mu_k N_B$

SUBSTITUTE INTO EQ(1): $N_B t + \mu_k(\mu_k N_B)t - Wt = 0$

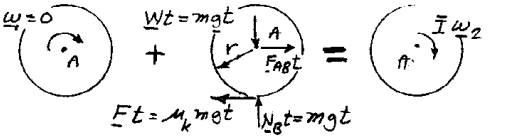
$N_B = \frac{1}{1+\mu_k^2}W$ $\quad$ EQ(3): $N_A = \frac{\mu_k}{1+\mu_k^2}W$

+↻ MOMENTS ABOUT G: $\bar{I}\omega_0 - r(\mu_k N_A t) - r(\mu_k N_B t) = 0$

$\frac{2}{5}\frac{W}{g}r^2\omega_0 - \frac{\mu_k^2}{1+\mu_k^2}rWt - \frac{\mu_k}{1+\mu_k^2}rWt = 0$

$\frac{2}{5}\frac{r}{g}\omega_0 - \frac{\mu_k + \mu_k^2}{1+\mu_k^2}t = 0$ $\quad t = \frac{1+\mu_k^2}{\mu_k + \mu_k^2}\cdot\frac{2}{5}\frac{r}{g}\omega_0$ ◀

17.57



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $0 + (\mu_k mgt)r = \bar{I}\omega_2$ (1)

FOR DISK: $\bar{I} = \frac{1}{2}mr^2$ $\quad$ FINAL ANGULAR VELOCITY: $\omega_2 = \frac{v}{r}$

SUBSTITUTE INTO EQ(1): $0 + (\mu_k mgt)r = \frac{1}{2}mr^2\left(\frac{v}{r}\right)$

$t = \frac{v}{2g\mu_k}$ ◀

17.58

SUBSTITUTE GIVEN DATA INTO EXPRESSION FOR t OBTAINED IN PROB. 17.57

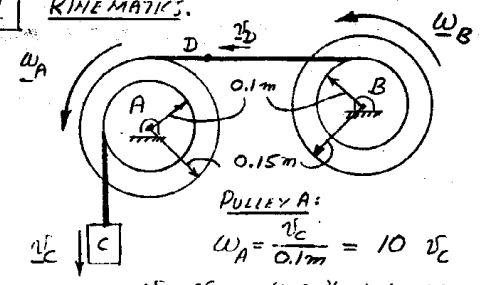
$g = 9.81\ m/s^2$ $v = 15\ m/s$ $\mu_k = 0.15$

$$t = \frac{v}{2g\mu_k} = \frac{15\ m/s}{2(9.81\ m/s^2)(0.15)} = 5.097\ s$$

$$t = 5.10\ s \blacktriangleleft$$

17.59

KINEMATICS.



PULLEY A:

$$\omega_A = \frac{v_C}{0.1\ m} = 10\ v_C$$

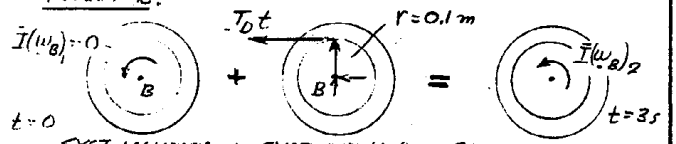
$$v_D = v_{cord} = (0.15\ m)(\omega_A) = (0.15)(10\ v_C) = 1.5\ v_C$$

PULLEY B:

$$\omega_B = \frac{v_D}{0.1\ m} = \frac{1.5\ v_C}{0.1\ m} = 15\ v_C$$

KINETICS. FOR EACH PULLEY $\bar{I} = 0.25\ kg \cdot m^2$

PULLEY B.



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

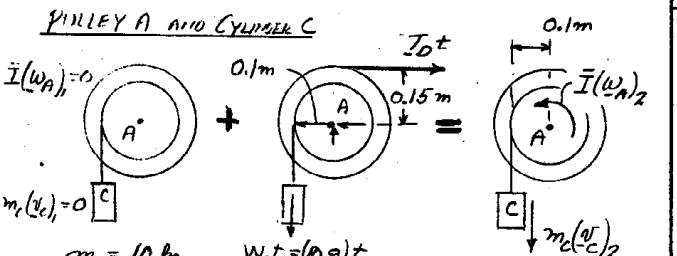
+) MOMENTS ABOUT B:

$$(T_D t)(0.1\ m) = \bar{I}(\omega_B)_2 \qquad (1)$$

$$T_D t(0.1\ m) = (0.25\ kg \cdot m^2)(15)(v_C)_2$$

$$T_D t = 37.5\,(v_C)_2 \qquad (2)$$

PULLEY A AND CYLINDER C



(a)

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ + SYST. MOMENTA$_2$

+) MOMENTS ABOUT A:

$$0 + (W_c t)(0.1\ m) - (T_D t)(0.15\ m) = m_c(v_C)_2(0.1\ m) + \bar{I}(\omega_A)_2$$

$$0 + (98.1t)(0.1\ m) - 0.15\,T_D t = (10)(v_C)_2(0.1) + (0.25)(10)(v_C)_2$$

$$9.81\,t - 0.15\,T_D t = 1.0\,(v_C)_2 + 2.5\,(v_C)_2$$

$$9.81\,t - 0.15\,T_D t = 3.5\,(v_C)_2 \qquad (3)$$

SUBSTITUTE $T_D t = 37.5\,(v_C)_2$ FROM EQ (2) INTO EQ (3)

$$9.81\,t - (0.15)(37.5)\,(v_C)_2 = 3.5\,(v_C)_2$$

$$9.81\,t - 5.625\,(v_C)_2 = 3.5\,(v_C)_2$$

$$9.81\,t = 9.125\,(v_C)_2$$

FOR $t = 3\ s$: $(9.81)(3) = 9.125\,(v_C)_2$

$$(v_C)_2 = 3.2252\ m/s \qquad (v_C)_2 = 3.23\ m/s \downarrow \blacktriangleleft$$

(CONTINUED)

17.59 CONTINUED

(b) WE NOW HAVE: $(v_C)_2 = 3.2252\ m/s$

RECALL EQ(2):

$$T_D t = 37.5\,(v_C)_2 = 37.5\,(3.2252)$$

$$T_D t = 120.945$$

FOR $t = 3\ s$: $T_D(3) = 120.945$

$$T_D = 40.315\ N \qquad T_D = 40.3\ N \blacktriangleleft$$

17.60

WE ADD BEARING FRICTION $M_f = 0.45\ N \cdot m$ TO THE TWO SYST. EXT. IMP$_{1-2}$ DIAGRAMS OF THE SOLUTION OF PROB. 17.59. WE THEN WRITE:

EQ(1): $(T_D t)(0.1\ m) - (0.45\ N \cdot m)t = \bar{I}(\omega_B)_2 \qquad (1')$

$$T_D t(0.1\ m) - (0.45\ N \cdot m)t = (0.25\ kg \cdot m^2)(15)(v_C)_2$$

$$T_D t = 4.5t + 37.5\,(v_C)_2 \qquad (2')$$

EQ(3): $9.81\,t - 0.15\,T_D t - (0.45\ N \cdot m)t = 3.5\,(v_C)_2 \qquad (3')$

SUBSTITUTE FROM (2') INTO (3'):

$$9.81\,t - 0.15\left[4.5t + 37.5\,(v_C)_2\right] - 0.45t = 3.5\,(v_C)_2$$

$$8.685t = 9.125\,(v_C)_2$$

FOR $t = 3\ s$: $8.685(3) = 9.125\,(v_C)_2$

(a) $(v_C)_2 = 2.8553\ m/s$ $\qquad (v_C)_2 = 2.86\ m/s \downarrow \blacktriangleleft$

(b) SUBSTITUTE FOR $(v_C)_2$ INTO EQ(2'):

$$T_D t = 4.5\,t + 37.5(2.8553)$$

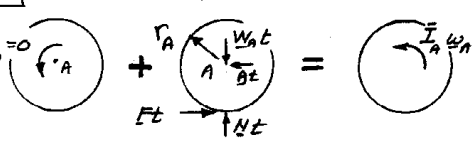
$$(T_D - 4.5)t = 107.075$$

FOR $t = 3\ s$: $(T_D - 4.5)(3\ s) = 107.075$

$$T_D - 4.5 = 35.69 \qquad T_D = 40.19\ N \qquad T_D = 40.2\ N \blacktriangleleft$$

17.61

DISK A: $(\omega_A)_0 = 0$

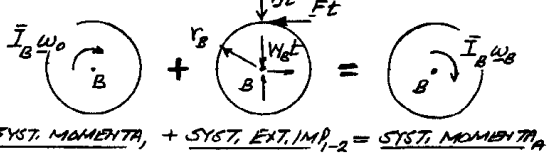


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ + SYST. MOMENTA$_2$

+) MOMENTS ABOUT A: $0 + (Ft)r_A = \bar{I}_A \omega_A$

$$Ft = \frac{1}{r_A}(I_A \omega_A) \qquad (1)$$

DISK B: $(\omega_B)_0 = \omega_0$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $\bar{I}_B \omega_0 - (Ft)r_B = \bar{I}_B \omega_B$

SUBSTITUTE FOR Ft:

$$\bar{I}_B \omega_0 - I_A \omega_A \frac{r_B}{r_A} = \bar{I}_B \omega_B \qquad (2)$$

FINAL ANGULAR VELOCITY OCCURS WHEN POINTS ON PERIMETER OF DISKS HAVE SAME VELOCITY.

$$r_A \omega_A = r_B \omega_B \quad \text{OR:} \quad \omega_A = \frac{r_B}{r_A}\omega_B$$

17.61 CONTINUED

THUS, WHEN NO SLIPPING OCCURS BETWEEN DISKS: $\omega_A = \frac{r_B}{r_A}\omega_B$

SUBSTITUTE INTO EQ(2):

$$\bar{I}_B\omega_0 - \bar{I}_A\omega_B\left(\frac{r_B}{r_A}\right)^2 = \bar{I}_B\omega_B$$

$$\omega_B = \frac{\omega_0}{1 + \frac{\bar{I}_A}{\bar{I}_B}\left(\frac{r_B}{r_A}\right)^2} \qquad (3)$$

BUT FOR UNIFORM DISKS $\frac{\bar{I}_A}{\bar{I}_B} = \frac{\frac{1}{2}m_A r_A^2}{\frac{1}{2}m_B r_B^2} = \frac{m_A}{m_B}\left(\frac{r_A}{r_B}\right)^2$

SUBSTITUTE INTO EQ(3)

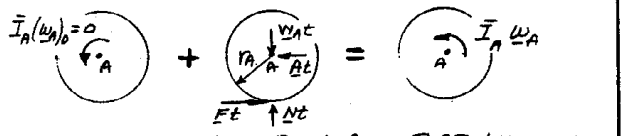
$$\omega_B = \frac{\omega_0}{1 + \frac{m_A}{m_B}}$$

THUS FINAL ANGULAR VELOCITY OF DISK B DEPENDS ONLY ON ω_0 AND THE RATIO $\frac{m_A}{m_B}$. (Q.E.D.)

17.62

DISK A: $(\omega_A)_0 = 0$ $m_A = 6$ kg $r_A = 135$ mm

$\bar{I}_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(6\text{ kg})(0.135\text{ m})^2 = 0.05468\text{ kg}\cdot\text{m}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

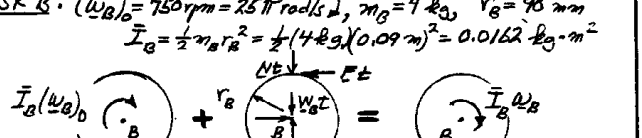
+) MOMENTS ABOUT A: $0 + Ft\,r_A = \bar{I}_A\omega_A$

$0 + (Ft)(0.135\text{ m}) = (0.05468\text{ kg}\cdot\text{m}^2)\omega_A$

$(Ft) = 0.405\,\omega_A \qquad (1)$

DISK B: $(\omega_B)_0 = 750$ rpm $= 25\pi$ rad/s, $m_B = 4$ kg, $r_B = 90$ mm

$\bar{I}_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(4\text{ kg})(0.09\text{ m})^2 = 0.0162\text{ kg}\cdot\text{m}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ + SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $\bar{I}_B(\omega_B)_0 - (Ft)r_B = \bar{I}_B\omega_B$

$(0.0162\text{ kg}\cdot\text{m}^2)(25\pi\text{ rad/s}) - (Ft)(0.09\text{ m}) = (0.0162\text{ kg}\cdot\text{m}^2)\omega_B$

$25\pi - 5.555(Ft) = \omega_B$

SUBSTITUTE (Ft) FROM (1): $25\pi - 5.555(0.405\,\omega_A) = \omega_B$

$\omega_B + 2.25\,\omega_A = 25\pi \qquad (2)$

FINAL ANGULAR VELOCITY OCCURS WHEN POINTS ON PERIMETER OF DISKS HAVE SAME VELOCITY. THAT IS, WHEN $r_A\omega_A = r_B\omega_B$

$$\omega_B = \frac{r_A}{r_B}\omega_A = \frac{135\text{ mm}}{90\text{ mm}}\omega_A \qquad \omega_B = 1.5\,\omega_A \qquad (3)$$

SUBSTITUTE FOR ω_B IN EQ(2)

$1.5\,\omega_A + 2.25\,\omega_A = 25\pi$

(a) $3.75\,\omega_A = 25\pi$ $\omega_A = 6.667\pi$ $\omega_A = 200$ rpm ↻ ◀

EQ(3) $\omega_B = 1.5\,\omega_A = 1.5(200\text{ rpm})$ $\omega_B = 300$ rpm ↺ ◀

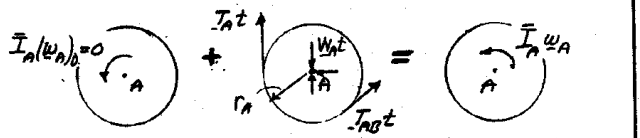
(b) EQ(1): $Ft = 0.405\,\omega_A = 0.405(6.667\pi) = 8.482$ N·s

IMPULSE ON DISK A: $Ft = 8.48$ N·s → ◀

17.63

DRUM A: $W_A = 1.4$ lb, $\bar{k}_A = 0.75$ in. $r_A = 0.9$ in

$\bar{I}_A = m_A\bar{k}_A^2 = \left(\frac{1.4\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(\frac{0.75}{12}\text{ ft}\right)^2 = 169.84\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT A: $0 - T_A t\,r_A + T_{AB} t\,r_A = \bar{I}_A\omega_A$

FOR $t = 0.24$ s AND $T_A = 0.75$ lb

$0 - (0.75\text{ lb})(0.24\text{ s})\left(\frac{0.9}{12}\text{ ft}\right) + T_{AB}(0.24\text{ s})\left(\frac{0.9}{12}\text{ ft}\right) = 169.84\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\omega_A$

$-0.75 + T_{AB} = 9.435\times10^{-3}\,\omega_A \qquad (1)$

SINCE THE FINAL VELOCITY OF THE TAPE IS TO BE 10 ft/s,

$\omega_A = \frac{v}{r_A} = \frac{10\text{ ft/s}}{\left(\frac{0.9}{12}\text{ ft}\right)} = 133.33$ rad/s

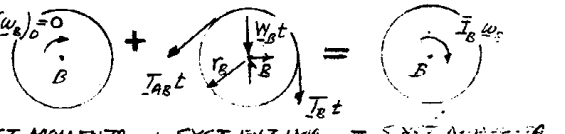
SUBSTITUTE INTO EQ(1)

$T_{AB} - 0.75 = (9.435\times10^{-3})(133.33) = 1.258$

$T_{AB} = 2.008$ lb $\qquad T_{AB} = 2.01$ lb ◀

DRUM B: $W_B = 3.5$ lb, $\bar{k}_B = 1.25$ in. $r_B = 1.5$ in.

$\bar{I}_B = m_B\bar{k}_B^2 = \left(\frac{3.5\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(\frac{1.25}{12}\text{ ft}\right)^2 = 1179.4\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $0 + T_B t\,r_B - T_{AB} t\,r_B = \bar{I}_B\omega_B$

FOR $t = 0.24$ s

$T_B(0.24\text{ s})\left(\frac{1.5}{12}\text{ ft}\right) - T_{AB}(0.24\text{ s})\left(\frac{1.5}{12}\text{ ft}\right) = (1179.4\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\omega_B$

$(T_B - T_{AB}) = 0.039314\,\omega_B \qquad (2)$

SINCE THE FINAL VELOCITY OF THE TAPE IS 10 ft/s,

$\omega_B = \frac{v}{r_B} = \frac{10\text{ ft/s}}{\left(\frac{1.5}{12}\text{ ft}\right)} = 80.0$ rad/s

SUBSTITUTE $\omega_B = 80.0$ rad/s AND $T_{AB} = 2.01$ lb INTO EQ(2).

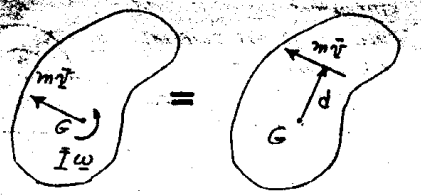
$T_B - 2.01\text{ lb} = 0.039314(80.0)$

$T_B - 2.01\text{ lb} = 3.1451\text{ lb}$

$T_B = 5.1551$ lb

$T_B = 5.16$ lb ◀

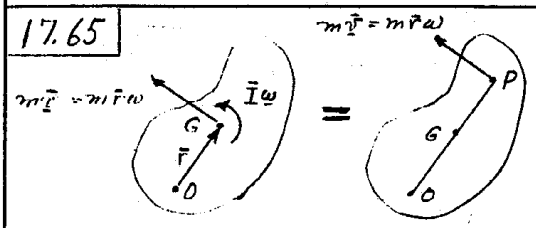
17.64



+) MOMENTS ABOUT G: $\bar{I}\omega = (m\bar{v})d$

$$d = \frac{\bar{I}}{m}\cdot\frac{\omega}{\bar{v}} = \frac{m\bar{k}^2}{m}\cdot\frac{\omega}{\bar{v}} \qquad d = \frac{\bar{k}^2\omega}{\bar{v}}$$ ◀

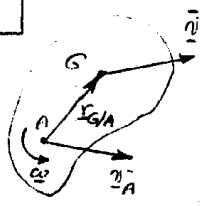
17.65



+) MOMENTS ABOUT G: $\bar{I}\omega = (m\bar{r}\omega)(GP)$

$$GP = \frac{\bar{I}\omega}{m\bar{r}\omega} = \frac{m\bar{k}^2}{m\bar{r}} \qquad GP = \frac{\bar{k}^2}{\bar{r}}$$ ◀

17.66



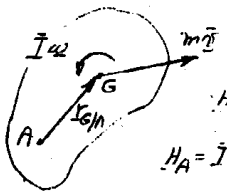
LET A BE ARBITRARY POINT WITH VELOCITY $\bar{v}_A$.

FOR GENERAL PLANE MOTION:

$$\bar{v} = \bar{v}_A + \bar{v}_{G/A}$$

$$\bar{v} = \bar{v}_A + \omega \times r_{G/A}$$

SYSTEM OF MOMENTA



WE SUM MOMENTS ABOUT A:

$$H_A = \bar{I}\omega + r_{G/A} \times m\bar{v}$$

$$H_A = \bar{I}\omega + m\,r_{G/A} \times (v_A + \omega \times r_{G/A})$$

$$H_A = \bar{I}\omega + m\,r_{G/A} \times v_A + m\,r_{G/A} \times (\omega \times r_{G/A})$$

WE OBSERVE THAT SINCE $\omega \perp r_{G/A}$, THE TRIPLE VECTOR PRODUCT MAY BE WRITTEN AS

$$r_{G/A} \times (\omega \times r_{G/A}) = r_{G/A}^2\,\omega$$

THUS: $H_A = \bar{I}\omega + m\,r_{G/A}^2\,\omega + m\,r_{G/A} \times v_A$

BY PARALLEL AXIS THEOREM, $I_A = \bar{I} + m\,r_{G/A}^2$

WE NOW HAVE: $H_A = I_A\omega + m\,r_{G/A} \times v_A$

THEREFORE, $H_A = I_A\omega$ IF AND ONLY IF $r_{G/A} \times v_A = 0$, WHICH OCCURS ONLY WHEN

(a) $r_{G/A} = 0$: POINT A COINCIDES WITH G.

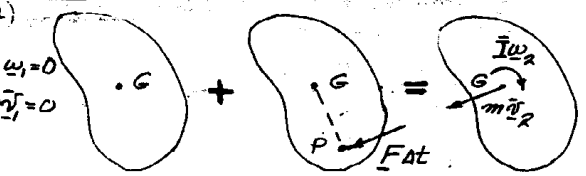
(b) $v_A = 0$: POINT A IS THE INSTANTANEOUS CENTER OF ROTATION OF THE SLAB

(c) $r_{G/A}$ AND v_A ARE COLINEAR: THE VELOCITY OF A IS DIRECTED ALONG LINE AG.

17.67

Δt = TIME OF APPLICATION OF F

(a)

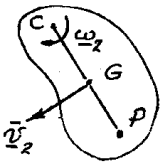


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↓ COMPONENTS: $F\Delta t = m\bar{v}_2 \qquad \bar{v}_2 = \frac{F\Delta t}{m}$ (1)

+↻ MOMENTS ABOUT G: $(F\Delta t)(GP) = \bar{I}\omega_2$

$$\omega_2 = \frac{F\Delta t}{\bar{I}}(GP) = \frac{F\Delta t}{m\bar{k}^2}(GP) \qquad (2)$$



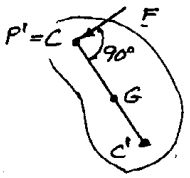
KINEMATICS: THE INSTANTANEOUS CENTER MUST BE LOCATED ON A LINE $\perp$ TO $\bar{v}_2$, THAT IS ON GP, ALSO,

$$\bar{v}_2 = (GC)\omega_2 \qquad GC = \frac{\bar{v}_2}{\omega_2}$$

SUBSTITUTE FROM (1) AND (2).

$$GC = \frac{\frac{F\Delta t}{m}}{\frac{F\Delta t}{m\bar{k}^2}(GP)} \qquad GC = \frac{\bar{k}^2}{GP}$$ ◀

(b) WE NOW ASSUME THAT F IS APPLIED SO THAT THE NEW CENTER OF PERCUSSION P' IS LOCATED AT C.

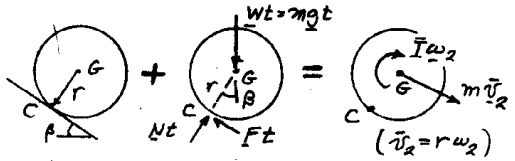


ACCORDING TO THE RESULT OBTAINED IN PART a, THE NEW INSTANTANEOUS CENTER WILL BE LOCATED ON LINE GC AT THE FOLLOWING DISTANCE FROM G.

$$GC' = \frac{\bar{k}^2}{GP'} = \frac{\bar{k}^2}{GC} = \frac{\bar{k}^2}{\bar{k}^2/GP} = GP$$

WE FIND THAT $GC' = GP$, THE NEW INSTANTANEOUS CENTER IS LOCATED AT P. ◀

17.68



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $Wt(r\sin\beta) = \bar{I}\omega_2 + m\bar{v}_2 r$

$$mgt(r\sin\beta) = m\bar{k}^2\omega_2 + mr^2\omega_2$$

$$\omega_2 = \frac{rgt\sin\beta}{r^2+\bar{k}^2}$$

(a) $\bar{v}_2 = r\omega_2$: $\bar{v}_2 = \frac{r^2}{r^2+\bar{k}^2}gt\sin\beta$ ◀β ◀

(b) +↗ ΣF: $Nt - mgt\cos\beta \qquad N = mg\cos\beta$

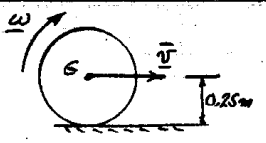
+↻ Σ MOMENTS ABOUT G:

$$(Ft)r = \bar{I}\omega_2$$

$$F = \frac{\bar{I}}{rt}\omega_2 = \frac{m\bar{k}^2}{rt}\cdot\frac{r^2gt}{r^2+\bar{k}^2}\sin\beta = \frac{\bar{k}^2}{r^2+\bar{k}^2}mg\sin\beta$$

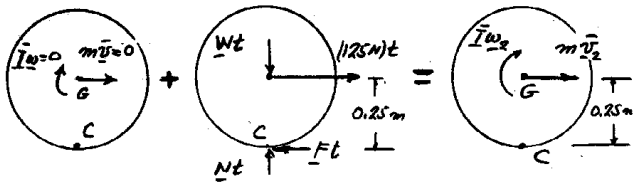
$$\mu_s = \frac{F}{N} = \frac{\bar{k}^2}{r^2+\bar{k}^2}\frac{mg\sin\beta}{mg\cos\beta} \qquad \mu_s = \frac{\bar{k}^2}{r^2+\bar{k}^2}\tan\beta$$ ◀

17.69



KINEMATICS
ROLLING WITHOUT SLIPPING
$\bar{v} = (0.25m)\omega$

CYLINDER: $\bar{I} = \frac{1}{2}mr^2$ $m = 15\ kg$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $0 + (125\ N)t(0.25m) = \bar{I}\omega_2 + m\bar{v}_2(0.25m)$

SUBSTITUTE $m = 15\ kg$, $\bar{v}_2 = 0.25\omega_2$, $t = 5\ s$

$0 + (125\ N)(5\ s)(0.25m) = \frac{1}{2}(15\ kg)(0.25m)^2\omega_2 + (15\ kg)(0.25m)^2\omega_2$

$156.25 = (0.46875 + 0.9375)\omega_2$

$156.25 = 1.40625\,\omega_2$ $\omega_2 = 111.11\ rad/s$

$\bar{v}_2 = (0.25m)(111.11\ rad/s) = 27.78\ m/s$

$\underline{\bar{v}_2 = 27.8\ m/s \rightarrow}$ ◀

+→ x COMPONENTS: $0 - Ft + (125\ N)t = m\bar{v}_2$

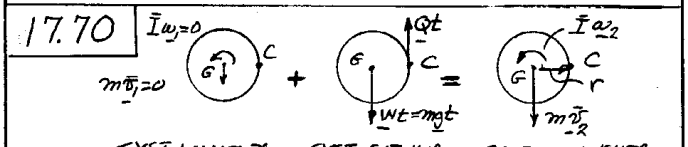
$0 - F(5\ s) + (125\ N)(5\ s) = (15\ kg)(27.78\ m/s)$

$-5F + 625 = 416.67$

$F = 41.67\ N$

$\underline{F = 41.7\ N \leftarrow}$ ◀

17.70



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $0 + (Nt)r = \bar{I}\omega_2 + m\bar{v}_2 r$

SINCE CORD IS INEXTENSIBLE $\bar{v}_c = 0$, AND $\bar{v}_2 = r\omega_2$

$mgt\,r = m\bar{k}^2\omega_2 + mr^2\omega_2$

$\omega_2 = \dfrac{rgt}{r^2 + \bar{k}^2}$

$\bar{v}_2 = r\omega_2$ $\bar{v}_2 = \dfrac{r^2gt}{r^2+\bar{k}^2} = \dfrac{gt}{1+\frac{\bar{k}^2}{r^2}}$

(a) FOR PIPE: $\bar{I} = m\bar{k}^2 = mr^2$

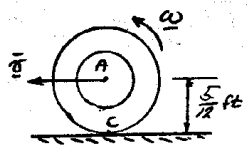
$\dfrac{\bar{k}^2}{r^2} = 1$ $\bar{v}_2 = \dfrac{gt}{1+1}$ $\underline{\bar{v}_2 = \frac{1}{2}gt\downarrow}$ ◀

(b) FOR CYLINDER: $\bar{I} = m\bar{k}^2 = \frac{1}{2}mr^2$

$\dfrac{\bar{k}^2}{r^2} = \dfrac{1}{2}$ $\bar{v}_2 = \dfrac{gt}{1+\frac{1}{2}}$ $\underline{\bar{v}_2 = \frac{2}{3}gt\downarrow}$ ◀

17.71

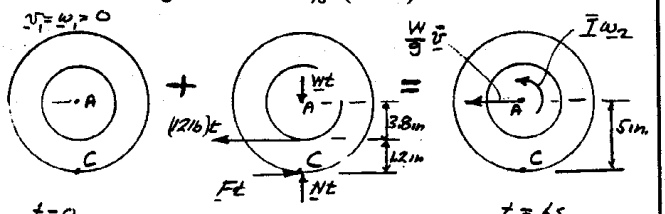
KINEMATICS
ROLLING WITHOUT SLIPPING
INST. CENTER AT C



$\bar{v} = \left(\frac{5}{12}\text{ ft}\right)\omega$

DRUM: $W = 18\ lb$, $\bar{k} = 3\ in.$

$\bar{I} = \frac{W}{g}\bar{k}^2 = \frac{18\ lb}{32.2\ ft/s^2}\left(\frac{3}{12}\text{ ft}\right)^2 = 0.03494\ lb\cdot ft\cdot s^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $(12\ lb)t\left(\frac{12}{12}\text{ ft}\right) = \bar{I}\omega_2 + \frac{W}{g}\bar{v}\left(\frac{5}{12}\text{ in.}\right)$

SUBSTITUTE $t = 6\ s$, $W = 18\ lb$, AND $\bar{I} = 0.03494\ lb\cdot ft\cdot s^2$

$12(6)\left(\frac{12}{12}\right) = 0.03494\,\omega_2 + \frac{18}{32.2}\left(\frac{5}{12}\right)^2\omega_2$

$72 = (0.03494 + 0.08705)\omega_2$

$72 = 0.13199\,\omega_2$ $\omega_2 = 54.55\ rad/s$

$\bar{v}_2 = \left(\frac{5}{12}\text{ ft}\right)(54.55\ rad/s) = 22.73\ ft/s$ $\bar{v} = 22.7\ ft/s \leftarrow$ ◀

+← x COMPONENTS: $0 + 12t - Ft = \frac{W}{g}\bar{a}$

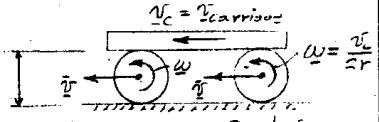
$0 + 12(6s) - F(6s) = \frac{18\ lb}{32.2}(22.73\ ft/s)$

$72 - 6F = 12.706$

$F = 9.88216$ $\underline{F = 9.88\ lb \rightarrow}$ ◀

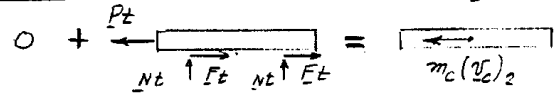
17.72

KINEMATICS



$\bar{v} = \frac{1}{2}v_c$

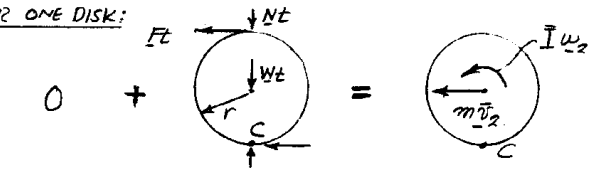
CARRIAGE:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

± COMPONENTS: $Pt - 2Ft = m_c(v_c)_2$ (1)

FOR ONE DISK:



+↻ MOMENTS ABOUT C: $(Ft)(2r) = \bar{I}\omega_2 + (m\bar{v}_2)r$

WE SUBSTITUTE: $\omega_2 = \dfrac{(v_c)_2}{2r}$, $\bar{v}_2 = \frac{1}{2}(v_c)_2$, $\bar{I} = \frac{1}{2}mr^2$

$(Ft)(2r) = \frac{1}{2}mr^2\left[\dfrac{(v_c)_2}{2r}\right] + m\left[\frac{1}{2}(v_c)_2\right]r$

$2Ft = \frac{3}{4}m(v_c)_2$ (2)

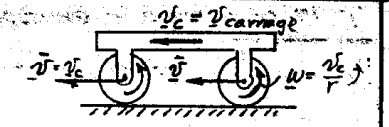
ADD EQS. (1) AND (2): $Pt = m_c(v_c)_2 + \frac{3}{4}m(v_c)_2$ (3)

DATA: $m_c = 6\ kg$; $m = 4\ kg$; $P = 10\ N$; $t = 2.5\ s$; $r = 75\ mm$

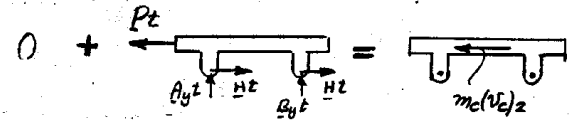
EQ (3): $(10)(2.5) = \left[6 + \frac{3}{4}(4)\right](v_c)_2 = 9(v_c)_2$

$(v_c)_2 = 2.778\ m/s$ $\underline{(v_c)_2 = 2.78\ m/s \leftarrow}$ ◀

17.73 KINEMATICS



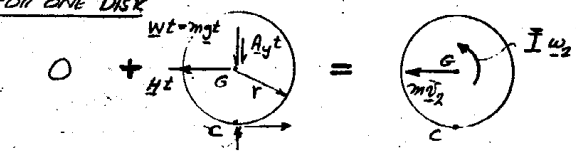
CARRIAGE



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\pm$ COMPONENTS: $Pt - 2Ht = m_c(v_c)_2$ (1)

FOR ONE DISK



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $(Ht)r = \bar{I}\omega_2 + (m\bar{v}_2)r$

WE SUBSTITUTE: $\omega_2 = \frac{(v_c)_2}{r}$, $\bar{v}_2 = (v_c)_2$, $\bar{I} = \frac{1}{2}mr^2$

$(Ht)r = \frac{1}{2}mr^2\left[\frac{(v_c)_2}{r}\right] + m(v_c)_2 r$

$Ht = \frac{3}{2}m(v_c)_2$ (2)

SUBSTITUTE FOR Ht INTO EQ(1): $Pt - 3m(v_c)_2 = m_c(v_c)_2$

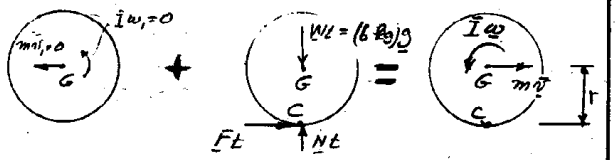
$Pt = (m_c + 3m)(v_c)_2$

DATA: $m_c = 6\,kg$, $m = 4\,kg$, $P = 10N$, $t = 2.5s$ $\Rightarrow$ $(10)(2.5) = [6 + 3(4)](v_c)_2$

$(v_c)_2 = 1.3889\,m/s$ $\quad (v_c)_2 = 1.389\,m/s$ ◀

17.74

PIPE: $m = 6\,kg$ $\quad \bar{I} = mr^2 = (6\,kg)r^2$ $\quad r = 80\,mm$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

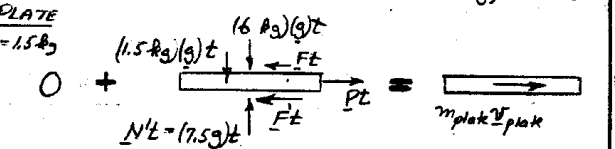
+↻ MOMENTS ABOUT C: $0 + 0 = m\bar{v}\,r - \bar{I}\omega$

$0 = m\bar{v}\,r - mr^2\omega$ $\quad \omega = \frac{\bar{v}}{r}$ (1)

$\pm\rightarrow$ X COMPONENTS: $Ft = m\bar{v}$ $\quad Ft = (6\,kg)\bar{v}$ (2)

PLATE

$m_{plate} = 1.5\,kg$



$F' = \mu_k N't = (0.20)(7.5)(g)t$

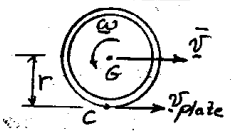
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\pm\rightarrow$ X COMPONENTS: $Pt - F't - Ft = m_{plate}v_{plate}$ (3)

$Pt - (0.20)(7.5g)t - 6\bar{v} = (1.5)v_{plate}$

$Pt - 1.5gt = 6\bar{v} + 1.5v_{plate}$ (4)

WE NOW ASSUME THAT PIPE ROLLS ON THE PLATE



$\bar{v} = v_C + v_{G/C} = [v_{plate}\rightarrow] + [r\omega\leftarrow]$

FROM EQ(1) $\omega = \frac{\bar{v}}{r}$ OR $r\omega = \bar{v}$

$\bar{v} = v_{plate} - \bar{v}$ $\quad v_{plate} = 2\bar{v}\rightarrow$ (5)

EQ(4): $Pt - 1.5gt = 6\bar{v} + 1.5(2\bar{v})$

$Pt - 1.5gt = 9\bar{v}$ (6)

(CONTINUED)

17.74 CONTINUED

DATA: $P = 25\,N$, $t = 0.75s$

EQ(6) $(25\,N)(0.75s) - 1.5(9.81\,m/s^2)(0.75s) = 9\bar{v}$

$\bar{v} = 0.8571\,m/s$ $\quad \bar{v} = 0.857\,m/s\rightarrow$

CHECK WHETHER PIPE ROLLS OR SLIDES ON PLATE.

EQ(3): $Ft = 6\bar{v}$ $\quad F(0.75) = 6(0.8571)$

$F = 6.857\,N$

AVAILABLE FRICTION: $F_{AVAIL.} = \mu_s N = (0.25)(6\,kg)g = 14.715\,N$

SINCE $F < F_{AVAILABLE}$, PIPE ROLLS WITHOUT SLIDING ON PLATE ◀

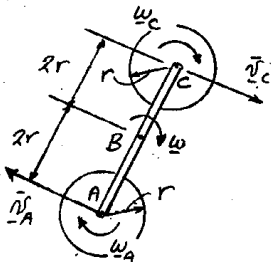
$\bar{v} = 0.857\,m/s\rightarrow$ ◀

EQ(1): $\omega = \frac{\bar{v}}{r} = \frac{0.8571\,m/s}{0.08\,m} = 10.714\,rad/s$ $\quad \omega = 10.71\,rad/s\,\circlearrowleft$ ◀

EQ(5): $v_{plate} = 2\bar{v} = 2(0.8571\,m/s)$ $\quad v_{plate} = 1.714\,m/s\rightarrow$ ◀

17.75 KINEMATICS OF MOTION

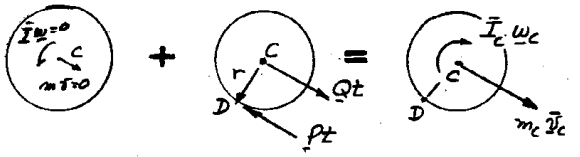
LET $\omega_{ABC} = \omega$



$\bar{v}_A = \bar{v}_C = (BC)\omega = 2r\omega$

SINCE GEARS A AND C ROLL ON THE FIXED GEAR B,

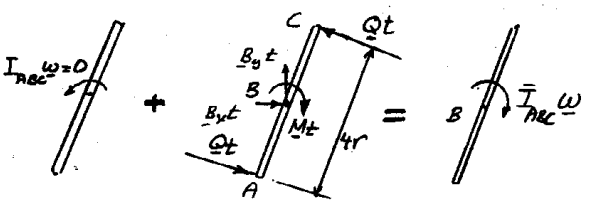
$\omega_A = \omega_C = \frac{\bar{v}_C}{r} = \frac{2r\omega}{r} = 2\omega$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT D: $0 + (Qt)r = m_c\bar{v}_c r + \bar{I}_C\omega_C$

$(Qt)r = m_c(2r\omega)r + \frac{1}{2}m_c r^2(2\omega)$ $\quad Qt = 3m_c r\omega$ (1)



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT B: $Mt - Qt(4r) = \bar{I}_{ABC}\omega$

$Mt - 4(Qt)r = \frac{1}{12}m_{ABC}(4r)^2\omega$

$Mt - 4(Qt)r = \frac{4}{3}m_{ABC}r^2\omega$ (2)

SUBSTITUTE FOR (Qt) FROM (1) INTO (2):

$Mt - 4(3m_c r\omega)r = \frac{4}{3}m_{ABC}r^2\omega$

$Mt = \frac{4}{3}r^2\omega(m_{ABC} + 9m_c)$ (3)

DATA: $t = 2.5s$, $r = \frac{2}{12}ft$, $m_{ABC} = \frac{4\,lb}{32.2\,ft/s^2}$, $m_c = \frac{2.5\,lb}{32.2\,ft/s^2}$

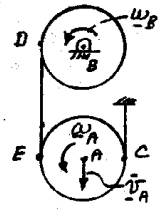
$\omega = 240\,rpm = 8\pi\,rad/s$

EQ(3): $M(2.5s) = \frac{4}{3}\left(\frac{2}{12}ft\right)^2(8\pi\,rad/s)\left[\frac{4}{32.2} + 9\left(\frac{2.5}{32.2}\right)\right]$

$2.5M = 0.76607$

$M = 0.3064\,lb\cdot ft$ $\quad \underline{M = 0.306\,lb\cdot ft}\,\circlearrowleft$ ◀

17.76



radii = r

KINEMATICS: INST. CENTER OF CYLINDER A IS AT C

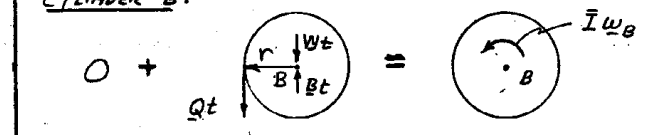
$\bar{v}_A = r\omega_A$

$\bar{v}_G = \bar{v}_D = 2r\omega_A$

CYLINDER B:

$a_B = \frac{\bar{v}_D}{r} = 2\omega_A$

CYLINDER B:



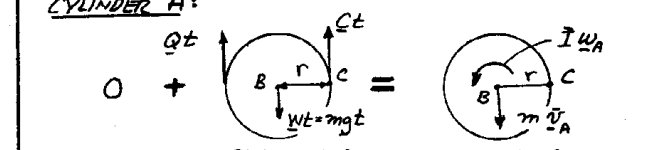
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $(Qt)r = \bar{I}\omega_B$

$(Qt)r = \frac{1}{2}mr^2(2\omega_A)$

$Qt = mr\omega_A$ (1)

CYLINDER A:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT C: $(mg)r - (Qt)2r = \bar{I}\omega_A + m\bar{v}_A r$

$mgr - (Qt)2r = \frac{1}{2}mr^2\omega_A + mr^2\omega_A$

$mg - 2Qt = \frac{3}{2}mr\omega_A$ (2)

SUBSTITUTE FOR Qt FROM EQ(1):

$mg - 2mr\omega_A = \frac{3}{2}mr\omega_A$

$\omega_A = \frac{2}{7}\frac{gt}{r}$

$\bar{v}_A = r\omega_A \qquad \bar{v}_A = \frac{2}{7}gt$

EQ(1): $Qt = mr\left(\frac{2}{7}\frac{gt}{r}\right)$

$Q = \frac{2}{7}mg \qquad Q = \frac{2}{7}W$

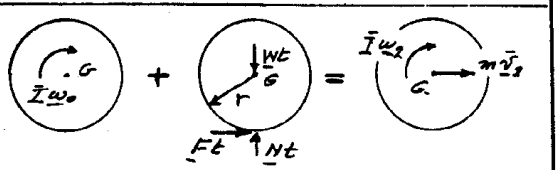
DATA $W = 2\text{ lb}$, $r = 5\text{ in.}$, $t = 2.5\text{ s}$

$\bar{v}_A = \frac{2}{7}gt = \frac{2}{7}(32.2\text{ ft/s}^2)(2.5\text{ s})$

(a) $\bar{v}_A = 23.0\text{ ft/s} \downarrow$ ◀

(b) TENSION IN CORD: $Q = \frac{2}{7}W = \frac{2}{7}(2\text{ lb})$ $Q = 6.00\text{ lb}$ ◀

17.77



SYST. MOMENTA$_1$ + SYST EXT IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↑ y COMPONENTS: $Nt - Wt = 0$ (1)

± x COMPONENTS: $0 + Ft = m\bar{v}_2$ (2)

+) MOMENTS ABOUT G: $\bar{I}\omega_0 - Ftr = \bar{I}\omega_2$ (3)

WE FIND: $N = W = mg \qquad F = \mu_k N = \mu_k mg$

EQ(2) $(\mu_k mg)t = m\bar{v}_2 \qquad \bar{v}_2 = \mu_k gt$ (4)

CONTINUED

17.77 CONTINUED

SUBSTITUTE $F = \mu_k mg$ AND $\bar{I} = \frac{2}{5}mr^2$

EQ(3): $\frac{2}{5}mr^2\omega_0 - (\mu_k mg)tr = \frac{2}{5}mr^2\omega_2$

SOLVE FOR ω_2 $\omega_2 = \omega_0 - \frac{5}{2}\frac{\mu_k gt}{r}$ (5)

SLIDING STOPS WHEN $\bar{v}_2 = r\omega_2$

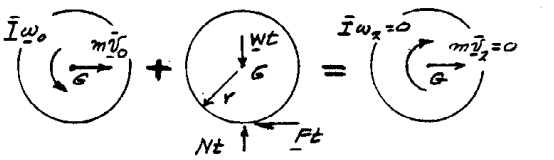
$\mu_k gt = r\omega_0 - \frac{5}{2}\mu_k gt$

$\frac{7}{2}\mu_k gt = r\omega_0 \qquad t = \frac{2}{7}\frac{r\omega_0}{\mu_k g}$ ◀

EQ(4): $\bar{v}_2 = \mu_k gt = \mu_k g\left(\frac{2}{7}\frac{r\omega_0}{\mu_k g}\right)$ $\bar{v}_2 = \frac{2}{7}r\omega_0 \rightarrow$ ◀

EQ(5): $\omega_2 = \omega_0 - \frac{5}{2}\frac{\mu_k g}{r}\left(\frac{2}{7}\frac{r\omega_0}{\mu_k g}\right)$ $\omega_2 = \frac{2}{7}\omega_0 \curvearrowright$ ◀

17.78



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↑ y COMPONENTS: $Nt - Wt = 0$ (1)

± x COMPONENTS: $m\bar{v}_0 - Ft = 0$ (2)

+) MOMENTS ABOUT G: $\bar{I}\omega_0 - Ftr = 0$ (3)

WE FIND: $N = W = mg \qquad F = \mu_k mg$

FROM EQ(2): $Ft = m\bar{v}_0$ SUBSTITUTE INTO EQ(3) WITH $\bar{I} = \frac{2}{5}mr^2$:

$\frac{2}{5}mr^2\omega_0 - (m\bar{v}_0)r = 0$

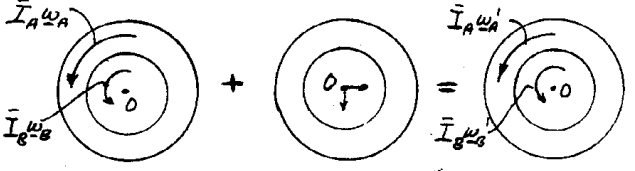
$\omega_0 = \frac{5}{2}\frac{\bar{v}_0}{r}$ ◀

FROM EQ(2): $t = \frac{m\bar{v}_0}{F} = \frac{m\bar{v}_0}{\mu_k mg} \qquad t = \frac{\bar{v}_0}{\mu_k g}$ ◀

17.79

FOR A DISK $m = \rho V = \rho\pi r^2 t$

$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi r^2 t)r^2 = \frac{1}{2}\rho\pi t r^4$



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) ΣM_O: $\bar{I}_A\omega_A + \bar{I}_B\omega_B = \bar{I}_A\omega_A' + \bar{I}_B\omega_B'$

$\frac{1}{2}\rho\pi t r_A^4\omega_A + \frac{1}{2}\rho\pi t r_B^4\omega_B = \frac{1}{2}\rho\pi t r_A^4\omega_A' + \frac{1}{2}\rho\pi t r_B^4\omega_B'$

$r_A^4\omega_A + r_B^4\omega_B = r_A^4\omega_A' + r_B^4\omega_B'$

FINAL VELOCITIES: $\omega_A' = \omega_B' = \omega_F$

$r_A^4\omega_A + r_B^4\omega_B = (r_A^4 + r_B^4)\omega_F$ (1)

(a) DATA: $r_A = 0.150\text{ m} \qquad r_B = 0.100\text{ m}$

$\omega_A = 400\text{ rpm} \qquad \omega_B = 0$

EQ(1): $(0.150\text{ m})^4(400\text{ rpm}) = \left[(0.150\text{ m})^4 + (0.100\text{ m})^4\right]\omega_F$

$\omega_F = 334.02\text{ rpm} \qquad \underline{\omega_F = 334\text{ rpm}}\ \curvearrowright$ ◀

(CONTINUED)

17.79 CONTINUED

(b) ENERGY LOSS

DISK A: $m_A = \rho \pi t r_A^2$ } $\frac{m_A}{m_B} = \frac{r_A^2}{r_B^2}$ (2)
DISK B: $m_B = \rho \pi t r_B^2$ }

DATA: $m_A = 4\ kg$; $r_A = 0.150\ m$; $r_B = 0.100\ m$

EQ.(2): $\frac{4\ kg}{m_B} = \frac{(0.150\ m)^2}{(0.100\ m)^2}$ $\quad m_B = 1.778\ kg$

INITIAL KINETIC ENERGY $\omega_A = 400\ rpm = 41.887\ rad/s$

$$T_1 = \frac{1}{2}\bar{I}_A\omega_A^2 = \frac{1}{2}\left[\frac{1}{2}(4\ kg)(0.150\ m)^2\right](41.887\ rad/s)^2$$

$$T_1 = 39.478\ J$$

FINAL KINETIC ENERGY $\omega_F = 334.02\ rpm = 34.978\ rad/s$

$$T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_F^2 = \frac{1}{2}\left[\frac{1}{2}(4\ kg)(0.150\ m)^2 + \frac{1}{2}(1.778\ kg)(0.1\ m)^2\right](34.978\ rad/s)$$

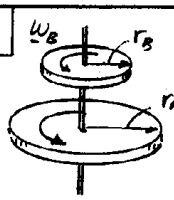
$$T_2 = 32.967\ J$$

ENERGY LOSS $\Delta T = T_2 - T_1 = 32.967\ J - 39.478\ J$

$$\Delta T = -6.51\ J$$ ◀

SEE SOLUTION OF PROB 17.80 FOR GENERAL EXPRESSION FOR ΔT. NOTE ΔT DEPENDS ONLY ON THE RELATIVE ANGULAR VELOCITY $\omega_{A/B} = \omega_A - \omega_B$.

17.80



FROM PROB 17.79

$$\bar{I}_A = \frac{1}{2}\rho\pi t r_A^4$$

$$\bar{I}_B = \frac{1}{2}\rho\pi t r_B^4$$

$$\therefore \bar{I}_B = \left(\frac{r_B}{r_A}\right)^4 \bar{I}_A = \gamma^4 \bar{I}_A$$

WHERE $\gamma = r_B/r_A$

FROM EQ.(1) OF PROB 17.79:

$$\omega_A + \left(\frac{r_B}{r_A}\right)^4 \omega_B = \left[1 + \left(\frac{r_B}{r_A}\right)^4\right]\omega_F$$

$$\omega_F = \frac{\omega_A + \left(\frac{r_B}{r_A}\right)^4 \omega_B}{1 + \left(\frac{r_B}{r_A}\right)^4} = \frac{\omega_A + \gamma^4\omega_B}{1+\gamma^4}$$

$$T_{INITIAL} = T_1 = \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}\bar{I}_B\omega_B^2 = \frac{1}{2}\bar{I}_A\left(\omega_A^2 + \gamma^4\omega_B^2\right)$$

$$T_{FINAL} = T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_F^2 = \frac{1}{2}\bar{I}_A(1+\gamma^4)\left(\frac{\omega_A + \gamma^4\omega_B}{1+\gamma^4}\right)^2$$

$$T_2 = \frac{1}{2}\bar{I}_A\frac{(\omega_A + \gamma^4\omega_B)^2}{1+\gamma^4}$$

$$\Delta T = T_2 - T_1 = \frac{1}{2}\bar{I}_A\frac{(\omega_A + \gamma^4\omega_B)^2}{1+\gamma^4} - \frac{1}{2}\bar{I}_A(\omega_A^2 + \gamma^4\omega_B^2)$$

$$= \frac{I_A}{2(1+\gamma^4)}\left[(\omega_A + \gamma^4\omega_B)^2 - (\omega_A^2 + \gamma^4\omega_B^2)(1+\gamma^4)\right]$$

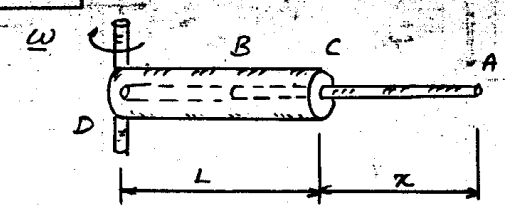
$$= \frac{I_A}{2(1+\gamma^4)}\left[\omega_A^2 + 2\gamma^4\omega_A\omega_B + \gamma^8\omega_B^2 - (\omega_A^2 + \gamma^4\omega_A^2 + \gamma^4\omega_B^2 + \gamma^8\omega_B^2)\right]$$

$$= \frac{I_A}{2(1+\gamma^4)}\left[\omega_A^2 - 2\omega_A\omega_B + \omega_B^2\right](-\gamma^4) = -\frac{1}{2}I_A\frac{\gamma^4}{1+\gamma^4}(\omega_A - \omega_B)^2$$

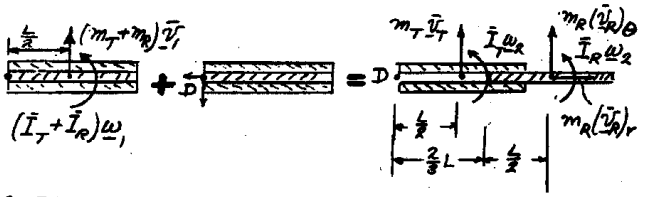
DATA: $\gamma = \frac{r_B}{r_A} = \frac{100\ mm}{150\ mm} = \frac{2}{3}$; $I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(4\ kg)(0.15\ m)^2 = 0.045\ kg\cdot m^2$

$$\Delta T = -\frac{1}{2}\frac{(2/3)^4}{1+(2/3)^4}(0.045)(\omega_{B/A})^2 \qquad \Delta T = -(3.71\times10^{-3}\ kg\cdot m^2)\omega_{B/A}^2$$ ◀

17.81



INITIALLY ROD AB IS ENTIRELY INSIDE TUBE, $x = 0$



SYST. MOMENTA, + SYST. EXT. IMP.$_{1-2}$ = SYST. MOMENTA

+↻ MOMENTS ABOUT SHAFT AT D

$$+(\bar{I}_T + \bar{I}_R)\omega_1 + (m_T + m_R)\bar{v}_1\left(\frac{L}{2}\right) = m_T\bar{v}_T\left(\frac{L}{2}\right) + m_R(\bar{v}_R)_B\left(\frac{2}{3}L + \frac{1}{2}L\right) + (\bar{I}_T + \bar{I}_R)\omega_2 \quad (1)$$

KINEMATICS: NOTE ROD MAY HAVE A RADIAL COMPONENT OF VELOCITY *.

$$\bar{v}_1 = \frac{L}{2}\omega_1 \qquad \bar{v}_T = \frac{L}{2}\omega_2 \qquad (\bar{v}_R)_B = \left(\frac{2}{3}L + \frac{1}{2}L\right)\omega_2 = \frac{7}{6}L\omega_2$$

SINCE ROD AND TUBE ARE EACH OF MASS m,

$$m_T = m_R = m \qquad \bar{I}_T = \bar{I}_R = \frac{1}{12}mL^2$$

SUBSTITUTE INTO (1):

$$2\left(\frac{1}{12}mL^2\right)\omega_1 + 2m\left(\frac{L}{2}\right)^2\omega_1 = m\left(\frac{L}{2}\right)^2\omega_2 + m\left(\frac{7}{6}L\right)^2\omega_2 + 2\left(\frac{1}{12}mL^2\right)\omega_2$$

$$\left(\frac{1}{6} + \frac{1}{2}\right)mL^2\omega_1 = \left(\frac{1}{4} + \frac{49}{36} + \frac{1}{6}\right)mL^2\omega_2$$

$$\frac{2}{3}\omega_1 = \frac{9+49+6}{36}\omega_2$$

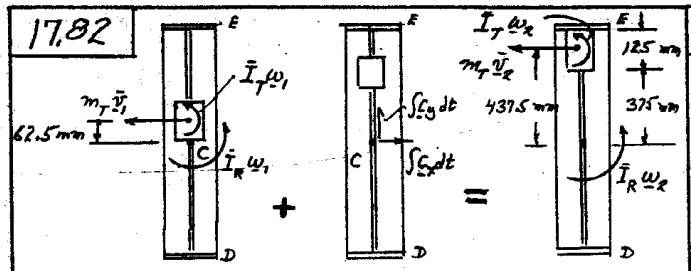
$$\frac{2}{3}\omega_1 = \frac{64}{36}\omega_2$$

$$\omega_2 = \frac{2}{3}\left(\frac{36}{64}\right)\omega_1$$

$$\omega_2 = \frac{3}{8}\omega_1$$ ◀

* THE RADIAL COMPONENT OF THE VELOCITY OF THE ROD WILL NOT BE DETERMINED HERE. IT WILL BE DETERMINED IN PROB. 17.91.

17.82



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$$m_T\bar{v}_1(0.0625\text{m}) + \bar{I}_T\omega_1 + \bar{I}_R\omega_1 = m_T\bar{v}_2(0.4375\text{m}) + \bar{I}_T\omega_2 + \bar{I}_R\omega_2 \quad (1)$$

DATA. TUBE: $m_T = 1.6\ \text{kg}$ $\bar{I}_T = 0.0025\ \text{kg}\cdot\text{m}^2$

ROD AND BRACKET: $\bar{I}_R = 0.30\ \text{kg}\cdot\text{m}^2$

KINEMATICS.

$\bar{v}_1 = (0.0625\text{m})\omega_1$ $\bar{v}_2 = (0.4375\text{m})\omega_2$ $\omega_1 = 5\ \text{rad/s}$

(a) SUBSTITUTE INTO EQ (1):

$$(1.6\text{kg})(0.0625\text{m})^2(5\ \text{rad/s}) + (0.0025\ \text{kg}\cdot\text{m}^2 + 0.30\ \text{kg}\cdot\text{m}^2)(5\ \text{rad/s}) = (1.6\text{kg})(0.4375\text{m})^2\omega_2 + (0.0025\ \text{kg}\cdot\text{m}^2 + 0.30\ \text{kg}\cdot\text{m}^2)\omega_2$$

$$0.03125 + 1.5125 = (0.30625 + 0.3025)\omega_2$$

$$1.54375 = 0.60875\,\omega_2$$

$$\omega_2 = 2.5359 \qquad \omega_2 = 2.54\ \text{rad/s} \blacktriangleleft$$

(b) ENERGY LOSS: $\Delta T = T_1 - T_2$

$$T_1 = \tfrac{1}{2}m_T\bar{v}_1^2 + \tfrac{1}{2}\bar{I}_T\omega_1^2 + \tfrac{1}{2}\bar{I}_R\omega_1^2 \qquad \bar{v}_1 = (0.0625)\omega_1$$

$$= \tfrac{1}{2}(1.6\text{kg})(0.0625\text{m})^2\omega_1^2 + \tfrac{1}{2}(0.0025\ \text{kg}\cdot\text{m}^2 + 0.30\ \text{kg}\cdot\text{m}^2)\omega_1^2$$

$$= \tfrac{1}{2}(0.00625 + 0.0025 + 0.30)\omega_1^2 = \tfrac{1}{2}(0.30875\ \text{kg}\cdot\text{m}^2)\omega_1^2$$

$$= \tfrac{1}{2}(0.30875)(5)^2 = 3.8594\ \text{J}$$

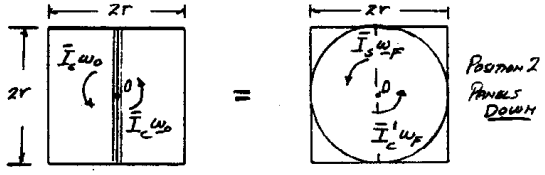
$$T_2 = \tfrac{1}{2}m_T\bar{v}_2^2 + \tfrac{1}{2}\bar{I}_T\omega_2^2 + \tfrac{1}{2}\bar{I}_R\omega_2^2 \qquad \bar{v}_2 = (0.375 + 0.0625)\omega = 0.4375\omega$$

$$= \tfrac{1}{2}(1.6\text{kg})(0.4375\text{m})^2\omega_2^2 + \tfrac{1}{2}(0.0025\ \text{kg}\cdot\text{m}^2 + 0.30\ \text{kg}\cdot\text{m}^2)\omega_2^2$$

$$= \tfrac{1}{2}(0.60875\ \text{kg}\cdot\text{m}^2)(2.5359\ \text{rad/s})^2 = 1.9574\ \text{J}$$

ENERGY LOSS: $\Delta T = T_1 - T_2 = 3.8594\ \text{J} - 1.9574\ \text{J}$ $\Delta T = 1.902\ \text{J} \blacktriangleleft$

17.83



POSITION 1: PANELS UP

POSITION 2 PANELS DOWN

SQUARE PLATE: $m = \rho t(2r)^2$; $\bar{I}_s = \frac{1}{6}m(2r)^2 = \frac{1}{6}\rho t(2r)^4 = \frac{16}{6}\rho t r^4$

SEMICIRCULAR PANELS

TWO PANELS UP: $m = \rho t\pi r^2$ $\bar{I}_c$ IS EQUIVALENT TO

$$\bar{I}_c = \frac{1}{4}mr^2 = \frac{1}{4}\rho t\pi r^4$$

TWO PANELS DOWN: $m = \rho t\pi r^2$; $\bar{I}_c'$ IS EQUIVALENT TO.

$$\bar{I}_c' = \frac{1}{2}mr^2 = \frac{1}{2}\rho t\pi r^4$$

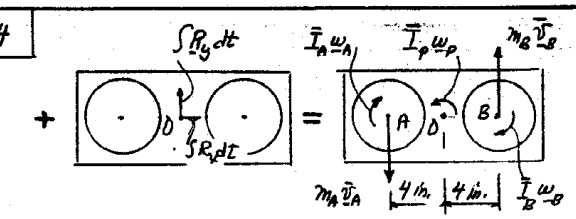
WE SHALL WRITE $+\circlearrowleft\Sigma M_O$, EXT. IMPULSES HAVE NO MOMENT

$+\circlearrowleft$ MOMENTS ABOUT O: $\bar{I}_c\omega_0 + \bar{I}_c\omega_0 = \bar{I}_s\omega_F + \bar{I}_c'\omega_F$

$$\left[\frac{16}{6}\rho t r^4 + \frac{1}{4}\rho t\pi r^4\right]\omega_0 = \left[\frac{16}{6}\rho t r^4 + \frac{1}{2}\rho t\pi r^4\right]\omega_F$$

$$\omega_F = \frac{\frac{16}{6} + \frac{\pi}{4}}{\frac{16}{6} + \frac{\pi}{2}}\omega_0 = \frac{32 + 3\pi}{22 + 6\pi}\omega_0 \qquad \omega_F = 0.815\,\omega_0 \blacktriangleleft$$

17.84



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+2 MOMENTS ABOUT O:

$$0 + 0 = \bar{I}_A\omega_A + \bar{I}_B\omega_B - m_A\bar{v}_A\left(\tfrac{4}{12}\text{ft}\right) - m_B\bar{v}_B\left(\tfrac{4}{12}\text{ft}\right) - \bar{I}_p\omega_p \quad (1)$$

DATA: $W_A = W_B = 10\ \text{lb}$ $W_{\text{PLATFORM}} = 15\ \text{lb}$

$$\bar{I}_A = \bar{I}_B = \tfrac{1}{2}m_A r_A^2 = \tfrac{1}{2}\left(\frac{10\text{lb}}{g}\right)\left(\frac{3}{12}\text{ft}\right)^2 = 0.3125/g$$

$$\bar{I}_p = \tfrac{1}{12}m_p(a^2 + b^2) = \tfrac{1}{12}\left(\frac{15\text{lb}}{g}\right)\left[\left(\frac{16}{12}\text{ft}\right)^2 + \left(\frac{6}{12}\text{ft}\right)^2\right] = 2.5347/g$$

$\bar{v}_A = \bar{v}_B = \left(\frac{4}{12}\text{ft}\right)\omega_p$ $\omega_A = \omega_B$

SUBSTITUTE INTO EQ (1):

$$(0.3125/g)\omega_B + (0.3125/g)\omega_B - \left(\frac{10\text{lb}}{g}\right)\left(\frac{4}{12}\omega_p\right)\left(\frac{4}{12}\right) - \left(\frac{10\text{lb}}{g}\right)\left(\frac{4}{12}\omega_p\right)\left(\frac{4}{12}\right) - (2.5347/g)\omega_p = 0$$

$$(0.625/g)\omega_B - (4.7569/g)\omega_p = 0$$

$$\omega_B = 7.611\,\omega_p \quad (2)$$

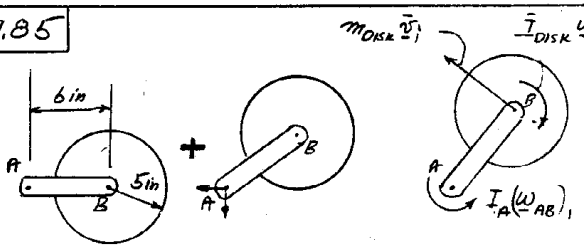
SINCE THE MOTOR SPEED IS 180 rpm, RELATIVE TO THE PLATE, WE HAVE $\omega_B + \omega_p = 180\ \text{rpm}$ (3)

$$7.611\omega_p + \omega_p = 180\ \text{rpm}$$

$$\omega_p = 20.903\ \text{rpm} \qquad \omega_p = 20.9\ \text{rpm} \circlearrowright \blacktriangleleft$$

$$\omega_B = 7.611\,\omega_p = 7.611(20.903\ \text{rpm}) \qquad \omega_A = \omega_B = 159.1\ \text{rpm} \circlearrowleft \blacktriangleleft$$

17.85



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowleft$ MOMENTS ABOUT A: $0 + 0 = I_A(\omega_{AB})_1 + m_{DISK}\bar{v}_1(AB) - \bar{I}_{DISK}\omega_1$ (1)

KINEMATICS: $\bar{v}_1 = (AB)(\omega_{AB})_1 = \left(\frac{6}{12}\text{ft}\right)(\omega_{AB})_1 = 0.5(\omega_{AB})_1$

DATA: $I_A = 0.032\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$$m_{DISK} = (10\ \text{lb})/32.2\ \text{ft/s}^2$$

$$\bar{I}_{DISK} = \tfrac{1}{2}m_{DISK}r^2 = \tfrac{1}{2}\frac{10\text{lb}}{32.2\ \text{ft/s}^2}\left(\frac{5}{12}\text{ft}\right)^2 = 0.02696\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

SUBSTITUTE INTO EQ(1):

$$0 = (0.032\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)(\omega_{AB})_1 + \frac{10\text{lb}}{32.2}\left(\frac{6}{12}\text{ft}\right)^2(\omega_{AB})_1 - (0.02696\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)\omega_1$$

$$(0.10964)(\omega_{AB})_1 = 0.02696\,\omega_1$$

$$(\omega_{AB})_1 = 0.2459\,\omega_1 \quad (2)$$

AT OPERATING SPEED: $\omega_1 + (\omega_{AB})_1 = 360\ \text{rpm}$ (3)

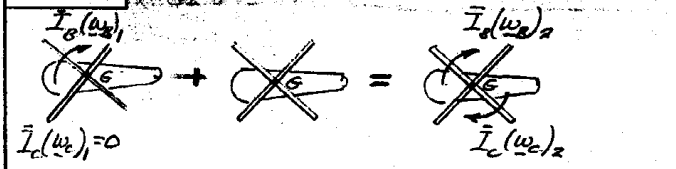
$$(1 + 0.2459)\omega_1 = 360\ \text{rpm}$$

$$\omega_1 = \omega_{DISK} = 288.95 \qquad \omega_{DISK} = 289\ \text{rpm} \circlearrowright \blacktriangleleft$$

EQ(2): $(\omega_{AB})_1 = \omega_{ARM} = (0.2459)(288.95) = 71.05$ $\omega_{ARM} = 71\ \text{rpm} \circlearrowleft \blacktriangleleft$

17.86

B = BLADES C = CAB



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT VERTICAL AXIS:

$$\bar{I}_B(\omega_B)_1 + 0 = \bar{I}_B(\omega_B)_2 + \bar{I}_C(\omega_C)_2 \quad (1)$$

DATA: $\bar{I}_B = 4\left[\frac{1}{3}(30\text{ kg})(5\text{ m})^2\right] = 1000\text{ kg}\cdot\text{m}^2$

$\bar{I}_C = 1200\text{ kg}\cdot\text{m}^2$

$(\omega_B)_1 = 200$ rpm $(\omega_B)_2 - (\omega_C)_2 = 300$ rpm

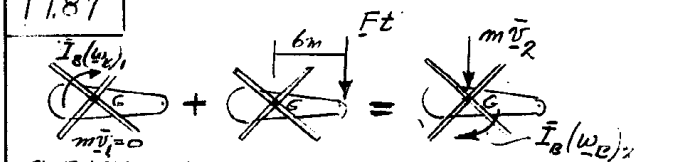
$(\omega_B)_2 = (\omega_C)_2 + 300$ rpm

EQ(1): $(1000)(200) = (1000)[(\omega_C)_2 + 300] + 1200(\omega_C)_2$

$-(1000)(100) = (1000+1200)(\omega_C)_2$

$(\omega_C)_2 = -45.454$ rpm $(\omega_C)_2 = 45.5$ rpm ◀

17.87



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$\bar{I}_B(\omega_B)_1 + (Ft)(6\text{ m}) = \bar{I}_B(\omega_B)_2 \quad (1)$$

+↑ COMPONENTS:

$$0 + Ft = m\bar{v}_2 \quad (2)$$

DATA: $\bar{I}_B = 4\left[\frac{1}{3}(30\text{ kg})(5\text{ m})^2\right] = 1000\text{ kg}\cdot\text{m}^2$

$m = 720\text{ kg} + 4(30\text{ kg}) = 840\text{ kg}$

$(\omega_B)_1 = 200$ rpm $= 20.944$ rad/s

$(\omega_B)_2 = 300$ rpm $= 31.416$ rad/s

EQ(1): $(1000)(20.944) + (Ft)(6) = (1000)(31.416)$

$6(Ft) = 10.4719\times10^3$

$Ft = 1745.3\text{ N}\cdot\text{s}$ (3)

SUBSTITUTE FOR Ft FROM (3) INTO (2)

$m\bar{v}_2 = Ft$

$(840\text{ kg})\bar{v}_2 = 1745.3\text{ N}\cdot\text{s}$

$\bar{v}_2 = 2.078$ m/s $\bar{v}_2 = 2.08$ m/s ◀

FOR $t = 12$ s, WE HAVE

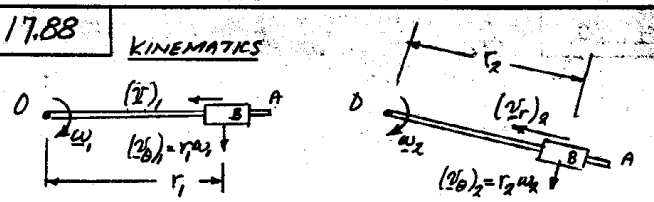
$Ft = 1745.3\text{ N}\cdot\text{s}$

$F(12\text{ s}) = 1745.3\text{ N}\cdot\text{s}$

$F = 145.4$ N ◀

17.88

KINEMATICS



KINETICS SINCE THE MOMENTS OF ALL FORCES ABOUT O ARE ZERO, WE HAVE

$$(H_O)_1 = (H_O)_2: \quad I_{OA}\omega_1 + m_B(v_B)_1 r_1 = I_{OA}\omega_2 + m(v_B)_2 r_2$$

$$(I_{OA} + m_B r_1^2)\omega_1 = (I_{OA} + m_B r_2^2)\omega_2 \quad (1)$$

GIVEN DATA: $I_{OA} = 0.35\text{ kg}\cdot\text{m}^2$ $m_B = 3$ kg

$\omega_1 = 1.8$ rad/s $(v_r)_1 = 1.5$ m/s

$r_1 = 0.5$ m $(v_B)_1 = r_1\omega_1 = (0.5)(1.8) = 0.9$ m/s

EQ(1): $[0.35 + 3(0.5)^2](1.8) = [0.35 + 3r_2^2]\omega_2$

$$1.980 = (0.35 + 3r_2^2)\omega_2 \quad (2)$$

CONSERVATION OF ENERGY. SINCE $V_1 = V_2$, WE HAVE $T_1 = T_2$.

$$T_1 = \tfrac{1}{2}I_{OA}\omega_1^2 + \tfrac{1}{2}m_B(v_B)_1^2 + \tfrac{1}{2}m_B(v_r)_1^2 \quad (3)$$

$$= \tfrac{1}{2}(0.35)(1.8)^2 + \tfrac{1}{2}(3)(0.9)^2 + \tfrac{1}{2}(3)(1.5)^2$$

$T_1 = 5.157$ J

$$T_2 = \tfrac{1}{2}I_{OA}\omega_2^2 + \tfrac{1}{2}m_B(v_B)_2^2 + \tfrac{1}{2}m_B(v_r)_2^2 \quad (4)$$

$$= \tfrac{1}{2}(0.35)\omega_2^2 + \tfrac{1}{2}(3)(r_2\omega_2)^2 + \tfrac{1}{2}(3)(v_r)_2^2$$

$$T_2 = \tfrac{1}{2}(0.35 + 3r_2^2)\omega_2^2 + (v_r)_2^2$$

$T_1 = T_2$: $5.157 = \tfrac{1}{2}(0.35 + 3r_2^2)\omega_2^2 + (v_r)_2^2$ (5)

SUBSTITUTE FOR ω_2 FROM EQ(2) INTO EQ(5):

$$5.157 = \tfrac{1}{2}(0.35 + 3r_2^2)\frac{(1.980)^2}{(0.35+3r_2^2)^2} + (v_r)_2^2$$

FOR $r_{MINIMUM}$ WE HAVE $(v_r)_2 = 0$

$$5.157 = \frac{1}{2}\frac{(1.980)^2}{(0.35+3r_2^2)} \quad (6)$$

$$(0.35 + 3r_2^2) = 0.380105 \quad (7)$$

$r_2^2 = 10.035\times10^{-3}\text{ m}^2$ $r_2 = 0.10017\text{ m} = 100.2$ mm ◀

SUBSTITUTING $(0.35 + 3r_2^2) = 0.380105$ FROM EQ(7) INTO EQ(2).

$1.980 = 0.380105\,\omega_2$ $\omega_2 = 5.21$ rad/s ◀

17.89

SEE SOLUTION OF PROB. 17.88

FOR $r_2 = r_{MIN} = 300\text{ mm} = 0.3\text{ m}$, EQ(2) BECOMES

$1.980 = [0.35 + 3(0.3)^2]\omega_2$ $\omega_2 = 3.1936$ rad/s

$(v_B)_2 = r_2\omega_2 = (0.3)(3.1936)$ $(v_B)_2 = 0.95806$ m/s

CONSERVATION OF ENERGY

EQ(3): $T_1 = \tfrac{1}{2}I_{OA}\omega_1^2 + \tfrac{1}{2}m_B(v_B)_1^2 + \tfrac{1}{2}m_B(v_r)_1^2$

$= \tfrac{1}{2}(0.35)(1.8)^2 + \tfrac{1}{2}(3)(0.9)^2 + \tfrac{1}{2}(3)(v_r)_1^2$

$T_1 = 1.782 + 1.5(v_r)_1^2$

SINCE $r_2 = r_{MIN}$, $(v_r)_2 = 0$

EQ(4) $T_2 = \tfrac{1}{2}I_{OA}\omega_2^2 + \tfrac{1}{2}m_B(v_B)_2^2$

$= \tfrac{1}{2}(0.35)(3.1936)^2 + \tfrac{1}{2}(3)(0.95806)^2 = 3.1617$ J

SINCE $V_1 = V_2$,

$T_1 = T_2$: $1.782 + (v_r)_1^2 = 3.1617$

$(v_r)_1^2 = 1.3797$ $(v_r)_1 = 1.175$ m/s ◀

17.90

SEE SOLUTION OF PROB. 17.82 WHERE WE FOUND THAT THE ENERGY LOST DURING THE PLASTIC IMPACT OF THE TUBE AND END E IS

$$\Delta T = 1.902\ \text{J}$$

BEFORE IMPACT THIS ENERGY WAS IN THE FORM OF KINETIC ENERGY ASSOCIATED WITH THE RADIAL VELOCITY OF THE TUBE, THAT IS WITH THE VELOCITY OF THE TUBE RELATIVE TO THE ROD. WE WRITE

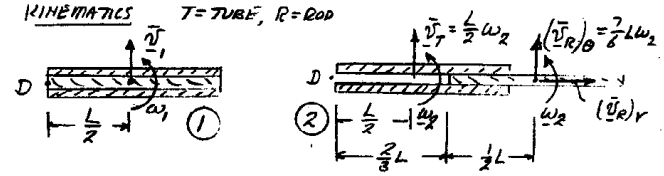
$$\tfrac{1}{2} m_T (v_{T/R})^2 = 1.902\ \text{J}$$

$$\tfrac{1}{2}(1.6\ \text{kg})(v_{T/R})^2 = 1.902\ \text{J} \qquad v_{T/R} = 1.542\ \text{m/s} \blacktriangleleft$$

17.91

FROM SOLUTION OF PROB. 17.81, WE HAVE $\omega_2 = \frac{3}{8}\omega_1$

KINEMATICS T = TUBE, R = ROD



CONSERVATION OF ENERGY

$$T_1 = \tfrac{1}{2}(m_T + m_R)\bar{v}_1^2 + \tfrac{1}{2}\bar{I}_T\omega_1^2 + \tfrac{1}{2}\bar{I}_R\omega_1^2$$

$$= \tfrac{1}{2}(m+m)\left(\tfrac{L}{2}\omega_1\right)^2 + 2\left[\tfrac{1}{2}\left(\tfrac{1}{12}mL^2\right)\right]\omega_1^2$$

$$= \tfrac{1}{2}\left(\tfrac{2}{3}mL^2\right)\omega_1^2 = \tfrac{1}{3}mL^2\omega_1^2$$

$$T_2 = \tfrac{1}{2}m_T\bar{v}_T^2 + \tfrac{1}{2}\bar{I}_T\omega_2^2 + \tfrac{1}{2}m_R\left[(\bar{v}_R)_\theta^2 + (\bar{v}_R)_r^2\right] + \tfrac{1}{2}\bar{I}_R\omega_2^2$$

$$= \tfrac{1}{2}m\left(\tfrac{L}{2}\omega_2\right)^2 + 2\left[\tfrac{1}{2}\left(\tfrac{1}{12}mL^2\right)\right]\omega_2^2 + \tfrac{1}{2}m\left[\left(\tfrac{7}{8}L\omega_2\right)^2 + (\bar{v}_R)_r^2\right]$$

$$T_2 = \tfrac{29}{96}mL^2\omega_2^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

RECALLING THAT $\omega_2 = \frac{3}{8}\omega_1$,

$$T_2 = \tfrac{29}{96}mL^2\left(\tfrac{3}{8}\omega_1\right)^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2 = \tfrac{1}{8}mL^2\omega_1^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

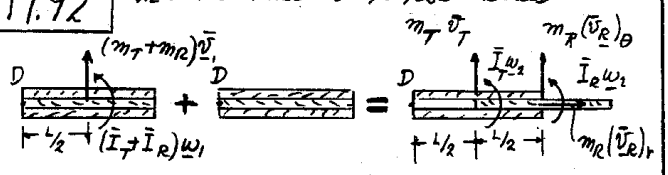
SINCE $V_1 = V_2$,

$$T_1 = T_2: \quad \tfrac{1}{3}mL^2\omega_1^2 = \tfrac{1}{8}mL^2\omega_1^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

$$(\bar{v}_R)_r^2 = \tfrac{5}{12}L^2\omega_1^2 \qquad (\bar{v}_R)_r = 0.645\,L\omega_1 \blacktriangleleft$$

17.92

VIEW FROM ABOVE. T = TUBE R = ROD



SYST. MOMENTA$_1$ + SYST. EXT. IMP.$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT SHAFT AT D.

$$(m_T+m_R)\bar{v}_1\tfrac{L}{2} + (\bar{I}_T+\bar{I}_R)\omega_1 = m_T\bar{v}_T\left(\tfrac{L}{2}\right) + m_R(\bar{v}_R)_\theta L + (\bar{I}_T+\bar{I}_R)\omega_2 \qquad (1)$$

KINEMATICS. $\bar{v}_1 = \frac{L}{2}\omega_1$, $\bar{v}_T = \frac{L}{2}\omega_2$, $(\bar{v}_R)_\theta = L\omega_2$

ROD AND TUBE ARE EACH OF MASS m.

$$m_T = m_R = m \qquad \bar{I}_R = \bar{I}_T = \tfrac{1}{12}mL^2$$

(CONTINUED)

17.92 CONTINUED

SUBSTITUTE INTO EQ(1):

$$2m\left(\tfrac{1}{2}\omega_1\right)\tfrac{1}{2} + 2\left(\tfrac{1}{12}mL^2\right)\omega_1 = m\left(\tfrac{1}{2}\omega_2\right)\tfrac{1}{2} + m(L\omega_2)L + 2\left(\tfrac{1}{12}mL^2\right)\omega_2$$

$$\tfrac{2}{3}mL^2\omega_1 = \tfrac{17}{12}mL^2\omega_2 \qquad \omega_2 = \tfrac{8}{17}\omega_1 \blacktriangleleft$$

CONSERVATION OF ENERGY

$$T_1 = \tfrac{1}{2}(m_T+m_R)\bar{v}_1^2 + \tfrac{1}{2}(\bar{I}_T+\bar{I}_R)\omega_1^2$$

$$= \tfrac{1}{2}(2m)\left(\tfrac{1}{2}\omega_1\right)^2 + \tfrac{1}{2}\left(\tfrac{2}{12}mL^2\right)\omega_1^2 = \tfrac{1}{3}mL^2\omega_1^2$$

$$T_2 = \tfrac{1}{2}m_T\bar{v}_T^2 + \tfrac{1}{2}(\bar{I}_T+\bar{I}_R)\omega_2^2 + \tfrac{1}{2}m_R\left[(\bar{v}_R)_\theta^2 + (\bar{v}_R)_r^2\right]$$

$$= \tfrac{1}{2}m\left(\tfrac{1}{2}\omega_2\right)^2 + \tfrac{1}{2}\left(\tfrac{2}{12}mL^2\right)\omega_2^2 + \tfrac{1}{2}m\left[(L\omega_2)^2 + (\bar{v}_R)_r^2\right]$$

$$= \tfrac{17}{24}mL^2\omega_2^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

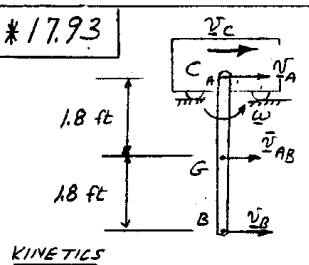
$$= \tfrac{17}{24}mL^2\left(\tfrac{8}{17}\omega_1\right)^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2 = \tfrac{8}{51}mL^2\omega_1^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

SINCE $V_1 = V_2$,

$$T_1 = T_2: \quad \tfrac{1}{3}mL^2\omega_1^2 = \tfrac{8}{51}mL^2\omega_1^2 + \tfrac{1}{2}m(\bar{v}_R)_r^2$$

$$(\bar{v}_R)_r^2 = \tfrac{6}{17}(L\omega_1)^2 \qquad (\bar{v}_R)_r = \sqrt{\tfrac{6}{17}}\,L\omega_1 = 0.5944\,L\omega_1 \blacktriangleleft$$

*17.93



KINEMATICS $\bar{v}_C = \bar{v}_A$

$$\bar{v}_C \rightarrow = \bar{v}_{AB} \rightarrow + (1.8\ \text{ft})\omega \leftarrow$$

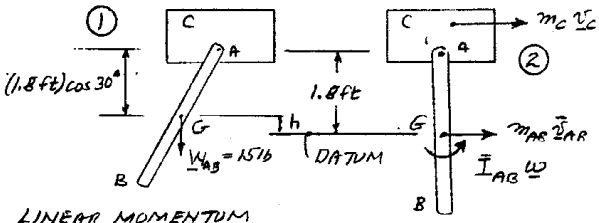
$$v_C = v_{AB} - 1.8\omega \qquad (1)$$

$AB = 3.6$ ft

$W_C = 25$ lb

$W_{AB} = 15$ lb

KINETICS



LINEAR MOMENTUM →

$$0 = m_C v_C + m_{AB}\bar{v}_{AB}$$

$$\bar{v}_{AB} = -\frac{m_C}{m_{AB}}v_C = -\frac{(25\ \text{lb})/g}{(15\ \text{lb})/g}v_C, \qquad \bar{v}_{AB} = -\tfrac{5}{3}v_C \qquad (2)$$

SUBSTITUTE INTO EQ(1): $v_C = -\frac{5}{3}v_C - 1.8\omega$

$$\tfrac{8}{3}v_C = -1.8\omega \qquad v_C = -0.675\omega \qquad (3)$$

SUBSTITUTE INTO EQ(2): $\bar{v}_{AB} = -\frac{5}{3}(-0.675\omega)$

$$\bar{v}_{AB} = 1.125\omega \qquad (4)$$

CONSERVATION OF ENERGY

$T_1 = 0$, $V_1 = W_{AB}\,h = (15\ \text{lb})(1.8\ \text{ft})(1-\cos 30°) = 3.6173\ \text{lb·ft}$, $V_2 = 0$

$$T_2 = \tfrac{1}{2}m_C v_C^2 + \tfrac{1}{2}m_{AB}\bar{v}_{AB}^2 + \tfrac{1}{2}\bar{I}_{AB}\omega^2$$

$$= \tfrac{1}{2}\left(\tfrac{25}{g}\right)(-0.675\omega)^2 + \tfrac{1}{2}\left(\tfrac{15}{g}\right)(1.125\omega)^2 + \tfrac{1}{2}\left[\tfrac{1}{12}\left(\tfrac{15}{g}\right)(3.6)^2\right]\omega^2$$

$$= \frac{\omega^2}{2g}(11.391 + 18.984 + 16.2) = \frac{\omega^2}{2(32.2)}(46.575) = 0.72321\,\omega^2$$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 3.6173 = 0.72321\,\omega^2$

$$\omega^2 = 5.0017 \qquad \omega = 2.2365\ \text{rad/s}$$

VELOCITY OF C: EQ.(3) $\quad v_C = -0.675(2.2365)$ $\qquad \bar{v}_C = 1.510\ \text{ft/s} \leftarrow \blacktriangleleft$

VELOCITY OF B: $\quad \bar{v}_B = \bar{v}_C + (3.6)\omega \rightarrow = (1.510\ \text{ft/s} \leftarrow) + \left[3.6(2.2365) \rightarrow\right]$

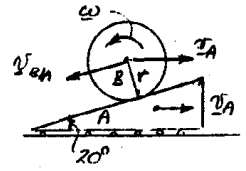
$$v_B = 1.510 \leftarrow + 8.051 \qquad \bar{v}_B = 6.54\ \text{ft/s} \rightarrow \blacktriangleleft$$

*17.94

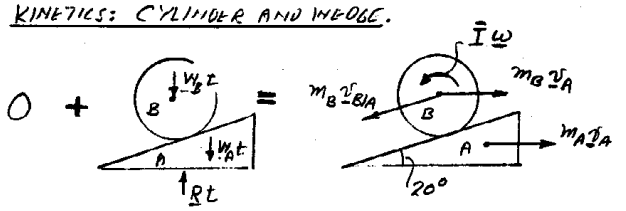
KINEMATICS WE RESOLVE $\underline{v}_B$ INTO $\underline{v}_A$ AND AND A COMPONENT $\underline{v}_{B/A}$ PARALLEL TO THE INCLINE:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

WHERE SINCE CYLINDER B ROLLS ON WEDGE A: $v_{B/A} = r\omega$ (1)



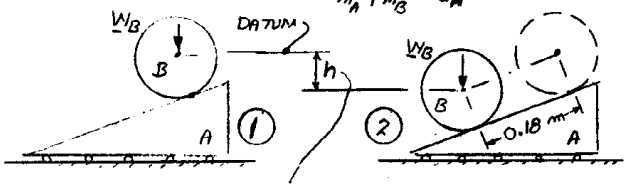
KINETICS: CYLINDER AND WEDGE.



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\pm\rightarrow$ X COMPONENTS: $0 + 0 = (m_A + m_B)v_A - m_B v_{B/A}\cos 20°$

$$v_A = \frac{m_B}{m_A + m_B} v_{B/A}\cos 20° \quad (2)$$



$h = (0.18\ m)\sin 20°$

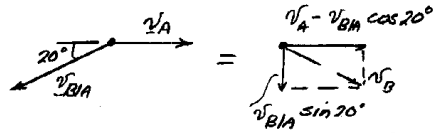
CONSERVATION OF ENERGY: $T_1 = 0$ $V_1 = 0$

$V_2 = -W_B h = -m_B g h$

$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2} m_B v_B^2$

$T_1 + V_1 = T_2 + V_2$: $0 = -m_B g h + \frac{1}{2} m_A v_A^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2} m_B v_B^2$ (3)

VELOCITY OF B: $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$$v_B^2 = (v_A - v_{B/A}\cos 20°)^2 + (v_{B/A}\sin 20°)^2 \quad (4)$$

DATA: $m_A = 3\ kg$, $m_B = 4\ kg$, $r = 0.12\ m$ $\bar{I} = \frac{1}{2} m_B r^2$

EQ(1): $v_{B/A} = r\omega = 0.12\omega$ (5)

EQ(2): $v_A = \frac{m_B}{m_A + m_B} v_{B/A}\cos 20° = \frac{4}{7}(0.12\omega)\cos 20°$

$$v_A = (0.06857\cos 20°)\omega \quad (6)$$

SUBSTITUTE FOR $v_{B/A}$ AND v_A FROM (5) AND (6) INTO (4)

$$v_B^2 = \left[(0.06857\cos 20°)\omega - (0.12\omega)\cos 20°\right]^2 + \left[0.12\omega\sin 20°\right]^2$$

$$= \left[-0.048327\omega\right]^2 + \left[0.041042\omega\right]^2$$

$$= 2.3355\times10^{-3}\omega^2 + 1.6845\times10^{-3}\omega^2$$

$$v_B^2 = 4.020\times10^{-3}\omega^2 \quad (7)$$

SUBSTITUTE INTO EQ (3)

$$0 = -(4\ kg)(9.81)(0.18\ m)\sin 20° + \frac{1}{2}(3\ kg)(0.06857\cos 20°)^2\omega^2 + \frac{1}{2}\left[\frac{1}{2}(4\ kg)(0.12\ m)^2\right]\omega^2 + \frac{1}{2}(4\ kg)(4.020\times10^{-3})\omega^2$$

$$0 = -2.4158 + (6.228\times10^{-3} + 14.4\times10^{-3} + 8.04\times10^{-3})\omega^2$$

(CONTINUED)

*17.94 CONTINUED

$$0 = -2.4158 + (28.668\times10^{-3})\omega^2$$

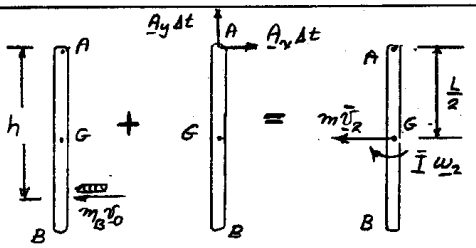
$\omega^2 = 84.248$ $\omega = 9.1798\ rad/s$

$\omega = 9.18\ rad/s$ ◀

EQ(6): $v_A = (0.06857\cos 20°)\omega = (0.06857\cos 20°)(9.1798)$

$v_A = 0.59149\ m/s$ $\underline{v}_A = 591\ mm/s \rightarrow$ ◀

17.95



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+⟲ MOMENTS ABOUT A: $m_B v_0 h = \bar{I}\omega_2 + m\bar{v}_2\frac{L}{2}$ (1)

$\pm\rightarrow$ X COMPONENTS: $m_B v_0 - A_x\Delta t = m\bar{v}_2$ (2)

KINEMATICS: ROTATION ABOUT A $\bar{v}_2 = \frac{L}{2}\omega_2$ (3)

DATA: $m_B = 0.030\ kg$, $v_0 = 400\ m/s$ $\Delta t = 1\ ms$

$m = 4\ kg$ $L = 0.75\ m$ $h = 0.6\ m$

EQ(1): $m_B v_0 h = \frac{1}{12} mL^2\omega_2 + m\left(\frac{L}{2}\right)^2\omega_2$

$$m_B v_0 h = \frac{1}{3} mL^2\omega_2 \quad (4)$$

$(0.030\ kg)(400\ m/s)(0.6\ m) = \frac{1}{3}(4\ kg)(0.75\ m)^2\omega_2$

$\omega_2 = 9.6\ rad/s$ $\underline{\omega}_2 = 9.6\ rad/s$ ↻

EQ(3) $\bar{v}_2 = \frac{L}{2}\omega_2 = \frac{1}{2}(0.75\ m)(9.6\ rad/s)$ $\underline{\bar{v}}_2 = 3.6\ m/s \leftarrow$ ◀

EQ(2) $(0.030\ kg)(400\ m/s) - A_x(1\ ms) = (4\ kg)(3.6\ m/s)$

$A_x = -2400\ N$ $\underline{A}_x = 2400\ N \leftarrow$ ◀

17.96

SEE SOLUTION OF PROB. 17.95 FOR DERIVATION OF EQS. (1) AND (2).

FOR $A_x = 0$:

EQ(1): $m_B v_0 h = \bar{I}\omega_2 + m\bar{v}_2\frac{L}{2}$

EQ(2): $m_B v_0 = m\bar{v}_2$

DIVIDE EQ(1) BY EQ(2):

$$h = \frac{\bar{I}\omega_2 + m\bar{v}_2\frac{L}{2}}{m\bar{v}_2}$$

FOR ROTATION ABOUT A: $\bar{v}_2 = \frac{L}{2}\omega_2$

$$h = \frac{\frac{1}{12} mL^2\omega_2 + m\left(\frac{L}{2}\right)^2\omega_2}{m\frac{L}{2}\omega_2} = \frac{\frac{1}{12} + \frac{1}{4}}{\frac{1}{2}} L = \frac{1/3}{1/2} L$$

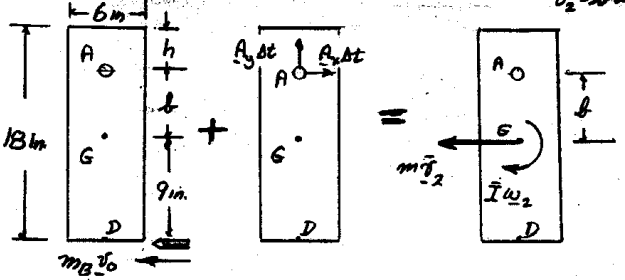
(a) $h = \frac{2}{3} L = \frac{2}{3}(750\ mm)$ $h = 500\ mm$ ◀

(b) EQ(2): $\bar{v}_2 = \frac{m_B}{m} v_0 = \frac{0.030\ kg}{4\ kg}(400\ m/s)$

$= 3.00\ m/s$ $\underline{\bar{v}}_2 = 3.00\ m/s \leftarrow$ ◀

17.97

ROTATION ABOUT A: $\bar{v}_2 = b\omega_2$



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$m_B = \frac{0.12}{9}$; $m = \frac{8}{9}$ lb NOTE: $h = 9\text{ in.} - b$ (1)

$I = \frac{1}{12}\frac{8}{9}\left[\left(\frac{18}{12}\text{ ft}\right)^2 + \left(\frac{6}{12}\text{ ft}\right)^2\right] = \frac{1.667}{9}$ $\quad v_0 = 900$ ft/s

+) MOMENTS ABOUT A: $m_B v_0\left(\frac{9}{12}\text{ ft} + b\right) = \bar{I}\omega_2 + m\bar{v}_2 b$

$\frac{0.12}{9}(900)(0.75 + b) = \frac{1.667}{9}\omega_2 + \frac{8}{9}b^2\omega_2$

$81 + 108b = (1.667 + 8b^2)\omega_2$ (2)

$\pm$ x COMPONENTS: $m_B v_0 - A_x\Delta t = m\bar{v}_2$

$\frac{0.12}{9}(900) - A_x\Delta t = \frac{8}{9}b\omega_2$ (3)

For $A_x = 0$: $108 = 8b\omega_2$ (4)

DIVIDE EQ(2) BY EQ(4): $0.75 + b = \frac{1.667 + 8b^2}{8b}$

$6b + 8b^2 = 1.667 + 8b^2$

$6b = 1.667$ $\quad b = 0.2778\text{ ft} = 3.333\text{ in.}$

(a) RECALL EQ(1): $h = 9 - b = 6.667$ in. $\quad h = 6.67$ in. ◀

EQ(4): $108 = 8(0.2778\text{ ft})\omega_2$ $\quad \omega_2 = 48.6$ rad/s

(b) VELOCITY OF D:

$v_D = \left(\frac{9}{12}\text{ ft} + b\right)\omega_2 = (0.75 + 0.2778)(48.6\text{ rad/s}) = 49.95$ ft/s

$\underline{v}_D = 50.0$ ft/s ◀

17.98

SEE PROB 17.97 FOR DERIVATION OF EQUATIONS

WE NOW HAVE $h = 5$ in. THUS, $b = 9 - 5 = 4$ in $= 0.333$ ft

EQ(2): $81 + 108(0.333) = \left[1.667 + 8(0.333)^2\right]\omega_2$

$81 + 36 = (2.555)\omega_2$

$\omega_2 = 45.78$ rad/s

(a) VELOCITY OF D

$v_D = \left(\frac{9}{12}\text{ ft} + \frac{4}{12}\text{ ft}\right)(\omega_2) = \frac{13}{12}(45.78) = 49.595$ ft/s

$\underline{v}_D = 49.6$ ft/s ◀

(b) IMPULSIVE REACTION AT A: GIVEN, $\Delta t = 0.002$ s

EQ(3): $\frac{0.12}{32.2}(900) - A_x(0.002) = \frac{8}{32.2}(0.333)(45.78)$

$3.354 - A_x(0.002) = 3.791$

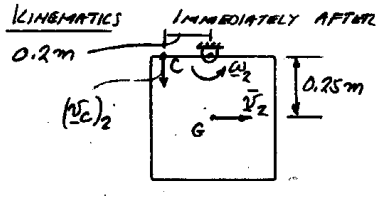
$A_x = -218.6$ lb $\quad \underline{A}_x = 219$ lb ◀

17.99

VELOCITY OF SPHERE AT C

$(v_C)_1 = \sqrt{2gy} = \sqrt{2(9.81\text{ m/s}^2)(0.25\text{ m})} = 2.2147$ m/s

KINEMATICS IMMEDIATELY AFTER IMPACT IN TERMS OF ω_2



$\bar{v}_2 = 0.25\omega_2$

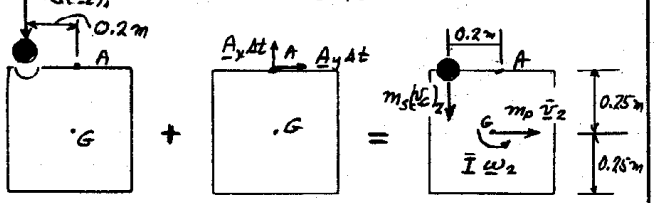
$(\underline{v}_C)_2 = 0.2\omega_2$

(CONTINUED)

17.99 CONTINUED

$m_s = 2$ kg $\quad m_p = 8$ kg

$\bar{I} = \frac{1}{6}m_p(0.5\text{ m})^2 = \frac{1}{6}(8)(0.5)^2 = 0.3333$ kg·m^2



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT A:

$m_s(v_C)_1(0.2\text{ m}) + 0 = m_s(\underline{v}_C)_2(0.2\text{ m}) + \bar{I}\omega_2 + m_p\bar{v}_2(0.25\text{ m})$

$(2\text{ kg})(2.2147\text{ m/s})(0.2\text{ m}) = (2\text{ kg})(0.2\omega_2)(0.2\text{ m}) + 0.33333\omega_2 + (8\text{ kg})(0.25\text{ m})^2\omega_2$

$0.88589 = (0.08 + 0.33333 + 0.500)\omega_2$

$\omega_2 = 0.96995$ rad/s $\quad \underline{\omega}_2 = 0.96995$ rad/s ↻

$\bar{v}_2 = (0.25\text{ m})\omega_2 = (0.25\text{ m})(0.96995\text{ rad/s})$

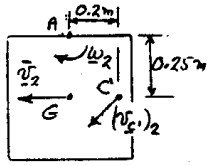
$\bar{v}_2 = 0.24249$ m/s $\quad \underline{\bar{v}}_2 = 242$ mm/s ◀

17.100

VELOCITY OF SPHERE AT C'.

$(v_{C'})_1 = \sqrt{2gy} = \sqrt{2(9.81\text{ m/s}^2)(0.5\text{ m})} = 3.1321$ m/s

KINEMATICS IMMEDIATELY AFTER IMPACT IN TERMS OF ω_2



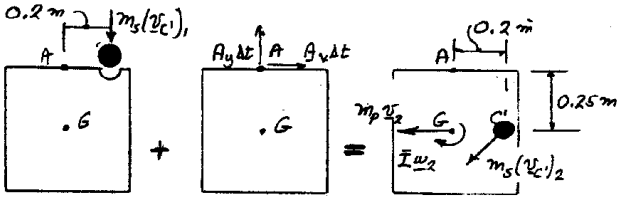
$AC' = \sqrt{(0.2)^2 + (0.25)^2} = 0.32016$ m

$(\underline{v}_{C'})_2 = AC'\omega_2$

$= 0.32016\omega_2$ ◀

$\bar{v}_2 = 0.25\omega_2$

KINETICS $m_s = 2$ kg, $m_p = 8$ kg, $\bar{I} = \frac{1}{6}m_p(0.5\text{ m})^2 = \frac{1}{6}(8)(0.5)^2 = 0.3333$ kg·m^2



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT A:

$m_s(v_{C'})_1(0.2\text{ m}) + 0 = m_s(v_{C'})_2(AC') + \bar{I}\omega_2 + m_p\bar{v}_2(0.25\text{ m})$

$(2\text{ kg})(3.1321\text{ m/s})(0.2\text{ m}) = (2\text{ kg})(0.32016\omega_2)(0.32016\text{ m}) + 0.3333\omega_2 + (8\text{ kg})(0.25\text{ m})^2\omega_2$

$1.25284 = (0.2050 + 0.3333 + 0.500)\omega_2$

$\omega_2 = 1.2066$ rad/s

$\bar{v}_2 = (0.25\text{ m})\omega_2 = (0.25\text{ m})(1.2066\text{ rad/s})$

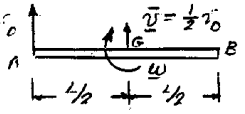
$= 0.3016$ m/s

$\underline{\bar{v}}_2 = 302$ mm/s ◀

17.101

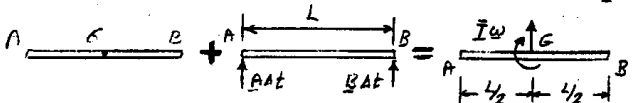
KINEMATICS: IMMEDIATELY AFTER ENGAGEMENT OF HOOK, ROD AB ROTATES ABOUT B

$\bar{v}_A = \bar{v}_0$; $\bar{v} = \frac{1}{2}\bar{v}_0$; $\omega = \frac{v_0}{L}$ ↻



KINETICS.

$m\bar{v} = m\frac{\bar{v}_0}{2}$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST MOMENTA$_2$

+↻ MOMENTS ABOUT B: $(A\,\Delta t)L = \bar{I}\omega + (m\frac{v_0}{2})(\frac{L}{2})$

$(A\,\Delta t)L = \frac{1}{12}mL^2(\frac{v_0}{L}) + \frac{1}{4}mv_0L = \frac{1}{3}mv_0L$

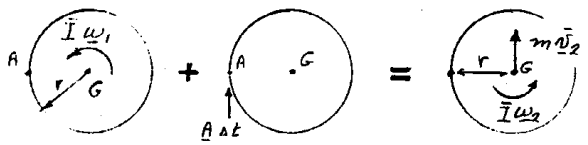
$A\,\Delta t = \frac{1}{3}mv_0$ ↑ ◀

+↑ y COMPS: $(A\,\Delta t) + (B\,\Delta t) = \frac{1}{2}mv_0$

$B\,\Delta t = \frac{1}{6}mv_0$ ↑ ◀

17.102

PLASTIC IMPACT: $(v_A)_2 = 0$, AND $\bar{v}_2 = r\omega_2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A:

$\bar{I}\omega_1 + 0 = \bar{I}\omega_2 + m\bar{v}_2 r$

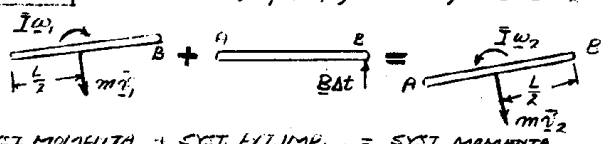
$\frac{1}{2}mr^2\omega_1 = \frac{1}{2}mr^2\omega_2 + m(r\omega_2)r$

$\frac{1}{2}mr^2\omega_1 = \frac{3}{2}mr^2\omega_2$ $\quad \omega_2 = \frac{1}{3}\omega_1$ ↻ ◀

$\bar{v}_2 = r\omega_2 = \frac{1}{3}r\omega_1$ $\quad \bar{v}_2 = \frac{1}{3}r\omega_1$ ↑ ◀

17.103

ROTATING ABOUT A, $\bar{v}_1 = \frac{L}{2}\omega_1$, ABOUT B, $\bar{v}_2 = \frac{L}{2}\omega_2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST MOMENTA$_2$

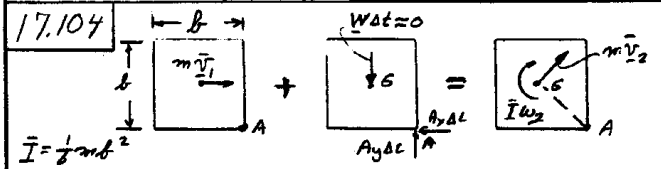
+↻ MOMENTS ABOUT B:

$m\bar{v}_1\frac{L}{2} - \bar{I}\omega_1 = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

$m(\frac{L}{2}\omega_1)\frac{L}{2} - \frac{1}{12}mL^2\omega_1 = m(\frac{L}{2}\omega_2)\frac{L}{2} + \frac{1}{12}mL^2\omega_2$

$\frac{1}{6}\omega_1 = \frac{1}{3}\omega_2$ $\quad \omega_2 = \frac{1}{2}\omega_1$ ↻ ◀

17.104



$\bar{I} = \frac{1}{6}mb^2$

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ M_A: $m\bar{v}_1\frac{b}{2} + 0 = \bar{I}\omega_2 + m\bar{v}_2(AG)$

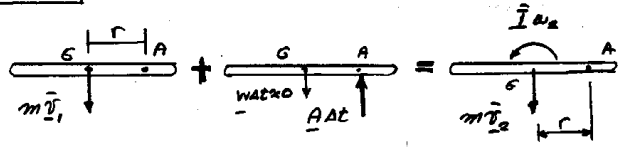
BLOCK ROTATES ABOUT A: $AG = b/\sqrt{2}$; $\bar{v}_2 = (\frac{b}{\sqrt{2}})\omega_2$

$m\bar{v}_1\frac{b}{2} = \frac{1}{6}mb^2\omega_2 + m(\frac{b}{\sqrt{2}})^2\omega_2$

$\frac{1}{2}\bar{v}_1 = \frac{2}{3}b\omega_2$ $\quad \omega_2 = \frac{3}{4}\frac{\bar{v}_1}{b}$ ↻ ◀

$\bar{v}_2 = \frac{b}{\sqrt{2}}\omega_2 = \frac{b}{\sqrt{2}}\cdot\frac{3}{4}\frac{\bar{v}_1}{b}$ $\quad \bar{v}_2 = \frac{3\sqrt{2}}{8}\bar{v}_1$, ∠45° ◀

17.105



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A:

$m\bar{v}_1 r + 0 = \bar{I}\omega_2 + m\bar{v}_2 r$ (1)

SINCE IMPACT IS PLASTIC, FINAL MOTION IS A ROTATION ABOUT A. THUS, $\bar{v}_2 = r\omega_2$

SUBSTITUTING $\omega_2 = \frac{\bar{v}_2}{r}$ AND $\bar{I} = m\bar{k}^2$ INTO (1)

$m\bar{v}_1 r = m\bar{k}^2\frac{\bar{v}_2}{r} + m\bar{v}_2 r$

SOLVING FOR $\bar{v}_2$, WE HAVE

$\bar{v}_2 = \bar{v}_1\frac{r^2}{r^2 + \bar{k}^2}$ ↓ ◀

17.106

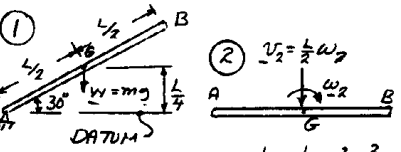
CONSERVATION OF ENERGY

POSITION 1: $T_1 = 0$, $V_1 = mg\frac{L}{4}$

POSITION 2: $V_2 = 0$

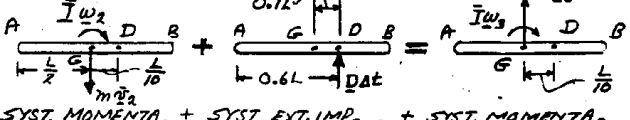
$T_2 = \frac{1}{2}m(\frac{1}{2}\omega_2)^2 + \frac{1}{2}(\frac{1}{12}mL^2)\omega_2^2$

$T_2 = \frac{1}{6}mL^2\omega_2^2$



$T_1 + V_1 = T_2 + V_2$: $0 + mg\frac{L}{4} = \frac{1}{6}mL^2\omega_2^2$ $\quad \omega_2 = \sqrt{3g/2L}$

PRINCIPLE OF IMPULSE – MOMENTUM.



SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2-3}$ + SYST. MOMENTA$_3$

+↻ MOMENTS ABOUT D:

$\bar{I}\omega_2 - m\bar{v}_2(\frac{L}{10}) = \bar{I}\omega_3 + m\bar{v}_3(\frac{L}{10})$ (1)

KINEMATICS: ROTATION ABOUT A, $\bar{v}_2 = \frac{L}{2}\omega_2$

ROTATION ABOUT D, $\bar{v}_3 = \frac{L}{10}\omega_3$

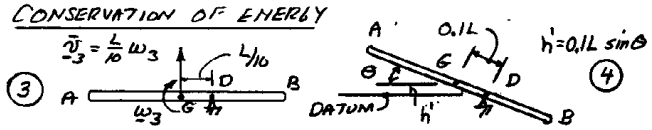
SUBSTITUTE INTO (1):

$\frac{1}{12}mL^2\omega_2 - m(\frac{L}{2}\omega_2)\frac{L}{10} = \frac{1}{12}mL^2\omega_3 + m(\frac{L}{10}\omega_3)\frac{L}{10}$

$(\frac{1}{12} - \frac{1}{20})\omega_2 = (\frac{1}{12} + \frac{1}{100})\omega_3$

$\omega_3 = \frac{5}{14}\omega_2 = \frac{5}{14}\sqrt{3g/2L}$ $\quad \omega_3 = 0.437\sqrt{g/L}$ ↻ ◀

CONSERVATION OF ENERGY



$V_3 = 0$, $T_3 = \frac{1}{2}m(\frac{L}{10}\omega_3)^2 + \frac{1}{2}(\frac{1}{12}mL^2)\omega_3^2 = \frac{14}{200}mL^2\omega_3^2 = \frac{14}{200}mL^2(\frac{5}{14}\sqrt{\frac{3g}{2L}})^2 = \frac{mgL}{112}$

$T_4 = 0$, $V_4 = mgh' = mg(0.1L)\sin\theta$

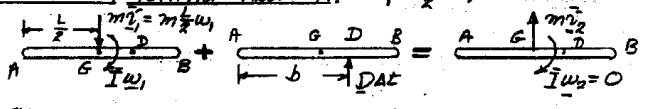
$T_3 + V_3 = T_4 + V_4$: $\frac{mgL}{112} + 0 = 0 + \frac{mgL}{10}\sin\theta$

$\sin\theta = \frac{10}{112}$ $\quad \theta = 5.12°$ ◀

MAKRON Books
CORTESIA
Marketing Universitário
(0xx11) 3845-3105
(0xx11) 3842-6729
fax: (0xx11) 3845-6095

17.107

ROTATION ABOUT A. $\bar{v}_1 = \frac{L}{2}\omega_1$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT D.

$$\bar{I}\omega_1 - m\left(\tfrac{L}{2}\omega_1\right)\left(b-\tfrac{L}{2}\right) + 0 = m\bar{v}_2\left(b-\tfrac{L}{2}\right)$$

$$\left(\tfrac{1}{12}mL^2 + \tfrac{1}{4}mL^2 - \tfrac{1}{2}mLb\right)\omega_1 = m\bar{v}_2\left(b-\tfrac{L}{2}\right)$$

$$\left(\tfrac{1}{3}L^2 - \tfrac{1}{2}Lb\right)\omega_1 = \left(b-\tfrac{L}{2}\right)\bar{v}_2$$

MULTIPLY BY $\frac{3}{L}$: $\left(1-\frac{3}{2}\frac{b}{L}\right)L\omega_1 = \left(3\frac{b}{L}-\frac{3}{2}\right)\bar{v}_2$ (1)

ELASTIC IMPACT: $e = 1$

VELOCITY OF POINT D BEFORE IMPACT = $v_D = b\omega_1$

SINCE $e = 1$, VELOCITY OF D HAS THE SAME MAGNITUDE AFTER IMPACT. SINCE $\omega_2 = 0$, WE HAVE

$$\bar{v}_2 = v_D' = b\omega_1$$

SUBSTITUTE INTO EQ(1): $\left(1-\frac{3}{2}\frac{b}{L}\right)L\omega_1 = \left(3\frac{b}{L}-\frac{3}{2}\right)b\omega_1$

$$1 - \tfrac{3}{2}\tfrac{b}{L} = 3\left(\tfrac{b}{L}\right)^2 - \tfrac{3}{2}\tfrac{b}{L}$$

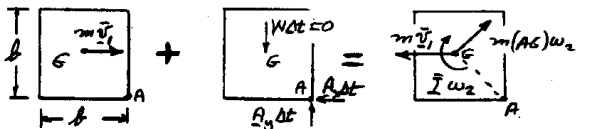
$$1 = 3\left(\tfrac{b}{L}\right)^2 \qquad \frac{b}{L} = \frac{1}{\sqrt{3}} \qquad b = \frac{L}{\sqrt{3}}$$ ◀

NOTE: FOR $b < L/\sqrt{3}$, ROD BOUNCES WITH ↻ ANG. VELOCITY. FOR $L/\sqrt{3} < b < \frac{2}{3}L$, ROD BOUNCES WITH ↺ ANG. VELOCITY. FOR $b \geq \frac{2}{3}L$, THERE IS AN ADDITIONAL IMPULSIVE REACTION AT A.

17.108

SINCE $e = 1$, AFTER IMPACT THE VELOCITY OF A WILL BE $\bar{v}_A = \bar{v}_1 \leftarrow$. THE VELOCITY $\bar{v}_2$ OF G AFTER IMPACT IS:

$$\bar{v}_2 = \bar{v}_A + \bar{v}_{G/A} = [v_1 \leftarrow] + [(AG)\omega_2 \angle 45°] \quad (1)$$



+↻ M_A: $m\bar{v}_1\frac{b}{2} + 0 = \bar{I}\omega_2 + m(AG)^2\omega_2 - m\bar{v}_1\frac{b}{2}$

$AG = \frac{b}{\sqrt{2}}$ $\quad \bar{I} = \frac{1}{6}mb^2$

THUS: $m\bar{v}_1 b = \frac{1}{6}mb^2\omega_2 + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$

$$v_1 b = \tfrac{2}{3}b^2\omega_2 \qquad \omega_2 = \tfrac{3}{2}\frac{\bar{v}_1}{b}\circlearrowright$$ ◀

VELOCITY OF G: EQ(1): $\bar{v}_2 = [\bar{v}_1 \leftarrow] + \left[\frac{b}{\sqrt{2}}\left(\frac{3}{2}\frac{\bar{v}_1}{b}\right)\angle 45°\right]$

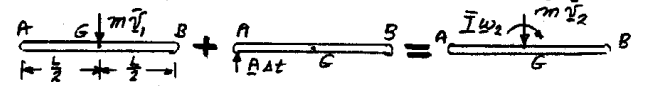
$$\bar{v}_2 = [\bar{v}_1 \leftarrow] + \left[\tfrac{3}{2\sqrt{2}}\bar{v}_1 \angle 45°\right] = [\bar{v}_1 \leftarrow] + \left[\tfrac{3}{4}\bar{v}_1 \rightarrow\right] + \left[\tfrac{3}{4}\bar{v}_1 \uparrow\right]$$

$$\bar{v}_2 = \left[\tfrac{1}{4}\bar{v}_1 \leftarrow\right] + \left[\tfrac{3}{4}\bar{v}_1 \uparrow\right] \qquad \bar{v}_2 = 0.791\,\bar{v}_1 \,\angle 71.6°$$ ◀

17.109

(a) ROD STRIKES SUPPORT AT A.

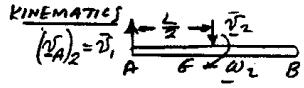
$(v_A)_1 = \bar{v}_1 \downarrow$; SINCE $e = 1$, $(v_A)_2 = \bar{v}_1 \uparrow$



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $m\bar{v}_1\frac{L}{2} = \bar{I}\omega_2 + m\bar{v}_2\frac{L}{2}$ (1)

KINEMATICS



$$\bar{v}_2 = (v_A)_2\uparrow + \tfrac{L}{2}\omega_2\downarrow$$

$$+\downarrow \quad \bar{v}_2 = \tfrac{L}{2}\omega_2 - \bar{v}_1 \quad (2)$$

(CONTINUED)

17.109 CONTINUED

SUBSTITUTE $\bar{v}_2 = \frac{L}{2}\omega_2 - \bar{v}_1$ INTO (1)

$$m\bar{v}_1\tfrac{L}{2} = \tfrac{1}{12}mL^2\omega_2 + m\left(\tfrac{L}{2}\omega_2 - \bar{v}_1\right)\tfrac{L}{2}$$

$$v_1 L = \tfrac{1}{3}L^2\omega_2 \qquad \omega_2 = 3\frac{\bar{v}_1}{L}\circlearrowright$$ ◀

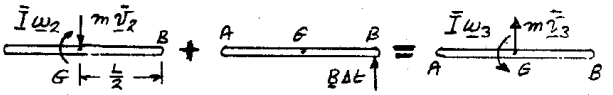
EQ(2) $\bar{v}_2 = \frac{L}{2}\left(3\frac{\bar{v}_1}{L}\right) - \bar{v}_1 = \frac{3}{2}\bar{v}_1 - \bar{v}_1$ $\qquad \bar{v}_2 = \frac{1}{2}\bar{v}_1 \downarrow$ ◀

FOR USE IN PART (b)

$$(v_B)_2\downarrow = \bar{v}_2\downarrow + \tfrac{L}{2}\omega_2\downarrow = \tfrac{1}{2}\bar{v}_1 + \tfrac{L}{2}\left(3\tfrac{\bar{v}_1}{L}\right) \qquad (v_B)_2 = 2\bar{v}_1\downarrow \quad (3)$$

(b) ROD STRIKES SUPPORT B.

EQ(3): $(v_B)_2 = 2\bar{v}_1\downarrow$; SINCE $e = 1$, $(v_B)_3 = 2\bar{v}_1\uparrow$

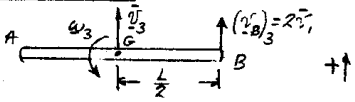


SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2-3}$ = SYST. MOMENTA$_3$

+↻ MOMENTS ABOUT B

$$\bar{I}\omega_2 - m\bar{v}_2\left(\tfrac{L}{2}\right) = -\bar{I}\omega_3 + m\bar{v}_3\left(\tfrac{L}{2}\right) \quad (4)$$

KINEMATICS



$$\bar{v}_3 = (v_B)_3\uparrow + \tfrac{L}{2}\omega_3\downarrow$$

$$+\uparrow \quad \bar{v}_3 = 2\bar{v}_1 - \tfrac{L}{2}\omega_3 \quad (5)$$

FROM PART (a) WE HAVE: $\omega_2 = 3\frac{\bar{v}_1}{L}$ AND $\bar{v}_2 = \frac{1}{2}\bar{v}_1$

SUBSTITUTE INTO EQ(4).

$$\tfrac{1}{12}mL^2\left(3\tfrac{\bar{v}_1}{L}\right) - m\left(\tfrac{1}{2}\bar{v}_1\right)\tfrac{L}{2} = -\tfrac{1}{12}mL^2\omega_3 + m\left(2\bar{v}_1 - \tfrac{L}{2}\omega_3\right)\tfrac{L}{2}$$

$$0 = -\tfrac{1}{3}mL^2\omega_3 + m\bar{v}_1 L$$

$$\omega_3 = 3\frac{\bar{v}_1}{L} \qquad \omega_3 = 3\frac{\bar{v}_1}{L}\circlearrowleft$$ ◀

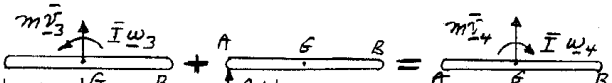
EQ(5): $\bar{v}_3 = 2\bar{v}_1 - \frac{L}{2}\omega_3 = 2\bar{v}_1 - \frac{L}{2}\left(3\frac{\bar{v}_1}{L}\right)$ $\qquad \bar{v}_3 = \frac{1}{2}\bar{v}_1\uparrow$ ◀

FOR USE IN PART (c)

$$(v_A)_3\downarrow = \bar{v}_3\uparrow + \tfrac{L}{2}\omega_3\downarrow = \tfrac{1}{2}\bar{v}_1\uparrow + \tfrac{L}{2}\left(3\tfrac{\bar{v}_1}{L}\right)\downarrow \qquad (v_A)_3 = \bar{v}_1\downarrow \quad (6)$$

(c) ROD AGAIN STRIKES SUPPORT A.

EQ(6): $(v_A)_3 = \bar{v}_1\downarrow$; SINCE $e = 1$, $(v_A)_4 = \bar{v}_1\uparrow$

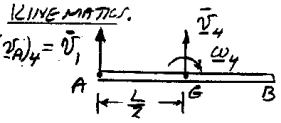


SYST. MOMENTA$_3$ + SYST. EXT. IMP$_{3-4}$ = SYST. MOMENTA$_4$

+↻ MOMENTS ABOUT A:

$$\bar{I}\omega_3 + m\bar{v}_3\left(\tfrac{L}{2}\right) = -\bar{I}\omega_4 + m\bar{v}_4\left(\tfrac{L}{2}\right) \quad (7)$$

KINEMATICS.



$$\bar{v}_4 = (v_A)_4\uparrow + \tfrac{L}{2}\omega_4\downarrow$$

$$+\uparrow \quad \bar{v}_4 = \bar{v}_1 - \tfrac{L}{2}\omega_4 \quad (8)$$

FROM PART (b) WE HAVE: $\omega_3 = 3\frac{\bar{v}_1}{L}$ AND $\bar{v}_3 = \frac{1}{2}\bar{v}_1$

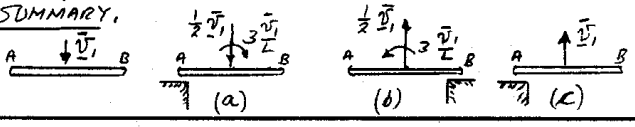
SUBSTITUTING INTO EQ(7):

$$\tfrac{1}{12}mL^2\left(3\tfrac{\bar{v}_1}{L}\right) + m\left(\tfrac{1}{2}\bar{v}_1\right)\tfrac{L}{2} = -\tfrac{1}{12}mL^2\omega_4 + m\left(\bar{v}_1 - \tfrac{L}{2}\omega_4\right)\tfrac{L}{2}$$

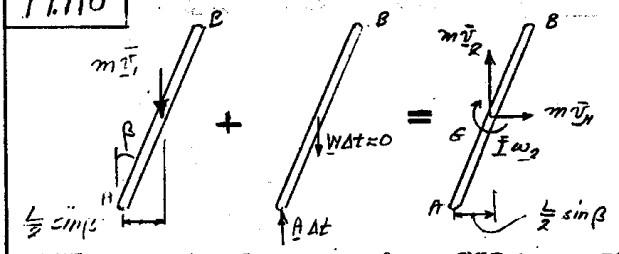
$$0 = -\tfrac{1}{3}mL^2\omega_4 \qquad \omega_4 = 0$$ ◀

EQ(8): $\bar{v}_4 = \bar{v}_1 - \frac{L}{2}\omega_4 = \bar{v}_1 + 0$ $\qquad \bar{v}_4 = \bar{v}_1\uparrow$ ◀

SUMMARY.



17.110



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

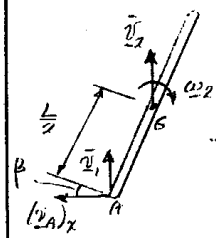
$\pm\rightarrow$ x COMPONENTS $\quad 0 = m\bar{v}_H \qquad \bar{v}_H = 0$

+) MOMENTS ABOUT A:

$$m\bar{v}_1\left(\frac{L}{2}\sin\beta\right) = \bar{I}\omega_2 - m\bar{v}_2\left(\frac{L}{2}\sin\beta\right) \quad (1)$$

IMPACT AT A: $(\underline{v}_A)_1 = \bar{v}_1\downarrow$; SINCE $e=1$, $\left[(\underline{v}_A)_2\right]_y = \bar{v}_1\uparrow$

KINEMATICS: $(\underline{v}_A)_2 = \bar{v}_1\uparrow + (\underline{v}_A)_x\leftarrow$



$$(\underline{v}_A)_2 = \underline{v}_G + \underline{v}_{A/G}$$

$$[\bar{v}_1\uparrow] + [(\underline{v}_A)_x\leftarrow] = [\bar{v}_2\uparrow] + [\tfrac{L}{2}\omega_2\ \angle\beta]$$

+↑ y COMPONENTS: $\bar{v}_1 = \bar{v}_2 + \frac{L}{2}\omega_2\sin\beta$

OR: $\bar{v}_2 = \bar{v}_1 - \frac{L}{2}\omega_2\sin\beta$

SUBSTITUTING INTO EQ (1).

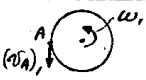
$$m\bar{v}_1\tfrac{L}{2}\sin\beta = \tfrac{1}{12}mL^2\omega_2 - m\left(\bar{v}_1 - \tfrac{L}{2}\omega_2\sin\beta\right)\left(\tfrac{L}{2}\sin\beta\right)$$

$$m\bar{v}_1 L\sin\beta = mL^2\left(\tfrac{1}{12} + \tfrac{1}{4}\sin^2\beta\right)\omega_2$$

$$\underline{\omega_2 = \frac{\bar{v}_1}{L}\cdot\frac{12\sin\beta}{3\sin^2\beta+1}}\ \curvearrowright$$

17.111

BEFORE IMPACT:



$(\underline{v}_A)_1 = r\omega_1\downarrow$

AFTER IMPACT SINCE $e=1$, $(\underline{v}_A)_2 = -(\underline{v}_A)_1$,

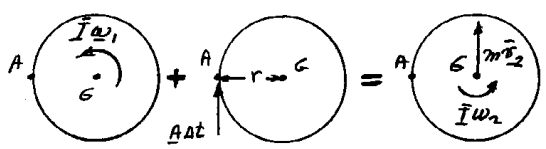
KINEMATICS

PLANE MOTION = TRANSLATION + ROTATION ABOUT A

$$\underline{\bar{v}_2 = (\underline{v}_A)_2\uparrow + \underline{v}_{G/A}\uparrow = r\omega_1\uparrow + r\omega_2\uparrow}$$

$$\underline{\bar{v}_2 = r(\omega_1+\omega_2)\uparrow} \quad (1)$$

PRINCIPLE OF IMPULSE AND MOMENTUM



+↶ M_A: $\quad \bar{I}\omega_1 + 0 = \bar{I}\omega_2 + m\bar{v}_2 r \quad (2)$

$\bar{I} = \frac{1}{2}mr^2$ AND FROM EQ(2) $\bar{v}_2 = r(\omega_1+\omega_2)$

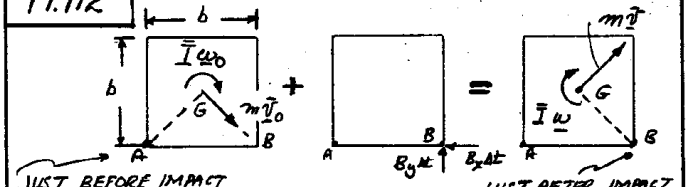
EQ(2): $\quad \frac{1}{2}mr^2\omega_1 = \frac{1}{2}mr^2\omega_2 + mr^2(\omega_1+\omega_2)$

$$-\tfrac{1}{2}\omega_1 = \tfrac{3}{2}\omega_2\ ;\quad \omega_2 = -\tfrac{1}{3}\omega_1, \qquad \underline{\omega_2 = \tfrac{1}{3}\omega_1}\ \curvearrowright$$

EQ(1):

$$\bar{v}_2 = r\left(\omega_1 - \tfrac{1}{3}\omega_1\right) = \tfrac{2}{3}r\omega_1, \qquad \underline{v_2 = \tfrac{2}{3}r\omega_1\uparrow}$$

17.112



SYST. MOMENTA$_0$ + SYST. EXT. IMP$_{0-1}$ = SYST. MOMENTA$_1$

+↷ MOMENTS ABOUT B.

$$\bar{I}\omega_0 + 0 = \bar{I}\omega + m\bar{v}(BG) \quad (1)$$

BUT: $BG = \sqrt{2}\,\frac{b}{2} = \frac{b}{\sqrt{2}}$, $\quad \bar{v} = (BG)\omega = \frac{b}{\sqrt{2}}\omega$

ALSO $\quad \bar{I} = \frac{1}{12}m(b^2+b^2) = \frac{1}{6}mb^2$

SUBSTITUTING INTO EQ (1)

$$\frac{1}{6}mb^2\omega_0 = \frac{1}{6}mb^2\omega + m\left(\frac{b}{\sqrt{2}}\omega\right)\left(\frac{b}{\sqrt{2}}\right)$$

(a) $\quad \frac{1}{6}mb^2\omega_0 = \frac{2}{3}mb^2\omega \qquad \underline{\omega = \frac{1}{4}\omega_0}\ \curvearrowright$

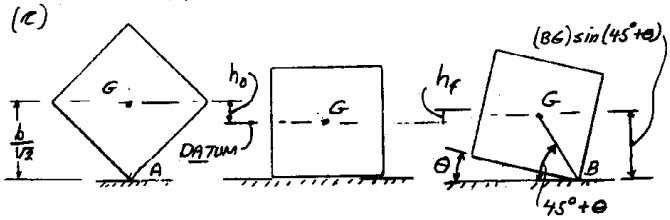
(b) KINETIC ENERGY LOST (SEE EQ 17.10, page 839)

$T_0 = \frac{1}{2}I_A\omega_0^2 \qquad T = \frac{1}{2}I_B\omega^2$ NOTE: $I_A = I_B$

$$\frac{\Delta T}{T_0} = \frac{T_0 - T}{T_0} = \frac{\omega_0^2-\omega^2}{\omega_0^2} = 1 - \left(\frac{\omega}{\omega_0}\right)^2 = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}$$

FRACTION OF KINETIC ENERGY LOST $= \frac{15}{16}$

(c)



INITIAL POSITION

$h_0 = \frac{b}{\sqrt{2}} - \frac{b}{2}$

$T_0 = 0$

$V_0 = Wh_0$

FINAL POSITION

$h_f = (BG)\sin(45°+\theta) - \frac{b}{2}$

$h_f = \frac{b}{\sqrt{2}}\sin(45°+\theta) - \frac{b}{2}$

$T_f = 0$

$V_f = Wh_f$

BUT, FROM PART (b) WE KNOW THAT $\frac{15}{16}$ OF THE ENERGY IS LOST. THUS

$$V_f = \tfrac{1}{16}V_h$$

AND $\quad Wh_f = \frac{1}{16}Wh_0$

$$h_f = \tfrac{1}{16}h_0$$

$$\frac{b}{\sqrt{2}}\sin(45°+\theta) - \frac{b}{2} = \frac{1}{16}\left(\frac{b}{\sqrt{2}} - \frac{b}{2}\right)$$

$$\frac{1}{\sqrt{2}}\sin(45°+\theta) = \frac{1}{16}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \frac{1}{2}$$

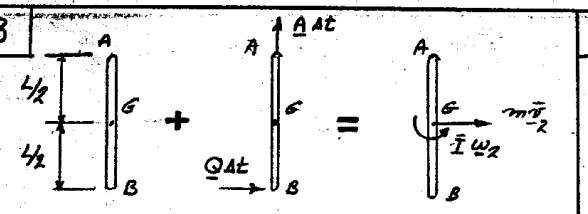
$$\sin(45°+\theta) = \frac{\sqrt{2}}{16}\left(\frac{1}{\sqrt{2}} - \frac{1}{2} + 8\right) = \frac{2+15\sqrt{2}}{32}$$

$$\sin(45°+\theta) = 0.72541$$

$$45°+\theta = 46.503°$$

$$\theta = 1.50°$$

17.113



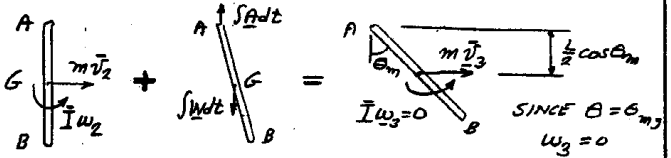
SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\pm$ χ COMPONENTS: $Q\Delta t = m\bar{v}_2$ (1)

$+\circlearrowleft$ MOMENTS ABOUT G: $(Q\Delta t)\frac{L}{2} = \bar{I}\omega_2$ (2)

SUBSTITUTE FOR $Q\Delta t$ FROM EQ(1) INTO EQ(2)

$$m\bar{v}_2\frac{L}{2} = \bar{I}\omega_2 = \frac{1}{12}mL^2\omega_2 \qquad \bar{v}_2 = \frac{1}{6}L\omega_2$$



SINCE $\theta = \theta_m$, $\omega_3 = 0$

SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2-3}$ = SYST. MOMENTA$_3$

$\pm$ χ COMPONENTS: $m\bar{v}_2 = m\bar{v}_3$ $\qquad \bar{v}_2 = \bar{v}_3$

CONSERVATION OF ENERGY CHOOSE DATUM AT ELEVATION OF A

POSITION 2: $V_2 = -mg\frac{L}{2}$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m\left(\frac{1}{6}L\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 = \frac{1}{18}mL^2\omega_2^2$$

POSITION 3: $V_3 = -mg\frac{L}{2}\cos\theta_m$

$\omega_3 = 0$ $\quad T_3 = \frac{1}{2}m\bar{v}_3^2 = \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}m\left(\frac{1}{6}L\omega_2\right)^2 = \frac{1}{72}mL^2\omega_2^2$

$T_2 + V_2 = T_3 + V_3$:

$$\frac{1}{18}mL^2\omega_2^2 - mg\frac{L}{2} = \frac{1}{72}mL^2\omega_2^2 - mg\frac{L}{2}\cos\theta_m$$

$$\left(\frac{1}{18} - \frac{1}{72}\right)L^2\omega_2^2 = \left(1 - \cos\theta_m\right)\frac{gL}{2}$$

$$\frac{1}{12}\frac{L}{g}\omega_2^2 = 1 - \cos\theta_m$$

FROM EQ(2): $\omega_2 = \frac{(Q\Delta t)L}{2I} = (Q\Delta t)\frac{L}{2\left(\frac{1}{12}mL^2\right)} = \frac{6(Q\Delta t)}{mL}$

$$\frac{1}{12}\frac{L}{g}\left[\frac{6(Q\Delta t)}{mL}\right]^2 = 1 - \cos\theta_m$$

$$\cos\theta_m = 1 - \frac{3}{gLm^2}(Q\Delta t)^2 \qquad (3)$$

DATA: $m = 1.5$ kg, $L = 0.5$ m, $Q\Delta t = 1.8$ N·s

$$\cos\theta_m = 1 - \frac{3}{(9.81\ \text{m/s}^2)(0.5\ \text{m})(1.5\ \text{kg})^2}(1.8\ \text{N·s})^2$$

$$\cos\theta_m = 0.11927 \qquad \theta_m = 83.2^\circ \blacktriangleleft$$

17.114

SEE PROB. 17.113 FOR DERIVATION OF EQ.(3).

$$\cos\theta_m = 1 - \frac{3}{gLm^2}(Q\Delta t)^2$$

$$(Q\Delta t)^2 = \frac{gLm^2}{3}(1 - \cos\theta_m)$$

DATA: $m = 1.5$ kg, $L = 0.5$ m

(a) FOR $\theta_m = 90^\circ$: $(Q\Delta t)^2 = \frac{(9.81)(0.5)(1.5)^2}{3}(1 - \cos 90^\circ) = 3.67875$

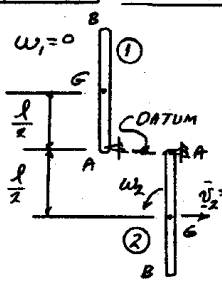
$Q\Delta t = 1.918$ N·s ◀

(b) FOR $\theta_m = 120^\circ$: $(Q\Delta t)^2 = \frac{(9.81)(0.5)(1.5)^2}{3}(1 - \cos 120^\circ) = 5.51813$

$Q\Delta t = 2.35$ N·s ◀

17.115

BEFORE IMPACT CONSERVATION OF ENERGY



POSITION 1: $T_1 = 0$ $\quad V_1 = W\frac{l}{2}$

POSITION 2:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\frac{W}{g}\left(\frac{l}{2}\right)^2\omega_2^2 + \frac{1}{2}\left(\frac{1}{12}\frac{W}{g}l^2\right)\omega_2^2$$

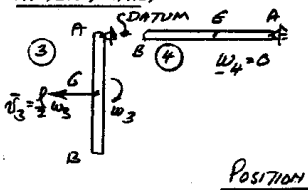
$$T_2 = \frac{1}{6}\frac{W}{g}l^2\omega_2^2 \qquad V_2 = -W\frac{l}{2}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + W\frac{l}{2} = \frac{1}{6}\frac{W}{g}l^2\omega_2^2 - W\frac{l}{2} \qquad \omega_2 = \sqrt{6g/l}$$

$$(v_K)_2 = r\omega_2 = r\sqrt{6g/l} \qquad (1)$$

AFTER IMPACT



POSITION 3: $V_3 = -W\frac{l}{2}$

$$T_3 = \frac{1}{2}m\bar{v}_3^2 + \frac{1}{2}\bar{I}\omega_3^2$$

SEE POSITION 2 ABOVE

$$T_3 = \frac{1}{6}\frac{W}{g}l^2\omega_3^2$$

POSITION 4: $T_4 = 0$; $V_4 = 0$

$T_3 + V_3 = T_4 + V_4$: $\quad \frac{1}{6}\frac{W}{g}l^2\omega_3^2 - W\frac{l}{2} = 0 \qquad \omega_3 = \sqrt{3g/l}$

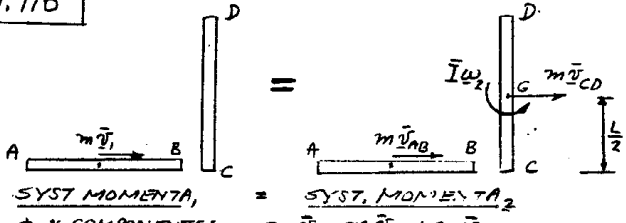
$$(v_K)_3 = r\omega_3 = r\sqrt{3g/l} \qquad (2)$$

DURING IMPACT

(a) $e = \frac{(v_K)_3}{(v_K)_2} = \frac{r\sqrt{3g/l}}{r\sqrt{6g/l}} = \sqrt{\frac{1}{2}} \qquad e = 0.707$ ◀

(b) RESULT IS INDEPENDENT OF LOCATION OF KNOB ◀

17.116



SYST MOMENTA$_1$ = SYST. MOMENTA$_2$

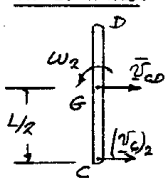
$\pm$ χ COMPONENTS: $m\bar{v}_1 = m\bar{v}_{AB} + m\bar{v}_{CD}$

$$\bar{v}_{CD} = \bar{v}_1 - \bar{v}_{AB}$$

$+\circlearrowleft$ MOMENTS ABOUT C: $0 = \bar{I}\omega_2 - m\bar{v}_{CD}\frac{L}{2}$ (1)

$$0 = \frac{1}{12}mL^2\omega_2 - m\bar{v}_{CD}\frac{L}{2} \qquad \omega_2 = 6\frac{\bar{v}_{CD}}{L} \qquad (2)$$

KINEMATICS:



$$(\bar{v}_C)_2 = \bar{v}_{CD} + \frac{L}{2}\omega_2 = \bar{v}_{CD} + \frac{L}{2}\left(6\frac{\bar{v}_{CD}}{L}\right)$$

$$(\bar{v}_C)_2 = 4\bar{v}_{CD}$$

IMPACT: BETWEEN B AND C

BEFORE IMPACT. $(v_B)_1 = \bar{v}_1$, $(v_C)_1 = 0$

AFTER IMPACT $(v_B)_2 = \bar{v}_{AB}$, $(v_C)_2 = 4\bar{v}_{CD}$

RELATIVE VELOCITIES: $\left[(v_C)_2 - (v_B)_2\right] = e\left[(v_B)_1 - (v_C)_1\right]$

$e = 1.00$: $\quad \left[4\bar{v}_{CD} - \bar{v}_{AB}\right] = 1.00\left[\bar{v}_1 - 0\right]$

SUBSTITUTE FOR $\bar{v}_{CD}$ FROM EQ(1):

$$\left[4(\bar{v}_1 - \bar{v}_{AB}) - \bar{v}_{AB}\right] = \bar{v}_1$$

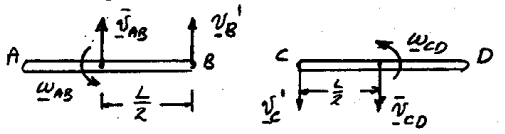
$$3\bar{v}_1 = 5\bar{v}_{AB} \qquad \bar{v}_{AB} = 0.6\,\bar{v}_1 \rightarrow \blacktriangleleft$$

EQ(1): $\bar{v}_{CD} = \bar{v}_1 - \bar{v}_{AB} = \bar{v}_1 - 0.6\,\bar{v}_1$ $\qquad \bar{v}_{CD} = 0.4\,\bar{v}_1 \rightarrow \blacktriangleleft$

EQ(2): $\omega_2 = 6\frac{\bar{v}_{CD}}{L} = 6\left(\frac{0.4\bar{v}_1}{L}\right)$ $\qquad \omega_2 = 2.4\frac{\bar{v}_1}{L}\ \circlearrowright \blacktriangleleft$

17.117

KINEMATICS: WE ASSUME VELOCITIES AFTER IMPACT ARE DIRECTED AS FOLLOWS:



$$\underline{v}_B' = \left[\bar{v}_{AB} + \tfrac{L}{2}\omega_{AB}\right]\uparrow \qquad \underline{v}_C' = \left[\bar{v}_{CD} + \tfrac{L}{2}\omega_{CD}\right]\downarrow$$

BEFORE IMPACT: $\underline{v}_B = \frac{L}{2}\omega_1 \downarrow$ AND $\underline{v}_C = 0$

COEFFICIENT OF RESTITUTION

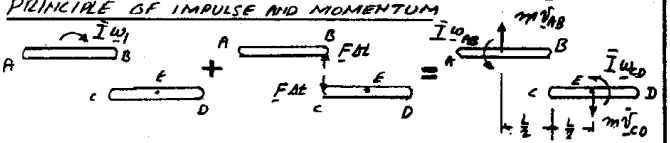
$$e = \frac{\text{Rel. Vel. After}}{\text{Rel. Vel. Before}} = \frac{\left[\bar{v}_{AB} + \tfrac{L}{2}\omega_{AB}\right] + \left[\bar{v}_{CD} + \tfrac{L}{2}\omega_{CD}\right]}{\tfrac{L}{2}\omega_1} \qquad (1)$$

FOR $e = 1$:

$$\tfrac{L}{2}\omega_1 = \bar{v}_{AB} + \bar{v}_{CD} + \tfrac{L}{2}(\omega_{AB} + \omega_{CD})$$

$$\omega_1 = \frac{2}{L}(\bar{v}_{AB} + \bar{v}_{CD}) + \omega_{AB} + \omega_{CD} \qquad (2)$$

PRINCIPLE OF IMPULSE AND MOMENTUM



SYST. MOMENTA$_0$ + SYST. EXT. IMP$_{0\text{-}1}$ = SYST. MOMENTA$_1$

ENTIRE SYSTEM: $+\uparrow$ COMPONENTS: $0 = m\bar{v}_{AB} - m\bar{v}_{CD}$; $\bar{v}_{AB} = \bar{v}_{CD}$ (3)

$+\circlearrowleft \Sigma M_E$: (NOTE: $m\bar{v}_{AB}$ AND $m\bar{v}_{CD}$ FORM A COUPLE)

$$\bar{I}\omega_1 = (m\bar{v}_{AB})L - \bar{I}\omega_{AB} - \bar{I}\omega_{CD}$$

$$\omega_1 = \frac{mL}{\bar{I}}\bar{v}_{AB} - \omega_{AB} - \omega_{CD} = \frac{mL}{\frac{1}{12}mL^2}\bar{v}_{AB} - \omega_{AB} - \omega_{CD}$$

$$\omega_1 = \frac{12}{L}\bar{v}_{AB} - \omega_{AB} - \omega_{CD} \qquad (4)$$

ROD AB: $+\circlearrowleft \Sigma M_B$: $\bar{I}\omega_1 = m\bar{v}_{AB}\frac{L}{2} - \bar{I}\omega_{AB}$

$$\omega_1 = \frac{mL}{2\bar{I}}\bar{v}_{AB} - \omega_{AB} = \frac{mL}{\frac{2}{12}mL^2}\bar{v}_{AB} - \omega_{AB}; \qquad \omega_1 = \frac{6}{L}\bar{v}_{AB} - \omega_{AB} \qquad (5)$$

EQ(2) WITH $\bar{v}_{AB} = \bar{v}_{CD}$: $\omega_1 = \frac{4}{L}\bar{v}_{AB} + \omega_{AB} + \omega_{CD}$ (6)

ADD EQ(4) AND EQ(6): $2\omega_1 = \frac{16}{L}\bar{v}_{AB}$; $\underline{\bar{v}_{AB} = +\frac{1}{8}\omega_1 L = \frac{1}{8}\omega_1 L \uparrow}$ ◀

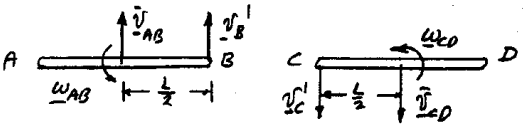
$\underline{\bar{v}_{CD} = -\bar{v}_{AB} = \frac{1}{8}\omega_1 L \downarrow}$ ◀

EQ(5): $\omega_1 = \frac{6}{L}\left(\frac{1}{8}\omega_1 L\right) - \omega_{AB}$; $\underline{\omega_{AB} = -\frac{1}{4}\omega_0 = \frac{1}{4}\omega_0 \circlearrowright}$ ◀

EQ(4): $\omega_1 = \frac{12}{L}\left(\frac{1}{8}\omega_1 L\right) - \left(-\frac{1}{4}\omega_1\right) - \omega_{CD}$; $\underline{\omega_{CD} = \frac{3}{4}\omega_1 = \frac{3}{4}\omega_1 \circlearrowright}$ ◀

17.118

KINEMATICS WE ASSUME VELOCITIES AFTER IMPACT ARE DIRECTED AS FOLLOWS:



$$\underline{v}_B' = \left[\bar{v}_{AB} + \tfrac{L}{2}\omega_{AB}\right]\uparrow \qquad \underline{v}_C' = \left[\bar{v}_{CD} + \tfrac{L}{2}\omega_{CD}\right]\downarrow$$

BEFORE IMPACT: $\underline{v}_B = -\bar{v}_1 = \bar{v}_1 \downarrow$ AND $v_C = 0$

COEFFICIENT OF RESTITUTION

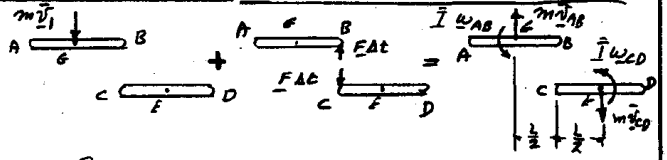
$$e = \frac{\text{Rel. Vel. After}}{\text{Rel. Vel. Before}} = \frac{\left[\bar{v}_{AB} + \tfrac{L}{2}\omega_{AB}\right] + \left[\bar{v}_{CD} + \tfrac{L}{2}\omega_{CD}\right]}{\bar{v}_1} \qquad (1)$$

FOR $e = 1$: $\bar{v}_1 = \bar{v}_{AB} + \bar{v}_{CD} + \frac{L}{2}(\omega_{AB} + \omega_{CD})$ (2)

(CONTINUED)

17.118 CONTINUED

PRINCIPLE OF IMPULSE AND MOMENTUM



NOTE POINTS G AND E ARE AT MASS CENTERS OF RODS

ROD AB: $+\circlearrowleft \Sigma M_G$: $0 + F\Delta t = \bar{I}\omega_{AB}$

ROD CD: $+\circlearrowleft \Sigma M_E$: $0 + F\Delta t = \bar{I}\omega_{CD}$ $\Big\}$ $\omega_{AB} = \omega_{CD}$ (3)

ROD CD: $+\circlearrowleft \Sigma M_C$: $0 = \bar{I}\omega_{CD} - m\bar{v}_{CD}\frac{L}{2}$

$$0 = \tfrac{1}{12}mL^2\omega_{CD} - m\bar{v}_{CD}\tfrac{L}{2} \qquad \omega_{CD} = \frac{6}{L}\bar{v}_{CD} \qquad (4)$$

ENTIRE SYSTEM $+\downarrow \Sigma F$: $m\bar{v}_1 = -m\bar{v}_{AB} + m\bar{v}_{CD}$

$$\bar{v}_1 = -\bar{v}_{AB} + \bar{v}_{CD} \qquad (5)$$

SUBSTITUTE $\omega_{AB} = \omega_{CD} = \frac{6}{L}\bar{v}_{CD}$ INTO EQ(2)

$$\bar{v}_1 = \bar{v}_{AB} + \bar{v}_{CD} + \frac{L}{2}\left(\frac{6}{L}\bar{v}_{CD} + \frac{6}{L}\bar{v}_{CD}\right)$$

$$\bar{v}_1 = \bar{v}_{AB} + 7\bar{v}_{CD} \qquad (6)$$

SOLVE EQS(6)+(6) SIMULTANEOUSLY: $\underline{\bar{v}_{CD} = +\frac{1}{4}\bar{v}_1 = \frac{1}{4}\bar{v}_1 \downarrow}$ ◀

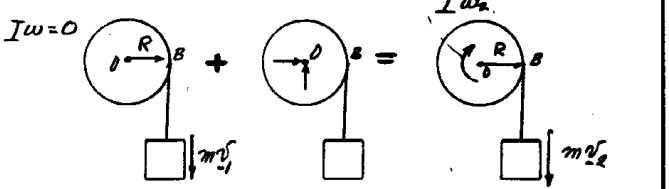
$\underline{\bar{v}_{AB} = -\frac{3}{4}\bar{v}_1 = \frac{3}{4}\bar{v}_1 \downarrow}$ ◀

EQ(3) AND EQ(4) $\underline{\omega_{AB} = \omega_{CD} = \frac{6}{L}\left(\frac{1}{4}\bar{v}_1\right) = +\frac{3}{2}\frac{\bar{v}_1}{L} = \frac{3}{2}\frac{\bar{v}_1}{L} \circlearrowright}$ ◀

17.119

BLOCK FALLS THROUGH DISTANCE h BEFORE CORD BECOMES TAUT, THUS $\underline{v_1 = \sqrt{2gh} \downarrow}$

PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\text{-}2}$ = SYST. MOMENTA$_2$

$+\circlearrowleft$ MOMENTS ABOUT O: $mv_1 R = \bar{I}\omega_2 + mv_2 R$ (1)

(a) PLASTIC IMPACT $e = 0$ AND $v_2 = R\omega_2$

SUBSTITUTING $\omega_2 = \frac{v_2}{R}$ AND $\bar{I} = \frac{1}{2}MR^2$

$$mv_1 R = \frac{1}{2}MR^2\left(\frac{v_2}{R}\right) + mv_2 R$$

$$mv_1 = \frac{1}{2}Mv_2 + mv_2$$

SINCE $v_1 = \sqrt{2gh}$ $\quad v_2 = \frac{2m}{2m+M}v_1$, $\quad \underline{v_2 = \frac{2m}{2m+M}\sqrt{2gh} \downarrow}$ ◀

$\omega_2 = \frac{v_2}{R}$, $\quad \underline{\omega_2 = \frac{2m}{2m+M}\frac{\sqrt{2gh}}{R} \circlearrowright}$ ◀

(b) ELASTIC IMPACT $e = 1$: $(v_B)_2 - (v_A)_2 = (v_A)_1 - (v_B)_1$,

$(v_B)_1 = 0$; $(v_B)_2 = R\omega_2$: $R\omega_2 - v_2 = v_1$ $\qquad \omega_2 = (v_1 + v_2)/R$ (2)

EQ(1): $mv_1 R = \frac{1}{2}MR^2\left(\frac{v_1 + v_2}{R}\right) + mv_2 R$

$$mv_1 R = \frac{1}{2}MRv_1 + \frac{1}{2}MRv_2 + mv_2 R$$

$$v_2 = \frac{2m - M}{2m + M}v_1 \qquad \underline{v_2 = \frac{2m - M}{2m + M}\sqrt{2gh} \downarrow}$$ ◀

EQ(2): $\omega_2 = \frac{v_1 + v_2}{R} = \left[1 + \frac{2m - M}{2m + M}\right]\frac{v_1}{R}$

$$\underline{\omega_2 = \frac{4m}{2m + M}\frac{\sqrt{2gh}}{R} \circlearrowright}$$ ◀

17.120

GIVEN DATA $M = 8\ kg$ $m = 3\ kg$

$R = 250\ mm = 0.25\ m$

$h = 900\ mm = 0.9\ m$ $v_1 = \sqrt{2gh} = \sqrt{2(9.81)(0.9)} = 4.202\ m/s$

SEE SOLUTION OF PROB. 17.119 FOR EQUATIONS OF v_2 AND ω_2

(a) PLASTIC IMPACT $v_2 = \frac{2m}{2m+M} v_1 = \frac{2(3)}{2(3)+8}(4.202) = 1.8009\ m/s$

$\underline{v}_2 = 1.801\ m/s \downarrow$ ◀

$\omega_2 = \frac{v_2}{R} = \frac{1.8009}{0.25} = 7.204\ rad/s$ $\underline{\omega}_2 = 7.20\ rad/s \curvearrowright$ ◀

(b) ELASTIC IMPACT

$v_2 = \frac{2m-M}{2m+M} v_1 = \frac{2(3)-8}{2(3)+8}(4.202) = -0.6003\ m/s$

$\underline{v}_2 = 0.600\ m/s \uparrow$ ◀

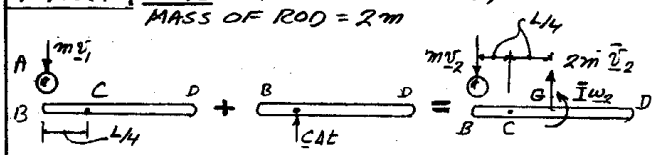
$\omega_2 = \frac{4m}{2m+M}\frac{v_1}{R} = \frac{4(3)}{2(3)+8}\left(\frac{4.202}{0.25}\right) = 14.407\ rad/s$

$\underline{\omega}_2 = 14.41\ rad/s \curvearrowright$ ◀

17.121

NOTE: MASS OF SPHERE = m

MASS OF ROD = $2m$



SYST. MOMENTA$_1$ + SYST EXT IMP$_{1-2}$ = SYST MOMENTA$_2$

+) MOMENTS ABOUT C:

$$m v_1 \left(\tfrac{L}{4}\right) = m v_2 \left(\tfrac{L}{4}\right) + \bar{I}\omega_2 + 2m\bar{v}_2 \tfrac{L}{4}$$

ROTATION ABOUT C: $\bar{v}_2 = \frac{L}{4}\omega_2$ (1)

FOR ROD: $\bar{I} = \frac{1}{12}(2m)L^2 = \frac{1}{6}mL^2$

$$m v_1 \tfrac{L}{4} = m v_2 \tfrac{L}{4} + \tfrac{1}{6} m L^2 \omega_2 + 2m\left(\tfrac{L}{4}\omega_2\right)\tfrac{L}{4}$$

$$\tfrac{1}{4} v_1 = \tfrac{1}{4} v_2 + \left(\tfrac{1}{6} + \tfrac{1}{8}\right) L\omega_2$$

$$\tfrac{1}{4}(v_1 - v_2) = \tfrac{7}{24} L\omega_2 \qquad \omega_2 = \tfrac{6}{7L}(v_1 - v_2) \quad (2)$$

EQ(1) $\bar{v}_2 = \frac{L}{4}\omega_2 = \frac{L}{4}\left(\frac{6}{7L}\right)(v_1 - v_2) = \frac{3}{14}(v_1 - v_2)$ (3)

ROTATION ABOUT C: $(v_B)_2 = \frac{L}{4}\omega_2 = \frac{3}{14}(v_1 - v_2)$ (4)

BEFORE IMPACT: $(v_A)_1 = v_1$ $(v_B)_1 = 0$

AFTER IMPACT: $(v_A)_2 = v_2$

IMPACT BETWEEN A AND B

RELATIVE VELOCITIES

$$\left[(v_B)_2 - (v_A)_2\right] = e\left[(v_A)_1 - (v_B)_1\right]$$

$e = 1.00$: $\left[\frac{3}{14}(v_1 - v_2) - v_2\right] = 1.00\left[v_1 - 0\right]$

$$-\tfrac{17}{14} v_2 = \tfrac{11}{14} v_1 \qquad v_2 = -\tfrac{11}{17} v_1$$

(a) FINAL VELOCITY OF SPHERE: $\underline{v}_2 = \frac{11}{17} v_1 \uparrow$ ◀

(b) EQ(2): $\omega_2 = \frac{6}{7L}(v_1 - v_2) = \frac{6}{7L}\left[v_1 - \left(-\frac{11}{17}\right)\right] = \frac{6}{7L} v_1 \left(1 + \frac{11}{17}\right)$

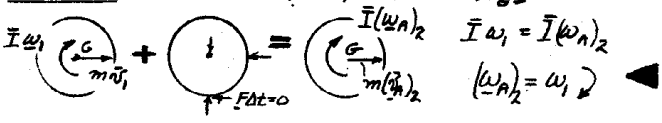
$= \frac{6}{7L} v_1 \left(\frac{28}{17}\right) = \frac{24}{17}\frac{v_1}{L}$ $\underline{\omega}_2 = \frac{24}{17}\frac{v_1}{L} \curvearrowright$ ◀

17.122

NOTE: DURING IMPACT FRICTION FORCES ARE NONIMPULSIVE.

(a) IMMEDIATELY AFTER IMPACT:

SPHERE A WAS ROLLING: $\bar{v}_1 = r\omega_1$ $\Sigma M_G \curvearrowright$



$\bar{I}\omega_1 = \bar{I}(\omega_A)_2$

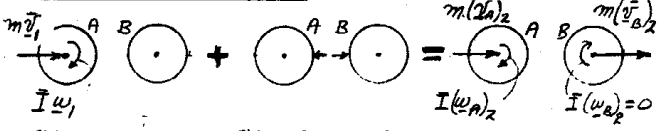
$(\underline{\omega}_A)_2 = \omega_1 \curvearrowright$ ◀

SPHERE B:

SIMILAR TO ANALYSIS OF SPHERE A, EXCEPT THAT INITIAL ANG. VELOCITY IS ZERO.

$(\underline{\omega}_B)_2 = 0$ ◀

CONSIDER BOTH SPHERES:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\pm$ x COMPONENTS: $m\bar{v}_1 = m(\bar{v}_A)_2 + m(\bar{v}_B)_2$

$$\bar{v}_1 = (\bar{v}_A)_2 + (\bar{v}_B)_2 \quad (1)$$

RELATIVE VELOCITIES: $(e = 1.00)$

$$(\bar{v}_B)_2 - (\bar{v}_A)_2 = e\,\bar{v}_1 = \bar{v}_1 \qquad \bar{v}_1 = (\bar{v}_B)_2 - (\bar{v}_A)_2 \quad (2)$$

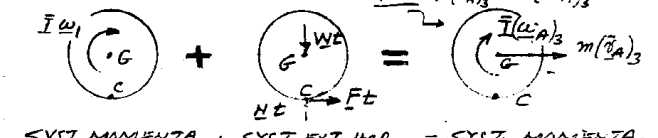
ADD EQS (1) AND (2): $2\bar{v}_1 = 2(\bar{v}_B)_2$ $(\underline{\bar{v}}_B)_2 = \bar{v}_1 \rightarrow$ ◀

SUBTRACT EQ(2) FROM EQ(1): $0 = 2(\bar{v}_A)_2$ $(\bar{v}_A)_2 = 0$ ◀

(b) MOTION AFTER SPHERES ROLL UNIFORMLY.

NOTE: THE TIME INTERVAL WILL NOT BE SMALL AND THE IMPULSES OF FRICTION FORCES MUST BE INCLUDED.

SPHERE A:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+2 MOMENTS ABOUT C: $\bar{I}\omega_1 = \bar{I}(\omega_A)_3 + m(\bar{v}_A)_3 r$

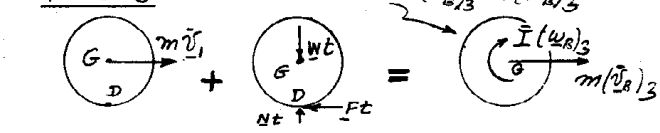
$$\tfrac{2}{5} m r^2 \omega_1 = \tfrac{2}{5} m r^2 (\omega_A)_3 + m r^2 (\omega_A)_3$$

Recall $\bar{v}_1 = r\omega_1$:

$(\underline{\omega}_A)_3 = \frac{2}{7}\omega_1 \curvearrowright$

$(\bar{v}_A)_3 = r(\omega_A)_3 = \frac{2}{7} r\omega_1 = \frac{2}{7}\bar{v}_1$ $(\underline{\bar{v}}_A)_3 = \frac{2}{7}\bar{v}_1 \rightarrow$ ◀

SPHERE B:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+2 MOMENTS ABOUT D: $m\bar{v}_1 r = \bar{I}(\omega_B)_3 + m(\bar{v}_B)_3 r$

$$m\bar{v}_1 r = \tfrac{2}{5} m r^2 (\omega_B)_3 + m r^2 (\omega_B)_3$$

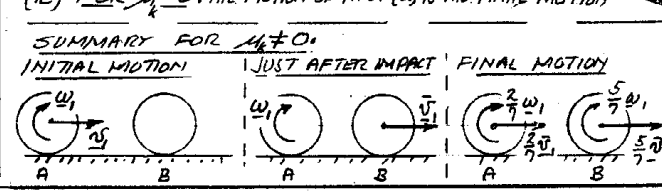
Recall $v_0 = r\omega_0$.

$(\omega_B)_3 = \frac{5}{7}\frac{\bar{v}_1}{r} = \frac{5}{7}\omega_1$ $(\underline{\omega}_B)_3 = \frac{5}{7}\omega_1 \curvearrowright$

$(\bar{v}_B)_3 = r(\omega_B)_3 = r\left(\frac{5}{7}\frac{\bar{v}_1}{r}\right) = \frac{5}{7}\bar{v}_1$ $(\underline{\bar{v}}_B)_3 = \frac{5}{7}\bar{v}_1 \rightarrow$ ◀

(c) FOR $\mu_k = 0$, THE MOTION OF PART (a) IS THE FINAL MOTION ◀

SUMMARY FOR $\mu_k \neq 0$.



17.123 (a) MOTION IMMEDIATELY AFTER IMPACT

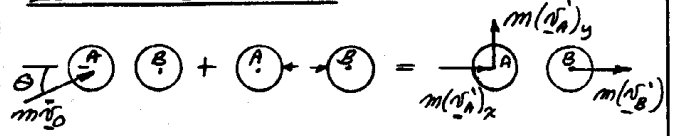
FRICTION FORCES ARE NON IMPULSIVE; THUS ANGULAR MOMENTUM (AND HENCE ALSO ANGULAR VELOCITIES) ARE NOT CHANGED.

WE HAVE $\underline{\omega}_B' = 0$ ◀

AND SINCE BALL A WAS ROLLING:

$\underline{\omega}_A' = \frac{\bar{v}_0}{r}\theta \qquad \underline{\omega}_A' = \frac{\bar{v}_0}{r}(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$ ◀

LOOKING DOWNWARD



SYST. MOMENTA$_0$ + SYST. EXT. IMP$_{0-1}$ = SYST MOMENTA$_1$,

$\pm$ COMPONENTS: $m\bar{v}_0\cos\theta = m(v_A')_x + m(v_B')$ (1)

$+\uparrow$ COMPONENTS OF BALL A: $m\bar{v}_0\sin\theta = m(v_A')_y$

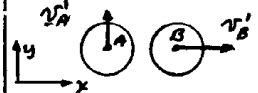
$(v_A')_y = \bar{v}_0\sin\theta$ (2)

FOR $e = 1$, $v_B' - (v_A')_x = \bar{v}_0\cos\theta$ (3)

SOLVING SIMULTANEOUSLY EQS (1) AND (2)

$(v_A')_x = 0 \qquad (v_B') = \bar{v}_0\cos\theta$ (4)

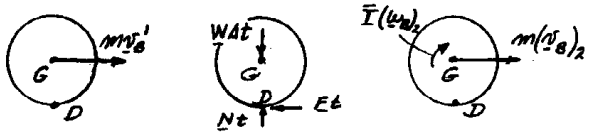
MOTION IMMEDIATELY AFTER IMPACT, FROM EQS. (2) AND (3), WE WRITE



$\underline{v}_A' = \bar{v}_0\sin\theta\,\underline{j}$ ◀

$\underline{v}_B' = \bar{v}_0\cos\theta\,\underline{i}$ ◀

(b) FINAL VELOCITY OF BALL B



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT D:

$(m\underline{v}_B')r = \bar{I}(\omega_B)_2 + m(\underline{v}_B)_2 r$ (1)

WE RECALL $\underline{v}_B' = \bar{v}_0\cos\theta$ AND $I = \frac{2}{5}mr^2$, SINCE BALL ROLLS: $(\underline{v}_B)_2 = r(\omega_B)_2$, WE FIND

$mr\bar{v}_0\cos\theta = \frac{2}{5}mr^2(\omega_B)_2 + mr(\omega_B)_2 r$

$(\omega_B)_2 = \frac{5}{7}\frac{\bar{v}_0}{r}\cos\theta \qquad (\underline{v}_B)_2 = r(\omega_B)_2 \rightarrow$

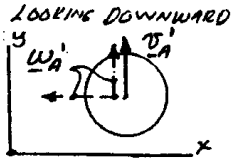
$(\underline{v}_B)_2 = \frac{5}{7}\bar{v}_0\cos\theta\,\underline{i}$ ◀

17.124 BALL A: FROM PROB. 17.123 THE MOTION IMMEDIATELY AFTER IMPACT IS

$\underline{v}_A' = \bar{v}_0\sin\theta\,\underline{j}$

$\underline{\omega}_A' = \frac{\bar{v}_0}{r}(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$

LOOKING DOWNWARD. WE NOTE THAT:



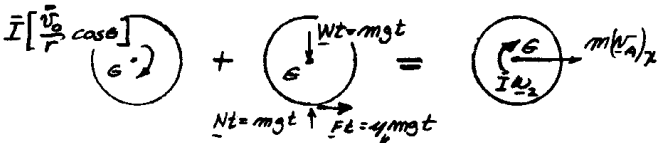
$\underline{v}_A' = \bar{v}_0\sin\theta\,\underline{j}$

$= \left(-\frac{\bar{v}_0}{r}\sin\theta\,\underline{i}\right) \times r\underline{k}$

THIS SHOWS THAN THE BALL ROLLS WITHOUT SLIPPING IN THE y DIRECTION. THUS, AT ANY TIME t, THE y COMPONENT OF THE VELOCITY IS CONSTANT. THE y COORDINATE AT ANY TIME t IS

$y = (v_A')t = (\bar{v}_0\sin\theta)\,t$ (1)

BALL ROLLS AND SLIDES IN THE x DIRECTION: PRINCIPLE OF IMPULSE-MOMENTUM



$\pm$ x COMPONENTS: $0 + \mu_k mgt = m(v_A)_x$

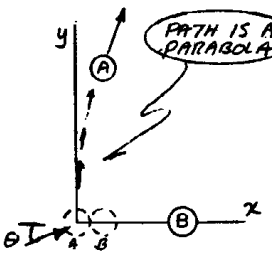
$(v_A)_x = \mu_k g\,t$

$(a_A)_x = \mu_k g$ = CONSTANT

THUS: $x = \frac{1}{2}at^2 \qquad x = \frac{1}{2}\mu_k g\,t^2$ (2)

ELIMINATE t BETWEEN (1) AND (2)

$y^2 = \left(2\bar{v}_0^2\sin^2\theta/\mu_k g\right)x$ ◀



SLIPPING ENDS AND UNIFORM MOTION BEGINS WHEN

$\omega_2 = \frac{(v_A)_x}{r} = \frac{\mu_k g\,t_F}{r}$

WE RETURN TO THE ABOVE SKETCHES WHICH EXPRESS PRINC. OF IMPULSE MOMENTUM AND WE NOW WRITE:

$+\circlearrowright$ MOMENTS ABOUT G:

$\bar{I}\left[\frac{\bar{v}_0}{r}\cos\theta\right] - (\mu_k mgt_F)r = \bar{I}\omega_2$

$\frac{2}{5}\frac{W}{g}r^2\left[\frac{\bar{v}_0}{r}\cos\theta\right] - \mu_k mgt_F r = \frac{2}{5}mr^2\left[\frac{\mu_k g\,t_F}{r}\right]$

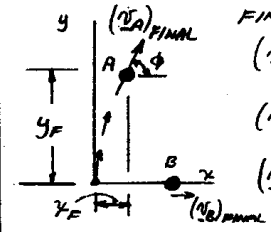
$\frac{2}{5}\bar{v}_0\cos\theta = \frac{2}{5}\mu_k g\,t_F$

ROLLING WITH NO SLIDING BEGINS WHEN: $t_F = \frac{2}{7}\frac{\bar{v}_0\cos\theta}{\mu_k g}$

THUS, MOTION BECOMES UNIFORM AT:

FROM (2). $x_F = \frac{1}{2}\mu_k g\,t_F^2 = \frac{2}{49}\frac{\bar{v}_0^2\cos^2\theta}{\mu_k g}$

FROM (1) $y_F = \bar{v}_0\sin\theta\,t_F = \frac{2}{7}\frac{\bar{v}_0^2\sin\theta\cos\theta}{\mu_k g}$



FINAL VELOCITY: (UNIFORM MOTION)

$(v_A)_x = \mu_k g\,t_F = \frac{2}{7}\bar{v}_0\cos\theta$

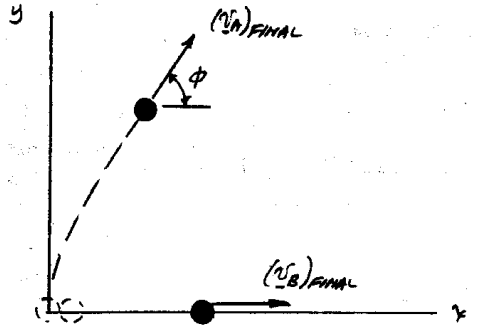
$(v_A)_y = \bar{v}_0\sin\theta$

$(\underline{v}_A)_{FINAL} = \left[\frac{2}{7}\bar{v}_0\cos\theta\right]\underline{i} + \left[\bar{v}_0\sin\theta\right]\underline{j}$

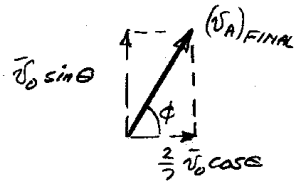
17.125

SEE SOLUTION OF PROB 17.124

WE FOUND THAT THE FINAL VELOCITIES ARE

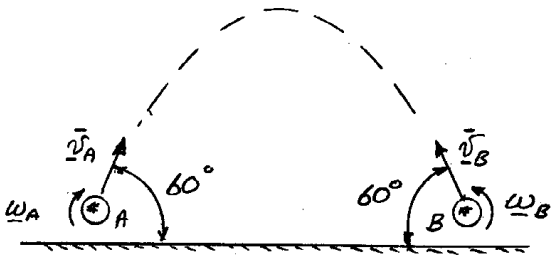


WHERE $(\underline{v}_A)_{FINAL} = (\frac{2}{7}\bar{v}_0\cos\theta)\underline{i} + (\bar{v}_0\sin\theta)\underline{j}$ ◀



$$\tan\phi = \frac{\bar{v}_0\sin\theta}{\frac{2}{7}\bar{v}_0\cos\theta} \qquad \tan\phi = \frac{7}{2}\tan\theta$$ ◀

17.126



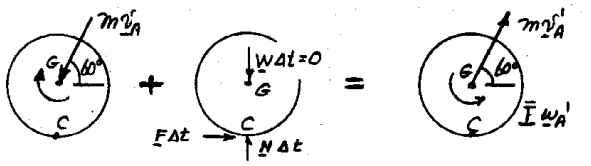
SINCE THE LINEAR AND ANGULAR VELOCITIES ARE CHANGED DURING THE INTERVAL (Δt) OF IMPACT, BOTH THE NORMAL AND FRICTION FORCES ARE IMPULSIVE. IT IS ASSUMED THAT THE COEFFICIENT OF FRICTION IS SUFFICIENT TO PREVENT SLIPPING DURING IMPACT. IF SLIPPING WERE TO OCCUR, ENERGY WOULD BE DISSIPATED AND PATH WOULD NOT REMAIN THE SAME FOR SUCCESSIVE BOUNCES.

(CONTINUED)

17.126 CONTINUED

FOR THE BALL TO BOUNCE A TO B TO A, ETC. THE VELOCITY MUST BE COMPLETELY REVERSED AT EACH IMPACT. FOR EXAMPLE, WE MUST HAVE AT POINT A:

IMPACT AT A:



SYST. MOMENTA$_1$ + SYST EXT IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$\bar{I}\omega_A - (mv_A\cos 60°)r = -\bar{I}\omega_A' + (mv_A'\cos 60°)r$$

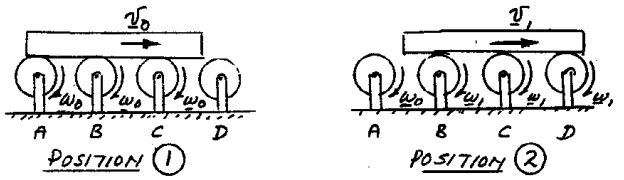
SUBSTITUTE $\omega_A' = \omega_A = \omega_0$, $v_A' = v_A = v_0$

$$2\bar{I}\omega_0 = 2(mv_0\cos 60°)r$$

$$\omega_0 = \frac{(mv_0\cos 60°)r}{\bar{I}} = \frac{mv_0(\frac{1}{2})r}{\frac{2}{5}mr^2} \qquad \omega_0 = \frac{5}{4}\frac{v_0}{r}$$ ◀

17.127

$\omega_0 = \frac{v_0}{r}$ $\qquad$ $\omega_1 = \frac{v_1}{r}$



(a)

CHANGES IN VELOCITY FROM POSITION (1) TO POSITION (2).

ROLLER A: NO CHANGE

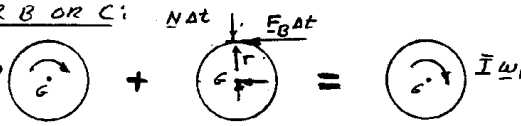
ROLLERS B AND C: $\underline{\omega}_0$ REDUCED TO $\underline{\omega}_1$

ROLLER D: INCREASES FROM REST TO $\underline{\omega}_1$

SLAB: $\underline{v}_0$ REDUCED TO $\underline{v}_1$

PRINCIPLE OF IMPULSE AND MOMENTUM:

ROLLER B OR C:

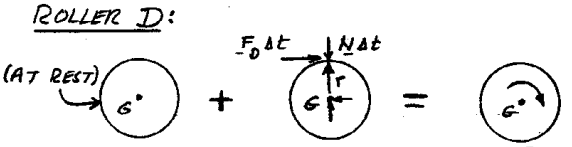


+↻ MOMENTS ABOUT G: $\bar{I}\omega_0 - (F_B\Delta t)r = \bar{I}\omega_1$

$$\tfrac{1}{2}m_R r^2\left(\frac{v_0}{r}\right) - (F_B\Delta t)r = \tfrac{1}{2}m_R r^2\left(\frac{v_1}{r}\right)$$

$$F_B\Delta t = \tfrac{1}{2}m_R(v_0 - v_1) \qquad (1)$$

ROLLER D:



+↻ MOMENTS ABOUT G: $0 + (F_D\Delta t)r = \bar{I}\omega_1$

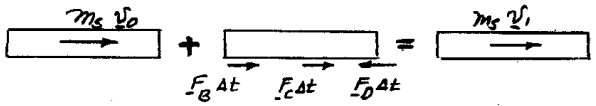
$$(F_D\Delta t)r = \tfrac{1}{2}m_R r^2\left(\frac{v_1}{r}\right) \qquad F_D\Delta t = \tfrac{1}{2}m_R v_1 \qquad (2)$$

(CONTINUED)

17.127 CONTINUED

SLAB: IMPULSES ACTING ON SLAB ARE

$\underline{F}_B\Delta t = \underline{F}_C\Delta t = \frac{1}{2}m_R(v_0 - v_1) \rightarrow$; $\underline{F}_D\Delta t = \frac{1}{2}m_R v_1 \leftarrow$



$\pm\rightarrow$ x COMPONENTS:

$$m_S v_0 + F_B\Delta t + F_C\Delta t - F_D\Delta t = m_S v_1 \qquad (3)$$

$$m_S v_0 + 2\left[\tfrac{1}{2}m_R(v_0 - v_1)\right] - \tfrac{1}{2}m_R v_1 = m_S v_1$$

$$(m_S + m_R)v_0 = (m_S + \tfrac{3}{2}m_R)v_1$$

$$\underline{v}_1 = \left[\frac{m_S + m_R}{m_S + \frac{3}{2}m_R}\right]v_0 \rightarrow$$ ◀

(b) AFTER ANOTHER ROLLER HAS BEEN ENGAGED THE VELOCITY OF THE SLAB IS:

$$v_2 = \left[\quad\right]v_1 = \left[\quad\right]^2 v_0 = \left[\frac{m_S + m_R}{m_S + \frac{3}{2}m_R}\right]^2 v_0$$

LIKEWISE AFTER MOVING n b AND ENGAGING n ROLLERS THE VELOCITY OF THE SLAB WILL BE.

$$\underline{v}_n = \left[\frac{m_S + m_R}{m_S + \frac{3}{2}m_R}\right]^n v_0 \rightarrow$$ ◀

17.128

SEE PROB. 17.127

DATA: $v_0 = 3$ m/s, $m_S = 18$ kg, $m_R = 3$ kg

$$\left[\frac{m_S + m_R}{m_S + \frac{3}{2}m_R}\right] = \left[\frac{18+3}{18 + \frac{3}{2}(3)}\right] = \frac{21}{22.5}$$

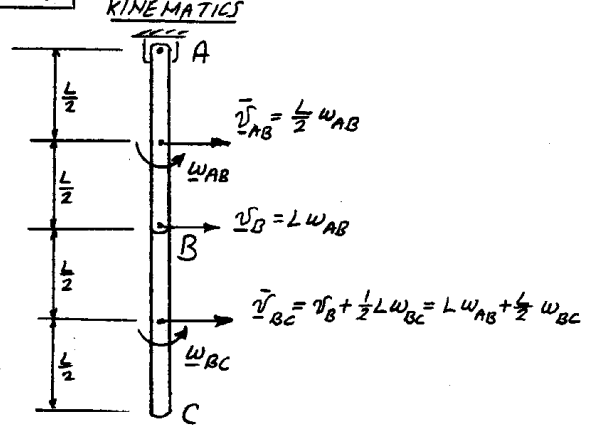
(a) $$v_1 = \left[\frac{m_S + m_R}{m_S + \frac{3}{2}m_R}\right]v_0 = \left[\frac{21}{22.5}\right](3\text{ m/s}) \qquad \underline{v}_1 = 2.8\text{ m/s} \rightarrow$$ ◀

(b) FOR $n = 8$

$$v_8 = \left[\frac{m_B + m_R}{m_B + \frac{3}{2}m_R}\right]^n v_0 = \left[\frac{21}{22.5}\right]^8(3\text{ m/s}) \qquad \underline{v}_8 = 1.727\text{ m/s} \rightarrow$$ ◀

✱17.129

KINEMATICS

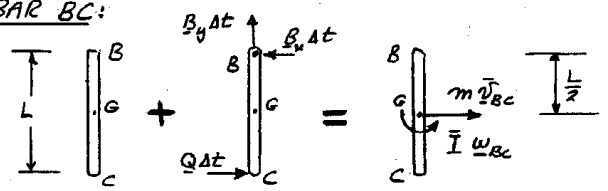


(CONTINUED)

✱17.129 CONTINUED

KINETICS:

BAR BC:



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $0 + (Q\Delta t)L = \bar{I}\omega_{BC} + m\bar{v}_{BC}\frac{L}{2}$

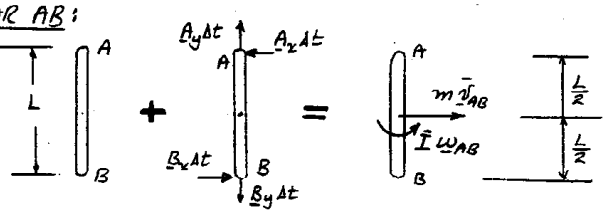
$$(Q\Delta t)L = \tfrac{1}{12}mL^2\omega_{BC} + m\left(L\omega_{AB} + \tfrac{1}{2}L\omega_{BC}\right)\tfrac{L}{2}$$

$$Q\Delta t = \tfrac{1}{2}mL\omega_{AB} + \tfrac{1}{3}mL\omega_{BC} \qquad (1)$$

$\pm\rightarrow$ x COMPONENTS: $Q\Delta t - B_x\Delta t = m\bar{v}_{BC}$

$$Q\Delta t - B_x\Delta t = m\left(L\omega_{AB} + \tfrac{L}{2}\omega_{BC}\right) \qquad (2)$$

BAR AB:



SYST. MOMENTA$_1$ + SYST. EYT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT A: $0 + (B_x\Delta t)L = \bar{I}\omega_{AB} + m\bar{v}_{AB}\frac{L}{2}$

$$(B_x\Delta t)L = \tfrac{1}{12}mL^2\omega_{AB} + m\left(\tfrac{L}{2}\omega_{AB}\right)\tfrac{L}{2}$$

$$B_x\Delta t = \tfrac{1}{3}mL\omega_{AB} \qquad (3)$$

ADD EQS. (2) AND (3):

$$Q\Delta t = \tfrac{4}{3}mL\omega_{AB} + \tfrac{1}{2}mL\omega_{BC} \qquad (4)$$

SUBTRACT EQ(1) FROM EQ(4):

$$0 = \tfrac{5}{6}mL\omega_{AB} + \tfrac{1}{6}mL\omega_{BC}$$

$$\omega_{BC} = -5\omega_{AB} \qquad (5)$$

SUBSTITUTE FOR ω_{BC} IN EQ(1):

$$Q\Delta t = \tfrac{1}{2}mL\omega_{AB} + \tfrac{1}{3}mL(-5\omega_{AB}) = -\tfrac{7}{6}mL\omega_{AB}$$

$$\omega_{AB} = -\tfrac{6}{7}\frac{Q\Delta t}{mL} \qquad (6)$$

SUBSTITUTING INTO EQ (5):

$$\omega_{BC} = -5\left(-\tfrac{6}{7}\frac{Q\Delta t}{mL}\right) \qquad \omega_{BC} = \tfrac{30}{7}\frac{Q\Delta t}{mL} \qquad (7)$$

GIVEN DATA: $L = 15$ in. $= 1.25$ ft

$Q\Delta t = 0.30$ lb·s

$W = 2.5$ lb $= 2.5$/g

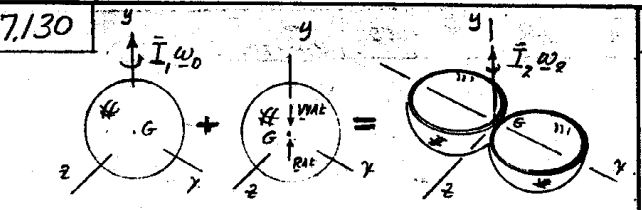
EQ(6): $$\omega_{AB} = -\frac{6}{7}\frac{Q\Delta t}{mL} = -\frac{6}{7}\frac{(0.30)(32.2)}{(2.5)(1.25)} = -2.6496\text{ rad/s}$$

$$\underline{\omega}_{AB} = 2.65\text{ rad/s}\circlearrowright$$ ◀

EQ(7): $$\omega_{BC} = \frac{30}{7}\frac{Q\Delta t}{mL} = \frac{30}{7}\frac{(0.30)(32.2)}{(2.5)(1.25)} = 13.248\text{ rad/s}$$

$$\underline{\omega}_{BC} = 13.25\text{ rad/s}\circlearrowright$$ ◀

17.130

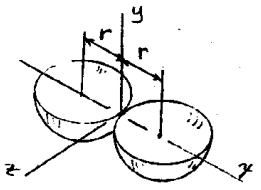


SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

MOMENTS ABOUT y AXIS: $\bar{I}_1\omega_0 = \bar{I}_2\omega_2$ (1)

FOR SPHERICAL SHELL OF MASS m, WE ARE GIVEN:

$$\bar{I}_1 = \frac{2}{3}mr^2$$



FOR EACH HEMISPHERICAL SHELL MASS $= \frac{1}{2}m$

$$\bar{I}_y = \frac{2}{3}\left(\frac{1}{2}m\right)r^2 = \frac{1}{3}mr^2$$

PARALLEL AXIS THEOREM

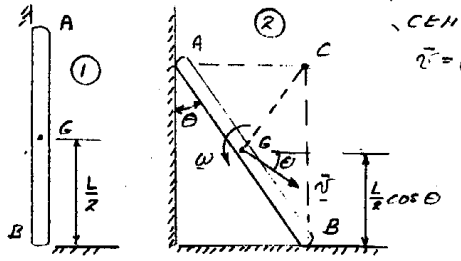
$$\bar{I}_2 = 2\left\{\bar{I}_y + \left(\frac{1}{2}m\right)r^2\right\}$$

$$= 2\left\{\frac{1}{3}mr^2 + \frac{1}{2}mr^2\right\} = \frac{5}{3}mr^2$$

EQ(1): $\bar{I}_1\omega_0 = \bar{I}_2\omega_2$

$$\frac{2}{3}mr^2\omega_0 = \frac{5}{3}mr^2\omega_2 \qquad \omega_2 = \frac{2}{5}\omega_0$$ ◀

17.131



INSTANTANEOUS CENTER AT C

$$\bar{v} = (CG)\omega = \frac{L}{2}\omega$$

$$T_1 = 0 \qquad T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2$$

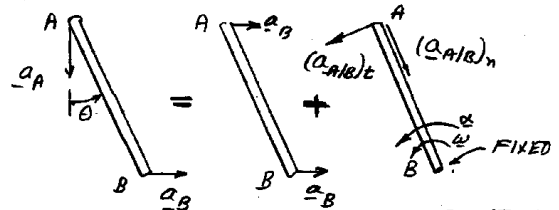
$$V_1 = mg\frac{L}{2} \qquad V_2 = mg\frac{L}{2}\cos\theta$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2}mgL = \frac{1}{6}mL^2\omega^2 + mg\frac{L}{2}\cos\theta$$

$$\omega^2 = 3\frac{g}{L}(1-\cos\theta) \qquad (1)$$

DIFFERENTIATE $\quad 2\omega\frac{d\omega}{dt} = 3\frac{g}{L}\sin\theta\frac{d\theta}{dt}$

SUBSTITUTE: $\alpha = \frac{d\omega}{dt}$, $\omega = \frac{d\theta}{dt}$: $\quad \alpha = \frac{d\omega}{dt} = \frac{3g}{2L}\sin\theta$ (2)



PLANE MOTION = TRANSLATION + ROTATION

$$\bar{a}_A = \bar{a}_B + (\bar{a}_{A/B})_t + (\bar{a}_{A/B})_n$$

$$[a_A\downarrow] = [a_B\rightarrow] + [L\alpha\ \nearrow\theta] + [L\omega^2\ \nwarrow\theta]$$

+↓ y COMPONENTS: $\quad a_A = L\alpha\sin\theta + L\omega^2\cos\theta$ (3)

(CONTINUED)

17.131 CONTINUED

SUBSTITUTE FOR ω AND α FROM EQS.(1) AND (2) INTO EQ.(3)

$$a_A = L\left(\frac{3g}{2L}\sin\theta\right)\sin\theta + L\left(3\frac{g}{L}\right)(1-\cos\theta)\cos\theta$$

WE WISH TO HAVE $a_A = 0$.

$$g = \frac{3}{2}g\sin^2\theta + 3g\cos\theta - 3g\cos^2\theta$$

$$1 = \frac{3}{2}(1-\cos^2\theta) + 3\cos\theta - 3\cos^2\theta$$

$$9\cos^2\theta - 6\cos\theta - 1 = 0$$

$$\cos\theta = \frac{6 \pm \sqrt{36+36}}{18} = \frac{1 \pm \sqrt{2}}{3}$$

$\cos\theta = -0.13807 \quad \theta = 97.94°$ (IMPOSSIBLE)

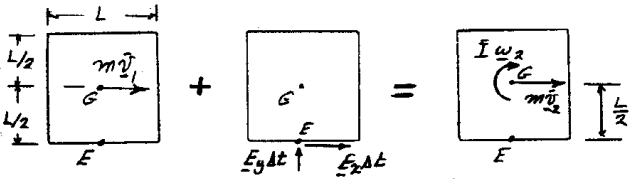
$\cos\theta = 0.80474 \quad \theta = 36.42°$

$\theta = 36.4°$ ◀

17.132

ROTATION ABOUT E: $\quad \bar{v}_2 = \frac{L}{2}\omega_2$

$$\bar{I} = \frac{1}{6}mL^2$$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT E:

$$m\bar{v}_1\frac{L}{2} + 0 = \bar{I}\omega_2 + m\bar{v}_2\frac{L}{2}$$

$$m\bar{v}_1\frac{L}{2} = \frac{1}{6}mL^2\omega_2 + m\left(\frac{L}{2}\right)^2\omega_2$$

$$m\bar{v}_1\frac{L}{2} = \frac{5}{12}mL^2\omega_2 \qquad \omega_2 = \frac{6}{5}\frac{\bar{v}_1}{L}\circlearrowright$$ ◀

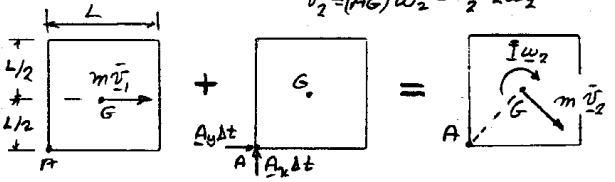
17.133

ROTATION ABOUT A:

$$\bar{I} = \frac{1}{6}mL^2$$

$$AG = \frac{\sqrt{2}}{2}L$$

$$\bar{v}_2 = (AG)\omega_2 = \frac{\sqrt{2}}{2}L\omega_2$$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A:

$$m\bar{v}_1\frac{L}{2} + 0 = \bar{I}\omega_2 + m\bar{v}_2(AG)$$

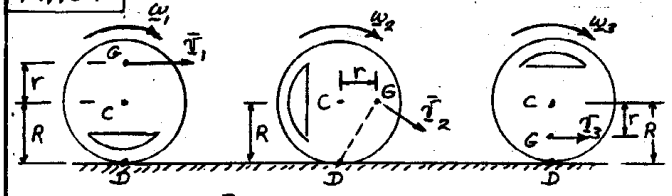
$$m\bar{v}_1\frac{L}{2} = \frac{1}{6}mL^2\omega_2 + m\left(\frac{\sqrt{2}}{2}L\omega_2\right)\left(\frac{\sqrt{2}}{2}L\right)$$

$$m\bar{v}_1\frac{L}{2} = \left(\frac{1}{6} + \frac{1}{2}\right)mL^2\omega_2$$

$$m\bar{v}_1\frac{L}{2} = \frac{2}{3}mL^2\omega_2 \qquad \omega_2 = \frac{3}{4}\frac{\bar{v}_1}{L}\circlearrowright$$ ◀

17.134

INSTANTANEOUS CENTER AT D:



KINEMATICS: $\bar{v} = \bar{v}_G = \bar{v}_C + \bar{v}_{G/C}$

$\bar{v}_1 = [R\omega_1 \rightarrow] + [r\omega_1 \rightarrow]$ $\quad \bar{v}_1 = (R+r)\omega_1$

$\bar{v}_2 = [R\omega_1 \rightarrow] + [r\omega_1 \downarrow]$ $\quad \bar{v}_2 = \sqrt{R^2+r^2}\,\omega_1$

$\bar{v}_3 = [R\omega_1 \rightarrow] + [r\omega_1 \leftarrow]$ $\quad \bar{v}_3 = (R-r)\omega_1$

KINETIC ENERGY:

$$T_1 = \tfrac{1}{2}m\bar{v}_1^2 + \tfrac{1}{2}(m\bar{k}^2)\omega_1^2 = \tfrac{1}{2}m\left[(R+r)^2 + \bar{k}^2\right]\omega_1^2 \quad (1)$$

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}(m\bar{k}^2)\omega_2^2 = \tfrac{1}{2}m\left[R^2 + r^2 + \bar{k}^2\right]\omega_2^2 \quad (2)$$

$$T_3 = \tfrac{1}{2}m\bar{v}_3^2 + \tfrac{1}{2}(m\bar{k}^2)\omega_3^2 = \tfrac{1}{2}m\left[(R-r)^2 + \bar{k}^2\right]\omega_3^2 \quad (3)$$

POTENTIAL ENERGY: PLACE DATUM AT C

$V_1 = mgr \qquad V_2 = 0 \qquad V_3 = -mgr \quad (4)$

CONSERVATION OF ENERGY:

$$\tfrac{1}{2}m\left[R^2+2Rr+r^2+\bar{k}^2\right]\omega_1^2 + mgr = \tfrac{1}{2}m\left[R^2+r^2+\bar{k}^2\right]\omega_2^2 + 0$$

$$(\omega_2^2 - \omega_1^2)(R^2+r^2+\bar{k}^2) = 2gr + 2Rr\omega_1^2 \quad (5)$$

$T_2 + V_2 = T_3 + V_3$:

$$\tfrac{1}{2}m\left[R^2+r^2+\bar{k}^2\right]\omega_2^2 = \tfrac{1}{2}m\left[R^2-2Rr+r^2+\bar{k}^2\right]\omega_3^2 - mgr$$

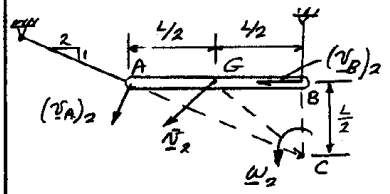
$$(\omega_3^2 - \omega_2^2)(R^2+r^2+\bar{k}^2) = 2gr + 2Rr\omega_3^2 \quad (6)$$

DIVIDE EQ(5) BY EQ(6) MEMBER BY MEMBER

$$\frac{\omega_2^2-\omega_1^2}{\omega_3^2-\omega_2^2} = \frac{2gr+2Rr\omega_1^2}{2gr+2Rr\omega_3^2} \qquad \frac{\omega_2^2-\omega_1^2}{\omega_3^2-\omega_2^2} = \frac{(3/2)+\omega_1^2}{(3/2)+\omega_3^2}$$ ◀

17.135

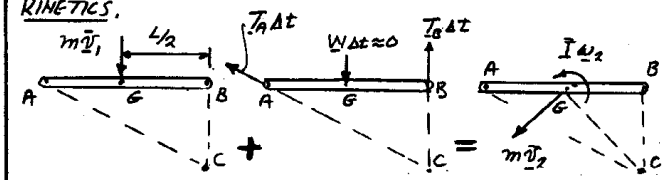
KINEMATICS. AFTER IMPACT POINTS A AND B MOVE IN DIRECTIONS ⊥ TO THE CORDS. WE DRAW LINES ⊥ TO $(\bar{v}_A)_2$ AND $(\bar{v}_B)_2$ TO LOCATE THE INSTANTANEOUS CENTER C.



$CG = \sqrt{(\frac{L}{2})^2+(\frac{L}{2})^2} = \frac{\sqrt{2}}{2}L$

$\bar{v}_2 = (CG)\omega_2 = \frac{\sqrt{2}}{2}L\omega_2$

KINETICS.



SYST. MOMENTA$_1$ + SYST. EXT IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$m\bar{v}_1(\tfrac{1}{2}L) + 0 = \bar{I}\omega_2 + m\bar{v}_2(CG)$$

$$m v_1 \tfrac{L}{2} + 0 = \tfrac{1}{12}mL^2\omega_2 + m\left(\tfrac{\sqrt{2}}{2}L\omega_2\right)\left(\tfrac{\sqrt{2}}{2}L\right)$$

$$mv_1\tfrac{L}{2} = \tfrac{7}{12}mL^2\omega_2 \qquad \omega_2 = \tfrac{6}{7}\tfrac{\bar{v}_1}{L}\ \circlearrowright$$ ◀

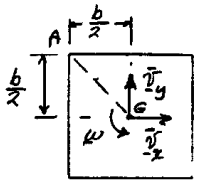
$$\bar{v}_2 = \tfrac{\sqrt{2}}{2}L\omega_2 = \tfrac{\sqrt{2}}{2}L\left(\tfrac{6}{7}\tfrac{\bar{v}_1}{L}\right) \qquad \bar{v}_2 = \tfrac{3\sqrt{2}}{7}\bar{v}_1\ \angle 45^\circ$$ ◀

17.136

ASSUME ω ↻ AND HENCE THAT WIRE B WILL GO SLACK. THUS $T_B = 0$.

KINEMATICS: VERTICAL COMPONENT OF $\bar{v}_A$ MUST BE ZERO

$$AG = \sqrt{2}\,\frac{b}{2}$$



$\bar{v}_A = \bar{v} + \bar{v}_{A/G}$

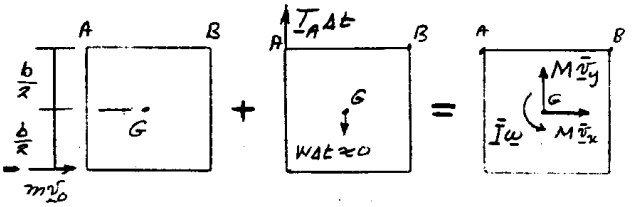
$[\bar{v}_A \rightarrow] = [\bar{v}_x \rightarrow] + [\bar{v}_y \uparrow] + \left[\left(\sqrt{2}\,\frac{b}{2}\right)\omega\ \angle 45^\circ\right]$

+↑ y COMPONENTS:

$$0 = \bar{v}_y - \left(\sqrt{2}\,\frac{b}{2}\right)\omega \sin 45^\circ$$

$$0 = \bar{v}_y - \tfrac{1}{2}b\omega \qquad \bar{v}_y = \tfrac{1}{2}b\omega \quad (1)$$

PRINCIPLE OF IMPULSE - MOMENTUM:



SYST. MOMENTA$_1$ + SYST. EXT IMP$_{1-2}$ = SYST. MOMENTA$_2$

+→ x COMPONENTS: $\quad m v_0 = M\bar{v}_x \qquad \bar{v}_x = \frac{m}{M}v_0 \quad (2)$

+↻ MOMENTS ABOUT A:

$$m v_0 b = \bar{I}\omega + M\bar{v}_x\tfrac{b}{2} + M\bar{v}_y\tfrac{b}{2} \quad (3)$$

WE SUBSTITUTE: $\bar{v}_x = \frac{m}{M}v_0$, $\bar{v}_y = \frac{1}{2}b\omega$,

$$\bar{I} = \tfrac{1}{12}M(b^2+b^2) = \tfrac{1}{6}Mb^2$$

$$m v_0 b = \tfrac{1}{6}Mb^2\omega + M\left(\tfrac{m}{M}v_0\right)\tfrac{b}{2} + M\left(\tfrac{1}{2}b\omega\right)\tfrac{b}{2}$$

$$\tfrac{1}{2}mv_0 b = \left(\tfrac{1}{6}+\tfrac{1}{4}\right)Mb^2\omega$$

$$\tfrac{1}{2}mv_0 b = \tfrac{5}{12}Mb^2\omega \qquad \omega = \tfrac{6}{5}\tfrac{mv_0}{Mb}\ \circlearrowright \quad (4)$$

EQ(1): $\bar{v}_y = \frac{1}{2}b\omega = \frac{1}{2}b\left(\frac{6}{5}\frac{mv_0}{Mb}\right) \qquad \bar{v}_y = \frac{3}{5}\frac{mv_0}{M}$

$$\bar{v} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j} = \frac{mv_0}{M}\left(\hat{i} + 0.6\hat{j}\right)$$

$$\bar{v} = 1.166\left(\frac{mv_0}{M}\right)\ \angle 31.0^\circ \quad (5)$$

GIVEN DATA:

$m = 25\,g = 0.025\,kg \qquad M = 3\,kg$

$v_0 = 400\,m/s \qquad b = 500\,mm = 0.5\,m$

EQ(4) $\quad \omega = \frac{6}{5}\frac{mv_0}{Mb} = \frac{6}{5}\frac{(0.025\,kg)(400\,m/s)}{(3\,kg)(0.5\,m)}$

$\omega = 8\,rad/s \qquad \underline{\omega = 8\,rad/s}\ \circlearrowright$ ◀

EQ(5) $\quad \bar{v} = 1.166\,\frac{(0.025\,kg)(400\,m/s)}{3\,kg}\ \angle 31.0^\circ$

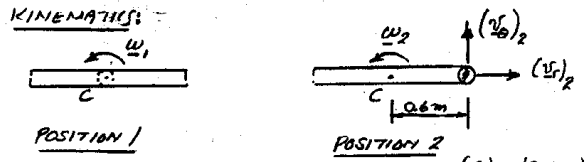
$\bar{v} = 3.89\,m/s\ \angle 31.0^\circ$ ◀

NOTE: ω IS ↻. THE ASSUMPTION MADE AT BEGINNING OF THE SOLUTION IS CORRECT

17.137

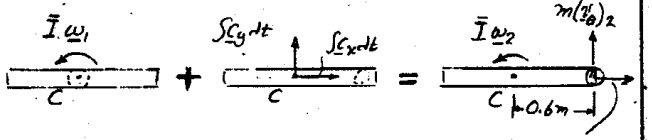
LOOKING FROM ABOVE.

KINEMATICS:



$(v_\theta)_2 = (0.6\,m)\omega_2$

PRINCIPLE OF IMPULSE - MOMENTUM:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$\bar{I}\omega_1 + 0 = \bar{I}\omega_2 + m(v_\theta)_2(0.6\,m) \qquad (1)$$

DATA: $m = 0.5\ kg$, $\omega_1 = 8\ rad/s$

$$\bar{I} = I_{rod} = \frac{1}{12}(1.5\,kg)(1.2\,m)^2 = 0.18\ kg\cdot m^2$$

EQ(1): $(0.18\ kg\cdot m^2)(8\ rad/s) = (0.18\ kg\cdot m^2)\omega_2 + (0.5\ kg)(0.6\,\omega_2)(0.6\,m)$

$$1.44 = 0.18\,\omega_2 + 0.18\,\omega_2$$

(a) $1.44 = 0.36\,\omega_2$ $\quad \omega_2 = 4.00\ rad/s$ $\quad \underline{\omega_2 = 4.00\ rad/s\ \circlearrowright}$ ◀

(b) AFTER THE BALL LEAVES THE TUBE, THE ANGULAR VELOCITY IS UNCHANGED. $\underline{\omega_2 = 4.00\ rad/s\ \circlearrowright}$ ◀

17.138

THE TRANSVERSE COMPONENT OF THE VELOCITY OF THE BALL IS THE SAME AS THAT OBTAINED IN PROB. 17.137.

$\omega_2 = 4.00\ rad/s$ $\quad (v_\theta)_2 = (0.6\,m)(4.00\ rad/s)$

$(v_\theta)_2 = 2.40\ m/s$ $\quad \underline{(v_\theta)_2 = 2.40\ m/s}$ ◀

CONSERVATION OF ENERGY:

$V_1 = V_2 = 0$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 + \frac{1}{2}m v_1^2$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\left[(v_\theta)_2^2 + (v_r)_2^2\right]$$

$T_1 + V_1 = T_2 + V_2$: $\frac{1}{2}\bar{I}\omega_1^2 + \frac{1}{2}m v_1^2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\left[(v_\theta)_2^2 + (v_r)_2^2\right]$

$$\bar{I}(\omega_1^2 - \omega_2^2) + m v_1^2 = m\left[(v_\theta)_2^2 + (v_r)_2^2\right] \qquad (1)$$

FROM DATA, PART (a), AND PROB. 17.137

$\bar{I} = 0.18\ kg\cdot m^2$, $\quad m = 0.5\ kg$, $\quad v_1 = 1.8\ m/s$

$\omega_1 = 8\ rad/s$, $\quad \omega_2 = 4\ rad/s$, $\quad (v_\theta)_2 = 2.40\ m/s$

SUBSTITUTE INTO (1)

$$(0.18\ kg\cdot m^2)\left[(8\ rad/s)^2 - (4\ rad/s)^2\right] + (0.5\ kg)(1.8\ m/s)^2 = (0.5\ kg)\left[(2.40\ m/s)^2 + (v_r)_2^2\right]$$

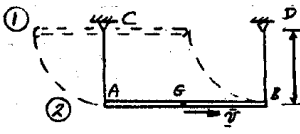
$$8.64 + 1.620 = 2.88 + 0.5(v_r)_2^2$$

$$(v_r)_2^2 = 14.76\ m^2/s^2$$

$(v_r)_2 = 3.842\ m/s$ $\quad \underline{(v_r)_2 = 3.84\ m/s}$ ◀

17.139

CURVILINEAR TRANSLATION $\quad \omega = 0$



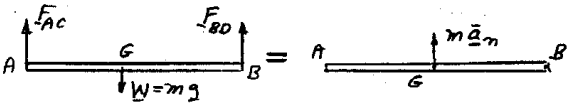
$T_1 = 0$ $\quad U_{1-2} = mgb$

$T_2 = \frac{1}{2}m\bar{v}^2$

$T_1 + U_{1-2} = T_2$

$0 + mgb = \frac{1}{2}m\bar{v}^2$ $\quad \bar{v}^2 = 2gb$

(a) JUST BEFORE BD BREAKS: $\quad \bar{a}_n = \frac{\bar{v}^2}{b} = \frac{2gb}{b} = 2g\uparrow$ $\quad \alpha = 0$



BY SYMMETRY: $\quad F_{AC} = F_{BD}$

$+\uparrow \Sigma F_y = m\bar{a}_n$: $\quad 2F_{AC} - W = m\bar{a}_n$

$$2F_{AC} - W = \frac{W}{g}(2g)$$

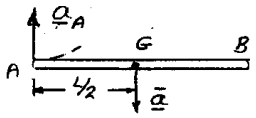
$2F_{AC} - W = 2W$ $\quad F_{AC} = F_{BD} = \frac{3}{2}W$ ◀

(b) JUST AFTER BD BREAKS:

END A CONTINUES TO MOVE ALONG CIRCLE OF RADIUS b WITH SPEED $\bar{v}$, THUS

$$\underline{a}_A = \frac{\bar{v}^2}{b} = \frac{2gb}{b} = 2g\uparrow$$

KINEMATICS:

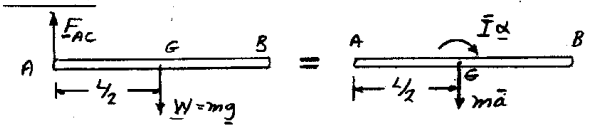


$\bar{a} = a_A + a_{G/A}$

$= [2g\uparrow] + [\frac{1}{2}\alpha\downarrow]$

$\bar{a} = (\frac{1}{2}\alpha - 2g)\downarrow$

KINETICS



$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad mg\frac{L}{2} = m\bar{a}\frac{L}{2} + \bar{I}\alpha$

$$mg\frac{L}{2} = m\left(\frac{L}{2}\alpha - 2g\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

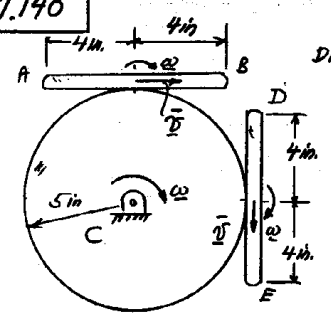
$\frac{3}{2}\frac{g}{L} = \frac{1}{3}\alpha$ $\quad \alpha = \frac{9}{2}\frac{g}{L}$

$+\circlearrowright \Sigma M_G = \bar{I}\alpha$: $\quad F_{AC}(\frac{L}{2}) = \bar{I}\alpha$

$$\frac{1}{2}F_{AC}L = \frac{1}{12}\frac{W}{g}L^2\left(\frac{9}{2}\frac{g}{L}\right)$$

$\frac{1}{2}F_{AC} = \frac{9}{24}W$ $\quad F_{AC} = \frac{3}{4}W$ ◀

17.140



KINETIC ENERGY FOR DISK + RODS ASSEMBLY

$W_{DISK} = 8\ lb$

$W_{ROD} = 5\ lb$

$\bar{I}_{ROD} = \frac{1}{12} m_{ROD} L^2$

$= \frac{1}{12}\left(\frac{5\ lb}{32.2\ ft/s^2}\right)\left(\frac{8}{12}\ ft\right)^2$

$= 5.751 \times 10^{-3}\ lb \cdot ft \cdot s^2$

$\bar{I}_{DISK} = \frac{1}{2} m_c r^2 = \frac{1}{2}\left(\frac{8\ lb}{32.2\ ft/s^2}\right)\left(\frac{5}{12}\ ft\right)^2 = 21.567 \times 10^{-3}\ lb \cdot ft \cdot s^2$

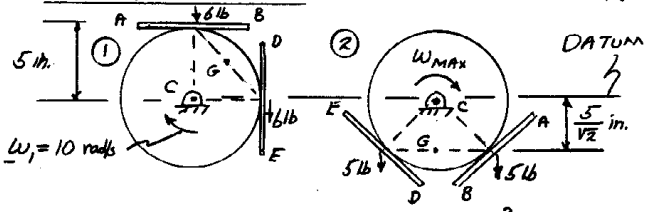
$\bar{v} = r\omega = \left(\frac{5}{12}\ ft\right)\omega$

$\bar{I}_C = \bar{I}_{DISK} + 2\left[\bar{I}_{ROD} + m_{ROD}\left(\frac{5}{12}\ ft\right)^2\right]$

$= 21.567 \times 10^{-3} + 2\left[5.751 \times 10^{-3} + \frac{5}{32.2}\left(\frac{5}{12}\right)^2\right] = 86.986 \times 10^{-3}\ lb \cdot ft \cdot s^2$

$T = \frac{1}{2}\bar{I}_C \omega^2 = (43.492 \times 10^{-3})\omega^2 \qquad (1)$

LARGEST ANGULAR VELOCITY: LET G BE MASS CENTER OF AB + DE



POSITION 1: $T_1 = (43.492 \times 10^{-3})(10\ rad/s)^2 = 4.3492\ ft \cdot lb$

$V_1 = (5\ lb)\left(\frac{5}{12}\ ft\right) = 2.0833\ ft \cdot lb$

POSITION 2: SINCE G IS AT ITS LOWEST ELEV. $\omega_2 = \omega_{MAX}$

$T_2 = (43.492 \times 10^{-3})\omega_{MAX}^2$

$V_2 = -2\left[(5\ lb)\left(\frac{5\ in}{\sqrt{2}} \times \frac{1\ ft}{12\ in}\right)\right] = -2.9463\ ft \cdot lb$

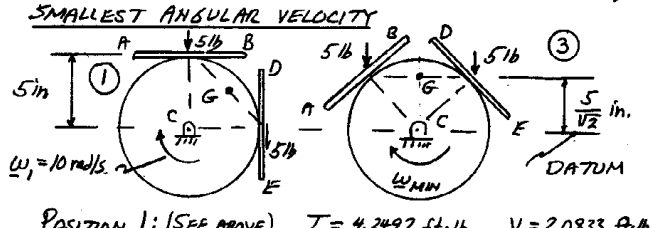
CONSERVATION OF ENERGY $T_1 + V_1 = T_2 + V_2$

$4.3492 + 2.0833 = 43.492 \times 10^{-3}\omega_{MAX}^2 - 2.9463$

$\omega_{MAX}^2 = 215.65 \qquad \omega_{MAX} = 14.684\ rad/s$

$\underline{\omega_{MAX} = 14.68\ rad/s\ \curvearrowright}$

SMALLEST ANGULAR VELOCITY



POSITION 1: (SEE ABOVE) $T_1 = 4.3492\ ft \cdot lb \qquad V_1 = 2.0833\ ft \cdot lb$

POSITION 3: SINCE G IS AT ITS HIGHEST ELEV. $\omega_3 = \omega_{MIN}$

$T_3 = (43.492 \times 10^{-3})\omega_{MIN}^2$

$V_3 = +2\left[(5\ lb)\left(\frac{5\ in.}{\sqrt{2}} \times \frac{1\ ft}{12\ in.}\right)\right] = +2.9463\ ft \cdot lb$

CONSERVATION OF ENERGY $T_1 + V_1 = T_3 + V_3$

$4.3492 + 2.0833 = (43.492 \times 10^{-3})\omega_{MIN}^2 + 2.9463$

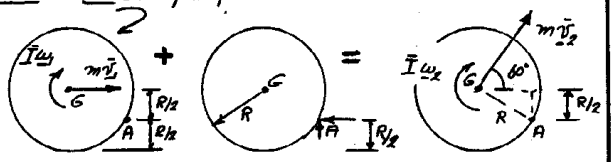
$\omega_{MIN}^2 = 80.158 \qquad \omega_{MIN} = 8.953\ rad/s$

$\underline{\omega_{MIN} = 8.95\ rad/s\ \curvearrowright}$

17.141

AFTER IMPACT THE SPHERE ROTATES ABOUT CORNER A. THUS: $\bar{v}_2 = R\omega_2 \nearrow$

KINETICS ROLLS: $\bar{v}_1 = R\omega_1$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A:

$\bar{I}\omega_1 + (m\bar{v}_1)\frac{R}{2} + 0 = \bar{I}\omega_2 + (m\bar{v}_2)R$

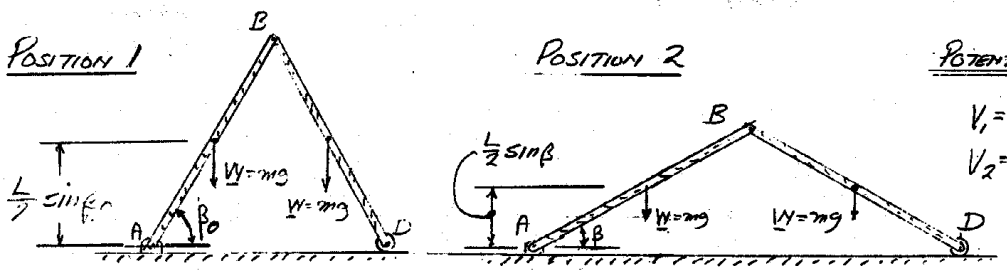
$\frac{2}{5}mR^2\omega_1 + m(R\omega_1)\frac{R}{2} = \frac{2}{5}mR^2\omega_2 + (mR\omega_2)R$

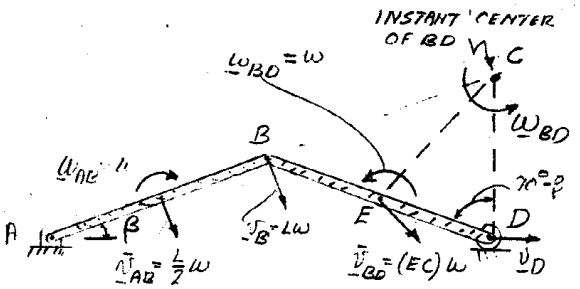
$\frac{9}{10}mR^2\omega_1 = \frac{7}{5}mR^2\omega_2$

$\omega_2 = \frac{9}{10}\cdot\frac{5}{7}\omega_1 \qquad \underline{\omega_2 = \frac{9}{14}\omega_1\ \curvearrowright}$

$\bar{v}_2 = R\omega_2 = \frac{9}{14}R\omega_1 = \frac{9}{14}\bar{v}_1 \qquad \underline{v_2 = \frac{9}{14}v_1\ \angle 60°}$

PROBLEM 17.C1





KINETIC ENERGY POSITION 1: (velocity = 0) $T_1 = 0$

POSITION 2: IN $\triangle ACD$: $CD = 2L\sin\beta$

IN $\triangle CED$: $(EC)^2 = (CD)^2 + \left(\frac{L}{2}\right)^2 - 2(CD)\left(\frac{L}{2}\right)\cos(90^\circ - \beta)$

$$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}\omega_{AB}^2 + \frac{1}{2}m\bar{v}_{BD}^2 + \frac{1}{2}\bar{I}\omega_{BD}^2$$

$$T_2 = \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2 + \frac{1}{2}m(EC)^2\omega^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2$$

$$= \left[\frac{1}{8} + \frac{1}{24} + \frac{1}{2}\left(\frac{EC}{L}\right)^2 + \frac{1}{24}\right]mL^2\omega^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2: \quad 0 + mgL\sin\beta_0 = \left[\frac{5 + 12(EC/L)^2}{24}\right]mL^2\omega^2 + mgL\sin\beta$$

$$\omega^2 = \frac{24g}{L}\cdot\frac{\sin\beta_0 - \sin\beta}{5 + 12(EC/L)^2} \qquad v_D = (CD)\omega$$

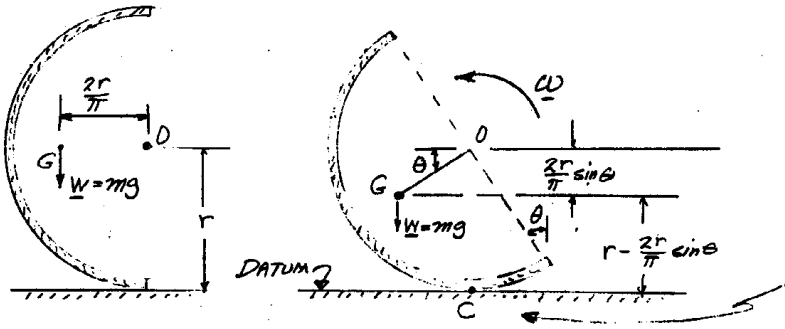
```
10 '*************************************************************
20 '*                      PROBLEM 17.C1                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         BETA = angle beta   (degrees)
90 '        BETAO = initial value of angle BETA
100 '          L = length of each rod     (meters)
110 '          M = mass of each rod        (kg)
111 '          E = mass center of rod BD
120 '         CD = distance from instantaneous center C to point D
130 '         ED = distance from instantaneous center C to point E
140 '        OMEGA = angular velocity of rods    (rad/s)
150 '          VD = velocity of point D    (m/s)
160 '
170 '     Data
180   LET L = .75
190   LET M = 6
200   LET G = 9.810001
210   LET BETAO = 60
220 '
230   K = 4*ATN(1)/180
240 '
250   PRINT " BETA          OMEGA          VD
260   PRINT "degrees        rad/s          m/s":PRINT
270 '
280   FOR BETA = 60 TO 0 STEP -5
290     CD = 2*L*SIN(K*BETA)
300     EC = SQR(CD^2 + (L/2)^2 - CD*L*COS(K*(90 - BETA)))
310     OMEGA = SQR((24*G/L)*(SIN(K*BETAO)-SIN(K*BETA))/(5+12*(EC/L)^2))
320     VD = CD*OMEGA
330     PRINT USING "  ##          ";BETA,
340     PRINT USING " #.###         ";OMEGA,VD
350   NEXT
```

BETA degrees	OMEGA rad/s	VD m/s
60	0.000	0.000
55	0.781	0.960
50	1.192	1.370
45	1.579	1.675
40	1.978	1.907
35	2.403	2.068
30	2.865	2.149
25	3.366	2.134
20	3.901	2.002
15	4.454	1.729
10	4.991	1.300
5	5.466	0.715
0	5.829	0.000

```
C*************************************************************
C*                      PROBLEM 17.C1                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C           Definition of Symbols

C                  SEE LIST AT LEFT

C     DATA
C
      REAL OMEGA, L, M, K
      L = 0.75
      M = 6.
      G = 9.81
      BETAO = 60.
      K = 4.*ATAN(1.)/180.
C
      WRITE (*,*) ' BETA            OMEGA           VD'
      WRITE (*,*) 'degrees          rad/s           m/s'
      WRITE (*,*) ' '
C
      DO 10 I = 60,0,-5
         BETA = I
         CD = 2.*L*SIN(K*BETA)
         EC = SQRT(CD**2 + (L/2.)**2 - CD*L*COS(K*(90.- BETA)))
C           Define DENOM as follows:
         DENOM = 5. + 12.*(EC/L)**2
         OMEGA = SQRT((24.*G/L)*(SIN(K*BETAO)-SIN(K*BETA))/DENOM)
         VD = CD*OMEGA
         WRITE (*,20) BETA,OMEGA,VD
   20    FORMAT (4X,F4.0,10X,F6.3,10X,F6.3)
   10 CONTINUE
      END
```

BETA degrees	OMEGA rad/s	VD m/s
60.	.000	.000
55.	.781	.960
50.	1.192	1.370
45.	1.579	1.675
40.	1.978	1.907
35.	2.403	2.068
30.	2.865	2.149
25.	3.366	2.134
20.	3.901	2.002
15.	4.454	1.729
10.	4.991	1.300
5.	5.466	.715
0.	5.829	.000

PROBLEM 17.C2



C = INSTANTANEOUS CENTER OF ROTATION

POSITION 1: $\theta = 0$

velocity $= 0$; $T_1 = 0$

$V_1 = (\text{POTENTIAL ENERGY})_{\theta=0}$

POSITION 2:

$T_2 = \frac{1}{2} I_c \omega^2$ WHERE I_c = MASS MOMENT OF INERTIA ABOUT INSTANT. CENTER C (see below)

$V_2 = (\text{POTENTIAL ENERGY})_{\theta=\theta} = mg\left(r - \frac{2r}{\pi}\sin\theta\right)$ ◁

CONSERVATION OF ENERGY:

$T_1 + V_1 = T_2 + V_2$: $0 + PE_{\theta=0} = \frac{1}{2} I_c \omega^2 + PE_{\theta=\theta}$

$$\omega = \sqrt{\frac{2}{I_c}\left(PE_{\theta=0} - PE_{\theta=\theta}\right)}$$ ◁

KINEMATICS: CONSIDER MOTION AS PIPE ROLLS THROUGH ANGLE $\Delta\theta$ FROM

$\theta = \theta_i$ WHERE $\omega = \omega_i$

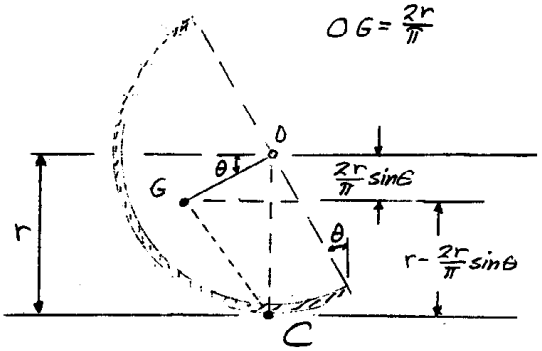
TO $\theta = \theta_{i+1}$ WHERE $\omega = \omega_{i+1}$

IF WE ASSUME ANGULAR ACCELERATION IS CONSTANT OVER TIME INTERVAL Δt, THE AVERAGE VELOCITY IS: $\omega_{AVG} = \frac{1}{2}(\omega_i + \omega_{i+1})$ ◁

AND TIME REQUIRED FOR PIPE TO ROLL THROUGH $\Delta\theta$ IS: $\Delta t = \frac{\Delta\theta}{\omega_{AVG}}$ ◁

WE THEN SUM Δt TO FIND TIME REQUIRED FOR PIPE TO ROLL THROUGH 180°.

DETERMINATION OF I_c: I_c IS MASS MOMENT INERTIA OF PIPE ABOUT INSTANTANEOUS CENTER OF ROTATION FOR POSITION $\theta = \theta$.



$OG = \frac{2r}{\pi}$

IN $\triangle COG$, FROM LAW OF COSINES:

$$(CG)^2 = (OG)^2 + (OC)^2 - 2(OG)(OC)\cos(90° - \theta)$$

$$(CG)^2 = \left(\frac{2r}{\pi}\right)^2 + r^2 - 2\left(\frac{2r}{\pi}\right)(r)\sin\theta$$

$$(CG)^2 = \frac{4r^2}{\pi^2} + r^2 - \frac{4r^2}{\pi}\sin\theta$$

$I_O = mr^2$

$I_O = \bar{I}_G + (OG)^2 m$; $mr^2 = \bar{I}_G + \left(\frac{2r}{\pi}\right)^2 m$

$$\bar{I}_G = mr^2 - \frac{4r^2}{\pi^2} m$$

$I_c = \bar{I}_G + (CG)^2 m$

$= mr^2 - \frac{4r^2}{\pi^2} m + \left(\frac{4r^2}{\pi^2} + r^2 - \frac{4r^2}{\pi}\sin\theta\right) m$

$= mr^2\left(2 - \frac{4}{\pi}\sin\theta\right)$

$$I_c = 2mr^2\left(1 - \frac{2}{\pi}\sin\theta\right)$$ ◁

NOTE: SINCE THE MASS OF THE PIPE APPEARS IN BOTH I_c AND POTENTIAL ENERGY, ITS VALUE IS IMMATERIAL. WE CHOOSE $m = 1$

CONTINUED

PROBLEM 17.C2 CONTINUED

```
10 '*****************************************************************
20 '*'                      PROBLEM 17.C2                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '              T = time    (seconds)
90 '             TH = angle theta formed by OG and horizontal  (degrees)
100 '          TH0 = initial value of angle theta
110 '          THF = final value of angle theta
120 '          DTH = increment of angle theta    (degrees)
130 '
140 '           OM = omega, angular velocity of section   (rad/s)
150 '         OMIN = value of OM at beginning of interval DTH
160 '         OMFL = value of OM at end of interval DTH
170 '         OMAV = average value of OM over interval DTH
180 '      FNPE(TH) = potential energy as a function of TH
190 '    FNINRT(TH) = moment of inertia about instantaneous center
200 '                 as a function of TH
210 '             Y = counter
220 '           NBR = number of increments in a printing interval
230 '
240 '       Data
250 '
260     LET TH0 = 0       'initial value of theta   (degrees)
270     LET THF = 90      'final value of theta   (degrees)
280     LET DTH = 1       'increment of theta   (degrees)
290 '
300     LET NBR = 5
310     LET R = 8/12      'ft
320     LET G = 32.2      'ft/s^2
330 '
340     PI = 4*ATN(1)
350     K = PI/180
360 '
370     PRINT " Theta              Time        Angular velocity"
380     PRINT "degrees            seconds          rad/s":PRINT
390 '
400     DEF FNINRT(TH) = 2*R^2*(1 - (2/PI)*SIN(K*TH))
410     DEF FNPE(TH) = G*(R - (2*R/PI)*SIN(K*TH))
420 '
430 '    Incremental Loop and Printing Instructions
440 '
450     WHILE Y <= THF/DTH
460        IF INT(Y/NBR) = Y/NBR THEN 470 ELSE 490
470        PRINT USING " ###            "; TH,
480        PRINT USING " ##.####        "; T, OMIN
490        Y = Y + 1
500        TH = TH0 + Y * DTH
510        OMFL = SQR((2/FNINRT(TH))*(FNPE(TH0) - FNPE(TH)))
520        OMAV = .5*(OMIN + OMFL)
530        T = T + (K*DTH/OMAV)
540        OMIN = OMFL
550     WEND
560     TIME = 2 * (T - K*DTH/OMAV): PRINT
570     PRINT USING "Time to roll through 180 degrees = #.### s";TIME
580     END
```

```
 Theta          Time        Angular velocity
degrees        seconds           rad/s

   0          0.0000          0.0000
   5          0.1054          1.6844
  10          0.1477          2.4501
  15          0.1793          3.0868
  20          0.2052          3.6666
  25          0.2273          4.2164
  30          0.2468          4.7498
  35          0.2643          5.2708
  40          0.2801          5.7840
  45          0.2945          6.2884
  50          0.3079          6.7806
  55          0.3203          7.2552
  60          0.3320          7.7040
  65          0.3430          8.1165
  70          0.3535          8.4804
  75          0.3636          8.7824
  80          0.3734          9.0096
  85          0.3830          9.1509
  90          0.3925          9.1988

Time to roll through 180 degrees = 0.785 s
```

```
C*****************************************************************
C*                      PROBLEM 17.C2                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C                T = time   (seconds)
C               TH = angle theta formed by OG and horizontal  (degrees)
C              TH0 = initial value of angle theta
C              THF = final value of angle theta
C              DTH = increment of angle theta   (degrees)
C
C               OM = omega, angular velocity of section   (rad/s)
C             OMIN = value of OM at the begining of interval DTH
C             OMFL = value of OM at end of interval DTH
C             OMAV = average value of OM over interval DTH
C          FNPE(TH) = potential energy as a function of TH
C        FNINRT(TH) = moment of inertia about instanteneous center
C                     as a function of TH
C                Y = counter
C              NRB = number of increments in a printing interval
C

      REAL  K, NBR
C
C     Define function
C
      FNINRT(TH) = 2.*R**2*(1.- (2./PI)*SIN(K*TH))
      FNPE(TH) = G*(R - (2.*R/PI)*SIN(K*TH))
C
      PI = 4.*ATAN(1.)
      K = PI/180.
C
      TH0 = 0.
      THF = 90.
      DTH = 1.
C
      NBR = 5.
      R = 8./12.
      G = 32.2
C
      WRITE (*,'(/,5X,A\)') 'Theta          Time       Angular Velocity'
      WRITE (*,'(/,5X,A\)') 'degrees       seconds          rad/s'
      WRITE (*,'(/,A\)')
C
C     Incremental loop and printing Instructions
C
  490 IF (INT(Y/NBR).NE.(Y/NBR)) THEN
                                    GOTO 530
      ENDIF
C
      WRITE (*,20) TH,T,OMIN
   20 FORMAT (7X,F3.0,9X,F7.4,9X,F7.4)
  530    Y = Y + 1
         TH = TH0 + Y * DTH
      OMFL = SQRT((2./FNINRT(TH))*(FNPE(TH0) -FNPE(TH)))
      OMAV = 0.5*(OMIN + OMFL)
         T = T + (K*DTH/OMAV)
      OMIN = OMFL
C
      IF (Y.LE.(THF/DTH)) GOTO 490
      TIME = 2 * (T - K*DTH/OMAV)
C     WRITE (*,'(/,A\)')
      WRITE (*,21) TIME
   21 FORMAT (/,T7,'Time to roll through 180 degrees = ',F5.3,' s')
      END
```

```
   Theta          Time       Angular Velocity
   degrees       seconds          rad/s

     0.           .0000           .0000
     5.           .1054          1.6844
    10.           .1477          2.4501
    15.           .1793          3.0868
    20.           .2052          3.6666
    25.           .2273          4.2164
    30.           .2468          4.7498
    35.           .2643          5.2708
    40.           .2801          5.7840
    45.           .2945          6.2884
    50.           .3079          6.7806
    55.           .3203          7.2552
    60.           .3320          7.7040
    65.           .3430          8.1165
    70.           .3535          8.4804
    75.           .3636          8.7824
    80.           .3734          9.0096
    85.           .3830          9.1509
    90.           .3925          9.1988

 Time to roll through 180 degrees =  .785 s
```

```
10 '*****************************************************************
20 '*'                      PROBLEM 17.C3                            *
30 '*                       BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '           MT = mass of tube    (kg)
90 '           MR = mass of rod     (kg)
100 '           L = length of tube and rod  (m)
110 '           X = distance rod has moved out of tube   (m)
120 '      OMEGA1 = initial angular velocity of assembly  (rad/s)
130 '      OMEGA2 = final angular velocity of assembly    (rad/s)
140 '          VR = velocity of rod relative to tube  (m/s)
150 '
160    LET L = .24       'length of rod  (meters)
170    LET MR = 1.5      'mass of rod    (kg)
180    LET OMEGA1 = 8    'initial angular velocity  (rad/s)
190 '
200    DATA 0.75, 1.5, 3, 0
210    READ MT:  IF MT = 0 GOTO 550
220    LET N = MT/MR
230 '
240    PRINT:PRINT
250    PRINT USING "    Mass of tube: MT = #.## kg";MT:PRINT
260    PRINT "  X         OMEGA 2          VR        "
270    PRINT "  mm         rad/s           m/s"
280    PRINT " "
290 '
300    FOR X = .04 TO .2 STEP .04
310 '
320 ' Conservation of angular momentum
330 '           ANGM1 = initial angular momentum = (COEF1)*OMEGA1
340      COEF1 = ((1 + N)/3)*MR*L^2
350 '           ANGM2 = final angular momentum = (COEF2)*OMEGA2
360      COEF2 = (N/3 + (1/12 + (X/L +.5)^2))*MR*L^2
370 '         ANGM1 = ANGM2:   (COEF1)*OMEGA1 = (COEF2)*OMEGA2
380      OMEGA2 = (COEF1/COEF2)*OMEGA1
390 '
400 ' Conservation of energy (Note that potential energy is constant)
410 '           T1 = initial kinetic energy = 0.5*(COEF1)*OMEGA1^2
420 '           T2 = final kinetic energy
430 '              = 0.5*(COEF2)*OMEGA2^2 + 0.5*MR*VR^2
440 '
450 '                          T1 = T2
460 '        0.5*(COEF1)*OMEGA1^2 = 0.5*(COEF2)*OMEGA2^2 +0.5*MR*VR^2
470 '
480      VR = SQR(((COEF1)*OMEGA1^2 - (COEF2)*OMEGA2^2)/MR)
490 '
500      DISTANCE = 1000*X     'express X in millimeters
510      PRINT USING " ###       ";DISTANCE,
520      PRINT USING " #.###         ";OMEGA2,VR
530    NEXT
540    GOTO 210
550    END
```

Mass of tube: MT = 0.75 kg

X mm	OMEGA 2 rad/s	VR m/s
40	5.760	0.718
80	4.235	0.931
120	3.200	1.052
160	2.483	1.127
200	1.973	1.178

X mm	OMEGA 2 rad/s	VR m/s
40	6.194	0.745
80	4.800	0.991
120	3.765	1.141
160	3.000	1.239
200	2.430	1.308

Mass of tube: MT = 3.00 kg

X mm	OMEGA 2 rad/s	VR m/s
40	6.698	0.775
80	5.538	1.065
120	4.571	1.257
160	3.789	1.393
200	3.165	1.493

```
C*****************************************************************
C*                      PROBLEM 17.C3                            *
C*                    FORTRAN    VERSION                        *
C*****************************************************************
C
C           Definition of Symbols
C
C               MT = mass of tube   (kg)
C               MR = mass of rod    (kg)
C                L = length of tube and rod (m)
C                X = distance rod has moved out of tube (m)
C           OMEGA1 = initial angular velocity of assembly   (rad/s)
C           OMEGA2 = final angular velocity of assembly   (rad/s)
C               VR = velocity of rod relative to tube   (m/s)
C
      REAL L, MT(4), MR, N, METERS
      DATA MT/0.75,1.5,3.0,0.0/
C
      L = 0.24
      MR = 1.5
      OMEGA1 = 8.
      I = 1
   99 N = MT(I)/MR
C
      WRITE (*,10) MT(I)
   10 FORMAT(///,5X,'Mass of tube:  MT = ',F4.2,' kg ',/)
      WRITE (*,*) '       X              OMEGA 2            VR'
      WRITE (*,*) '       mm              rad/s             m/s'
      WRITE (*,*) ' '
C
      DO 50 J=1,5
         X = 0.04*J
C           ANGM1 = Initial angular momentum = (COEF1)*OMEGA1
   11 COEF1 = ((1.+N)/3.)*MR*L**2
C           ANGM2 = Final angular momentum = (COEF2)*OMEGA2
      COEF2 = (N/3.+ (1./12.+ (X/L + 0.5)**2))*MR*L**2
C           ANGM1 = ANGM2:   (COEF1)*OMEGA1 = (COEF2)*OMEGA2
      OMEGA2 = (COEF1/COEF2)*OMEGA1
C
C     Conservation of energy (Note that potential energy is constant)
C         T1 = Initial kinetic energy = 0.5*(COEF1)*OMEGA1**2
C         T2 = Final kinetic energy
C            = 0.5*(COEF2)*OMEGA2**2 - 0.5*MR*VR**2
C
C                          T1 = T2
C       0.5*(COEF1)*OMEGA1**2 = 0.5*(COEF2)*OMEGA2**2 + 0.5*MR*VR**2
C
      VR = SQRT(((COEF1)*OMEGA1**2 - (COEF2)*OMEGA2**2)/MR)
C
C         Express X in millimeters
      METERS = 1000.* X
      WRITE (*,20) METERS,OMEGA2,VR
   20 FORMAT (7X,F4.0,12X,F5.3,12X,F5.3)
   50 CONTINUE
      I = I + 1
      IF (MT(I).GT.0.0) GOTO 99
      END
```

Mass of tube: MT = .75 kg

X mm	OMEGA 2 rad/s	VR m/s
40.	5.760	.718
80.	4.235	.931
120.	3.200	1.052
160.	2.483	1.127
200.	1.973	1.178

Mass of tube: MT = 1.50 kg

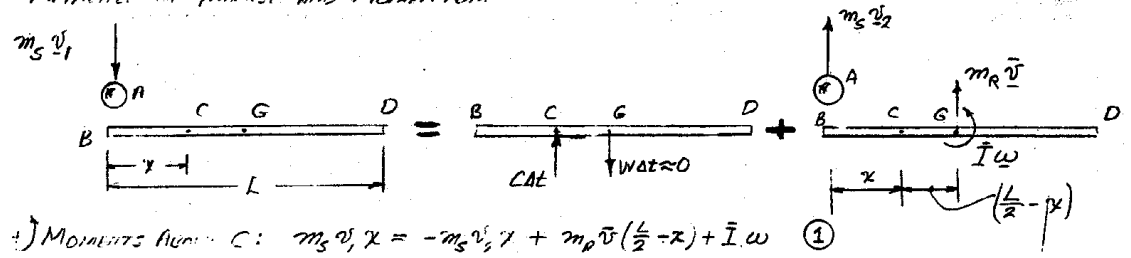
X mm	OMEGA 2 rad/s	VR m/s
40.	6.194	.745
80.	4.800	.991
120.	3.765	1.141
160.	3.000	1.239
200.	2.430	1.308

Mass of tube: MT = 3.00 kg

X mm	OMEGA 2 rad/s	VR m/s
40.	6.698	.775
80.	5.538	1.065
120.	4.571	1.257
160.	3.789	1.393
200.	3.165	1.493

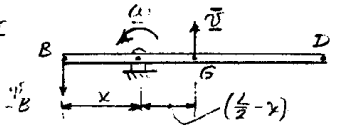
PROBLEM 17.C4

PRINCIPLE OF IMPULSE AND MOMENTUM



+) MOMENTS ABOUT C: $m_S v_1 x = -m_S v_2 x + m_R \bar{v}\left(\frac{L}{2} - x\right) + \bar{I}\omega$ (1)

KINEMATICS



$v_B = x\omega \downarrow$ $\qquad \bar{v} = \left(\frac{L}{2} - x\right)\omega \uparrow$

$\bar{I} = \frac{1}{12} m_R L^2$

SUBSTITUTE FOR $\bar{v}$ AND $\bar{I}$ INTO (1)

$$m_S v_1 x = -m_S v_2 x + m_R \left(\frac{L}{2} - x\right)^2 \omega + \frac{1}{12} m_R L^2 \omega \quad ; \quad \omega = \frac{m_S x (v_1 + v_2)}{\frac{1}{12} m_R L^2 + m_R \left(\frac{L}{2} - x\right)^2}$$

LET: $K = \frac{1}{12} m_R L^2 + m_R \left(\frac{L}{2} - x\right)^2$ ◁

THUS: $\omega = \frac{m_S x}{K}(v_1 + v_2)$ ◁

RELATIVE VELOCITIES

AT POINT B:

$$e = \frac{v_B + v_2}{v_1} \quad ; \quad e = \frac{x\omega + v_2}{v_1} \quad ; \quad e = \frac{x\left[\frac{m_S x}{K}(v_1 + v_2)\right] + v_2}{v_1}$$

$$e v_1 = \frac{m_S x^2}{K}(v_1 + v_2) + v_2$$

$$v_2\left(1 + \frac{m_S x^2}{K}\right) = \left(e - \frac{m_S x^2}{K}\right) v_1$$

FOR PERFECTLY PLASTIC IMPACT $e = 1$:

$$v_2 = v_1 \frac{1 - \frac{m_S x^2}{K}}{1 + \frac{m_S x^2}{K}}$$ ◁

SEE NEXT PAGE FOR PROGRAMS

Definition of Symbols

MS = mass of rod (kg)
MR = mass of sphere (kg)
L = length of rod (m)
V1 = velocity of sphere before impact [+ down] (m/s)
V2 = velocity of sphere after impact [+ up] (m/s)
OMEGA = angular velocity of rod after impact [+ ccwise] (rad/s)

CONTINUED

PROBLEM 17.C4 CONTINUED

```
10 '*************************************************************
20 '*                    PROBLEM 17.C4                          *
30 '*                    BASIC VERSION                          *
40 '*************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '       MS = mass of sphwere     (kg)
90 '       MR = mass of rod         (kg)
100 '       L = length of rod       (m)
110 '      V1 = velocity of sphere before impact [+ down]  (m/s)
120 '      V2 = velocity of sphere after impact [+ up]  (m/s)
130 '   OMEGA = angular velocity of rod after impact [+ ccwise]  (rad/s)
140 '
150 '   Data
160     LET MS = .8       'kg
170     LET MR = 1.6      'kg
180     LET  L = .9       'm
190     LET V1 = 10       'm/s
200 '
210     PRINT
220     PRINT "   X          OMEGA            V2"
230     PRINT "   mm         rad/s            m/s
240     PRINT
250 '
260     FOR X = 0 TO .4 STEP .025
270        K = MR*L^2/12 + MR*(.5*L - X)^2
280        V2 = V1*(1 - MS*X^2/K)/(1 + MS*X^2/K)
290        OMEGA = MS*(X/K)*(V1 + V2)
300 '
310        MILLI= 1000*X    'express X in millimeters
320        PRINT USING "  ###      ";MILLI,
330        PRINT USING "  ###.###        ";OMEGA,V2
340     NEXT
350 '
360     PRINT
370     PRINT "   Positive directions are:
380     PRINT "        OMEGA:  + counterclockwise.      V2:  + up.
```

X mm	OMEGA rad/s	V2 m/s
0	0.000	10.000
25	1.006	9.975
50	2.186	9.891
75	3.556	9.733
100	5.128	9.487
125	6.908	9.136
150	8.889	8.667
175	11.045	8.067
200	13.333	7.333
225	15.686	6.471
250	18.018	5.495
275	20.230	4.437
300	22.222	3.333
325	23.908	2.230
350	25.225	1.171
375	26.144	0.196

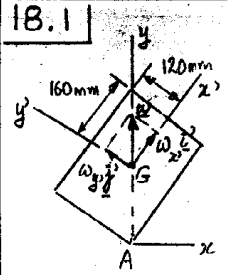
```
Positive directions are:
    OMEGA:  + counterclockwise.      V2:  + up.
```

```
C*************************************************************
C*                    PROBLEM 17.C4                          *
C*                    FORTRAN VERSION                        *
C*************************************************************
C
C           Definition of Symbols
C
C          MS = mass of rod      (kg)
C          MR = mass of sphere   (kg)
C           L = length of rod    (m)
C          V1 = velocity of sphere before impact [+ down]  (m/s)
C          V2 = velocity of sphere after impact [+ up]  (m/s)
C       OMEGA = angular velocity of rod after impact [+ ccwise] (rad/s)
C
      REAL MS, MR, MILLI, L, K
C
      MS = 0.8
      MR = 1.6
       L = 0.9
      V1 = 10.
C
      WRITE (*,*) '        X             OMEGA 2            V2'
      WRITE (*,*) '        mm             rad/s             m/s'
      WRITE (*,*) ' '
C
      X = 0.
      STEP = 0.025
   22 K = MR*L**2/12.+ MR*(0.5*L - X)**2
      V2 = V1*(1. - MS*X**2/K)/(1.+ MS*X**2/K)
      OMEGA = MS*(X/K)*(V1 + V2)
C
C           Express X in millimeters
      MILLI = 1000.* X
      WRITE (*,20) MILLI,OMEGA,V2
   20 FORMAT (7X,F4.0,10X,F7.3,10X,F7.3)
      X = X + STEP
      IF (X.LT.0.40) GOTO 22
C
      WRITE (*,*) ' '
      WRITE (*,*) '    Positive directions are:'
      WRITE (*,*) '         OMEGA:  +counterclockwise.   V2:   + up.'
      END
```

X mm	OMEGA 2 rad/s	V2 m/s
0.	.000	10.000
25.	1.006	9.975
50.	2.186	9.891
75.	3.556	9.733
100.	5.128	9.487
125.	6.908	9.136
150.	8.889	8.667
175.	11.045	8.067
200.	13.333	7.333
225.	15.686	6.471
250.	18.018	5.495
275.	20.230	4.437
300.	22.222	3.333
325.	23.908	2.230
350.	25.225	1.171
375.	26.144	.196

```
Positive directions are:
    OMEGA:  +counterclockwise.   V2:   + up.
```

18.1



Resolving $\underline{\omega}$ along the principal axes x', y', z:

$\omega_{x'} = \frac{160}{200}\omega = 0.8(5\text{ rad/s}) = 4\text{ rad/s}$

$\omega_{y'} = \frac{120}{200}\omega = 0.6(5\text{ rad/s}) = 3\text{ rad/s}$

$\omega_z = 0$

Moments of inertia:

$I_{x'} = \frac{1}{12}(6\text{ kg})(0.24\text{m})^2 = 0.0288\text{ kg.m}^2$

$I_{y'} = \frac{1}{12}(6\text{ kg})(0.32\text{m})^2 = 0.0512\text{ kg.m}^2$

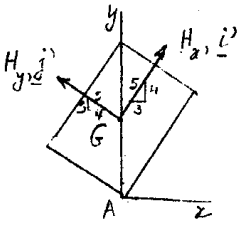
From Eqs. (18.10):

$H_{x'} = I_{x'}\omega_{x'} = (0.0288)(4) = 0.1152\text{ kg.m}^2/\text{s}$

$H_{y'} = I_{y'}\omega_{y'} = (0.0512)(3) = 0.1536\text{ kg.m}^2/\text{s}$

$H_z = I_z\omega_z = 0$

$\underline{H}_G = (115.2\text{ g.m}^2/\text{s})\underline{i}' + (153.6\text{ g.m}^2/\text{s})\underline{j}'$



Components along x and y axes:

$H_x = \frac{3}{5}H_{x'} - \frac{4}{5}H_{y'}$

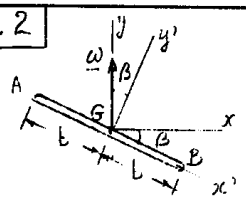
$= \frac{3}{5}(115.2) - \frac{4}{5}(153.6) = -53.76$

$H_y = \frac{4}{5}H_{x'} + \frac{3}{5}H_{y'}$

$= \frac{4}{5}(115.2) + \frac{3}{5}(153.6) = 184.32$

$\underline{H}_G = -(53.8\text{ g.m}^2/\text{s})\underline{i} + (184.3\text{ g.m}^2/\text{s})\underline{j}$ ◀

18.2



Resolving $\underline{\omega}$ along principal axes x', y', z:

$\omega_{x'} = -\omega\sin\beta$

$\omega_{y'} = \omega\cos\beta$

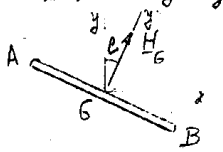
$\omega_z = 0$

Moments of Inertia:

$\bar{I}_{x'} = 0 \quad \bar{I}_{y'} = \bar{I}_z = \frac{m}{12}(2b)^2 = \frac{1}{3}mb^2$

From Eqs. (18.10):

$H_{x'} = \bar{I}_{x'}\omega_{x'} = 0,\ H_{y'} = \bar{I}_{y'}\omega_{y'} = \frac{1}{3}mb^2\omega\cos\beta,\ H_z = \bar{I}_z\omega_z = 0$



Thus: $\underline{H}_G = \frac{1}{3}mb^2\omega\cos\beta\,\underline{j}'$

where $\underline{j}'$ is unit vector $\perp$ to AB

Returning to axes $Gxyz$:

$H_x = H_G\sin\beta = \frac{1}{3}mb^2\omega\cos\beta\sin\beta$

$= \frac{1}{6}mb^2\omega\sin 2\beta$

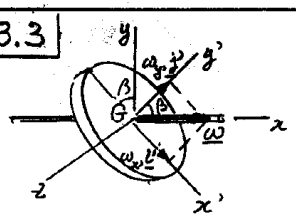
$H_y = H_G\cos\beta = \frac{1}{3}mb^2\omega\cos^2\beta$

$H_z = 0$

We have therefore:

$\underline{H}_G = \frac{1}{6}mb^2\omega(\sin 2\beta\,\underline{i} + 2\cos^2\beta\,\underline{j})$ ◀

18.3



Resolving $\underline{\omega}$ along the principal axes x', y', z:

$\omega_{x'} = \omega\sin\beta$

$\omega_{y'} = \omega\cos\beta$

$\omega_z = 0$

Moments of inertia:

$\bar{I}_{x'} = \frac{1}{4}mr^2, \qquad \bar{I}_{y'} = \frac{1}{2}mr^2$

$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_z\omega_z\underline{k}$

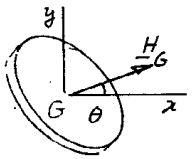
$= \frac{1}{4}mr^2\omega\sin\beta\,\underline{i}' + \frac{1}{2}mr^2\omega\cos\beta\,\underline{j}'$

But $\underline{i}' = \sin\beta\,\underline{i} - \cos\beta\,\underline{j}$; $\underline{j}' = \cos\beta\,\underline{i} + \sin\beta\,\underline{j}$

Thus: $\underline{H}_G = \frac{1}{4}mr^2\omega\sin\beta(\sin\beta\,\underline{i} - \cos\beta\,\underline{j})$

$+ \frac{1}{2}mr^2\omega\cos\beta(\cos\beta\,\underline{i} + \sin\beta\,\underline{j})$

$\underline{H}_G = \frac{1}{4}mr^2\omega[(\sin^2\beta + 2\cos^2\beta)\underline{i} + \sin\beta\cos\beta\,\underline{j}]$



Thus: $H_x = \frac{1}{4}mr^2\omega(\sin^2\beta + 2\cos^2\beta)$

$= \frac{1}{4}mr^2\omega(1 + \cos^2\beta)$

$= \frac{1}{4}mr^2\omega(1 + \frac{1}{2} + \frac{1}{2}\cos 2\beta)$

$= \frac{1}{8}mr^2\omega(3 + \cos 2\beta)$

$H_y = \frac{1}{4}mr^2\omega\sin\beta\cos\beta$

$= \frac{1}{8}mr^2\omega(2\sin\beta\cos\beta)$

$= \frac{1}{8}mr^2\omega\sin 2\beta$

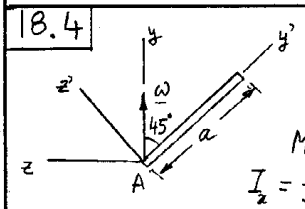
$\tan\theta = \frac{H_y}{H_x} = \frac{\sin 2\beta}{3 + \cos 2\beta}$ (1)

Making $\beta = 20°$ in Eq. (1):

$\tan\theta = \frac{\sin 40°}{3 + \cos 40°} = 0.17068 \qquad \theta = 9.686°$

$\theta = 9.7°$ ◀

18.4



Resolving $\underline{\omega}$ along the principal axes x, y', z':

$\omega_x = 0, \quad \omega_{y'} = \omega_{z'} = \frac{\omega}{\sqrt{2}}$

Moments of inertia:

$I_{x'} = \frac{1}{3}ma^2, \qquad I_{y'} = \frac{1}{12}ma^2$

$I_{z'} = I_x + I_{y'} = \frac{1}{3}ma^2 + \frac{1}{12}ma^2 = \frac{5}{12}ma^2$

$\underline{H}_A = I_x\omega_x\underline{i} + I_{y'}\omega_{y'}\underline{j}' + I_{z'}\omega_{z'}\underline{k}'$

$= 0 + \frac{1}{12}ma^2\frac{\omega}{\sqrt{2}}\underline{j}' + \frac{5}{12}ma^2\frac{\omega}{\sqrt{2}}\underline{k}'$

But $\underline{j}' = \frac{1}{\sqrt{2}}(\underline{j} - \underline{k}) \qquad \underline{k}' = \frac{1}{\sqrt{2}}(\underline{j} + \underline{k})$

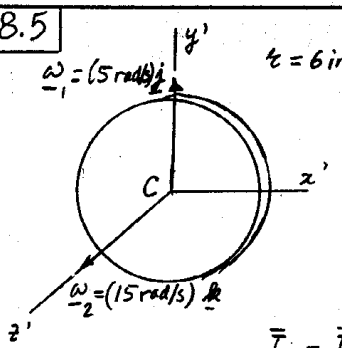
Thus:

$\underline{H}_A = \frac{1}{12}ma^2\frac{\omega}{\sqrt{2}}\frac{1}{\sqrt{2}}(\underline{j} - \underline{k}) + \frac{5}{12}ma^2\frac{\omega}{\sqrt{2}}\frac{1}{\sqrt{2}}(\underline{j} + \underline{k})$

$= \frac{1}{24}ma^2\omega(\underline{j} - \underline{k} + 5\underline{j} + 5\underline{k})$

$\underline{H}_A = \frac{1}{12}ma^2\omega(3\underline{j} + 2\underline{k})$ ◀

18.5



Angular velocity of disk:

$$\underline{\omega} = \omega_1 \underline{j} + \omega_2 \underline{k}$$
$$= (5\ \text{rad/s})\underline{j} + (15\ \text{rad/s})\underline{k}$$

Moments of inertia:

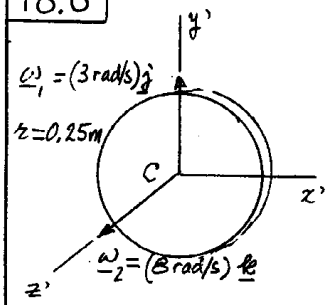
$$\bar{I}_{x'} = \bar{I}_{y'} = \tfrac{1}{4} m r^2$$
$$= \tfrac{1}{4}\,\frac{10}{32.2}\left(\frac{6}{12}\ \text{ft}\right)^2$$
$$= 0.019410\ \text{ft·lb·s}^2$$

$$I_{z'} = \tfrac{1}{2} m r^2 = 0.038820\ \text{ft·lb·s}^2$$

$$\underline{H}_C = \bar{I}_{x'}\,\omega_{x'}\,\underline{i} + \bar{I}_{y'}\,\omega_{y'}\,\underline{j} + I_{z'}\,\omega_{z'}\,\underline{k}$$
$$= 0 + (0.019410)\,5\,\underline{j} + (0.038820)\,15\,\underline{k}$$
$$= (0.09705\ \text{ft·lb·s})\underline{j} + (0.58230\ \text{ft·lb·s})\underline{k}$$

$$\underline{H}_C = (0.0970\ \text{ft·lb·s})\underline{j} + (0.582\ \text{ft·lb·s})\underline{k}$$ ◀

18.6



Angular velocity of disk:

$$\underline{\omega} = \omega_1 \underline{j} + \omega_2 \underline{k}$$
$$= (3\ \text{rad/s})\underline{j} + (8\ \text{rad/s})\underline{k}$$

Moments of inertia:

$$\bar{I}_{x'} = \bar{I}_{y'} = \tfrac{1}{4} m r^2$$
$$= \tfrac{1}{4}(5\ \text{kg})(0.25\ \text{m})^2$$
$$= 78.125 \times 10^{-3}\ \text{kg·m}^2$$

$$\bar{I}_{z'} = \tfrac{1}{2} m r^2 = \tfrac{1}{2}(5\ \text{kg})(0.25\ \text{m})^2$$
$$= 156.25 \times 10^{-3}\ \text{kg·m}^2$$

$$\underline{H}_C = \bar{I}_{x'}\,\omega_{x'}\,\underline{i} + \bar{I}_{y'}\,\omega_{y'}\,\underline{j} + I_{z'}\,\omega_{z'}\,\underline{k}$$
$$= 0 + (78.125 \times 10^{-3})(3)\underline{j} + (156.25 \times 10^{-3})(8)\underline{k}$$

$$\underline{H}_C = (0.234\ \text{kg·m}^2/\text{s})\underline{j} + (1.250\ \text{kg·m}^2/\text{s})\underline{k}$$ ◀

18.7

Because of the symmetry of the projectile, the x and y axes are principal axes of inertia.

We have:

$$\bar{I}_x = m\bar{k}_x^2 = (25\ \text{kg})(0.072\ \text{m})^2 = 0.1296\ \text{kg·m}^2$$
$$\bar{I}_y = m\bar{k}_y^2 = (25\ \text{kg})(0.300\ \text{m})^2 = 2.25\ \text{kg·m}^2$$

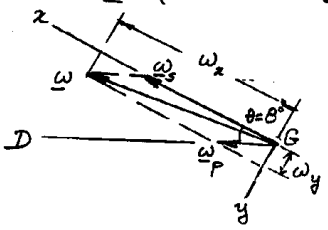
Given: $H_x = 0.600\ \text{kg·m}^2/\text{s}$, $H_y = 0.016\ \text{kg·m}^2/\text{s}$

From Eqs. (18.10):

$$\omega_x = \frac{H_x}{\bar{I}_x} = \frac{0.600\ \text{kg·m}^2/\text{s}}{0.1296\ \text{kg·m}^2} = 4.6296\ \text{rad/s}$$

$$\omega_y = \frac{H_y}{\bar{I}_y} = \frac{0.016\ \text{kg·m}^2/\text{s}}{2.25\ \text{kg·m}^2} = 7.1111 \times 10^{-3}\ \text{rad/s}$$

$$\underline{\omega} = (4.6296\ \text{rad/s})\,\underline{i} + (7.1111 \times 10^{-3}\ \text{rad/s})\,\underline{j}$$



We note that ω_x and ω_y are rectangular components of $\underline{\omega}$ along the x and y axes, while the desired components $\underline{\omega}_s$ (spin) and $\underline{\omega}_p$ (precession) are oblique components along GD and GD. We have

$$\omega_p = \frac{\omega_y}{\sin\theta} = \frac{7.1111 \times 10^{-3}}{\sin 8°} = 51.095 \times 10^{-3}\ \text{rad/s}$$

$$\omega_s = \omega_x - \omega_p \cos\theta = 4.6296 - (51.095 \times 10^{-3}) \cos 8°$$
$$= 4.5790\ \text{rad/s}$$

(a) $\omega_s = 4.58\ \text{rad/s}$, (b) $\omega_p = 0.0511\ \text{rad/s}$ ◀

18.8

We use Eq. (18.11):

$$\underline{H}_A = \underline{r} \times m\underline{\bar{v}} + \underline{H}_G \qquad (1)$$

where $\underline{r} = \underline{r}_{G/A} = (0.330\ \text{m})\,\underline{i}$

$$m\underline{\bar{v}} = m\bar{v}(\cos\theta_x\,\underline{i} + \sin\theta_x\,\underline{j})$$
$$= (25\ \text{kg})(750\ \text{m/s})(\cos 8°\,\underline{i} + \sin 8°\,\underline{j})$$
$$= (18.5675 \times 10^3\ \text{kg·m/s})\,\underline{i} + (2.6095 \times 10^3\ \text{kg·m/s})\,\underline{j}$$

and from statement of Prob. 18.7:

$$\underline{H}_G = (600\ \text{g·m}^2/\text{s})\,\underline{i} + (16\ \text{g·m}^2/\text{s})\,\underline{j}$$

Substituting into (1):

$$\underline{H}_A = (0.330\ \text{m})\,\underline{i} \times [(18.5675 \times 10^3\ \text{kg·m/s})\,\underline{i} + (2.6095 \times 10^3\ \text{kg·m/s})\,\underline{j}]$$
$$+ (0.600\ \text{kg·m}^2/\text{s})\,\underline{i} + (16 \times 10^{-3}\ \text{kg·m}^2/\text{s})\,\underline{j}$$
$$= (861.14\ \text{kg·m}^2/\text{s})\,\underline{k} + (0.600\ \text{kg·m}^2/\text{s})\,\underline{i}$$
$$+ (16 \times 10^{-3}\ \text{kg·m}^2/\text{s})\,\underline{j}$$

$$\underline{H}_A = (0.600\ \text{kg·m}^2/\text{s})\,\underline{i} + (0.016\ \text{kg·m}^2/\text{s})\,\underline{j}$$
$$+ (861\ \text{kg·m}^2/\text{s})\,\underline{k}$$ ◀

18.9 We use Eq. (18.11):

$$\underline{H}_A = \underline{r} \times m\underline{\bar{v}} + \underline{H}_C \qquad (1)$$

where $\underline{r} = \underline{r}_C = (0.4\text{m})\underline{i} + (0.3\text{m})\underline{k}$

$$\underline{\bar{v}} = \underline{v}_C = \underline{\omega}_1 \times \underline{r}_C = (3\text{rad/s})\underline{j} \times [(0.4\text{m})\underline{i} + (0.3\text{m})\underline{k}]$$
$$= -(1.2\text{ m/s})\underline{k} + (0.9\text{m/s})\underline{i}$$

$$m\underline{\bar{v}} = (5\text{kg})\underline{\bar{v}} = (4.5\text{ kg·m/s})\underline{i} - (6\text{ kg·m/s})\underline{k}$$

and from the solution of Prob. 18.6:

$$\underline{H}_C = (0.234\text{ kg·m}^2\text{/s})\underline{j} + (1.250\text{ kg·m}^2\text{/s})\underline{k}$$

Substituting into (1):

$$\underline{H}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.4 & 0 & 0.3 \\ 4.5 & 0 & -6 \end{vmatrix} + 0.234\,\underline{j} + 1.250\,\underline{k}$$

$$= (1.35 + 2.40)\underline{j} + 0.234\underline{j} + 1.250\underline{k}$$

$$\underline{H}_A = (3.98\text{ kg·m}^2\text{/s})\underline{j} + (1.250\text{ kg·m}^2\text{/s})\underline{k} \blacktriangleleft$$

18.10 We use Eq. (18.11):

$$\underline{H}_A = \underline{r} \times m\underline{\bar{v}} + \underline{H}_C \qquad (1)$$

where $\underline{r} = \underline{r}_C = \left(\frac{12}{17}\text{ft}\right)\underline{i} + \left(\frac{9}{12}\text{ft}\right)\underline{j} = (1.5\text{ft})\underline{i} + (0.75\text{ft})\underline{j}$

$$\underline{\bar{v}} = \underline{v}_C = \underline{\omega}_1 \times \underline{r}_C = (5\text{ rad/s})\underline{j} \times [(1.5\text{ ft})\underline{i} + (0.75\text{ ft})\underline{j}]$$
$$= -(7.5\text{ ft/s})\underline{k}$$

$$m\underline{\bar{v}} = \left(\frac{10}{32.2}\right)(-7.5\underline{k}) = -(2.3292\text{ lb·s})\underline{k}$$

and from the solution of Prob. 18.5:

$$\underline{H}_C = (0.0970\text{ ft·lb·s})\underline{j} + (0.582\text{ ft·lb·s})\underline{k}$$

Substituting into (1):

$$\underline{H}_A = (1.5\underline{i} + 0.75\underline{j}) \times (-2.3292\underline{k}) + 0.0970\,\underline{j} + 0.582\,\underline{k}$$

$$= 3.4938\underline{j} - 1.7469\,\underline{i} + 0.0970\,\underline{j} + 0.582\,\underline{k}$$

$$\underline{H}_A = -(1.747\text{ ft·lb·s})\underline{i} + (3.59\text{ ft·lb·s})\underline{j} + (0.582\text{ ft·lb·s})\underline{k} \blacktriangleleft$$

18.11 We use Eq. (18.11):

$$\underline{H}_O = \underline{r} \times m\underline{\bar{v}} + \underline{H}_G \qquad (1)$$

From Sample Prob. 18.2: $\underline{\bar{r}} = L\underline{i}$

$$\underline{\bar{v}} = (-\omega_2\underline{j}) \times \underline{\bar{r}} = \left(-\frac{r\omega_1}{L}\underline{j}\right) \times (L\underline{i}) = r\omega_1\underline{k}$$

$$\underline{H}_G = \frac{1}{2}mr^2\omega_1\left(\underline{i} - \frac{r}{2L}\underline{j}\right)$$

Substituting into (1):

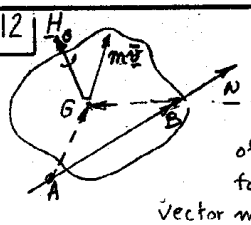
$$\underline{H}_O = L\underline{i} \times mr\omega_1\underline{k} + \frac{1}{2}mr^2\omega_1\left(\underline{i} - \frac{r}{2L}\underline{j}\right)$$

$$= -mLr\omega_1\underline{j} + \frac{1}{2}mr^2\omega_1\underline{i} - \frac{1}{4}mr^2\frac{r\omega_1}{L}\underline{j}$$

$$\underline{H}_O = \frac{1}{2}mr^2\omega_1\underline{i} - m\left(L^2 + \frac{1}{4}r^2\right)\frac{r\omega_1}{L}\underline{j} \blacktriangleleft$$

Result obtained is same as obtained in part b of Sample Prob. 18.2

18.12



Body rotates with angular velocity $\underline{\omega}$ about fixed axis through A and B. We represent the system of the momenta of the particles forming the rigid body by the vector $m\underline{\bar{v}}$ and the couple $\underline{H}_G$ at G.

We use Eq. (18.11):

$$\underline{H}_A = \underline{r}_{G/A} \times m\underline{\bar{v}} + \underline{H}_G$$

$$\underline{H}_B = \underline{r}_{G/B} \times m\underline{\bar{v}} + \underline{H}_G$$

$$\underline{H}_A - \underline{H}_B = (\underline{r}_{G/A} - \underline{r}_{G/B}) \times m\underline{\bar{v}}$$

But $\underline{r}_{G/A} = \underline{r}_{G/B} + \underline{r}_{B/A}$. Thus:

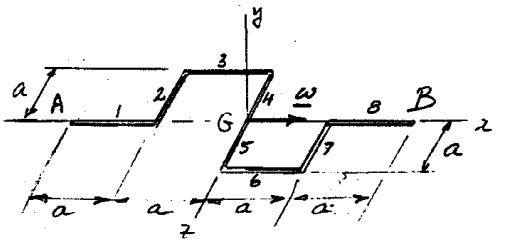
$$\underline{H}_A - \underline{H}_B = \underline{r}_{B/A} \times m\underline{\bar{v}}$$

or, since $\underline{\bar{v}} = \underline{\omega} \times \underline{r}_{G/A}$,

$$\underline{H}_A - \underline{H}_B = m[\underline{r}_{B/A} \times (\underline{\omega} \times \underline{r}_{G/A})]$$

Assuming $\omega \neq 0$ and noting that $\underline{r}_{B/A}$ is perpendicular to $(\underline{\omega} \times \underline{r}_{G/A})$ and, thus, not parallel to it, we conclude that $\underline{H}_A - \underline{H}_B = 0$ only if $\underline{r}_{G/A}$ is parallel to $\underline{\omega}$, that is, only if G is located on the axis AB. (Q.E.D.)

18.13 Moments and Products of Inertia.



Each of the numbered elements has a mass equal to $m'a$

$$I_x = 4[I_x \text{ of } ②] + 2[I_x \text{ of } ⑤] = 4\left(\frac{1}{3}m'a\,a^2\right) + 2(m'a\,a^2)$$

$$I_x = \frac{10}{3}m'a^3$$

Noting that for each element $\bar{I}_{x'z'} = 0$, we have

$$I_{xz} = m'a(-a)\left(-\frac{a}{2}\right) + m'a\left(-\frac{a}{2}\right)(-a) + m'a\left(\frac{a}{2}\right)a + m'a(a)\left(\frac{a}{2}\right)$$

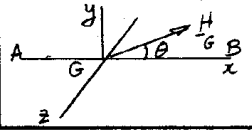
$$I_{xz} = +2m'a^3$$

Also: $I_{xy} = I_{yz} = 0$

Noting that $\underline{\omega} = \omega\underline{i}$, we apply Eqs. (18.7):

$$H_x = I_x\omega_x = \frac{10}{3}m'a^3\omega$$
$$H_y = -I_{xy}\omega_x = 0$$
$$H_z = -I_{xz}\omega_x = -2m'a^3\omega$$

(a) Therefore: $\underline{H}_G = \frac{2}{3}m'a^3\omega(5\underline{i} - 3\underline{k})$ ◀



(b) $\tan\theta = \frac{-H_z}{H_x} = \frac{2m'a^3\omega}{\frac{10}{3}m'a^3\omega}$

$\tan\theta = 0.6$ $\quad \theta = 31.0°$ ◀

18.14 (a) Angular momentum $\underline{H}_A$.

We first compute $\underline{H}_G$ (see solution of Prob. 18.13):

$$\underline{H}_G = \frac{2}{3} m' a^3 \omega (5\underline{i} - 3\underline{k})$$

and note that $\underline{v} = \underline{v}_G = 0$. Using Eq. (18.11):

$$\underline{H}_A = \underline{r}_{G/A} \times m\underline{v} + \underline{H}_G = 0 + \frac{2}{3} m' a^3 \omega (5\underline{i} - 3\underline{k})$$

$$\underline{H}_A = \frac{2}{3} m' a^3 \omega (5\underline{i} - 3\underline{k}) \blacktriangleleft$$

(b) Angular momentum $\underline{H}_B$. We find in a similar way

$$\underline{H}_B = \underline{r}_{G/B} \times m\underline{v} + \underline{H}_G = 0 + \frac{2}{3} m' a^3 \omega (5\underline{i} - 3\underline{k})$$

$$\underline{H}_B = \frac{2}{3} m' a^3 \omega (5\underline{i} - 3\underline{k}) \blacktriangleleft$$

Note: This result could have been predicted from the property derived in Prob. 18.12.

18.15 Angular velocity:

$$\omega_x = 0, \quad \omega_y = \omega, \quad \omega_z = 0$$

Substituting in Eqs. (18.7), we have

$$H_x = -I_{xy}\omega, \quad H_y = I_y \omega, \quad H_z = -I_{zx}\omega \qquad (1)$$

Moment and Products of inertia for one plate:

Referring to back of book cover,

$$(I_y)_{Area} = \frac{1}{12} b h^3$$

$$(I_y)_{Mass} = \frac{m}{A}(I_y)_{Area} = \frac{m}{\frac{1}{2}bh}\left(\frac{1}{12} b h^3\right) = \frac{1}{6} m h^2$$

Referring to Sample Prob. 9.6 (page 390 of Statics):

$$(I_{xy})_{Area} = -\frac{1}{24} b^2 h^2$$

$$(I_{xy})_{Mass} = \frac{m}{A}(I_{xy})_{Area} = \frac{m}{\frac{1}{2}bh}\left(-\frac{1}{24} b^2 h^2\right) = -\frac{1}{12} m b h$$

Thus, for system consisting of two plates:

$$I_y = 2\left(\frac{1}{6} m h^2\right) = \frac{1}{3} m h^2$$

$$I_{xy} = 2\left(-\frac{1}{12} m b h\right) = -\frac{1}{6} m b h$$

where m = mass of one plate

We also note that

$$I_{zx} = 0$$

Substituting into Eqs. (1):

$$H_x = +\frac{1}{6} m b h \omega, \quad H_y = \frac{1}{3} m h^2 \omega, \quad H_z = 0$$

$$\underline{H}_G = H_x \underline{i} + H_y \underline{j} + H_z \underline{k}$$

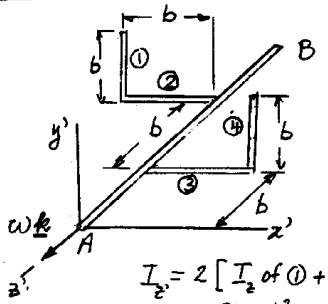
$$= \frac{1}{6} m b h \omega \underline{i} + \frac{1}{3} m h^2 \omega \underline{j}$$

Given Data: $m = 8$ kg, $b = h = 0.75$ m, $\omega = 6$ rad/s

$$\underline{H}_G = \frac{1}{6}(8\text{ kg})(0.75\text{ m})^2(6\text{ rad/s})\underline{i} + \frac{1}{3}(8\text{ kg})(0.75\text{ m})^2(6\text{ rad/s})\underline{j}$$

$$\underline{H}_G = (4.50\text{ kg}\cdot\text{m}^2/\text{s})\underline{i} + (9.00\text{ kg}\cdot\text{m}^2/\text{s})\underline{j} \blacktriangleleft$$

18.16 (a) Angular momentum $\underline{H}_A$. We select axes centered at A as shown.



Angular velocity:

$$\omega_{x'} = \omega_{y'} = 0, \quad \omega_{z'} = \omega$$

Moments & Products of Inertia

For each of the numbered elements:

$$b = 8\text{ in.} = \frac{2}{3}\text{ ft}$$

$$m = \frac{1}{2}\frac{4\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112$$

$$I_{z'} = 2\left[I_{z'} \text{ of } ① + I_{z'} \text{ of } ②\right]$$

$$= 2\left[\frac{1}{12} m b^2 + m\left(b^2 + \frac{b^2}{4}\right)\right] + 2\left[\frac{1}{12} m b^2 + m\left(\frac{b^2}{4}\right)\right]$$

$$I_{z'} = \frac{10}{3} m b^2$$

Since $\bar{I}_{yz} = \bar{I}_{zx} = 0$ for each element:

$$I_{y'z'} = \left[I_{y'z'} \text{ of } ①\right] + \left[I_{y'z'} \text{ of } ④\right] = m\bar{y}_1'\bar{z}_1' + m\bar{y}_4'\bar{z}_4'$$

$$= m\left(\frac{b}{2}\right)(-2b) + m\left(\frac{b}{2}\right)(-b) = -\frac{3}{2} m b^2$$

$$I_{z'x'} = \left[I_{z'x'} \text{ of } ①,②,③,④\right] = m\bar{z}_1'\bar{x}_1' + m\bar{z}_2'\bar{x}_2' + m\bar{z}_3'\bar{x}_3' + m\bar{z}_4'\bar{x}_4'$$

$$= m(-2b)(-b) + m(-2b)\left(-\frac{b}{2}\right) + m(-b)\left(\frac{b}{2}\right) + m(-b)(b)$$

$$= +\frac{3}{2} m b^2$$

Substituting into Eqs. (18.13):

$$H_{x'} = -I_{z'x'}\omega_{z'} = -\frac{3}{2} m b^2 \omega$$

$$H_{y'} = -I_{y'z'}\omega_{z'} = +\frac{3}{2} m b^2 \omega$$

$$H_{z'} = I_{z'}\omega_{z'} = \frac{10}{3} m b^2 \omega$$

$$\underline{H}_A = H_{x'}\underline{i} + H_{y'}\underline{j} + H_{z'}\underline{k}$$

$$\underline{H}_A = m b^2 \omega\left(-\frac{3}{2}\underline{i} + \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right) \qquad (1)$$

Given Data: $m = 0.062112$ lb·s²/ft, $b = \frac{2}{3}$ ft

$\omega = 240$ rpm $= 25.133$ rad/s

$$\underline{H}_A = 0.062112\left(\frac{2}{3}\right)^2(25.133)\left(-\frac{3}{2}\underline{i} + \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right)$$

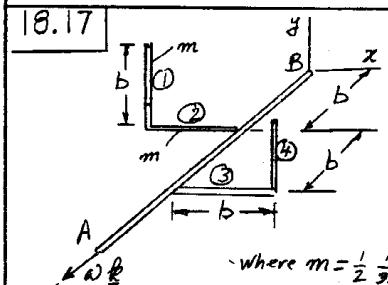
$$\underline{H}_A = -(1.041\text{ ft·lb·s})\underline{i} + (1.041\text{ ft·lb·s})\underline{j} + (2.31\text{ ft·lb·s})\underline{k} \blacktriangleleft$$

(b) Since the unit vector of AB is $-\underline{k}$, we have, recalling (1):

$$\cos\theta = \frac{\underline{H}_A \cdot (-\underline{k})}{|\underline{H}_A|} = \frac{\left(-\frac{3}{2}\underline{i} + \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right)\cdot(-\underline{k})}{\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{10}{3}\right)^2}} = \frac{-10/3}{3.951}$$

$$\cos\theta = -0.8437 \qquad \theta = 147.5^\circ \blacktriangleleft$$

18.17



(a) Ang. Momentum $\underline{H}_B$

Angular velocity:

$$\omega_x = \omega_y = 0, \quad \omega_z = \omega$$

Moment of Inertia I_z:

Same as I_z in Prob. 18.16:

$$I_z = \frac{10}{3} m b^2$$

where $m = \frac{1}{2}\frac{4\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112$ lb·s²/ft

and $b = 8\text{ in.} = \frac{2}{3}$ ft

Products of Inertia. Since $\bar{I}_{yz} = \bar{I}_{zx} = 0$ for each element,

$$I_{yz} = m\bar{y}_1\bar{z}_1 + m\bar{y}_4\bar{z}_4 = m\left(\frac{b}{2}\right)(b) + m\left(\frac{b}{2}\right)(2b) = +\frac{3}{2} m b^2$$

$$I_{zx} = m\bar{z}_1\bar{x}_1 + m\bar{z}_2\bar{x}_2 + m\bar{z}_3\bar{x}_3 + m_4\bar{z}_4\bar{x}_4$$

$$= mb(-b) + mb\left(-\frac{b}{2}\right) + m(2b)\left(\frac{b}{2}\right) + m(2b)b = +\frac{3}{2} m b^2$$

(CONTINUED)

18.17 CONTINUED We have $\omega_x = \omega_y = 0$, $\omega_z = \omega$

$$I_z = \frac{10}{3} m b^2, \quad I_{yz} = I_{zx} = +\frac{3}{2} m b^2$$

Substituting into Eqs. (18.13):

$$H_x = -I_{zx}\omega_z = -\frac{3}{2} m b^2 \omega$$
$$H_y = -I_{yz}\omega_z = -\frac{3}{2} m b^2 \omega$$
$$H_z = I_z \omega_z = \frac{10}{3} m b^2 \omega$$
$$\underline{H}_B = H_x \underline{i} + H_y \underline{j} + H_z \underline{k}$$
$$\underline{H}_B = m b^2 \omega \left(-\frac{3}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right) \quad (2)$$

Given data: $m = 0.062112$ lb·s²/ft, $b = \frac{2}{3}$ ft

$$\omega = 240 \text{ rpm} = 25.133 \text{ rad/s}$$

$$\underline{H}_B = 0.062112\left(\frac{2}{3}\right)^2(25.133)\left(-\frac{3}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right)$$

$$\underline{H}_B = -(1.041 \text{ ft·lb·s})\underline{i} - (1.041 \text{ ft·lb·s})\underline{j} + (2.31 \text{ ft·lb·s})\underline{k}$$ ◀

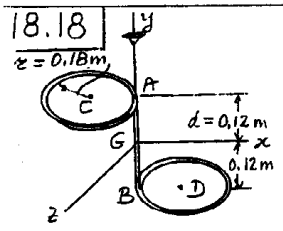
(b) Since the unit vector of BA is $\underline{k}$, we have, recalling (2):

$$\cos\theta = \frac{\underline{H}_B \cdot \underline{k}}{|\underline{H}_B|} = \frac{\left(-\frac{3}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{10}{3}\underline{k}\right)\cdot \underline{k}}{\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{10}{3}\right)^2}} = \frac{10/3}{3.951} = 0.8437$$

$$\theta = 32.5^\circ$$ ◀

Note. The fact that the expression found for $\underline{H}_B$ is different from that obtained for $\underline{H}_A$ in Prob. 18.16 could have been anticipated, since the mass center G of the body is not located on axis AB. (cf. Prob. 18.12).

18.18



Moments and Products of Inertia

Rod AB: $\ell = 0.24$ m

$m = (2.5 \text{ kg/m})(0.24 \text{ m}) = 0.6 \text{ kg}$

$$I_x = I_z = \frac{1}{12} m \ell^2 = \frac{1}{12}(0.6)(0.24)^2$$
$$= 2.88 \times 10^{-3} \text{ kg·m}^2$$

$I_y = 0$, $I_{xy} = I_{yz} = I_{zx} = 0$

For each ring: $m = 2\pi(0.18\text{m})(2.5 \text{ kg/m}) = 0.9\pi$ kg

$$I_x = \bar{I}_{x'} + md^2 = \frac{1}{2} m r^2 + md^2 = 0.9\pi\left(\frac{0.18^2}{2} + 0.12^2\right) = 0.02754\pi$$
$$I_y = \bar{I}_{y'} + md^2 = mr^2 + mr^2 = 0.9\pi(0.18^2 + 0.18^2) = 0.05832\pi$$
$$I_z = \bar{I}_{z'} + md^2 = \frac{1}{2} mr^2 + m(r^2 + d^2) = 0.9\pi\left[\frac{3}{2}(0.18)^2 + (0.12)^2\right] = 0.05670\pi$$
$$P_{xy} = m\bar{x}\bar{y} = -(0.9\pi)(0.18)(0.12) = -0.01944\pi, \quad P_{yz} = P_{zx} = 0$$

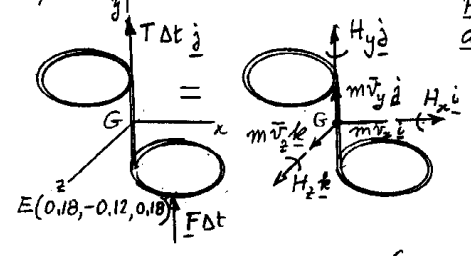
For entire figure: $m = 0.6 + 2(0.9\pi) = 6.2549$ kg

$I_x = 0.00288 + 2(0.02754\pi) = 0.17592$ kg·m²

$I_y = 2(0.05832\pi) = 0.36644$ kg·m²

$I_z = 0.00288 + 2(0.05670\pi) = 0.35914$ kg·m²

$P_{xy} = -2(0.01944\pi) = -0.12215$ kg·m²



Principle of impulse and momentum (Weight is omitted, since it is nonimpulsive)

(CONTINUED)

18.18 CONTINUED We note that wire becomes slack. Therefore: $T\Delta t\, \underline{j} = 0$

(a) Velocity of mass center

Equating vectors in diagram:

$$\underline{F}\Delta t = m\underline{\bar{v}} \qquad \underline{\bar{v}} = \frac{(3 \text{ N·s})\underline{j}}{6.2549 \text{ kg}}, \quad \underline{\bar{v}} = (0.480 \text{ m/s})\underline{j}$$ ◀

(b) Angular velocity. Equating moments about G:

$$\underline{H}_G = \underline{r}_{E} \times \underline{F}\Delta t = (0.18\underline{i} - 0.12\underline{j} + 0.18\underline{k}) \times 3\underline{j}$$
$$= 0.54\underline{k} - 0.54\underline{i}$$

Thus: $H_x = -0.54$, $H_y = 0$, $H_z = 0.54$

Substituting these values and those of the moments and products of inertia into Eqs. (18.7):

$$-0.54 = 0.17592\omega_x + 0.12215\omega_y - 0 \quad (1)$$
$$0 = +0.12215\omega_x + 0.36644\omega_y - 0 \quad (2)$$
$$0.54 = -0 - 0 + 0.35914\omega_z \quad (3)$$

Solving (1) and (2) simultaneously for ω_x and ω_y:

$\omega_x = -3.9940$ $\qquad$ $\omega_y = +1.3314$

Solving (3) for ω_z: $\omega_z = +1.5036$

Thus: $\underline{\omega} = -(3.99 \text{ rad/s})\underline{i} + (1.331 \text{ rad/s})\underline{j} + (1.504 \text{ rad/s})\underline{k}$ ◀

Note that $\omega_y \neq 0$, even though $\underline{F}\Delta t$ is // y axis.

18.19 See solution of Prob. 18.18 for computation of moments and products of inertia. Also, use impulse-momentum diagram at bottom of LH column with $\underline{F}\Delta t = -(3 \text{ N·s})\underline{k}$.

(a) Velocity of mass center:

From constraint imposed by wire: $\bar{v}_y = 0$.

Equating vectors in diagram:

x comp: $0 = m\bar{v}_x$ $\qquad \bar{v}_x = 0$

y comp: $T\Delta t = m\bar{v}_y = 0$ $\qquad T\Delta t = 0$

z comp: $-3 \text{ N·s} = m\bar{v}_z$ $\qquad \bar{v}_z = -\frac{3 \text{ N·s}}{6.2549 \text{ kg}} = -0.4796 \text{ m/s}$

$$\underline{\bar{v}} = -(0.480 \text{ m/s})\underline{k}$$ ◀

(b) Angular velocity. Equating moments about G:

$$\underline{H}_G = \underline{r}_E \times \underline{F}\Delta t = (0.18\underline{i} - 0.12\underline{j} + 0.18\underline{k}) \times (-3\underline{k}) = 0.54\underline{j} + 0.36\underline{i}$$

Thus: $H_x = 0.36$, $H_y = 0.54$, $H_z = 0$

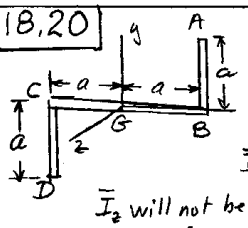
Substituting into Eqs. (18.7) and recalling values of moments and products of inertia:

$$0.36 = 0.17592\omega_x + 0.12215\omega_y \quad (1)$$
$$0.54 = 0.12215\omega_x + 0.36644\omega_y \quad (2)$$
$$0 = 0.35914\omega_z \qquad \omega_z = 0$$

Solving (1) and (2) simultaneously: $\omega_x = 1.3313$, $\omega_y = 1.0299$

Thus: $\underline{\omega} = (1.331 \text{ rad/s})\underline{i} + (1.030 \text{ rad/s})\underline{j}$ ◀

18.20



Moments and Products of Inertia

Mass of AB = Mass of CD = $\frac{1}{4}m$

Mass of BC = $\frac{1}{2}m$

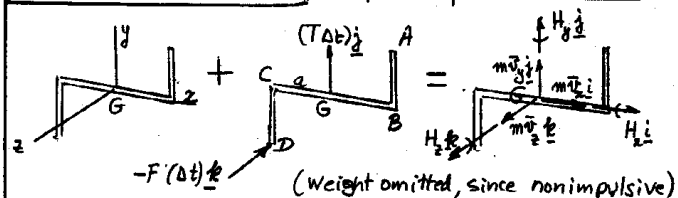
$$\bar{I}_x = 2\left[\frac{1}{3}\left(\frac{m}{4}\right)a^2\right] = \frac{1}{6}ma^2$$
$$\bar{I}_y = \frac{1}{12}\left(\frac{m}{2}\right)(2a)^2 + 2\left(\frac{m}{4}\right)a^2 = \frac{2}{3}ma^2$$

$\bar{I}_z$ will not be needed, but for the record:

$$\bar{I}_z = 2\left[\frac{1}{12}\left(\frac{m}{4}\right)a^2 + \frac{m}{4}\left(a^2 + \frac{a^2}{4}\right)\right] + \frac{1}{12}\left(\frac{m}{2}\right)(2a)^2 = \frac{5}{6}ma^2$$
$$\bar{I}_{xy} = 2\left[\left(\frac{m}{4}\right)\bar{x}\bar{y}\right] = 2\left(\frac{m}{4}\right)(a)\left(\frac{a}{2}\right) = \frac{1}{4}ma^2$$
$$\bar{I}_{yz} = \bar{I}_{zx} = 0$$

(CONTINUED)

18.20 CONTINUED | Principle of Impulse & Momentum



(Weight omitted, since nonimpulsive)

(a) Velocity of mass center

From given constraints: $\bar{v}_y = 0$

Equating components of impulse and momentum:

x comp.: $0 = m\bar{v}_x$ $\quad \bar{v}_x = 0$

y comp.: $T\Delta t = m\bar{v}_y = 0$ $\quad T\Delta t = 0$

z comp.: $-F\Delta t = m\bar{v}_z$ $\quad \bar{v}_z = -\dfrac{F\Delta t}{m}$

$$\underline{\bar{v}} = -(F\Delta t/m)\underline{k}$$ ◀

(b) Angular velocity. Equating moments about G:

$$\underline{r}_D \times (-F\Delta t)\underline{k} = \underline{H}_G$$

$$\underline{H}_G = -(a\underline{i} + a\underline{j}) \times (-F\Delta t)\underline{k} = aF\Delta t\,\underline{i} - aF\Delta t\,\underline{j}$$

$$H_x = aF\Delta t, \quad H_y = -aF\Delta t, \quad H_z = 0$$

Recalling Eqs. (18.7) and the values obtained for the moments and products of inertia on preceding page:

$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z$: $aF\Delta t = \frac{1}{6}ma^2\omega_x - \frac{1}{4}ma^2\omega_y$ (1)

$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z$: $-aF\Delta t = -\frac{1}{4}ma^2\omega_x + \frac{2}{3}ma^2\omega_y$ (2)

$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z$: $0 = \bar{I}_z\omega_z$ $\quad \omega_z = 0$ (3)

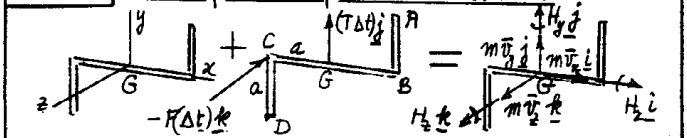
Multiply (1) by 8/3 and add (2):

$$\left(\frac{8}{3} - 1\right)aF\Delta t = \left(\frac{4}{9} - \frac{1}{4}\right)ma^2\omega_x \qquad \omega_x = \frac{60}{7}\frac{F\Delta t}{ma}$$

Eq. (1): $aF\Delta t = \frac{1}{6}ma^2\left(\frac{60}{7}\frac{F\Delta t}{ma}\right) - \frac{1}{4}ma^2\omega_y$ $\quad \omega_y = \frac{12}{7}\frac{F\Delta t}{ma}$

$$\underline{\omega} = \frac{12}{7}\frac{F\Delta t}{ma}(5\underline{i} + \underline{j})$$ ◀

18.21' Principle of Impulse & Momentum



(Weight omitted since nonimpulsive)

(a) Velocity of mass center

Same eqs. as for Prob. 18.20a: $\underline{\bar{v}} = -(F\Delta t/m)\underline{k}$ ◀

(b) Angular velocity. Equating moments about G:

$$\underline{r}_C \times (-F\Delta t)\underline{k} = \underline{H}_G$$

$$\underline{H}_G = -a\underline{i} \times (-F\Delta t)\underline{k} = -aF\Delta t\,\underline{j}$$

$$H_x = 0, \quad H_y = -aF\Delta t, \quad H_z = 0$$

Recalling Eqs. (18.7) and the values obtained for the moments and products of inertia in solution of Prob. 18.20:

$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z$: $0 = \frac{1}{6}ma^2\omega_x - \frac{1}{4}ma^2\omega_y$ (1)

$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z$: $-aF\Delta t = -\frac{1}{4}ma^2\omega_x + \frac{2}{3}ma^2\omega_y$ (2)

$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z$: $0 = \bar{I}_z\omega_z$ $\quad \omega_z = 0$ (3)

(CONTINUED)

18.21 CONTINUED | Multiply (1) by 8/3 and add (2):

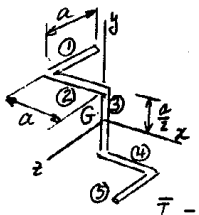
$$-aF\Delta t = \left(\frac{4}{9} - \frac{1}{4}\right)ma^2\omega_x \qquad \omega_x = -\frac{36}{7}\frac{F\Delta t}{ma}$$

Eq. (1): $0 = \frac{1}{6}ma^2\left(-\frac{36}{7}\frac{F\Delta t}{ma}\right) - \frac{1}{4}ma^2\omega_y$

$$-\frac{6}{7}aF\Delta t = \frac{1}{4}ma^2\omega_y \qquad \omega_y = -\frac{24}{7}\frac{F\Delta t}{ma}$$

$$\underline{\omega} = \frac{12}{7}\frac{F\Delta t}{ma}(-3\underline{i} - 2\underline{j})$$ ◀

18.22 Moments and Products of Inertia.



They will be computed for each element and in the order indicated.

$$\bar{I}_x = \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right] + \frac{m}{5}\left(\frac{a}{2}\right)^2$$
$$+ \frac{m}{5}\left(\frac{1}{12}a^2\right) + \frac{m}{5}\left(\frac{a}{2}\right)^2 + \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]$$
$$\bar{I}_x = 0.35ma^2$$

$$\bar{I}_y = \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + a^2\right] + \frac{m}{5}\left(\frac{1}{3}a^2\right) + 0 + \frac{m}{5}\left(\frac{1}{3}a^2\right)$$
$$+ \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + a^2\right] = 0.6667ma^2$$

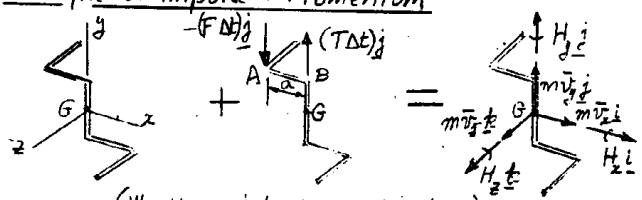
$$\bar{I}_z = \frac{m}{5}\left[a^2 + \left(\frac{a}{2}\right)^2\right] + \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right] + \frac{m}{5}\left(\frac{1}{12}a^2\right)$$
$$+ \frac{m}{5}\left[\frac{1}{12}a^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right] + \frac{m}{5}\left[a^2 + \left(\frac{a}{2}\right)^2\right] = 0.75ma^2$$

$$\bar{I}_{xy} = \frac{m}{5}(-a)\frac{a}{2} + \frac{m}{5}\left(-\frac{a}{2}\right)\left(\frac{a}{2}\right) + 0 + \frac{m}{5}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + \frac{m}{5}a\left(-\frac{a}{2}\right) = -0.3ma^2$$

$$\bar{I}_{yz} = \frac{m}{5}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + 0 + 0 + 0 + \frac{m}{5}\left(-\frac{a}{2}\right)\left(\frac{a}{2}\right) = -0.1ma^2$$

$$\bar{I}_{zx} = \frac{m}{5}\left(-\frac{a}{2}\right)(-a) + 0 + 0 + 0 + \frac{m}{5}\left(\frac{a}{2}\right)a = +0.2ma^2$$

Principle of Impulse & Momentum



(Weight omitted since nonimpulsive)

(a) Velocity of mass center. From given constraints: $\bar{v}_y = 0$

Equating components of impulse and momentum:

x comp.: $0 = m\bar{v}_x$ $\quad \bar{v}_x = 0$

y comp.: $T\Delta t - F\Delta t = m\bar{v}_y = 0$ $\quad T\Delta t = F\Delta t$

z comp.: $0 = m\bar{v}_z$ $\quad \bar{v}_z = 0$

$$\underline{\bar{v}} = 0$$ ◀

(b) Angular velocity. Equating moments about G:

$$\underline{r}_A \times (-F\Delta t)\underline{j} = \underline{H}_G$$

$$\underline{H}_G = (-a\underline{i}) \times (-F\Delta t\,\underline{j}) = aF\Delta t\,\underline{k}$$

$$H_x = 0, \quad H_y = 0, \quad H_z = aF\Delta t$$

Recalling Eqs. (18.17) and the values of the moments and products of inertia:

$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z$:
$$0 = 0.35ma^2\omega_x + 0.3ma^2\omega_y - 0.2ma^2\omega_z \quad (1)$$

$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z$:
$$0 = 0.3ma^2\omega_x + 0.6667ma^2\omega_y + 0.1ma^2\omega_z \quad (2)$$

$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z$:
$$aF\Delta t = -0.2ma^2\omega_x + 0.1ma^2\omega_y + 0.75ma^2\omega_z \quad (3)$$

(CONTINUED)

18.22 CONTINUED We rewrite Eqs. (1), (2), (3), dividing all terms by ma^2:

$$0.35\omega_x + 0.3\omega_y - 0.2\omega_z = 0 \quad (1')$$
$$0.3\omega_x + 0.6667\omega_y + 0.1\omega_z = 0 \quad (2')$$
$$-0.2\omega_x + 0.1\omega_y + 0.75\omega_z = \frac{F\Delta t}{ma} \quad (3')$$

Multiply (2') by 2 and add (1):

$$0.95\omega_x + 1.6333\omega_y = 0, \quad \omega_y = -0.581633\omega_x \quad (4)$$

Multiply (2') by 0.45 and subtract from (1):

$$0.215\omega_x - 0.245\omega_z = 0 \qquad \omega_z = 0.877551\omega_x \quad (5)$$

Substituting from (4) and (5) into (3'):

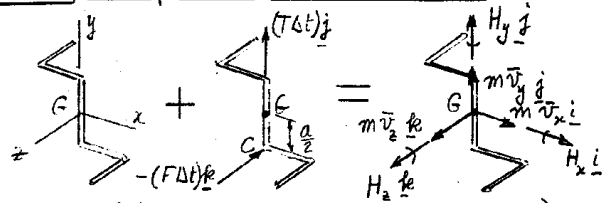
$$-0.2\omega_x - 0.0581633\omega_x + 0.6581633\omega_x = \frac{F\Delta t}{ma}, \quad \omega_x = \frac{2.5F\Delta t}{ma}$$

Eq. (4): $\omega_y = -0.581633\,\frac{2.5F\Delta t}{ma} = -1.454082\,\frac{F\Delta t}{ma}$

Eq. (5): $\omega_z = 0.877551\,\frac{2.5F\Delta t}{ma} = 2.193878\,\frac{F\Delta t}{ma}$

$$\underline{\omega} = \frac{F\Delta t}{ma}(2.5\,\underline{i} - 1.454\,\underline{j} + 2.194\,\underline{k})$$

18.23 Principle of Impulse & Momentum



(Weight omitted since nonimpulsive)

(a) Velocity of mass center. From given constraints: $\bar{v}_y = 0$

Equating components of impulse and momentum:

x comp.: $0 = m\bar{v}_x$ $\quad \bar{v}_x = 0$

y comp.: $T\Delta t = m\bar{v}_y = 0$ $\quad T\Delta t = 0$

z comp.: $-F\Delta t = m\bar{v}_z$, $\quad \bar{v}_z = -\frac{F\Delta t}{m}$, $\quad \underline{\bar{v}} = -\frac{F\Delta t}{m}\underline{k}$

(b) Angular velocity. Equating moments about G:

$$\underline{r}_C \times (-F\Delta t)\underline{k} = \underline{H}_G \qquad \underline{H}_G = (-\tfrac{a}{2}\underline{j}) \times (-F\Delta t\,\underline{k}) = \tfrac{a}{2}F\Delta t\,\underline{i}$$

$$H_x = \tfrac{1}{2}aF\Delta t \qquad H_y = 0 \qquad H_z = 0$$

We note that the moments and products of inertia are the same as for Prob. 18.22. Eqs. (18.17) lead to equations similar to Eqs. (1'), (2'), (3'), except for the right-hand members:

$$0.35\omega_x + 0.3\omega_y - 0.2\omega_z = \tfrac{1}{2}\,\frac{F\Delta t}{ma} \quad (1')$$
$$0.3\omega_x + 0.6667\omega_y + 0.1\omega_z = 0 \quad (2')$$
$$-0.2\omega_x + 0.1\omega_y + 0.75\omega_z = 0 \quad (3')$$

Multiply (2') by 7.5 and subtract (3'):

$$2.45\omega_x + 4.9\omega_y = 0 \qquad \omega_y = -\tfrac{1}{2}\omega_x \quad (4)$$

Multiply (2') by 0.15 and subtract (3'):

$$0.245\omega_x - 0.735\omega_z = 0 \qquad \omega_z = \tfrac{1}{3}\omega_x \quad (5)$$

Substitute from (4) and (5) into (1'):

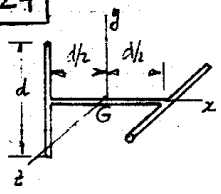
$$0.35\omega_x - 0.15\omega_x - 0.0667\omega_x = \tfrac{1}{2}\,\frac{F\Delta t}{ma} \qquad \omega_x = \frac{30}{8}\,\frac{F\Delta t}{ma}$$

Eq. (4): $\omega_y = -\frac{1}{2}\left(\frac{30}{8}\,\frac{F\Delta t}{ma}\right) = -\frac{15}{8}\,\frac{F\Delta t}{ma}$

Eq. (5): $\omega_z = \frac{1}{3}\left(\frac{30}{8}\,\frac{F\Delta t}{ma}\right) = \frac{10}{8}\,\frac{F\Delta t}{ma}$

$$\underline{\omega} = \frac{F\Delta t}{8ma}(30\,\underline{i} - 15\,\underline{j} + 10\,\underline{k})$$

18.24 Moments & Products of Inertia



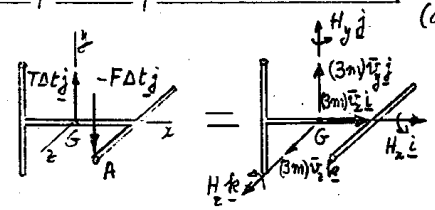
$$I_x = 2\left(\tfrac{1}{12}md^2\right) = \tfrac{1}{6}md^2$$

$$I_y = I_z = m\left(\tfrac{d}{2}\right)^2 + \tfrac{1}{12}md^2 + \left[\tfrac{1}{12}md^2 + m\left(\tfrac{d}{2}\right)^2\right]$$

$$I_y = I_z = \tfrac{2}{3}md^2$$

$$P_{xy} = P_{yz} = P_{zx} = 0$$

Principle of Impulse and Momentum:



(a) From constraints: $\bar{v}_y = 0$

Weight is omitted, since nonimpulsive

$\rightarrow$ x comp.: $0 = (3m)\bar{v}_x$ $\quad \bar{v}_x = 0$

$+\uparrow$ y comp.: $T\Delta t - F\Delta t = (3m)\bar{v}_y = 0$ $\quad T\Delta t = F\Delta t$

$\not\leftarrow$ z comp.: $0 = (3m)\bar{v}_z$ $\quad \bar{v}_z = 0$ $\quad \underline{\bar{v}} = 0$

(b) Equating moments about G:

$$\underline{r}_A \times (-F\Delta t\,\underline{j}) = \underline{H}_G$$

$$\underline{H}_G = \left(\tfrac{d}{2}\underline{i} + \tfrac{d}{2}\underline{k}\right) \times (-F\Delta t\,\underline{j}) = \tfrac{1}{2}F\Delta t\,d\,\underline{i} - \tfrac{1}{2}F\Delta t\,d\,\underline{k}$$

Thus: $H_x = \tfrac{1}{2}F\Delta t\,d$, $H_y = 0$, $H_z = -\tfrac{1}{2}F\Delta t\,d$

Since the products of inertia are zero, we have

$H_x = I_x\omega_x$, $\quad \omega_x = \dfrac{H_x}{I_x} = \dfrac{\frac{1}{2}F\Delta t\,d}{\frac{1}{6}md^2} = \dfrac{3F\Delta t}{md}$

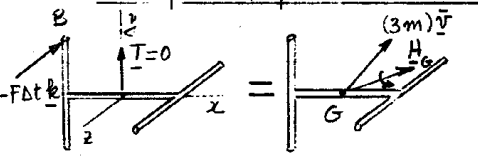
$H_y = I_y\omega_y$, $\quad \omega_y = 0$

$H_z = I_z\omega_z$, $\quad \omega_z = \dfrac{H_z}{I_z} = \dfrac{-\frac{1}{2}F\Delta t\,d}{\frac{2}{3}md^2} = -\dfrac{3F\Delta t}{4md}$

$$\underline{\omega} = \frac{3F\Delta t}{md}\left(\underline{i} - \tfrac{1}{4}\underline{k}\right)$$

18.25 See solution of Prob. 18.24 for moments of inertia.

Principle of Impulse and Momentum:



Weight is omitted, since nonimpulsive

(a) Equating vectors:

$$-F\Delta t\,\underline{k} = (3m)\,\underline{\bar{v}} \qquad \underline{\bar{v}} = -\left(\frac{F\Delta t}{3m}\right)\underline{k}$$

(b) Equating moments about G:

$$\underline{r}_B \times (-F\Delta t\,\underline{k}) = \underline{H}_G$$

$$\underline{H}_G = \left(-\tfrac{d}{2}\underline{i} + \tfrac{d}{2}\underline{j}\right) \times (-F\Delta t\,\underline{k}) = -\tfrac{1}{2}F\Delta t\,d\,\underline{i} - \tfrac{1}{2}F\Delta t\,d\,\underline{j}$$

Thus: $H_x = -\tfrac{1}{2}F\Delta t\,d$, $\quad H_y = -\tfrac{1}{2}F\Delta t\,d$, $\quad H_z = 0$

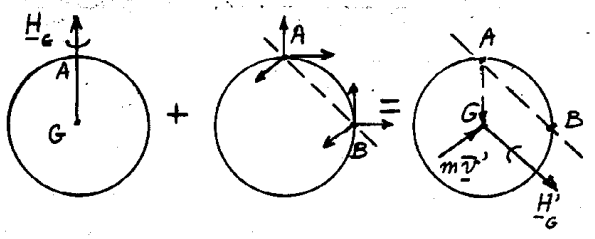
Since the products of inertia are zero, we have

$H_x = I_x\omega_x$, $\quad \omega_x = \dfrac{H_x}{I_x} = \dfrac{-\frac{1}{2}F\Delta t\,d}{\frac{1}{6}md^2} = -\dfrac{3F\Delta t}{md}$

$H_y = I_y\omega_y$, $\quad \omega_y = \dfrac{H_y}{I_y} = \dfrac{-\frac{1}{2}F\Delta t\,d}{\frac{2}{3}md^2} = -\dfrac{3F\Delta t}{4md}$

$$\underline{\omega} = \frac{3F\Delta t}{4md}(-4\underline{i} - \underline{j})$$

18.26 Principle of Impulse and Momentum



We note that

$$\underline{H}_G = I_D \omega_0 \underline{j}, \text{ where } I_D = I_{\text{Diameter}} = \tfrac{1}{4} m a^2$$

$$\underline{H}'_G = I_D \omega' \underline{\lambda}_{AB}, \text{ where } \underline{\lambda}_{AB} = \tfrac{1}{\sqrt{2}}(\underline{i} - \underline{j})$$

$$\underline{\bar{v}}' = \underline{\omega}' \times \overrightarrow{AG} = \omega' \underline{\lambda}_{AB} \times (-a\underline{j})$$

$$= \frac{\omega'}{\sqrt{2}}(\underline{i} - \underline{j}) \times (-a\underline{j}) = -\frac{a}{\sqrt{2}} \omega' \underline{k}$$

(a) Equating the moments about AB of all vectors and couples shown:

$$\underline{\lambda}_{AB} \cdot \underline{H}_G + 0 = \underline{\lambda}_{AB} \cdot (\overrightarrow{AG} \times m\underline{\bar{v}}') + \underline{\lambda}_{AB} \cdot \underline{H}'_G,$$

$$\frac{1}{\sqrt{2}}(\underline{i} - \underline{j}) \cdot I_D \omega_0 \underline{j} = \frac{1}{\sqrt{2}}(\underline{i} - \underline{j}) \cdot \left[-a\underline{j} \times m\left(-\frac{a}{\sqrt{2}}\omega' \underline{k}\right)\right] + \underline{\lambda}_{AB} \cdot I_D \omega' \underline{\lambda}_{AB}$$

$$\frac{1}{\sqrt{2}} I_D \omega_0 (0 - 1) = \frac{1}{\sqrt{2}}(\underline{i} - \underline{j}) \cdot \frac{1}{\sqrt{2}} m a^2 \omega' \underline{i} + I_D \omega' (1)$$

$$-\frac{1}{\sqrt{2}} \frac{1}{4} m a^2 \omega_0 = \frac{1}{2} m a^2 \omega' (1 - 0) + \frac{1}{4} m a^2 \omega'$$

$$-\frac{1}{4\sqrt{2}} m a^2 \omega_0 = \frac{3}{4} m a^2 \omega'$$

$$\omega' = -\frac{1}{3\sqrt{2}} \omega_0$$

$$\underline{\omega}' = -\frac{1}{3\sqrt{2}} \omega_0 \underline{\lambda}_{AB}$$

Since $\underline{\lambda}_{AB} = \frac{1}{\sqrt{2}}(\underline{i} - \underline{j})$:

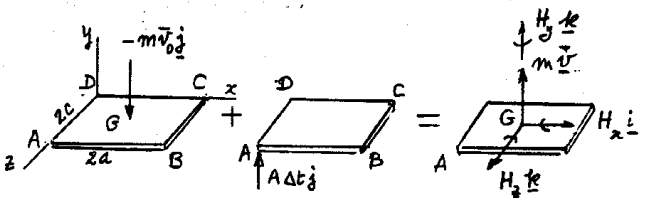
$$\underline{\omega}' = -\frac{1}{3\sqrt{2}} \omega_0 \frac{1}{\sqrt{2}}(\underline{i} - \underline{j}) \qquad \underline{\omega}' = \frac{1}{6} \omega_0 (-\underline{i} + \underline{j}) \blacktriangleleft$$

(b) Recalling that $\underline{\bar{v}}' = -\frac{a}{\sqrt{2}} \omega' \underline{k}$:

$$\underline{\bar{v}}' = -\frac{a}{\sqrt{2}} \left(-\frac{1}{3\sqrt{2}} \omega_0\right) \underline{k}$$

$$\underline{\bar{v}}' = \frac{1}{6} \omega_0 a \underline{k} \blacktriangleleft$$

18.27 Principle of Impulse & Momentum



Equating x components: $\bar{v}_x = 0$

Equating z components: $\bar{v}_z = 0$ Thus: $\underline{\bar{v}} = \bar{v}\underline{j}$

Equating moments about A:

$$(a\underline{i} - c\underline{k}) \times (-m\bar{v}_0 \underline{j}) + 0 = (a\underline{i} - c\underline{k}) \times m\bar{v}\underline{j} + H_x \underline{i} + H_y \underline{j} + H_z \underline{k} \quad (1)$$

Since plate rotates about fixed point A:

$$\bar{v}\underline{j} = \underline{\omega} \times \underline{r}_{G/A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ a & 0 & -c \end{vmatrix} = -\omega_y c \underline{i} + (\omega_z a + \omega_x c)\underline{j} - \omega_y a \underline{k}$$

Equating the coeff. of the unit vectors:

$$\omega_y = 0 \qquad \bar{v} = \omega_z a + \omega_x c \quad (2)$$

Thus: $(a\underline{i} - c\underline{k}) \times m\bar{v}\underline{j} = (a\underline{i} - c\underline{k}) \times m(\omega_z a + \omega_x c)\underline{j}$

$$= m(a^2\omega_z + ac\omega_x)\underline{k} + m(ac\omega_z + c^2\omega_x)\underline{i}$$

On the other hand:

$$H_x = \bar{I}_x \omega_x = \tfrac{1}{12} m(2c)^2 \omega_x = \tfrac{1}{3} m c^2 \omega_x$$

$$H_z = \bar{I}_z \omega_z = \tfrac{1}{12} m(2a)^2 \omega_z = \tfrac{1}{3} m a^2 \omega_z$$

$$H_y = \bar{I}_y \omega_y$$

Substituting into Eq. (1):

$$m\bar{v}_0(-c\underline{i} - a\underline{k}) = m(a^2\omega_z + ac\omega_x)\underline{k} + m(ac\omega_z + c^2\omega_x)\underline{i} + \tfrac{1}{3} m c^2 \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \tfrac{1}{3} m a^2 \omega_z \underline{k}$$

Equating the coefficients of the unit vectors:

$\underline{j}$: $0 = \bar{I}_y \omega_y \qquad \omega_y = 0$

$\underline{i}$: $-m\bar{v}_0 c = \frac{4}{3} m c^2 \omega_x + mac\omega_z \quad (3)$

$\underline{k}$: $-m\bar{v}_0 a = mac\omega_x + \frac{4}{3} m a^2 \omega_z \quad (4)$

Multiply (3) by 4a: $-4m\bar{v}_0 ac = \frac{16}{3} m c^2 \omega_x + 4ma^2 c\omega_z$

Multiply (4) by -3c: $+3m\bar{v}_0 ac = -3mac^2\omega_x - 4ma^2 c\omega_z$

Add: $-m\bar{v}_0 ac = \frac{16-9}{3} mac^2 \omega_x$

$$\omega_x = -\frac{3}{7}\frac{\bar{v}_0}{c}$$

Substitute into (3):

$$-m\bar{v}_0 c = \frac{4}{3} m c^2 \left(-\frac{3}{7}\frac{\bar{v}_0}{c}\right) + mac\omega_z$$

$$\omega_z = \left(\frac{4}{7} - 1\right)\frac{\bar{v}_0}{a} = -\frac{3}{7}\frac{\bar{v}_0}{a}$$

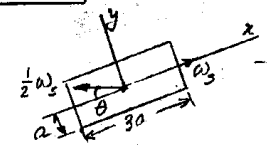
(a) $$\underline{\omega} = -\frac{3}{7}\bar{v}_0\left(\frac{1}{c}\underline{i} + \frac{1}{a}\underline{k}\right) \blacktriangleleft$$

(b) Substitute for ω_x and ω_z into Eq. (2):

$$\bar{v} = \omega_z a + \omega_x c = -\frac{3}{7}\bar{v}_0 - \frac{3}{7}\bar{v}_0 = -\frac{6}{7}\bar{v}_0$$

$$\underline{\bar{v}} = -\frac{6}{7}\bar{v}_0 \underline{j} \blacktriangleleft$$

18.28 Angular momentum of cylinder A.



We resolve angular velocities into components along principal axes of inertia:

$\omega_x = \omega_s - \frac{1}{2}\omega_s\cos\theta$

$\omega_y = \frac{1}{2}\omega_s\sin\theta$

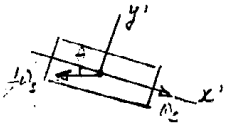
Moments of inertia: $I_x = \frac{1}{2}ma^2$

$I_y = \frac{1}{12}m(3a^2+L^2) = \frac{1}{12}m(3a^2+9a^2) = ma^2$

$H_x = I_x\omega_x = \frac{1}{2}ma^2\omega_s(1-\frac{1}{2}\cos\theta)$

$H_y = I_y\omega_y = ma^2(\frac{1}{2}\omega_s\sin\theta) = \frac{1}{2}ma^2\omega_s\sin\theta$

Angular momentum of cylinder B.

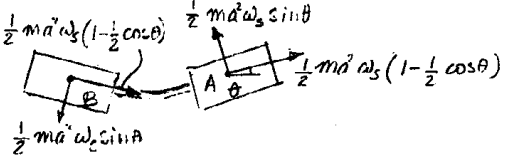


We find in a similar way

$H_{x'} = \frac{1}{2}ma^2\omega_s(1-\frac{1}{2}\cos\theta)$

$H_{y'} = -\frac{1}{2}ma^2\omega_s\sin\theta$

Angular momentum of entire model



We check that the vertical components of the angular-momentum vectors add up to zero while their horizontal components add up to

$H_h = 2\left[\frac{1}{2}ma^2\omega_s(1-\frac{1}{2}\cos\theta)\right]\cos\theta - 2\left[\frac{1}{2}ma^2\omega_s\sin\theta\right]\sin\theta$

$= ma^2\omega_s(\cos\theta - \frac{1}{2}\cos^2\theta - \sin^2\theta)$

$= ma^2\omega_s(\cos\theta - \frac{1}{2}\cos^2\theta - 1 + \cos^2\theta)$

$= \frac{1}{2}ma^2\omega_s(\cos^2\theta + 2\cos\theta - 2)$

Conservation of angular momentum. Since the system starts from rest, we must have

$H_h = 0 \qquad \cos^2\theta + 2\cos\theta - 2 = 0$

$\cos\theta = -1 \pm \sqrt{3}$

$\cos\theta = -1-\sqrt{3} = -2.732$ (impossible)

$\cos\theta = -1+\sqrt{3} = +0.73205 \qquad \theta = 42.9°$ ◀

18.29 $m = 1200$ kg

$\bar{k}_x = 1.120$ m, $\bar{k}_y = 1.200$ m, $\bar{k}_z = 0.900$ m

$\underline{\omega}_0 = (0.050\ \text{rad/s})\underline{i} + (0.075\ \text{rad/s})\underline{k}$

Initial ang. momentum about G:

$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$

with $\bar{I}_x = m\bar{k}_x^2 = 1200(1.120)^2 = 1505.28$ kg·m²

$\bar{I}_y = m\bar{k}_y^2 = 1200(1.200)^2 = 1728.0$ kg·m²

$\bar{I}_z = m\bar{k}_z^2 = 1200(0.900)^2 = 972.0$ kg·m²

$\underline{H}_G = (1505.28)(0.050\underline{i}) + 0 + (972.0)(0.075)\underline{k}$

$\underline{H}_G = 75.264\underline{i} + 72.900\underline{k}$

(CONTINUED)

18.29 CONTINUED Principle of Impulse and Momentum

Final angular momentum will be zero if $\omega = 0$: $\underline{H}'_G = 0$

Syst. Init. Mom. + Syst. Impulses = Syst. Final Mom.:

(a) Equating moments about G:

$\underline{H}_G + \overrightarrow{GA}\times(-50\text{N})(\Delta t_A)\underline{j} + \overrightarrow{GB}\times(-50\text{N})(\Delta t_B)\underline{j} = 0$

$\underline{H}_G + (1.2\underline{i} + 1.8\underline{k})\times(-50\,\Delta t_A\,\underline{j}) + (1.2\underline{i} - 1.6\underline{k})\times(-50\,\Delta t_B\,\underline{j}) = 0$

$75.264\underline{i} + 72.9\underline{k} - 60\,\Delta t_A\,\underline{k} + 90\,\Delta t_A\,\underline{i} - 60\,\Delta t_B\,\underline{k} - 80\,\Delta t_B\,\underline{i} = 0$

Equating to zero the coeffs. of $\underline{i}$ and $\underline{k}$:

(i) $75.264 + 90\,\Delta t_A - 80\,\Delta t_B = 0$ $\qquad 1.125\,\Delta t_A - \Delta t_B = -0.9408$

(k) $72.9 - 60\,\Delta t_A - 60\,\Delta t_B = 0$ $\qquad \Delta t_A + \Delta t_B = 1.215$

Solving simultaneously:

$\Delta t_A = 0.1290$ s $\qquad \Delta t_B = 1.086$ s ◀

(b) Equating vectors:

$m\underline{\bar{v}}_0 + (-50\text{N})(\Delta t_A)\underline{j} + (-50\text{N})(\Delta t_B)\underline{j} = m\underline{\bar{v}}'$

$\Delta\underline{\bar{v}} = \underline{\bar{v}}' - \underline{\bar{v}}_0 = -\frac{50}{1200}(\Delta t_A + \Delta t_B)\underline{j} = -\frac{50}{1200}(0.129 + 1.086)\underline{j}$

$\Delta\underline{\bar{v}} = -(50.62\times10^{-3}\ \text{m/s})\underline{j} = -(50.6\ \text{mm/s})\underline{j}$ ◀

18.30 See solution of Prob. 18.29 for determination of initial angular momentum about G:

$\underline{H}_G = 75.264\underline{i} + 72.900\underline{k}$

Principle of Impulse and Momentum

We want $\omega_z = 0$, therefore $(H'_G)_z = 0$. Thus, final ang. momentum $\underline{H}'_G$ will be

$\underline{H}'_G = \bar{I}_z\omega_z\underline{k} = 972\,\omega_z\underline{k}$

Equating moments about G:

$\underline{H}_G + \overrightarrow{GB}\times(-50\text{N})(\Delta t_B)\underline{j} = \underline{H}'_G$

$75.264\underline{i} + 72.9\underline{k} + (1.2\underline{i} - 1.6\underline{k})\times(-50\,\Delta t_B\,\underline{j}) = 972\omega_z\underline{k}$

$75.264\underline{i} + 72.9\underline{k} - 60\,\Delta t_B\,\underline{k} - 80\,\Delta t_B\,\underline{i} = 972\,\omega_z\underline{k}$

(a) Operating time for jet B:

Equate coeffs. of $\underline{i}$: $75.264 - 80\,\Delta t_B = 0$

$\Delta t_B = 0.9408$ s $\qquad \Delta t_B = 941$ ms ◀

(b) Final angular velocity:

Equate coeff. of $\underline{k}$: $72.9 - 60\,\Delta t_B = 972\omega_z$

$972\,\omega_z = 72.9 - 60(0.9408), \quad \omega_z = 16.926\times10^{-3}$ rad/s

$\underline{\omega} = (0.01693\ \text{rad/s})\underline{k}$ ◀

(c) Change in velocity of mass center:

Equate vectors:

$m\underline{\bar{v}}_0 + (-50\text{N})(\Delta t_B)\underline{j} = m\underline{\bar{v}}'$

$\Delta\underline{\bar{v}} = \underline{\bar{v}}' - \underline{\bar{v}}_0 = -\frac{50}{1200}\Delta t_B = -\frac{50}{1200}(0.9408)$

$\Delta\underline{\bar{v}} = -(39.20\times10^{-3}\ \text{m/s})\underline{j} = -(39.2\ \text{mm/s})\underline{j}$ ◀

18.31 Moments of Inertia of satellite

$$\bar{I}_x = \bar{I}_z = \frac{800\text{ lb}}{g}\left(\frac{28.8}{12}\text{ ft}\right)^2 = \frac{4608}{g}$$

$$\bar{I}_y = \frac{800\text{ lb}}{g}\left(\frac{32.4}{12}\text{ ft}\right)^2 = \frac{5832}{g}$$

Since the x, y, z axes are principal axes of inertia, all products of inertia are zero

Principle of Impulse and Momentum for Satellite and Meteorite.

Since there is no external impulse, the initial and final momenta must be equipollent. Equating moments about G we write

$$(\underline{H}_G)_0 + \underline{r}_A \times m\underline{v}_0 = \underline{H}_G \qquad (1)$$

where $\underline{r}_A = \left(\frac{42}{12}\text{ ft}\right)\underline{i}$

$$m\underline{v}_0 = \frac{6/16}{g}\left(v_x\underline{i} + v_y\underline{j} + v_z\underline{k}\right) = \frac{0.375}{g}\left(v_x\underline{i} + v_y\underline{j} + v_z\underline{k}\right)$$

$$\underline{r}_A \times m\underline{v}_0 = \frac{42}{12}\underline{i} \times \frac{0.375}{g}\left(v_x\underline{i} + v_y\underline{j} + v_z\underline{k}\right)$$

$$= \frac{1.3125}{g}\left(v_y\underline{k} - v_z\underline{j}\right) \qquad (2)$$

$(\underline{H}_G)_0$ = Initial ang. mom. of satellite

$$= \bar{I}_y\omega_0\underline{j} = \frac{5832}{g}(1.5\text{ rad/s})\underline{j} = \frac{8748}{g}\underline{j} \qquad (3)$$

$\underline{H}_G$ = Final ang. mom. of satellite

$$= \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$$

$$= 0 + \frac{5832}{g}(0.6\text{ rad/s})\underline{j} + \frac{4608}{g}(0.37\text{ rad/s})\underline{k}$$

$$= \frac{3499.2}{g}\underline{j} + \frac{1704.96}{g}\underline{k} \qquad (4)$$

Substituting from (2), (3), (4) into Eq. (1) and multiplying through by g:

$$8748\underline{j} + 1.3125v_y\underline{k} - 1.3125v_z\underline{j} = 3499.2\underline{j} + 1704.96\underline{k}$$

Equating the coefficients of the unit vectors:

(i) $8748 - 1.3125v_z = 3499.2$

$$v_z = 4000\text{ ft/s}$$

(k) $1.3125v_y = 1704.96$

$$v_y = 1300\text{ ft/s}$$

Recalling that $v_0 = 4500$ ft/s:

$$v_0^2 = v_x^2 + v_y^2 + v_z^2$$

$$(4500)^2 = v_x^2 + (1300)^2 + (4000)^2$$

$$v_x = \pm 1600\text{ ft/s}$$

But, for meteorite to hit satellite at A, we need $v_x < 0$. Thus $v_x = -1600$ ft/s

$$\underline{v}_0 = -(1600\text{ ft/s})\underline{i} + (1300\text{ ft/s})\underline{j} + (4000\text{ ft/s})\underline{k}$$ ◀

18.32 Principle of Impulse and Momentum for Satellite and Meteorite.

Since there is no external impulse, the initial and final momenta must be equipollent. Equating moments about G we write

$$(\underline{H}_G)_0 + \underline{r}_B \times m\underline{v}_0 = \underline{H}_G \qquad (1)$$

where $\underline{r}_B = \left(\frac{42}{12}\text{ ft}\right)\underline{i} - b\underline{j}$

$$m\underline{v}_0 = \frac{6/16}{g}\left[-(3360\text{ ft/s})\underline{i} + (3480\text{ ft/s})\underline{j} - (2400\text{ ft/s})\underline{k}\right]$$

$$\underline{r}_B \times m\underline{v}_0 = \frac{6/16}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{42}{12} & -b & 0 \\ -3360 & 3480 & -2400 \end{vmatrix}$$

$$= \frac{1}{g}\left[900b\underline{i} + 3150\underline{j} + (4567.5 - 1260b)\underline{k}\right] \qquad (2)$$

$(\underline{H}_G)_0$ = Initial ang. mom. of satellite

$$= \bar{I}_y\omega_0\underline{j} \quad \text{or, since } \bar{I}_y = \frac{5832}{g} \text{ (see Prob. 18.31)}$$

$$= \frac{5832}{g}(1.8\text{ rad/s})\underline{j} = \frac{10,497.6}{g}\underline{j} \qquad (3)$$

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\underline{k}$$

or, since $\bar{I}_x = \bar{I}_z = \frac{4608}{g}$ and $\bar{I}_y = \frac{5832}{g}$ (see Prob. 18.31)

and $\omega_x = 0.36$ rad/s

$$\underline{H}_G = \frac{4608}{g}(0.36\text{ rad/s})\underline{i} + \frac{5832}{g}\omega_y\underline{j} + \frac{4608}{g}\omega_z\underline{k}$$

$$= \frac{1}{g}\left(1658.88\underline{i} + 5832\omega_y\underline{j} + 4608\omega_z\underline{k}\right) \qquad (4)$$

Substituting from (2), (3), (4) into Eq. (1) and multiplying through by g:

$$10,497.6\underline{j} + 900b\underline{i} + 3150\underline{j} + (4567.5 - 1260b)\underline{k} = 1658.88\underline{i} + 5832\omega_y\underline{j} + 4608\omega_z\underline{k}$$

Equating the coefficients of the unit vectors:

(i) $900b = 1658.88$ $\quad b = 1.8432$ ft

(j) $10,497.6 + 3150 = 5832\omega_y$ $\quad \omega_y = 2.34$ rad/s

(k) $4567.5 - 1260b = 4608\omega_z$

$4567.5 - 1260(1.8432\text{ ft}) = 4608\omega_z$

$\omega_z = 0.487$ rad/s

(a) Distance b

$b = 1.8432\text{ ft} = 1.8432 \times 12 = 22.12\text{ in.}$

$b = 22.1$ in. ◀

(b) Components of $\underline{\omega}$

$\omega_y = 2.34$ rad/s, $\omega_z = 0.487$ rad/s ◀

Thus: $\underline{\omega} = (0.36\text{ rad/s})\underline{i} + (2.34\text{ rad/s})\underline{j} + (0.487\text{ rad/s})\underline{k}$

18.33 (a) We first compute the scalar product $\underline{H}_O \cdot \underline{\omega}$:

$$\underline{H}_O \cdot \underline{\omega} = (H_x \underline{i} + H_y \underline{j} + H_z \underline{k}) \cdot (\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k})$$
$$= H_x \omega_x + H_y \omega_y + H_z \omega_z \qquad (1)$$

We may assume that the principal axes have been selected as coordinate axes. We then have

$$H_x = I_x \omega_x \qquad H_y = I_y \omega_y \qquad H_z = I_z \omega_z$$

Substituting into Eq. (1):

$$\underline{H}_O \cdot \underline{\omega} = (I_x \omega_x)\omega_x + (I_y \omega_y)\omega_y + (I_z \omega_z)\omega_z$$
$$= I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 \qquad (2)$$

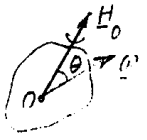
But, from Eq. (18.20)

$$T = \tfrac{1}{2}(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$$

Comparing with Eq. (2) we conclude that

$$\underline{H}_O \cdot \underline{\omega} = 2T \qquad (Q.E.D.)$$

(b) The scalar product $\underline{H}_O \cdot \underline{\omega}$ may be expressed as



$$\underline{H}_O \cdot \underline{\omega} = H_O \omega \cos\theta$$

Thus: $\cos\theta = \dfrac{\underline{H}_O \cdot \underline{\omega}}{H_O \omega} = \dfrac{2T}{H_O \omega}$

Since T is always positive, $\cos\theta > 0$, and $\theta < 90°$ (Q.E.D.)

18.34 (a) Derivation from Eqs. (9.46) and (18.19).

Eq. (18.11): $T = \tfrac{1}{2}(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 - 2I_{xy}\omega_x\omega_y - 2I_{yz}\omega_y\omega_z - 2I_{zx}\omega_z\omega_x)$

Setting $\omega_x = \omega \cos\theta_x = \omega \lambda_x$
$\omega_y = \omega \cos\theta_y = \omega \lambda_y$
$\omega_z = \omega \cos\theta_z = \omega \lambda_z$

we have

$$T = \tfrac{1}{2}(I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x)\omega^2$$

But, by Eq. (9.46) of Sec. 9.17, the expression in parentheses is I_{OL}. Thus:

$$T = \tfrac{1}{2} I_{OL} \omega^2 \qquad (Q.E.D.)$$

(b) Considering T as the sum of the kinetic energies of the particles:

$$T = \tfrac{1}{2}\sum_i (\Delta m_i) v_i^2 = \tfrac{1}{2}\sum_i (\Delta m_i) \rho_i^2 \omega^2$$
$$T = \tfrac{1}{2}\left(\sum_i \rho_i^2 \Delta m_i\right)\omega^2$$

But $\sum_i \rho_i^2 \Delta m_i = I_{OL}$. Thus:

$$T = \tfrac{1}{2} I_{OL} \omega^2 \qquad (Q.E.D.)$$

18.35

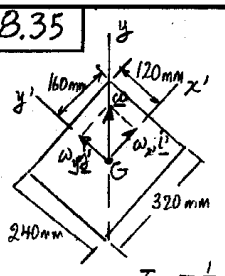


Plate of Prob. 18.2

Resolving $\underline{\omega}$ along the principal axes x', y', z':

$$\omega_{x'} = \frac{160}{200}\omega = 0.8(5 \text{ rad/s}) = 4 \text{ rad/s}$$
$$\omega_{y'} = \frac{120}{200}\omega = 0.6(5 \text{ rad/s}) = 3 \text{ rad/s}$$
$$\omega_{z'} = 0$$

Moments of Inertia:

$$I_{x'} = \tfrac{1}{12}(6 \text{ kg})(0.24 \text{ m})^2 = 0.0288 \text{ kg}\cdot\text{m}^2$$
$$I_{y'} = \tfrac{1}{12}(6 \text{ kg})(0.32 \text{ m})^2 = 0.0512 \text{ kg}\cdot\text{m}^2$$

From Eqs. (18.20):

$$T = \tfrac{1}{2}(I_{x'}\omega_{x'}^2 + I_{y'}\omega_{y'}^2 + I_{z'}\omega_{z'}^2)$$
$$= \tfrac{1}{2}[0.0288(4)^2 + 0.0512(3)^2 + 0] = 0.4608 \text{ J}$$
$$T = 0.461 \text{ J}$$

18.36

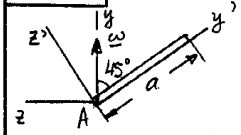


Plate of Prob. 18.4

Resolving $\underline{\omega}$ along the principal axes x, y', z':

$$\omega_x = 0 \qquad \omega_{y'} = \omega_{z'} = \frac{\omega}{\sqrt{2}}$$

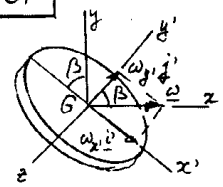
Moments of Inertia:

$$I_{x'} = \tfrac{1}{3}ma^2, \qquad I_{y'} = \tfrac{1}{12}ma^2$$
$$I_{z'} = I_{x'} + I_{y'} = \tfrac{1}{3}ma^2 + \tfrac{1}{12}ma^2 = \tfrac{5}{12}ma^2$$

From Eqs. (18.20):

$$T = \tfrac{1}{2}(I_{x'}\omega_{x'}^2 + I_{y'}\omega_{y'}^2 + I_{z'}\omega_{z'}^2)$$
$$= \tfrac{1}{2}\left(0 + \tfrac{1}{12}ma^2\frac{\omega^2}{2} + \tfrac{5}{12}ma^2\frac{\omega^2}{2}\right) = \tfrac{1}{8}ma^2\omega^2$$

18.37



Disk of Prob. 18.3

Resolving $\underline{\omega}$ along the principal axes x', y', z':

$$\omega_{x'} = \omega\sin\beta, \quad \omega_{y'} = \omega\cos\beta, \quad \omega_{z'} = 0$$

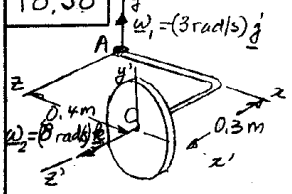
Moments of inertia:

$$I_{x'} = \tfrac{1}{4}mr^2 \qquad I_{y'} = \tfrac{1}{2}mr^2$$

From Eqs. (18.20):

$$T = \tfrac{1}{2}(I_{x'}\omega_{x'}^2 + I_{y'}\omega_{y'}^2 + I_{z'}\omega_{z'}^2) = \tfrac{1}{8}mr^2\omega^2(\sin^2\beta + 2\cos^2\beta)$$
$$= \tfrac{1}{8}mr^2\omega^2(1 + \cos^2\beta) = \tfrac{1}{8}mr^2\omega^2(1 + \cos^2 20°)$$
$$T = 0.235 mr^2\omega^2$$

18.38



Disk of Prob. 18.6

From Eqs. (18.17):

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_{z'}\omega_{z'}^2)$$

where $\bar{v} = \omega_1 \times r_{G/A}$

$$\bar{v} = 3\underline{j} \times (0.4\underline{i} + 0.3\underline{k})$$
$$= 0.9\underline{i} - 1.2\underline{k}$$

$r = 0.25$ m

$\omega_{x'} = 0, \quad \omega_{y'} = 3 \text{ rad/s}, \quad \omega_{z'} = 8 \text{ rad/s}$

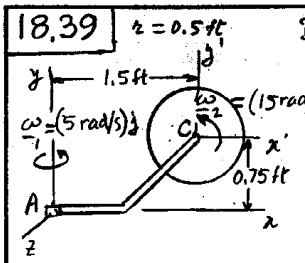
$\bar{I}_{x'} = \bar{I}_{y'} = \tfrac{1}{4}mr^2 = \tfrac{1}{4}(5)(0.25)^2 = 0.078125, \quad \bar{I}_{z'} = \tfrac{1}{2}mr^2 = 0.15625$

Thus: $T = \tfrac{1}{2}(5)[(0.9)^2 + (1.2)^2] + \tfrac{1}{2}[0 + 0.078125(3)^2 + (0.15625(8)^2]$

$= 5.625 + 5.352 = 10.977$ J

$T = 10.98$ J

18.39 $r = 0.5$ ft Disk of Prob. 18.5.



Eq. (18.17):

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\left(\bar{I}_{x'} \omega_{x'}^2 + \bar{I}_{y'} \omega_{y'}^2 + \bar{I}_{z'} \omega_{z'}^2\right)$$

where $\underline{\bar{v}} = \underline{\omega}_1 \times \underline{r}_C$

$= (5 \text{ rad/s})\underline{j} \times [(1.5 \text{ ft})\underline{i} + (0.75 \text{ ft})\underline{j}]$

$= -(7.5 \text{ ft/s})\underline{k}$

$\omega_x = 0, \quad \omega_y = 5 \text{ rad/s}, \quad \omega_z = 15 \text{ rad/s}$

$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2 = \frac{1}{4} \frac{10}{32.2} (0.5 \text{ ft})^2 = 0.019410 \text{ ft·lb·s}^2$

$\bar{I}_{z'} = \frac{1}{2} m r^2 = 0.038820 \text{ ft·lb·s}^2$

Thus: $T = \frac{1}{2} \frac{10}{32.2} (7.5)^2 + \frac{1}{2}[0 + 0.019410(5)^2 + 0.038820(15)^2]$

$= 8.7345 + 4.6099 \qquad T = 13.34 \text{ ft·lb}$

18.40 Body of Prob. 18.16.

We have $\omega_x = \omega_y = 0, \ \omega_z = \omega = 240 \text{ rpm} = 25.133 \text{ rad/s}$

Eq. (18.14) reduces to $T = \frac{1}{2} \bar{I}_z \omega_z^2$

From the solution of Prob. 18.17 we recall that

$$\bar{I}_z = \bar{I}_{z'} = \frac{10}{3} m a^2$$

where $a = 8 \text{ in.} = \frac{2}{3} \text{ ft}, \quad m = \text{mass of one element of rod}$

$= \frac{1}{2} \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.062112$

Thus: $T = \frac{1}{2}\left(\frac{10}{3} m a^2\right)\omega_z^2 = \frac{5}{3} m a^2 \omega_z^2$

$= \frac{5}{3}(0.062112)\left(\frac{2}{3}\right)^2 (25.133)^2 = 29.06 \text{ ft·lb}$

$T = 29.1 \text{ ft·lb}$ ◀

18.41 Plate of Prob. 18.27.

Before impact: $T_0 = \frac{1}{2} m \bar{v}_0^2$

After impact: Since centroidal axes x', y', z' parallel to the coordinate axes are principal axes of inertia, Eq. (18.17) reduces to

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\left(\bar{I}_{x'} \omega_{x'}^2 + \bar{I}_{y'} \omega_{y'}^2 + \bar{I}_{z'} \omega_{z'}^2\right)$$

But, from Prob. 18.27 we have

$$\underline{\bar{v}} = -\frac{6}{7} \bar{v}_0 \underline{j} \qquad \bar{v} = \frac{6}{7} \bar{v}_0$$

$$\omega_x = -\frac{3}{7}\frac{\bar{v}_0}{c} \qquad \omega_y = 0 \qquad \omega_z = -\frac{3}{7}\frac{\bar{v}_0}{a}$$

$$\bar{I}_{x'} = \frac{1}{3} m c^2 \qquad \bar{I}_{z'} = \frac{1}{3} m a^2$$

Thus: $T = \frac{1}{2} m\left(\frac{6}{7}\bar{v}_0\right)^2 + \frac{1}{2}\left[\frac{1}{3} m c^2 \left(-\frac{3}{7}\frac{\bar{v}_0}{c}\right)^2 + 0 + \frac{1}{3} m a^2 \left(-\frac{3}{7}\frac{\bar{v}_0}{a}\right)^2\right]$

$= \frac{1}{2} m \bar{v}_0^2 \left(\frac{36}{49} + \frac{3}{49} + \frac{3}{49}\right) = \frac{1}{2}\frac{42}{49} m \bar{v}_0^2$

$\Delta T = T - T_0 = \frac{1}{2}\frac{42}{49} m \bar{v}_0^2 - \frac{1}{2} m \bar{v}_0^2 = -\frac{1}{2}\frac{7}{49} m \bar{v}_0^2$

Kinetic Energy Lost $= \frac{1}{14} m \bar{v}_0^2$ ◀

18.42 Plate of Prob. 18.26

Before impact: $T_0 = \frac{1}{2} \bar{I}_D \omega_0^2 = \frac{1}{2}\frac{ma^2}{4}\omega_0^2 = \frac{1}{8} m a^2 \omega_0^2$

After impact: Eq. (18.17):

$$T = \frac{1}{2} m \bar{v}'^2 + \frac{1}{2}\left[\bar{I}_{x'} \omega_{x'}^2 + \bar{I}_{y'} \omega_{y'}^2 + \bar{I}_{z'} \omega_{z'}^2\right]$$

From Prob. 18.26:

$$\underline{\bar{v}}' = \frac{1}{6}\omega_0 a \underline{k}, \qquad \underline{\omega}' = \frac{1}{6}\omega_0(-\underline{i} + \underline{j})$$

$$\bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_D = \frac{ma^2}{4}$$

Therefore:

$$T = \frac{1}{2} m\left(\frac{1}{6}\omega_0 a\right)^2 + \frac{1}{2}\left[\frac{ma^2}{4}\left(\frac{1}{6}\omega_0\right)^2 + \frac{ma^2}{4}\left(\frac{1}{6}\omega_0\right)^2\right]$$

$$= \frac{1}{2} m a^2 \omega_0^2 \left(\frac{1}{36} + \frac{1}{144} + \frac{1}{144}\right) = \frac{1}{48} m a^2 \omega_0^2$$

$$\Delta T = T - T_0 = \frac{1}{48} m a^2 \omega_0^2 - \frac{1}{8} m a^2 \omega_0^2 = -\frac{5}{48} m a^2 \omega_0^2$$

Kinetic Energy Lost $= \frac{5}{48} m a^2 \omega_0^2$ ◀

18.43 Satellite of Prob. 18.29. See solution of that problem.

Before jets operate:

$$T_0' = \frac{1}{2}\left[\bar{I}_x \omega_x^2 + \bar{I}_y \omega_y^2 + \bar{I}_z \omega_z^2\right]$$

$$= \frac{1}{2}\left[1505.28(0.050)^2 + 0 + 972.0(0.075)^2\right]$$

$$= 4.61535 \text{ J}$$

After jets have operated: $\underline{\omega} = 0 \qquad T' = 0$

Thus:

$$\Delta T' = T' - T_0' = 0 - 4.61535 \text{ J}$$

$$\Delta T = -4.62 \text{ J}$$ ◀

Note: T_0' and T' represent the kinetic energy of the satellite in its motion relative to centroidal axes.

18.44 Satellite of Prob. 18.32

Moments of Inertia: $\bar{I}_x = \bar{I}_z = \frac{800 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{28.8}{12} \text{ ft}\right)^2 = 143.106 \text{ lb·ft·s}^2$

$$\bar{I}_y = \frac{800 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{32.4}{12} \text{ ft}\right)^2 = 181.118 \text{ lb·ft·s}^2$$

Before collision: $\underline{\omega}_0 = (1.8 \text{ rad/s})\underline{j}$

Kinetic energy of satellite relative to centroidal axes is

$$T_0' = \frac{1}{2} \bar{I}_y \omega_0^2 = \frac{1}{2}(181.118)(1.8)^2 = 293.41 \text{ ft·lb}$$

After collision: $\omega_x = 0.36 \text{ rad/s}, \ \omega_y = 2.34 \text{ rad/s}, \ \omega_z = 0.487 \text{ rad/s}$

$$T' = \frac{1}{2}\left(\bar{I}_x \omega_x^2 + \bar{I}_y \omega_y^2 + \bar{I}_z \omega_z^2\right)$$

$$= \frac{1}{2}\left[143.106(0.36)^2 + 181.118(2.34)^2 + 143.106(0.487)^2\right]$$

$$= 522.11 \text{ ft·lb}$$

$$\Delta T' = T' - T_0 = 522.11 - 293.41 = +228.70 \text{ ft·lb}$$

$$\Delta T' = +229 \text{ ft·lb}$$ ◀

18.45 From Prob. 18.1:

$$\underline{H}_G = -(53.8\ g\cdot m^2/s)\underline{i} + (184.3\ g\cdot m^2/s)\underline{j}$$

where the frame $Axyz$ rotates with the plate with the angular velocity

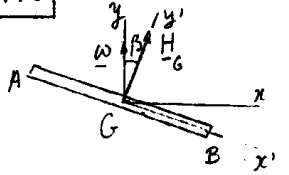
$$\underline{\Omega} = \underline{\omega} = (5\ rad/s)\underline{j}$$

We have $(\dot{\underline{H}}_G)_{Axyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Axyz} + \underline{\Omega}\times\underline{H}_G = 0 + 5\underline{j}\times(-53.8\underline{i} + 184.3\underline{j})$$

$$= (269\ g\cdot m^2/s^2)\underline{k} \qquad \dot{\underline{H}}_G = (0.269\ N\cdot m)\underline{k}$$ ◀

18.46 From Prob. 18.2:



$$\underline{H}_G = \tfrac{1}{6}mb^2\omega(\sin 2\beta\,\underline{i} + 2\cos^2\beta\,\underline{j})$$

where the frame $Gxyz$ rotates with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega\underline{j}$$

Since ω and β are constant, we have $(\dot{\underline{H}}_G)_{Gxyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega\underline{j}\times\tfrac{1}{6}mb^2\omega(\sin 2\beta\,\underline{i} + 2\cos^2\beta\,\underline{j})$$

$$\dot{\underline{H}}_G = -\tfrac{1}{6}mb^2\omega^2\sin 2\beta\,\underline{k}$$ ◀

18.47 From Prob. 18.3:

$$\underline{H}_G = \tfrac{1}{4}mr^2\omega[(\sin^2\beta + 2\cos^2\beta)\underline{i} + \sin\beta\cos\beta\,\underline{j}]$$

where the frame $Gxyz$ rotates with the disk with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega\,\underline{i}$$

Since ω = constant, we have $(\dot{\underline{H}}_G)_{Gxyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G$$

$$= 0 + \omega\underline{i}\times\tfrac{1}{4}mr^2\omega[(\sin^2\beta + 2\cos^2\beta)\underline{i} + \sin\beta\cos\beta\,\underline{j}]$$

$$= \tfrac{1}{4}mr^2\omega^2\sin\beta\cos\beta\,\underline{k}, \qquad \dot{\underline{H}}_G = \tfrac{1}{8}mr^2\omega^2\sin 2\beta\,\underline{k}$$ ◀

18.48 From Prob. 18.4:

$$\underline{H}_A = \tfrac{1}{12}ma^2\omega(3\underline{j} + 2\underline{k})$$

where the frame $Axyz$ rotates with the plate with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega\underline{j}$$

Since ω = constant, we have $(\dot{\underline{H}}_A)_{Axyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A = 0 + \omega\underline{j}\times\tfrac{1}{12}ma^2\omega(3\underline{j} + 2\underline{k})$$

$$\dot{\underline{H}}_A = \tfrac{1}{6}ma^2\omega^2\underline{i}$$ ◀

18.49 From Prob. 18.5:

$$\underline{H}_C = (0.0970\ ft\cdot lb\cdot s)\underline{j} + (0.582\ ft\cdot lb\cdot s)\underline{k}$$

where the frame $Oxyz$ rotates with arm AC with the angular velocity

$$\underline{\Omega} = \underline{\omega}_1 = (5\ rad/s)\underline{j}$$

We have $(\dot{\underline{H}}_C)_{Oxyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Oxyz} + \underline{\Omega}\times\underline{H}_C = 0 + 5\underline{j}\times(0.0970\underline{j} + 0.582\underline{k})$$

$$\dot{\underline{H}}_C = (2.91\ lb\cdot ft)\underline{i}$$ ◀

18.50 From Prob. 18.6:

$$\underline{H}_C = (0.234\ kg\cdot m^2/s)\underline{j} + (1.250\ kg\cdot m^2/s)\underline{k}$$

where the frame $Axyz$ rotates with the bent axle ABC with the angular velocity

$$\underline{\Omega} = \underline{\omega}_1 = (3\ rad/s)\underline{j}$$

We have $(\dot{\underline{H}}_C)_{Axyz} = 0$. Substituting into Eq. (18.22):

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Axyz} + \underline{\Omega}\times\underline{H}_C = 0 + 3\underline{j}\times(0.234\underline{j} + 1.250\underline{k})$$

$$= (3.75\ kg\cdot m^2/s^2)\underline{i} \qquad \dot{\underline{H}}_C = (3.75\ N\cdot m)\underline{i}$$ ◀

18.51 From Prob. 18.3:

$$\underline{H}_G = \tfrac{1}{4}mr^2\omega[(\sin^2\beta + 2\cos^2\beta)\underline{i} + \sin\beta\cos\beta\,\underline{j}]$$

or, since $\sin^2\beta + 2\cos^2\beta = 1 + \cos^2\beta = \tfrac{1}{2}(3 + \cos 2\beta)$ and $\sin\beta\cos\beta = \tfrac{1}{2}\sin 2\beta$

$$\underline{H}_G = \tfrac{1}{8}mr^2\omega[(3 + \cos 2\beta)\underline{i} + \sin 2\beta\,\underline{j}]$$

where the frame $Gxyz$ rotates with the disk with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega\underline{i}$$

Since $\dot{\underline{\omega}} = \underline{\alpha} = \alpha\underline{i}$, we have $\dot{\omega} = \alpha$ and

$$(\dot{\underline{H}}_G)_{Gxyz} = \tfrac{1}{8}mr^2\alpha[(3 + \cos 2\beta)\underline{i} + \sin 2\beta\,\underline{j}]$$

Substituting into Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = \tfrac{1}{8}mr^2\alpha[(3 + \cos 2\beta)\underline{i} + \sin 2\beta\,\underline{j}]$$

$$+ \omega\underline{i}\times\tfrac{1}{8}mr^2\omega[(3 + \cos 2\beta)\underline{i} + \sin 2\beta\,\underline{j}]$$

$$\dot{\underline{H}}_G = \tfrac{1}{8}mr^2[\alpha(3 + \cos 2\beta)\underline{i} + \alpha\sin 2\beta\,\underline{j} + \omega^2\sin 2\beta\,\underline{k}]$$ ◀

18.52 From Prob. 18.4:

$$\underline{H}_A = \tfrac{1}{12}ma^2\omega(3\underline{j} + 2\underline{k})$$

where the frame $Axyz$ rotates with the plate with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega\underline{j}$$

Since $\dot{\underline{\omega}} = \underline{\alpha} = \alpha\underline{j}$, we have $\dot{\omega} = \alpha$ and

$$(\dot{\underline{H}}_A)_{Axyz} = \tfrac{1}{12}ma^2\alpha(3\underline{j} + 2\underline{k})$$

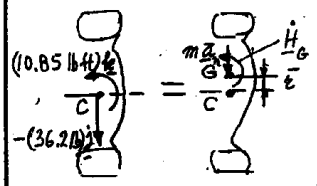
Substituting into Eq. (18.22):

$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A = \tfrac{1}{12}ma^2\alpha(3\underline{j} + 2\underline{k})$$

$$+ \omega\underline{j}\times\tfrac{1}{12}ma^2\omega(3\underline{j} + 2\underline{k})$$

$$\dot{\underline{H}}_A = \tfrac{1}{12}ma^2(2\omega^2\underline{i} + 3\alpha\,\underline{j} + 2\alpha\,\underline{k})$$ ◀

18.53 (a) The forces exerted on the wheel are equivalent to the force and couple shown in left-hand diagram. The effective forces are shown in right-hand diagram.



$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$:

$-(36.2\text{ lb})\underline{j} = -m a_m \underline{j}$

$a_m = \frac{36.2}{40/32.2} = 29.141\text{ ft/s}^2$

But $a_m = \bar{r}\omega^2$

Since $\omega = 750\text{ rpm} = 78.540\text{ rad/s}$:

$\bar{r} = \frac{a_m}{\omega^2} = \frac{29.141}{(78.540)^2} = 4.724\times10^{-3}\text{ ft}$, $\bar{r} = 0.0567\text{ in.}$ ◀

$\Sigma \underline{M}_G = \Sigma (\underline{M}_G)_{eff}$: $(10.85\text{ lb-ft})\underline{k} = \dot{\underline{H}}_G$ (1)

But, since $\underline{\omega} = \omega \underline{i}$, we have $\underline{H}_G = I_x \omega \underline{i} - I_{xy}\omega \underline{j} - I_{xz}\omega \underline{k}$

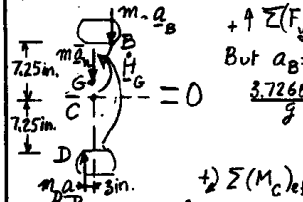
and $\dot{\underline{H}}_G = \underline{\omega} \times \underline{H}_G = \omega \underline{i} \times (I_x \omega \underline{i} - I_{xy}\omega \underline{j} - I_{xz}\omega \underline{k})$

$= -I_{xy}\omega^2 \underline{k} + I_{xz}\omega^2 \underline{j}$

Substituting into (1): $10.85\,\underline{k} = -I_{xy}\omega^2 \underline{k} + I_{xz}\omega^2 \underline{j}$

Thus: $I_{xy} = -\frac{10.85}{(78.540)}$, $I_{xy} = -1.759\times10^{-3}\text{ lb-ft-s}^2$, $I_{xz} = 0$ ◀

(b) With corrective weights, the forces exerted on the wheel are equivalent to zero. For the effective forces to also be equivalent to zero, the weights must be placed at B and D:



$+\uparrow \Sigma (F_y)_{eff} = 0$: $m_D a_D - m_B a_B - m\bar{a}_n = 0$

But $a_B = a_D = r\omega^2 = \frac{7.25}{12}(78.540)^2 = 3.7268\times10^3$

$\frac{3.7268\times10^3}{g}(w_D - w_B) = \frac{40}{g}(29.141)$

$w_D - w_B = 0.31277\text{ lb}$ (2)

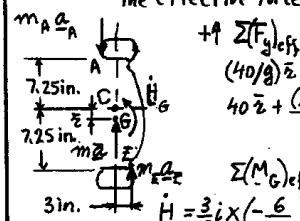
$+\circlearrowleft \Sigma (M_C)_{eff} = 0$: $m_D a_D (\frac{3}{12}) + m_B a_B (\frac{3}{12}) - 10.85 = 0$

$\frac{3.7268\times10^3}{32.2}(w_D + w_B)\frac{3}{12} = 10.85$ $w_D + w_B = 0.37498\text{ lb}$ (3)

Solving (2) and (3) simultaneously:

$w_B = 0.0311\text{ lb}$, $w_D = 0.3439\text{ lb}$; $w_B = 0.500\text{ oz}$, $w_D = 5.50\text{ oz}$ ◀

18.54 (a) After the corrective weights have been added, the effective forces are equivalent to zero:



$+\uparrow \Sigma (F_y)_{eff} = 0$: $m\bar{a} + m_E a_E - m_A a_A = 0$

$(40/g)\bar{r}\omega^2 + (2/16g)(7.25/12)\omega^2 - (6/16g)(7.25/12)\omega^2 = 0$

$40\bar{r} + \frac{(2-6)7.25}{16(12)} = 0$, $\bar{r} = 3.776\times10^{-3}\text{ ft}$

$\bar{r} = 0.0453\text{ in. (below axis)}$ ◀

$\Sigma (M_G)_{eff} = 0$: $\frac{3}{12}\underline{i} \times m_E a_E - \frac{3}{12}\underline{i} \times m_A a_A + \dot{\underline{H}}_G = 0$

$\dot{\underline{H}}_G = \frac{3}{12}\underline{i} \times (-\frac{6}{16g}\frac{7.25}{12}\omega^2 \underline{j}) - \frac{3}{12}\underline{i} (\frac{2}{16g}\frac{7.25}{12}\omega^2 \underline{j}) = -2.345\times10^{-3}\omega^2 \underline{k}$

But, since $\underline{\omega} = \omega \underline{j}$, where $\omega = 900\text{ rpm} = 94.248\text{ rad/s}$, we have

$\underline{H}_G = I_y \omega \underline{j} - I_{xy}\omega \underline{j} - I_{yz}\omega \underline{k}$ and

$\dot{\underline{H}}_G = \underline{\omega} \times \underline{H}_G = \omega \underline{i} \times (I_x \omega \underline{i} - I_{xy}\omega \underline{j} - I_{yz}\omega \underline{k}) = -I_{xy}\omega^2 \underline{k} + I_{yz}\omega^2 \underline{j}$

Equating the two expressions for $\dot{\underline{H}}_G$, we have

$-I_{xy}\omega^2 \underline{k} + I_{yz}\omega^2 \underline{j} = -2.345\times10^{-3}\omega^2 \underline{k}$

$I_{xy} = -2.35\times10^{-3}\text{ lb-ft-s}^2$, $I_{yz} = 0$ ◀

(b) Force-couple system exerted before weights have been added:

The only effective forces are $m\bar{a}$ and $\dot{H}_G$. Thus the forces exerted by the machine on the wheel are equivalent to the force and couple

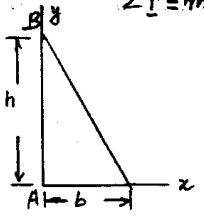
$\underline{F} = m\bar{a} = \frac{40}{g}\bar{r}\omega^2 \underline{j} = \frac{40}{32.2}(3.776\times10^{-3}\text{ ft})(94.248)^2 \underline{j} = (41.7\text{ lb})\underline{j}$ and

$\underline{M}'_C = \dot{\underline{H}}_G = -2.345\times10^{-3}\omega^2 \underline{k} = -2.345\times10^{-3}(94.248)^2 \underline{k} = -(20.8\text{ lb-ft})\underline{k}$

Force-couple on machine: $\underline{F} = -(41.8\text{ lb})\underline{j}$, $\underline{M}_C = (20.8\text{ lb-ft})\underline{k}$ ◀

18.55 We shall use Eqs. (18.1) and (18.2B):

$\Sigma \underline{F} = m\bar{\underline{a}}$ $\Sigma \underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega} \times \underline{H}_A$



Computation of $\underline{H}_A$:

$\underline{\omega} = \omega \underline{j}$

$\underline{H}_A = -I_{xy}\omega \underline{i} + I_y \omega \underline{j} - I_{yz}\omega \underline{k}$

But, from back cover of book:

$(I_y)_{Area} = \frac{1}{12}b^3 h$ $A = \frac{1}{2}bh$

$(I_y)_{mass} = \frac{m}{A}(I_y)_{Area} = \frac{1}{6}m b^2$

From Sample Prob. 9.6 (page 360 of 5 tatics): $(I_{xy})_{Area} = \frac{1}{24}b^2 h^2$.

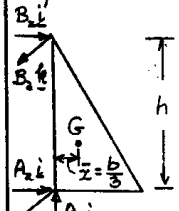
$(I_{xy})_{mass} = \frac{m}{A}(I_{xy})_{Area} = \frac{1}{12}m\,bh$ Also: $(I_{yz})_{mass} = 0$

Thus: $\underline{H}_A = -\frac{1}{12}mbh\omega \underline{i} + \frac{1}{6}m b^2 \omega \underline{j}$ (1)

where the frame of reference Axyz rotates with the plate with the angular velocity

$\underline{\Omega} = \underline{\omega} = \omega \underline{j}$

Equations of motion (Weight is omitted for dynamic reactions)



Eq. (18.2B):

$\Sigma \underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega} \times \underline{H}_A$

Since ω = constant, it follows from (1) that $(\dot{\underline{H}}_A)_{Axyz} = 0$

Thus:

$h\underline{j} \times (B_x \underline{i} + B_z \underline{k}) = 0 + \omega \underline{j} \times (-\frac{mbh}{12}\omega \underline{i} + \frac{mb^2}{6}\omega \underline{j})$

$-hB_x \underline{k} + hB_z \underline{i} = +\frac{1}{12}mbh\omega^2 \underline{k}$

Equating coeff. of unit vectors: $B_x = -\frac{1}{12}mb\omega^2$, $B_z = 0$

$\underline{B} = -\frac{1}{12}mb\omega^2 \underline{i}$

The dynamic reactions must also satisfy Eq. (18.1):

$\Sigma \underline{F} = m\bar{\underline{a}}$ $\underline{A} + \underline{B} = -m\bar{x}\omega^2 \underline{i} = -m(\frac{b}{3})\omega^2 \underline{i}$

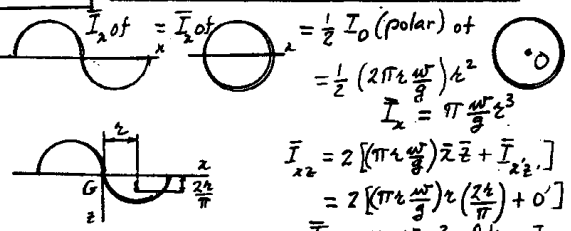
$\underline{A} = -\frac{1}{3}mb\omega^2 \underline{i} - (-\frac{1}{12}mb\omega^2 \underline{i})$, $\underline{A} = -\frac{1}{4}mb\omega^2 \underline{i}$

Given data: $m = 2.5\text{ kg}$, $b = 0.3\text{ m}$, $h = 0.6\text{ m}$, $\omega = 8\text{ rad/s}$

$\underline{A} = -\frac{1}{4}(2.5\text{ kg})(0.3\text{ m})(8\text{ rad/s})^2 \underline{i}$ $\underline{A} = -(12.00\text{ N})\underline{i}$ ◀

$\underline{B} = -\frac{1}{12}(2.5\text{ kg})(0.3\text{ m})(8\text{ rad/s})^2 \underline{i}$ $\underline{B} = -(4.00\text{ N})\underline{i}$ ◀

18.56 Moments and Products of Inertia:



$\bar{I}_{xz} = 2[(\pi r\frac{w}{g})\bar{x}\bar{z} + \bar{I}_{x'z'}]$

$= 2[(\pi r\frac{w}{g})r(\frac{2r}{\pi}) + 0']$

$\bar{I}_{xz} = 4\frac{w}{g}r^3$, Also: $I_{xy} = I_{yz} = 0$

Angular Momentum about G:

Angular velocity: $\omega_x = \omega$, $\omega_y = \omega_z$.

We use Eqs. (18.7):

$H_x = \bar{I}_x \omega_x + 0 + 0 = \pi\frac{w}{g}r^3\omega$

$H_y = 0 + 0 + 0 = 0$

$H_z = -\bar{I}_{xz}\omega_x + 0 + 0 = -4\frac{w}{g}r^3\omega$

$\underline{H}_G = \frac{w}{g}r^3\omega(\pi \underline{i} - 4\underline{k})$

(CONTINUED)

18.56 CONTINUED We found on the preceding page the following expression for the angular momentum about G:

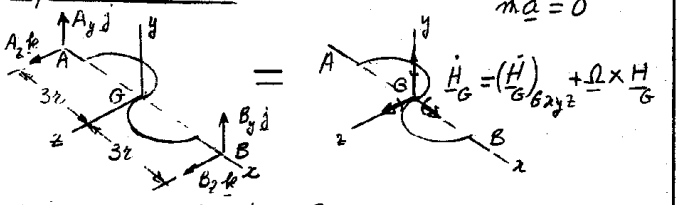
$$\underline{H}_G = \frac{w}{g} r^3 \omega (\pi \underline{i} - 4\underline{k})$$

where the frame Gxyz rotates with the rod with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega \underline{i}$$

Equations of motion

$$m\underline{\bar{a}} = 0$$



Equating moments about B:

$$\underline{r}_{A/B} \times (A_y \underline{j} + A_z \underline{k}) = (\dot{\underline{H}})_{Gxyz} + \underline{\Omega} \times \underline{H}_G$$

But $(\dot{\underline{H}})_{Gxyz} = 0$, since ω = constant. Thus

$$-6r\underline{i} \times (A_y \underline{j} + A_z \underline{k}) = 0 + \omega \underline{i} \times \frac{w}{g} r^3 \omega (\pi \underline{i} - 4\underline{k})$$

$$-6r A_y \underline{k} + 6r A_z \underline{j} = +4 \frac{w}{g} r^3 \omega^2 \underline{j}$$

$A_y = 0$, $A_z = \frac{2}{3} \frac{w}{g} r^2 \omega^2$ $\quad \underline{A} = \frac{2}{3} \frac{w}{g} r^2 \omega^2 \underline{k}$ ◀

$\underline{A} + \underline{B} = m\underline{\bar{a}} = 0$ $\quad \underline{B} = -\underline{A}$ $\quad \underline{B} = -\frac{2}{3} \frac{w}{g} r^2 \omega^2 \underline{k}$ ◀

18.57 Equations of motion. We first shall use Eq. (18.28):

$$\Sigma \underline{M}_B = (\dot{\underline{H}}_B)_{Bxyz} + \underline{\Omega} \times \underline{H}_B \qquad (1)$$

But, from Eq. (2) in solution of Prob. 18.17:

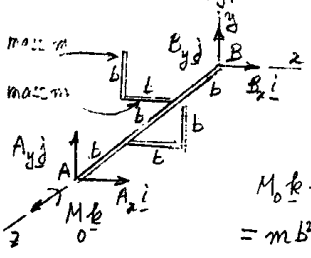
$$\underline{H}_B = mb^2\omega\left(-\tfrac{2}{3}\underline{i} - \tfrac{2}{3}\underline{j} + \tfrac{10}{3}\underline{k}\right) \qquad (2)$$

where the frame Bxyz rotates with assembly with

$$\underline{\Omega} = \omega \underline{k} \qquad (3)$$

Differentiating both members of (2) with respect to rotating frame:

$$(\dot{\underline{H}}_B)_{Bxyz} = mb^2\alpha\left(-\tfrac{2}{3}\underline{i} - \tfrac{2}{3}\underline{j} + \tfrac{10}{3}\underline{k}\right) \qquad (4)$$



We substitute from (2), (3), and (4) into (1), observing that $\omega = 0$, and compute $\Sigma \underline{M}_B$ from adjoining diagram:

$$M_0\underline{k} + 3b\underline{k} \times (A_x\underline{i} + A_y\underline{j}) = mb^2\alpha\left(-\tfrac{2}{3}\underline{i} - \tfrac{2}{3}\underline{j} + \tfrac{10}{3}\underline{k}\right) + 0$$

$$M_0\underline{k} + 3bA_z\underline{j} - 3bA_y\underline{i} = -\tfrac{2}{3}mb^2\alpha\underline{i} - \tfrac{2}{3}mb^2\alpha\underline{j} + \tfrac{10}{3}mb^2\alpha\underline{k}$$

Equating the coefficients of the unit vectors:

(k) $M_0 = \frac{10}{3} mb^2\alpha$ $\quad \alpha = \frac{3}{10}\frac{M_0}{mb^2}$ (5)

(j) $3bA_z = -\frac{2}{3}mb^2\alpha$ $\quad A_z = -\frac{1}{2}mb\alpha = -\frac{3}{20}\frac{M_0}{b}$ (6)

(i) $-3bA_y = -\frac{2}{3}mb^2\alpha$ $\quad A_y = \frac{1}{2}mb\alpha = \frac{3}{20}\frac{M_0}{b}$ (7)

(CONTINUED)

18.57 CONTINUED Given data:

m = mass of one straight element of rod

$= \frac{1}{2}\frac{4\,\text{lb}}{32.2\,\text{ft/s}^2} = 0.062112\ \text{lb}\cdot\text{s}^2/\text{ft}$

$b = 8\,\text{in.} = \frac{2}{3}\,\text{ft}$ $\quad M_0 = 20\,\text{lb}\cdot\text{in.} = \frac{20}{12} = \frac{5}{3}\,\text{lb}\cdot\text{ft}$

Substituting into (5), (6), (7):

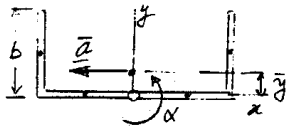
(a) $\alpha = \frac{3}{10}\frac{5/3}{0.062112(2/3)^2} = 18.11$ $\quad \underline{\alpha} = (18.11\,\text{rad/s}^2)\underline{k}$ ◀

(b) $A_z = -\frac{3}{20}\frac{5/3}{2/3} = -0.375\,\text{lb}$, $\quad A_y = \frac{3}{20}\frac{5/3}{2/3} = +0.375\,\text{lb}$

$\underline{A} = -(0.375\,\text{lb})\underline{i} + (0.375\,\text{lb})\underline{j}$ ◀

We shall now apply Eq. (18.1): $\Sigma \underline{F} = (\text{mass})\underline{\bar{a}}$ (8)

Considering only the L-shaped arms (since mass center of shaft has no acceleration) we locate their mass center G:



We have $\bar{y} = \frac{1}{4}b$

$\underline{\bar{a}} = -\bar{y}\alpha\underline{i} = -\frac{1}{4}b\alpha\underline{i}$

Mass of both L-shaped arms $= 4m$

where m = mass of one straight element

Substituting into Eq. (8):

$\underline{A} + \underline{B} = (4m)\underline{\bar{a}}$ $\quad \underline{A} + \underline{B} = 4m\left(-\frac{1}{4}b\alpha\underline{i}\right)$

$\underline{B} = -mb\alpha\underline{i} - (A_x\underline{i} + A_y\underline{j})$

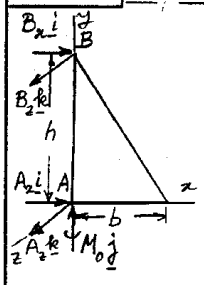
Substituting for α from (5) and A_x and A_y from (6) and (7):

$$\underline{B} = -mb\left(\frac{3}{10}\frac{M_0}{mb^2}\right)\underline{i} + \frac{3}{20}\frac{M_0}{b}\underline{i} - \frac{3}{20}\frac{M_0}{b}\underline{j} = -\frac{3}{20}\frac{M_0}{b}(\underline{i} + \underline{j})$$

Substituting the data:

$$\underline{B} = -\frac{3}{20}\frac{5/3}{2/3}(\underline{i} + \underline{j}), \quad \underline{B} = -(0.375\,\text{lb})\underline{i} - (0.375\,\text{lb})\underline{j}$$ ◀

18.58 Equations of motion. We first use Eq. (18.28):



$$\Sigma \underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega} \times \underline{H}_A$$

where $\underline{H}_A$ was obtained in Eq. (1) of Prob. 18.56:

$$\underline{H}_A = -\frac{1}{12}mbh\omega\underline{i} + \frac{1}{6}mb^2\omega\underline{j}$$

Differentiating with respect to the rotating frame:

$$(\dot{\underline{H}}_A)_{Axyz} = -\frac{1}{12}mbh\alpha\underline{i} + \frac{1}{6}mb^2\alpha\underline{j}$$

Substituting for $\underline{H}_A$ and $(\dot{\underline{H}}_A)_{Axyz}$ into Eq. (18.28), noting that $\omega = 0$, and computing $\Sigma \underline{M}_A$ from diagram:

$$M_0\underline{j} + h\underline{j} \times (B_x\underline{i} + B_z\underline{k}) = -\frac{1}{12}mbh\alpha\underline{i} + \frac{1}{6}mb^2\alpha\underline{j} + 0$$

$$M_0\underline{j} - hB_x\underline{k} + hB_z\underline{i} = -\frac{1}{12}mbh\alpha\underline{i} + \frac{1}{6}mb^2\alpha\underline{j}$$

Equating the coefficients of the unit vectors:

(j) $M_0 = \frac{1}{6}mb^2\alpha$ $\quad \alpha = \frac{6M_0}{mb^2}$

$$\alpha = \frac{6(0.75\,\text{N.m})}{(2.5\,\text{kg})(0.3\,\text{m})^2} = 20.0\,\text{rad/s}^2 \quad \underline{\alpha} = (20.0\,\text{rad/s}^2)\underline{j}$$ ◀

(k) $-hB_x = 0$ $\quad B_x = 0$

(i) $hB_z = -\frac{1}{12}mbh\alpha$, $\quad B_z = -\frac{1}{12}mb\alpha = -\frac{1}{12}(2.5)(0.3)(20.0)$

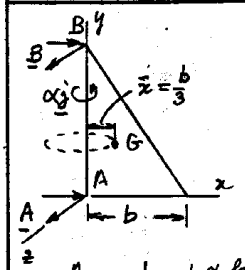
$B_z = -1.250\,\text{N}$ $\quad \underline{B} = -(1.250\,\text{N})\underline{k}$ ◀

(CONTINUED)

18.58 CONTINUED

We shall now apply Eq. (18.1):

$$\Sigma \underline{F} = m\underline{\bar{a}}$$



Since $\omega = 0$:

$$\underline{\bar{a}} = \underline{\bar{a}}_t = \alpha \underline{k} \times \frac{b}{3}\underline{i} = -\frac{1}{3}b\alpha \underline{k}$$

Substituting in Eq. (18.1):

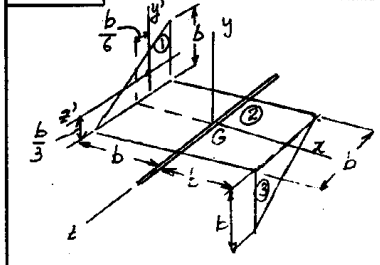
$$\underline{A} + \underline{B} = -\frac{1}{3}mb\alpha \underline{k}$$

$$\underline{A} = -\frac{1}{3}mb\alpha \underline{k} - \underline{B} = -\frac{1}{3}mb\alpha \underline{k} + \frac{1}{12}mb\alpha \underline{k}$$

$$\underline{A} = -\frac{1}{4}mb\alpha \underline{k} = -\frac{1}{4}(2.5\text{ kg})(0.3\text{ m})(20\text{ rad/s}^2)\underline{k}$$

$$\underline{A} = -(3.75\text{ N})\underline{k} \quad \blacktriangleleft$$

18.59 Moments and Products of Inertia



Let m = total mass

Rectangle (2):

$$\text{mass} = \frac{2}{3}m$$

$$I_z = \frac{1}{12}\left(\frac{2}{3}m\right)(2b)^2 = \frac{2}{9}mb^2$$

$$I_{xz} = I_{yz} = 0$$

Triangle (1): mass $= \frac{m}{6}$

From back corner: $(\bar{I}_{z'})_{\text{Area}} = \frac{1}{36}b^4$

$$(\bar{I}_{z'})_{\text{mass}} = \frac{\text{mass}}{\text{Area}}(\bar{I}_{z'})_{\text{Area}} = \frac{m/6}{b^2/2}\frac{b^4}{36} = \frac{1}{108}mb^2$$

From Sample Prob. 9.6 (page 360 of Statics):

$$(\bar{I}_{y'z'})_{\text{Area}} = -\frac{1}{72}b^4, \quad (\bar{I}_{y'z'})_{\text{mass}} = \frac{m/6}{b^2/2}\left(-\frac{1}{72}b^4\right) = -\frac{1}{216}mb^2$$

Therefore:

$$\bar{I}_z = \bar{I}_{z'} + \frac{m}{6}\left[b^2 + \left(\frac{b}{3}\right)^2\right] = \left(\frac{1}{108} + \frac{10}{54}\right)mb^2 = \frac{7}{36}mb^2$$

$$\bar{I}_{yz} = \bar{I}_{y'z'} + \frac{m}{6}\bar{y}\bar{z} = -\frac{1}{216}mb^2 + \frac{m}{6}\left(\frac{b}{3}\right)\left(-\frac{b}{6}\right) = -\frac{1}{72}mb^2$$

$$\bar{I}_{xz} = \bar{I}_{x'z'} + \frac{m}{6}\bar{x}\bar{z} = 0 + \frac{m}{6}(-b)\left(-\frac{b}{6}\right) = +\frac{1}{36}mb^2$$

Triangle (2): By symmetry:

$$I_z = \frac{7}{36}mb^2, \quad I_{yz} = -\frac{1}{72}mb^2, \quad I_{xz} = +\frac{1}{36}mb^2$$

For entire sheet-metal component:

$$\bar{I}_z = \frac{2}{9}mb^2 + 2\left(\frac{7}{36}mb^2\right) = \frac{11}{18}mb^2$$

$$\bar{I}_{yz} = 0 + 2\left(-\frac{1}{72}mb^2\right) = -\frac{1}{36}mb^2$$

$$\bar{I}_{xz} = 0 + 2\left(+\frac{1}{36}mb^2\right) = +\frac{1}{18}mb^2$$

Angular momentum

We use Eqs. (18.7) with the values of $\bar{I}_z$, $\bar{I}_{yz}$, $\bar{I}_{xz}$ found above and

$$\omega_x = \omega_y = 0 \qquad \omega_z = \omega$$

$$H_x = -\bar{I}_{xz}\omega_z = -\frac{1}{18}mb^2\omega$$

$$H_y = -\bar{I}_{yz}\omega_z = +\frac{1}{36}mb^2\omega$$

$$H_z = \bar{I}_z\omega_z = \frac{11}{18}mb^2\omega$$

$$\underline{H}_G = \frac{1}{36}mb^2\omega(-2\underline{i} + \underline{j} + 22\underline{k}) \qquad (1)$$

(CONTINUED)

18.59 CONTINUED

Rate of change of ang. momentum

From Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G \qquad (2)$$

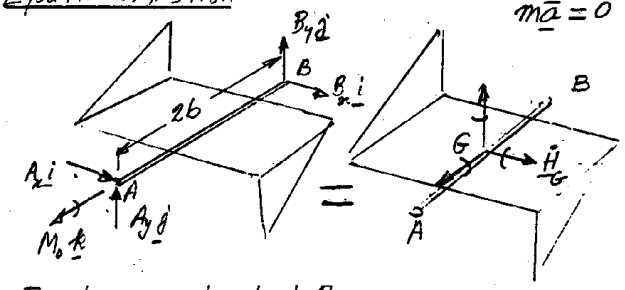
Where the reference frame $Gxyz$ rotates with the sheet-metal component with the angular velocity $\underline{\Omega} = \omega\underline{k}$. Differentiating (1) with respect to that frame, we obtain

$$(\dot{\underline{H}}_G)_{Gxyz} = \frac{1}{36}mb^2\alpha(-2\underline{i} + \underline{j} + 22\underline{k}) \qquad (3)$$

Since $\omega = 0$, the rate of change $\dot{\underline{H}}_G$ reduces to this expression.

Equations of motion

$$m\underline{\bar{a}} = 0$$



Equating moments about B:

$$M_0\underline{k} + 2b\underline{k} \times (A_x\underline{i} + A_y\underline{j}) = \frac{1}{36}mb^2\alpha(-2\underline{i} + \underline{j} + 22\underline{k})$$

$$M_0\underline{k} + 2bA_x\underline{j} - 2bA_y\underline{i} = \frac{1}{36}mb^2\alpha(-2\underline{i} + \underline{j} + 22\underline{k})$$

Equating coefficients of the unit vectors:

$(\underline{k})$ $M_0 = \frac{22}{36}mb^2\alpha = \frac{22}{36}(0.6\text{ kg})(0.075\text{ m})^2(12\text{ rad/s}^2)$

$$M_0 = 24.75 \times 10^{-3}\text{ N}\cdot\text{m}, \qquad \underline{M}_0 = (24.75\text{ mN}\cdot\text{m})\underline{k} \quad \blacktriangleleft$$

$(\underline{i})$ $2bA_z = \frac{1}{36}mb^2\alpha$, $A_z = \frac{1}{72}mb\alpha = \frac{1}{72}(0.6)(0.075)(12)$

$$A_x = +7.5 \times 10^{-3}\text{ N}, \qquad A_x = +7.5\text{ mN}$$

$(\underline{j})$ $-2bA_y = -\frac{2}{36}mb^2\alpha$, $A_y = \frac{1}{36}mb\alpha = \frac{1}{36}(0.6)(0.075)(12)$

$$A_y = +15 \times 10^{-3}\text{ N}, \qquad A_y = +15\text{ mN}$$

$$\underline{A} = (7.5\text{ mN})\underline{i} + (15\text{ mN})\underline{j} \quad \blacktriangleleft$$

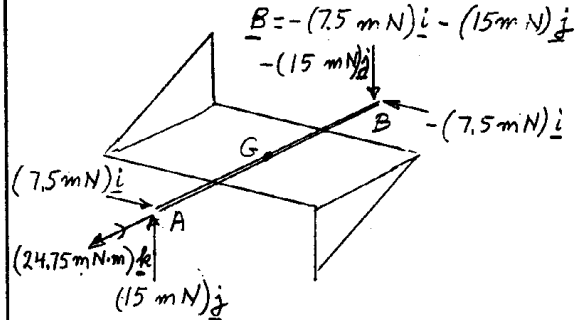
Equating x components:

$$A_x + B_x = 0 \qquad B_x = -A_x = -7.5\text{ mN}$$

Equating y components:

$$A_y + B_y = 0 \qquad B_y = -A_y = -15\text{ mN}$$

$$\underline{B} = -(7.5\text{ mN})\underline{i} - (15\text{ mN})\underline{j} \quad \blacktriangleleft$$



18.60 From the solution of Prob. 18.56:

$$\underline{H}_G = \frac{W}{g} r^3 \omega (\pi \underline{i} - 4 \underline{k}) \quad (1)$$

where the frame $Gxyz$ rotates with the rod with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega \underline{i}$$

Rate of change of angular momentum.

Differentiating (1) with respect to the rotating frame:

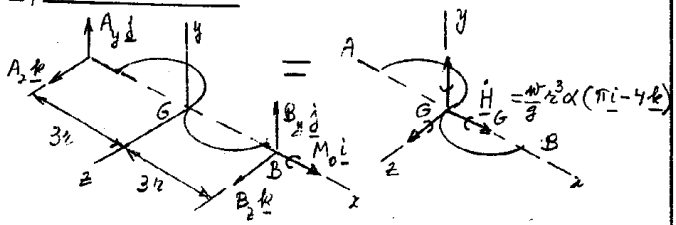
$$(\dot{\underline{H}}_G)_{Gxyz} = \frac{W}{g} r^3 \alpha (\pi \underline{i} - 4 \underline{k}) \quad (2)$$

The rate of change $\dot{\underline{H}}_G$ of $\underline{H}_G$ with respect to a newtonian frame is, by Eq. (18.22),

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G \quad (3)$$

and reduces to the expression (2) since $\omega = 0$

Equations of motion



Equating moments about B:

$$M_0 \underline{i} - 6r \underline{i} \times (A_y \underline{j} + A_z \underline{k}) = \frac{W}{g} r^3 \alpha (\pi \underline{i} - 4 \underline{k})$$

$$M_0 \underline{i} - 6r A_y \underline{k} + 6r A_z \underline{j} = \frac{W}{g} r^3 \alpha (\pi \underline{i} - 4 \underline{k})$$

Equating the coefficients of the unit vectors:

(1) $M_0 = \pi \frac{W}{g} r^3 \alpha$ $\qquad \underline{M}_0 = \pi \frac{W}{g} r^3 \alpha \underline{i}$ ◀

(2) $6r A_z = 0$ $\qquad A_z = 0$

(3) $-6r A_y = -4 \frac{W}{g} r^3 \alpha$ $\qquad A_y = \frac{2}{3} \frac{W}{g} r^2 \alpha$

Thus: $\qquad \underline{A} = \frac{2}{3} \frac{W}{g} r^2 \alpha \underline{j}$ ◀

Equating y components:

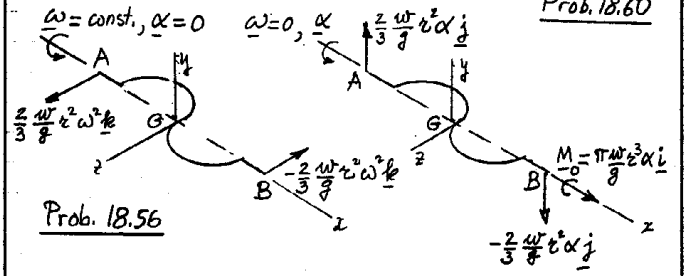
$$A_y + B_y = 0 \qquad B_y = -A_y = -\frac{2}{3} \frac{W}{g} r^2 \alpha$$

Equating z components:

$$A_z + B_z = 0 \qquad B_z = -A_z = 0$$

$$\underline{B} = -\frac{2}{3} \frac{W}{g} r^2 \alpha \underline{j}$$ ◀

Comparison of reactions in Probs. 18.56 and 18.60:



18.61 See solution of Prob. 18.59 for the computation of $\underline{H}_G$:

$$\underline{H}_G = \frac{1}{36} m b^2 \omega (-2 \underline{i} + \underline{j} + 22 \underline{k}) \quad (1)$$

where the reference frame $Gxyz$ rotates with the sheet-metal component with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega \underline{k} \quad (2)$$

Differentiating (1) with respect to the rotating frame:

$$(\dot{\underline{H}}_G)_{Gxyz} = \frac{1}{36} m b^2 \alpha (-2 \underline{i} + \underline{j} + 22 \underline{k}) \quad (3)$$

To obtain the rate of change of $\underline{H}_G$ with respect to a newtonian frame of reference, we use Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G \quad (4)$$

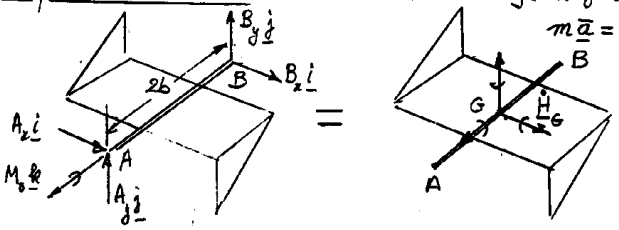
Substituting from (1), (2), (3) into (4):

$$\dot{\underline{H}}_G = \frac{1}{36} m b^2 \alpha (-2 \underline{i} + \underline{j} + 22 \underline{k}) + \omega \underline{k} \times \frac{1}{36} m b^2 \omega (-2 \underline{i} + \underline{j} + 22 \underline{k})$$

$$= \frac{1}{36} m b^2 \alpha (-2 \underline{i} + \underline{j} + 22 \underline{k}) + \frac{1}{36} m b^2 \omega^2 (-2 \underline{j} - \underline{i})$$

$$\dot{\underline{H}}_G = \frac{1}{36} m b^2 [(-2\alpha - \omega^2) \underline{i} + (\alpha - 2\omega^2) \underline{j} + 22 \alpha \underline{k}] \quad (5)$$

Equations of motion. We assume the following configuration:



Equating moments about B:

$$M_0 \underline{k} + 2b \underline{k} \times (A_x \underline{i} + A_y \underline{j}) = \frac{1}{36} m b^2 [(-2\alpha - \omega^2) \underline{i} + (\alpha - 2\omega^2) \underline{j} + 22\alpha \underline{k}]$$

$$M_0 \underline{k} + 2b A_x \underline{j} - 2b A_y \underline{i} = \frac{1}{36} m b^2 [(-2\alpha - \omega^2) \underline{i} + (\alpha - 2\omega^2) \underline{j} + 22\alpha \underline{k}]$$

Equating the coefficients of the unit vectors $\underline{j}$ and $\underline{i}$:

(1) $2b A_x = \frac{1}{36} m b^2 (\alpha - 2\omega^2)$, $A_x = \frac{1}{72} m b (\alpha - 2\omega^2)$ (6)

(2) $-2b A_y = -\frac{1}{36} m b^2 (2\alpha + \omega^2)$, $A_y = \frac{1}{72} m b (2\alpha + \omega^2)$ (7)

Data: $m = 0.6$ kg, $b = 0.075$ m, $\alpha = 12$ rad/s^2

(a) After 0.6 s: $\omega = \alpha t = (12 \text{ rad/s}^2)(0.6 \text{ s}) = 7.2$ rad/s

$$\underline{\omega} = (7.20 \text{ rad/s}) \underline{k}$$ ◀

(b) Eq. (6): $A_x = \frac{1}{72}(0.6)(0.075)[12 - 2(7.2)^2] = -57.3 \times 10^{-3}$ N

Eq. (7): $A_y = \frac{1}{72}(0.6)(0.075)[2(12) + (7.2)^2] = 47.4 \times 10^{-3}$ N

Thus: $\qquad \underline{A} = -(57.3 \text{ mN}) \underline{i} + (47.4 \text{ mN}) \underline{j}$ ◀

Equating x components:

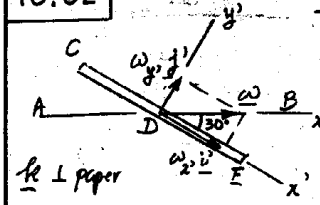
$$A_x + B_x = 0 \qquad B_x = -A_x = 57.3 \times 10^{-3} \text{ N}$$

Equating y components:

$$A_y + B_y = 0 \qquad B_y = -A_y = -47.4 \times 10^{-3} \text{ N}$$

$$\underline{B} = (57.3 \text{ mN}) \underline{i} - (47.4 \text{ mN}) \underline{j}$$ ◀

18.62



$\underline{k} \perp$ paper

Angular momentum. Only rod CE contributes to the angular momentum. With principal axes $x'y'$, we have:

$$\underline{H}_D = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}'$$
$$= 0 + \frac{1}{12}m\ell^2\omega\sin 30^\circ\,\underline{j}'$$
$$= \frac{1}{24}m\ell^2\omega\,\underline{j}'$$

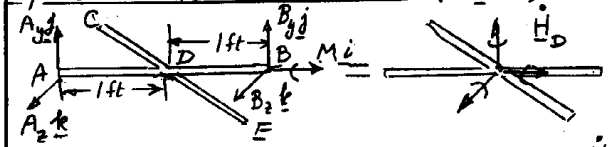
Rate of change of $\underline{H}_D$:

$$\dot{\underline{H}}_D = (\dot{\underline{H}}_D)_{Ax'y'z} + \underline{\omega}\times\underline{H}_D = \frac{1}{24}m\ell^2\alpha\,\underline{j}' + \omega\underline{i}\times\frac{1}{24}m\ell^2\omega\,\underline{j}'$$

But $\underline{j}' = \sin 30^\circ\,\underline{i} + \cos 30^\circ\,\underline{j}$ and $\underline{i}\times\underline{j}' = \cos 30^\circ\,\underline{k}$

Thus: $\dot{\underline{H}}_D = \frac{1}{24}m\ell^2(\alpha\sin 30^\circ\,\underline{i} + \alpha\cos 30^\circ\,\underline{j} + \omega^2\cos 30^\circ\,\underline{k})$

Equivalence of ext. and eff. forces: $(m\bar{\underline{a}}=0)$



$$\Sigma\underline{M}_A = \Sigma(\underline{M}_A)_{eff}:\quad 2\underline{i}\times B_y\underline{j} + 2\underline{i}\times B_z\underline{k} + M\underline{i} = \dot{\underline{H}}_D$$

$$M\underline{i} - 2B_z\underline{j} + 2B_y\underline{k} = \frac{1}{24}m\ell^2(\alpha\sin 30^\circ\,\underline{i} + \alpha\cos 30^\circ\,\underline{j} + \omega^2\cos 30^\circ\,\underline{k})$$

Data: $m = (5\text{ lb})/32.2 = 0.15528$, $\ell = \frac{18\text{ in.}/12}{\cos 30^\circ} = \frac{1.5}{\cos 30^\circ}$

$\alpha = 45\text{ rad/s}^2$, $\omega = 12\text{ rad/s}$

Coeff. of $\underline{i}$: $M = \frac{1}{24}m\ell^2\alpha\sin 30^\circ = \frac{1}{24}(0.15528)\left(\frac{1.5}{\cos 30^\circ}\right)^2(45)\sin 30^\circ$

$= 0.43671\text{b}\cdot\text{ft}$ $\quad \underline{M} = (0.4371\text{b}\cdot\text{ft})\underline{i}$ ◀

Coeff. of $\underline{j}$: $-2B_z = \frac{1}{24}m\ell^2\alpha\cos 30^\circ = \frac{1}{24}(0.15528)\left(\frac{1.5}{\cos 30^\circ}\right)^2(45)\cos 30^\circ$

$= 0.75643$ $\quad B_z = -0.3781\text{b}$

Coeff. of $\underline{k}$: $2B_y = \frac{1}{24}m\ell^2\omega^2\cos 30^\circ = \frac{1}{24}(0.15528)\left(\frac{1.5}{\cos 30^\circ}\right)^2(12)^2\cos 30^\circ$

$= 2.4206$ $\quad B_y = 1.210\text{ lb}$

Thus: $\underline{B} = (1.210\text{lb})\underline{j} - (0.378\text{ lb})\underline{k}$ ◀

Since $\Sigma\underline{F} = m\bar{\underline{a}} = 0$: $\underline{A} = -\underline{B} = -(1.210\text{lb})\underline{j} + (0.378\text{ lb})\underline{k}$ ◀

18.63

Angular momentum

$$\underline{H}_G = \bar{I}_x\omega_1\underline{i} + \bar{I}_y\omega_2\underline{j}$$

where the reference frame Gxyz rotates with the motor casing with the angular velocity

$$\underline{\Omega} = \omega_2\underline{j}$$

Rate of change of ang. momentum

Since ω_1 and ω_2 are constant, $(\dot{\underline{H}}_G)_{Gxyz} = 0$ and Eq. (18.22) yields

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_2\underline{j}\times(\bar{I}_x\omega_1\underline{i} + \bar{I}_y\omega_2\underline{j})$$

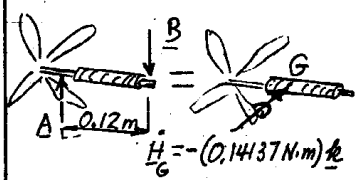
$$\dot{\underline{H}}_G = -\bar{I}_z\omega_1\omega_2\,\underline{k}$$

But $\bar{I}_z = m\bar{k}_z^2 = (0.2\text{ kg})(0.075\text{ m})^2 = 1.125\times10^{-3}\text{ kg}\cdot\text{m}^2$

$\omega_1 = 2400\text{ rpm} = 251.33\text{ rad/s}$, $\omega_2 = 0.5\text{ rad/s}$

Thus: $\dot{\underline{H}}_G = -(1.125\times10^{-3})(251.33)(0.5)\underline{k} = -(0.14137\text{ N}\cdot\text{m})\underline{k}$

Equivalence of external and effective forces.



$\dot{\underline{H}}_G = -(0.14137\text{ N}\cdot\text{m})\underline{k}$

The reactions $\underline{A}$ and $\underline{B}$ are equivalent to the couple $\dot{\underline{H}}_G$:

$A = B$

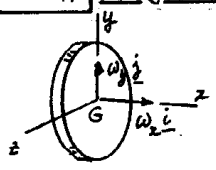
$(0.12\text{m})A = 0.14137\text{ N}\cdot\text{m}$

$\underline{A} = (1.178\text{ N})\underline{j}$ ◀

$\underline{B} = -(1.178\text{ N})\underline{j}$ ◀

18.64

Angular velocity: $\omega_z = 12\,000\text{ rpm} = 1256.6\text{ rad/s}$



Since plane turns right, $\omega_y < 0$:

$$\omega_y = -\frac{v}{\rho} = -\frac{(720/3.6)\text{ m/s}}{800\text{ m}} = -0.250\text{ rad/s}$$

Rate of change of ang. momentum

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j}$$

where the reference frame Gxyz rotates with the airplane with the angular velocity

$$\underline{\Omega} = \omega_y\underline{j}$$

Since ω_x and ω_y are constant, $(\dot{\underline{H}}_G)_{Gxyz} = 0$ and Eq. (18.22) yields

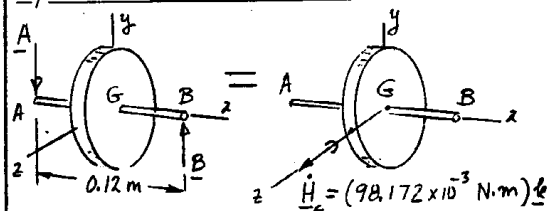
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_y\underline{j}\times(\bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j})$$

$$= -\bar{I}_x\omega_x\omega_y\,\underline{k} = -\left(\frac{1}{2}m r^2\right)\omega_x\omega_y\,\underline{k}$$

$$= -\tfrac{1}{2}(0.25\text{ kg})(0.05\text{ m})^2(1256.6\text{ rad/s})(-0.250)\,\underline{k}$$

$$\dot{\underline{H}}_G = +(98.172\times10^{-3}\text{ N}\cdot\text{m})\underline{k}$$

Equivalence of external and effective forces.



The forces $\underline{A}$ and $\underline{B}$ exerted by the springs must be equivalent to the couple $\dot{\underline{H}}_G$. They must therefore be directed as shown, which means that the spring at A will stretch and **A will move up**. ◀

We have $A = B$, $(0.12\text{m})A = 98.172\times10^{-3}\text{ N}\cdot\text{m}$

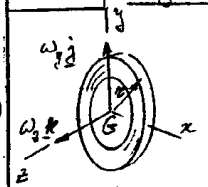
$F = A = 0.81810\text{ N}$

Defl. of spring $= x = \frac{F}{k} = \frac{0.81810\text{ N}}{600\text{ N/m}} = 1.3635\times10^{-3}\text{ m}$

Angle of rotation $= \frac{x}{GA} = \frac{1.3635\times10^{-3}\text{ m}}{0.06\text{ m}} = 0.022725\text{ rad} = 1.30^\circ$ ◀

18.65

Angular velocity of each wheel



Ang. vel. due to spin about axle:

$$\omega_z = \frac{v}{r} = \frac{88\text{ ft/s}}{11.5/12} = 91.826\text{ rad/s}$$

Ang. vel. due to turn (assume left turn):

$$\omega_y = \frac{v}{\rho} = \frac{88\text{ ft/s}}{500\text{ ft}} = 0.1760\text{ rad/s}$$

$$\underline{\omega} = (0.1760\text{ rad/s})\underline{j} + (91.826\text{ rad/s})\underline{k}$$

Rate of change of ang. momentum for each wheel:

$$\underline{H}_G = \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$$

where reference frame Gxyz rotates with $\underline{\Omega} = \omega_y\underline{j}$.

Since ω_y and ω_z are constant, $(\dot{\underline{H}}_G)_{Gxyz} = 0$ and Eq. (18.22) yields

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_y\underline{j}\times(\bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k})$$

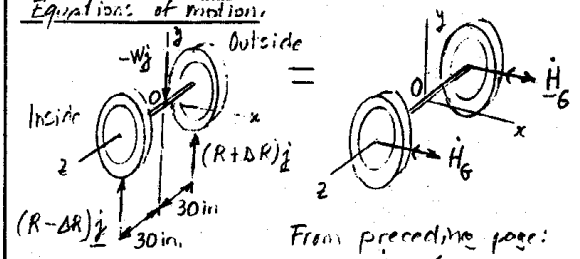
$$= \bar{I}_z\omega_y\omega_z\,\underline{i} = \left(\frac{W}{g}\bar{k}_z^2\right)\omega_y\omega_z\,\underline{i} = \frac{50}{32.2}\left(\frac{9}{12}\right)^2(0.1760)(91.826)$$

$$\dot{\underline{H}}_G = (14.116\text{ lb}\cdot\text{ft})\underline{i}$$

(CONTINUED)

18.65 CONTINUED Consider 2 wheels and axle:

Equations of motion:



From preceding page: $\underline{H}_G = (14.116\ \text{lb·ft})\underline{i}$

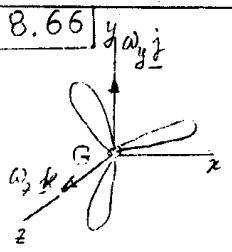
Equating moments about O:

$$\left(\tfrac{30}{12}\text{ ft}\right)\underline{k} \times (R-\Delta R)\underline{j} + \left(-\tfrac{30}{12}\text{ ft}\right)\underline{k} \times (R+\Delta R)\underline{j} = 2\dot{\underline{H}}_G$$

$$-2.5R\,\underline{i} + 2.5\Delta R\,\underline{i} + 2.5\Delta R\,\underline{i} + 2.5\Delta R\,\underline{i} = 2(14.116)\underline{i}$$

$$2.5(4\Delta R) = 2(14.116) \qquad \Delta R = 5.65\ \text{lb} \uparrow$$ ◀

18.66



Selection of axes

We select the z axis along the axis of the propeller

Angular velocity of airplane:

$$\underline{\Omega} = \omega_y \underline{j} = \frac{v}{\rho}\underline{j}$$

$$= \frac{350(44/30)}{1200}\underline{j}$$

$$\underline{\Omega} = \omega_y \underline{j} = (0.4278\ \text{rad/s})\underline{j}$$

Angular velocity of propeller:

$$\underline{\omega} = \omega_y \underline{j} + \omega_z \underline{k}$$

with $\omega_z = 1500\ \text{rpm} = 157.08\ \text{rad/s}$

Angular momentum of propeller:

$$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k}$$

$$= 0 + \bar{I}_y(0.4278)\underline{j} + \bar{I}_z(157.08)\underline{k}$$

Rate of change of angular momentum:

Eq. (18.11):

$$\dot{\underline{H}}_G = (\dot{H}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \underline{\Omega} \times \underline{H}_G$$

$$= (0.4278\underline{j}) \times (0.4278\bar{I}_y\underline{j} + 157.08\bar{I}_z\underline{k})$$

$$= (0.4278)(157.08)\bar{I}_z\,\underline{i}$$

But $\bar{I}_z = m\bar{k}_z^2 = \frac{300}{32.2}(3)^2 = 83.851\ \text{lb·ft·s}^2$

Thus: $\dot{\underline{H}}_G = (0.4278)(157.08)(83.851)\underline{i}$

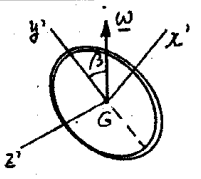
$$= (5634\ \text{lb·ft})\underline{i}$$

Equivalence of external and effective forces:

$$\underline{M} = \dot{\underline{H}}_G = (5634\ \text{lb·ft})\underline{i}$$

$$M = 5630\ \text{lb·ft}$$ ◀

18.67 Angular momentum about G:



We use the principal axes $Gx'y'z'$, with x' ⊥ plane of ring.

$\omega_{x'} = \omega\sin\beta$ $\quad \bar{I}_{x'} = ma^2$

$\omega_{y'} = \omega\cos\beta$ $\quad \bar{I}_{y'} = \frac{1}{2}ma^2$

$\omega_{z'} = 0$ $\quad \bar{I}_{z'} = \frac{1}{2}ma^2$

$$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_{z'}\omega_{z'}\underline{k}'$$

$$\underline{H}_G = ma^2\omega\left(\sin\beta\,\underline{i}' + \tfrac{1}{2}\cos\beta\,\underline{j}'\right)$$

Rate of change of $\underline{H}_G$:

$$\dot{\underline{H}}_G = (\dot{H}_G)_{Gx'y'z'} + \underline{\Omega} \times \underline{H}_G$$

where $(\dot{H}_G)_{Gx'y'z'} = 0$, since ω = constant,

and $\underline{\Omega} = \underline{\omega} = \omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}'$

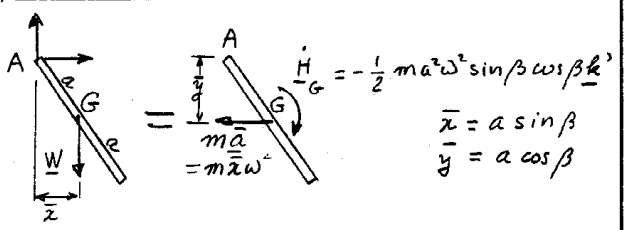
Thus:

$$\dot{\underline{H}}_G = (\omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}') \times ma^2\omega\left(\sin\beta\,\underline{i}' + \tfrac{1}{2}\cos\beta\,\underline{j}'\right)$$

$$= \tfrac{1}{2}ma^2\omega^2\sin\beta\cos\beta\,\underline{k}' - ma^2\omega^2\sin\beta\cos\beta\,\underline{k}'$$

$$= -\tfrac{1}{2}ma^2\omega^2\sin\beta\cos\beta\,\underline{k}'$$

Equivalence of external and effective forces:



(a) $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$$-\bar{x}W = -\bar{y}(m\bar{x}\omega^2) - \tfrac{1}{2}ma^2\omega^2\sin\beta\cos\beta$$

$$-a\sin\beta\,mg = -a\cos\beta\,ma\sin\beta\omega^2 - \tfrac{1}{2}ma^2\omega^2\sin\beta\cos\beta$$

$$g = a\omega^2\cos\beta + \tfrac{1}{2}a\omega^2\cos\beta$$

$$g = \tfrac{3}{2}a\omega^2\cos\beta$$

$$\cos\beta = \frac{2}{3}\frac{g}{a\omega^2}$$ ◀

(b) For $\beta = 0$, $\cos\beta = 1$

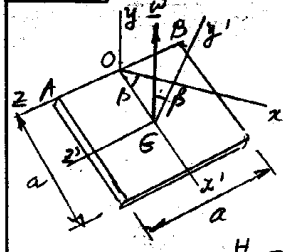
$$1 = \frac{2}{3}\frac{g}{a\omega^2}$$

$$\omega^2 = \frac{2}{3}\frac{g}{a}$$

$$\omega = \sqrt{2g/3a}$$ ◀

For $\omega \le \sqrt{2g/3a}$, the ring remains vertical.

18.68



Angular momentum $\underline{H}_G$:

Resolving $\underline{\omega}$ into components along the principal axes $Gx'y'z'$:

$\omega_{x'} = -\omega\sin\beta,\ \omega_{y'} = \omega\cos\beta,\ \omega_{z'} = 0$

Moments of inertia:

$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{12}ma^2$

$\bar{I}_{y'} = \bar{I}_{x'} + \bar{I}_{z'} = \frac{1}{6}ma^2$

$$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_{z'}\omega_{z'}\underline{k}'$$
$$= -\frac{1}{12}ma^2\omega\sin\beta\,\underline{i}' + \frac{1}{6}ma^2\omega\cos\beta\,\underline{j}'$$

where the reference frame $Gx'y'z'$ rotates with the plate with the angular velocity

$$\underline{\Omega} = \underline{\omega} = -\omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}'$$

Rate of change of $\underline{H}_G$:

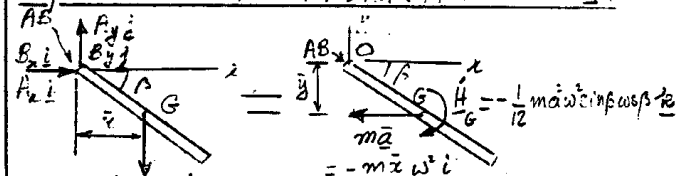
Since ω and β are constant, $\left(\dot{\underline{H}}_G\right)_{Gx'y'z'} = 0$ and Eq. (18.22) reduces to

$$\dot{\underline{H}}_G = \left(\dot{\underline{H}}_G\right)_{Gx'y'z'} + \underline{\Omega}\times\underline{H}_G$$
$$= 0 + \left(-\omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}'\right)\times\left(-\frac{1}{12}ma^2\omega\sin\beta\,\underline{i}' + \frac{1}{6}ma^2\omega\cos\beta\,\underline{j}'\right)$$

and, since $\underline{i}'\times\underline{j}' = \underline{k}' = \underline{k}$:

$$\dot{\underline{H}}_G = -\frac{1}{6}ma^2\omega^2\sin\beta\cos\beta\,\underline{k} + \frac{1}{12}ma^2\omega^2\cos\beta\sin\beta\,\underline{k}$$
$$\dot{\underline{H}}_G = -\frac{1}{12}ma^2\omega^2\sin\beta\cos\beta\,\underline{k}$$

Equivalence of external and effective forces:



Equating moments about O:

$$\left(\tfrac{1}{2}a\cos\beta\,\underline{i}\right)\times(-mg\,\underline{j}) = \left(-\tfrac{1}{2}a\sin\beta\,\underline{j}\right)\times\left(-m\tfrac{a}{2}\cos\beta\,\omega^2\,\underline{i}\right) - \frac{1}{12}ma^2\omega^2\sin\beta\cos\beta\,\underline{k}$$

$$-\frac{1}{2}mga\cos\beta\,\underline{k} = -\frac{1}{4}ma^2\omega^2\sin\beta\cos\beta\,\underline{k} - \frac{1}{12}ma^2\omega^2\sin\beta\cos\beta\,\underline{k}$$

$$\frac{1}{2}\frac{g}{a}\cos\beta = \frac{1}{3}\omega^2\sin\beta\cos\beta$$

$\therefore\ \cos\beta = 0$ and $\sin\beta = \frac{3}{2}\frac{g}{a\omega^2}$

(a) For $\beta = 40°$: $\sin 40° = \frac{3}{2}\frac{g}{a\omega^2}$

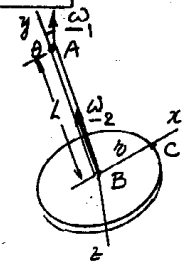
$$\omega^2 = \frac{3g}{2a\sin 40°} \qquad \omega = 1.528\sqrt{g/a}$$ ◀

(b) For $\beta = 90°$ to be the only solution, we need to have $\frac{3}{2}\frac{g}{a\omega^2} \ge 1$, that is

$$0 \le \omega \le 1.225\sqrt{g/a}$$ ◀

Note: When $0 \le \omega \le 1.225\sqrt{g/a}$, $\beta = 90°$ is the only possible solution and this position of equilibrium is stable. When $\omega > 1.225\sqrt{g/a}$ $\beta = 90°$ is still a position of equilibrium, but it is unstable.

18.69



Angular velocity of disk

We use axes $Bxyz$ with y along BA and z through C. The ang. velocity of this frame is

$$\underline{\Omega} = \underline{\omega}_1 = \omega_1(\sin\theta\,\underline{i} + \cos\theta\,\underline{j}) \quad (1)$$

We denote by $\underline{\omega}_2$ the ang. veloc. of the disk with respect to that frame. The total ang. velocity of the disk is

$$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2$$
$$\underline{\omega} = \omega_1\sin\theta\,\underline{i} + (\omega_1\cos\theta + \omega_2)\,\underline{j} \quad (2)$$

To determine ω_2, we write that $\underline{v}_C = 0$:

$$\underline{v}_C = \underline{\omega}\times\underline{r}_{C/A} = \left[\omega_1\sin\theta\,\underline{i} + (\omega_1\cos\theta + \omega_2)\,\underline{j}\right]\times(r\,\underline{i} - L\,\underline{j}) = 0$$
$$-L\omega_1\sin\theta\,\underline{k} - r(\omega_1\cos\theta + \omega_2)\,\underline{k} = 0$$
$$\omega_2 = -\omega_1\cos\theta - \frac{L}{r}\omega_1\sin\theta \quad (3)$$

Substituting for ω_2 into (2):

$$\underline{\omega} = \omega_1\sin\theta\,\underline{i} - \frac{L}{r}\omega_1\sin\theta\,\underline{j} \quad (4)$$

Angular momentum about B:

$$\underline{H}_B = \bar{I}_x\omega_x\,\underline{i} + \bar{I}_y\omega_y\,\underline{j} + \bar{I}_z\omega_z\,\underline{k}$$

Recalling (4): $\underline{H}_B = \left(\frac{1}{4}mr^2\right)\omega_1\sin\theta\,\underline{i} - \left(\frac{1}{2}mr^2\right)\frac{L}{r}\omega_1\sin\theta\,\underline{j}$

$$\underline{H}_B = \frac{1}{4}mr\omega_1\sin\theta(r\,\underline{i} - 2L\,\underline{j}) \quad (5)$$

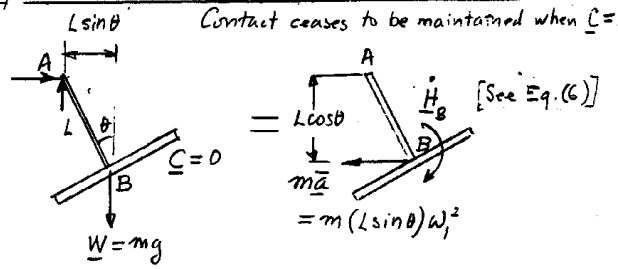
Rate of change of $\underline{H}_B$:

From Eq. (18.22), using Eqs. (1) and (5), and noting that θ is constant:

$$\dot{\underline{H}}_B = \left(\dot{\underline{H}}_B\right)_{Bxyz} + \underline{\Omega}\times\underline{H}_B = 0 + \omega_1(\sin\theta\,\underline{i} + \cos\theta\,\underline{j})\times\frac{1}{4}mr\omega_1\sin\theta(r\,\underline{i} - 2L\,\underline{j})$$
$$\dot{\underline{H}}_B = -\frac{1}{4}mr\omega_1^2\sin\theta(2L\sin\theta + r\cos\theta)\,\underline{k} \quad (6)$$

Equivalence of external and effective forces:

Contact ceases to be maintained when $\underline{C} = 0$.



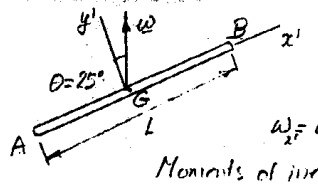
$+\circlearrowleft\ \Sigma M_A = \Sigma(M_A)_{eff}$:

$$-L\sin\theta(mg) = -L\cos\theta(mL\sin\theta)\omega_1^2 - \frac{1}{4}mr\omega_1^2\sin\theta(2L\sin\theta + r\cos\theta)$$
$$g = \omega_1^2L\cos\theta + \frac{1}{2}\omega_1^2r\sin\theta + \frac{1}{4}\omega_1^2\frac{r^2}{L}\cos\theta$$
$$\frac{g}{L} = \omega_1^2\left[1 + \left(\frac{r}{2L}\right)^2\right]\cos\theta + \omega_1^2\left(\frac{r}{2L}\right)\sin\theta$$

$$\omega_1 = \sqrt{\frac{g/L}{\left[1 + \left(\frac{r}{2L}\right)^2\right]\cos\theta + \left(\frac{r}{2L}\right)\sin\theta}}$$ ◀

18.70

Angular momentum $\underline{H}_G$ of rod AB:



Resolving the angular velocity $\underline{\omega}$ into components along the principal axes $Gx'y'z'$:

$$\omega_{x'} = \omega\sin\theta,\quad \omega_{y'} = \omega\cos\theta,\quad \omega_{z'} = 0$$

Moments of inertia: $\bar{I}_{x'} = 0,\ \bar{I}_{y'} = \frac{1}{12}mL^2$

$$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_{z'}\omega_{z'}\underline{k}'$$
$$= \frac{1}{12}mL^2\omega\cos\theta\,\underline{j}'$$

Where the reference frame $Gx'y'z'$ rotates with the rod with the angular velocity

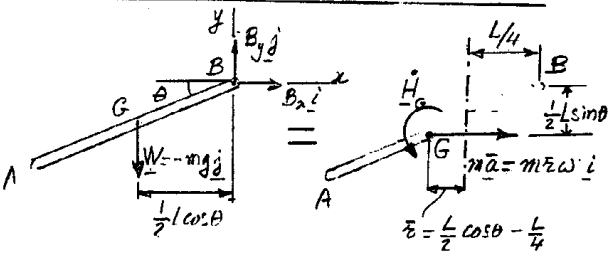
$$\underline{\Omega} = \underline{\omega} = \omega\sin\theta\,\underline{i}' + \omega\cos\theta\,\underline{j}'$$

Rate of change of $\underline{H}_G$:

Since ω and θ are constant, $(\dot{\underline{H}}_G)_{Gx'y'z'} = 0$ and Eq. (18.22) reduces to

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gx'y'z'} + \underline{\Omega}\times\underline{H}_G = 0 + (\omega\sin\theta\,\underline{i}' + \omega\cos\theta\,\underline{j}')\times\tfrac{1}{12}mL^2\omega\cos\theta\,\underline{j}'$$
$$\dot{\underline{H}}_G = \frac{1}{12}mL^2\omega^2\sin\theta\cos\theta\,\underline{k}'$$

Equivalence of external and effective forces:



Using the axes $Bxyz$ and noting that $\underline{k}' \equiv \underline{k}$ we equate moments about B:

$$\left(-\tfrac{1}{2}L\cos\theta\,\underline{i}\right)\times\left(-mg\underline{j}\right) = \left(-\tfrac{1}{2}L\sin\theta\,\underline{j}\right)\times m\bar{r}\omega^2\,\underline{i} + \dot{\underline{H}}_G$$
$$\tfrac{1}{2}mgL\cos\theta\,\underline{k} = \tfrac{1}{2}mL\sin\theta\left(\tfrac{L}{2}\cos\theta - \tfrac{L}{4}\right)\omega^2\underline{k} + \tfrac{1}{12}mL^2\omega^2\sin\theta\cos\theta\,\underline{k}$$

Dividing by $\frac{1}{2}mL^2$:

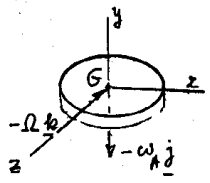
$$\frac{g}{L}\cos\theta = \left(\tfrac{1}{2}\sin\theta\cos\theta - \tfrac{1}{4}\sin\theta + \tfrac{1}{6}\sin\theta\cos\theta\right)\omega^2$$
$$\frac{g}{L} = \left(\tfrac{2}{3}\sin\theta - \tfrac{1}{4}\tan\theta\right)\omega^2$$

For $\theta = 20°$:

$$\frac{g}{L} = \left(\tfrac{2}{3}\sin 20° - \tfrac{1}{4}\tan 20°\right)\omega^2$$
$$\frac{g}{L} = (0.22801 - 0.09099)\,\omega^2$$
$$\omega^2 = \frac{1}{0.13702}\frac{g}{L}$$
$$\omega = 2.70\sqrt{g/L}$$ ◀

18.71

Disk A



Ang. velocity: $\omega_x = 0,\ \omega_y = -\omega_A,\ \omega_z = -\Omega$

Moments of inertia:

$$\bar{I}_y = \tfrac{1}{2}mr^2,\quad \bar{I}_z = \tfrac{1}{4}mr^2$$

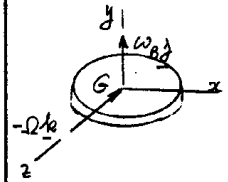
Ang. momentum:

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$$
$$= 0 - \tfrac{1}{2}mr^2\omega_A\underline{j} - \tfrac{1}{4}mr^2\Omega\,\underline{k}$$

Where the reference frame $Gx'y'z'$ rotates with the angular velocity $-\Omega\underline{k}$.

Rate of change of $\underline{H}_G$: Since ω_A and Ω are constant, $(\dot{\underline{H}}_G)_{Gxyz} = 0$ and Eq. (18.22) reduces to

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 - \Omega\underline{k}\times\left(-\tfrac{1}{2}mr^2\omega_A\underline{j} - \tfrac{1}{4}mr^2\Omega\underline{k}\right)$$
$$(\dot{\underline{H}}_G)_A = -\tfrac{1}{2}mr^2\omega_A\Omega\,\underline{i}$$



Disk B (with ω_B as described in part a)

Ang. velocity: $\omega_x = 0,\ \omega_y = \omega_A,\ \omega_z = -\Omega$

Moments of inertia:

$$\bar{I}_y = \tfrac{1}{2}mr^2,\quad \bar{I}_z = \tfrac{1}{4}mr^2$$

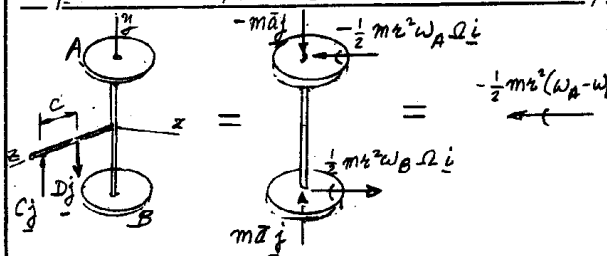
Ang. momentum:

$$\underline{H}_G = \tfrac{1}{2}mr^2\omega_B\underline{j} - \tfrac{1}{4}mr^2\Omega\underline{k}$$

Rate of change of $\underline{H}_G$:

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 - \Omega\underline{k}\times\left(\tfrac{1}{2}mr^2\omega_B\underline{j} - \tfrac{1}{4}mr^2\Omega\underline{k}\right)$$
$$(\dot{\underline{H}}_G)_B = +\tfrac{1}{2}mr^2\omega_B\Omega\,\underline{i}$$

Equivalence of external and effective forces for system:



We note that the effective forces reduce to a single couple of moment $-\frac{1}{2}mr^2(\omega_A - \omega_B)\Omega\,\underline{i}$. Thus the reactions at C and D must also form a couple as shown, with

$$C = D = \frac{1}{2}\frac{mr^2}{c}(\omega_A - \omega_B)\Omega$$

(a) **Disks rotating as originally described.**

Since $\omega_A = \omega_B = 1200$ rpm, $\underline{C} = \underline{D} = 0$ ◀

(b) **Direction of spin of B reversed:**

We have $\omega_A = 1200\text{ rpm} = 125.66$ rad/s

$\omega_B = -1200\text{ rpm} = -125.66$ rad/s

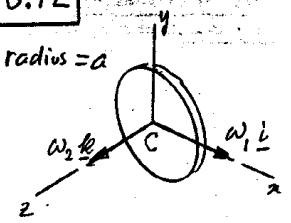
$\Omega = 60\text{ rpm} = 6.283$ rad/s

$m = \frac{10}{32.2} = 0.31056$, $r = 12\text{ in.} = 1\text{ ft}$, $c = 2\text{ ft}$

$$C = D = \frac{1}{2}\frac{0.31056}{2}(1)^2(125.66 + 125.66)(6.283) = 122.6\text{ lb}$$

$\underline{C} = (122.6\text{ lb})\underline{j}$; $\underline{D} = -(122.6\text{ lb})\underline{j}$ ◀

18.72



Angular momentum of disk

Resolving the angular velocity $\underline{\omega}$ of the disk into components along the principal axes $Cxyz$:

$$\underline{\omega} = \omega_1 \underline{i} + \omega_2 \underline{k}$$

Moments of inertia:

$$\bar{I}_x = \tfrac{1}{4} m a^2, \quad \bar{I}_z = \tfrac{1}{2} m a^2$$

Angular momentum:

$$\underline{H}_C = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k} = \tfrac{1}{4} m a^2 \omega_1 \underline{i} + 0 + \tfrac{1}{2} m a^2 \omega_2 \underline{k}$$

where the reference frame $Cxyz$ rotates with the horizontal rod ABC with the angular velocity

$$\underline{\Omega} = \omega_1 \underline{i}$$

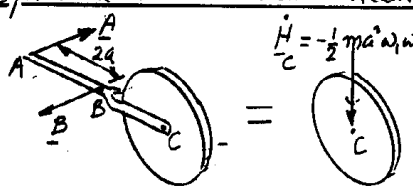
Rate of change of $\underline{H}_C$.

Since ω_1 and ω_2 are constant, $(\dot{\underline{H}}_C)_{Cxyz} = 0$ and Eq. (18.22) reduces to

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Cxyz} + \underline{\Omega} \times \underline{H}_C = 0 + \omega_1 \underline{i} \times \left(\tfrac{1}{4} m a^2 \omega_1 \underline{i} + \tfrac{1}{2} m a^2 \omega_2 \underline{k}\right)$$

$$\dot{\underline{H}}_C = -\tfrac{1}{2} m a^2 \omega_1 \omega_2 \underline{j}$$

Equivalence of external and effective forces:



$\dot{\underline{H}}_C = -\tfrac{1}{2} m a^2 \omega_1 \omega_2 \underline{j}$ $\qquad m\underline{\bar{a}} = 0$

Since the dynamic reactions $\underline{A}$ and $\underline{B}$ are equivalent to the couple $\dot{\underline{H}}_C$, they must form themselves a couple. Thus $A = B$ and

$$A(2a) = \tfrac{1}{2} m a^2 \omega_1 \omega_2 \qquad A = B = \tfrac{1}{4} m a^2 \omega_1 \omega_2$$

$$\underline{A} = -\tfrac{1}{4} m a \omega_1 \omega_2 \underline{k}; \quad \underline{B} = \tfrac{1}{4} m a \omega_1 \omega_2 \underline{k}$$

18.73

See solutions of Probs. 18.6 and 18.50 for computation of rate of change of $\underline{H}_C$:

$$\dot{\underline{H}}_C = (3.75 \text{ N·m}) \underline{i}$$

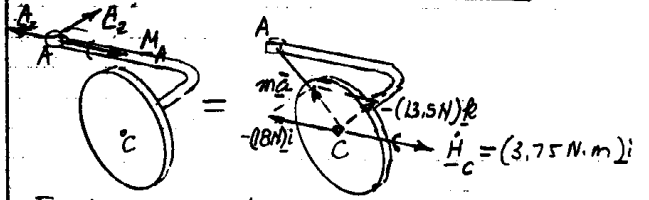
For point C, since $\underline{\alpha}_1 = 0$ and $\underline{\omega}_1 \perp \underline{r}_{C/A}$:

$$\underline{a}_C = \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{C/A}) = -\omega_1^2 \underline{r}_{C/A}$$

$$m\underline{\bar{a}} = m\underline{a}_C = -(5 \text{ kg})(3 \text{ rad/s})^2 [(0.4 \text{ m})\underline{i} + (0.3 \text{ m})\underline{k}]$$

$$= -(18 \text{ N})\underline{i} - (13.5 \text{ N})\underline{k} \qquad \text{(along } CA\text{)}$$

Equivalence of external and effective forces:



Equating components:

$$\underline{A}_x = -(18\text{N})\underline{i}, \quad \underline{A}_z = -(13.5\text{N})\underline{k}; \quad \underline{A} = -(18\text{N})\underline{i} - (13.5\text{N})\underline{k}$$

Since $m\underline{\bar{a}}$ passes through A, it has zero moment about A and

$$\underline{M}_A = \dot{\underline{H}}_C = (3.75 \text{ N·m})\underline{i}, \qquad \underline{M}_A = (3.75 \text{ N·m})\underline{i}$$

18.74

See solutions of Probs. 18.5 and 18.49 for computation of rate of change of $\dot{\underline{H}}_C$:

$$\dot{\underline{H}}_C = (2.91 \text{ lb·ft})\underline{i}$$

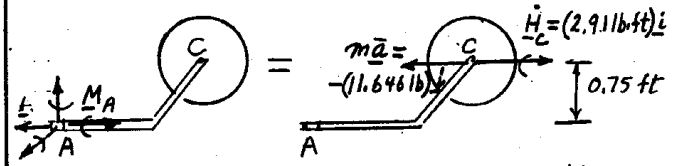
For point C of ABC, since $\underline{\alpha}_1 = 0$:

$$\underline{a}_C = \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{C/A}) = \omega_1 \underline{j} \times \left[\omega_1 \underline{j} \times \left(\tfrac{18}{12}\underline{i} + \tfrac{9}{12}\underline{j}\right)\right]$$

$$= \omega_1 \underline{j} \times (-1.5\omega_1 \underline{k}) = -1.5\omega_1^2 \underline{i}$$

$$m\underline{\bar{a}} = m\underline{a}_C = -\frac{10}{g}(1.5)(5 \text{ rad/s})^2 \underline{i} = -\frac{375}{g}\underline{i} = -(11.646 \text{ lb})\underline{i}$$

Equivalence of external and effective forces:



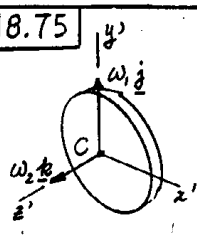
Equating components: $\underline{A} = m\underline{\bar{a}}$ $\qquad A = -(11.65 \text{ lb})\underline{i}$

Equating moments about A:

$$\underline{M}_A = \dot{\underline{H}}_C + 0.75\underline{j} \times m\underline{\bar{a}} = 2.91\underline{i} + 0.75\underline{j} \times (-11.646\underline{i})$$

$$\underline{M}_A = (2.91 \text{ lb·ft})\underline{i} + (8.73 \text{ lb·ft})\underline{k}$$

18.75



Angular momentum of disk

Using reference frame $Cx'y'z'$:

$$\omega_{x'} = 0, \quad \omega_{y'} = \omega_1, \quad \omega_{z'} = \omega_2$$

$$\bar{I}_{y'} = \tfrac{1}{4} m r^2, \quad \bar{I}_{z'} = \tfrac{1}{2} m r^2$$

$$\underline{H}_C = \bar{I}_{x'} \omega_{x'} \underline{i} + \bar{I}_{y'} \omega_{y'} \underline{j} + \bar{I}_{z'} \omega_{z'} \underline{k}$$

$$= 0 + \tfrac{1}{4} m r^2 \omega_1 \underline{j} + \tfrac{1}{2} m r^2 \omega_2 \underline{k}$$

where reference frame rotates with ABC with angular velocity

$$\underline{\Omega} = \omega_1 \underline{j}$$

Rate of change of $\underline{H}_C$. Using Eq. (18.22):

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Cx'y'z'} + \underline{\Omega} \times \underline{H}_C = \tfrac{1}{4} m r^2 \dot{\omega}_1 \underline{j} + \tfrac{1}{2} m r^2 \dot{\omega}_2 \underline{k}$$

$$+ \omega_1 \underline{j} \times \left(\tfrac{1}{4} m r^2 \omega_1 \underline{j} + \tfrac{1}{2} m r^2 \omega_2 \underline{k}\right)$$

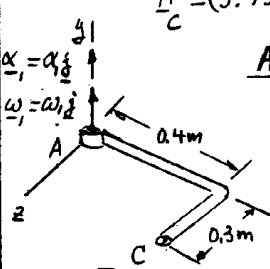
$$\dot{\underline{H}}_C = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} + \tfrac{1}{4} m r^2 \alpha_1 \underline{j} + \tfrac{1}{2} m r^2 \alpha_2 \underline{k} \qquad (1)$$

Given data: $m = 5 \text{ kg}$, $r = 0.25 \text{ m}$; $\omega_1 = 3 \text{ rad/s}$, $\alpha_1 = 7.5 \text{ rad/s}^2$, $\omega_2 = 8 \text{ rad/s}$, $\alpha_2 = 0$

$$\dot{\underline{H}}_C = \tfrac{1}{2}(5)(0.25)^2(3)(8)\underline{i} + \tfrac{1}{4}(5)(0.25)^2(7.5)\underline{j} + 0$$

$$\dot{\underline{H}}_C = (3.75 \text{ N·m})\underline{i} + (0.58594 \text{ N·m})\underline{j} \qquad (2)$$

Acceleration of mass center C



$$\underline{\bar{a}} = \underline{a}_C = \underline{\alpha}_1 \times \underline{r}_{C/A} + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{C/A})$$

$$= \alpha_1 \underline{j} \times (0.4\underline{i} + 0.3\underline{k}) + \omega_1 \underline{j} \times [\omega_1 \underline{j} \times (0.4\underline{i} + 0.3\underline{k})$$

$$= -0.4\alpha_1 \underline{k} + 0.3\alpha_1 \underline{i} + \omega_1 \underline{j} \times (-0.4\omega_1 \underline{k} + 0.3\omega_1 \underline{i})$$

$$\underline{\bar{a}} = (0.3\alpha_1 - 0.4\omega_1^2)\underline{i} - (0.4\alpha_1 + 0.3\omega_1^2)\underline{k} \qquad (3)$$

(CONTINUED)

18.75 CONTINUED We found the following expression for the accel. of the mass center G:

$$\bar{a} = (0.3\alpha_1 - 0.4\omega_1^2)\,\underline{i} - (0.4\alpha_1 + 0.3\omega_1^2)\,\underline{k} \quad (3)$$

With given data, $\omega_1 = 3$ rad/s, $\alpha_1 = 7.5$ rad/s^2:

$$\bar{a} = [0.3(7.5) - 0.4(3)^2]\,\underline{i} - [0.4(7.5) + 0.3(3)^2]\,\underline{k}$$

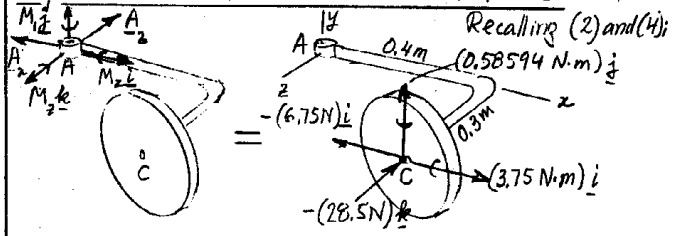
$$\bar{a} = -(1.35 \text{ m/s}^2)\,\underline{i} - (5.7 \text{ m/s}^2)\,\underline{k}$$

and $m\bar{a} = (5 \text{ kg})\bar{a} = -(6.75 \text{ N})\,\underline{i} - (28.5 \text{ N})\,\underline{k}$ (4)

We also recall the expression found for $\dot{\underline{H}}_C$:

$$\dot{\underline{H}}_C = (3.75 \text{ N·m})\,\underline{i} + (0.58594 \text{ N·m})\,\underline{j} \quad (2)$$

Equivalence of external and effective forces:



Recalling (2) and (4):

Equating moments about A:

$$M_1\underline{j} + M_2\underline{i} + M_2\underline{k} = 0.4\underline{i} \times (-28.5\underline{k}) + (0.3\underline{k}) \times (-6.75\underline{i}) + 3.75\underline{i} + 0.58594\underline{j}$$

$$= 11.4\underline{j} - 2.025\underline{j} + 3.75\underline{i} + 0.58594\underline{j}$$

Equating the coefficients of the unit vectors:

(j) $M_1 = 11.4 - 2.025 + 0.58594 = 9.961$ N·m

(a) Required couple: $M_1\underline{j} = (9.96 \text{ N·m})\,\underline{j}$ ◀

(i) $M_2 = 3.75$ N·m

(k) $M_2 = 0$

Equating components: $A_x = -6.75$ N, $A_y = 0$, $A_z = -28.5$ N

(b) Dynamic reaction at A:

$$\underline{A} = -(6.75 \text{ N})\,\underline{i} - (28.5 \text{ N})\,\underline{k}, \quad \underline{M}_A = (3.75 \text{ N·m})\,\underline{i}$$ ◀

18.76 See solution of Prob. 18.75 for derivation of Eqs. (1) and (3):

$$\dot{\underline{H}}_C = \tfrac{1}{2} m r^2 \omega_1 \omega_2\,\underline{i} + \tfrac{1}{4} m r^2 \alpha_1\,\underline{j} + \tfrac{1}{2} m r^2 \alpha_2\,\underline{k} \quad (1)$$

$$\bar{a} = (0.3\alpha_1 - 0.4\omega_1^2)\,\underline{i} - (0.4\alpha_1 + 0.3\omega_1^2)\,\underline{k} \quad (3)$$

With given data: $m = 5$ kg, $r = 0.25$ m, $\omega_1 = 3$ rad/s, $\alpha_1 = 0$, $\omega_2 = 8$ rad/s, $\alpha_2 = -1.8$ rad/s^2;

Eq. (1): $\dot{\underline{H}}_C = \tfrac{1}{2}(5)(0.25)^2(3)(8)\,\underline{i} + 0 + \tfrac{1}{2}(5)(0.25)^2(-1.8)\,\underline{k}$

$$\dot{\underline{H}}_C = (3.75 \text{ N·m})\,\underline{i} - (0.28125 \text{ N·m})\,\underline{k} \quad (2)$$

Eq. (3): $\bar{a} = -0.4\omega_1^2\,\underline{i} - 0.3\omega_1^2\,\underline{k}$

$$= -0.4(3)^2\,\underline{i} - 0.3(3)^2\,\underline{k}$$

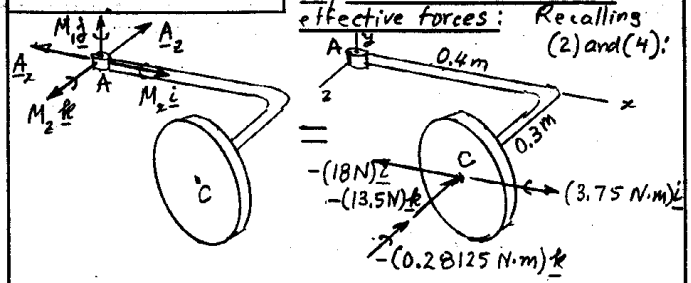
$$= -(3.6 \text{ m/s}^2)\,\underline{i} - (2.7 \text{ m/s}^2)\,\underline{k}$$

$$m\bar{a} = (5 \text{ kg})\bar{a} = -(18 \text{ N})\,\underline{i} - (13.5 \text{ N})\,\underline{k} \quad (4)$$

(We may note that $m\bar{a}$ is directed along CA and will have zero moment about A.)

(CONTINUED)

18.76 CONTINUED Equivalence of external and effective forces: Recalling (2) and (4):



Equating moments about A:

$$M_1\underline{j} + M_2\underline{i} + M_2\underline{k} = 0.4\underline{i} \times (-13.5\underline{k}) + (0.3\underline{k}) \times (-18\underline{i}) + 3.75\underline{i} - 0.28125\underline{k}$$

$$= 5.4\underline{j} - 5.4\underline{j} + 3.75\underline{i} - 0.28125\underline{k}$$

Equating the coefficients of the unit vectors:

(j) $M_1 = 0$ (a) Required couple: $M_1\underline{j} = 0$ ◀

(i) $M_2 = 3.75$ N·m

(k) $M_2 = -0.28125$ N·m

Equating components: $A_x = -18.00$ N, $A_y = 0$, $A_z = -13.50$ N

(b) Dynamic reaction at A:

$$\underline{A} = -(18.00 \text{ N})\,\underline{i} - (13.50 \text{ N})\,\underline{k}, \quad \underline{M}_A = (3.75 \text{ N·m})\,\underline{i} - (0.281 \text{ N·m})\,\underline{k}$$ ◀

18.77 See solution of Prob. 18.72 for the determination of the angular momentum $\underline{H}_C$:

$$\underline{H}_C = \tfrac{1}{4} m a^2 \omega_1\,\underline{i} + \tfrac{1}{2} m a^2 \omega_2\,\underline{k}$$

where the reference frame rotates with $\underline{\Omega} = \omega_1\,\underline{i}$

Rate of change of $\underline{H}_C$. Using Eq. (18.22):

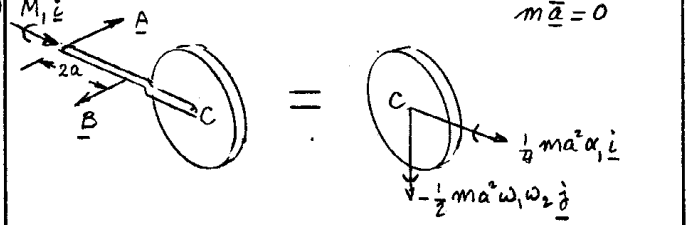
$$\dot{\underline{H}}_C = \left(\dot{\underline{H}}_C\right)_{Oxyz} + \underline{\Omega} \times \underline{H}_C = \tfrac{1}{4} m a^2 \dot{\omega}_1\,\underline{i} + \tfrac{1}{2} m a^2 \dot{\omega}_2\,\underline{k} + \omega_1\,\underline{i} \times \left(\tfrac{1}{4} m a^2 \omega_1\,\underline{i} + \tfrac{1}{2} m a^2 \omega_2\,\underline{k}\right)$$

$$\dot{\underline{H}}_C = \tfrac{1}{4} m a^2 \alpha_1\,\underline{i} - \tfrac{1}{2} m a^2 \omega_1 \omega_2\,\underline{j} + \tfrac{1}{2} m a^2 \alpha_2\,\underline{k}$$

But $\omega_2 =$ constant and, thus, $\alpha_2 = 0$. We have

$$\dot{\underline{H}}_C = \tfrac{1}{4} m a^2 \alpha_1\,\underline{i} - \tfrac{1}{2} m a^2 \omega_1 \omega_2\,\underline{j}$$

Equivalence of external and effective forces:



$m\bar{a} = 0$

(a) Required couple. We must have

$$M_1\,\underline{i} = \tfrac{1}{4} m a^2 \alpha_1\,\underline{i}$$ ◀

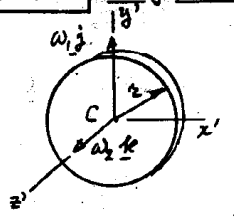
(b) Dynamic reactions at A and B.

The reactions at A and B must form a couple equivalent to the couple $-\tfrac{1}{2} m a^2 \omega_1 \omega_2\,\underline{j}$. They must be directed as shown and we must have

$$A = B, \quad A(2a) = \tfrac{1}{2} m a^2 \omega_1 \omega_2 \qquad A = B = \tfrac{1}{4} m a \omega_1 \omega_2$$

$$\underline{A} = -\tfrac{1}{4} m a \omega_1 \omega_2\,\underline{k}; \quad \underline{B} = \tfrac{1}{4} m a \omega_1 \omega_2\,\underline{k}$$ ◀

18.78 **Angular momentum about C:**



Using the reference frame $Cx'y'z'$:

$\omega_{x'} = 0,\ \omega_{y'} = \omega_1,\ \omega_{z'} = \omega_2$

$\bar{I}_{y'} = \frac{1}{4} m r^2 \qquad \bar{I}_{z'} = \frac{1}{2} m r^2$

$\underline{H}_C = \bar{I}_{x'}\omega_{x'}\underline{i} + \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k}$

$\underline{H}_C = \frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m r^2 \omega_2 \underline{k}$

where the reference frame rotates with arm ABC with

$\underline{\Omega} = \omega_1 \underline{j}$

Rate of change of $\underline{H}_C$: From Eq. (18.22):

$$\dot{\underline{H}}_C = \left(\dot{\underline{H}}_C\right)_{Cx'y'z'} + \underline{\Omega} \times \underline{H}_C = \frac{1}{4} m r^2 \dot{\omega}_1 \underline{j} + \frac{1}{2} m r^2 \dot{\omega}_2 \underline{k} + \omega_1 \underline{j} \times \left(\frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m r^2 \omega_2 \underline{k}\right)$$

$$\dot{\underline{H}}_C = \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} + \frac{1}{4} m r^2 \alpha_1 \underline{j} + \frac{1}{2} m r^2 \alpha_2 \underline{k}$$

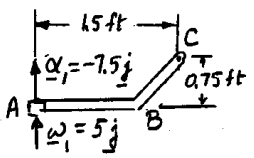
Given data: $m = \frac{10}{32.2}$, $r = 0.5$ ft, $\omega_1 = 5$ rad/s, $\omega_2 = 15$ rad/s,

$\alpha_1 = -7.5$ rad/s^2, $\alpha_2 = 0$

$$\dot{\underline{H}}_C = \frac{1}{2}\frac{10}{32.2}(0.5)^2(5)(15)\underline{i} + \frac{1}{4}\frac{10}{32.2}(0.5)^2(-7.5)\underline{j} + 0$$

$$\dot{\underline{H}}_C = (2.9115 \text{ lb·ft})\underline{i} - (0.14557 \text{ lb·ft})\underline{j} \qquad (1)$$

Acceleration of mass center C:



$$\underline{a}_C = \underline{\alpha}_1 \times \underline{r}_{C} + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{C})$$

$$= -7.5\underline{j} \times (1.5\underline{i} + 0.75\underline{j}) + 5\underline{j} \times [5\underline{j} \times (1.5\underline{i} + 0.75\underline{j})]$$

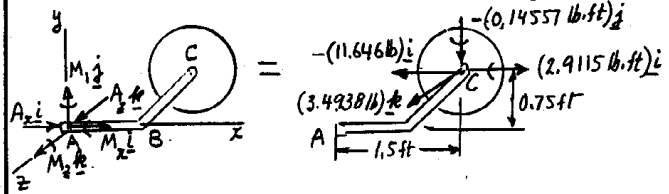
$$= 11.25\underline{k} + 5\underline{j} \times (-7.5\underline{k})$$

$$\underline{a} = \underline{a}_C = -(37.5 \text{ ft/s}^2)\underline{i} + (11.25 \text{ ft/s}^2)\underline{k}$$

$$m\underline{a} = \frac{10}{32.2}(-37.5\underline{i} + 11.25\underline{k}) = -(11.646 \text{ lb})\underline{i} + (3.4938 \text{ lb})\underline{k} \qquad (2)$$

Equivalence of external and effective forces

Recalling (1) and (2):



Equating moments about A:

$$M_x\underline{i} + M_1\underline{j} + M_z\underline{k} = 2.9115\underline{i} - 0.14557\underline{j} + (1.5\underline{i} + 0.75\underline{j}) \times (-11.646\underline{i} + 3.4938\underline{k})$$

$$= 2.9115\underline{i} - 0.14557\underline{j} - 5.2407\underline{j} + 8.7345\underline{k} + 2.6204\underline{i}$$

Equating coeff. of unit vectors:

(i) $M_1 = -0.14557 - 5.2407 = -5.3863$ lb·ft

(a) **Required couple:** $M_1\underline{j} = -(5.39 \text{ lb·ft})\underline{j}$ ◀

(i) $M_x = 2.9115 + 2.6204 = 5.5319$ lb·ft

(k) $M_z = 8.7345$ lb·ft

Equating components:

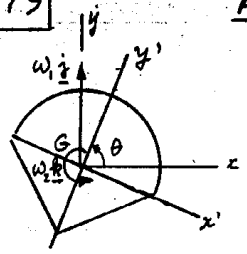
$A_x = -11.646$ lb, $A_y = 0$, $A_z = 3.4938$ lb

(b) **Dynamic reaction at A:**

$$\underline{A} = -(11.65 \text{ lb})\underline{i} + (3.49 \text{ lb})\underline{k}$$ ◀

$$\underline{M}_A = (5.53 \text{ lb·ft})\underline{i} + (8.73 \text{ lb·ft})\underline{k}$$ ◀

***18.79**



Angular momentum $\underline{H}_G$:

We resolve the angular velocity $\underline{\omega} = \omega_1\underline{i} + \omega_2\underline{k}$ into components along the principal axes $Gx'y'z$:

$\omega_{x'} = -\omega_1 \cos\theta$

$\omega_{y'} = \omega_1 \sin\theta$

$\omega_z = \omega_2$

Moments of inertia. We denote by I the moment of inertia about the axis of symmetry y' and by I' the moment of inertia about any transverse axis through G. Thus:

$\bar{I}_{x'} = \bar{I}_z = I'$ and $\bar{I}_{y'} = I$

We have

$H_{x'} = \bar{I}_{x'}\omega_{x'} = -I'\omega_1 \cos\theta$

$H_{y'} = \bar{I}_{y'}\omega_{y'} = I\omega_1 \sin\theta$

$H_z = \bar{I}_z\omega_z = I'\omega_2$

Computing the components of $\underline{H}_G$ along the x, y, z axes:

$$H_x = H_{x'}\sin\theta + H_{y'}\cos\theta = -I'\omega_1 \cos\theta \sin\theta + I\omega_1 \sin\theta \cos\theta$$

$$= (I - I')\omega_1 \sin\theta \cos\theta$$

$$H_y = -H_{x'}\cos\theta + H_{y'}\sin\theta = I'\omega_1 \cos^2\theta + I\omega_1 \sin^2\theta$$

$$= (I'\cos^2\theta + I\sin^2\theta)\omega_1$$

$$H_z = I'\omega_2$$

$$\underline{H}_G = (I - I')\omega_1 \sin\theta\cos\theta\,\underline{i} + (I'\cos^2\theta + I\sin^2\theta)\omega_1\underline{j} + I'\omega_2\underline{k}$$

where the reference frame $Gxyz$ rotates with the angular velocity $\underline{\Omega} = \omega_1\underline{j}$.

Rate of change of $\underline{H}_G$. We note that ω_1 and ω_2 are constant, while θ varies with t, with $\dot{\theta} = \omega_2$. Eq. (18.22) yields

$$\dot{\underline{H}}_G = \left(\dot{\underline{H}}_G\right)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = (I - I')\omega_1(\cos^2\theta - \sin^2\theta)\dot{\theta}\,\underline{i} + 2\sin\theta\cos\theta\,\dot{\theta}(I - I')\omega_1\underline{j} + \omega_1\underline{j} \times [(I - I')\omega_1 \sin\theta\cos\theta\,\underline{i} + I'\omega_2\underline{k}]$$

$$= (I - I')\omega_1\omega_2 \cos 2\theta\,\underline{i} + (I - I')\omega_1\omega_2 \sin 2\theta\,\underline{j} - \frac{1}{2}(I - I')\omega_1^2 \sin 2\theta\,\underline{k}$$

$$\dot{\underline{H}}_G = [I' + (I - I')\cos 2\theta]\omega_1\omega_2\underline{i} + (I - I')\omega_1\omega_2 \sin 2\theta\,\underline{j} - \frac{1}{2}(I - I')\omega_1^2 \sin 2\theta\,\underline{k} \qquad (1)$$

Given data: $I = (30 \text{ Mg})(12 \text{ m})^2 = 4.32 \times 10^6$ kg·m^2

$I' = (30 \text{ Mg})(10 \text{ m})^2 = 3 \times 10^6$ kg·m^2; $I - I' = 1.32 \times 10^6$ kg·m^2

$\omega_1 = 0.25$ rad/s, $\omega_2 = 0.20$ rad/s, $\theta = 60°$. Eq. (1) yields

$$\dot{\underline{H}}_G = [3 \times 10^6 + (1.32 \times 10^6)\cos 120°](0.25)(0.20)\underline{i} + (1.32 \times 10^6)(0.25)(0.20)\sin 120°\,\underline{j} - \frac{1}{2}(1.32 \times 10^6)(0.25)^2 \sin 120°\,\underline{k}$$

$$\dot{\underline{H}}_G = (117.00 \times 10^3 \text{ N·m})\underline{i} + (57.158 \times 10^3 \text{ N·m})\underline{j} - (35.724 \times 10^3 \text{ N·m})\underline{k} \qquad (2)$$

Vector $m\underline{\bar{a}}$

Since the mass center G is fixed, we have

$$m\underline{\bar{a}} = 0$$

The system of the effective forces thus reduces to the couple $\dot{\underline{H}}_G$ defined by Eq. (2).

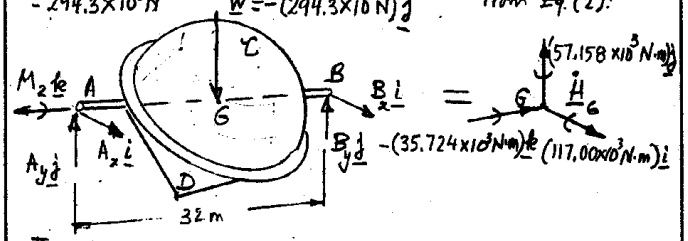
(CONTINUED)

*18.79 CONTINUED

Equivalence of external and effective forces. (Weight included)

$W = (30\,\text{Mg})(9.81\,\text{m/s}^2)$
$= 294.3\times10^3\,\text{N}$ $\quad \underline{W} = -(294.3\times10^3\,\text{N})\,\underline{j}$

From Eq. (2):



Equating moments about B:

$$32\,\underline{k}\times(A_x\,\underline{i}+A_y\,\underline{j}) + 16\,\underline{k}\times(-294.3\times10^3\,\underline{j}) + M_2\,\underline{k} = \dot{\underline{H}}_G$$

$$32A_x\,\underline{j} - 32A_y\,\underline{i} + 4.7088\times10^6\,\underline{i} + M_2\,\underline{k} = 117.00\times10^3\,\underline{i} + 57.158\times10^3\,\underline{j} - 35.724\times10^3\,\underline{k}$$

Equating the coefficients of the unit vectors:

(k) $M_2 = -35.724\times10^3\,\text{N}\cdot\text{m}$ $\quad \underline{M}_2 = -(35.7\,\text{kN}\cdot\text{m})\,\underline{k}$ ◀

(j) $32A_x = 57.158\times10^3$ $\quad A_x = 1.786\,\text{kN}$

(i) $-32A_y + 4.7088\times10^6 = 117.00\times10^3$ $\quad A_y = 143.5\,\text{kN}$

$$\underline{A} = (1.786\,\text{kN})\,\underline{i} + (143.5\,\text{kN})\,\underline{j}$$ ◀

Equating components:

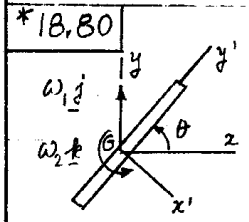
$$A_x\,\underline{i} + A_y\,\underline{j} - W\,\underline{j} + B_x\,\underline{i} + B_y\,\underline{j} = Q$$

$$\underline{B} = B_x\,\underline{i} + B_y\,\underline{j} = W\,\underline{j} - A_x\,\underline{i} - A_y\,\underline{j}$$

$$= (294.3\,\text{kN})\,\underline{j} - (1.786\,\text{kN})\,\underline{i} - (143.5\,\text{kN})\,\underline{j}$$

$$\underline{B} = -(1.786\,\text{kN})\,\underline{i} + (150.8\,\text{kN})\,\underline{j}$$ ◀

*18.80



Angular momentum $\underline{H}_G$: We resolve the angular velocity $\underline{\omega} = \omega_1\,\underline{j}' + \omega_2\,\underline{k}$ into components along the principal axes $Gx'y'z$:

$$\omega_{x'} = -\omega_1\cos\theta$$
$$\omega_{y'} = \omega_1\sin\theta$$
$$\omega_z = \omega_2$$

Moments of inertia

$$\bar{I}_{x'} = \bar{I}_z = \frac{1}{12}mL^2 \qquad \bar{I}_{y'} = 0$$

We have $H_{x'} = \bar{I}_{x'}\omega_{x'} = -\frac{1}{12}mL^2\omega_1\cos\theta$

$H_{y'} = \bar{I}_{y'}\omega_{y'} = 0$

$H_z = \bar{I}_z\omega_2 = \frac{1}{12}mL^2\omega_2$

Computing the components of $\underline{H}_G$ along the x, y, z axes:

$$H_x = H_{x'}\sin\theta = -\frac{1}{12}mL^2\omega_1\cos\theta\sin\theta = -\frac{1}{24}mL^2\omega_1\sin2\theta$$

$$H_y = -H_{x'}\cos\theta = +\frac{1}{12}mL^2\omega_1\cos^2\theta$$

$$H_z = \frac{1}{12}mL^2\omega_2$$

(CONTINUED)

*18.80 CONTINUED

The angular momentum is therefore

$$\underline{H}_G = -\frac{1}{24}mL^2\omega_1\sin2\theta\,\underline{i} + \frac{1}{12}mL^2\omega_1\cos^2\theta\,\underline{j} + \frac{1}{12}mL^2\omega_2\,\underline{k}$$

where the reference frame $Gxyz$ rotates with the angular velocity

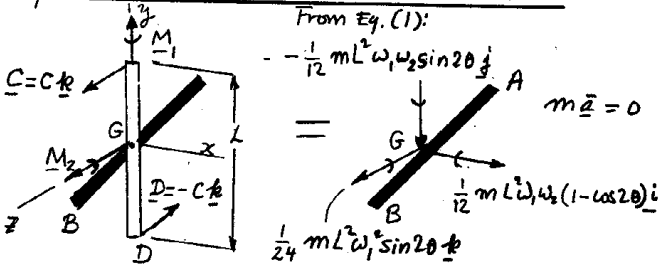
$$\underline{\Omega} = \omega_1\,\underline{j}$$

Rate of change of $\underline{H}_G$. We note that ω_1 and ω_2 are constant, while θ varies with t, with $\dot{\theta} = \omega_2$. Eq. (18.22) yields:

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = -\frac{1}{24}mL^2\omega_1(2\cos2\theta\,\dot{\theta})\,\underline{i}$$
$$+\frac{1}{12}mL^2\omega_1(-2\cos\theta\sin\theta\,\dot{\theta})\,\underline{j}$$
$$+\omega_1\,\underline{j}\times\left(-\frac{1}{24}mL^2\omega_1\sin2\theta\,\underline{i} + \frac{1}{12}mL^2\omega_2\,\underline{k}\right)$$
$$= -\frac{1}{12}mL^2\omega_1\omega_2\cos2\theta\,\underline{i} - \frac{1}{12}mL^2\omega_1\omega_2\sin2\theta\,\underline{j}$$
$$+\frac{1}{24}mL^2\omega_1^2\sin2\theta\,\underline{k} + \frac{1}{12}mL^2\omega_1\omega_2\,\underline{i}$$

$$\dot{\underline{H}}_G = \frac{1}{12}mL^2\omega_1\omega_2(1-\cos2\theta)\,\underline{i} - \frac{1}{12}mL^2\omega_1\omega_2\sin2\theta\,\underline{j}$$
$$+\frac{1}{24}mL^2\omega_1^2\sin2\theta\,\underline{k} \qquad (1)$$

Equivalence of external and effective forces



Equating the moments of the various couples:

$$L\,\underline{j}\times C\,\underline{k} + M_1\,\underline{j} + M_2\,\underline{k} = \dot{\underline{H}}_G$$

$$LC\,\underline{i} + M_1\,\underline{j} + M_2\,\underline{k} = \frac{1}{12}mL^2\omega_1\omega_2(1-\cos2\theta)\,\underline{i} - \frac{1}{12}mL^2\omega_1\omega_2\sin2\theta\,\underline{j} + \frac{1}{24}mL^2\omega_1^2\sin2\theta\,\underline{k}$$

Equating the coefficients of the unit vectors:

(j) $M_1 = -\frac{1}{12}mL^2\omega_1\omega_2\sin2\theta$

(a) Couple M_1: $\quad \underline{M}_1 = -\frac{1}{12}mL^2\omega_1\omega_2\sin2\theta\,\underline{j}$ ◀

(k) $M_2 = \frac{1}{24}mL^2\omega_1^2\sin2\theta$

(b) Couple M_2: $\quad \underline{M}_2 = \frac{1}{24}mL^2\omega_1^2\sin2\theta\,\underline{k}$ ◀

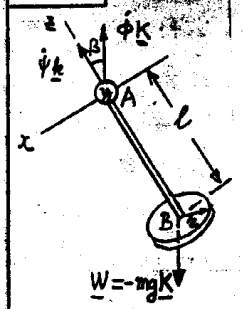
(i) $LC = \frac{1}{12}mL^2\omega_1\omega_2(1-\cos2\theta)$

$$C = \frac{1}{12}mL\omega_1\omega_2(1-\cos2\theta) = \frac{1}{6}mL\omega_1\omega_2\sin^2\theta$$

(c) Dynamic Reactions

$$\underline{C} = \frac{1}{6}mL\omega_1\omega_2\sin^2\theta\,\underline{k}\,;\quad \underline{D} = -\frac{1}{6}mL\omega_1\omega_2\sin^2\theta\,\underline{k}$$ ◀

18.81



Angular Velocity:

$$\underline{\omega} = -\dot\phi\sin\beta\,\underline{i} + (\dot\psi+\dot\phi\cos\beta)\underline{k}$$

Angular Momentum $\underline{H}_A$:

$$\underline{H}_A = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$
$$= -I'\dot\phi\sin\beta\,\underline{i} + I(\dot\psi+\dot\phi\cos\beta)\underline{k}$$

where frame $Axyz$ rotates with

$$\underline{\Omega} = -\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}$$

Rate of change of $\underline{H}_A$:

Since we have steady precession:

$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A = 0 + \underline{\Omega}\times\underline{H}_A$$

$$\dot{\underline{H}}_A = (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k})\times[-I'\dot\phi\sin\beta\,\underline{i} + I(\dot\psi+\dot\phi\cos\beta)\underline{k}]$$

$$\dot{\underline{H}}_A = [I(\dot\psi+\dot\phi\cos\beta) - I'\dot\phi\cos\beta]\dot\phi\sin\beta\,\underline{j}$$

$$\Sigma\underline{M}_A = \dot{\underline{H}}_A: \quad \Sigma\underline{M}_A = [I(\dot\psi+\dot\phi\cos\beta) - I'\dot\phi\cos\beta]\dot\phi\sin\beta\,\underline{j} \quad (1)$$

[Note that Eq. (1) could have been obtained from Eq. (18.44)].

But $\Sigma\underline{M}_A = -\ell\underline{k}\times(-mg\underline{K}) = -mg\ell\sin\beta\,\underline{j}$

Substituting into Eq. (1):

$$[I(\dot\psi+\dot\phi\cos\beta) - I'\dot\phi\cos\beta]\dot\phi\sin\beta = -mg\ell\sin\beta$$
$$[I\dot\psi + (I-I')\dot\phi\cos\beta]\dot\phi = -mg\ell$$
$$\dot\psi = \frac{I'-I}{I}\dot\phi\cos\beta - \frac{mg\ell}{I\dot\phi} \quad (2)$$

Given Data: $r = 0.075\text{m}$ $\ell = 0.6\text{ m}$

$$I = \tfrac{1}{2}mr^2 = \tfrac{1}{2}m(0.075)^2 = (2.8125\times10^{-3})m$$
$$I' = \tfrac{1}{4}mr^2 + m\ell^2 = \tfrac{1}{4}m(0.075)^2 + m(0.6)^2 = (0.36141)m$$
$$\frac{I'-I}{I} = \frac{0.36141-0.00281}{2.8125\times10^{-3}} = 127.5$$

$\dot\phi = -36\text{ rpm} = -3.7699\text{ rad/s}$ $\beta = 60^\circ$

Eq. (2): $\dot\psi = 127.5(-3.7699)\cos60^\circ - \dfrac{m(9.81)(0.6)}{(2.8125\times10^{-3})m(-3.7699)}$

$\dot\psi = -240.33 + 555.13 = 314.80\text{ rad/s}$

$\dot\psi = 3010\text{ rpm}$ ◀

18.82 See solution of Prob. 18.81 for derivation of Eq. (2).

Given Data: $r = 0.075\text{m}$ $\ell = 0.6\text{ m}$, $\beta = 30^\circ$

Moments of inertia are same as for Prob. 18.82;

$I = (2.8125\times10^{-3})m$, $I' = (0.36141)m$, $\dfrac{I'-I}{I} = 127.5$

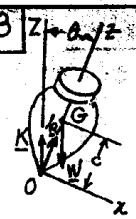
Eq. (2): $\dot\psi = \dfrac{I'-I}{I}\dot\phi\cos\beta - \dfrac{mg\ell}{I\dot\phi}$

$\dot\psi = 127.5(-3.7699)\cos30^\circ - \dfrac{m(9.81)(0.6)}{(2.8125\times10^{-3})m(-3.7699)}$

$= -416.27 + 555.13$

$= 138.86\text{ rad/s}$ $\dot\psi = 1326\text{ rpm}$ ◀

18.83



We use the principal axes $Oxyz$ (with y pointing into the paper). Following the same procedure as in Sec. 18.10, we obtain Eq. (18.44):

$$\Sigma\underline{M}_O = (I\omega_z - I'\dot\phi\cos\theta)\dot\phi\sin\theta\,\underline{j}$$

We note from diagram that

$$\Sigma\underline{M}_O = \underline{r}_G\times\underline{W} = c\underline{k}\times(-W\underline{K}) = Wc(\underline{K}\times\underline{k})$$

But $\underline{K}\times\underline{k} = \sin\theta\,\underline{j}$ (into the paper). Substituting into (18.44):

$$Wc\sin\theta\,\underline{j} = (I\omega_z - I'\dot\phi\cos\theta)\dot\phi\sin\theta\,\underline{j}$$
$$(I\omega_z - I'\dot\phi\cos\theta)\dot\phi = Wc \quad (1)$$

Recalling from page 920 that

$$\omega_z = \dot\psi + \dot\phi\cos\theta$$

we have

$$[I(\dot\psi+\dot\phi\cos\theta) - I'\dot\phi\cos\theta]\dot\phi = Wc \quad (2)$$

Rearranging terms:

$$(I'-I)\cos\theta\,\dot\phi^2 - I\dot\psi\,\dot\phi + Wc = 0 \quad (3)$$

Given Data: $c = 1.5\text{ in.} = 0.125\text{ ft}$

$$I = \frac{W}{g}k^2 = \frac{W}{g}\left(\frac{0.84}{12}\right)^2 = 0.00490\,\frac{W}{g}$$
$$I' = \frac{W}{g}k'^2 = \frac{W}{g}\left(\frac{1.80}{12}\right)^2 = 0.02250\,\frac{W}{g}$$

$\dot\psi = 1800\text{ rpm} = 188.496\text{ rad/s}$ $\theta = 30^\circ$

Substituting into Eq. (3):

$$(0.02250-0.00490)\frac{W}{g}\cos30^\circ\,\dot\phi^2 - 0.00490\,\frac{W}{g}(188.496)\dot\phi + 0.125W = 0$$
$$0.015242\,\dot\phi^2 - 0.92363\,\dot\phi + 0.125(32.2) = 0$$
$$\dot\phi^2 - 60.598\,\dot\phi + 264.07 = 0$$

$\dot\phi = 4.726\text{ rad/s}$ and $\dot\phi = 55.87\text{ rad/s}$

$\dot\phi = 45.1\text{ rpm}$ and $\dot\phi = 534\text{ rpm}$ ◀

18.84 (a) See solution of Prob. 18.83 for derivation of Eq. (1):

$$(I\omega_z - I'\dot\phi\cos\theta)\dot\phi = Wc$$ ◀ (Q.E.D.)

(b) See solution of Prob. 18.83 for derivation of Eq. (2). Rearranging terms:

$$[I\dot\psi - (I'-I)\dot\phi\cos\theta]\dot\phi = Wc \quad (4)$$

For $\dot\psi \gg \dot\phi$, we have $I\dot\psi \gg (I'-I)\dot\phi\cos\theta$ and Eq. (4) reduces to

$$I\dot\psi\dot\phi \approx Wc \quad (5)$$ ◀ (Q.E.D.)

(c) Recalling data of Prob. 18.83, and solving (5) for $\dot\phi$:

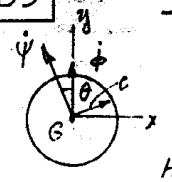
$$\dot\phi = \frac{Wc}{I\dot\psi} = \frac{W(0.125\text{ft})}{(0.00490\frac{W}{g})(188.496)} = \frac{0.125(32.2)}{0.92363} = 4.3578\text{ rad/s}$$

$\dot\phi = 4.3578\text{ rad/s} = 41.6\text{ rpm}$

Percent error when compared to actual value from Prob. 18.83:

$$= \frac{41.6-45.1}{45.1}(100)$$ Percent error $= -7.8\%$ ◀

18.85



Angular Momentum about G:

Resolving $\underline{\omega}$ along x and y axes:

$$\omega_x = -\dot\psi\sin\theta, \quad \omega_y = \dot\phi + \dot\psi\cos\theta$$

Observing that $\bar I_x = \bar I_y = \frac{2}{5}mc^2$,

$$H_x = \bar I_x\,\omega_x = -\frac{2}{5}mc^2\dot\psi\sin\theta$$

$$H_y = \bar I_y\,\omega_y = \frac{2}{5}mc^2(\dot\phi + \dot\psi\cos\theta)$$

Therefore $\underline{H}_G = -\frac{2}{5}mc^2\dot\psi\sin\theta\,\underline{i} + \frac{2}{5}mc^2(\dot\phi+\dot\psi\cos\theta)\underline{j}$

where the reference frame $Gxyz$ rotates with the angular velocity $\underline{\Omega} = \dot\phi\,\underline{j}$

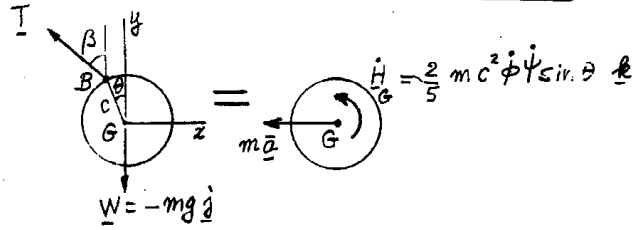
Rate of Change of $\underline{H}_G$: Noting that $(\dot{\underline{H}}_G)_{Gxyz} = 0$, we obtain from Eq. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \dot\phi\underline{j}\times\left(-\frac{2}{5}mc^2\dot\psi\sin\theta\,\underline{i} + H_y\underline{j}\right)$$

$$\dot{\underline{H}}_G = \frac{2}{5}mc^2\dot\phi\dot\psi\sin\theta\,\underline{k}$$

where unit vector $\underline{k}$ points out of the paper.

Equivalence of External and Effective Forces



$$\pm\Sigma F_x = \Sigma(F_x)_{eff}: \quad T\sin\beta = m\bar a \qquad (1)$$

$$+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}: \quad T\cos\beta - mg = 0$$

$$T\cos\beta = mg \qquad (2)$$

Dividing (1) by (2): $\tan\beta = \frac{\bar a}{g}$ $\qquad \bar a = g\tan\beta$ (3)

$$+\circlearrowleft\Sigma M_B = \Sigma(M_B)_{eff}:$$

$$-mg(c\sin\theta) = \frac{2}{5}mc^2\dot\phi\dot\psi\sin\theta - m\bar a(c\cos\theta)$$

Substituting for $\bar a$ from (3):

$$-mgc\sin\theta = \frac{2}{5}mc^2\dot\phi\dot\psi\sin\theta - mg\tan\beta(c\cos\theta)$$

Dividing by $mgc\cos\theta$:

$$-\tan\theta = \frac{2}{5}\frac{c}{g}\dot\phi\dot\psi\tan\theta - \tan\beta$$

$$\tan\theta = \frac{\tan\beta}{1+\frac{2}{5}\frac{c}{g}\dot\phi\dot\psi}$$ ◀

18.86

We use the answer to Prob. 18.85 with the given data: $c = 0.04$ m, $\beta = 30°$, $\dot\phi = 5$ rad/s, $g = 9.81$ m/s^2:

$$\tan\theta = \frac{\tan 30°}{1+\frac{2}{5}\left(\frac{0.04}{9.81}\right)(5)\dot\psi} = \frac{\tan 30°}{1+(8.1549\times10^{-3})\dot\psi}$$

(a) $\dot\psi = 0$: $\tan\theta = \tan 30°$ $\qquad \theta = 30.0°$ ◀

(b) $\dot\psi = 30$ rad/s $\quad \tan\theta = 0.46387$ $\qquad \theta = 24.9°$ ◀

(c) $\dot\psi = -30$ rad/s $\quad \tan\theta = 0.76435$ $\qquad \theta = 37.4°$ ◀

18.87

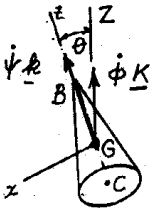
Angular velocity:

Since θ = constant:

$$\underline{\omega} = \dot\phi\,\underline{K} + \dot\psi\,\underline{k}$$

$$= -\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k} + \dot\psi\,\underline{k}$$

$$\underline{\omega} = -\dot\phi\sin\theta\,\underline{i} + (\dot\psi+\dot\phi\cos\theta)\underline{k}$$



Angular momentum about G:

$$\underline{H}_G = \bar I'\omega_x\,\underline{i} + 0 + \bar I\,\omega_z\,\underline{k}$$

$$\underline{H}_G = -\bar I'\dot\phi\sin\theta\,\underline{i} + \bar I(\dot\psi+\dot\phi\cos\theta)\underline{k} \qquad (1)$$

where the reference frame $Gxyz$ (with y pointing out of the paper) has the angular velocity

$$\underline{\Omega} = \dot\phi\underline{K} = -\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k}$$

Rate of Change of $\underline{H}_G$: Since $(\dot{\underline{H}}_G)_{Gxyz} = 0$, Eq. (18.22) yields

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G$$

$$= 0 + (-\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k})\times[-\bar I'\dot\phi\sin\theta\,\underline{i} + \bar I(\dot\psi+\dot\phi\cos\theta)\underline{k}]$$

$$= \bar I\dot\phi\sin\theta(\dot\psi+\dot\phi\cos\theta)\underline{j} - \bar I'\dot\phi^2\sin\theta\cos\theta\,\underline{j}$$

$$= [\bar I\dot\psi + \bar I\dot\phi\cos\theta - \bar I'\dot\phi\cos\theta]\dot\phi\sin\theta\,\underline{j}$$

$$\dot{\underline{H}}_G = [\bar I\dot\psi - (\bar I' - \bar I)\dot\phi\cos\theta]\dot\phi\sin\theta\,\underline{j} \qquad (2)$$

From back cover of book, for circular cone:

$$\bar I = \frac{3}{10}ma^2 = \frac{3}{10}m\left(\frac{d}{2}\right)^2 = \frac{3}{40}md^2$$

$$\bar I' = I_y - m\left(\frac{3h}{4}\right)^2 = \frac{3}{5}m\left(\frac{a^2}{4}+h^2\right) - \frac{9}{16}mh^2$$

$$= \frac{3}{20}m\left(\frac{d}{2}\right)^2 + \frac{3}{80}mh^2 = \frac{3}{80}m(d^2+h^2)$$

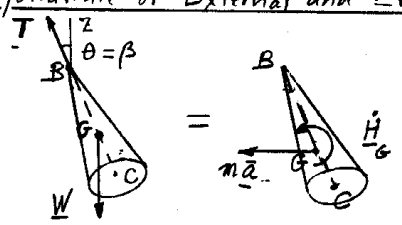
$$\bar I' - \bar I = \frac{3}{80}m(d^2+h^2) - \frac{3}{40}md^2 = \frac{3}{80}m(h^2-d^2)$$

Substituting into Eq. (2):

$$\dot{\underline{H}}_G = \frac{3}{80}m\left[2d^2\dot\psi - (h^2-d^2)\dot\phi\cos\theta\right]\dot\phi\sin\theta\,\underline{j} \qquad (3)$$

where the unit vector $\underline{j}$ points out of the paper.

Equivalence of External and Effective Forces



Equating moments about G and noting that force $\underline{T}$ is directed along CB:

$$0 = \dot{\underline{H}}_G \quad \text{or, recalling Eq. (3) with } \theta = \beta,$$

$$2d^2\dot\psi - (h^2-d^2)\dot\phi\cos\beta = 0, \qquad \cos\beta = \frac{2d^2}{h^2-d^2}\frac{\dot\psi}{\dot\phi}$$ ◀

18.88 **Rate of Change of $\underline{H}_G$:** See solution of Prob. 18.87 for the derivation of Eq. (3):

$$\dot{\underline{H}}_G = \frac{3}{80} m\left[2d^2\ddot{\psi} - (l^2 - d^2)\dot{\phi}\cos\theta\right]\dot{\phi}\sin\theta\,\underline{j} \qquad (3)$$

where the unit vector $\underline{j}$ points out of the paper

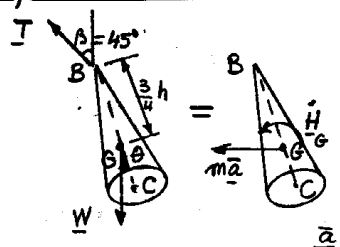
Given Data: $h = 12\text{ in.} = 1\text{ ft}$, $d = 6\text{ in.} = 0.5\text{ ft}$, $\dot{\phi} = 8\text{ rad/s}$, $\beta = 45°$, $\theta = 30°$

Substitute in (3):

$$\dot{\underline{H}}_G = \frac{3}{80} m\left[2(0.5)^2\ddot{\psi} - (1^2 - 0.5^2)8\cos 30°\right]8\sin 30°\,\underline{j}$$

$$\dot{\underline{H}}_G = \frac{3}{40} m(\ddot{\psi} - 10.392)\,\underline{j} \qquad (4)$$

Equivalence of External and Effective forces:



$\pm$ Horiz. components:

$$T\sin 45° = m\bar{a} \qquad (5)$$

+↑ Vertical components:

$$T\cos 45° - W = 0$$

$$T\cos 45° = mg \qquad (6)$$

Dividing (5) by (6):

$$\frac{\bar{a}}{g} = \tan 45° = 1 \qquad \bar{a} = g \qquad (7)$$

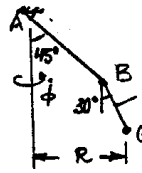
(a) Equating moments about B:

$$+\circlearrowleft \quad (W\sin\theta)\left(\tfrac{3}{4}h\right) = m\bar{a}\left(\tfrac{3}{4}h\right)\cos\theta - \dot{H}_G$$

$$mg\sin 30°(0.75) = mg(0.75)\cos 30° - \frac{3}{40}m(\ddot{\psi} - 10.392)$$

$$\ddot{\psi} = 10.392 + \frac{40}{3}(32.2)(0.75)(\cos 30° - \sin 30°) = 10.392 + 117.860$$

$$\ddot{\psi} = 128.3\text{ rad/s}$$ ◀



(b) Length AB

$\bar{a} = R\dot{\phi}^2$ Recalling (7):

$32.2\text{ ft/s}^2 = R(8\text{ rad/s})^2$, $R = 0.50313\text{ ft}$

But $R = (AB)\sin 45° + (0.75\text{ ft})\sin 30°$

$(AB)\sin 45° = 0.50313 - 0.375 = 0.12813\text{ ft}$

$AB = 0.18120\text{ ft}$ $\qquad AB = 2.17\text{ in.}$ ◀

18.89 We recall Eq. (18.44), page 922:

$$\Sigma\underline{M}_O = (I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi}\sin\theta\,\underline{j}$$

where the unit vector points *into* the paper. Comparing Fig. 18.18 and Fig. P18.89, we note that

$$\Sigma\underline{M}_O = -\underline{M}$$

$$I = I' = \frac{2}{5}mR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2 = \frac{8}{15}\pi\rho R^5$$

$$= \frac{8}{15}\pi\frac{(5.51)(62.4)}{32.2}(3960\times 5280)^5 = 71.5\times 10^{36}$$

$$\omega_z = \frac{2\pi\text{ rad}}{(23.93\text{ h})(3600\text{ s/h})} = 72.9\times 10^{-6}\text{ rad/s}$$

$$\dot{\phi} = \frac{2\pi\text{ rad}}{(25\,800\text{ yr})(365.24\text{ day/yr})(24\text{ h/day})(3600\text{ s/h})} = 7.72\times 10^{-12}\text{ rad/s}$$

$\theta = 23.45°$, $\sin\theta = 0.39795$, $\cos\theta = 0.91741$

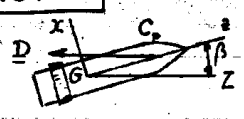
Noting that the second term in the parentheses in Eq. (18.44) may be neglected, we have

$$-\underline{M} = I\omega_z\dot{\phi}\sin\theta\,\underline{j}$$

$$= (71.5\times 10^{36})(72.9\times 10^{-6})(7.72\times 10^{-12})(0.39795)\,\underline{j}$$

Magnitude: $M = 16.01\times 10^{21}\text{ lb·ft}$ ◀

18.90



We use principal axes xyz which precess with the projectile, but do not spin. Comparing sketch with Fig. P18.83 and P18.84 on page 926, we note that the motion is the same as that of the top, with drag $\underline{D}$ replacing weight $\underline{W}$ (both have a fixed direction parallel to the precession axis).

(a) In first approximation, we use the relation derived in Prob. 18.84b:

$$I\dot{\psi}\dot{\phi} \approx Dc \qquad \dot{\phi} \approx \frac{Dc}{I\dot{\psi}}$$

Given: $I = m\bar{k}^2 = (18\text{ kg})(0.040\text{ m})^2 = 0.0288\text{ kg·m}^2$

$D = 150\text{ N}$, $c = 0.120\text{ m}$

$\dot{\psi} = 5000\text{ rpm} = 523.6\text{ rad/s}^*$

$$\dot{\phi} \approx \frac{(150\text{ N})(0.120\text{ m})}{(0.0288\text{ kg·m}^2)(523.6\text{ rad/s})} = 1.1937\text{ rad/s}$$

$$\dot{\phi} \approx 11.40\text{ rpm}$$ ◀

*Note: Rad/s units must be used in this computation.

(b) We now use the relation derived in Prob. 18.84a:

$$(I\omega_z - I'\dot{\phi}\cos\beta)\dot{\phi} = Dc$$

Substituting $\omega_z = \dot{\psi} + \dot{\phi}\cos\beta$ (see page 922):

$$[I(\dot{\psi} + \dot{\phi}\cos\beta) - I'\dot{\phi}\cos\beta]\dot{\phi} = Dc$$

$$(I' - I)\cos\beta\,\dot{\phi}^2 - I\dot{\psi}\dot{\phi} + Dc = 0 \qquad (1)$$

Given: $I = 0.0288$, $I' = (18\text{ kg})(0.16\text{ m})^2 = 0.4608$, $I' - I = 0.432\text{ kg·m}^2$

$\beta = 4°$, $\cos\beta = 0.99756$, $D = 150\text{ N}$, $c = 0.120\text{ m}$

$\dot{\psi} = 5000\text{ rpm} = 523.6\text{ rad/s}$

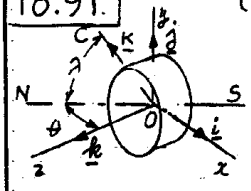
Substituting into Eq. (1):

$$0.43095\dot{\phi}^2 - 15.08\dot{\phi} + 18 = 0$$

Solving: $\dot{\phi} = 1.2374\text{ rad/s}$ and $\dot{\phi} = 33.755\text{ rad/s}$

$\dot{\phi} = 11.82\text{ rpm}$ and $\dot{\phi} = 322\text{ rpm}$ ◀

18.91



(a) We choose a frame of reference $Oxyz$ attached to the gimbal, with y along the vertical OA and z along the axis of the rotor. The angular velocity of this frame with respect to a Newtonian frame of reference is

$$\underline{\Omega} = \omega_e\underline{K} + \dot{\theta}\,\underline{j}, \quad \text{where } \underline{K} \text{ is // earth axis}$$

Resolving $\underline{K}$ into components along x, y, z:

$$\underline{K} = -\cos\lambda\sin\theta\,\underline{i} + \sin\lambda\,\underline{j} + \cos\lambda\cos\theta\,\underline{k}$$

Substituting:

$$\underline{\Omega} = -\omega_e\cos\lambda\sin\theta\,\underline{i} + (\dot{\theta} + \omega_e\sin\lambda)\underline{j} + \omega_e\cos\lambda\cos\theta\,\underline{k} \qquad (1)$$

The angular velocity of the rotor is obtained by adding the spin $\dot{\psi}\underline{k}$ to $\underline{\Omega}$. Setting $\dot{\psi} + \omega_e\cos\lambda\cos\theta = \omega_z$, we have:

(CONTINUED)

18.91 CONTINUED

$$\underline{\omega} = -\omega_e \cos\lambda \sin\theta\, \underline{i} + (\dot\theta + \omega_e \sin\lambda)\underline{j} + \omega_z \underline{k} \qquad (2)$$

The angular momentum of the rotor is

$$\underline{H}_O = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

where $I_x = I_y = I'$ and $I_z = I$. Using (2), we have

$$\underline{H}_O = -I'\omega_e \cos\lambda \sin\theta\, \underline{i} + I'(\dot\theta + \omega_e \sin\lambda)\underline{j} + I\omega_z \underline{k} \qquad (3)$$

We now write

$$\Sigma \underline{M}_O = \left(\dot{\underline{H}}_O\right)_{Oxyz} + \underline{\Omega} \times \underline{H}_O \qquad (4)$$

Substituting from (1) and (3) into (4):

$$\Sigma \underline{M}_O = -I'\omega_e \cos\lambda \cos\theta\, \dot\theta\, \underline{i} + I'\ddot\theta\, \underline{j} + I\dot\omega_z \underline{k}$$
$$+ \left[-\omega_e \cos\lambda \sin\theta\, \underline{i} + (\dot\theta + \omega_e \sin\lambda)\underline{j} + \omega_z \underline{k}\right]$$
$$\times \left[-I'\omega_e \cos\lambda \sin\theta\, \underline{i} + I'(\dot\theta + \omega_e \sin\lambda)\underline{j} + I\omega_z \underline{k}\right]$$

$$\Sigma \underline{M}_O = \left[-I'\omega_e \cos\lambda \cos\theta\, \dot\theta + (\dot\theta + \omega_e \sin\lambda)(I\omega_z - I'\omega_e \cos\lambda \cos\theta)\right]\underline{i}$$
$$+ \left[I'\ddot\theta + I\omega_z \omega_e \cos\lambda \sin\theta - I'\omega_e^2 \cos^2\lambda \sin\theta \cos\theta\right]\underline{j}$$
$$+ \left[I\dot\omega_z - \omega_e \cos\lambda \sin\theta\, I'(\dot\theta + \omega_e \sin\lambda) + (\dot\theta + \omega_e \sin\lambda) I'\omega_e \cos\lambda \sin\theta\right]\underline{k}$$

We observe that the rotor is free to spin about the z axis and free to rotate about the y axis. (The only constraint imposed is a couple about the x axis.) It follows that the y and z components of $\Sigma \underline{M}_O$ must be zero. Thus:

$$I'\ddot\theta + I\omega_z \omega_e \cos\lambda \sin\theta - I'\omega_e^2 \cos^2\lambda \sin\theta \cos\theta = 0$$

$$I\dot\omega_z = 0 \qquad (Q.E.D.)$$

(b) The second equation yields $\omega_z = \text{constant}$. We rewrite the first equation as follows:

$$I'\ddot\theta + (I\omega_z - I'\omega_e \cos\lambda \cos\theta)\omega_e \cos\lambda \sin\theta = 0$$

It is evident that $\omega_z \gg\gg \omega_e$ and we may thus neglect the second term in the parentheses:

$$I'\ddot\theta + I\omega_z \omega_e \cos\lambda \sin\theta = 0$$

$$\ddot\theta + \frac{I\omega_z \omega_e \cos\lambda}{I'} \sin\theta = 0$$

which shows that the rotor will oscillate as a simple pendulum about the N-S direction (which corresponds to $\theta = 0$) and thus make it possible to determine this direction. For small values of θ,

$$\ddot\theta + \frac{I\omega_z \omega_e \cos\lambda}{I'}\theta = 0 \qquad (Q.E.D.)$$

which is simple harmonic motion. We note that the period of oscillation is

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I'}{I\omega_z \omega_e \cos\lambda}}$$

and becomes infinite for $\lambda = \pm 90°$. Thus, the gyrocompass may not be used to determine the N-S direction in the polar regions.

18.92

From Fig. 18.21, page 923:

$$-\omega_x = \dot\phi \sin\theta$$

But, from the first of Eqs. (18.48):

$$\omega_x = -\frac{H_G \sin\theta}{I'}$$

Eliminating ω_x between the above two equations:

$$\dot\phi \sin\theta = \frac{H_G \sin\theta}{I'}$$

$$\dot\phi = \frac{H_G}{I'} \qquad (1) \quad (Q.E.D.)$$

From Fig. 18.21, we also have

$$\dot\psi = \omega_z - \dot\phi \cos\theta \qquad (2)$$

But, from the last of Eqs. (18.48):

$$\omega_z = \frac{H_G \cos\theta}{I} \qquad (3)$$

and from Eq. (1) that we have just derived:

$$\dot\phi \cos\theta = \frac{H_G \cos\theta}{I'} \qquad (4)$$

Substituting from (3) and (4) into (2):

$$\dot\psi = \frac{H_G \cos\theta}{I} - \frac{H_G \cos\theta}{I'}$$

$$\dot\psi = \frac{H_G \cos\theta\,(I' - I)}{II'} \qquad (Q.E.D.)$$

18.93

(a) See Prob. 18.92 for the derivation of Eq (1):

$$\dot\phi = \frac{H_G}{I'} \qquad (1)$$

From the last of Eqs. (18.48), we have

$$H_G = \frac{I\omega_z}{\cos\theta}$$

Substituting into (1):

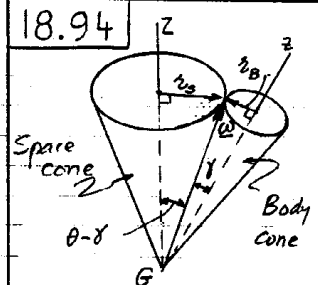
$$\dot\phi = \frac{I\omega_z}{I'\cos\theta} \qquad (Q.E.D.)$$

(b) From the relation we have just obtained,

$$I\omega_z - I'\dot\phi\cos\theta = 0$$

which means that, for an axisymmetrical body under no force, the right-hand member of Eq. (18.44) is zero. But, since we have no force, we also have $\Sigma \underline{M}_O = 0$. Thus, the left-hand member of Eq. (18.44) is also zero and the equation is satisfied. (Q.E.D.)

18.94



Assuming direct precession ($I' > I$), we consider the space and body cones. The plane ZGz rotates about the Z axis at the rate $\dot\phi$; so will the vector $\underline{\omega}$ contained in that plane. Thus, the tip of $\underline{\omega}$ will describe an arc of circle of length $r_s \dot\phi\, \Delta t$ in the time Δt. But, according to the definition of n, the vector $\underline{\omega}$ is observed to rotate at the rate n with respect to the body. Thus, the tip of $\underline{\omega}$ will describe an arc of circle of length $r_B n\, \Delta t$ in the time Δt.

(CONTINUED)

18.94 CONTINUED

Since the body cone rolls on the space cone:

$$r_s \dot{\phi}\,\Delta t = r_B n\,\Delta t \qquad (1)$$

But, from the figure: $r_s = \omega \sin(\theta-\gamma)$ and $r_B = \omega \sin\gamma$. Substituting into (1):

$$\dot{\phi}\sin(\theta-\gamma) = n\sin\gamma$$

$$n = \dot{\phi}\,\frac{\sin(\theta-\gamma)}{\sin\gamma} \qquad (2)$$

We recall the relation derived in Prob. 18.94:

$$\dot{\phi} = \frac{I\omega_z}{I'\cos\theta}$$

Substituting into (2), and expanding $\sin(\theta-\gamma)$:

$$n = \frac{I\omega_z}{I'\cos\theta}\,\frac{\sin\theta\cos\gamma - \sin\gamma\cos\theta}{\sin\gamma}$$

$$= \frac{I\omega_z}{I'}\left(\frac{\tan\theta}{\tan\gamma} - 1\right)$$

Recalling from Eq. (18.49) that $\frac{\tan\theta}{\tan\gamma} = \frac{I'}{I}$, we have

$$n = \frac{I'-I}{I'}\,\omega_z \qquad (Q.E.D.)$$

Note: For $I > I'$ (retrograde precession), we would find

$$n = \frac{I-I'}{I'}\,\omega_z$$

18.95

(a) We recall the relation derived in Prob. 18.93:

$$\dot{\phi} = \frac{I\omega_z}{I'\cos\theta}$$

or $$I'\dot{\phi}\cos\theta = I\omega_z$$

Substituting $\omega_z = \dot{\psi} + \dot{\phi}\cos\theta$, we write

$$I'\dot{\phi}\cos\theta = I(\dot{\psi} + \dot{\phi}\cos\theta)$$

Solving for $\dot{\phi}$:

$$\dot{\phi} = -\frac{I}{I-I'}\,\frac{\dot{\psi}}{\cos\theta}$$

or $$\dot{\phi} = -\frac{\sec\theta}{1-(I'/I)}\,\dot{\psi}$$

For retrograde precession, we must have $I' < I$. On the other hand, the smallest possible value of I'/I is $1/2$ (it corresponds to the case when the body is in the shape of a flat disk or annulus). Thus

$$\frac{1}{2} \le \frac{I'}{I} < 1, \qquad \frac{1}{2} \ge 1 - \frac{I'}{I} \ge 0$$

$$\frac{1}{1-(I'/I)} \ge 2$$

Since, on the other hand, $\sec\theta \ge 1$, we have for retrograde precession

$$|\dot{\phi}| \ge 2|\dot{\psi}| \qquad (Q.E.D.)$$

(b) We recall Eq. (18.49):

$$\tan\gamma = \frac{I}{I'}\tan\theta$$

Since $\frac{I'}{I} \ge \frac{1}{2}$, we have $\frac{I}{I'} \le 2$ and

$$\tan\gamma \le 2\tan\theta \qquad (1)$$

(CONTINUED)

18.95 CONTINUED

We write the trigonometric identity

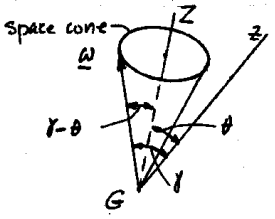
$$\tan(\gamma-\theta) = \frac{\tan\gamma - \tan\theta}{1+\tan\gamma\tan\theta}$$

Since $\gamma < \frac{\pi}{2}$ and $\theta < \frac{\pi}{2}$, we have $1+\tan\gamma\tan\theta \ge 1$ and, from (1),

$$\tan\gamma - \tan\theta \le \tan\theta$$

Thus:

$$\tan(\gamma-\theta) \le \frac{\tan\theta}{1}$$

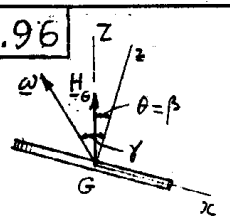


$$\tan(\gamma-\theta) \le \tan\theta$$

$$\gamma - \theta \le \theta$$

The z axis cannot lie within the space cone (see diagram). (Q.E.D.)

18.96



(a) We conclude from Prob. 18.93 that the normal z to the coin forms a constant angle $\theta = \beta$ with the vertical Z. The angular momentum $\underline{H}_G$ is along the fixed Z direction. We have:

$$\omega_x = \omega\sin\gamma, \quad \omega_y = 0, \quad \omega_z = \omega\cos\gamma$$

$$H_x = I'\omega\sin\gamma, \quad H_y = 0, \quad H_z = I\omega\cos\gamma$$

$$\tan\beta = \tan\theta = \frac{H_x}{H_z} = \frac{I'}{I}\tan\gamma$$

and, thus, $$\tan\gamma = \frac{I}{I'}\tan\beta$$ (this is Eq. 18.49)

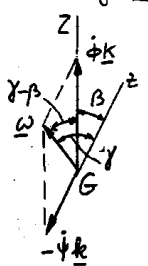
Since $I = \frac{1}{2}mr^2$ and $I' = \frac{1}{4}mr^2$:

$$\tan\gamma = 2\tan\beta \qquad (1)$$

$$\tan(\gamma-\beta) = \frac{\tan\gamma - \tan\beta}{1+\tan\gamma\tan\beta} = \frac{2\tan\beta - \tan\beta}{1+2\tan^2\beta}$$

$$\tan(\gamma-\beta) = \frac{\tan\beta}{1+2\tan^2\beta}$$ ◀

(b) Resolving $\underline{\omega}$ into components along Gz and GZ



Law of sines:

$$\frac{\dot{\phi}}{\sin\gamma} = \frac{-\dot{\psi}}{\sin(\gamma-\beta)}$$

$$\dot{\phi} = -\frac{\dot{\psi}\sin\gamma}{\sin(\gamma-\beta)}$$

$$= -\dot{\psi}\,\frac{\sin\gamma}{\sin\gamma\cos\beta - \sin\beta\cos\gamma}$$

$$= -\dot{\psi}\,\frac{1}{\sin\beta(\cot\beta - \cot\gamma)}$$

But, from (1): $\cot\gamma = \frac{1}{2}\cot\beta$

Thus: $$\dot{\phi} = -\dot{\psi}\,\frac{2}{\sin\beta\cot\beta}$$

$$\dot{\phi} = -2\dot{\psi}\sec\beta \qquad (2)$$ ◀

(CONTINUED)

18.96 CONTINUED (c) For $\beta = 20°$:

From Eq. (1): $\tan\gamma = 2\tan\beta = 2\tan 20° = 0.72794$

$\gamma = 36.05°$

$\gamma - \beta = 36.05° - 20°$ $\qquad \gamma - \beta = 16.05°$ ◀

From Eq. (2): $\dot\phi = -2\dot\psi \sec\beta = -2\dot\psi \sec 20°$

$\dot\phi = -2.13\,\dot\psi$ ◀

Note: The minus sign indicates that the precession is retrograde.

18.97 See solution of Prob. 18.94. Since $I > I'$, we write

$$n = \frac{I - I'}{I'}\omega_z$$

Period of precession $= \dfrac{2\pi}{n} = \dfrac{I'}{I - I'}\dfrac{2\pi}{\omega_z}$

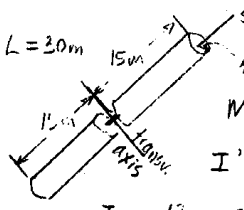
But $\dfrac{2\pi}{\omega_z}$ = period of spin = 1 day

and $\dfrac{I'}{I - I'} = \dfrac{0.9967\,I}{(1 - 0.9967)I} = \dfrac{0.9967}{0.0033} = 302$

Thus: Period of precession = 302 days ◀

18.98 We recall Eq. (18.49), page 923:

$$\tan\gamma = \frac{I}{I'}\tan\theta$$



We have $I = \frac{1}{2}ma^2$

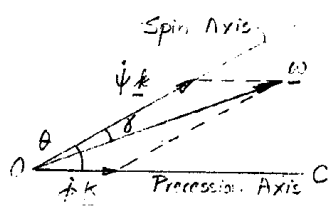
$I' = \frac{1}{12}m(3a^2 + L^2)$

$$\frac{I}{I'} = \frac{12}{2}\frac{a^2}{3a^2 + L^2} = \frac{6a^2}{3a^2 + L^2} = \frac{6(3)^2}{2(3)^2 + (30)^2}$$

$$= \frac{54}{927} = \frac{6}{103}$$

Angle between spin and precession axes $= \theta = 30°$

Thus: $\tan\gamma = \dfrac{6}{103}\tan 30° = 0.03363$ $\qquad \gamma = 1.926°$



We resolve $\vec\omega$ into components along the precession and spin axes.

$\theta - \gamma = 30° - 1.93° = 28.1°$

Law of sines:

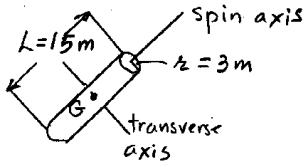
$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\theta - \gamma)}\ ; \qquad \frac{\omega}{\sin 30°} = \frac{1\ \text{rev/h}}{\sin 1.926°} = \frac{\dot\psi}{\sin 28.1°}$$

$\omega = 14.019$ rev/h $\qquad$ Rate of spin $= \dot\psi = 14.01$ rev/h ◀

18.99 We recall Eq. (18.49), page 923:

$$\tan\gamma = \frac{I}{I'}\tan\theta$$

where I and I' are the axial and transverse centroidal moments of inertia of portion B.



As in Prob. 18.98, we have

$$I = \frac{1}{2}ma^2 \qquad I' = \frac{1}{12}m(3a^2 + L^2)$$

But now, $a = 3$ m and $L = 15$ m. Therefore

$$\frac{I}{I'} = \frac{12}{2}\frac{a^2}{3a^2 + L^2} = \frac{6a^2}{3a^2 + L^2} = \frac{6(3)^2}{3(3)^2 + (15)^2} = \frac{54}{252} = \frac{3}{14}$$

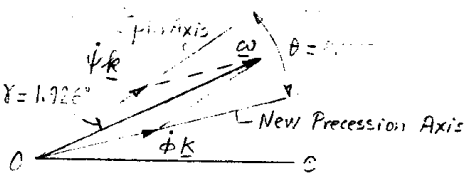
Thus: $\tan\gamma = \dfrac{3}{14}\tan\theta$, or $\tan\theta = \dfrac{14}{3}\tan\gamma$

At instant when connection is severed, portion B has angular velocity $\vec\omega$ determined in Prob. 18.98 which forms with spin axis the angle

$\gamma = 1.926°$

The new axis of precession will form with spin axis an angle θ defined by

$\tan\theta = \frac{14}{3}\tan 1.926° = 0.1569$ $\qquad \theta = 8.92°$



We resolve $\vec\omega$ into components along the spin axis and the new precession axis. We know from Solution to Prob. 18.98 that

$\omega = 14.88$ rev/h

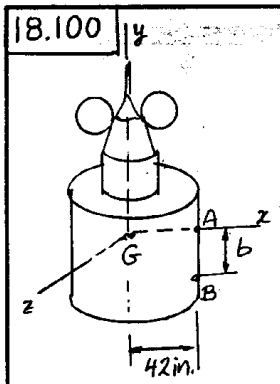
Law of sines:

$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\theta - \gamma)}$$

$$\frac{14.88\ \text{rev/h}}{\sin 8.92°} = \frac{\dot\phi}{\sin 1.926°} = \frac{\dot\psi}{\sin 7.0°}$$

$\dot\phi = 3.23$ rev/h ; $\qquad \dot\psi = 11.70$ rev/h ◀

18.100



Moments of inertia of satellite:

$$\bar{I}_x = m\bar{k}_x^2 = \frac{80.0\text{lb}}{g}\left(\frac{28.8}{12}\text{ ft}\right)^2 = \frac{460.8}{g}$$

$$\bar{I}_y = m\bar{k}_y^2 = \frac{80.0\text{lb}}{g}\left(\frac{32.4}{12}\text{ ft}\right)^2 = \frac{583.2}{g}$$

$$\bar{I}_z = \bar{I}_x = \frac{460.8}{g}$$

Angular velocity of satellite before impact:

$$\underline{\omega}_0 = (1.5\text{ rad/s})\underline{j}$$

Angular momentum of satellite before impact:

$$(\underline{H}_G)_0 = \bar{I}_y\omega_0\underline{j} = \frac{583.2}{g}(1.5\text{ rad/s})\underline{j}$$

$$(\underline{H}_G)_0 = \frac{874.8}{g}\underline{j} \qquad (1)$$

Angular momentum of meteorite about G:

$$\underline{r}_B \times m_0\underline{v}_0 = m_0(\underline{r}_B \times \underline{v}_0) = \frac{6/16}{g}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{42}{12} & -\frac{20}{12} & 0 \\ -1600 & 1300 & 4000 \end{vmatrix}$$

$$\underline{r}_B \times m_0\underline{v}_0 = \frac{1}{g}(-2500\underline{i} - 5250\underline{j} + 706.25\underline{k}) \qquad (2)$$

Principle of impulse and momentum for satellite-meteorite system:

Since there is no external impulse, the angular momentum of the satellite after impact is

$$\underline{H}_G = (\underline{H}_G)_0 + \underline{r}_B \times m_0\underline{v}_0 = \frac{1}{g}[-2500\underline{i} + (8748-5250)\underline{j} + 706.25\underline{k}]$$

$$\underline{H}_G = \frac{1}{g}(-2500\underline{i} + 3498\underline{j} + 706.25\underline{k}) \qquad (3)$$

(a) Precession axis.

Since $\underline{H}_G$ is directed along the precession axis, the unit vector $\underline{K}$ along that axis is

$$\underline{K} = \frac{\underline{H}_G}{H_G} = \frac{-2500\underline{i} + 3498\underline{j} + 706.25\underline{k}}{4357.15}$$

$$\underline{K} = -0.57377\underline{i} + 0.80282\underline{j} + 0.16209\underline{k}$$

The angles that the precession axis forms with x, y, z are therefore

$$\theta_x = 125.0°,\ \theta_y = 36.6°,\ \theta_z = 80.7° \blacktriangleleft$$

(b) Rates of precession and spin.

We recall that $\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$

$$\underline{H}_G = \frac{1}{g}(460.8\omega_x\underline{i} + 583.2\omega_y\underline{j} + 460.8\omega_z\underline{k}) \qquad (4)$$

(CONTINUED)

18.100 CONTINUED

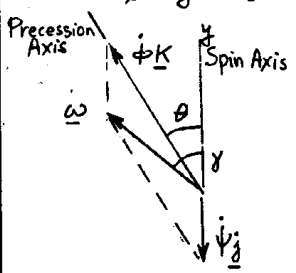
Equating the coefficients of the unit vectors in (3) and (4):

$460.8\,\omega_x = -2500$ $\quad\omega_x = -0.54253$ rad/s

$583.2\,\omega_y = 3498$ $\quad\omega_y = 0.59979$ rad/s

$460.8\,\omega_z = 706.25$ $\quad\omega_z = 0.15327$ rad/s

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} = 0.82316\text{ rad/s}$$



Since the spin axis is along the y axis, we have

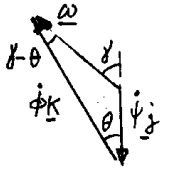
$$\cos\gamma = \frac{\omega_y}{\omega} = \frac{0.59979}{0.82316}$$

$$\gamma = 43.227°$$

On the other hand,

$$\theta = \theta_y = 36.6°$$

$$\gamma - \theta = 43.227° - 36.6° = 6.627°$$



Law of sines:

$$\frac{\omega}{\sin\theta} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)}$$

$$\frac{0.82316}{\sin 36.6°} = \frac{\dot{\phi}}{\sin 43.227°} = \frac{\dot{\psi}}{\sin 6.627°}$$

Rate of precession: $\dot{\phi} = 0.946$ rad/s ◀

Rate of spin: $\dot{\psi} = 0.1593$ rad/s ◀

Since $\gamma > \theta$, precession is retrograde ◀

18.101 Impulse-Momentum Principle for Satellite and Meteorite.

See solution of Prob. 18.100 for computation of moments of inertia:

$$\bar{I}_x = \bar{I}_z = \frac{460.8}{g} \qquad \bar{I}_y = \frac{583.2}{g}$$

and for angular momentum of satellite before impact:

$$(\underline{H}_G)_0 = \frac{874.8}{g}\underline{j} \qquad (1)$$

The angular momentum of the meteorite about G is

$$\underline{r}_A \times m_0\underline{v}_0 = \left(\frac{42}{12}\text{ft}\right)\underline{i} \times \frac{6/16}{g}(-1600\underline{i} + 1300\underline{j} + 4000\underline{k})$$

$$= \frac{1}{g}(-5250\underline{j} + 1706.25\underline{k}) \qquad (2)$$

Since there is no external impulse, the angular momentum of the satellite after impact is

$$\underline{H}_G = (\underline{H}_G)_0 + \underline{r}_A \times m_0\underline{v}_0 = \frac{1}{g}[(8748-5250)\underline{j} + 1706.25\underline{k}]$$

$$\underline{H}_G = \frac{1}{g}(3498\underline{j} + 1706.25\underline{k}) \qquad (3)$$

Since $\underline{H}_G$ is directed along the precession axis, the unit vector $\underline{K}$ along that axis is

$$\underline{K} = \frac{\underline{H}_G}{H_G} = \frac{3498\underline{j} + 1706.25\underline{k}}{3891.95} = 0.89878\underline{j} + 0.43840\underline{k}$$

The angles that the precession axis forms with x, y, z are therefore

$$\theta_x = 90°,\ \theta_y = 26.002°,\ \theta_z = 63.998°$$

(CONTINUED)

$$\theta_x = 90°,\ \theta_y = 26.0°,\ \theta_z = 64.0° \blacktriangleleft$$

18.101 CONTINUED We recall from preceding page:

$$\underline{H}_G = \frac{1}{g}(3498\,\underline{j} + 1706.25\,\underline{k}) \quad (3)$$

We recall that

$$\underline{H}_G = \bar{I}_x\omega_x\,\underline{i} + \bar{I}_y\omega_y\,\underline{j} + \bar{I}_z\omega_z\,\underline{k}$$

or, substituting the values of the moments of inertia,

$$\underline{H}_G = \frac{1}{g}(4608\omega_x\,\underline{i} + 5832\omega_y\,\underline{j} + 4608\omega_z\,\underline{k}) \quad (4)$$

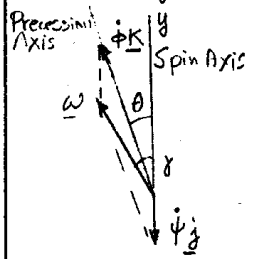
Equating the coefficients of the unit vectors in (3) and (4):

$4608\,\omega_x = 0$ $\quad\omega_x = 0$

$5832\,\omega_y = 3498$ $\quad\omega_y = 0.59979$ rad/s

$4608\,\omega_z = 1706.25$ $\quad\omega_z = 0.37028$ rad/s

$$\omega = \omega_x^2 + \omega_y^2 + \omega_z^2 = 0.70488 \text{ rad/s}$$



Since the spin axis is along the y axis, we have

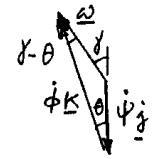
$$\cos\gamma = \frac{\omega_y}{\omega} = \frac{0.59979}{0.70488}$$

$$\gamma = 31.689°$$

On the other hand,

$$\theta = \theta_y = 26.002°$$

$$\gamma - \theta = 31.689° - 26.002° = 5.687°$$



Law of sines:

$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\gamma-\theta)}$$

$$\frac{0.70488}{\sin 26.002°} = \frac{\dot\phi}{\sin 31.689°} = \frac{\dot\psi}{\sin 5.687°}$$

Rate of precession: $\dot\phi = 0.845$ rad/s ◀

Rate of spin: $\dot\psi = 0.1593$ rad/s ◀

Since $\gamma > \theta$, precession is retrograde ◀

18.102 (a) From Sample Prob. 18.6: $I = \frac{1}{2}ma^2$, $I' = \frac{5}{4}ma^2$

After impact:

$$\underline{H}_G = \underline{r}\times m_0\underline{v}_0 + \bar{I}\,\underline{\omega}_0$$

$$= (-a\,\underline{j} - a\,\underline{k})\times m_0 v_0\,\underline{i} + I\omega_0\,\underline{k}$$

$$\underline{H}_G = -m_0 v_0 a\,\underline{j} + (I\omega_0 + m_0 v_0 a)\,\underline{k} \quad (1)$$

But $$\underline{H}_G = I'\omega_x\,\underline{i} + I'\omega_y\,\underline{j} + I\omega_z\,\underline{k} \quad (2)$$

Equating the coefficients of the unit vectors in (1) and (2):

$$\omega_x = 0,\quad \omega_y = -\frac{m_0 v_0 a}{I'} = -\frac{4}{5}\frac{m_0 v_0}{ma}$$

$$\omega_z = \omega_0 + \frac{m_0 v_0 a}{I} = \omega_0 + 2\frac{m_0 v_0}{ma}$$

Given Data: $\omega_0 = 60$ rpm $= 6.283$ rad/s, $\frac{m_0}{m} = 0.001$

$a = 0.8$ m $\quad v_0 = 2000$ m/s

We find

$$\omega_x = 0,\quad \omega_y = -\frac{4}{5}(0.001)\frac{2000}{0.8} = -2 \text{ rad/s}$$

$$\omega_z = 6.283 + 2(0.001)\frac{2000}{0.8} = 11.283 \text{ rad/s}$$

$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 11.459 \text{ rad/s},\quad \omega = 109.4 \text{ rpm}$$ ◀

$$\cos\gamma_x = 0,\quad \cos\gamma_y = \omega_y/\omega = -0.17454,\quad \gamma_y = 100.05°$$

$$\cos\gamma_z = \omega_z/\omega = 0.98464,\quad \gamma_z = 10.05°$$

(CONTINUED) $\gamma_x = 90°,\ \gamma_y = 100.05°,\ \gamma_z = 10.05°$ ◀

18.102 CONTINUED (b) Precession Axis

Since it is directed along $\underline{H}_G$, we determine from (1)

$H_x = 0$, $H_y = -m_0 v_0 a = -\frac{m}{1000}(2000)(0.8) = -(1.6)$ m

$$H_z = I\omega_0 + m_0 v_0 a = \frac{1}{2}ma^2\omega_0 + m_0 v_0 a$$

$$= \frac{1}{2}m(0.8)^2(6.283) + \frac{m}{1000}(2000)(0.8) = (3.6106)\,m$$

$$H = \sqrt{H_y^2 + H_z^2} = (3.9492)\,m$$

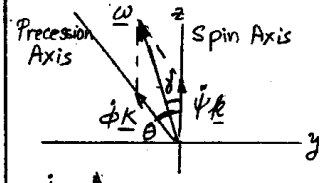
$\cos\theta_x = 0$, $\cos\theta_y = H_y/H = -0.40515$, $\theta_y = 113.90°$

$\cos\theta_z = H_z/H = 0.91426$, $\theta_z = 23.90°$

Direction of precession axis is

$\theta_x = 90°$, $\theta_y = 113.9°$, $\theta_z = 23.9°$ ◀

(c) Rates of Precession and Spin

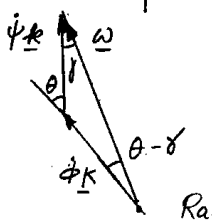


We have

$\theta = \theta_z = 23.90°$

$\gamma = \gamma_z = 10.05°$

$\theta - \gamma = 23.90° - 10.05° = 13.85°$



Law of sines:

$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\theta-\gamma)}$$

$$\frac{109.4 \text{ rpm}}{\sin 23.90°} = \frac{\dot\phi}{\sin 10.05°} = \frac{\dot\psi}{\sin 13.85°}$$

Rate of precession $= \dot\phi = 47.1$ rpm ◀

Rate of spin $= \dot\psi = 64.6$ rpm ◀

18.103 Immediately after rod breaks, angular velocity of disk A is still

$\omega = 63.0$ rpm, $\gamma = 17.7°$

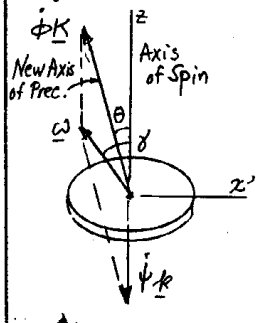
(a) Precession Axis.

For disk A: $I = \frac{1}{2}ma^2$, $I' = \frac{1}{4}ma^2$

From Eq. (18.49):

$$\tan\theta = \frac{I'}{I}\tan\gamma = \frac{1}{2}(0.3183),\quad \theta = 9.05°$$

The precession axis, therefore, forms the following angles with the coordinate axes:

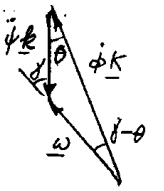


$\theta_x = 90° + 9.05° = 99.05°$ ◀

$\theta_y = 90°,\ \theta_z = \theta = 9.05°$ ◀

(b) Rates of Precession and Spin

Since $\gamma > \theta$, the precession is retrograde. Resolving $\underline{\omega}$ into precession and spin and applying the law of sines:



$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\gamma-\theta)}$$

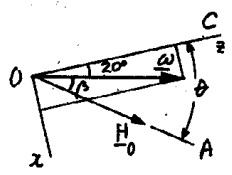
$$\frac{63.0 \text{ rpm}}{\sin 9.05°} = \frac{\dot\phi}{\sin 17.7°} = \frac{\dot\psi}{\sin 8.65°}$$

Rate of precession $= \dot\phi = 121.8$ rpm ◀

Rate of spin $= \dot\psi = 60.2$ rpm ◀

18.104

(a) Using the reference frame Oxyz with x and z as shown:



$\omega_x = \omega \sin 20°$
$\omega_y = 0$
$\omega_z = \omega \cos 20°$
$H_x = I'\omega_x = I'\omega \sin 20°$
$H_y = I'\omega_y = 0$
$H_z = I\omega_z = I\omega \cos 20°$

$$\tan\theta = \frac{H_x}{H_z} = \frac{I'\omega \sin 20°}{I\omega \cos 20°} = \frac{I'}{I}\tan 20°$$

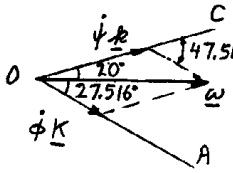
(This result could be obtained directly from Eq. (18.49) where $\gamma = 20°$.)

$$\tan\theta = \frac{1}{1/3}\tan 20° = 3\tan 20° = 1.09191$$

$\theta = 47.516°$ $\quad \beta = 47.516° - 20° = 27.516°$

$\beta = 27.5°$ ◀

(b) Using the oblique components of $\underline{\omega}$ along OA and OC:



Law of sines:

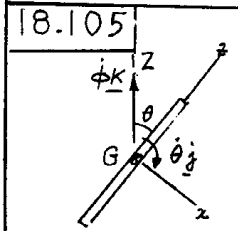
$$\frac{\omega}{\sin 47.516°} = \frac{\dot\phi}{\sin 20°} = \frac{\dot\psi}{\sin 27.516°}$$

Making $\omega = 180$ rpm, we find:

Rate of Precession $= \dot\phi = 83.5$ rpm ◀

Rate of Spin $= \dot\psi = 112.8$ rpm ◀

18.105



Angular Velocity

Using the coordinate axes x, y, z shown (with y pointing into the paper), we have

$$\underline{\omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k} \qquad (1)$$

Moments of Inertia

For slender rod of length L and mass m:

$$\bar I_x = \bar I_y = \frac{1}{12}mL^2, \quad \bar I_z = 0 \qquad (2)$$

Conservation of Energy.

Since the x, y, z axes are principal axes, we use Eq. (18.20), page 897:

$$T = \frac{1}{2}\left(\bar I_x\omega_x^2 + \bar I_y\omega_y^2 + \bar I_z\omega_z^2\right)$$

$$= \frac{1}{2}\left(\frac{1}{12}mL^2\dot\phi^2\sin^2\theta + \frac{1}{12}mL^2\dot\theta^2 + 0\right)$$

$$T = \frac{1}{24}mL^2\left(\dot\phi^2\sin^2\theta + \dot\theta^2\right) \qquad (3)$$

Using a datum through G, we have $V = 0$

$T + V = \text{const.}$: $\quad \frac{1}{24}mL^2\left(\dot\phi^2\sin^2\theta + \dot\theta^2\right) = \text{const.}$

Recalling the initial conditions $\theta = \theta_0$, $\dot\theta = 0$, $\dot\phi = \dot\phi_0$, we determine the constant and write

$$\dot\phi^2\sin^2\theta + \dot\theta^2 = \dot\phi_0^2\sin^2\theta_0 \qquad (4)$$

(CONTINUED)

18.105 CONTINUED

Conservation of angular momentum.

Since the only forces exerted on the rod are its weight and the reaction at G, we have $\Sigma\underline{M}_G = 0$. Using a fixed reference frame GXYZ, with Z directed vertically upward, we have from Eq. (18.2)

$$\dot{\underline{H}}_G = \Sigma\underline{M}_G = 0$$

or, integrating with respect to the frame GXYZ,

$$\underline{H}_G = \text{const.}$$

Considering the vertical component of $\underline{H}_G$:

$$H_Z = \text{const.}$$

But $H_Z = H_z\cos\theta - H_x\sin\theta = \bar I_z\omega_z\cos\theta - \bar I_x\omega_x\sin\theta$

$$= 0 - \frac{1}{12}mL^2\left(-\dot\phi\sin\theta\right)\sin\theta$$

Thus: $\quad H_Z = \frac{1}{12}mL^2\dot\phi\sin^2\theta = \text{const.}$

Recalling the initial conditions, we obtain

$$\dot\phi\sin^2\theta = \dot\phi_0\sin^2\theta_0 \qquad (5)$$

Solving Eq. (5) for $\dot\phi$ and substituting into (4):

$$\left(\frac{\dot\phi_0\sin^2\theta_0}{\sin^2\theta}\right)^2\sin^2\theta + \dot\theta^2 = \dot\phi_0^2\sin^2\theta_0$$

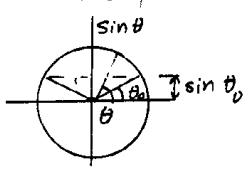
$$\dot\theta^2 = \dot\phi_0^2\sin^2\theta_0\left(1 - \frac{\sin^2\theta_0}{\sin^2\theta}\right) \qquad (6)$$

(a) Range of values of θ

Since $\dot\theta^2 \geq 0$, we must have

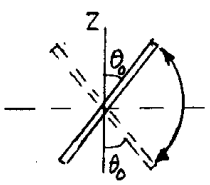
$$1 - \frac{\sin^2\theta_0}{\sin^2\theta} \geq 0 \qquad \sin^2\theta \geq \sin^2\theta_0$$

$$|\sin\theta| \geq |\sin\theta_0|$$



From trigonometric circle, we conclude that the range is

$$\theta_0 \leq \theta \leq 180° - \theta_0$$ ◀



Rod oscillates about axle and about horizontal line (dashed line)

(b) Maximum value of $\dot\theta$

Referring to Eq. (6), we note that this occurs when $\sin\theta = 1$, that is, when $\theta = 90°$. We have

$$\dot\theta_{max}^2 = \dot\phi_0^2\sin^2\theta_0\left(1 - \sin^2\theta_0\right) = \dot\phi_0^2\sin^2\theta_0\cos^2\theta_0$$

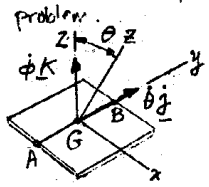
$$\dot\theta_{max} = \dot\phi_0\sin\theta_0\cos\theta_0$$ ◀

(c) Minimum value of $\dot\phi$

Referring to Eq. (5), we note that $\dot\phi$ is minimum when $\sin\theta = 1$, that is, when $\theta = 90°$. We have:

$$\dot\phi_{min} = \dot\phi_0\sin^2\theta_0$$ ◀

18.106 Since this problem is similar to Prob. 18.105, except for different values of the moments of inertia, the reader may find it useful to refer to the discussion of that problem.



Angular Velocity

Using the coordinate axes x, y, z shown (with $z \perp$ to plate) we have

$$\underline{\omega} = -\dot{\phi}\sin\theta\,\underline{i} + \dot{\theta}\,\underline{j} + \dot{\phi}\cos\theta\,\underline{k} \quad (1)$$

Moments of Inertia: For square plate of side a:

$$\bar{I}_x = \bar{I}_y = \frac{1}{12}m(a^2+a^2) = \frac{1}{6}ma^2, \qquad \bar{I}_z = \bar{I}_x + \bar{I}_y = \frac{1}{3}ma^2 \quad (2)$$

Conservation of Energy. From Eq. (18.20), page 897:

$$T = \frac{1}{2}\left(\bar{I}_x\omega_x^2 + \bar{I}_y\omega_y^2 + \bar{I}_z\omega_z^2\right)$$
$$= \frac{1}{2}\left(\frac{1}{6}ma^2\dot{\phi}^2\sin^2\theta + \frac{1}{6}ma^2\dot{\theta}^2 + \frac{1}{3}ma^2\dot{\phi}^2\cos^2\theta\right)$$
$$= \frac{1}{12}ma^2\left[\dot{\phi}^2(\sin^2\theta + 2\cos^2\theta) + \dot{\theta}^2\right]$$
$$T = \frac{1}{12}ma^2\left[\dot{\phi}^2(1+\cos^2\theta) + \dot{\theta}^2\right] \quad (3)$$

Since V = const., $T+V$ = const. yields T = const. Referring to initial conditions $\theta=\theta_0$, $\dot{\theta}=0$, $\dot{\phi}=\dot{\phi}_0$,

$$\dot{\phi}^2(1+\cos^2\theta) + \dot{\theta}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0) \quad (4)$$

Conservation of angular momentum. Since plate may move freely about G, we have $\Sigma \underline{M}_G = 0$ and $\underline{\dot{H}}_G = 0$ or $\underline{H}_G$ = const. (see solution of Prob. 18.104). We write H_Z = const.

But $H_Z = H_z\cos\theta - H_x\sin\theta = \bar{I}_z\omega_z\cos\theta - \bar{I}_x\omega_x\sin\theta$

$$= \frac{1}{3}ma^2(\dot{\phi}\cos\theta)\cos\theta - \frac{1}{6}ma^2(-\dot{\phi}\sin\theta)\sin\theta$$
$$= \frac{1}{6}ma^2\dot{\phi}(2\cos^2\theta + \sin^2\theta) = \frac{1}{6}ma^2\dot{\phi}(1+\cos^2\theta)$$

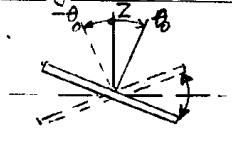
Recalling the initial conditions, H_Z = const. yields

$$\dot{\phi}(1+\cos^2\theta) = \dot{\phi}_0(1+\cos^2\theta_0) \quad (5)$$

Solving (5) for $\dot{\phi}$ and substituting in (4):

$$\frac{\dot{\phi}_0^2(1+\cos^2\theta_0)^2}{(1+\cos^2\theta)^2}(1+\cos^2\theta) + \dot{\theta}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0)$$
$$\dot{\theta}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0)\left(1 - \frac{1+\cos^2\theta_0}{1+\cos^2\theta}\right)$$
$$\dot{\theta}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0)\,\frac{\cos^2\theta - \cos^2\theta_0}{1+\cos^2\theta} \quad (6)$$

(a) Range of values of θ. Since $\dot{\theta}^2 \geq 0$, we must have



$$\cos^2\theta - \cos^2\theta_0 \geq 0$$
$$|\cos\theta| \geq |\cos\theta_0|$$
$$-\theta_0 \leq \theta \leq \theta_0$$ ◀

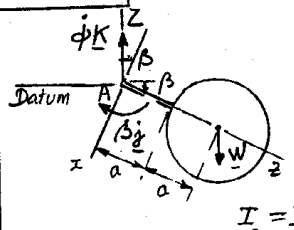
(b) Maximum value of $\dot{\theta}$. This occurs when $\theta=0$, that is, when plate is horizontal. Making $\theta=0$ in (6):

$$\dot{\theta}_{max}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0)\,\frac{1-\cos^2\theta_0}{2}, \qquad \dot{\theta}_{max} = \dot{\phi}_0\sin\theta_0\sqrt{\frac{1+\cos^2\theta_0}{2}}$$ ◀

(c) Minimum value of $\dot{\phi}$. From (5), we note that $\dot{\phi}$ is minimum when $\theta=0$:

$$\dot{\phi}_{min} = \frac{1}{2}\dot{\phi}_0(1+\cos^2\theta_0)$$ ◀

18.107



Angular Velocity

Using coordinate axes x, y, z (with y pointing into the paper).

$$\underline{\omega} = -\dot{\phi}\cos\beta\,\underline{i} + \dot{\beta}\,\underline{j} - \dot{\phi}\sin\beta\,\underline{k} \quad (1)$$

Moments of Inertia of Sphere and Rod about A:

$$I_z = \frac{2}{5}ma^2$$
$$I_x = I_y = \frac{2}{5}ma^2 + m(2a)^2 = \frac{22}{5}ma^2 \quad (2)$$

Conservation of Energy. Since axes are principal axes of inertia, we use Eq. (18.20), page 897:

$$T = \frac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right)$$
$$= \frac{1}{2}\left(\frac{22}{5}ma^2\right)(\dot{\phi}^2\cos^2\beta + \dot{\beta}^2) + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\dot{\phi}^2\sin^2\beta$$

For $\beta=0$, $\dot{\beta}=0$, $\dot{\phi}=\dot{\phi}_0$:

$$T_0 = \frac{1}{2}\left(\frac{22}{5}ma^2\right)\dot{\phi}_0^2 = 2.2ma^2\dot{\phi}_0^2 \qquad V_0 = 0$$

For $\beta=30°$, $\dot{\beta}=0$ (since β is maximum), $\dot{\phi}=\dot{\phi}_1$:

$$T_1 = \frac{1}{2}\left(\frac{22}{5}ma^2\right)\dot{\phi}_1^2\cos^2 30° + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\dot{\phi}_1^2\sin^2 30°$$
$$T_1 = 1.7ma^2\dot{\phi}_1^2 \qquad V_1 = -2mga\sin 30° = -mga$$

$T_1+V_1 = T_0+V_0$: $\quad 1.7ma^2\dot{\phi}_1^2 - mga = 2.2ma^2\dot{\phi}_0^2$ (3)

Conservation of angular momentum about Z axis

Using a fixed frame of reference $AXYZ$, we have $\Sigma \underline{M}_A = \underline{\dot{H}}_A$ or, considering only the Z components, $\Sigma M_Z = \dot{H}_Z$. But the only external forces are the weight $\underline{W}$ and the reaction at A, and both have zero moment about the Z axis. Thus $\dot{H}_Z = 0$ and H_Z = const.

But $H_Z = -H_z\sin\beta - H_x\cos\beta = -I_z\omega_z\sin\beta - I_x\omega_x\cos\beta$

$$= -\frac{2}{5}ma^2(-\dot{\phi}\sin\beta)\sin\beta - \frac{22}{5}ma^2(-\dot{\phi}\cos\beta)\cos\beta$$
$$H_Z = \frac{2}{5}ma^2\dot{\phi}(\sin^2\beta + 11\cos^2\beta)$$

For $\beta=0$, $\dot{\phi}=\dot{\phi}_0$: $\quad (H_Z)_0 = \frac{22}{5}ma^2\dot{\phi}_0$

For $\beta=30°$, $\dot{\phi}=\dot{\phi}_1$: $\quad (H_Z)_1 = \frac{2}{5}ma^2\dot{\phi}_1\left(\frac{17}{2}\right)$

$(H_Z)_1 = (H_Z)_0$: $\quad \frac{17}{5}\dot{\phi}_1 = \frac{22}{5}\dot{\phi}_0$ $\quad$ or $\quad \dot{\phi}_1 = \frac{22}{17}\dot{\phi}_0$ (4)

(a) Initial Angular Velocity

Substituting for $\dot{\phi}_1$ from (4) into (3):

$$1.7ma^2\left(\frac{22}{17}\dot{\phi}_0\right)^2 - mga = 2.2ma^2\dot{\phi}_0^2$$
$$17\left(\frac{22}{17}\right)^2\dot{\phi}_0^2 - 22\dot{\phi}_0^2 = 10\frac{g}{a}$$
$$\frac{22}{17}(22-17)\dot{\phi}_0^2 = \frac{10g}{a} \qquad \dot{\phi}_0 = \sqrt{\frac{17}{11}\frac{g}{a}}$$ ◀

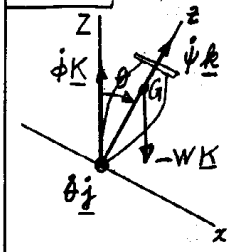
(b) Value of $\dot{\phi}$ when $\beta = 30°$

Substituting for $\dot{\phi}_0$ into (4):

$$\dot{\phi}_1 = \frac{22}{17}\sqrt{\frac{17}{11}\frac{g}{a}} = \sqrt{\frac{(22)^2}{17(11)}\frac{g}{a}} \qquad \dot{\phi}_1 = \sqrt{\frac{44}{17}\frac{g}{a}}$$ ◀

Note: The constraints imposed on the sphere prevented it from spinning about AB. Compare the results obtained with those of Probs. 18.112 & 18.113 where spin is permitted.

***18.108**



We use the reference frame xyz with z along axis of symmetry, x in plane Zoz and y pointing into the paper. This frame has same precession and nutation as top, but does not spin. Its angular velocity is

$$\underline{\Omega} = \dot\phi\,\underline{K} + \dot\theta\,\underline{j}$$

or $$\underline{\Omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k} \qquad (A)$$

The angular velocity of the top itself is obtained by adding the spin $\dot\psi\,\underline{k}$ to $\underline{\Omega}$:

$$\underline{\omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + (\dot\psi+\dot\phi\cos\theta)\,\underline{k} \qquad (B)$$

Multiplying the components of $\underline{\omega}$ respectively by I', I', and I, we obtain the angular momentum:

$$\underline{H}_O = -I'\dot\phi\sin\theta\,\underline{i} + I'\dot\theta\,\underline{j} + I(\dot\psi+\dot\phi\cos\theta)\,\underline{k} \qquad (C)$$

(a) We now recall Eq. (18.27), page 897:

$$\Sigma \underline{M}_O = \dot{\underline{H}}_O \qquad \text{(Eq. 18.27)}$$

Observing that the external forces (reaction at O and weight $-W\underline{K}$) have zero moment about the Z axis, we write $\Sigma M_Z = 0$ and, from Eq. (18.27),

$$\frac{dH_Z}{dt} = 0, \qquad H_Z = \text{constant} = \alpha$$

But $H_Z = \underline{H}_O \cdot \underline{K}$ or, substituting for $\underline{H}_O$ from (C) and noting that $\underline{i}\cdot\underline{K} = -\sin\theta$, $\underline{j}\cdot\underline{K} = 0$, $\underline{k}\cdot\underline{K} = \cos\theta$,

$$H_Z = \left[-I'\dot\phi\sin\theta\,\underline{i} + I'\dot\theta\,\underline{j} + I(\dot\psi+\dot\phi\cos\theta)\,\underline{k}\right]\cdot\underline{K} = \alpha$$

$$I'\dot\phi\sin^2\theta + I(\dot\psi+\dot\phi\cos\theta)\cos\theta = \alpha \qquad (1)$$

We now observe that the external forces have zero moment about the z axis, that is $\Sigma M_z = 0$. We cannot, however, conclude that H_z is constant, since the z axis is not part of a newtonian frame of reference. We must use Eq. (18.28):

$$\Sigma \underline{M}_O = (\dot{\underline{H}}_O)_{Oxyz} + \underline{\Omega}\times\underline{H}_O \qquad \text{(Eq. 18.28)}$$

Substituting from (A) and (C):

$$\Sigma \underline{M}_O = -I'\frac{d}{dt}(\dot\phi\sin\theta)\,\underline{i} + I'\ddot\theta\,\underline{j} + I\frac{d}{dt}(\dot\psi+\dot\phi\cos\theta)\,\underline{k}$$
$$+\left(-\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k}\right)\times\left[-I'\dot\phi\sin\theta\,\underline{i} + I'\dot\theta\,\underline{j} + I(\dot\psi+\dot\phi\cos\theta)\,\underline{k}\right]$$

Considering only the z components (i.e., the coefficients of $\underline{k}$ after the vector product has been computed), we have

$$\Sigma M_z = I\frac{d}{dt}(\dot\psi+\dot\phi\cos\theta) - I'\dot\phi\,\dot\theta\sin\theta + I'\dot\phi\,\dot\theta\sin\theta = 0$$

But the 2nd and 3d terms cancel out. (This is due to the symmetry of the top and is not a general property). Thus

$$\Sigma M_z = I\frac{d}{dt}(\dot\psi+\dot\phi\cos\theta) = 0$$

and $$I(\dot\psi+\dot\phi\cos\theta) = \text{const} = \beta \qquad (2)$$

(CONTINUED)

***18.108 CONTINUED**

(b) We recall from (B) that

$$\omega_z = \dot\psi + \dot\phi\cos\theta$$

Using the result obtained in (2):

$$\omega_z = \dot\psi + \dot\phi\cos\theta = \frac{\beta}{I} = \text{constant} \qquad (3)$$

Substituting for $I(\dot\psi+\dot\phi\cos\theta)$ from (2) into (1):

$$I'\dot\phi\sin^2\theta + \beta\cos\theta = \alpha$$

$$\dot\phi = \frac{\alpha-\beta\cos\theta}{I'\sin^2\theta} \qquad (4)$$

***18.109** (a) Since the weight of the top is a conservative force, we have $T+V = E = \text{constant}$

Recalling Eq. (18.20), p. 897, and making use of the expressions found for the components of $\underline{\omega}$ in Eq. (B) of Prob. 18.108, we write

$$T = \tfrac{1}{2}\left(I'\omega_x^2 + I'\omega_y^2 + I\omega_z^2\right)$$

$$T = \tfrac{1}{2}\left[I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + I(\dot\psi+\dot\phi\cos\theta)^2\right]$$

On the other hand, $V = mg\bar{Z} = mgc\cos\theta$

Thus: $T+V = E$:

$$\tfrac{1}{2}\left[I'(\dot\phi^2\sin^2\theta + \dot\theta^2) + I(\dot\psi+\dot\phi\cos\theta)^2\right] + mgc\cos\theta = E \qquad (5)$$

(b) The equations (1) and (2) of Prob. 18.108 and (5) just obtained form a system of 3 diff. equations in 3 unknowns. We shall now use Eqs (1) and (2) of Prob. 18.108 to eliminate $\dot\phi$ and $\dot\psi$ from Eq (5). Actually, we shall use the equivalent Eqs (3) and (4) of Prob. 18.108. Substituting for $(\dot\psi+\dot\phi\cos\theta)$ from (3) and for $\dot\phi$ from (4) into (5), and multiplying by 2, we have

$$I'\left(\frac{\alpha-\beta\cos\theta}{I'\sin^2\theta}\right)^2\sin^2\theta + I'\dot\theta^2 + I\left(\frac{\beta}{I}\right)^2 + 2mgc\cos\theta = 2E$$

$$\frac{(\alpha-\beta\cos\theta)^2}{I'\sin^2\theta} + I'\dot\theta^2 + \frac{\beta^2}{I} + 2mgc\cos\theta = 2E$$

and, solving for $\dot\theta^2$:

$$\dot\theta^2 = f(\theta)$$

where

$$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgc\cos\theta\right) - \left(\frac{\alpha-\beta\cos\theta}{I'\sin\theta}\right)^2$$

Note: This equation may be transformed into a standard type of equation which may be integrated by means of elliptic functions.

Setting $\cos\theta = x$, $-\sin\theta\,\dot\theta = \dot x$, we have

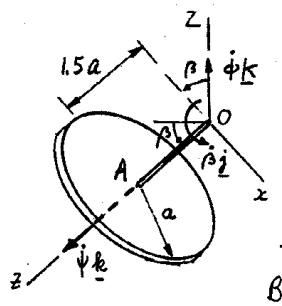
$$\frac{\dot x^2}{1-x^2} = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgcx\right) - \frac{(\alpha-\beta x)^2}{I'^2(1-x^2)}$$

or:

$$\dot x^2 = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1-x^2) - \frac{1}{I'^2}(\alpha-\beta x)^2$$

***18.110** Conservation of angular momentum about the Z and z axes.

Since the only external forces are the weight of the disk and the reaction at O, a reasoning similar to that indicated in Prob. 18.109 shows that the angular momentum is conserved about the Z and z axes.



Choosing the principal axes Oxyz shown (with y horizontal and pointing out of the paper), we have

$$\underline{\omega} = \dot{\phi}\underline{K} + \dot{\beta}\underline{j} + \dot{\psi}\underline{k}$$

$$\underline{\omega} = -\dot{\phi}\cos\beta\,\underline{i} + \dot{\beta}\underline{j} + (\dot{\psi} - \dot{\phi}\sin\beta)\underline{k}$$

$$\underline{H}_O = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$

But $I_x = I_y = \frac{1}{4}ma^2 + m(1.5a)^2 = \frac{5}{2}ma^2$

$I_z = \frac{1}{2}ma^2$

Therefore:

$$\underline{H}_O = -\tfrac{5}{2}ma^2\dot{\phi}\cos\beta\,\underline{i} + \tfrac{5}{2}ma^2\dot{\beta}\,\underline{j} + \tfrac{1}{2}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)\underline{k}$$

We write $H_Z = \text{const.}$, or $\underline{H}_O \cdot \underline{K} = \text{const.}$

Since $\underline{i}\cdot\underline{K} = -\cos\beta$, $\underline{j}\cdot\underline{K} = 0$, $\underline{k}\cdot\underline{K} = -\sin\beta$,

$$\underline{H}_O \cdot \underline{K} = -\tfrac{5}{2}ma^2\dot{\phi}\cos\beta(-\cos\beta) + \tfrac{1}{2}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)(-\sin\beta) = \text{constant}$$

With the initial conditions $\dot{\psi} = \dot{\psi}_0$, $\dot{\phi} = \dot{\beta} = 0$, $\beta = 0$ we find that the constant is zero. Thus

$$5\dot{\phi}\cos^2\beta - (\dot{\psi} - \dot{\phi}\sin\beta)\sin\beta = 0 \qquad (1)$$

We now write $H_z = \text{constant}$:

$$H_z = \tfrac{1}{2}ma^2(\dot{\psi} - \dot{\phi}\sin\beta) = \text{constant}$$

and, from the initial conditions, find that the constant is $\frac{1}{2}ma^2\dot{\psi}_0$. Thus

$$\dot{\psi} - \dot{\phi}\sin\beta = \dot{\psi}_0 \qquad (2)$$

Conservation of Energy.

We have $T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$

$$T = \tfrac{1}{2}\left[\tfrac{5}{2}ma^2\dot{\phi}^2\cos^2\beta + \tfrac{5}{2}ma^2\dot{\beta}^2 + \tfrac{1}{2}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right]$$

and selecting the datum at $\beta = 0$:

$$V = -mg(1.5a)\sin\beta$$

$T + V = \text{const.}$:

$$\tfrac{1}{2}\left[\tfrac{5}{2}ma^2\dot{\phi}^2\cos^2\beta + \tfrac{5}{2}ma^2\dot{\beta}^2 + \tfrac{1}{2}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right] - 1.5mga\sin\beta = \text{constant}$$

From the initial conditions $\dot{\psi} = \dot{\psi}_0$, $\dot{\phi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\frac{1}{4}ma^2\dot{\psi}_0^2$. Thus

$$5\dot{\phi}^2\cos^2\beta + 5\dot{\beta}^2 + (\dot{\psi} - \dot{\phi}\sin\beta)^2 - \frac{6g}{a}\sin\beta = \dot{\psi}_0^2 \qquad (3)$$

(CONTINUED)

***18.110 CONTINUED** Since $\beta_{max} = 30°$, we make $\beta = 30°$ and $\beta = 0$ in (1), (2), (3):

Eq. (1): $5\dot{\phi}\cos^2 30° - (\dot{\psi} - \dot{\phi}\sin 30°)\sin 30° = 0$

$$\tfrac{15}{4}\dot{\phi} - \tfrac{1}{2}(\dot{\psi} - \tfrac{1}{2}\dot{\phi}) = 0$$

$$8\dot{\phi} - \dot{\psi} = 0 \qquad (1')$$

Eq. (2): $\dot{\psi} - \frac{1}{2}\dot{\phi} = \dot{\psi}_0 \qquad (2')$

Eq. (3): $5\dot{\phi}^2\cos^2 30° + (\dot{\psi} - \dot{\phi}\sin 30°)^2 - \frac{6g}{a}\sin 30° = \dot{\psi}_0^2$

$$15\dot{\phi}^2 + 4(\dot{\psi} - \tfrac{1}{2}\dot{\phi})^2 - 12\frac{g}{a} = 4\dot{\psi}_0^2 \qquad (3')$$

Solving (1') and (2') simultaneously for $\dot{\phi}$ and $\dot{\psi}$:

$$\dot{\phi} = \frac{2}{15}\dot{\psi}_0 \qquad \dot{\psi} = \frac{16}{15}\dot{\psi}_0 \qquad (4)$$

Substituting for $\dot{\phi}$ and $\dot{\psi}$ from (4) into (3'):

$$\frac{4}{15}\dot{\psi}_0^2 + 4\dot{\psi}_0^2 - \frac{12g}{a} = 4\dot{\psi}_0^2, \qquad \dot{\psi}_0 = \sqrt{\frac{45g}{a}}$$

(a) For given data, $a = 0.200\text{ m}$, $g = 9.81\text{ m/s}^2$:

$$\dot{\psi}_0 = \sqrt{\frac{45(9.81)}{0.2}} = 46.981\text{ rad/s}, \qquad \dot{\psi}_0 = 47.0\text{ rad/s}$$ ◀

(b) Making $\dot{\psi}_0 = 46.981$ in Eqs. (4):

$$\dot{\phi} = \frac{2}{15}(46.981) = 6.2642 \qquad \dot{\phi} = 6.26\text{ rad/s}$$ ◀

$$\dot{\psi} = \frac{16}{15}(46.981) = 50.113 \qquad \dot{\psi} = 50.1\text{ rad/s}$$ ◀

***18.111** See solution of Prob. 18.110 for the derivation of Eqs. (1), (2), and (3).

Substituting for $\dot{\psi} - \dot{\phi}\sin\beta$ from (2) into (1):

$$5\dot{\phi}\cos^2\beta - \dot{\psi}_0\sin\beta = 0$$

$$\dot{\phi} = \frac{1}{5}\frac{\dot{\psi}_0\sin\beta}{\cos^2\beta} \qquad (4)$$

(a) Substituting from (2) and (4) into (3) and making $\dot{\beta} = 0$, since β is to be maximum, we have

$$5\left(\frac{1}{5}\frac{\dot{\psi}_0\sin\beta}{\cos^2\beta}\right)^2\cos^2\beta + \dot{\psi}_0^2 - \frac{4g}{a}\sin\beta = \dot{\psi}_0^2$$

$$\frac{1}{5}\dot{\psi}_0^2\frac{\sin^2\beta}{\cos^2\beta} = \frac{4g}{a}\sin\beta$$

Dividing by $\frac{4g}{a}\sin\beta$ and multiplying by $\cos^2\beta = 1 - \sin^2\beta$:

$$\sin^2\beta + \frac{a\dot{\psi}_0^2}{20g}\sin\beta - 1 = 0$$

Given data: $a = 0.200\text{ m}$, $\dot{\psi}_0 = 20\text{ rad/s}$, $g = 9.81\text{ m/s}^2$

$$\sin^2\beta + 0.40775\sin\beta - 1 = 0$$

Solving the quadratic:

$\sin\beta = -1.2244$ and $\sin\beta = 0.81670$

(impossible) $\beta = 54.756°$, $\beta = 54.8°$ ◀

(b) Substituting $\dot{\psi}_0 = 20\text{ rad/s}$ and $\beta = 54.756°$ into (4):

$$\dot{\phi} = \frac{1}{5}\frac{(20)\sin 54.756°}{\cos^2 54.756°} = 9.8100$$

$$\dot{\phi} = 9.81\text{ rad/s}$$ ◀

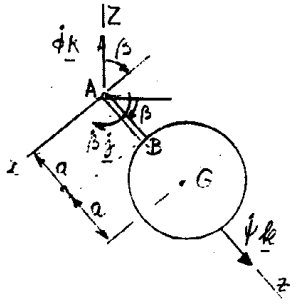
Substituting into (2): $\dot{\psi} - 9.81\sin\beta = 20$

$$\dot{\psi} = 20 + 9.81(0.81670) = 28.01\text{ rad/s}$$

$$\dot{\psi} = 28.0\text{ rad/s}$$ ◀

*18.112 Conservation of angular momentum about the Z and z axes.

Since the only external forces are the weight of the sphere and the reaction at A, a reasoning similar to that used in Prob. 18.107 shows that the angular momentum is conserved about the Z and z axes.



Choosing the principal axes Axyz shown (with $\underline{j}$ horizontal and pointing into the paper), we have

$$\underline{\omega} = -\dot{\phi}\cos\beta\,\underline{i} + \dot{\beta}\,\underline{j} + (\dot{\psi} - \dot{\phi}\sin\beta)\,\underline{k}$$

The moments of inertia are

$$I_z = \tfrac{2}{5}ma^2$$

$$I_x = I_y = \tfrac{2}{5}ma^2 + m(2a)^2 = \tfrac{22}{5}ma^2$$

Angular momentum about A:

$$\underline{H}_A = I_x\omega_x\,\underline{i} + I_y\omega_y\,\underline{j} + I_z\omega_z\,\underline{k}$$

$$\underline{H}_A = -\tfrac{22}{5}ma^2\dot{\phi}\cos\beta\,\underline{i} + \tfrac{22}{5}ma^2\dot{\beta}\,\underline{j} + \tfrac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)\,\underline{k}$$

We write $H_Z = \text{constant}$, or $\underline{H}_A \cdot \underline{K} = \text{const.}$

Since $\underline{i}\cdot\underline{K} = -\cos\beta$, $\underline{j}\cdot\underline{K} = 0$, $\underline{k}\cdot\underline{K} = -\sin\beta$,

$$\underline{H}_0 \cdot \underline{K} = -\tfrac{22}{5}ma^2\dot{\phi}\cos\beta(-\cos\beta) + \tfrac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)(-\sin\beta) = \text{constant}$$

With the initial conditions $\dot{\phi} = \dot{\phi}_0$, $\dot{\psi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\tfrac{22}{5}ma^2\dot{\phi}_0$. Thus

$$11\dot{\phi}\cos^2\beta - (\dot{\psi} - \dot{\phi}\sin\beta)\sin\beta = 11\dot{\phi}_0 \qquad (1)$$

We now write $H_z = \text{constant}$:

$$H_z = \tfrac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta) = \text{constant}$$

and, from the initial conditions, we find that the constant is zero. Thus

$$\dot{\psi} - \dot{\phi}\sin\beta = 0 \qquad (2)$$

Conservation of Energy

We have

$$T = \tfrac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

$$T = \tfrac{1}{2}\left[\tfrac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \tfrac{22}{5}ma^2\dot{\beta}^2 + \tfrac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right]$$

and, selecting the datum at $\beta = 0$:

$$V = -2mga\sin\beta$$

$$T + V = \text{const.}: \quad \tfrac{1}{2}\left[\tfrac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \tfrac{22}{5}ma^2\dot{\beta}^2 + \tfrac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right] - 2mga\sin\beta = \text{constant}$$

From the initial conditions $\dot{\phi} = \dot{\phi}_0$, $\dot{\psi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\tfrac{11}{5}ma^2\dot{\phi}_0^2$. Thus

$$11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 + (\dot{\psi} - \dot{\phi}\sin\beta)^2 - 10\tfrac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \qquad (3)$$

(CONTINUED)

*18.112 CONTINUED

Substituting for $\dot{\psi} - \dot{\phi}\sin\beta$ from (2) into (1) and (3):

Eq. (1): $11\dot{\phi}\cos^2\beta = 11\dot{\phi}_0$ (1')

Eq. (3): $11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 - 10\tfrac{g}{a}\sin\beta = 11\dot{\phi}_0^2$ (3')

Solving (1') for $\dot{\phi}$:

$$\dot{\phi} = \dot{\phi}_0\sec^2\beta \qquad (4)$$

Substituting for $\dot{\phi}$ from (4) into (3'):

$$11(\dot{\phi}_0\sec^2\theta)^2\cos^2\beta + 11\dot{\beta}^2 - 10\tfrac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \qquad (5)$$

For the maximum value of β, we have $\dot{\beta} = 0$ and Eq. (5) yields

$$\dot{\phi}_0^2\left(\frac{1}{\cos^4\beta} - 1\right) = \frac{10}{11}\frac{g}{a}\sin\beta$$

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \qquad (6)$$

Given Data: $\dot{\phi}_0^2 = \tfrac{17}{11}\tfrac{g}{a}$. Substituting into Eq. (6):

$$\frac{17}{11}\frac{g}{a} = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \qquad \cos^2\beta = 1.7\sin\beta$$

Letting $\cos^2\beta = 1 - \sin^2\beta$, we have

$$\sin^2\beta + 1.7\sin\beta - 1 = 0$$

Solving the quadratic:

$\sin\beta = -2.162$ and $\sin\beta = 0.46244$

(impossible) $\qquad \beta_{max} = 27.5^\circ$ ◀

Note: In Prob. 18.107, where the sphere was prevented from spinning, the same initial conditions yielded $\beta_{max} = 30.0^\circ$.

*18.113

See solution of Prob. 18.112 for derivation of Eqs. (2), (4), and (6).

$$\dot{\psi} - \dot{\phi}\sin\beta = 0 \qquad (2)$$

$$\dot{\phi} = \dot{\phi}_0\sec^2\beta \qquad (4)$$

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \qquad (6)$$

(a) Making $\beta = 30^\circ$ in Eq. (6), we have

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{0.75}{0.5} = \frac{15}{11}\frac{g}{a} \qquad \dot{\phi}_0 = \sqrt{\frac{15}{11}\frac{g}{a}}$$ ◀

(b) Substituting for $\dot{\phi}_0$ in Eq. (4), and making $\beta = 30^\circ$:

$$\dot{\phi} = \dot{\phi}_0\sec^2 30^\circ = \sqrt{\frac{15}{11}\frac{g}{a}}\left(\frac{4}{3}\right) = \sqrt{\frac{15(16)}{11(9)}\frac{g}{a}}$$

$$\dot{\phi} = 2\sqrt{\frac{20}{33}\frac{g}{a}}$$ ◀

Substituting for $\dot{\phi}$ in Eq. (2), and making $\beta = 30^\circ$:

$$\dot{\psi} = \dot{\phi}\sin 30^\circ = \tfrac{1}{2}\dot{\phi} \qquad \dot{\psi} = \sqrt{\frac{20}{33}\frac{g}{a}}$$ ◀

Note: Compare the results obtained for $\dot{\phi}_0$ and $\dot{\phi}$ with those obtained in Prob. 18.107. Even though in both cases the sphere is released with no spin and no nutation, the answers are different. This is due to the fact that the constraints in Prob. 18.113 allow the sphere to acquire spin, while those in Prob. 18.107 did not.

***18.114** (a) We recall Eq. (18.27): $\Sigma \underline{M}_O = \dot{\underline{H}}_O$

Since $\Sigma \underline{M}_O = 0$: $\underline{H}_O = \text{constant}$ (1) ◀

This means that the vector $\underline{H}_O$ has a constant magnitude and a fixed direction with respect to a Newtonian frame of reference

We recall $T + V = \text{const.}$ Since $V = \text{const.}$, we have $T = \text{constant}$ (2) ◀

We now recall from Prob. 18.3 that

$\underline{H}_O \cdot \underline{\omega} = 2T$

$H_O \omega \cos\beta = 2T$

But $\omega\cos\beta$ represents the projection of $\underline{\omega}$ on $\underline{H}_O$ and we have, in view of (1) and (2):

$$\omega \cos\beta = \frac{2T}{H_O} = \text{constant} \quad (3)$$ ◀

(b) Since the projection of $\underline{\omega}$ on $\underline{H}_O$ is constant and equal to $2T/H_O$, the tip of $\underline{\omega}$ must remain on a plane $\perp \underline{H}_O$ and at distance $2T/H_O$ from O. (This is the <u>invariable plane</u>.)

(c) Using the principal axes of inertia, we may use Eq. (18.20) to express the kinetic energy:

$$T = \tfrac{1}{2}\left(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2\right)$$

In view of (2), we must have therefore

$$I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 = 2T = \text{const.} \quad (4)$$ ◀

This is the equation of a quadric described by the point of coordinates $\omega_x, \omega_y, \omega_z$, that is, by the tip of the vector $\underline{\omega}$. Comparing (4) with Eq. (9.49), we observe that this quadric is an ellipsoid of the <u>same shape as the ellipsoid of inertia</u>. Since the right-hand member is equal to $2T$ instead of 1, however, this ellipsoid is of a <u>different size</u>. (This ellipsoid is the <u>Poinsot ellipsoid</u>.)

***18.115** (a) The tip of $\underline{\omega}$ is both on the invariable plane and on the Poinsot ellipsoid. At that point, the direction of the normal to the ellipsoid is that of <u>grad</u> $F(\omega_x, \omega_y, \omega_z)$, where F denotes the left-hand member of Eq. (4). We have (Sec. 13.7)

$$\underline{\text{grad}}\, F = \frac{\partial F}{\partial \omega_x}\underline{i} + \frac{\partial F}{\partial \omega_y}\underline{j} + \frac{\partial F}{\partial \omega_z}\underline{k}$$

$$= 2I_x \omega_x \underline{i} + 2I_y \omega_y \underline{j} + 2I_z \omega_z \underline{k}$$

But, from Eqs. (18.10):

$$\underline{H}_O = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

Thus $\underline{\text{grad}}\, F = 2\underline{H}_O$. This means that the normal to the ellipsoid is parallel to $\underline{H}_O$.

(CONTINUED)

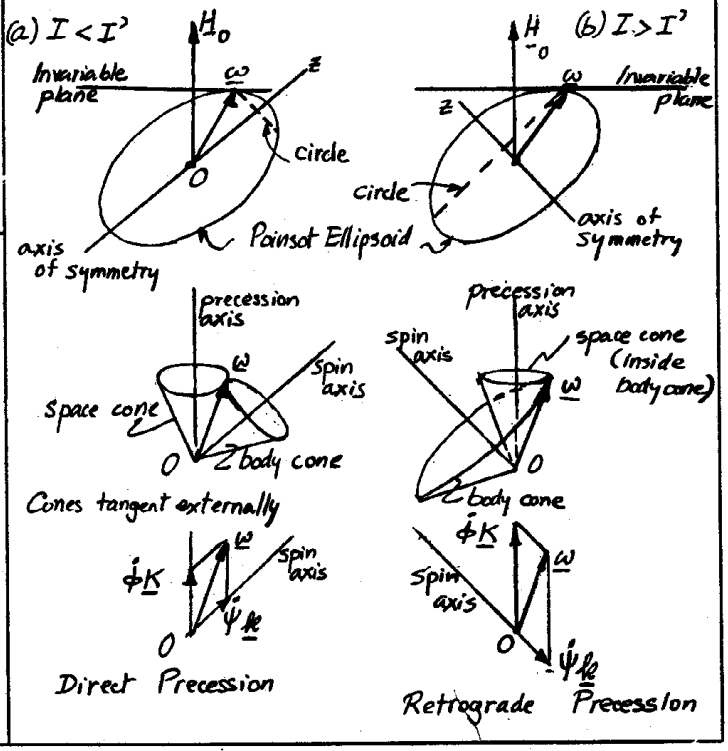
***18.115 CONTINUED** Thus, the plane tangent to the ellipsoid at the tip of $\underline{\omega}$ is perpendicular to $\underline{H}_O$. This plane, therefore, is the invariable plane.

(b) We already know that the Poinsot ellipsoid is tangent to the invariable plane. Now, we recall that any point of a rigid body which happens to be on the instantaneous axis of rotation has zero velocity at that instant. The Poinsot ellipsoid is part of the rigid body whose motion is being analyzed, and the point where it touches the invariable plane is at the tip of the vector $\underline{\omega}$ (which defines the inst. axis). Thus the point of contact has zero velocity, and the Poinsot ellipsoid rolls on the invariable plane (with its center O remaining fixed).

***18.116** We choose the z axis along the axis of symmetry. We have $I_x = I_y = I'$ and $I_z = I$. Substituting into Eq. (4) of Prob. 18.114 we have

$$I'(\omega_x^2 + \omega_y^2) + I\omega_z^2 = \text{const.}$$

which is the equation of an ellipsoid of revolution. Because of the symmetry, the point of contact between the Poinsot ellipsoid and the invariable plane will remain at a constant distance from the axis of precession and also at a constant distance from the axis of symmetry. Thus, the tip of $\underline{\omega}$ will describe a circle on the invariable plane and a circle on the Poinsot ellipsoid. The vector $\underline{\omega}$ itself will therefore describe circular space and body cones. The Poinsot ellipsoid, the invariable plane, and the space and body cones are shown below for the cases (a) $I < I'$ and (b) $I > I'$



*18.117 (a) We recall Eq.(4) of Prob. 18.114 This is the Eq.(1) to be shown. It expresses conservation of energy:

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{const.} \quad (1)$$

We now recall from Prob. 18.114 that the magnitude H_0 of the ang. momentum is constant. Using the relations (18.10), p. 893, we have

$$H_0^2 = H_x^2 + H_y^2 + H_z^2 = (I_x\omega_x)^2 + (I_y\omega_y)^2 + (I_z\omega_z)^2$$

or

$$I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2 = H_0^2 = \text{const.} \quad (2)$$

The tip of $\underline{\omega}$ must therefore describe the curve (polhode) obtained by intersecting the Poinsot ellipsoid (1) with the ellipsoid (2).

(b) We now write the equations of the two ellipsoids in the standard form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

We have

Poinsot ellipsoid: $$\frac{\omega_x^2}{2T/I_x} + \frac{\omega_y^2}{2T/I_y} + \frac{\omega_z^2}{2T/I_z} = 1 \quad (3)$$

Other ellipsoid: $$\frac{\omega_x^2}{H_0^2/I_x^2} + \frac{\omega_y^2}{H_0^2/I_y^2} + \frac{\omega_z^2}{H_0^2/I_z^2} = 1 \quad (4)$$

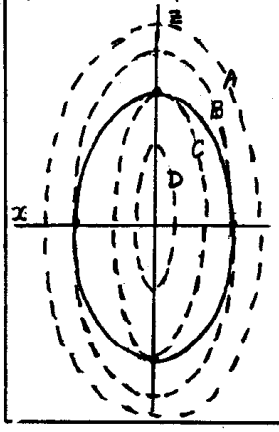
Assuming $I_z > I_y > I_x$, we check that

$$\frac{2T}{I_z} < \frac{2T}{I_y} < \frac{2T}{I_x}$$

that is, the x axis is the minor axis of the Poinsot ellipsoid, the y axis the intermediate axis, and the z axis the major axis, as shown in Fig. P18.117. We check similarly that

$$\frac{H_0^2}{I_x^2} < \frac{H_0^2}{I_y^2} < \frac{H_0^2}{I_z^2}$$

that is, the second ellipsoid has the same minor, intermediate, and major axes as the Poinsot ellipsoid. We note, however, that the ratio of major to minor axes for this ellipsoid is the square of the corresponding ratio for the Poinsot ellipsoid. Thus, the shape of the second ellipsoid is "more pronounced" than that of the Poinsot ellipsoid.



Depending upon the value of the constant $H_0^2/2T$, the Poinsot ellipsoid (solid line) and the second ellipsoid (dashed line) will occupy one of the relative positions shown. In case A, the second ellipsoid lies entirely outside Poinsot's. There is no intersection (and no polhode); this case cannot exist. In case D, the second ellipsoid lies entirely within Poinsot's. This is also impossible. Curve B shows the largest possible second ellipsoid; it touches Poinsot's at 2 points along minor axis. Curve C shows the smallest possible second ellipsoid; it touches Poinsot's at 2 points along major axis. All other possible second ellipsoids are comprised between B and C and lie partly within and partly outside Poinsot's, forming one of the intersection curves shown in Fig. P18.117. Note that the second ellipsoid which has the same intermediate axis as Poinsot's does not "touch" Poinsot's. It intersects it along two ellipses whose planes intersect along the y axis. These curves are not actually polhodes since the tip of $\underline{\omega}$ will never describe them, but they separate the polhodes into 4 groups: two groups loop around the minor x axis, the other two groups around the major z axis.

Note that each polhode consists of all the points of the Poinsot ellipsoid which will come in contact with the invariable plane during a particular motion. In the case of an axisymmetrical body, the Poinsot ellipsoid is of revolution (cf. Prob. 18.116) and the polhodes are circles.

(c) If the body is set to spin about one of its principal axes, the Poinsot ellipsoid remains tangent to the invariable plane at the same point (on the x, y, or z axes); the rotation is steady. In any other case, the point of tangency will be located on one of the polhodes and the tip of $\underline{\omega}$ will start describing that polhode, while the Poinsot ellipsoid rolls on the invariable plane.

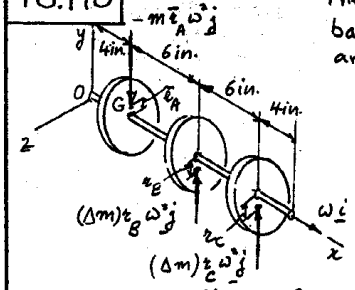
It should now be noted that, while steady rotation about either of the 3 principal axes is theoretically possible, it can be maintained in practice only about the minor and the major axes. Steady rotation about the intermediate axis is unstable. This is shown as follows:

Consider steady rotation about the <u>minor axis</u> (x axis). If the motion is slightly disturbed, the value of the constant $H_0^2/2T$ will be slightly changed and the new ellipsoids will intersect along a small polhode circling the x axis. As a result, the body will only slightly wobble. The motion is <u>stable</u>.

Same result for steady rotation about z axis. Rotation about <u>major axis</u> is also <u>stable</u>.

Consider now rotation about the y axis (<u>intermediate axis</u>). The vector $\underline{\omega}$ lies along the y axis. A slight disturbance will change the value of $H_0^2/2T$ and place the tip of $\underline{\omega}$ on one of the polhodes which pass close to the y axis. The tip of $\underline{\omega}$ will start describing this polhode, with $\underline{\omega}$ departing completely from its original position and the body will tumble. The motion is <u>unstable</u>.

18.118



The system is dynamically balanced if the effective forces are equivalent to zero.

For disk A: $\underline{H}_G = \bar{I}\omega\underline{i}$

Since ω = constant: $\dot{\underline{H}}_G = 0$

Thus, effective forces for disk A reduce to vector $-m\bar{r}_A\omega^2\underline{j}$.

We write that the 3 vectors representing the effective forces of disk A and of the 2 weights are equivalent to zero:

$$\Sigma(\underline{M}_O)_{eff}=0: \quad 4\underline{i}\times(-m\bar{r}_A\omega^2\underline{j}) + 10\underline{i}\times(\Delta m r_B\omega^2\underline{j}) + 16\underline{i}\times(\Delta m r_C\omega^2\underline{j}) = 0$$

$$-4m\bar{r}_A\underline{k} + 10\Delta m r_B\underline{k} + 16\Delta m r_C\underline{k} = 0$$

$$5r_B + 8r_C = 2\frac{m}{\Delta m}\bar{r}_A \qquad (1)$$

$$\Sigma\underline{F}_{eff}=0: \quad -m\bar{r}_A\omega^2\underline{j} + \Delta m r_B\omega^2\underline{j} + \Delta m r_C\omega^2\underline{j} = 0$$

$$r_B + r_C = \frac{m}{\Delta m}\bar{r}_A \qquad (2)$$

Multiply (2) by 5 and subtract from (1):

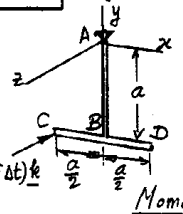
$$3r_C = -3\frac{m}{\Delta m}\bar{r}_A \qquad r_C = -\frac{m}{\Delta m}\bar{r}_A = -\frac{25}{2}\left(\tfrac{1}{4}\text{ in.}\right) = -3.125\text{ in.}$$

From (2): $r_B = \frac{m}{\Delta m}\bar{r}_A - r_C = \frac{25}{2}\left(\tfrac{1}{4}\text{ in.}\right) - (-3.125) = +6.25\text{ in.}$

On B: $6\tfrac{1}{4}$ in. below shaft. On C: $3\tfrac{1}{8}$ in. above shaft ◀

18.119

Angular momentum about A



H_A is equal to the moment about A of the impulse $\underline{F}\Delta t$:

$$\underline{H}_A = \underline{r}_{C/A}\times\underline{F}\,\Delta t = \left(-\frac{a}{2}\underline{j} - \frac{a}{2}\underline{i}\right)\times(-F\Delta t\,\underline{k})$$

$$= aF\Delta t\left(\underline{i} - \tfrac{1}{2}\underline{j}\right)$$

$H_x = aF\Delta t$, $H_y = -\frac{1}{2}aF\Delta t$, $H_z = 0$

Moments of Inertia $\left(m_{AB} = m_{CD} = \frac{m}{2}\right)$

$$I_x = (I_x)_{AB} + (I_x)_{CD} = \frac{1}{3}\left(\frac{m}{2}\right)a^2 + \left(\frac{m}{2}\right)a^2 = \frac{2}{3}ma^2$$

$$I_y = 0 + \frac{1}{12}\left(\frac{m}{2}\right)a^2 = \frac{1}{24}ma^2$$

I_z will not be required; also $I_{xy} = I_{yz} = I_{zx} = 0$ since x, y, z axes are principal axes of inertia due to symmetry.

(a) Angular velocity. Since A is a fixed point, we may use Eq. (18.13), which reduces to

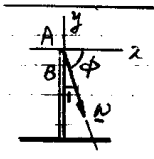
$H_x = I_x\omega_x$: $\quad aF\Delta t = \frac{2}{3}ma^2\omega_x \qquad \omega_x = \frac{3}{2}\frac{F\Delta t}{ma}$

$H_y = I_y\omega_y$: $\quad -\frac{1}{2}aF\Delta t = \frac{1}{24}ma^2\omega_y \qquad \omega_y = -12\frac{F\Delta t}{ma}$

$H_z = I_z\omega_z$: $\quad 0 = I_z\omega_z \qquad \omega_z = 0$

$$\underline{\omega} = \frac{3}{2}\frac{F\Delta t}{ma}\left(\underline{i} - 8\underline{j}\right)$$ ◀

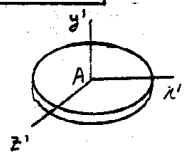
(b) Instantaneous axis of rotation. It passes through the fixed point A and is directed along $\underline{\omega}$ in the xy plane ◀



$\tan\phi = 8 \qquad \phi = 82.9°$ ◀

18.120

Angular momentum about mass center A.



$\omega_x = \omega_1 = 10\text{ rad/s}$, $\omega_y = \omega_z = 20\text{ rad/s}$, $\omega_z = 0$

$$\bar{I}_{x'} = \tfrac{1}{4}mr^2 = \tfrac{1}{4}(800\text{ g})(0.1\text{ m})^2$$

$$\bar{I}_{x'} = 2\text{ g}\cdot\text{m}^2$$

$$\bar{I}_{y'} = \tfrac{1}{2}mr^2 = 4\text{ g}\cdot\text{m}^2$$

Since the centroidal axes x'y'z' are principal axes:

$$\underline{H}_A = \bar{I}_{x'}\omega_{x'}\underline{i} + \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k}$$

$$= (2\text{ g}\cdot\text{m}^2)(10\text{ rad/s})\underline{i} + (4\text{ g}\cdot\text{m}^2)(20\text{ rad/s})\underline{j}$$

$$\underline{H}_A = (20\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (80\text{ g}\cdot\text{m}^2/\text{s})\underline{j} \qquad (1)$$

Linear momentum.

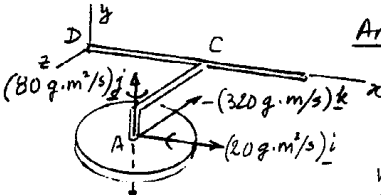
We have $\bar{\underline{v}}_A = (\omega_1\underline{i})\times\underline{r}_A$

where $\underline{r}_A = (0.18\text{ m})\underline{i} - (0.04\text{ m})\underline{j} + (0.1\text{ m})\underline{k}$

Thus: $\bar{\underline{v}}_A = 10\underline{i}\times(0.18\underline{i} - 0.04\underline{j} + 0.1\underline{k})$

$$\bar{\underline{v}}_A = -\underline{j} + 0.4\underline{k}$$

and $m\bar{\underline{v}}_A = 800\,\bar{\underline{v}}_A = -(800\text{ g}\cdot\text{m/s})\underline{j} - (320\text{ g}\cdot\text{m/s})\underline{k}$ (2)



Ang. momentum about C

Taking moments about C:

$$\underline{H}_C = \underline{H}_A + \overrightarrow{CA}\times m\bar{\underline{v}}_A \qquad (3)$$

where $\overrightarrow{CA} = -(0.04\text{ m})\underline{j} + (0.1\text{ m})\underline{k}$

Recalling (1) and (2) and substituting into (3):

$$\underline{H}_C = 20\underline{i} + 80\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -0.04 & 0.1 \\ 0 & -800 & -320 \end{vmatrix} = 20\underline{i} + 80\underline{j} + 72.8\underline{i}$$

$$\underline{H}_C = (112.8\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (80\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$$ ◀

18.121

Angular momentum about mass center A.

See solution of Prob. 18.120. We found

$$\underline{H}_A = (20\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (80\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$$

or: $\underline{H}_A = (0.020\text{ kg}\cdot\text{m}^2/\text{s})\underline{i} + (0.080\text{ kg}\cdot\text{m}^2/\text{s})\underline{j}$

Rate of change of $\underline{H}_A$

Assuming that the frame Dxyz rotates with the shaft and arm, we have

$$(\dot{\underline{H}}_A)_{Dxyz} = 0$$

and $\underline{\Omega} = \underline{\omega}_1 = (10\text{ rad/s})\underline{i}$

From Eq. (18.22):

$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Dxyz} + \underline{\Omega}\times\underline{H}_A$$

$$= 0 + 10\underline{i}\times(0.020\underline{i} + 0.080\underline{j})$$

$$\dot{\underline{H}}_A = (0.8\text{ N}\cdot\text{m})\underline{k} \qquad (1)$$

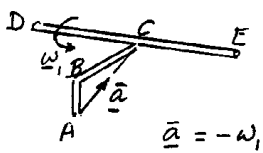
(CONTINUED)

18.121 CONTINUED

We recall from previous page:

$$\underline{\dot{H}}_A = (0.8\ \text{N·m})\,\underline{k} \qquad (1)$$

Effective force $m\underline{\bar{a}}$



Since the mass center A of the disk describes a circle centered at C at the constant ang. vel. ω_1:

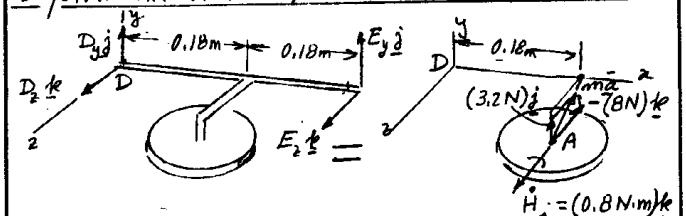
$$\underline{\bar{a}} = -\omega_1^2\,\underline{r}_{A/C} = -\omega_1^2\,\overrightarrow{CA} = \omega_1^2\,\overrightarrow{AC}$$

$$= (10\ \text{rad/s})^2[(0.04\ \text{m})\underline{i} - (0.1\ \text{m})\underline{k}]$$

$$= (4\ \text{m/s}^2)\,\underline{i} - (10\ \text{m/s}^2)\,\underline{k}$$

and: $m\underline{\bar{a}} = (0.8\ \text{kg})\,\underline{\bar{a}} = (3.2\ \text{N})\,\underline{i} - (8\ \text{N})\,\underline{k}$

Equivalence of the external and effective forces



$\Sigma\underline{M}_D = \Sigma(\underline{M}_D)_{\text{eff}}$: Since $m\underline{\bar{a}}$ passes through C:

$$0.36\,\underline{i} \times (E_y\,\underline{j} + E_z\,\underline{k}) = 0.8\,\underline{k} + 0.18\,\underline{i} \times (3.2\,\underline{i} - 8\,\underline{k})$$

$$0.36\,E_y\,\underline{k} - 0.36\,E_z\,\underline{j} = 0.8\,\underline{k} + 0.576\,\underline{k} + 1.44\,\underline{j}$$

Equating the coefficients of $\underline{k}$ and $\underline{j}$:

$\underline{k}$: $0.36\,E_y = 0.8 + 0.576 = 1.376$ $\quad E_y = 3.822\ \text{N}$

$\underline{j}$: $-0.36\,E_z = 1.44$ $\quad E_z = -4.00\ \text{N}$

$$\underline{E} = (3.82\ \text{N})\,\underline{j} - (4.00\ \text{N})\,\underline{k} \blacktriangleleft$$

$\Sigma\underline{F} = \Sigma\underline{F}_{\text{eff}}$: $\underline{D} + 3.822\,\underline{j} - 4\,\underline{k} = 3.2\,\underline{j} - 8\,\underline{k}$

$$\underline{D} = -(0.622\ \text{N})\,\underline{i} - (4.00\ \text{N})\,\underline{k} \blacktriangleleft$$

18.122 CONTINUED

Moments of Inertia.

$$I_{x'} = \tfrac{1}{4}ma^2 + mL^2 = m\left(L^2 + \tfrac{a^2}{4}\right), \qquad I_{y'} = \tfrac{1}{2}ma^2 \qquad (3)$$

(a) *Angular Momentum* $\underline{H}_D$

$$\underline{H}_D = I_{x'}\omega_{x'}\underline{i}' + I_{y'}\omega_{y'}\underline{j}' + I_{z'}\omega_{z'}\underline{k}'$$

$$= m\left[\left(L^2 + \tfrac{a^2}{4}\right)(\omega_1 \sin\beta)\underline{i}' + \tfrac{1}{2}ma^2\left(\tfrac{L}{a}\omega_1 \sin\beta\right)\underline{j}' + 0\right]$$

$$= m\omega_1 a^2 \sin\beta\left\{\left[\left(\tfrac{L}{a}\right)^2 + \tfrac{1}{4}\right]\underline{i}' + \tfrac{1}{2}\tfrac{L}{a}\underline{j}'\right\}$$

$$= 2(4)(0.1)^2 \sin 30°\,(25.25\,\underline{i}' + 2.5\,\underline{j}') = 1.01\,\underline{i}' + 0.1\,\underline{j}'$$

Reverting to the original axes:

$$\underline{i}' = \cos 30°\,\underline{i} + \sin 30°\,\underline{j}, \qquad \underline{j}' = -\sin 30°\,\underline{i} + \cos 30°\,\underline{j}$$

$$\underline{H}_D = 1.01\,(\cos 30°\,\underline{i} + \sin 30°\,\underline{j}) + 0.1\,(-\sin 30°\,\underline{i} + \cos 30°\,\underline{j})$$

$$\underline{H}_D = (0.825\ \text{kg·m}^2/\text{s})\,\underline{i} + (0.592\ \text{kg·m}^2/\text{s})\,\underline{j} \blacktriangleleft$$

(b) *Kinetic Energy.*

From (2): $\omega_{x'} = \omega_1 \sin\beta$, $\omega_{y'} = \frac{L}{a}\omega_1 \sin\beta$, $\omega_{z'} = 0$

Recalling (3):

$$T = \tfrac{1}{2}\left(I_{x'}\omega_{x'}^2 + I_{y'}\omega_{y'}^2 + I_{z'}\omega_{z'}^2\right)$$

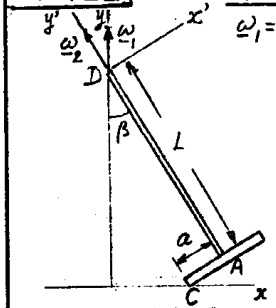
$$= \tfrac{1}{2}\left[m\left(L^2 + \tfrac{a^2}{4}\right)\omega_1^2 \sin^2\beta + \tfrac{1}{2}ma^2\left(\tfrac{L}{a}\right)^2\omega_1^2 \sin^2\beta\right]$$

$$= \tfrac{1}{8}ma^2\omega_1^2 \sin^2\beta\left[1 + 6\left(\tfrac{L}{a}\right)^2\right]$$

$$= \tfrac{1}{8}(2)(0.1)^2(4)^2 \sin^2 30°\,[1 + 6(5)^2] = 0.01\,(151)$$

$$T = 1.510\ \text{J} \blacktriangleleft$$

18.122



Given: $L = 0.5\ \text{m}$, $a = 0.1\ \text{m}$, $\beta = 30°$

$\underline{\omega}_1 = (4\ \text{rad/s})\,\underline{j}$, $m = 2\ \text{kg}$

Angular Velocity

$$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2$$

Introducing the auxiliary principal axes $Dx'y'z'$:

$$\underline{\omega}_1 = \omega_1 \sin\beta\,\underline{i}' + \omega_1 \cos\beta\,\underline{j}'$$

and

$$\underline{\omega} = \omega_1 \sin\beta\,\underline{i}' + (\omega_2 + \omega_1 \cos\beta)\,\underline{j}' \qquad (1)$$

Since the velocity of point C is ~~zero~~:

$$\underline{v}_C = \underline{\omega} \times \underline{r}_{C/D} = 0$$

$$\left[\omega_1 \sin\beta\,\underline{i}' + (\omega_2 + \omega_1 \cos\beta)\,\underline{j}'\right] \times (-a\,\underline{i}' - L\,\underline{j}') = 0$$

$$\left[a(\omega_2 + \omega_1 \cos\beta) - L\omega_1 \sin\beta\right]\underline{k}' = 0$$

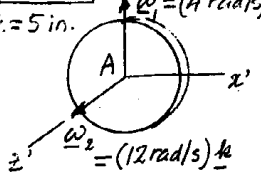
$$\omega_2 + \omega_1 \cos\beta = \frac{L}{a}\omega_1 \sin\beta$$

Substituting into (1):

$$\underline{\omega} = \frac{\omega_1}{a}\sin\beta\,(a\,\underline{i}' + L\,\underline{j}') \qquad (2)$$

(CONTINUED)

18.123



Angular velocity of disk:

$$\underline{\omega} = \omega_1\,\underline{j} + \omega_2\,\underline{k}$$

$$= (4\ \text{rad/s})\,\underline{j} + (12\ \text{rad/s})\,\underline{k}$$

Moments of inertia:

$$\bar{I}_{z'} = \bar{I}_{y'} = \tfrac{1}{4}mr^2$$

$$= \frac{1}{4}\frac{8}{32.2}\left(\frac{5}{12}\text{ft}\right)^2 = 0.010783\ \text{ft·lb·s}^2$$

$$I_{z'} = \tfrac{1}{2}mr^2 = 0.021567\ \text{ft·lb·s}^2$$

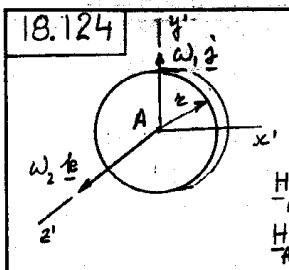
$$\underline{H}_A = \bar{I}_{x'}\omega_{x'}\underline{i} + \bar{I}_{y'}\omega_{y'}\underline{j} + I_{z'}\omega_{z'}\underline{k}$$

$$= 0 + (0.010783)(4)\,\underline{j} + (0.021567)(12)\,\underline{k}$$

$$= (0.04313\ \text{ft·lb·s})\,\underline{j} + (0.2588\ \text{ft·lb·s})\,\underline{k}$$

$$\underline{H}_A = (0.0431\ \text{ft·lb·s})\,\underline{j} + (0.259\ \text{ft·lb·s})\,\underline{k} \blacktriangleleft$$

18.124



Angular momentum $\underline{H}_A$:

Using the reference frame Ax'y'z':

$\omega_{x'} = 0,\ \omega_{y'} = \omega_1,\ \omega_{z'} = \omega_2$

$\bar{I}_{y'} = \frac{1}{4} m r^2 \qquad \bar{I}_{z'} = \frac{1}{2} m r^2$

$\underline{H}_A = \bar{I}_{x'}\omega_{x'}\underline{i} + \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k}$

$\underline{H}_A = \frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m r^2 \omega_2 \underline{k}$

where the reference frame rotates with the arm OA with the angular velocity $\underline{\Omega} = \omega_1 \underline{j}$

Rate of change of $\underline{H}_A$: We use Eq. (18.22):

$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A = \frac{1}{4} m r^2 \dot{\omega}_1 \underline{j} + \frac{1}{2} m r^2 \dot{\omega}_2 \underline{k} + \omega_1 \underline{j} \times \left(\frac{1}{4} m r^2 \omega_1 \underline{j} + \frac{1}{2} m r^2 \omega_2 \underline{k}\right)$

$\dot{\underline{H}}_A = \frac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} + \frac{1}{4} m r^2 \alpha_1 \underline{j} + \frac{1}{2} m r^2 \alpha_2 \underline{k}$

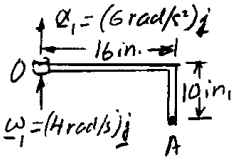
Given data: $m = \frac{8}{32.2}$, $r = \frac{5}{12}$ ft, $\omega_1 = 4$ rad/s, $\alpha_1 = 6$ rad/s^2

$\omega_2 = 12$ rad/s = constant; thus $\alpha_2 = 0$

$\dot{\underline{H}}_A = \frac{1}{2}\left(\frac{8}{32.2}\right)\left(\frac{5}{12}\right)^2 (4)(12)\underline{i} + \frac{1}{4}\left(\frac{8}{32.2}\right)\left(\frac{5}{12}\right)^2 (6)\underline{j} + 0$

$\dot{\underline{H}}_A = (1.0352 \text{ lb·ft})\underline{i} + (0.06470 \text{ lb·ft})\underline{j} \qquad (1)$

Acceleration of mass center A:



$\underline{a}_A = \underline{\alpha}_1 \times \underline{r}_A + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_A)$

$= 6\underline{j} \times \left(\frac{16}{12}\underline{i} - \frac{10}{12}\underline{j}\right) + 4\underline{j} \times \left[4\underline{j} \times \left(\frac{16}{12}\underline{i} - \frac{10}{12}\underline{j}\right)\right]$

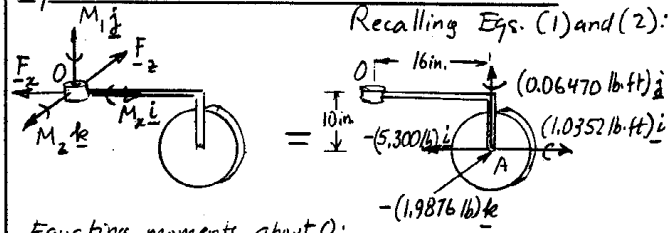
$= -8\underline{k} + 4\underline{j} \times (-5.333\underline{k})$

$\bar{\underline{a}} = \underline{a}_A = -(21.333 \text{ ft/s}^2)\underline{i} - (8 \text{ ft/s}^2)\underline{k}$

$m\bar{\underline{a}} = \frac{8}{32.2}(-21.333\underline{i} - 8\underline{k}) = -(5.300 \text{ lb})\underline{i} - (1.9876 \text{ lb})\underline{k} \qquad (2)$

Equivalence of external and effective forces

Recalling Eqs. (1) and (2):



Equating moments about O:

$M_1\underline{j} + M_x\underline{i} + M_z\underline{k} = \left(\frac{16}{12}\underline{i} - \frac{10}{12}\underline{j}\right) \times (-5.300\underline{i} - 1.9876\underline{k}) + 1.0352\underline{i} + 0.06470\underline{j}$

$= 2.6501\underline{j} - 4.4167\underline{k} + 1.6563\underline{i} + 1.0352\underline{i} + 0.0647\underline{j}$

Equating the coefficients of the unit vectors:

(j) $M_1 = 2.6501 + 0.0647 = 2.7148$ lb·ft

(a) **Required couple:** $M_1\underline{j} = (2.71 \text{ lb·ft})\underline{j}$ ◀

(i) $M_x = 1.6563 + 1.0352 = 2.6915$ lb·ft

(k) $M_z = -4.4167$ lb·ft

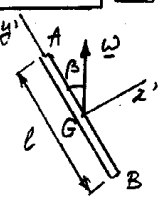
Equating components: $F_x = -5.30$ lb, $F_y = 0$, $F_z = -1.988$ lb

(b) **Dynamic Reaction at O:**

$\underline{F} = -(5.30 \text{ lb})\underline{i} - (1.988 \text{ lb})\underline{k}$; $\underline{M}_O = (2.69 \text{ lb·ft})\underline{i} - (4.42 \text{ lb·ft})\underline{k}$ ◀

18.125

Angular momentum $\underline{H}_G$.



Resolving $\underline{\omega}$ into components along the principal axes Gx'y'z':

$\omega_{x'} = \omega\sin\beta, \quad \omega_{y'} = \omega\cos\beta, \quad \omega_{z'} = 0$

Moments of inertia:

$\bar{I}_{x'} = \frac{1}{12} m l^2, \qquad \bar{I}_{y'} = 0$

$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_{z'}\omega_{z'}\underline{k}'$

$= \frac{1}{12} m l^2 \omega \sin\beta \underline{i}' + 0 + 0$

where the reference frame Gx'y'z' rotates with the rod with the angular velocity

$\underline{\Omega} = \underline{\omega} = \omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}'$

Rate of change of $\underline{H}_G$:

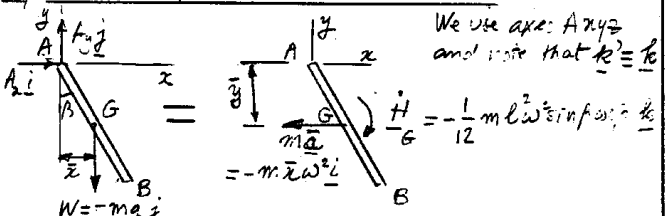
Since ω and β are constant, $(\dot{\underline{H}}_G)_{Gx'y'z'} = 0$ and Eq. (18.22) reduces to

$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gx'y'z'} + \underline{\Omega} \times \underline{H}_G$

$= 0 + (\omega\sin\beta\,\underline{i}' + \omega\cos\beta\,\underline{j}') \times \frac{1}{12} m l^2 \omega \sin\beta\,\underline{i}'$

$= -\frac{1}{12} m l^2 \omega^2 \sin\beta\cos\beta\,\underline{k}'$

Equivalence of external and effective forces:



We use axes Axyz and note that $\underline{k}' \equiv \underline{k}$

$\bar{z} = \frac{1}{2} l\sin\beta \qquad \bar{y} = -\frac{1}{2} l\cos\beta$

Equating moments about A:

$\left(\frac{1}{2} l\sin\beta\,\underline{i}\right) \times (-mg\,\underline{j}) = \left(-\frac{1}{2} l\cos\beta\,\underline{j}\right) \times \left(-m\frac{l}{2}\sin\beta\omega^2\underline{i}\right) - \frac{1}{12} m l^2 \omega^2 \sin\beta\cos\beta\,\underline{k}$

$-\frac{1}{2} mgl\sin\beta\,\underline{k} = -\frac{1}{4} m l^2 \omega^2 \sin\beta\cos\beta\,\underline{k} - \frac{1}{12} m l^2 \omega^2 \sin\beta\cos\beta\,\underline{k}$

$\frac{1}{2}\frac{g}{l}\sin\beta = \frac{1}{3}\omega^2 \sin\beta\cos\beta$

(a) $\sin\beta = 0$ and $\cos\beta = \frac{3}{2}\frac{g}{l\omega^2}$

$\beta = 0$ and $\beta = \cos^{-1}(3g/2l\omega^2)$ ◀

(b) For rod to remain vertical, we need $\beta = 0$ to be the only possible solution. This will occur if $(3g/2l\omega^2) \geq 1$ since $\cos\beta$ cannot be larger than 1 and for $\cos\beta = 1$, we have $\beta = 0$.

$\omega^2 \leq 3g/2l \qquad 0 \leq \omega \leq \sqrt{3g/2l}$ ◀

Note. When $0 \leq \omega \leq \sqrt{3g/2l}$, $\beta = 0$ is the only possible solution and this position of equilibrium is **stable**. When $\omega > \sqrt{3g/2l}$, $\beta = 0$ is still a position of equilibrium but it is **unstable**.

18.126

Initial angular momentum of satellite:

$$(\underline{H}_G)_0 = \bar{I}_x \omega_0 \underline{i} = (240\ \text{kg})(0.3\ \text{m})^2(1.5\ \text{rad/s})\underline{i}$$

$$(\underline{H}_G)_0 = (32.4\ \text{kg}\cdot\text{m}^2/\text{s})\underline{i} \qquad (1)$$

Final angular momentum of satellite:

$$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k}$$

$$= (240\ \text{kg})[(0.3\ \text{m})^2\omega_x \underline{i} + (0.4\ \text{m})^2\omega_y \underline{j} + (0.4\ \text{m})^2\omega_z \underline{k}]$$

$$\underline{H}_G = 21.6\,\omega_x \underline{i} + 38.4\,\omega_y \underline{j} + 38.4\,\omega_z \underline{k} \qquad (2)$$

Angular momentum about G of meteorite

We have $\underline{r}_A = (0.3\ \text{m})\underline{i} + (0.375\ \text{m})\underline{j}$

$$m\underline{v}_0 = (0.030\ \text{kg})(-576\underline{i} - 432\underline{j} + 960\underline{k})$$

$$= -17.28\underline{i} - 12.96\underline{j} + 28.8\underline{k}$$

Using determinant form for vector product:

$$\underline{r}_A \times m\underline{v}_0 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.3 & 0.375 & 0 \\ -17.28 & -12.96 & 28.8 \end{vmatrix}$$

$$= 0.375(28.8)\underline{i} - 0.3(28.8)\underline{j} + [0.3(-17.96) - 0.375(-17.28)]\underline{k}$$

$$\underline{r}_A \times m\underline{v}_0 = 10.80\underline{i} - 8.64\underline{j} + 2.592\underline{k} \qquad (3)$$

Principle of Impulse and Momentum for Satellite and Meteorite.

Since there is no external impulse, the angular momentum of the system about G is conserved:

$$(\underline{H}_G)_0 + \underline{r}_A \times m\underline{v}_0 = \underline{H}_G \qquad (4)$$

Substituting from (1), (2), and (3) into Eq. (4):

$$32.4\underline{i} + 10.80\underline{i} - 8.64\underline{j} + 2.592\underline{k} = 21.6\,\omega_x \underline{i} + 38.4\,\omega_y \underline{j} + 38.4\,\omega_z \underline{k}$$

Equating the coefficients of the unit vectors:

(i)	$32.4 + 10.80 = 21.6\,\omega_x$	$\omega_x = 2.00$ rad/s
(j)	$-8.64 = 38.4\,\omega_y$	$\omega_y = -0.225$ rad/s
(k)	$2.592 = 38.4\,\omega_z$	$\omega_z = 0.0675$ rad/s

$$\underline{\omega} = (2.00\ \text{rad/s})\underline{i} - (0.225\ \text{rad/s})\underline{j} + (0.0675\ \text{rad/s})\underline{k}$$

18.127

See the solution of Prob. 18.126 for the determination of the angular velocity of the satellite after impact:

$$\underline{\omega} = (2.00\ \text{rad/s})\underline{i} - (0.225\ \text{rad/s})\underline{j} + (0.0675\ \text{rad/s})\underline{k}$$

Angular momentum of satellite after impact:

$$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k}$$

$$= (240\ \text{kg})[(0.3\ \text{m})^2\omega_x \underline{i} + (0.4\ \text{m})^2\omega_y \underline{j} + (0.4\ \text{m})^2\omega_z \underline{k}]$$

$$= 21.6(2.00)\underline{i} + 38.4(-0.225)\underline{j} + 38.4(0.0675)\underline{k}$$

$$\underline{H}_G = 43.2\underline{i} - 8.64\underline{j} + 2.592\underline{k}$$

(a) *Precession Axis*

The precession axis is directed along $\underline{H}_G$. Its unit vector $\underline{K}$, therefore, is

$$\underline{K} = \frac{\underline{H}_G}{H_G} = \frac{43.2\underline{i} - 8.64\underline{j} + 2.592\underline{k}}{44.1317}$$

$$\underline{K} = 0.97889\underline{i} - 0.19578\underline{j} + 0.058733\underline{k}$$

The angles the precession axis forms with the coordinate axes x, y, z, are therefore

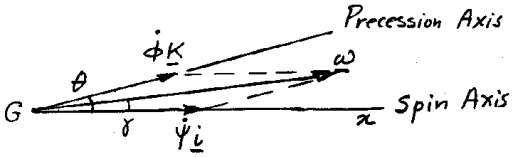
$$\theta_x = 11.794^\circ \quad \theta_y = 101.290^\circ \quad \theta_z = 86.633^\circ$$

$$\theta_x = 11.8^\circ\ ; \quad \theta_y = 101.3^\circ, \quad \theta_z = 86.6^\circ$$

(b) *Rates of precession and spin*

The magnitude of $\underline{\omega}$ is

$$\omega = \sqrt{(2)^2 + (-0.225)^2 + (0.0675)^2} = 2.01375\ \text{rad/s}$$

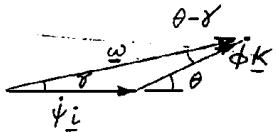


Since the spin axis is along the x axis, we have

$$\cos\gamma = \frac{\omega_x}{\omega} = \frac{2}{2.01375} = 0.99317, \quad \gamma = 6.699^\circ$$

On the other hand, $\theta = \theta_x = 11.794^\circ$

$$\theta - \gamma = 11.794^\circ - 6.699^\circ = 5.095^\circ$$



Law of sines:

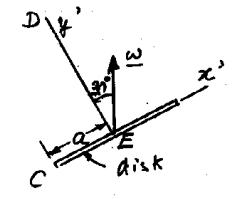
$$\frac{\omega}{\sin(180^\circ - \theta)} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\theta - \gamma)}$$

$$\frac{2.01375}{\sin(180^\circ - 11.794^\circ)} = \frac{\dot{\phi}}{\sin 6.699^\circ} = \frac{\dot{\psi}}{\sin 5.095^\circ}$$

Rate of precession $= \dot{\phi} = 1.149$ rad/s

Rate of spin $= \dot{\psi} = 0.875$ rad/s

18.128 Angular momentum about mass center $\bar{H}_E$



Resolving $\underline{\omega}$ into components along the principal axes Ex'y'z':

$\omega_{x'} = \omega \sin 30°$, $\omega_{y'} = \omega \cos 30°$, $\omega_{z'} = 0$

Moments of inertia

$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} ma^2$

$\bar{I}_{y'} = \frac{1}{2} ma^2$

$$\underline{H}_E = \bar{I}_{x'} \omega_{x'} \underline{i}' + \bar{I}_{y'} \omega_{y'} \underline{j}' + \bar{I}_{z'} \omega_{z'} \underline{k}'$$

$$= \tfrac{1}{4} ma^2 \omega \sin 30° \underline{i}' + \tfrac{1}{2} ma^2 \omega \cos 30° \underline{j}'$$

where the reference frame Ex'y'z' rotates with the disk with the angular velocity

$$\underline{\Omega} = \underline{\omega} = \omega \sin 30° \underline{i}' + \omega \cos 30° \underline{j}'$$

Rate of change of $\underline{H}_E$

Since ω is constant, $\left(\dot{\underline{H}}_E\right)_{Ex'y'z'} = 0$ and Eq. (18.22) reduces to

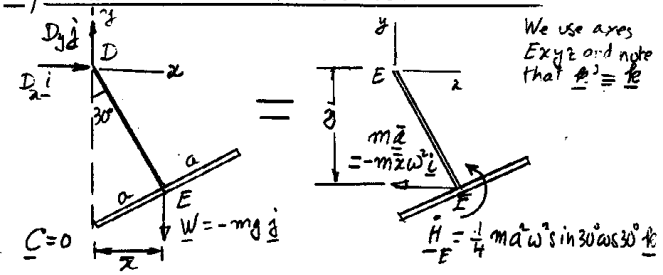
$$\dot{\underline{H}}_E = \left(\dot{\underline{H}}_E\right)_{Ex'y'z'} + \underline{\Omega} \times \underline{H}_E$$

$$= 0 + \left(\omega \sin 30° \underline{i}' + \omega \cos 30° \underline{j}'\right) \times \left(\tfrac{1}{4} ma^2 \omega \sin 30° \underline{i}' + \tfrac{1}{2} ma^2 \omega \cos 30° \underline{j}'\right)$$

$$\dot{\underline{H}}_E = \tfrac{1}{2} ma^2 \omega^2 \sin 30° \cos 30° \underline{k}' - \tfrac{1}{4} ma^2 \omega^2 \cos 30° \sin 30° \underline{k}'$$

$$\underline{H}'_E = \tfrac{1}{4} ma^2 \omega^2 \sin 30° \cos 30° \underline{k}'$$

Equivalence of external and effective forces:



$$\bar{x} = a \cos 30° \qquad \bar{y} = (DE) \cos 30° = \frac{a}{\tan 30°} \cos 30°$$

Equating moments about D:

$$(a \cos 30° \underline{i}) \times (-mg \underline{j}) = \left(-\frac{a}{\tan 30°} \cos 30° \underline{j}\right) \times \left(-ma \cos 30° \omega^2 \underline{i}\right) + \tfrac{1}{4} ma^2 \omega^2 \sin 30° \cos 30° \underline{k}$$

Computing the vector products and dividing by $ma \cos 30°$

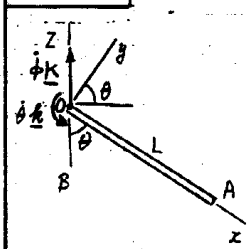
$$-g \underline{k} = -\frac{a\omega^2}{\tan 30°} \cos 30° \underline{k} + \tfrac{1}{4} a\omega^2 \sin 30° \underline{k}$$

$$g = \frac{a\omega^2}{\sin 30°} \left(\cos^2 30° - \tfrac{1}{4} \sin^2 30°\right)$$

$$g = 2a\omega^2 \left(\tfrac{3}{4} - \tfrac{1}{16}\right) = \tfrac{11}{8} a\omega^2$$

$$\omega = \sqrt{\frac{8}{11} \frac{g}{a}}$$

18.129



We choose a reference frame Oxyz attached to the rod, with x along the rod, y in the vertical plane of the rod, and z horizontal (out of the plane of the figure).

We have $\underline{\omega} = \dot{\phi} \underline{K} + \dot{\theta} \underline{k}$

$$\underline{\omega} = -\dot{\phi} \cos\theta \underline{i} + \dot{\phi} \sin\theta \underline{j} + \dot{\theta} \underline{k}$$

$$\underline{H}_O = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + I_z \omega_z \underline{k}$$

$$\underline{H}_O = 0 + \tfrac{1}{3} mL^2 \dot{\phi} \sin\theta \underline{j} + \tfrac{1}{3} mL^2 \dot{\theta} \underline{k}$$

$$H_Z = \underline{H}_O \cdot \underline{K} = \left(\tfrac{1}{3} mL^2 \dot{\phi} \sin\theta \underline{j} + \tfrac{1}{3} mL^2 \dot{\theta} \underline{k}\right) \cdot \underline{K}$$

Since $\underline{j} \cdot \underline{K} = \sin\theta$ and $\underline{k} \cdot \underline{K} = 0$:

$$H_Z = \tfrac{1}{3} mL^2 \dot{\phi} \sin^2\theta$$

Since $H_Z = \text{const.}$: $\dot{\phi} \sin^2\theta = \text{const.}$

or, using the initial conditions $\dot{\phi}_0$ and $\theta_0 = 90°$:

$$\dot{\phi} \sin^2\theta = \dot{\phi}_0 \qquad (1)$$

We now apply the principle of conservation of energy

We have $T = \tfrac{1}{2}\left(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2\right)$

$$T = \tfrac{1}{2}\left(0 + \tfrac{1}{3} mL^2 \dot{\phi}^2 \sin^2\theta + \tfrac{1}{3} mL^2 \dot{\theta}^2\right)$$

and $V = -W\left(\tfrac{1}{2} L \cos\theta\right)$ (with $V = 0$ for $\theta = 90°$)

$T + V = \text{const.}$: $\tfrac{1}{6} mL^2 \left(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2\right) - \tfrac{1}{2} mgL \cos\theta = \text{const.}$

or, with the initial conditions $\dot{\phi}_0$, $\theta_0 = 90°$, $\dot{\theta}_0 = 0$:

$$\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2 - 3\frac{g}{L} \cos\theta = \dot{\phi}_0^2 \qquad (2)$$

Substituting for $\dot{\phi}$ from (1) into (2) and solving for $\dot{\theta}^2$:

$$\dot{\theta}^2 = 3\frac{g}{L} \cos\theta + \dot{\phi}_0^2 - \frac{\dot{\phi}_0^2}{\sin^4\theta} \sin^2\theta$$

$$\dot{\theta}^2 = 3\frac{g}{L} \cos\theta - \dot{\phi}_0^2 \frac{\cos^2\theta}{\sin^2\theta} \qquad (3)$$

(a) For θ minimum, we have $\dot{\theta} = 0$, and (3) yields

$$\dot{\phi}_0^2 = 3\frac{g}{L} \frac{\sin^2\theta_m}{\cos\theta_m} \qquad (4)$$

Setting $\sin^2\theta_m = 1 - \cos^2\theta_m$ and $\dot{\phi}_0^2 = Bg/L$ in (4):

$$B = 3\left(\frac{1 - \cos^2\theta_m}{\cos\theta_m}\right)$$

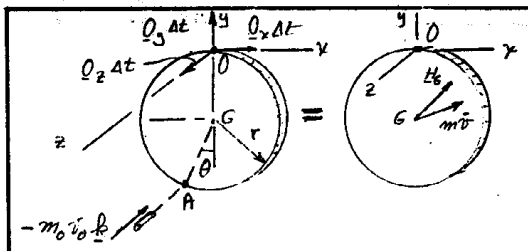
$$\cos^2\theta_m + \frac{B}{3} \cos\theta_m - 1 = 0$$

Solving: $\cos\theta_m = \frac{1}{3}$ $\qquad \theta_m = 70.5°$ ◀

(b) From (1):

$$\dot{\phi} = \frac{\dot{\phi}_0}{\sin^2\theta_m} = \frac{\dot{\phi}_0}{1 - \cos^2\theta_m} = \frac{\sqrt{Bg/L}}{1 - (1/3)^2} = \frac{\sqrt{Bg/L}}{8/9}$$

$$\dot{\phi} = 9\sqrt{g/8L}$$ ◀



PROBLEM 18.C1

PRINC. IMPULSE-MOMENTUM

$\underline{r}_{A/O} \times (-m_0 v_0 \underline{k}) = \underline{H}_O$

$[-r\sin\theta\,\underline{i} - (r + r\cos\theta)\underline{j}] \times (-m_0 v_0 \underline{k}) = \underline{H}_O$

$\underline{H}_O = +m_0 v_0 r(1+\cos\theta)\underline{i} - m_0 v_0 r \sin\theta\,\underline{j}$

$\begin{cases} (H_O)_x = m_0 v_0 r(1+\cos\theta) \\ (H_O)_y = -m_0 v_0 r \sin\theta \\ (H_O)_z = 0 \quad (\text{THUS } \omega_z = 0) \end{cases}$

$\underline{r}_{A/O} = -r\sin\theta\,\underline{i} - (r + r\cos\theta)\underline{j}$

$I_x = \frac{1}{4} m r^2 + m r^2 = \frac{5}{4} m r^2$

$I_y = \frac{1}{4} m r^2$

LET: CONST $= \frac{m_0 v_0}{m r}$ ◁

$(H_O)_x = I_x \omega_x$: $m_0 v_0 r(1+\cos\theta) = \frac{5}{4} m r^2 \omega_x$; $\omega_x = \frac{4}{5}(1+\cos\theta)$(CONST) ◁

$(H_O)_y = I_y \omega_y$: $-m_0 v_0 r \sin\theta = \frac{1}{4} m r^2 \omega_y$; $\omega_y = -4\sin\theta$ (CONST) ◁

$\omega = \sqrt{\omega_x^2 + \omega_y^2}$ ◁

ω LIES IN xy PLANE AND FORMS ANGLE PHI WITH THE x AXIS.

$\phi = \tan^{-1}(\omega_y / \omega_x)$ ◁

```
10 '*****************************************************************
20 '*                        PROBLEM 18.C1                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '              M = mass of disk          (kg)
90 '              R = radius of disk        (m)
100 '            MO = mass of bullet        (kg)
110 '            VO = velocity of bullet    (m/s)
120 '            TH = angle theta           (degrees)
130 '         OMEGA = angular velocity of disk after impact  (rad/s)
140 '        OMEGAX = x component of OMEGA
150 '        OMEGAY = y component of OMEGA
160 '        OMEGAZ = z component of OMEGA
170 '           PHI = angle formed by OMEGA and x axis     (degrees)
180 '
190     LET M = 8        'kg
200     LET MO = .02     'kg
210     LET R = .2       'm
220     LET VO = 250     'm/s
230     PI = 4*ATN(1)
240     K = PI/180
250 '
260     PRINT " THETA         OMEGAX          OMEGAY  ",
270     PRINT "   OMEGA             PHI"
280     PRINT "degrees         rad/s           rad/s ",
290     PRINT "   rad/s           rad/s"
300     PRINT
310 '
320     CONST = (MO/M)*(VO/R)
330     FOR TH = 0 TO 180 STEP 10
340         OMEGAX = .8*(1 + COS(K*TH))*CONST
350         OMEGAY = -4*SIN(K*TH)*CONST
360         OMEGA = SQR(OMEBAX^2 + OMEBAY^2)
370         IF OMEGAX <> 0 THEN 410
380           PRINT " 180           0.000          0.000",
390           PRINT "   0.000      indeterminate"
400           GOTO 460
410         PHI = (ATN(OMEGAY/OMEGAX))/K
420         PRINT USING " ###         ";TH,
430         PRINT USING " ###.###       ";OMEGAX,OMEGAY,
440         PRINT USING " ###.###       ";OMEBA,PHI
450     NEXT
460     PRINT:PRINT " Instantaneous axis of rotation passes through O,
470     PRINT "   lies in the xy plane and forms angle PHI with x axis.
```

THETA degrees	OMEGAX rad/s	OMEGAY rad/s	OMEGA rad/s	PHI rad/s
0	5.000	0.000	5.000	0.000
10	4.962	-2.171	5.416	-23.627
20	4.849	-4.275	6.465	-41.401
30	4.665	-6.250	7.799	-53.262
40	4.415	-8.035	9.168	-61.211
50	4.107	-9.576	10.419	-66.785
60	3.750	-10.825	11.456	-70.893
70	3.355	-11.746	12.216	-74.059
80	2.934	-12.310	12.655	-76.594
90	2.500	-12.500	12.748	-78.690
100	2.066	-12.310	12.482	-80.473
110	1.645	-11.746	11.861	-82.028
120	1.250	-10.825	10.897	-83.413
130	0.893	-9.576	9.617	-84.672
140	0.585	-8.035	8.056	-85.837
150	0.335	-6.250	6.259	-86.932
160	0.151	-4.275	4.278	-87.980
170	0.038	-2.171	2.171	-88.998
180	0.000	0.000	0.000	indeterminate

Instantaneous axis of rotation passes through O,
lies in the xy plane and forms angle PHI with x axis.

```
C*****************************************************************
C*                        PROBLEM 18.C1                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C           Definition of Symbols
```

SEE LIST AT LEFT

```
      REAL  M, MO, K
C       Data  (using units of kg, m, and s)
      M = 8.
      MO = 0.02
      R = 0.2
      VO = 250.
      PI = 4.*ATAN(1.)
      K = PI/180.
C
      WRITE (*,'(/,2X,2A\)') '  THETA          OMEGAX          OMEGAY',
     +    '          OMEGAZ          PHI'
      WRITE (*,'(/,2X,2A\)') ' degrees          rad/s           rad/s',
     +    '           rad/s           rad/s'
      WRITE (*,'(/,A\)')
C
      CONST = (MO/M)*(VO/R)
C
      DO 10 I = 0,180,10
          TH = I
      OMEGAX  = 0.8*(1.+ COS(K*TH))*CONST
      OMEGAY  = -4.*SIN(K*TH)*CONST
      OMEGA = SQRT(OMEGAX **2 + OMEGAY **2)
      IF (OMEGAX.NE.0.) GO TO 21
          WRITE (*,20)
   20     FORMAT (4X,'180.',3(12X,'.000'),6X,'indeterminate')
          GO TO 10
   21 PHI = (ATAN(OMEGAY /OMEGAX ))/K
      WRITE (*,30) TH,OMEGAX ,OMEGAY ,OMEGA, PHI
   30 FORMAT (4X,F4.0,4(9X,F7.3))
   10 CONTINUE
C
      WRITE (*,*)' '
      WRITE (*,*)'Instantaneous axis of rotation passes through O,'
      WRITE (*,*)'lies in the xy plane and forms angle PHI with x axis.'
      END
```

THETA degrees	OMEGAX rad/s	OMEGAY rad/s	OMEGAZ rad/s	PHI rad/s
0.	5.000	.000	5.000	.000
10.	4.962	-2.171	5.416	-23.627
20.	4.849	-4.275	6.465	-41.401
30.	4.665	-6.250	7.799	-53.262
40.	4.415	-8.035	9.168	-61.211
50.	4.107	-9.576	10.419	-66.785
60.	3.750	-10.825	11.456	-70.893
70.	3.355	-11.746	12.216	-74.059
80.	2.934	-12.310	12.655	-76.594
90.	2.500	-12.500	12.748	-78.690
100.	2.066	-12.310	12.482	-80.473
110.	1.645	-11.746	11.861	-82.028
120.	1.250	-10.825	10.897	-83.413
130.	.893	-9.576	9.617	-84.672
140.	.585	-8.035	8.056	-85.837
150.	.335	-6.250	6.259	-86.932
160.	.151	-4.275	4.278	-87.980
170.	.038	-2.171	2.171	-88.998
180.	.000	.000	.000	indeterminate

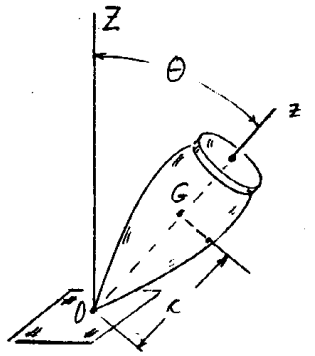
Instantaneous axis of rotation passes through O,
lies in the xy plane and forms angle PHI with x axis.

```
10 '*****************************************************************
20 '*                      PROBLEM 18.C2                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '          Definition of Symbols
70 '
80 '           W = weight of satellite   (lb)
90 '          KX = radius of gyration with respect to x axis  (in.)
100 '         KY = radius of gyration with respect to y axis  (in.)
110 '         KZ = radius of gyration with respect to z axis  (in.)
120 '
130 '     OMEGAO = initial angular velocity of satellite about y axis
140 '     OMEGAX = x component of final angular velocity
150 '     OMEGAY = y component of final angular velocity
160 '     OMEGAZ = z component of final angular velocity
170 '
180 '         WO = weight of meteorite  (oz)
190 '         VX = x component of velocity of meteorite  (ft/s)
200 '         VY = y component of velocity of meteorite  (ft/s)
210 '         VZ = z component of velocity of meteorite  (ft/s)
220 '
230 '          B = vertical distance to point of impact   (in)
240 '         RX = x coordinate of point of impact  (in)
250 '
260 '   Data
270     W = 800      'lb
280     G = 32.2       /-^2
290     KX = 28.8    'in.
300     KY = 32.4    'in.
310     KZ = 28.8    'in.
320 '
330     OMEGAO = 1.5        'rad/s
340 '
350     WO = 6       'oz
360      WO = WO/16  'express WO in pounds
370     VX = -2800   'ft/s
380     VY = +2900   'ft/s
390     VZ = -2000   'ft/s
400     RX = 42      'in.
410         RX = RX/12  'express RX in feet
420 '
430 '     Centroidal mass moments of inertia of satellite
440     IX = (W/G)*(KX/12)^2
450     IY = (W/G)*(KY/12)^2
460     IZ = (W/G)*(KZ/12)^2
470 '
480     PRINT "  b      omega x       omega y       omega z",
490     PRINT "       omega         delta T"
500     PRINT " in.      rad/s         rad/s         rad/s",
510     PRINT "       rad/s          ft.lb";PRINT
520 '
530     FOR B = 0 TO 30 STEP 2
540         B = B/12      'express B in feet
550 '         'Angular momentum of system
560         HX = -(WO/G)*VZ*B
570         HY = -(WO/G)*VZ*RX + IY*OMEGAO
580         HZ = (WO/G)*(VY*RX + VX*B)
590 '
600         OMEGAX = HX/IX
610         OMEGAY = HY/IY
620         OMEGAZ = HZ/IZ
630         OMEGA = SQR(OMEGAX^2 + OMEGAY^2 + OMEGAZ^2)
640 '
650 '         'Change in kinetic energy of rotation
660         T1 = .5*IY*(OMEGAO^2)
670         T2 = .5*(IX*(OMEGAX^2) +IY*(OMEGAY^2) + IZ*(OMEGAZ^2))
680         DELTAT = T2 - T1
690         B = B*12    'express B in inches for printing answers
700         PRINT USING " ##     ";B,
710         PRINT USING "##.####       ";OMEGAX,OMEGAY,OMEGAZ,
720         PRINT USING " ##.####        ";OMEGA,
730         PRINT USING "###.##        ";DELTAT
740     NEXT
```

```
RUN
  b        omega.x       omega.y       omega.z        omega        delta T
 in         rad/s         rad/s         rad/s         rad/s          ft.lb

  0       0.0000        1.9501        0.8260        2.1178         189.45
  2       0.0271        1.9501        0.7880        2.1035         185.12
  4       0.0543        1.9501        0.7501        2.0901         181.09
  6       0.0814        1.9501        0.7121        2.0776         177.38
  8       0.1085        1.9501        0.6741        2.0662         173.99
 10       0.1356        1.9501        0.6361        2.0557         170.90
 12       0.1628        1.9501        0.5981        2.0463         168.12
 14       0.1899        1.9501        0.5602        2.0378         165.66
 16       0.2170        1.9501        0.5222        2.0304         163.51
 18       0.2441        1.9501        0.4842        2.0241         161.67
 20       0.2713        1.9501        0.4462        2.0188         160.14
 22       0.2984        1.9501        0.4083        2.0146         158.93
 24       0.3255        1.9501        0.3703        2.0115         158.02
 26       0.3526        1.9501        0.3323        2.0094         157.43
 28       0.3798        1.9501        0.2943        2.0084         157.15
 30       0.4069        1.9501        0.2563        2.0085         157.18
```

```
C*****************************************************************
C*                      PROBLEM 18.C2                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C          Definition of Symbols
C
C               W = weight of satellite  (lb)
C              KX = radius of gyration with respect to x axis  (in.)
C              KY = radius of gyration with respect to y axis  (in.)
C              KZ = radius of gyration with respect to z axis  (in.)
C
C          OMEGAO = initial angular velocity of satellite about y axis
C          OMEGAX = x component of final angular velocity
C          OMEGAY = y component of final angular velocity
C          OMEGAZ = z component of final angular velocity
C
C               WO = weight of meteorite  (oz)
C               VX = x component of velocity of meteorite   (ft/s)
C               VY = y component of velocity of meteorite   (ft/s)
C               VZ = z component of velocity of meteorite   (ft/s)
C
C                B = vertical distance to point of impact  (in)
C               RX = x coordinate of point of impact  (in)
C
      REAL OMEGAO, OMEGAX, OMEGAY, OMEGAZ, KX, KY, KZ, IX, IY, IZ
C       Data:
      W = 800.
      G = 32.2
      KX = 28.8
      KY = 32.4
      KZ = 28.8
      OMEGAO = 1.5
      WO = 6.
C        Express WO in pounds
      WO = WO/16.
      VX = -2800.
      VY = +2900.
      VZ = -2000.
      RX = 42.
C        Express RX in feet
      RX = RX/12.
C        Centroidal mass moments of inertia of satellite
      IX = (W/G)*(KX/12.)**2
      IY = (W/G)*(KY/12.)**2
      IZ = (W/G)*(KZ/12.)**2
C
      WRITE (*,'(/,2X,2A\)') ' b        omega x       omega y      ',
     +' omega z       omega         delta T'
      WRITE (*,'(/,2X,2A\)') 'in.         rad/s         rad/s        ',
     +'rad/s         rad/s         ft.lb'
      WRITE (*,'(/,A\)')
C
      DO 10 I = 0,30,2
         B = I
C           Express B in feet
         B = B/12.
C           Angular momentum of system
         HX = -(WO/G)*VZ*B
         HY = -(WO/G)*VZ*RX + IY*OMEGAO
         HZ = (WO/G)*(VY*RX + VX*B)
         OMEGAX = HX/IX
         OMEGAY = HY/IY
         OMEGAZ = HZ/IZ
         OMEGA = SQRT(OMEGAX**2 + OMEGAY**2 + OMEGAZ**2)
C           Change in kinetic energy of rotation
         T1 = 0.5*IY*(OMEGAO**2)
         T2 = 0.5*(IX*(OMEGAX**2) + IY*(OMEGAY**2) + IZ*(OMEGAZ**2))
         DELTAT = T2 - T1
C           Express B in inches for printing answers
         B = B*12.
         WRITE (*,20) B,OMEGAX,OMEGAY,OMEGAZ,OMEGA,DELTAT
   20    FORMAT (2X,F3.0,4(7X,F6.4),7X,F6.2)
   10 CONTINUE
      END
```

```
  b        omega x       omega y       omega z        omega        delta T
 in.         rad/s         rad/s         rad/s         rad/s          ft.lb

  0.        .0000        1.9501        .8260        2.1178         189.45
  2.        .0271        1.9501        .7880        2.1035         185.12
  4.        .0543        1.9501        .7501        2.0901         181.09
  6.        .0814        1.9501        .7121        2.0776         177.38
  8.        .1085        1.9501        .6741        2.0662         173.99
 10.        .1356        1.9501        .6361        2.0557         170.90
 12.        .1628        1.9501        .5981        2.0463         168.12
 14.        .1899        1.9501        .5602        2.0378         165.66
 16.        .2170        1.9501        .5222        2.0304         163.51
 18.        .2441        1.9501        .4842        2.0241         161.67
 20.        .2713        1.9501        .4462        2.0188         160.14
 22.        .2984        1.9501        .4083        2.0146         158.93
 24.        .3255        1.9501        .3703        2.0115         158.02
 26.        .3526        1.9501        .3323        2.0094         157.43
 28.        .3798        1.9501        .2943        2.0084         157.15
 30.        .4069        1.9501        .2563        2.0085         157.18
```



PROBLEM 18.C3

WITH RESPECT TO AXIS OF SYMMETRY, Z AXIS.

$$I = mk^2$$

WITH RESPECT TO TRANSVERSE AXIS THROUGH O.

$$I' = mk'^2$$

FROM STATEMENT OF PROB. 18.85:

$$(I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi} = Wr$$

RECALLING FROM PAGE 836 THAT $\omega_z = \dot{\psi} + \dot{\phi}\cos\theta$, WE WRITE

$$[I(\dot{\psi} + \dot{\phi}\cos\theta) - I'\dot{\phi}\cos\theta]\dot{\phi} = mgr$$

REARRANGING TERMS

$$(I' - I)\cos\theta\,\dot{\phi}^2 - I\dot{\psi}\dot{\phi} + mgr = 0$$

$$(mk'^2 - mk^2)\cos\theta\,\dot{\phi}^2 - mk^2\dot{\psi}\dot{\phi} + mgr = 0$$

DIVIDE BY m:

$$\left[(k'^2 - k^2)\cos\theta\right]\dot{\phi}^2 + \left[-k^2\dot{\psi}\right]\dot{\phi} + gr = 0$$

WRITE QUADRATIC FORMULA AS: $A\dot{\phi}^2 + B\dot{\phi} + (SEE) = 0$

$$\begin{cases} A = (k'^2 - k^2)\cos\theta \\ B = -k^2\dot{\psi} \\ SEE = gr \end{cases}$$

LOWER VALUE OF $\dot{\phi}$ IS:

$$\dot{\phi} = \left(-B - \sqrt{B^2 - 4A(SEE)}\right)/(2A)$$

FROM PROB. 18.85(b) THE APPROXIMATE VALUE OF $\dot{\phi}$ IS:

$$\dot{\phi}_{APPROX} = (Wr)/(I\dot{\psi}) = (mgr)/(mk^2\dot{\psi})$$

$$\dot{\phi}_{APPROX} = gr/k^2\dot{\psi}$$

$$\%\,ERROR = \frac{\dot{\phi}_{APPROX} - \dot{\phi}}{\dot{\phi}}(100)$$

```
10 '*****************************************************************
20 '*                        PROBLEM 18.C3                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '          C = distance from tip of top to its mass center
90 '          K = radius of gyration with respect to axis of symmetry
100 '        K1 = radius of gyration with respect to transverse axis
110 '       PSI. = psi dot = rate of spin of top
120 '       PHI. = phi dot = rate of precession
130 '
140   LET C = 1.5/12      'in.
150   LET K = .84/12      'in.
160   LET K1 = 1.8/12     'in.
170   LET G = 32.2        'ft/s^2
180 '
190 ' Read rates of spin in rpm  (PSI.)
200   DATA 1050,1200,1500,1800,2400,6000,0
210   READ  PSI.
220   IF PSI. = 0 GOTO 570
230 '
240   PI = 4*ATN(1)
250   J = PI/180
260 '
270   PRINT "-------------------------------------------------------------"
280   PRINT USING "      Rate of spin = #### rpm";PSI.:PRINT
290   PRINT "             - - - - Rate of Precession - - - -              "
300   PRINT "  Theta      exact          approx        % error
310   PRINT "(degrees)   (rpm)           (rpm)":PRINT
320 '
330   PSI. = (PSI.)*PI/30      'express spin in rad/s
340 '
```

CONTINUED

```
C*****************************************************************
C*                        PROBLEM 18.C3                          *
C*                        FORTRAN VERSION                        *
C*****************************************************************
C
C           Definition of Symbols
C
C             C = distance from tip of top to its mass center
C             K = radius of gyration with respect to axis of symmetry
C            K1 = radius of gyration with respect to transverse axis
C        PSIDOT = psi dot = rate of spin of top
C        PHIDOT = phi dot = rate of pecession
C
      REAL K, K1, I, J, PSIDOT(7)
C
      C = 1.5/12
      K = 0.84/12.
      K1 = 1.8/12.
      G = 32.2
C        Rates of spin in rpm.
      DATA PSIDOT/1050.,1200.,1500.,1800.,2400.,6000.,0.0/
      PI = 4.*ATAN(1.)
      J = PI/180.
C
      M = 1
    9 WRITE (*,11) PSIDOT(M)
   11 FORMAT (65('-'),//,3X,'Rate of spin  = ',F5.0,' rpm',/,16X,'- - -
     +- Rate of Precession - - -')
C
      WRITE (*,*)' Theta        exact         approx       % error'
      WRITE (*,*)'degrees       (rpm)          (rpm)'
      WRITE (*,*) ' '
C
C        Express spin in rad/s; denote spin in rad/s by PSII
      PSII = PSIDOT(M)*PI/30.
```

CONTINUED

PROBLEM 18.C3 (CONTINUED)

BASIC VERSION CONTINUED

```
350   FOR TH = 15 TO 45 STEP 15
360 '
370 '       'Solve quadratic for rate of precession, (PHI.)
380       A = (K1^2 - K^2)*COS(J*TH)
390       B = -K^2*PSI.
400       SEE = G*C
410       PHI.  = (-B-SQR(B^2 - 4*A*SEE))/(2*A)
420 '
430 '       'Solve for approximate rate of precession, (APPROX)
440       APPROX = (G*C)/(K^2*(PSI.))
450 '
460 '       'Express rates of precession in rpm
470       APPROX = APPROX*30/PI
480       PHI. = (PHI.)*30/PI
490 '
500 '       'Calculate percent error introduced by approximation
510       PERCENT = ((APPROX - PHI.)/PHI.)*100
520       PRINT USING "   ##      "¦TH,
530       PRINT USING " ###.##          "¦PHI.,APPROX,
540       PRINT USING " ###.##      "¦PERCENT
550   NEXT
560   GOTO 210
570   END
```

Rate of spin = 1050 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	115.15	71.34	-38.05
30	102.41	71.34	-30.34
45	91.66	71.34	-22.17

Rate of spin = 1200 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	81.74	62.42	-23.63
30	78.32	62.42	-20.30
45	74.02	62.42	-15.67

Rate of spin = 1500 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	57.61	49.94	-13.33
30	56.57	49.94	-11.73
45	55.07	49.94	-9.32

Rate of spin = 1800 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	45.63	41.61	-8.79
30	45.13	41.61	-7.80
45	44.40	41.61	-6.26

Rate of spin = 2400 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	32.76	31.21	-4.74
30	32.59	31.21	-4.22
45	32.32	31.21	-3.42

Rate of spin = 6000 rpm

Theta (degrees)	exact (rpm)	approx (rpm)	% error
15	12.58	12.48	-0.73
30	12.57	12.48	-0.65
45	12.55	12.48	-0.53

FORTRAN VERSION CONTINUED

```
C
      DO 10 N = 15,45,15
         TH = N
C
C        Solve quadratic for rate of precession, (PHI)
           A = (K1**2 - K**2)*COS(J*TH)
           B = -K**2*PSII
         SEE = G*C
         PHIDOT = (-B-SQRT(B**2 - 4.*A*SEE))/(2.*A)
C
C        Solve for approximate rate of precession, (APPROX)
       APPROX = (G*C)/(K**2*(PSII))
C
C        Express rates of precession in rpm
       APPROX = APPROX*30./PI
         PHIDOT = PHIDOT*30./PI
C
C        Calculate percent error introduced by approximation
      PERCENT = ((APPROX - PHIDOT)/PHIDOT)*100.
      WRITE (*,20) TH,PHIDOT,APPROX,PERCENT
   20 FORMAT (4X,F3.0,3(7X,F6.2))
   10 CONTINUE
      M = M + 1
      IF (PSIDOT(M).GT.0) GOTO 9
      END
```

Rate of spin = 1050. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	115.15	71.34	-38.05
30.	102.41	71.34	-30.34
45.	91.66	71.34	-22.17

Rate of spin = 1200. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	81.74	62.42	-23.63
30.	78.32	62.42	-20.30
45.	74.02	62.42	-15.67

Rate of spin = 1500. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	57.61	49.94	-13.33
30.	56.57	49.94	-11.73
45.	55.07	49.94	-9.32

Rate of spin = 1800. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	45.63	41.61	-8.79
30.	45.13	41.61	-7.80
45.	44.40	41.61	-6.26

Rate of spin = 2400. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	32.76	31.21	-4.74
30.	32.59	31.21	-4.22
45.	32.32	31.21	-3.42

Rate of spin = 6000. rpm

Theta degrees	exact (rpm)	approx (rpm)	% error
15.	12.58	12.48	-.73
30.	12.57	12.48	-.65
45.	12.55	12.48	-.53

PROBLEM 18.C4

FROM ANSWER TO PROB 18.110: $\dot{\theta}^2 = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgk\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2$ ①

$$E = \frac{1}{2}\left[I'(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2) + I(\dot{\psi} + \dot{\phi}\cos\theta)^2\right] + mgk\cos\theta$$

WHERE $\beta = I(\dot{\psi} + \dot{\phi}\cos\phi)$ $\alpha = H_z = \text{CONSTANT} = I'\dot{\phi}\sin^2\theta + I(\dot{\psi} + \dot{\phi}\cos\theta)\cos\theta$

WE CHOOSE TO WRITE ① AS: $\dot{\theta}^2 = f(\theta)$ ②

WHERE, $f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgk\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2$ ③

INITIAL CONDITIONS: $\theta = \theta_0$, $\dot{\phi} = \dot{\phi}_0 = 0$ $\dot{\theta} = \dot{\theta}_0 = 0$, $\dot{\psi} = \dot{\psi}_0$. THESE VALUES YIELD THE FOLLOWING.

$$\beta = I\dot{\psi}_0, \quad \alpha = I\dot{\psi}_0\cos\theta_0, \quad E = \frac{1}{2}I\dot{\psi}_0^2 + mgk\cos\theta_0$$

SUBSTITUTE β, α, AND E INTO ③:

$$f(\theta) = \frac{1}{I'}\left(I\dot{\psi}_0^2 + 2mgk\cos\theta_0 - \frac{I^2\dot{\psi}_0^2}{I} - 2mgk\cos\theta\right) - \left(\frac{I\dot{\psi}_0\cos\theta_0 - I\dot{\psi}_0\cos\theta}{I'\sin\theta}\right)^2$$

$$= \frac{1}{I'}\left[2mgk(\cos\theta_0 - \cos\theta)\right] - \frac{I^2}{I'^2}\frac{\dot{\psi}_0^2}{\sin^2\theta}(\cos\theta_0 - \cos\theta)^2$$

$$f(\theta) = (\cos\theta_0 - \cos\theta)\left(\frac{2mgk}{I'} - \frac{I^2}{I'^2}\dot{\psi}_0^2\frac{\cos\theta_0 - \cos\theta}{\sin^2\theta}\right)$$

WE SHALL SET: $I = mk^2$ $I' = mk'^2$ $u = \frac{\cos\theta_0 - \cos\theta}{\sin^2\theta}$ ④ ◁

$$f(\theta) = (\cos\theta_0 - \cos\theta)\left(\frac{2gk}{k'^2} - \frac{k^4}{k'^4}\dot{\psi}_0^2 u\right)$$ ⑤

FOR MAX. VALUE OF θ: $\theta = \theta_m$ AND THUS $\dot{\theta} = 0$.

FROM ② WE HAVE $f(\theta) = 0$, AND FROM ⑤, WE FIND $\frac{2gk}{k'^2} - \frac{k^4}{k'^4}\dot{\psi}_0^2 u_m = 0$

$$\dot{\psi}_0^2 = \frac{2gk}{u_m}\cdot\frac{k'^2}{k^4}$$ ◁ WHERE, $u_m = \frac{\cos\theta_0 - \cos\theta_m}{\sin^2\theta_m}$

SUBSTITUTE FOR $\dot{\psi}_0^2$ INTO ⑤: $f(\theta) = (\cos\theta_0 - \cos\theta)\left(\frac{2gk}{k'^2} - \frac{k^4}{k'^4}\cdot\frac{2gk}{u_m}\cdot\frac{k'^2}{k^4}u\right)$

REARRANGING, WE HAVE $f(\theta) = (\cos\theta_0 - \cos\theta)\frac{2gk}{k'^2}\left(1 - \frac{u}{u_m}\right)$ ◁

FROM ② $\dot{\theta} = \sqrt{f(\theta)}$ OR DENOTING θ BY w, WE HAVE: $w = \sqrt{f(\theta)}$ ◁

KINEMATICS CONSIDER MOTION OF TOP AS THE ANGLE OF NUTATION θ MOVES THROUGH ANGLE $\Delta\theta$ FROM

$\theta = \theta_i$ WHERE $w = w_i$

TO $\theta = \theta_{i+1}$ WHERE $w = w_{i+1}$

IF WE ASSUME CONSTANT ANGULAR ACCELERATION DURING TIME INTERVAL Δt, THE AVERAGE ANGULAR VELOCITY IS:

$$w_{AVG} = \frac{1}{2}(w_i + w_{i+1})$$ ◁

AND THE TIME REQUIRED FOR THE ANGLE OF NUTATION TO INCREASE BY $\Delta\theta$ IS:

$$\Delta t = \frac{\Delta\theta}{w_{Avg}}$$ ◁

(CONTINUED)

WE THEN SUM Δt TO FIND TIME FOR θ TO VARY FROM 30° TO 90° AND BACK TO 30°. THIS TIME INTERVAL IS THE PERIOD OF OSCILLATION OF THE CUSPIDAL MOTION.

PROBLEM 18.C4 CONTINUED

```
10 '*****************************************************************
20 '*'                        PROBLEM 18.C4                          *
30 '*                        BASIC VERSION                          *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '          T = time  (seconds)
90 '         TH = theta, angle of nutation  (degrees)
100 '       TH0 = initial value of angle of nutation
110 '       THM = maximum value of angle of nutation
120 '     OMEGA = rate of change of angle of nutation (dTH/dt)
130 '       DTH = increment of angle theta  (degrees)
140 '    OMINIT = value of OMEGA at beginning of interval DTH
150 '    OMFINL = value of OMEGA at end of interval DTH
160 '     OMAVG = average value of OMEGA over interval DTH
170 '   FNF(TH) = function F(TH)
180 '
190 '         C = distance from tip of top to its mass center
200 '         K = radius of gyration with respect to axis of symmetry
210 '        K1 = radius of gyration with respect to transverse axis
220 '
230 '         Y = counter
240 '       NBR = number of increments in a printing interval
250 '
260 '     Data
270     G = 9.810001    'acceleration of gravity  m/s^2
280     C = .036    'meters
290     K = .02     'meters
300     K1 = .045   'meters
310     TH0 = 30    'degrees
320     THM = 40    'degrees
330     NBR = 5
340     DTH = .1    'degrees
350     PI = 4*ATN(1)
360     J = PI/180
370 '
380     PRINT "  theta          time           omega   "
390     PRINT " degrees        seconds        rad/s   ";PRINT
400 '
410     DEF FNA(TH) = COS(J*TH0) - COS(J*TH)
420     DEF FNB(TH) = (FNA(TH))/(SIN(J*TH))^2   'This is "u".
430     DEF FNF(TH) = FNA(TH)*(2*G*C/K1^2)*(1 - FNB(TH)/FNB(THM))
440 '
450     LET TH = TH0
460     WHILE Y <= (THM - TH0)/DTH
470        IF INT(Y/NBR) = Y/NBR THEN 480 ELSE 500
480        PRINT USING "  ##.##      ";TH,
490        PRINT USING "   #.####       ";T,OMINIT
500        LET Y = INT(Y+1)
510        LET TH = TH0 + Y * DTH
520        LET OMFINL = SQR(ABS(FNF(TH)))
530        OMAVG = .5*(OMINIT + OMFINL)
540        T = T + (J*DTH/OMAVG)
550        OMINIT = OMFINL
560     WEND
570     T = T - (J*DTH/OMAVG)
580     PERIOD = 2*T:PRINT
590     PRINT USING " Period of oscillation = #.#### seconds";PERIOD
600     PRINT
610 '   Computation of initial rate of spin
620     TH = THM
630     SPIN0 = SQR(2*G*C/(FNB(TH)))*K1/K^2
640     SPIN0 = SPIN0 *60/(2*PI)  'to express spin in rpm
650     PRINT
660     PRINT USING " Initial rate of spin = ####.# rpm"; SPIN0
```

theta degrees	time seconds	omega rad/s
30.00	0.0000	0.0000
30.50	0.0143	1.1938
31.00	0.0285	1.6320
31.50	0.0254	1.9295
32.00	0.0297	2.1475
32.50	0.0336	2.3102
33.00	0.0372	2.4301
33.50	0.0408	2.5145
34.00	0.0442	2.5680
34.50	0.0476	2.5936
35.00	0.0509	2.5929
35.50	0.0543	2.5667
36.00	0.0578	2.5150
36.50	0.0613	2.4370
37.00	0.0649	2.3306
37.50	0.0688	2.1924
38.00	0.0729	2.0165
38.50	0.0775	1.7926
39.00	0.0828	1.5001
39.50	0.0895	1.0855
40.00	0.1054	0.0000

Period of oscillation = 0.2108 seconds

Initial rate of spin = 1835.4 rpm

```
C*****************************************************************
C*                        PROBLEM 18.C4                          *
C*                        FORTRAN VERSION.                       *
C*****************************************************************
C
C           Definition of Symbols
C
```

SEE LIST AT LEFT

```
C
      REAL OMEGA, OMINIT, OMFINL, OMAVG, K, K1, NBR, J
C     Define functions:
      FNA(TH) = COS(J*TH0) - COS(J*TH)
      FNB(TH) = (FNA(TH))/(SIN(J*TH))**2
C                     Note:  FNB(TH) is "u"
      FNF(TH) = FNA(TH)*(2.*G*C/K1**2)*(1.- FNB(TH)/FNB(THM))
C
      G = 9.81
      C = 0.036
      K = 0.02
      K1 = 0.045
      TH0 = 30.
      THM = 40.
      NBR = 5.
      DTH = 0.1
      PI = 4.*ATAN(1.)
      J = PI/180.
C
      WRITE (*,*) ' theta          time          omega'
      WRITE (*,*) 'degrees        seconds       rad/s'
      WRITE (*,*) ' '
C
      TH = TH0
C
   99 IF (INT(Y/NBR).EQ.(Y/NBR)) THEN
         WRITE(*,20) TH,T,OMINIT
   20    FORMAT (3X,F5.2,2(8X,F6.4))
      ENDIF
      Y = INT(Y + 1)
      TH = TH0 + Y * DTH
      OMFINL = SQRT(ABS(FNF(TH)))
      OMAVG = 0.5*(OMINIT + OMFINL)
      T = T + (J*DTH/OMAVG)
      OMINIT = OMFINL
      IF (Y.LE.((THM - TH0)/DTH)) GOTO 99
C
      T = T - (J*DTH/OMAVG)
      PERIOD = 2.*T
      WRITE (*,30) PERIOD
   30 FORMAT (/,2X,'Period of oscillation  = ',F6.4,' seconds')
C
      WRITE (*,*) ' '
C     Computation of initial rate of spin
      TH = THM
      SPIN0 = SQRT(2*G*C/(FNB(TH)))*K1/K**2
      SPIN0 = SPIN0 * 60/(2*PI)
      WRITE (*,31) SPIN0
   31 FORMAT (/,2X,'Initial rate of spin = ',F6.1,' rpm')
      END
```

theta degrees	time seconds	omega rad/s
30.00	.0000	.0000
30.50	.0143	1.1938
31.00	.0285	1.6320
31.50	.0254	1.9295
32.00	.0297	2.1475
32.50	.0336	2.3102
33.00	.0372	2.4301
33.50	.0408	2.5145
34.00	.0442	2.5680
34.50	.0476	2.5936
35.00	.0509	2.5929
35.50	.0543	2.5667
36.00	.0578	2.5150
36.50	.0613	2.4370
37.00	.0649	2.3305
37.50	.0688	2.1923
38.00	.0729	2.0165
38.50	.0775	1.7926
39.00	.0828	1.5000
39.50	.0895	1.0855
40.00	.1054	.0000

Period of oscillation = .2108 seconds

Initial rate of spin = 1835.4 rpm

19.1 $f = 20$ Hz $\quad f = \frac{p}{2\pi}$; $p = 2\pi(20) = 125.66$ rad/s

AMPLITUDE: $x_m = 3$ mm

MAX. VELOCITY: $v_m = x_m p = (3\text{mm})(125.66)$ $\quad v_m = 0.377$ m/s

MAX ACCELERATION: $a_m = x_m p^2 = (3\text{mm})(125.66)^2$ $\quad a_m = 47.4$ m/s^2

19.2 EQ (19.15): $v_m = x_m p$ $\quad a_m = x_m p^2$

GIVEN DATA $\quad v_m = 0.2$ m/s $\quad a_m = 4$ m/s^2

$v_m = x_m p$: $\quad 0.2$ m/s $= x_m p$ $\quad (1)$

$a_m = x_m p^2$: $\quad 4$ m/s$^2 = x_m p^2$ $\quad (2)$

DIVIDE EQ(2) BY EQ(1): $\quad p = \frac{4\text{ m/s}^2}{0.2\text{ m/s}} = 20$ rad/s

EQ(1): $\quad 0.2$ m/s $= x_m (20$ rad/s$)$

$x_m = 0.01$ m $\quad x_m = 10$ mm

FREQUENCY $\quad f = \frac{p}{2\pi} = \frac{20\text{ rad/s}}{2\pi}$ $\quad f = 3.18$ Hz

19.3 $\tau = 1.2$ s $\quad p = \frac{2\pi}{\tau} = \frac{2\pi}{1.2\text{ s}} = 5.236$ rad/s

$x_m = 0.135$ m; EQ(19.15) $\quad v_m = x_m p = (0.135\text{ m})(5.236$ rad/s$)$

$v_m = 0.707$ m/s

$a_m = x_m p^2 = (0.135\text{ m})(5.236\text{ rad/s})^2$ $\quad a_m = 3.70$ m/s^2

19.4 $k = 5$ lb/in. $= 60$ lb/ft $\quad x_m = 3$ in. $\quad W = 7.5$ lb

$F = ma$; $\quad -kx = m\ddot{x}$ $\quad \ddot{x} + \frac{k}{m} x = 0$

THUS: $\quad p^2 = \frac{k}{m} = \frac{60}{7.5/32.2} = 257.6$ $\quad p = 16.05$ rad/s

EQ(19.15): $\quad v_m = x_m p = (3\text{ in.})(16.05\text{ rad/s}) = 48.15$ in/s

$v_m = 4.01$ ft/s

$a_m = x_m p^2 = (3\text{ in})\left(16.05\ \frac{\text{rad}^2}{\text{s}^2}\right)^2 = 772.8$ in/s^2

$a_m = 64.4$ ft/s^2

19.5 $k = 6$ lb/in. $= 72$ lb/ft, $v_m = 55$ in./s, $W = 4$ lb.

$F = ma$: $\quad -kx = m\ddot{x}$ $\quad \ddot{x} + \frac{k}{m} x = 0$

THUS: $\quad p^2 = \frac{k}{m} = \frac{72}{4/32.2} = 579.6$ $\quad p = 24.075$ rad/s

EQ(19.15): $\quad v_m = x_m p$

55 in./s $= x_m (24.075$ rad/s$)$

$x_m = 2.2845$ in. $\quad x_m = 2.28$ in.

$a_m = x_m p^2 = (2.2845\text{ in})\left(579.6\ \frac{\text{rad}^2}{\text{s}^2}\right)$

$a_m = 1324.1$ in./s^2 $\quad a_m = 110.3$ ft/s^2

19.6 AT BOTH 600 rpm AND 1200 rpm, THE MAXIMUM ACCELERATION IS JUST EQUAL TO g.

(a) $p = 600$ rpm $= 62.832$ rad/s

EQ(19.15): $\quad a_m = x_m p^2$ $\quad x_m = \frac{g}{(62.832)^2}$

SI: $\quad x_m = \frac{9.81}{(62.832)^2} = 2.4849 \times 10^{-3}$ m $\quad x_m = 2.48$ mm

US: $\quad x_m = \frac{32.2}{(62.832)^2} = 0.008156$ ft $\quad x_m = 0.0979$ in.

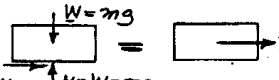
(b) $p = 1200$ rpm $= 125.664$ rad/s

EQ(19.15): $\quad a_m = x_m p^2$ $\quad x_m = \frac{g}{(125.664)^2}$

SI: $\quad x_m = \frac{9.81}{(125.664)^2} = 621.2 \times 10^{-6}$ m $\quad x_m = 0.621$ mm

US: $\quad x_m = \frac{32.2}{(125.664)^2} = 0.002039$ ft $\quad x_m = 0.0245$ in.

19.7 MAXIMUM ALLOWABLE ACCELERATION OF B



$F_m = \mu_s N$ $\quad N = W = mg$ $\quad \mu_s = 0.40$

$\pm \Sigma F = ma$: $\quad F_m = ma_m$

$\mu_s mg = ma_m$

$a_m = \mu_s g$ $\quad a_m = 0.40\,g$

SIMPLE HARMONIC MOTION

$f = 3$ Hz $= \frac{p}{2\pi}$ $\quad p = 6\pi$ rad/s

EQ(19.15): $\quad a_m = x_m p^2$

$0.40g = x_m (6\pi$ rad/s$)^2$

$x_m = 1.1258 \times 10^{-3}\, g$

SI: $\quad x_m = 1.1258 \times 10^{-3}(9.81) = 11.044 \times 10^{-3}$ m

$x_m = 11.04$ mm

US: $\quad x_m = 1.1258 \times 10^{-3}(32.2) = 0.03625$ ft

$x_m = 0.435$ in.

19.8 IF MOTION IS TO BE SIMPLE HARMONIC, THE COLLAR MUST REMAIN IN CONTACT WITH THE SPRING. THUS $a_m \le g$

FOR $a_m = g$ AND $x_m = 75$ mm $= 0.075$ m

EQ. (19.15): $\quad a_m = x_m p^2$, $\quad 9.81$ m/s$^2 = (0.075$ m$)\, p^2$

$p^2 = 130.8$ rad^2/s^2 $\quad p = 11.437$ rad/s

(a) FROM $F = ma$: $\quad -kx = m\ddot{x}$ $\quad \ddot{x} + \frac{k}{m} x = 0$

THUS $\quad \frac{k}{m} = p^2$ $\quad k = p^2 m = (130.8\text{ rad}^2/\text{s}^2)(2.5\text{ kg})$

$k = 327.0$ N/m $\quad k = 327$ N/m

(b) MEASURE THE DISPLACEMENT $x \downarrow +$ TO CONFORM WITH FIG 19.2 OF TEXT.

AT $t = 0$: $\quad x = x_m = 0.075$ m, AND $pt = 0$

THUS FROM FIG. 19.2: $\quad \phi = \frac{\pi}{2} = 90°$

AT $t = 0.12$ s: $\quad pt = (11.437\text{ rad/s})(0.12\text{ s}) = 1.3724$ rad

OR $\quad pt = 78.634°$

$pt + \phi = 78.634° + 90° = 168.634°$

FROM EQ. (19.10):

$x = x_m \sin(pt + \phi) = (0.075\text{ m}) \sin 168.634° = +14.781 \times 10^{-3}$ m

$x = 14.78$ mm

(BELOW EQUIL. POSITION)

FROM EQ. (19.11):

$v = x_m p \cos(pt + \phi) = (0.075\text{ m})(11.437\text{ rad/s}) \cos 168.634°$

$= -0.8409$ m/s

$v = 841$ mm/s ↑

FROM EQ(19.12):

$a = -x_m p^2 \sin(pt + \phi) = -(0.075\text{ m})(130.8\text{ rad}^2/\text{s}^2) \sin 168.634°$

$= -1.9333$ m/s^2

$a = 1.933$ m/s^2 ↑

19.9

FOR MASS-SPRING SYSTEM:

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{900\ N/m}{5\ kg}} = 13.416\ rad/s$$

MEASURING THE DISPLACEMENT x ↓+ TO CONFORM WITH FIG. 19.2, WE HAVE:

AT $t = 0$: $x = x_m = 50\ mm = 0.05\ m$

$pt = 0$, THUS, FROM FIG. 19.2: $\phi = \frac{1}{2}\pi = 90°$

(a) AFTER COLLAR HAS MOVED 75 mm UPWARD,

↑+ $x = x_m - 0.075\ m = 0.05\ m - 0.075\ m = -0.025\ m$

FROM EQ. (19.10): $x = x_m \sin(pt+\phi)$

$-0.025\ m = (0.05\ m)\sin(pt+\phi)$

$\sin(pt+\phi) = -0.5$

$pt + \phi = 210°$ $\quad pt = 210° - \phi = 210° - 90° = 120° = \frac{2\pi}{3}$ rad

$$t = \frac{2\pi/3}{p} = \frac{2\pi/3}{13.416} = 0.15611\ s \qquad t = 0.1561\ s$$

(b) FROM EQ. (19.11):

$v = x_m p \cos(pt+\phi) = (0.05\ m)(13.416\ rad/s)\cos 210°$

$= -0.5809\ m/s$ $\quad v = 0.581\ m/s$ ↑

FROM EQ. (19.12):

$a = -x_m p^2 \sin(pt+\phi) = -(0.05\ m)(13.416\ rad/s)^2 \sin 210°$

$= +4.50\ m/s^2$ $\quad a = 4.5\ m/s^2$ ↓

19.10

FOR MASS-SPRING SYSTEM:

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{(6\ lb/in)(12\ in/ft)}{(4\ lb)/(32.2\ ft/s^2)}} = 24.075\ rad/s$$

MEASURING THE DISPLACEMENT x ↓+ TO CONFORM WITH FIG. 19.2, WE HAVE:

AT $t = 0$: $x = x_m = 2.5\ in. = \frac{2.5}{12}\ ft$

$pt = 0$, THUS, FROM FIG. 19.2: $\phi = \frac{1}{2}\pi = 90°$

(a) AFTER COLLAR HAS MOVED 2 in. UPWARD,

↓+ $x = x_m - 2\ in. = 2.5\ in. - 2\ in. = 0.5\ in.$

FROM EQ. (19.10): $x = x_m \sin(pt+\phi)$

$0.5\ in. = (2.5\ in.)\sin(pt+\phi)$

$\sin(pt+\phi) = 0.2$

$pt + \phi = 11.54°$ $\quad pt = 11.54° - \phi = 11.54° - 90°$ (IMPOSSIBLE)

OR $pt + \phi = 168.46°$ $\quad pt = 168.46° - \phi = 168.46° - 90°$

$pt = 78.46° = 1.3694$ rad

$$t = \frac{1.3694\ rad}{24.075\ rad/s} = 0.05688\ s \qquad t = 0.0569\ s$$

(b) FROM EQ. (19.11):

$v = x_m p \cos(pt+\phi) = \left(\frac{2.5}{12}\ ft\right)(24.075\ rad/s)\cos 168.46°$

$= -4.914\ ft/s$ $\quad v = 4.91\ ft/s$ ↑

FROM EQ. (19.12):

$a = -x_m p^2 \sin(pt+\phi) = -\left(\frac{2.5}{12}\ ft\right)(24.075\ rad/s)^2 \sin 168.46°$

$= -24.16\ ft/s^2$ $\quad a = 24.2\ ft/s^2$ ↑

19.11

FROM SOLUTION OF PROB. 19.10:

$p = 24.075\ rad/s$, $\phi = \frac{\pi}{2} = 90°$

$x = x_m = 2.5\ in. = \frac{2.5}{12}\ ft$

AT $t = 0.15\ s$:

$pt + \phi = (24.075\ rad/s)(0.15\ s) + \frac{\pi}{2}\ rad$

$= 5.182\ rad = 296.9°$

(CONTINUED)

19.11 CONTINUED

FROM EQ. (19.10):

$x = x_m \sin(pt+\phi) = (2.5\ in.)\sin 296.9° = -2.229\ in.$

$x = 2.23\ in.$ ABOVE EQUIL. POSITION

FROM EQ. (19.11):

$v = x_m p \cos(pt+\phi) = \left(\frac{2.5}{12}\ ft\right)(24.075\ rad/s)\cos 296.9°$

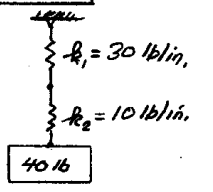
$= 2.270\ ft/s$ $\quad v = 2.27\ ft/s$ ↓

FROM EQ. (19.12):

$a = -x_m p^2 \sin(pt+\phi) = -\left(\frac{2.5}{12}\ ft\right)(24.075\ rad/s)^2 \sin 296.9°$

$= +107.68\ ft/s^2$ $\quad a = 107.7\ ft/s^2$ ↑

19.12



FOR A STATIC LOAD P THE TOTAL ELONGATION OF THE SPRINGS IS:

$$\delta = \frac{P}{k_1} + \frac{P}{k_2} = \frac{P}{30\ lb/in} + \frac{P}{10\ lb/in}$$

$$\delta = P\,\frac{1+3}{30} = P\left(\frac{4}{30}\right) = \frac{P}{7.5\ lb/in.}$$

$$k = \frac{P}{\delta} = 7.5\ lb/in. = 90\ lb/ft.$$

(a) $$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{90\ lb/ft}{(40\ lb)/(32.2\ ft/s^2)}} = 8.512\ rad/s$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{8.512} = 0.7382\ s \qquad \tau = 0.738\ s$$

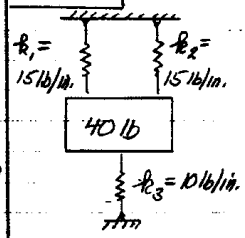
$$f = \frac{p}{2\pi} = \frac{8.512}{2\pi} = 1.3547\ Hz \qquad f = 1.355\ Hz$$

(b) FOR $x_m = 1.25\ in. = \frac{1.25}{12}\ ft$

$v_m = x_m p = \left(\frac{1.25}{12}\ ft\right)(8.512\ rad/s)$ $\quad v_m = 0.887\ ft/s$

$a_m = x_m p^2 = \left(\frac{1.25}{12}\ ft\right)(8.512\ rad/s)^2$ $\quad a_m = 7.55\ ft/s^2$

19.13



LET P BE STATIC FORCE REQUIRED TO CAUSE STATIC DEFLECTION δ

$P = k_1\delta + k_2\delta + k_3\delta$

$= (15\ lb/in. + 15\ lb/in. + 10\ lb/in.)\,\delta$

$P = (40\ lb/in.)\,\delta$

$$k = \frac{P}{\delta} = 40\ lb/in. = 480\ lb/ft$$

(a) $$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{480\ lb/ft}{(40\ lb)/(32.2\ ft/s^2)}} = 19.657\ rad/s$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{19.657} = 0.3196\ s \qquad \tau = 0.320\ s$$

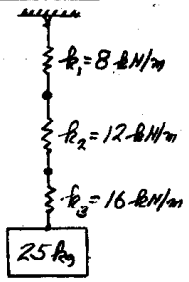
$$f = \frac{p}{2\pi} = \frac{19.657}{2\pi} = 3.1285\ Hz \qquad f = 3.13\ Hz$$

(b) FOR $x_m = 1.25\ in. = \frac{1.25}{12}\ ft$

$v_m = x_m p = \left(\frac{1.25}{12}\ ft\right)(19.657\ rad/s)$ $\quad v_m = 2.05\ ft/s$

$a_m = x_m p^2 = \left(\frac{1.25}{12}\ ft\right)(19.657\ rad/s)^2$ $\quad a_m = 40.3\ ft/s^2$

19.14



FOR A STATIC LOAD P THE TOTAL ELONGATION OF THE SPRINGS IS,

$$\delta = \frac{P}{k_1} + \frac{P}{k_2} + \frac{P}{k_3}$$

$$= P\left(\frac{1}{8\text{ kN/m}} + \frac{1}{12\text{ kN/m}} + \frac{1}{16\text{ kN/m}}\right)$$

$$\delta = P\left(\frac{6+4+3}{48}\right) = P\,\frac{13}{48}$$

$$k = \frac{P}{\delta} = \frac{48}{13} = 3.6923\text{ kN/m}$$

(a) $$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.6923\times10^3\text{ N/m}}{25\text{ kg}}} = 12.153\text{ rad/s}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{12.153} = 0.517\text{ s} \qquad \tau = 0.517\text{ s}$$

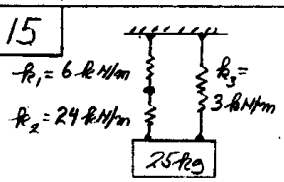
$$f = \frac{p}{2\pi} = \frac{12.153}{2\pi} = 1.9342\text{ Hz} \qquad f = 1.934\text{ Hz}$$

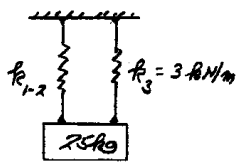
(b) FOR $r_m = 30\text{ mm} = 0.03\text{ m}$

$$v_m = r_m p = (0.03\text{ m})(12.153\text{ rad/s}) \qquad v_m = 0.365\text{ m/s}$$

$$a_m = r_m p^2 = (0.03\text{ m})(12.153\text{ rad/s})^2 \qquad a_m = 4.43\text{ m/s}^2$$

19.15





WE SHALL REPLACE SPRINGS 1 AND 2 BY AN EQUIVALENT SPRING 1-2. FOR A STATIC LOAD P THE TOTAL ELONGATION OF SPRINGS 1 AND 2 IS

$$\delta = \frac{P}{k_1} + \frac{P}{k_2} = P\left(\frac{1}{6\text{ kN/m}} + \frac{1}{24\text{ kN/m}}\right) = P\,\frac{4+1}{24} = P\,\frac{5}{24}$$

$$k_{1-2} = \frac{P}{\delta} = \frac{24}{5} = 4.8\text{ kN/m}$$

WE NOW CONSIDER THE EQUIVALENT SPRING SYSTEM SHOWN ABOVE AT THE RIGHT.

LET Q BE THE STATIC FORCE REQUIRED TO CAUSE STATIC DEFLECTION Δ.

$$Q = k_{1-2}\Delta + k_3\Delta = (4.8\text{ kN/m} + 3\text{ kN/m})\Delta$$

$$k = \frac{Q}{\Delta} = 7.8\text{ kN/m}$$

(a) $$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{7800\text{ N/m}}{25\text{ kg}}} = 17.6635\text{ rad/s}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{17.6635} = 0.3557\text{ s} \qquad \tau = 0.3565$$

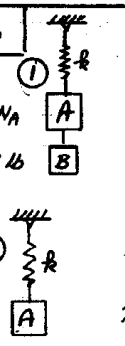
$$f = \frac{p}{2\pi} = \frac{17.6635}{2\pi} = 2.811\text{ Hz} \qquad f = 2.81\text{ Hz}$$

(b) FOR $r_m = 30\text{ mm} = 0.03\text{ m}$

$$v_m = r_m p = (0.03\text{ m})(17.6635\text{ rad/s}) \qquad v_m = 0.530\text{ m/s}$$

$$a_m = r_m p^2 = (0.03\text{ m})(17.6635\text{ rad/s})^2 \qquad a_m = 9.36\text{ m/s}^2$$

19.16



$$m_1 = \frac{W_A + 3}{g} \qquad \tau_1 = 0.65$$

$$\tau_1 = \frac{2\pi}{p_1} \qquad p_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.6} = 3.333\pi\text{ rad/s}$$

$$p_1^2 = \frac{k}{m_1} \qquad k = m_1 p_1^2 = \left(\frac{W_A+3}{g}\right)(3.333\pi)^2 \quad (1)$$

$$m_2 = \frac{W_A}{g} \qquad \tau_2 = 0.5\text{ s}$$

$$\tau_2 = \frac{2\pi}{p_2} \qquad p_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{0.5} = 4\pi\text{ rad/s}$$

$$p_2^2 = \frac{k}{m_2} \qquad k = m_2 p_2^2 = \frac{W_A}{g}(4\pi)^2 \quad (2)$$

(a) EQUATING THE EXPRESSIONS FOUND FOR k IN EQS. (1) AND (2):

$$\frac{W_A+3}{g}(3.333\pi)^2 = \frac{W_A}{g}(4\pi)^2$$

$$(11.111)(W_A+3) = 16\,W_A$$

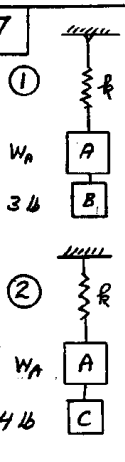
$$4.889\,W_A = 33.333$$

$$W_A = 6.818\text{ lb} \qquad W_A = 6.82\text{ lb}$$

(b) EQ.(1): $$k = \frac{6.818\text{ lb} + 3\text{ lb}}{32.2\text{ ft/s}^2}(3.333\pi\text{ rad/s})^2$$

$$k = 33.44\text{ lb/ft} \qquad k = 33.4\text{ lb/ft}$$

19.17



$$m_1 = \frac{W_A+3}{g} \qquad \tau_1 = 2.4\text{ s}$$

$$\tau_1 = \frac{2\pi}{p_1} \qquad p_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{2.4} = \frac{5}{6}\pi\text{ rad/s}$$

$$p_1^2 = \frac{k}{m_1} \qquad k = m_1 p_1^2 = \left(\frac{W_A+3}{g}\right)\left(\frac{5}{6}\pi\right)^2 \quad (1)$$

$$m_2 = \frac{W_A+4}{g} \qquad \tau_2 = 2.6\text{ s}$$

$$\tau_2 = \frac{2\pi}{p_2} \qquad p_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{2.6} = \frac{10}{13}\pi\text{ rad/s}$$

$$p_2^2 = \frac{k}{m_2} \qquad k = m_2 p_2^2 = \left(\frac{W_A+4}{g}\right)\left(\frac{10}{13}\pi\right)^2 \quad (2)$$

(a) EQUATING THE EXPRESSIONS FOUND FOR k IN EQS.(1) AND (2):

$$\left(\frac{W_A+3}{g}\right)\left(\frac{5}{6}\pi\right)^2 = \left(\frac{W_A+4}{g}\right)\left(\frac{10}{13}\pi\right)^2$$

$$W_A+3 = (W_A+4)\left(\frac{10}{13}\right)^2\left(\frac{5}{6}\right)^2 = (W_A+4)(0.85207)$$

$$W_A+3 = 0.85207\,W_A + 3.4083$$

$$0.14793\,W_A = 0.4083$$

$$W_A = 2.76\text{ lb} \qquad W_A = 2.76\text{ lb}$$

(b) EQ(1):

$$k = \frac{2.76\text{ lb} + 3\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{5}{6}\pi\right)^2 = \qquad k = 1.226\text{ lb/ft}$$

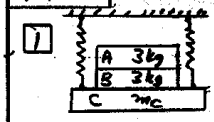
ALTERNATE SOLUTION:

$$p^2 = \frac{k}{m}, \quad \tau = \frac{2\pi}{p} \qquad \tau^2 = \frac{4\pi^2}{p^2} = 4\pi^2\frac{m}{k} = 4\pi^2\frac{W}{g\,k}$$

THUS $\tau^2 \sim W$, OR

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{W_1}{W_2} : \left(\frac{2.4\text{ s}}{2.6\text{ s}}\right)^2 = \frac{W_A+3}{W_A+4}.$$ SOLVE FOR: $W_A = 4.26\text{ lb}$

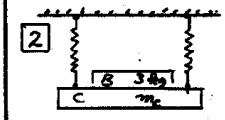
19.18



$m_1 = m_c + 6\ kg$ $\quad \tau_1 = 0.8\ s$

$p_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.8} = \frac{2\pi}{0.8}$ rad/s

$p_1^2 = \frac{k}{m_1}$; $\quad k = m_1 p_1^2 = (m_c+6)\left(\frac{2\pi}{0.8}\right)^2$ (1)



$m_2 = m_c + 3\ kg$ $\quad \tau_2 = 0.7\ s$

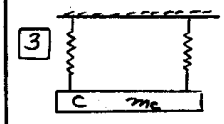
$p_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{0.7}$ rad/s

$p_2^2 = \frac{k}{m_2}$; $\quad k = m_2 p_2^2 = (m_c+3)\left(\frac{2\pi}{0.7}\right)^2$ (2)

EQUATING THE EXPRESSIONS FOUND FOR k IN EQS.(1) AND (2):

$$(m_c+6)\left(\frac{2\pi}{0.8}\right)^2 = (m_c+3)\left(\frac{2\pi}{0.7}\right)^2$$

$\frac{m_c+6}{m_c+3} = \left(\frac{0.8}{0.7}\right)^2$; SOLVE FOR m_c: $\quad m_c = 6.80\ kg$ ◀



$p_3 = \frac{2\pi}{\tau_3}$

$p_3^2 = \frac{k}{m_c}$; $\quad k = m_c p_3^2 = m_c\left(\frac{2\pi}{\tau_3}\right)^2$ (3)

EQUATING EXPRESSIONS FOR k FROM EQS.(2) + (3)

$$(m_c+3)\left(\frac{2\pi}{0.7}\right)^2 = m_c\left(\frac{2\pi}{\tau_3}\right)^2$$

RECALL $m_c = 6.8\ kg$: $\quad (6.8+3)\left(\frac{2\pi}{0.7}\right)^2 = 6.8\left(\frac{2\pi}{\tau_3}\right)^2$

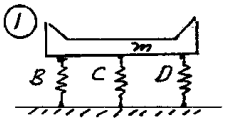
$\left(\frac{\tau_3}{0.7}\right)^2 = \frac{6.8}{9.8}$; $\quad \frac{\tau_3}{0.7} = 0.833$ $\quad \tau_3 = 0.583\ s$ ◀

19.19

SPRING CONSTANTS:

$k_{BD} = k_B + k_D$

$k_C = 125\ N/m$



$m_1 = m$ $\quad \tau_1 = 0.5\ s$

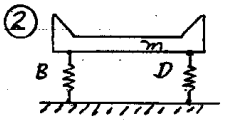
$\tau_1 = \frac{2\pi}{p_1}$ $\quad p_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.5} = 4\pi$ rad/s

$k_1 = k_{BD} + k_C = k_{BD} + 125\ N/m$

$p_1^2 = \frac{k_1}{m_1}$ $\quad k_1 = m_1 p_1^2$

$k_{BD} + 125 = m(4\pi)^2$

$k_{BD} = m(4\pi)^2 - 125$ (1)



$m_2 = m$ $\quad \tau_2 = 0.6\ s$

$\tau_2 = \frac{2\pi}{p_2}$ $\quad p_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{0.6} = 3.333\pi$ rad/s

$k_2 = k_B + k_D = k_{BD}$

$p_2^2 = \frac{k_2}{m_2}$ $\quad k_2 = m_2 p_2^2$

$k_{BD} = m(3.333\pi)^2$ (2)

WE EQUATE THE EXPRESSIONS FOUND FOR k_{BD} IN EQS.(1) AND (2).

$m(3.333\pi)^2 = m(4\pi)^2 - 125$

$m = \frac{125}{\pi^2(4^2 - 3.333^2)} = 2.591\ kg$ $\quad m = 2.59\ kg$ ◀

ALTERNATE SOLUTION.

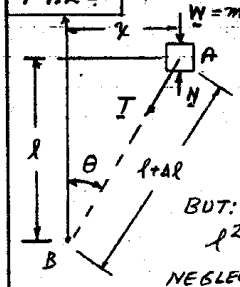
$p^2 = \frac{k}{m}$, $\quad \tau = \frac{2\pi}{p}$, $\quad \tau^2 = \frac{4\pi^2}{p^2} = 4\pi^2\frac{m}{k}$

THUS, $\tau^2 \sim 1/k$

$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{k_2}{k_1}$; $\quad \left(\frac{0.5}{0.6}\right)^2 = \frac{k_{BD}}{k_{BD}+125}$; $\quad k_{BD} = 284.1\ N/m$

EQ.(2): $\quad 284.1 = m(3.333\pi)^2$ $\quad m = 2.59\ kg$ ◀

19.20



$+\rightarrow \Sigma F = ma$

$-T\sin\theta = m\ddot{x}$

FOR SMALL OSCILLATIONS:

$\sin\theta \approx \tan\theta \approx \frac{x}{l}$

THUS: $\quad \ddot{x} + \frac{T}{ml}x = 0$ (1)

BUT: $\quad (l+\Delta l)^2 = l^2 + x^2$

$l^2 + 2l\Delta l + (\Delta l)^2 = l^2 + x^2$

NEGLECTING $(\Delta l)^2$

$\Delta l = \frac{x^2}{2l}$

DENOTING BY T_0 THE TENSION IN THE EQUILIBRIUM POSITION,

$T = T_0 + k(\Delta l) = T_0 + \frac{k}{2l}x^2$ (2)

(a) WE HAVE $T_0 = 0$ AND $T = \frac{k}{2l}x^2$.

SUBSTITUTING INTO EQ.(1):

$\ddot{x} + \frac{k}{2ml^2}x^3 = 0$

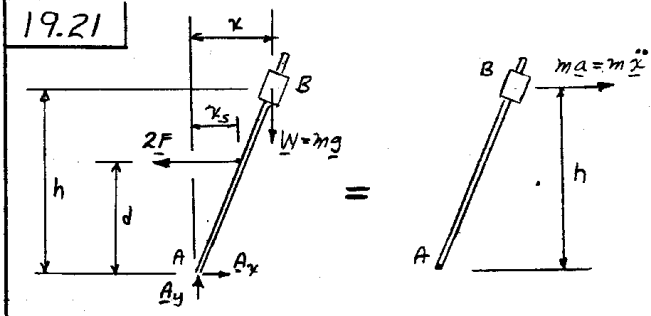
THIS EQUATION DOES NOT REPRESENT SHM ◀

(b) WHEN $T_0 \neq 0$, WE MAY NEGLECT THE SECOND TERM IN EQ.(2) FOR SMALL VALUES OF x. SUBSTITUTING $T = T_0$ INTO EQ.(1):

$\ddot{x} + \frac{T_0}{ml}x = 0$

THIS IS SHM WITH $p^2 = \frac{T_0}{ml}$, $\quad \tau = 2\pi\sqrt{\frac{ml}{T_0}}$ ◀

19.21



$x_s = x\frac{d}{h}$ $\quad 2F = 2kx_s = 2k\frac{d}{h}x$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad 2Fd - mgx = -(m\ddot{x})h$

$2k\left(\frac{d}{h}x\right)d - mgx = -m\ddot{x}h$

$\ddot{x} + \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]x = 0$

$p^2 = \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]$ $\quad p^2 = \frac{2k}{m}\left(\frac{d}{h}\right)^2 - \frac{g}{h}$ (1)

DATA: $d = 0.3\ m$, $\quad h = 0.7\ m$, $\quad m = 20\ kg$

(a) FOR $\tau = 1\ s$: $\quad \tau = \frac{2\pi}{p}$; $\quad 1\ s = \frac{2\pi}{p}$ $\quad p^2 = 4\pi^2$

EQ.(1): $\quad 4\pi^2 = \frac{2k}{20}\left(\frac{0.3}{0.7}\right)^2 - \frac{9.81}{0.7}$

$k = 2912.4\ N/m$ $\quad k = 2.91\ kN/m$ ◀

(b) FOR $\tau = \text{INFINITE}$: $\quad \tau = \frac{2\pi}{p}$ $\quad p = 0$

EQ.(1): $\quad p^2 = 0 = \frac{2k}{20}\left(\frac{0.3}{0.7}\right)^2 - \frac{9.81}{0.7}$

$k = 763\ N/m$ $\quad k = 763\ N/m$ ◀

19.22

SEE SOLUTION OF PROB 19.21 FOR DERIVATION OF EQ.(1)

$$p^2 = \frac{2k}{m}\left(\frac{d}{h}\right)^2 - \frac{g}{h} \qquad (1)$$

DATA: $d = 0.5$ m, $h = 0.7$ m, $k = 600$ N/m

(a) FOR $\tau = 0.5$ s: $\tau = \frac{2\pi}{p}$; $0.5 = \frac{2\pi}{p}$ $\quad p = 4\pi$

EQ.(1): $(4\pi)^2 = \frac{2(600)}{m}\left(\frac{0.5}{0.7}\right)^2 - \frac{9.81}{0.7}$

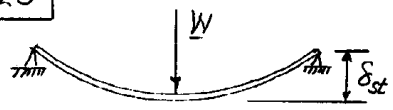
$m = 3.561$ kg $\quad$ $m = 3.56$ kg ◀

(b) FOR $\tau =$ INFINITE: $\tau = \frac{2\pi}{p}$ $\quad p = 0$

EQ.(1): $0 = \frac{2(600)}{m}\left(\frac{0.5}{0.7}\right)^2 - \frac{9.81}{0.7}$

$m = 43.69$ kg $\quad$ $m = 43.7$ kg ◀

19.23



$k = \frac{W}{\delta_{st}}$ $\quad m = \frac{W}{g}$

EQ.(19.4): $p^2 = \frac{k}{m} = \frac{W/\delta_{st}}{W/g} = \frac{g}{\delta_{st}}$ $\quad p = \sqrt{\frac{g}{\delta_{st}}}$

$f = \frac{p}{2\pi}$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$ ◀

*19.24

$$\tau = 4\sqrt{\frac{l}{g}}\int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \sin^2\left(\frac{\theta_m}{2}\right)\sin^2\phi}}$$

USING THE BINOMIAL THEOREM, WE WRITE

$$\frac{1}{\sqrt{1 - \sin^2\left(\frac{\theta_m}{2}\right)\sin^2\phi}} = \left[1 - \sin^2\left(\frac{\theta_m}{2}\right)\sin\phi\right]^{-1/2}$$

$$= 1 + \frac{1}{2}\sin^2\left(\frac{\theta_m}{2}\right)\sin^2\phi + \cdots$$

WHERE WE NEGLECT TERMS OF ORDER HIGHER THAN 2. SETTING $\sin^2\phi = \frac{1}{2}(1 - \cos 2\phi)$, WE HAVE

$$\tau = 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\left\{1 + \frac{1}{2}\sin^2\frac{\theta_m}{2}\left[\frac{1}{2}(1 - \cos 2\phi)\right]\right\}d\phi$$

$$= 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\left[1 + \frac{1}{4}\sin^2\frac{\theta_m}{2} - \frac{1}{4}\sin^2\frac{\theta_m}{2}\cos 2\phi\right]d\phi$$

$$= 4\sqrt{\frac{l}{g}}\left[\phi + \frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\phi - \frac{1}{8}\sin^2\frac{\theta_m}{2}\sin 2\phi\right]_0^{\pi/2}$$

$$= 4\sqrt{\frac{l}{g}}\left[\frac{\pi}{2} + \frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\frac{\pi}{2} + 0\right]$$

$$\tau = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right) \quad \text{(Q.E.D.)}$$

*19.25

FOR SMALL OSCILLATIONS: $\tau_0 = 2\pi\sqrt{\frac{l}{g}}$

WE WANT $\tau = 1.01\,\tau_0 = (1.01)2\pi\sqrt{\frac{l}{g}}$

USING THE FORMULA OF PROB 19.24, WE WRITE

$$\tau = \tau_0\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right) = 1.01\,\tau_0$$

$\sin^2\frac{\theta_m}{2} = 4(1.01 - 1) = 0.04$; $\sin\frac{\theta_m}{2} = 0.2$; $\frac{\theta_m}{2} = 11.54°$

$\theta_m = 23.1°$ ◀

*19.26

EQ.(19.20) $\quad \tau = \frac{2K}{\pi}\left(2\pi\sqrt{\frac{l}{g}}\right)$

FOR $l = 0.8$ m:

$$2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.8\text{ m}}{9.81\text{ m/s}^2}} = 1.7943\text{ s}$$

THUS:

$$\tau = \frac{2K}{\pi}(1.7943\text{ s})$$

USING TABLE 19.1:

(a) FOR SMALL OSCILLATIONS: $\theta_m = 0$; $\frac{2K}{\pi} = 1$

$\tau = 1.794$ s ◀

(b) FOR $\theta_m = 30°$: $\frac{2K}{\pi} = 1.017$

$\tau = (1.017)(1.7943\text{ s})$ $\quad \tau = 1.825$ s ◀

(c) FOR $\theta_m = 90°$: $\frac{2K}{\pi} = 1.180$

$\tau = (1.180)(1.7943\text{ s})$ $\quad \tau = 2.12$ s. ◀

*19.27

EQ.(19.20): $\quad \tau = \frac{2K}{\pi}\left(2\pi\sqrt{\frac{l}{g}}\right)$

FOR $l = 0.8$ m

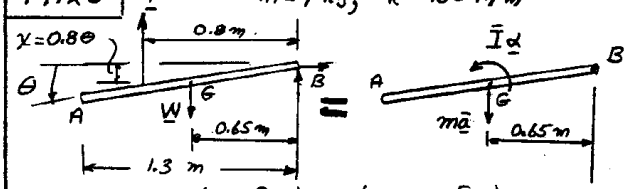
$$2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.8\text{ m}}{9.81\text{ m/s}^2}} = 1.7943\text{ s}$$

FROM "STANDARD MATHEMATICAL TABLES."

$\theta_m = 40°$; $\theta = \frac{1}{2}\theta_m = 20°$; $K = 1.620$

$\tau = \frac{2K}{\pi}(1.7943\text{ s}) = \frac{2(1.620)}{\pi}(1.7943\text{ s})$ $\quad \tau = 1.850$ s ◀

19.28



WHERE $F = k(x + \delta_{st}) = k(0.8\theta + \delta_{st})$

$m\bar{a} = m\bar{r}\alpha = (4\text{ kg})(0.65\text{ m})\ddot{\theta}$

$\bar{I}\alpha = \frac{1}{12}(4\text{ kg})(1.3\text{ m})^2\ddot{\theta}$

(a) $+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$W(0.65\text{ m}) - F(0.8\text{ m}) = \bar{I}\alpha + m\bar{a}(0.65\text{ m})$

$W(0.65) - k(0.8\theta + \delta_{st})(0.8\text{ m}) = \frac{1}{12}(4)(1.3)^2\ddot{\theta} + (4)(0.65)^2\ddot{\theta}$

BUT, IN EQUILIBRIUM, WE HAVE

$W(0.65) - k\delta_{st}(0.8) = 0$

THUS $\quad -(750)(0.8)^2\theta = \left[\frac{4}{12}(1.3)^2 + 4(0.65)^2\right]\ddot{\theta}$

$2.2533\,\ddot{\theta} + 480\theta = 0$ $\quad \ddot{\theta} + 213.02\,\theta = 0$

THUS: $\quad p^2 = 213.02$ $\quad p = 14.595$ rad/s

$\tau = \frac{2\pi}{p} = \frac{2\pi}{14.595} = 0.4305$ s $\quad \tau = 0.430$ s ◀

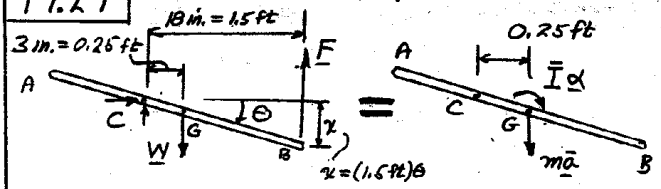
(b) AT END A: $\quad x_m = 40$ mm

$v_m = x_m p = (40\text{ mm})(14.595\text{ rad/s}) = 583.8$ mm/s

$v_m = 0.584$ m/s ◀

19.29

$k = 4$ lb/in. $= 48$ lb/ft; $W = 15$ lb



WHERE $F = k(x + \delta_{st}) = k(1.5\theta + \delta_{st})$

$m\bar{a} = m\bar{r}\alpha = \frac{15}{g}(0.25\text{ ft})\ddot{\theta} = \frac{3.75}{g}\ddot{\theta}$

$\bar{I}\alpha = \frac{1}{12}\left(\frac{15}{g}\right)(2.5\text{ ft})^2\ddot{\theta} = \frac{7.8125}{g}\ddot{\theta}$

(a) $+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}$:

$W(0.25\text{ ft}) - F(1.5\text{ ft}) = \bar{I}\alpha + m\bar{a}(0.25\text{ ft})$

$W(0.25) - k(1.5\theta + \delta_{st})(1.5\text{ ft}) = \frac{7.8125}{g}\ddot{\theta} + \frac{3.75}{g}\ddot{\theta}(0.25)$

BUT, IN EQUILIBRIUM, WE HAVE

$+\downarrow \quad W(0.25\text{ ft}) - k\delta_{st}(1.5\text{ ft}) = 0$

THUS: $-k(1.5)^2\theta = \left[\frac{7.8125}{g} + \frac{0.9375}{g}\right]\ddot{\theta}$

$-(48\text{ ft/s}^2)(1.5)^2\theta = \frac{8.75}{32.2}\ddot{\theta}$

$\ddot{\theta} + (397.44)\theta = 0$

$p^2 = 364.11 \qquad p = 19.936$ rad/s

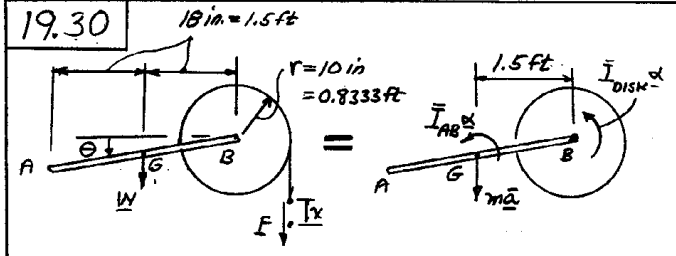
$\tau = \frac{2\pi}{p} = \frac{2\pi}{19.936\text{ rad/s}} \qquad \tau = 0.315$ s ◀

(b) AT END B. $x_m = 0.4$ in.

$v_m = x_m p = (0.4\text{ in.})(19.936\text{ rad/s}) = 7.974$ in./s

$v_m = 0.665$ ft/s ◀

19.30



SPRING: $k = 30$ lb/in. $= 360$ lb/ft

$x = r\theta + \delta_{st} = 0.8333\theta + \delta_{st}$

$F = kx = k(0.8333\theta + \delta_{st})$

ROD AB: $W_{AB} = 15$ lb $\quad L_{AB} = 3$ ft

$m\bar{a} = m(1.5\text{ ft})\alpha = \frac{15}{g}(1.5)\ddot{\theta} = \frac{22.5}{g}\ddot{\theta}$

$\bar{I}_{AB}\alpha = \frac{1}{12}\left(\frac{15}{g}\right)(3)^2\ddot{\theta} = \frac{11.25}{g}\ddot{\theta}$

DISK: $W_{DISK} = 12$ lb $\quad r = 0.8333$ ft

$\bar{I}_{DISK}\alpha = \frac{1}{2}\frac{12\text{ lb}}{g}(0.8333\text{ ft})^2\ddot{\theta} = \frac{4.1667}{g}\ddot{\theta}$

(CONTINUED)

19.30 CONTINUED

$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{eff}$:

$W(1.5\text{ ft}) - F(0.8333\text{ ft}) = \bar{I}_{AB}\alpha + m\bar{a}(1.5\text{ ft}) + \bar{I}_{DISK}\alpha$

$W(1.5) - k(0.8333\theta + \delta_{st})(0.8333\text{ ft}) = \frac{11.25}{g}\ddot{\theta} + \frac{22.5}{g}\ddot{\theta}(1.5) + \frac{4.1667}{g}\ddot{\theta}$

BUT, IN EQUILIBRIUM, WE HAVE

$+\circlearrowright \quad W(1.5\text{ ft}) - k\delta_{st}(0.8333) = 0$

THUS: $-k(0.8333)^2\theta = \left[11.25 + 33.75 + 4.1667\right]\frac{\ddot{\theta}}{g}$

$-(360\text{ lb/ft})(0.8333\text{ ft})^2\ddot{\theta} = \frac{49.1667}{32.2}\ddot{\theta}$

(a) $\ddot{\theta} + 163.729\,\theta = 0$

$p^2 = 163.729 \qquad p = 12.796$ rad/s

$\tau = \frac{2\pi}{p} = \frac{2\pi}{12.796\text{ rad/s}} = 0.4910$ s

$\tau = 0.491$ s ◀

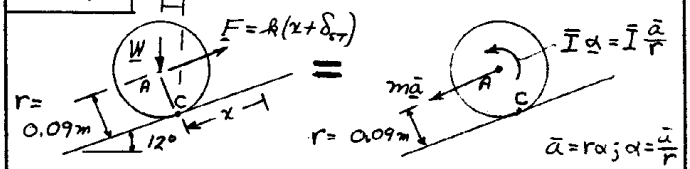
(b) AT END A: $x_m = 0.75$ in.

$v_m = x_m p = (0.75\text{ in.})(12.796\text{ rad/s}) = 9.597$ in./s

$v_m = 9.60$ in./s ◀

19.31

$r\sin 12°$ $\quad m = 5$ kg $\quad k = 750$ N/m



$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}$:

$W(r\sin 12°) - Fr = \bar{I}\frac{\bar{a}}{r} + m\bar{a}\,r$

$W(r\sin 12°) - k(x + \delta_{st})r = \frac{1}{2}mr^2\frac{\bar{a}}{r} + m\bar{a}\,r$

BUT, IN EQUILIBRIUM, WE HAVE

$+\circlearrowright \quad W(r\sin 12°) - k\delta_{st}r = 0$

THUS: $-kxr = \left(\frac{1}{2}mr + mr\right)\bar{a}$

$-kx = \frac{3}{2}m\ddot{x}$

$\ddot{x} + \frac{2}{3}\frac{k}{m}x = 0$

$p^2 = \frac{2k}{3m} \qquad p = \sqrt{\frac{2k}{3m}}$

$p = \sqrt{\frac{2(750\text{ N/m})}{3(5\text{ kg})}} = \sqrt{100} = 10$ rad/s

(a) $\tau = \frac{2\pi}{p} = \frac{2\pi}{10\text{ rad/s}} = 0.6283$ s $\qquad \tau = 0.628$ s ◀

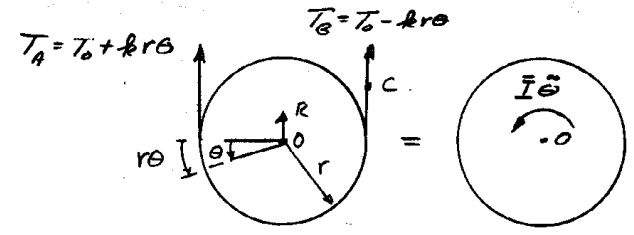
(b) AT CENTER OF DISK: $x_m = 10$ mm

$v_m = x_m p = (10\text{ mm})(10\text{ rad/s}) \qquad v_m = 100.0$ mm/s ◀

NOTE RESULTS ARE INDEPENDENT OF RADIUS r OF DISK AND OF THE SLOPE θ OF THE INCLINE.

19.32

DENOTE THE INITIAL TENSION BY T_0.



$+\circlearrowleft \Sigma M_O = 0$: $\quad -T_A r + T_B r = \bar{I}\ddot{\theta}$

$$-(T_0 + kr\theta)r + (T_0 - kr\theta)r = \bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{2kr^2}{\bar{I}}\theta = 0$$

$$p^2 = \frac{2kr^2}{\bar{I}} \qquad (1)$$

DATA

$m = 240$ kg $\qquad k = 15$ kN/m

$\tau = 0.5$ s $\qquad r = 0.45$ m

$$\tau = \frac{2\pi}{p}; \quad p = \frac{2\pi}{\tau} = \frac{2\pi}{0.5} = 4\pi \text{ rad/s}$$

EQ(1): $\quad p^2 = \frac{2kr^2}{\bar{I}}$; $\quad (4\pi \text{ rad/s})^2 = \frac{2(15 \text{ kN/m})(0.45 \text{ m})^2}{\bar{I}}$

$$\bar{I} = 38.47 \text{ kg}\cdot\text{m}^2$$

OR SINCE $\bar{I} = m\bar{k}^2$

$$(240 \text{ kg})\bar{k}^2 = 38.47 \text{ kg}\cdot\text{m}^2$$

$$\bar{k} = 0.4004 \text{ m} \qquad \bar{k} = 400 \text{ mm}$$ ◀

MAX. ANGULAR VELOCITY: IF POINT C IS PULLED DOWN 40 mm AND RELEASED,

$$\theta_m = \theta_{max} = (40 \text{ mm})/(450 \text{ mm}) = 88.89\times10^{-3} \text{ rad}$$

$$\omega_m = \theta_m p = (88.89\times10^{-3} \text{ rad})(4\pi \text{ rad/s})$$

$$\omega_m = 1.117 \text{ rad/s}$$ ◀

19.33

SEE SOLUTION OF PROB 19.32.

DATA: $\quad r = 0.45$ m $\qquad k = 15$ kN/m

$m = 240$ kg

RADIUS OF GYRATION: $\bar{k} = 0.38$ m

$$\bar{I} = m\bar{k}^2 = (240 \text{ kg})(0.38 \text{ m})^2 = 34.656 \text{ kg}\cdot\text{m}^2$$

EQ(1) $\quad p^2 = \frac{2kr^2}{\bar{I}} = \frac{2(15 \text{ kN/m})(0.45 \text{ m})^2}{(34.656 \text{ kg}\cdot\text{m}^2)}$

$$p^2 = 175.294 \qquad p = 13.24 \text{ rad/s}$$

PERIOD OF VIBRATION $\quad \tau = \frac{2\pi}{p} = \frac{2\pi}{13.24 \text{ rad/s}} = 0.47456 \text{ s}$

$$\tau = 0.475 \text{ s}$$ ◀

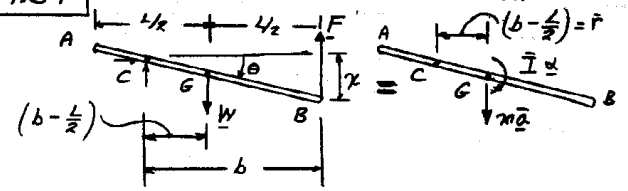
MAX. ANGULAR VELOCITY

$$\theta_m = (40 \text{ mm})/(450 \text{ mm}) = 88.89\times10^{-3} \text{ rad}$$

$$\omega_m = \theta_m p = (88.89\times10^{-3} \text{ rad})(13.24 \text{ rad/s}) \qquad \omega_m = 1.177 \text{ rad/s}$$ ◀

19.34

DENOTE BY L THE LENGTH OF ROD AB.



WHERE, $\quad F = k(\chi + \delta_{st}) = k(b\theta + \delta_{st})$

$$m\bar{a} = m\bar{r}\alpha = m\left(b - \tfrac{L}{2}\right)\ddot{\theta}$$

$$\bar{I}\alpha = \tfrac{1}{12}mL^2\ddot{\theta}$$

(a) $+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$$W\left(b - \tfrac{L}{2}\right) - Fb = \bar{I}\alpha + m\bar{a}\left(b - \tfrac{L}{2}\right)$$

$$W\left(b - \tfrac{L}{2}\right) - k(b\theta + \delta_{st})b = \tfrac{1}{12}mL^2\ddot{\theta} + m\left(b - \tfrac{L}{2}\right)^2\ddot{\theta}$$

BUT, IN EQUILIBRIUM, WE HAVE

$+\uparrow \; W\left(b - \tfrac{L}{2}\right) - k\delta_{st}b = 0$

THUS: $\quad -kb^2\theta = \left[\tfrac{1}{12}mL^2 + m\left(b - \tfrac{L}{2}\right)^2\right]\ddot{\theta}$

$$\ddot{\theta} + \left[\frac{kb^2}{\frac{1}{12}mL^2 + m\left(b - \frac{L}{2}\right)^2}\right]\theta = 0$$

$$p^2 = [\;] = \frac{k}{m}\frac{b^2}{\frac{1}{12}L^2 + \left(b - \frac{L}{2}\right)^2} \qquad (1)$$

(a) FOR MINIMUM τ:

$$\frac{d(p^2)}{db} = \frac{k}{m}\frac{\left[\frac{1}{12}L^2 + \left(b - \frac{L}{2}\right)^2\right]2b - b^2\left[2\left(b - \frac{L}{2}\right)\right]}{\left[\frac{1}{12}L^2 + \left(b - \frac{L}{2}\right)^2\right]^2} = 0$$

$$\frac{1}{12}L^2 + \left(b - \frac{L}{2}\right)^2 - b\left(b - \frac{L}{2}\right) = 0$$

$$\frac{1}{12}L^2 + b^2 - bL + \frac{1}{4}L^2 - b^2 + \frac{1}{2}bL = 0$$

$$\frac{1}{3}L^2 - \frac{1}{2}bL = 0 \; ; \quad b = \frac{2}{3}L = \frac{2}{3}(30 \text{ in}) = 20 \text{ in.}$$ ◀

(b) SUBSTITUTE $b = \frac{2}{3}L$ INTO EQ(1):

$$p^2 = \frac{k}{m}\frac{\left(\frac{2}{3}L\right)^2}{\frac{1}{12}L^2 + \left(\frac{L}{6}\right)^2} = \frac{k}{m}\frac{\left(\frac{2}{3}\right)^2}{\frac{1}{9}} = 4\frac{k}{m}$$

DATA: $\quad W = 15$ lb, $\quad k = 4$ lb/in. $= 48$ lb/ft

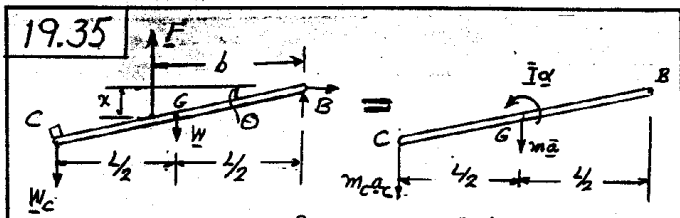
$$p^2 = 4\frac{48 \text{ lb/ft}}{(15 \text{ lb})/(32.2 \text{ ft/s}^2)} = 412.16$$

$$p = 20.302 \text{ rad/s}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{20.302 \text{ rad/s}} = 0.30949 \text{ s}$$

$$\tau = 0.309 \text{ s}$$ ◀

19.35



WHERE $F = k(x + \delta_{cr}) = k(b\theta + \delta_{st})$

$m\bar{a} = m\frac{L}{2}\ddot{\theta}$ $\quad m_c a_c = m_c L\ddot{\theta}$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$W_c L + W\frac{L}{2} - Fb = m_c a_c L + \bar{I}\alpha + m\bar{a}\frac{L}{2}$

$W_c L + W\frac{L}{2} - k(b\theta + \delta_{st})b = m_c L^2\ddot{\theta} + \frac{1}{12}mL^2\ddot{\theta} + m\left(\frac{L}{2}\right)^2\ddot{\theta}$

BUT, IN EQUILIBRIUM, WE HAVE:

$+\circlearrowleft \; W_c L + W\frac{L}{2} - k\delta_{st}b = 0$

THUS: $-kb^2\theta = \left(m_c + \frac{1}{3}m\right)L^2\ddot{\theta}$

$$\ddot{\theta} + \left[\left(\frac{b}{L}\right)^2 \frac{k}{m_c + \frac{1}{3}m}\right]\theta = 0$$

$$p^2 = \left(\frac{b}{L}\right)^2 \frac{k}{m_c + \frac{1}{3}m} \qquad p = \frac{b}{L}\sqrt{\frac{k}{m_c + \frac{1}{3}m}}$$

DATA: $L = 1.3\text{ m}$, $b = 0.8\text{ m}$, $k = 750\text{ N/m}$

$m_c = 0.5\text{ kg}$, $m = 4\text{ kg}$

$$p = \frac{0.8\text{ m}}{1.3\text{ m}}\sqrt{\frac{750\text{ N/m}}{0.5\text{ kg} + \frac{1}{3}(4\text{ kg})}} = 12.447\text{ rad/s}$$

(a) $\tau = \frac{2\pi}{p} = \frac{2\pi}{12.447\text{ rad/s}} = 0.5048$ $\qquad \tau = 0.505\text{ s}$ ◀

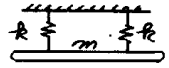
(b) FOR BLOCK C TO REMAIN IN CONTACT $(a_c)_m \le g$

$(a_c)_m = |\delta_0 p^2|$ $\qquad 9.81\text{ m/s}^2 = \delta_0(12.447\text{ rad/s})^2$

$\delta_0 = 0.06332\text{ m}$ $\qquad \delta_0 = 63.3\text{ mm}$ ◀

19.36

(a) VERTICAL DISPLACEMENT

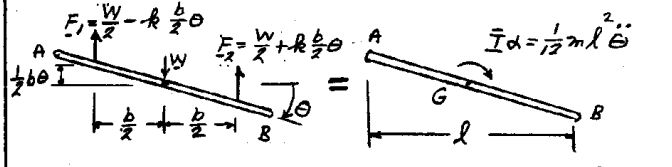


SPRING-MASS SYSTEM

EQ(19.4) $p^2 = \frac{2k}{m}$

$f = \frac{p}{2\pi}$ $\qquad f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ ◀

(b) ROTATION ABOUT G.



$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $\left(\frac{W}{2} - k\frac{b}{2}\theta\right)\frac{b}{2} - \left(\frac{W}{2} + k\frac{b}{2}\theta\right)\frac{b}{2} = \frac{1}{12}ml^2\ddot{\theta}$

$$\ddot{\theta} + \left[\frac{6k}{m}\left(\frac{b}{l}\right)^2\right]\theta = 0$$

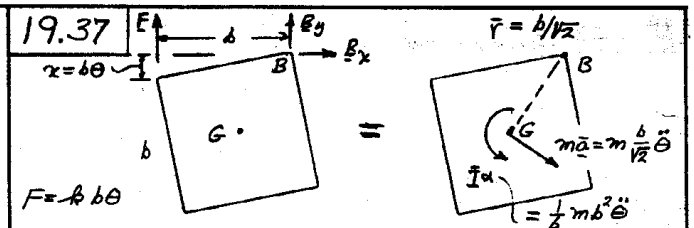
$$p^2 = 6\frac{k}{m}\left(\frac{b}{l}\right)^2$$

$f = \frac{p}{2\pi}$ $\qquad f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}\left(\frac{b}{l}\right)^2}$ $\qquad f = \frac{1}{2\pi}\frac{b}{l}\sqrt{\frac{6k}{m}}$ ◀

(c) FOR FREQUENCIES TO BE EQUAL

$f = f$: $\frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\frac{b}{l}\sqrt{\frac{6k}{m}}$; $\frac{b}{l} = \sqrt{\frac{2}{6}}$; $\frac{b}{l} = \frac{1}{\sqrt{3}}$ ◀

19.37



$F = kb\theta$

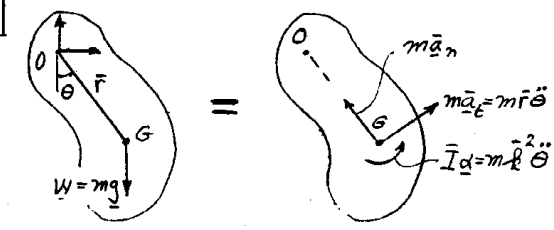
$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $-Fb = \bar{I}\alpha + m\bar{a}\bar{r}$

$-(kb\theta)b = \frac{1}{6}mb^2\ddot{\theta} + m\frac{b}{\sqrt{2}}\ddot{\theta}\frac{b}{\sqrt{2}}$

$-k\theta = \frac{2}{3}m\ddot{\theta}$ $\qquad \ddot{\theta} + \left[\frac{3}{2}\frac{k}{m}\right]\theta = 0$

$p^2 = \frac{3}{2}\frac{k}{m}$ $\qquad \tau = \frac{2\pi}{p}$ $\qquad \tau = 2\pi\sqrt{\frac{2m}{3k}}$ ◀

19.38

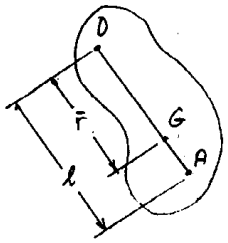


$+\circlearrowleft \Sigma M_O = \Sigma(M_O)_{eff}$: $-W\bar{r}\sin\theta = \bar{I}\alpha + ma_t\bar{r}$

$-mg\bar{r}\sin\theta = m\bar{k}^2\ddot{\theta} + m\bar{r}^2\ddot{\theta}$

$$\ddot{\theta} + \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0 \qquad (1)$$

BUT, FOR SIMPLE PENDULUM OF LENGTH $OA = l$



$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0 \qquad (2)$$

COMPARING EQS. (1) AND (2):

$l = \frac{\bar{r}^2 + \bar{k}^2}{\bar{r}} = \bar{r} + \frac{\bar{k}^2}{\bar{r}}$

$GA = l - \bar{r} = \frac{\bar{k}^2}{\bar{r}}$ $\qquad$ (Q.E.D.)

19.39

SEE SOLUTION OF PROB 19.38 FOR DERIVATION OF

$$\ddot{\theta} + \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0 \qquad (1)$$

FOR SMALL OSCILLATIONS: $\ddot{\theta} + \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\theta = 0$

THUS, $p^2 = \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2} = \frac{g}{\bar{r} + \frac{\bar{k}^2}{\bar{r}}}$

$\tau = \frac{2\pi}{p}$ $\qquad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{\bar{r} + \frac{\bar{k}^2}{\bar{r}}}$

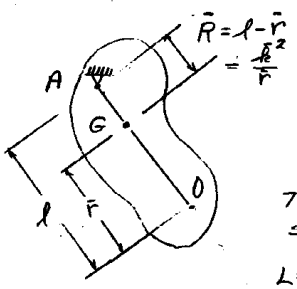
FOR SMALLEST τ WE MUST HAVE $\bar{r} + \frac{\bar{k}^2}{\bar{r}}$ MINIMUM:

$\frac{d}{d\bar{r}}\left(\bar{r} + \frac{\bar{k}^2}{\bar{r}}\right) = 1 - \frac{\bar{k}^2}{\bar{r}^2} = 0$,

$\bar{r}^2 = \bar{k}^2$

WE FIND THAT $r = \bar{k}$ $\qquad$ (Q.E.D.)

19.40



SAME DERIVATION AS IN PROB 19.38 WITH $\bar{r}$ REPLACED BY $\bar{R}$. THUS, WE OBTAIN

$$\ddot{\theta}+\frac{g\bar{R}}{\bar{R}^2+\bar{k}^2}\sin\theta=0 \qquad (1')$$

THE LENGTH OF THE EQUIVALENT SIMPLE PENDULUM IS:

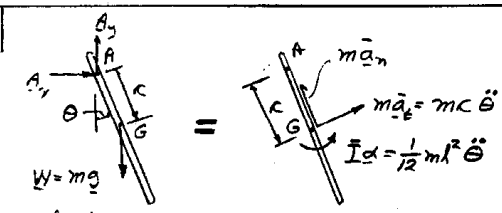
$$L=\frac{\bar{R}^2+\bar{k}^2}{\bar{R}}=\bar{R}+\frac{\bar{k}^2}{\bar{R}}$$

$$=(l-\bar{r})+\frac{\bar{k}^2}{\bar{k}^2/\bar{r}}=l$$

THUS, THE LENGTH OF THE EQUIVALENT SIMPLE PENDULUM IS THE SAME AS IN PROB 19.38.

IT FOLLOWS THAT THE PERIOD OF OSCILLATION IS THE SAME AND THAT THE NEW CENTER OF OSCILLATION IS AT O. (Q.E.D.)

19.41



$$+\circlearrowleft \Sigma M_A=\Sigma(M_A)_{eff}:$$

$$-W\bar{k}\sin\theta=\bar{I}\alpha+(m\bar{a}_t)\bar{k}$$

$$-mg\bar{k}\sin\theta=\frac{1}{12}ml^2\ddot{\theta}+m\bar{k}^2\ddot{\theta}$$

$$\ddot{\theta}+\frac{g\bar{k}}{\bar{k}^2+\frac{1}{12}l^2}\sin\theta=0$$

FOR SMALL OSCILLATIONS $\sin\theta\approx\theta$: $\ddot{\theta}+\frac{g\bar{k}}{\bar{k}^2+\frac{1}{12}l^2}\theta=0$

$$p^2=\frac{g\bar{k}}{\bar{k}^2+\frac{1}{12}l^2} \qquad (1)$$

(a) FOR $\bar{k}=\frac{1}{2}l$: $p^2=\frac{g(l/2)}{(l/2)^2+\frac{1}{12}l^2}=\frac{gl}{\frac{1}{2}+\frac{1}{6}}=\frac{3}{2}\frac{g}{l}$

$$f=\frac{p}{2\pi} \qquad f=\frac{1}{2\pi}\sqrt{\frac{3g}{2l}}$$ ◀

(b) MAKING $p^2=\frac{3}{2}\frac{g}{l}$ IN EQ.(1):

$$\frac{3}{2}\frac{g}{l}=\frac{g\bar{k}}{\bar{k}^2+\frac{1}{12}l^2} \qquad \bar{k}^2-\frac{2l}{3}\bar{k}+\frac{l^2}{12}=0$$

SOLVING FOR $\bar{k}$: $\bar{k}=\frac{l}{3}\pm\frac{l}{6}$, $\left(\bar{k}=\frac{l}{2}\right)$ AND $\bar{k}=\frac{l}{6}$ ◀

19.42

SEE SOLUTION OF PROB 19.41 FOR DERIVATION OF EQ.(1):

$$p^2=\frac{g\bar{k}}{\bar{k}^2+\frac{1}{12}l^2} \qquad (1)$$

(a)

$$p^2=\frac{g}{\bar{k}+\frac{1}{12}\frac{l^2}{\bar{k}}} \qquad \tau=\frac{2\pi}{p}=\frac{2\pi}{\sqrt{g}}\sqrt{\bar{k}+\frac{1}{12}\frac{l^2}{\bar{k}}}$$

FOR SMALLEST τ WE MUST HAVE $\bar{k}+\frac{1}{12}\frac{l^2}{\bar{k}}$ MINIMUM.

$$\frac{d}{d\bar{k}}\left(\bar{k}+\frac{1}{12}\frac{l^2}{\bar{k}}\right)=1-\frac{1}{12}\frac{l^2}{\bar{k}^2}=0 \qquad \bar{k}^2=\frac{l^2}{12}$$

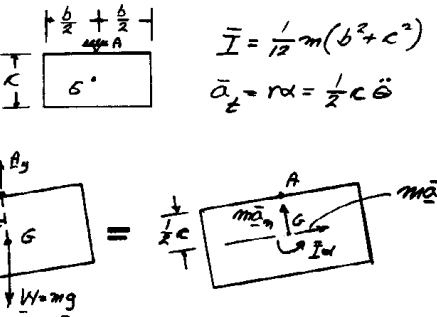
$$\bar{k}=l/\sqrt{12}$$ ◀

(b) FOR $\bar{k}=l/\sqrt{12}$ OR $\bar{k}^2=l^2/12$

EQ.(1): $p^2=\frac{g\,l/\sqrt{12}}{\frac{l^2}{12}+\frac{l^2}{12}}=\frac{6}{\sqrt{12}}\frac{g}{l}=\sqrt{3}\frac{g}{l}=1.7321\frac{g}{l}$

$$\tau=\frac{2\pi}{p}=2\pi\sqrt{\frac{l}{1.7321g}}=4.774\sqrt{\frac{l}{g}} \qquad \tau=4.77\sqrt{\frac{l}{g}}$$ ◀

19.43



$$+\circlearrowleft \Sigma M_A=\Sigma(M_A)_{eff}: \quad -mg\left(\tfrac{1}{2}\bar{k}\sin\theta\right)=\bar{I}\alpha+m\bar{a}_t\left(\tfrac{1}{2}c\right)$$

$$-\frac{1}{2}mg\bar{k}\sin\theta=\frac{m}{12}(b^2+c^2)\ddot{\theta}+m\left(\tfrac{1}{2}c\right)^2\ddot{\theta}$$

$$\ddot{\theta}+\frac{g}{2}\left[\frac{c}{\frac{c^2}{3}+\frac{b^2}{12}}\right]\sin\theta=0$$

$$p^2=\frac{g}{2}\frac{c}{\frac{c^2}{3}+\frac{b^2}{12}}=\frac{g}{b}\frac{6(c/b)}{\left[4(c/b)^2+1\right]} \qquad (1)$$

(a) SINCE $\tau=2\pi/p$, MINIMUM τ OCCURS WHEN p^2 IS MAXIMUM. WE CONSIDER (c/b) AS THE VARIABLE.

$$\frac{d(p^2)}{d(c/b)}=\frac{6g}{b}\frac{\left[4(c/b)^2+1\right]-(c/b)\left[8(c/b)\right]}{\left[4(c/b)^2+1\right]^2}$$

$$4\left(\frac{c}{b}\right)^2+1-8\left(\frac{c}{b}\right)^2=0$$

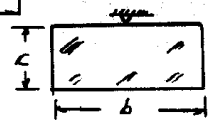
$$\left(\frac{c}{b}\right)^2=\frac{1}{4} \qquad \frac{c}{b}=\frac{1}{2}$$ ◀

(b) FOR SIMPLE PENDULUM OF LENGTH $\bar{k}$, WE HAVE $p^2=g/\bar{k}$. WE EQUATE THIS VALUE OF p^2 TO THAT OF EQ.(1).

$$\frac{g}{b}\frac{6(c/b)}{\left[4(c/b)^2+1\right]}=\frac{g}{\bar{k}}$$

$$6(c/b)^2=4(c/b)^2+1 \qquad \frac{c^2}{b^2}=\frac{1}{2} \qquad \frac{c}{b}=0.707$$ ◀

19.44



FROM PROB 19.43, EQ(1):

$$p^2 = \frac{g}{b}\,\frac{6(c/b)}{\left[4(c/b)^2+1\right]} \qquad (1)$$

(a) FOR $c = b$:

$$p^2 = \frac{g}{b}\,\frac{6}{[4+1]} = \frac{6g}{5b}$$

$$\tau = \frac{2\pi}{p} \qquad \tau = 2\pi\sqrt{\frac{5b}{6g}}$$ ◀

(b) WE SEEK A SECOND VALUE FOR WHICH $p^2 = 6g/5b$

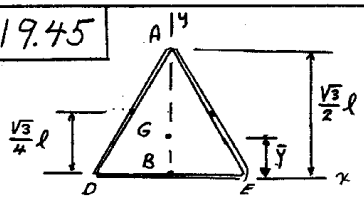
$$\frac{6g}{5b} = \frac{g}{b}\,\frac{6(c/b)}{\left[4(c/b)^2+1\right]}$$

$$\frac{6}{5}\left[4(c/b)^2+1\right] = 6(c/b) \;;\quad 24(c/b)^2 - 30(c/b) + 6 = 0$$

$$6\left[(c/b)-1\right]\left[4(c/b)-1\right] = 0$$

$c = b$; $c = \frac{1}{4}b$ ◀

19.45



FOR 3 IDENTICAL SIDES EACH OF MASS $\frac{1}{3}m$.

$$\bar{y} = \frac{2\left[\frac{1}{3}m\left(\frac{\sqrt{3}}{4}\ell\right)\right]}{m}$$

$$\bar{y} = \frac{\ell}{2\sqrt{3}}$$

THUS: $BG = \bar{y} = \dfrac{\ell}{2\sqrt{3}}$

$$AG = \frac{\sqrt{3}}{2}\ell - \bar{y} = \frac{\sqrt{3}}{2}\ell - \frac{\ell}{2\sqrt{3}} = \frac{3-1}{2\sqrt{3}}\ell = \frac{\ell}{\sqrt{3}}$$

MASS MOMENT OF INERTIA ABOUT G OF SIDE DE.

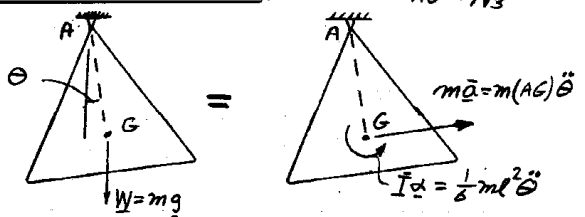
$$I_G = \frac{1}{12}\left(\frac{m}{3}\right)\ell^2 + \frac{m}{3}(BG)^2 = \frac{1}{36}m\ell^2 + \frac{m}{3}\left(\frac{\ell}{2\sqrt{3}}\right)^2$$

$$= \left(\frac{1}{36}+\frac{1}{36}\right)m\ell^2 = \frac{1}{18}m\ell^2$$

BY SYMMETRY WE HAVE THE SAME VALUE FOR THE MOMENT OF INERTIA ABOUT G OF SIDES AD AND AE THUS FOR WHOLE WIRE FIGURE

$$\bar{I} = 3I_G = 3\left(\frac{1}{18}m\ell^2\right) = \frac{1}{6}m\ell^2$$

(a) SUSPENDED FROM VERTEX $\qquad AG = \ell/\sqrt{3}$



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}: \quad -mg(AG)\sin\theta = \bar{I}\alpha + (m\bar{a})(AG)$$

$$-mg\left(\frac{\ell}{\sqrt{3}}\right)\sin\theta = \frac{1}{6}m\ell^2\ddot{\theta} + m\left(\frac{\ell}{\sqrt{3}}\right)^2\ddot{\theta}$$

$$-\frac{g}{\sqrt{3}\ell}\sin\theta = \left(\frac{1}{6}+\frac{1}{3}\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{2}{\sqrt{3}}\frac{g}{\ell}\sin\theta = 0 \qquad (1)$$

FOR SMALL OSCILLATIONS $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{2}{\sqrt{3}}\frac{g}{\ell}\theta = 0 \qquad p = \sqrt{\frac{2}{\sqrt{3}}\frac{g}{\ell}} = \sqrt{\frac{g}{0.86603\ell}}$$

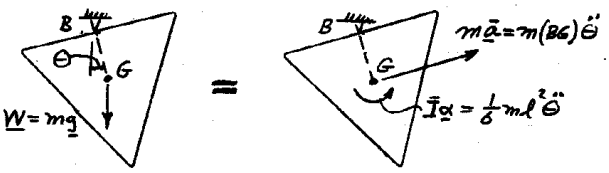
$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{0.86603\,\ell/g}$$

FOR $\ell = 0.3$ m: $\quad \tau = 2\pi\sqrt{0.86603(0.3)/9.81} \qquad \tau = 1.023$ s ◀

(CONTINUED)

19.45 CONTINUED

(b) SUSPENDED FROM MIDPOINT OF A SIDE $\qquad BG = \dfrac{\ell}{2\sqrt{3}}$



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}: \quad -mg(BG)\sin\theta = \bar{I}\alpha + (m\bar{a})(BG)$$

$$-mg\left(\frac{\ell}{2\sqrt{3}}\right)\sin\theta = \frac{1}{6}m\ell^2\ddot{\theta} + m\left(\frac{\ell}{2\sqrt{3}}\right)^2\ddot{\theta}$$

$$-\frac{g}{2\sqrt{3}\ell}\sin\theta = \left(\frac{1}{6}+\frac{1}{12}\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{2}{\sqrt{3}}\frac{g}{\ell}\sin\theta = 0$$

WE NOTE THAT THIS EQUATION IS IDENTICAL TO EQ.(1) OF PART (a). THUS WE AGAIN HAVE

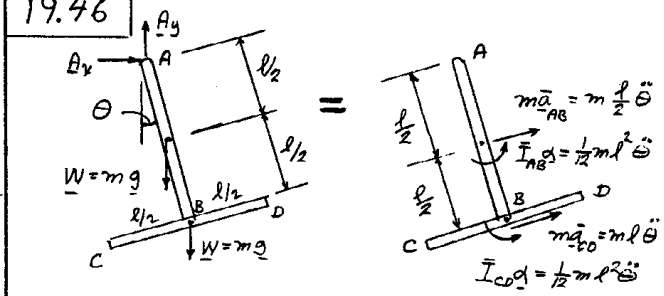
$$\tau = 2\pi\sqrt{0.86603\,\ell/g} \qquad \text{FOR } \ell = 0.3\text{ m} \qquad \tau = 1.023\text{ s}$$ ◀

NOTE: THE SAME ANSWER IS OBTAINED IN PARTS (a) AND (b) BECAUSE THE VERTEX A OF THE WIRE TRIANGLE IS THE CENTER OF OSCILLATION CORRESPONDING TO THE MIDPOINT B OF THE SIDE.

INDEED: $AG = \dfrac{\ell}{\sqrt{3}}$; $BG = \dfrac{\ell}{2\sqrt{3}}$, $\bar{k}^2 = \dfrac{\bar{I}}{m} = \dfrac{1}{6}\ell^2$

WE CHECK THAT $(AG)(BG) = \bar{k}^2$ (cf. PROB 19.38)

19.46



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}:$$

$$-mg\frac{\ell}{2}\sin\theta - mg\ell\sin\theta = \bar{I}_{AB}\alpha + m\bar{a}_{AB}\frac{\ell}{2} + \bar{I}_{CD}\alpha + m\bar{a}_{CD}\ell$$

$$-\frac{3}{2}mg\ell\sin\theta = \frac{1}{12}m\ell^2\ddot{\theta} + m\left(\frac{\ell}{2}\right)^2\ddot{\theta} + \frac{1}{12}m\ell^2\ddot{\theta} + m\ell^2\ddot{\theta}$$

$$-\frac{3}{2}mg\ell\sin\theta = \frac{17}{12}m\ell^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{18}{17}\frac{g}{\ell}\sin\theta = 0$$

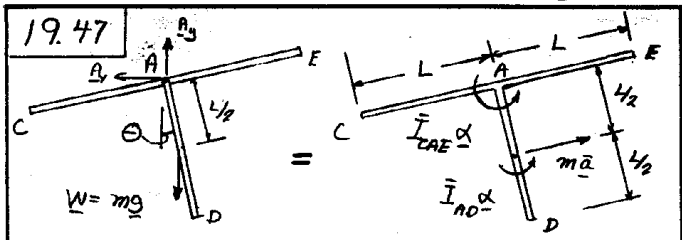
FOR SMALL OSCILLATIONS, $\sin\theta \approx \theta$, WE HAVE

$$\ddot{\theta} + \frac{18}{17}\frac{g}{\ell}\theta = 0$$

$$p^2 = \frac{18}{17}\frac{g}{\ell} \qquad p = \sqrt{\frac{18g}{17\ell}}$$

$$f = \frac{p}{2\pi} \qquad f = \frac{1}{2\pi}\sqrt{\frac{18g}{17\ell}}$$ ◀

19.47



m = MASS OF EACH ARM OF LENGTH L.

$\bar{I}_{AD} = \frac{1}{12}mL^2 \qquad \bar{I}_{CAE} = \frac{1}{12}(2m)(2L)^2 = \frac{2}{3}mL^2$

$\alpha = \ddot{\theta} \qquad \bar{a} = \frac{L}{2}\alpha = \frac{L}{2}\ddot{\theta}$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}: \quad -mg\frac{L}{2}\sin\theta = \bar{I}_{CAE}\alpha + \bar{I}_{AD}\alpha + m\bar{a}\frac{L}{2}$

$$-mg\frac{L}{2}\sin\theta = \frac{2}{3}mL^2\ddot{\theta} + \frac{1}{12}mL^2\ddot{\theta} + m\left(\frac{L}{2}\right)^2\ddot{\theta}$$

$$\ddot{\theta} + \left[\frac{g}{2L}\right]\sin\theta = 0$$

FOR SMALL OSCILLATIONS, $\sin\theta \approx \theta$, WE HAVE

$$\ddot{\theta} + \left[\frac{g}{2L}\right]\theta = 0 \qquad p^2 = \frac{g}{2L}$$

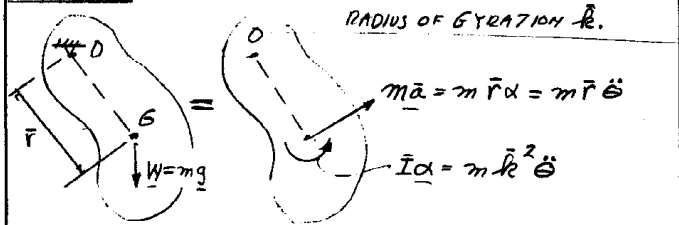
$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{2L}{g}}$$

DATA: $L = 120$ ft

$$\tau = 2\pi\sqrt{\frac{2(120\text{ ft})}{32.2\text{ ft/s}^2}} = 17.154\text{ s} \qquad \tau = 17.15\text{ s}$$ ◀

19.48

CONSIDER GENERAL PENDULUM OF CENTROIDAL RADIUS OF GYRATION $\bar{k}$.



$+\circlearrowleft \Sigma M_O = \Sigma(M_O)_{eff}: \quad -mg\bar{r}\sin\theta = (m\bar{r}\ddot{\theta})\bar{r} + m\bar{k}^2\ddot{\theta}$

$$\ddot{\theta} + \left[\frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\right]\sin\theta = 0$$

FOR SMALL OSCILLATIONS, $\sin\theta \approx \theta$, WE HAVE

$$\ddot{\theta} + \left[\frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\right]\theta = 0 \qquad p^2 = \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}}$$

CONNECTING ROD SUSPENDED AT A: $\tau = 1.03$ s

$\bar{r} = r_a = 160$ mm $= 0.16$ m

THUS:

$$1.03 = 2\pi\sqrt{\frac{(0.16)^2 + \bar{k}^2}{(9.81)(0.16)}}$$

$$(0.16)^2 + \bar{k}^2 = (9.81)(0.16)\left(\frac{1.03}{2\pi}\right)^2 = 0.04218$$

$$\bar{k}^2 = 0.04218 - 0.0256 = 0.01658$$

$$\bar{k} = 0.12876\text{ m} \qquad \bar{k} = 128.8\text{ mm}$$ ◀

19.49

FROM SOLUTION OF PROB 19.48:

$$\tau = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}}$$

FOR ROD SUSPENDED AT A: $\tau = 0.87$ s $\qquad \bar{r} = r_a$

$$0.87\text{ s} = 2\pi\sqrt{\frac{r_a^2 + \bar{k}^2}{g r_a}} \qquad (1)$$

FOR ROD SUSPENDED AT B: $\tau = 0.78$ s $\qquad \bar{r} = r_b$

$$0.78\text{ s} = 2\pi\sqrt{\frac{r_b^2 + \bar{k}^2}{g r_b}} \qquad (2)$$

WE ALSO KNOW THAT $r_a + r_b = 0.255$ m (3)

FROM (1): $r_a^2 + \bar{k}^2 = g r_a\left(\frac{0.87}{2\pi}\right)^2$ (1')

FROM (2): $r_b^2 + \bar{k}^2 = g r_b\left(\frac{0.78}{2\pi}\right)^2$ (2')

(a) SUBTRACTING (2') FROM (1'):

$$r_a^2 - r_b^2 = \frac{g}{4\pi^2}(0.7569\, r_a - 0.6084\, r_b) \qquad (4)$$

DIVIDING (4) BY (3), MEMBER BY MEMBER:

$$r_a - r_b = \frac{1}{0.255}\left(\frac{g}{4\pi^2}\right)(0.7569\, r_a - 0.6084\, r_b)$$

$$r_a - r_b = 0.73758\, r_a - 0.59287\, r_b$$

$$0.26242\, r_a = 0.40713\, r_b$$

$$r_b = 0.64456\, r_a \qquad (5)$$

SUBSTITUTING FOR r_b FROM (5) INTO (3):

$r_a + 0.64456\, r_a = 0.255$, $\qquad r_a = 0.15506\text{ m} = 155.1\text{ mm}$ ◀

EQ (5): $r_b = (0.64456)(0.15506\text{ m}) \qquad r_b = 99.9\text{ mm}$

(b) FROM (1'): $(0.15506)^2 + \bar{k}^2 = 9.81(0.15506)\left(\frac{0.87}{2\pi}\right)^2$

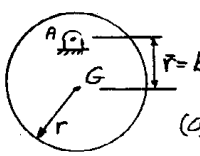
$$\bar{k}^2 = 0.029164 - 0.024044 = 0.00512$$

$$\bar{k} = 0.07156 \qquad \bar{k} = 71.6\text{ mm}$$ ◀

19.50

FOR CIRCULAR DISK: $\bar{I} = \frac{1}{2}mr^2 \qquad \bar{k}^2 = \frac{1}{2}r^2$

FROM SOLUTION OF PROB 19.48:



$$\tau = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}}$$

(a) FOR $\bar{r} = b = r$

$$\tau = 2\pi\sqrt{\frac{r^2 + \frac{1}{2}r^2}{gr}} \qquad \tau = 2\pi\sqrt{\frac{3r}{2g}}$$ ◀

(b) SECOND VALUE b_2, FOR SAME PERIOD

$$\tau_2 = \tau_1: \quad 2\pi\sqrt{\frac{b_2^2 + \frac{1}{2}r^2}{g b_2}} = 2\pi\sqrt{\frac{3r}{2g}}$$

$$\frac{b_2^2 + \frac{1}{2}r^2}{g b_2} = \frac{3r}{2g}; \qquad 2b_2^2 + r^2 = 3rb_2$$

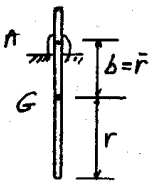
$$b_2^2 - \frac{3}{2}rb_2 + \frac{1}{2}r^2 = 0$$

$$(b_2 - r)\left(b_2 - \tfrac{1}{2}r\right) = 0: \qquad b_2 = r \text{ OR } b_2 = \tfrac{1}{2}r$$ ◀

19.51

FOR CIRCULAR DISK: $I_{DIAMETER} = \frac{1}{4}mr^2$

$\bar{k}^2 = \frac{1}{4}r^2$



FROM SOLUTION OF PROB 19.48

$$T = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}}$$

(a) FOR $\bar{r} = b = r$:

$$T = 2\pi\sqrt{\frac{r^2 + \frac{1}{4}r^2}{gr}} \qquad T = 2\pi\sqrt{\frac{5r}{4g}}$$ ◀

(b) SECOND VALUE, b_2, FOR SAME PERIOD

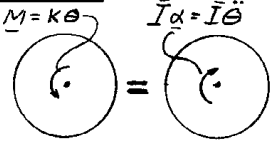
$T_2 = T_1$: $2\pi\sqrt{\frac{b_2^2 + \frac{1}{4}r^2}{g b_2}} = 2\pi\sqrt{\frac{5r}{4g}}$

$$\frac{b_2^2 + \frac{1}{4}r^2}{g b_2} = \frac{5r}{4g}; \quad 4b_2^2 + r^2 = 5rb_2$$

$$b_2^2 - \frac{5}{4}rb_2 + \frac{1}{4}r^2 = 0$$

$(b_2 - r)(b_2 - \frac{1}{4}r) = 0$; $b_2 = r$ or $b_2 = \frac{1}{4}r$ ◀

19.52



$+\circlearrowright \Sigma M = \Sigma M_{eff}$

$-K\theta = \bar{I}\ddot{\theta}$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

$p^2 = \frac{K}{\bar{I}}$ $\quad T = 2\pi\sqrt{\frac{\bar{I}}{K}}$ (1)

FOR 50-mm-DIAMETER SPHERE, $r = 0.025$ m

$$\bar{I}_S = \frac{2}{5}mr^2 = \frac{2}{5}\left(\frac{4}{3}\pi r^3\rho\right)r^2 = \frac{8}{15}\pi r^5\rho$$

$$= \frac{8}{15}\pi(0.025)^5(7850\ kg/m^3) = 128.45 \times 10^{-6}\ kg\cdot m^2$$

SOLVE EQ(1) FOR $\bar{I}$:

$$\bar{I} = \frac{K}{4\pi^2}T^2; \qquad \frac{\bar{I}_R}{\bar{I}_S} = \left(\frac{T_R}{T_S}\right)^2 \quad (2)$$

DATA: $T_R = 4.1$ s, $T_S = 6.2$ s, $m_R = 0.45$ kg

EQ(2): $\frac{\bar{I}_R}{\bar{I}_S} = \left(\frac{4.1\ s}{6.2\ s}\right)^2 = 0.4373$

$$\bar{I}_R = 0.4373\,\bar{I}_S = 0.4373(128.45 \times 10^{-6}\ kg\cdot m^2)$$

$$\bar{I}_R = 56.17 \times 10^{-6}\ kg\cdot m^2$$

$$\bar{k}_R^2 = \frac{\bar{I}_R}{m_R} = \frac{56.17 \times 10^{-6}\ kg\cdot m^2}{0.45\ kg}$$

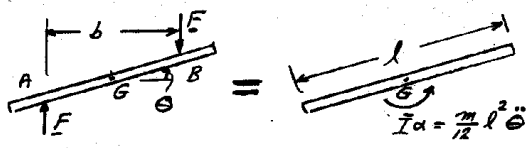
$$\bar{k}_R^2 = 124.82 \times 10^{-6}\ m^2$$

$$\bar{k}_R = 11.173 \times 10^{-3}\ m$$

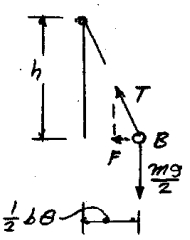
$\bar{k}_R = 11.17$ mm ◀

19.53

(a) SMALL OSCILLATIONS ABOUT VERTICAL AXIS



$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{eff}$: $\quad -Fb = \frac{1}{12}ml^2\ddot{\theta}$ (1)



BUT WE NOTE THAT F IS HORIZONTAL COMPONENT OF TENSION T IN THE WIRE

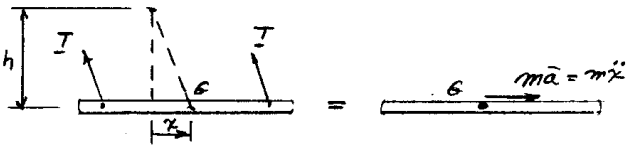
$$F = \frac{\frac{1}{2}b\theta}{h}T = \frac{b\theta}{2h}\left(\frac{mg}{2}\right) = \frac{mgb}{4h}\theta$$

EQ(1): $\quad -\left(\frac{mgb}{4h}\theta\right)b = \frac{1}{12}ml^2\ddot{\theta}$

$$\ddot{\theta} + \frac{3b^2g}{l^2h}\theta = 0$$

THUS: $p^2 = \frac{3b^2g}{l^2h}$ $\quad T = \frac{2\pi}{p} = 2\pi\frac{l}{b}\sqrt{\frac{h}{3g}}$ ◀

(b) SMALL OSCILLATIONS IN VERTICAL PLANE



$+\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $\quad -2\left(\frac{x}{h}T\right) = m\ddot{x}$

BUT $T = \frac{1}{2}W = \frac{1}{2}mg$: $\quad -2\frac{x}{h}\frac{mg}{2} = m\ddot{x}$

$$\ddot{x} + \frac{g}{h}x = 0$$

THUS: $p^2 = \frac{g}{h}$ $\quad T = \frac{2\pi}{p} = 2\pi\sqrt{\frac{h}{g}}$ ◀

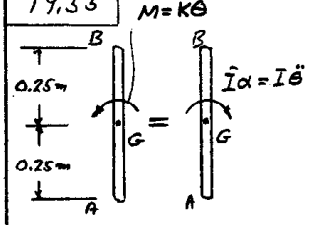
19.54

DATA: $h = 0.3$ m, $b = 0.4$ m, $l = 0.6$ m

(a) $T = 2\pi\frac{l}{b}\sqrt{\frac{h}{3g}} = 2\pi\frac{0.6}{0.4}\sqrt{\frac{0.3}{3(9.81)}}$ $\quad T = 0.952$ s ◀

(b) $T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{0.3}{9.81}}$ $\quad T = 1.099$ s ◀

19.55



$+\circlearrowright \Sigma M = \Sigma(M)_{eff}$

$-K\theta = \bar{I}\ddot{\theta}$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

$p^2 = \frac{K}{\bar{I}}$ (1)

$T = 2\pi\sqrt{\frac{\bar{I}}{K}}$ (2)

DATA: $K = 1.95$ N·m/rad

$m = 3$ kg

$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(3\ kg)(0.5\ m)^2 = 62.5 \times 10^{-3}\ kg\cdot m^2$

(a) EQ(1) $\quad T = 2\pi\sqrt{\frac{62.5 \times 10^{-3}\ kg\cdot m^2}{1.95\ N\cdot m/rad}}$ $\quad T = 1.125$ s ◀

(CONTINUED)

19.55 CONTINUED

(b) MAX. VELOCITY

EQ(1) $p^2 = \frac{k}{\bar{I}} = \frac{1.95\ N\cdot m/rad}{62.5\times10^{-3}\ kg\cdot m^2} = 31.2$

$p = 5.586\ rad/s$

SIMPLE HARMONIC MOTION

$\omega_m = \theta_m p \qquad \theta_m = 180^\circ = \pi\ rad$

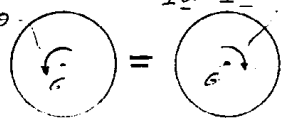
$\omega_m = (\pi\ rad)(5.586\ rad/s) = 17.548\ rad/s$

$(v_A)_m = (AG)\omega_m = (0.25m)(17.548\ rad/s) = 4.387\ m/s$

$(v_A)_m = 4.39\ m/s$ ◀

19.56

θ = ANGLE OF ROTATION ABOUT VERTICAL



$\bar{I}\alpha = \bar{I}\ddot{\theta}$

$+\circlearrowleft \Sigma M = \Sigma M_{eff}$

$-K\theta = \bar{I}\ddot{\theta}$

$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$

$p^2 = \frac{K}{\bar{I}} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\bar{I}}{K}}$

SPRING CONSTANT

$M = 120\ lb\cdot in.$ FOR $\theta = 5^\circ$:

$$K = \frac{M}{\theta} = \frac{(120\ lb\cdot in.)\left(\frac{1\ ft}{12\ in.}\right)}{5^\circ\left(\frac{\pi\ rad}{180^\circ}\right)} = 114.592\ lb\cdot ft/rad$$

Disk. $W = 20\ lb$, $r = 12\ in. = 1\ ft$

$$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}\frac{20\ lb}{32.2\ ft/s^2}(1\ ft)^2 = 0.31056\ lb\cdot ft\cdot s^2$$

(a) $p^2 = \frac{k}{\bar{I}} = \frac{114.592}{0.31056} = 368.99 \qquad p = 19.209\ rad/s$

$\tau = \frac{2\pi}{p} = \frac{2\pi}{19.209\ rad/s} = 0.3271\ s \qquad \tau = 0.327\ s$ ◀

(b) FOR SIMPLE HARMONIC MOTION, WITH $\theta_m = 3^\circ$

$\omega_m = p\,\theta_m = (19.209\ rad/s)\left(3^\circ\left(\frac{\pi\ rad}{180^\circ}\right)\right) = 1.0058\ rad/s$

FOR POINT ON RIM:

$v_m = r\omega_m = (12in.)(1.0058\ rad/s) \qquad v_m = 12.07\ in/s$ ◀

19.57

SEE SOLUTION OF PROB 19.56 FOR DERIVATION OF

$p^2 = \frac{K}{\bar{I}}$ AND $\tau = 2\pi\sqrt{\frac{\bar{I}}{K}}$

SOLVING FOR $\bar{I}$: $\bar{I} = \frac{K}{4\pi^2}\tau^2 \qquad (1)$

FOR DISK ALONE (SEE PROB 19.56):

$\bar{I}_D = \bar{I}_{DISK} = 0.31056\ lb\cdot ft\cdot s^2 \qquad \tau_D = 0.3271\ s$

FOR DISK AND CASTING:

$\bar{I}_{D+C} = \bar{I}_D + \bar{I}_C = 0.31056 + \bar{I}_C$ AND $\tau_{D+C} = 0.410\ s$

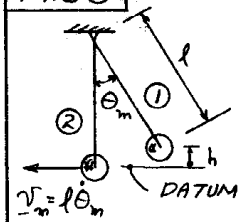
FROM EQ(1) IT FOLLOWS THAT:

$$\frac{\bar{I}_{D+C}}{\bar{I}_D} = \left(\frac{\tau_{D+C}}{\tau_D}\right)^2; \qquad \frac{0.31056+\bar{I}_C}{0.31056} = \left(\frac{0.410\ s}{0.3271\ s}\right)^2$$

$1 + 3.22\,\bar{I}_C = 1.5711 \qquad \bar{I}_C = 0.17736\ lb\cdot ft\cdot s^2$

$\bar{I}_C = 0.1774\ lb\cdot ft\cdot s^2$ ◀

19.58



$v_m = l\dot{\theta}_m$

POSITION 1: $T_1 = 0$

$V_1 = Wh = mgl(1-\cos\theta_m)$

FOR SMALL OSCILLATIONS, $\cos\theta_m = 1 - \frac{1}{2}\theta_m^2$

THUS: $V_1 = mgl\left(\frac{1}{2}\theta_m^2\right) = \frac{1}{2}mgl\theta_m^2$

POSITION 2: $V_2 = 0, \quad T_2 = \frac{1}{2}mv_m^2 = \frac{1}{2}m(l\dot{\theta}_m)^2$

$T_2 = \frac{1}{2}ml^2\dot{\theta}_m^2$

CONSERVATION OF ENERGY $T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}mgl\theta_m^2 = \frac{1}{2}ml^2\dot{\theta}_m^2 + 0$

BUT $\dot{\theta}_m = \theta_m p$: $\frac{1}{2}mgl\theta_m^2 = \frac{1}{2}ml^2\theta_m^2 p^2$

SOLVING FOR p^2:

$p^2 = \frac{g}{l} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{l}{g}}$ ◀

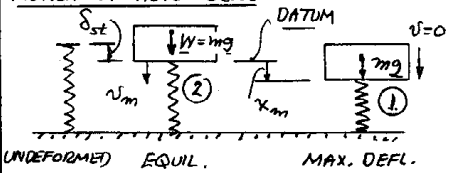
19.59

m = MASS OF AUTOMOBILE

k = TOTAL SPRING CONSTANT OF FOUR SPRINGS

$$\delta_{ST} = \frac{W}{k} = \frac{mg}{k} = 0.180\ mm \qquad (1)$$

MOTION OF AUTOMOBILE



FOR S.H.M. $v_m = p\,x_m$

POSITION 1: $T_1 = 0, \quad V_1 = \frac{1}{2}k(\delta_{st}+x_m)^2 - mgx_m$

POSITION 2: $T_2 = \frac{1}{2}mv_m^2 = \frac{1}{2}m(px_m)^2$

$V_2 = \frac{1}{2}k\delta_{st}^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}k(\delta_{st}+x_m)^2 - mgx_m = \frac{1}{2}m(px_m)^2 + \frac{1}{2}k\delta_{st}^2$

$\frac{1}{2}k\delta_{st}^2 + k\delta_{st}x_m + \frac{1}{2}kx_m^2 - mgx_m = \frac{1}{2}mp^2x_m^2 + \frac{1}{2}k\delta_{st}^2$

SINCE $mg = \delta_{st}k$, WE HAVE

$k\delta_{st}x_m + \frac{1}{2}kx_m^2 - (\delta_{st}k)x_m = \frac{1}{2}mp^2x_m^2$

$\frac{1}{2}kx_m^2 = \frac{1}{2}mp^2x_m^2$

$p^2 = \frac{k}{m}$ RECALL FROM (1) THAT $k = \frac{mg}{\delta_{st}}$

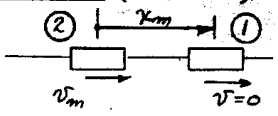
$p^2 = \frac{1}{m}\left(\frac{mg}{\delta_{st}}\right) = \frac{g}{\delta_{st}} = \frac{9.81\ m/s^2}{0.180\ m} = 54.5$

$p = 7.382\ rad/s$

$f = \frac{p}{2\pi} = \frac{7.382\ rad/s}{2\pi} = 1.1749\ s^{-1}$

$f = 1.175\ Hz$ ◀

19.60 (Prob 19.4)



Position 1: MAX. DISPLACEMENT
$T_1 = 0, \quad V_1 = \frac{1}{2} k x_m^2$

Position 2: EQUIL. POSITION
$V_1 = 0; \quad T_2 = \frac{1}{2} m v_m^2$

CONSERVATION OF ENERGY
$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2} k x_m^2 = \frac{1}{2} m v_m^2 + 0$

$k x_m^2 = m v_m^2$

SETTING $v_m = x_m p$:

$k x_m^2 = m x_m^2 p^2 \qquad p^2 = \frac{k}{m}$

DATA $W = 7.5$ lb $\quad k = 5$ lb/in. $= 60$ lb/ft, $\quad x_m = 3$ in.

$p^2 = \frac{k}{m} = \frac{60 \text{ lb/ft}}{(7.5 \text{ lb})/(32.2 \text{ ft/s}^2)} = 257.6 \qquad p = 16.05$ rad/s

$v_m = p x_m = (16.05 \text{ rad/s})(\frac{3}{12} \text{ ft}) \qquad v_m = 4.01$ ft/s ◀

$a_m = p^2 x_m = (257.6)(\frac{3}{12} \text{ ft}) \qquad a_m = 64.4$ ft/s^2 ◀

19.61 (Prob. 19.5)

SEE SOLUTION OF PROB 19.60 FOR DERIVATION OF $p^2 = \frac{k}{m}$

DATA: $W = 4$ lb, $\quad k = 6$ lb/in. $= 72$ lb/ft, $\quad v_m = 55$ in/s

$p^2 = \frac{k}{m} = \frac{72 \text{ lb/ft}}{(4 \text{ lb})/(32.2 \text{ ft/s}^2)} = 579.6 \qquad p = 24.075$ rad/s

$v_m = p x_m; \quad (55 \text{ in./s}) = (24.075 \text{ rad/s}) x_m$

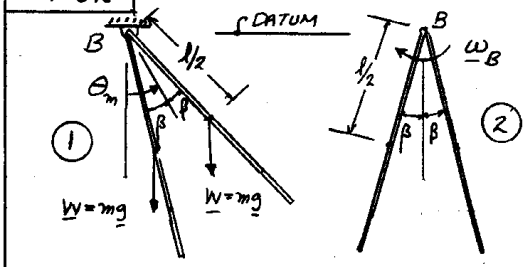
$x_m = 2.285$ in. $\qquad x_m = 2.28$ in ◀

$a_m = p^2 x_m; \quad a_m = (579.6)(2.285 \text{ in}) = 1324.1 \text{ in./s}^2$

$a_m = 110.3$ ft/s^2 ◀

19.62

WE DENOTE BY m THE MASS OF HALF THE WIRE



Position 1 (MAX DEFLECTION)

$T_1 = 0, \quad V_1 = -mg\frac{l}{2}\cos(\theta_m - \beta) - mg\frac{l}{2}\cos(\theta_m + \beta)$

$= -mg\frac{l}{2}(\cos\theta_m\cos\beta + \sin\theta_m \sin\beta + \cos\theta_m\cos\beta - \sin\theta_m\sin\beta)$

$V_1 = -mgl\cos\beta\cos\theta_m$

FOR SMALL OSCILLATIONS $\cos\theta_m \approx 1 - \frac{1}{2}\theta_m^2$

$V_1 = -mgl\cos\beta + \frac{1}{2}mgl\cos\beta\,\theta_m^2$

(CONTINUED)

19.62 CONTINUED

Position 2. MAX VELOCITY

$T_2 = \frac{1}{2} I_B \omega_m^2 \qquad$ BUT $I_B = 2(\frac{1}{3} m l^2)$

THUS $T_2 = \frac{1}{2}(\frac{2}{3} m l^2)\omega_m^2$

$V_2 = -2mg(\frac{l}{2}\cos\beta) = -mgl\cos\beta$

CONSERVATION OF ENERGY $T_1 + V_1 = T_2 + V_2$

$-mgl\cos\beta + \frac{1}{2}mgl\cos\beta\,\theta_m^2 = \frac{1}{3}ml^2\omega_m^2 - mgl\cos\beta$

SETTING $\omega_m = \theta_m p$: $\quad \frac{1}{2}mgl\cos\beta\,\theta_m^2 = \frac{1}{3}ml^2\theta_m^2 p^2$

$p^2 = \frac{3}{2}\frac{g}{l}\cos\beta \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{2l}{3g\cos\beta}} \qquad (1)$

BUT, FOR $\beta = 0, \quad \tau_0 = 2\pi\sqrt{\frac{2l}{3g}}$

FOR $\tau = 2\tau_0$: $\quad 2\pi\sqrt{\frac{2l}{3g\cos\beta}} = 2\left(2\pi\sqrt{\frac{2l}{3g}}\right)$

SQUARING AND REDUCING:

$\frac{1}{\cos\beta} = 4 \qquad \cos\beta = \frac{1}{4} \qquad \beta = 75.5°$ ◀

19.63

SEE SOLUTION OF PROB 19.62 FOR DERIVATION OF EQ(1) $\quad \tau = 2\pi\sqrt{\frac{2l}{3g\cos\beta}}$

DATA: $l = 0.6$ m $\qquad \beta = 30°$

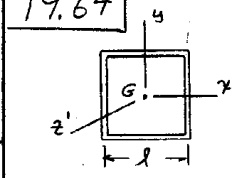
$\tau = 2\pi\sqrt{\frac{2(0.6 \text{ m})}{3(9.8) \text{ m/s}^2)\cos 30°}} = 1.3634$ s

$\tau = 1.363$ s ◀

19.64

CENTROIDAL MASS MOMENTS OF INERTIA

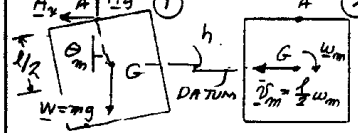
m = TOTAL MASS; $\frac{1}{4}m$ = MASS OF ONE SIDE



$\bar{I}_{x'} = 2(\frac{1}{12}\frac{m}{4}l^2) + 2\frac{m}{4}(\frac{l}{2})^2 = \frac{1}{6}ml^2$

$\bar{I}_{z'} = 4\left[\frac{1}{12}\frac{m}{4}l^2 + \frac{m}{4}(\frac{l}{2})^2\right] = \frac{1}{3}ml^2$

(a) OSCILLATIONS IN PLANE OF SQUARE



$h = \frac{l}{2}(1 - \cos\theta_m) = \frac{l}{2}(\frac{\theta_m^2}{2})$

$h = \frac{1}{4}l\theta_m^2$

FOR SHM. $\quad \omega_m = p\,\theta_m$

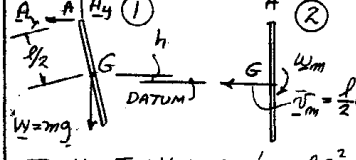
Position 1: $\quad T_1 = 0, \quad V_1 = mgh = \frac{1}{4}mgl\theta_m^2$

Position 2: $\quad V_2 = 0, \quad T_2 = \frac{1}{2}\bar{I}_{z'}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}(\frac{1}{3}ml^2)\omega_m^2 + \frac{1}{2}m(\frac{l}{2}\omega_m)^2$

$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{4}mgl\theta_m^2 = \frac{7}{24}ml^2\omega_m^2$

$\theta_m^2 = \frac{7}{6}\frac{l}{g}(p\theta_m)^2 \qquad p^2 = \frac{6g}{7l} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{7l}{6g}}$ ◀

(b) OSCILLATIONS IN DIRECTION PERPENDICULAR TO SQUARE



Position 1: $\quad T_1 = 0$

$V_1 = mgh = \frac{1}{4}mgl\theta_m^2$

Position 2: $\quad V_2 = 0$

$T_2 = \frac{1}{2}\bar{I}_{x'}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2$

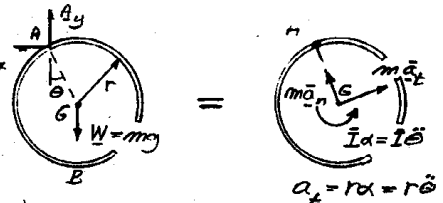
$= \frac{1}{2}(\frac{1}{6}ml^2)\omega_m^2 + \frac{1}{2}m(\frac{l}{2}\omega_m)^2$

$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{4}mgl\theta_m^2 = \frac{5}{24}ml^2\omega_m^2$

$\theta_m^2 = \frac{5}{6}\frac{l}{g}(p\theta_m)^2 \qquad p^2 = \frac{6g}{5l} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{5l}{6g}}$ ◀

19.65

(a) OSCILLATIONS IN THE PLANE OF THE HOOP



$a_t = r\alpha = r\ddot{\theta}$

$+\circlearrowleft \sum M_A = \sum (M_A)_{eff}$:

$-mg(r\sin\theta) = \bar{I}\alpha + (m\bar{a}_t)r$

$-mgr\sin\theta = mr^2\ddot{\theta} + mr^2\ddot{\theta}$

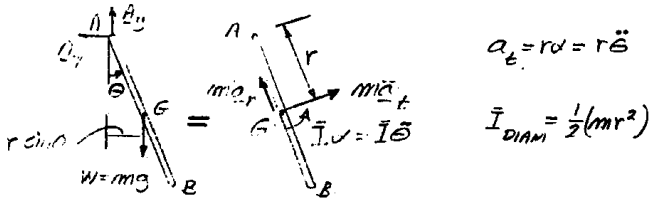
$$\ddot{\theta} + \left[\frac{g}{2r}\right]\sin\theta = 0$$

For small oscillations, $\sin\theta \approx \theta$, we have

$$\ddot{\theta} + \left[\frac{g}{2r}\right]\theta = 0 \qquad p^2 = \frac{g}{2r}$$

$$f = \frac{p}{2\pi} \qquad f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{g}{2r}}$$ ◀

(b) OSCILLATIONS IN DIRECTION $\perp$ TO HOOP



$a_t = r\alpha = r\ddot{\theta}$

$\bar{I}_{DIAM} = \frac{1}{2}(mr^2)$

$+\circlearrowleft \sum M_A = \sum (M_A)_{eff}$: $-mg(r\sin\theta) = \bar{I}\alpha + (m\bar{a}_t)r$

$-mgr\sin\theta = \frac{1}{2}mr^2\ddot{\theta} + mr^2\ddot{\theta}$

$$\ddot{\theta} + \left[\frac{3}{2}\frac{g}{r}\right]\sin\theta = 0$$

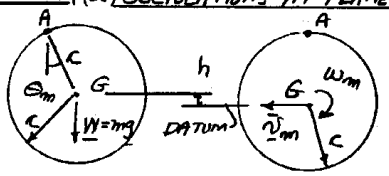
For small oscillations, $\sin\theta \approx \theta$, we have

$$\ddot{\theta} + \left[\frac{3}{2}\frac{g}{r}\right]\theta = 0 \qquad p^2 = \frac{3g}{2r}$$

$$f = \frac{p}{2\pi} \qquad f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{3g}{2r}}$$ ◀

19.66

(a) OSCILLATIONS IN PLANE OF DISK



$h = c(1-\cos\theta_m)$

$h = \frac{1}{2}c\theta_m^2$

$\bar{v}_m = c\omega_m$

$\bar{I} = \frac{1}{2}mc^2$

POSITION 1: $T_1 = 0$, $V_1 = mgh = \frac{1}{2}mgc\theta_m^2$

POSITION 2: $V_2 = 0$, $T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2$

$= \frac{1}{2}\left(\frac{1}{2}mc^2\right)\omega_m^2 + \frac{1}{2}m(c\omega_m)^2 = \frac{3}{4}mc^2\omega_m^2$

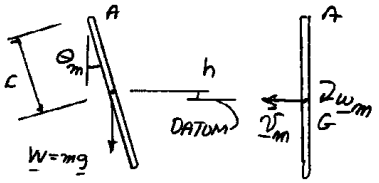
CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + \frac{1}{2}mgc\theta_m^2 = \frac{3}{4}mc^2\omega_m^2$

BUT, FOR SHM: $\omega_m = p\theta_m$: $\frac{1}{2}mgc\theta_m^2 = \frac{3}{4}mc^2(p\theta_m)^2$

$$p^2 = \frac{2}{3}\frac{g}{c} \qquad f = \frac{p}{2\pi} \qquad f = \frac{1}{2\pi}\sqrt{\frac{2g}{3c}}$$ ◀

(b) OSCILLATIONS IN DIRECTION PERPENDICULAR TO DISK



$h = c(1-\cos\theta_m)$

$h = \frac{1}{2}c\theta_m^2$

$\bar{v}_m = c\omega_m$

$\bar{I} = I_{DIAMETER} = \frac{1}{4}mc^2$

POSITION 1: $T_1 = 0$, $V_1 = mgh = \frac{1}{2}mgc\theta_m^2$

POSITION 2: $V_2 = 0$, $T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2$

$= \frac{1}{2}\left(\frac{1}{4}mc^2\right)\omega_m^2 + \frac{1}{2}m(c\omega_m)^2 = \frac{5}{8}mc^2\omega_m^2$

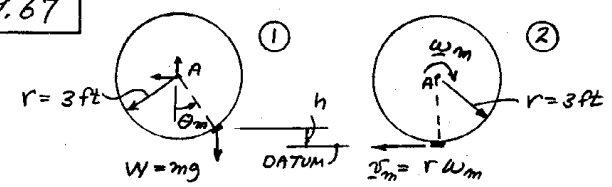
CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + \frac{1}{2}mgc\theta_m^2 = \frac{5}{8}mc^2\omega_m^2$

BUT, FOR SHM: $\omega_m = p\theta_m$: $\frac{1}{2}mgc\theta_m^2 = \frac{5}{8}mc^2(p\theta_m)^2$

$$p^2 = \frac{4}{5}\frac{g}{c} \qquad f = \frac{p}{2\pi} \qquad f = \frac{1}{2\pi}\sqrt{\frac{4}{5}\frac{g}{c}}$$ ◀

19.67



$h = r(1-\cos\theta_m) = r\left(\frac{1}{2}\theta_m^2\right)$

POSITION 1: $T_1 = 0$, $V_1 = W\cdot h = \frac{1}{2}mgr\theta_m^2$

POSITION 2: $V_2 = 0$,

$T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m(r\omega_m)^2 = \frac{1}{2}(\bar{I} + mr^2)\omega_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $0 + \frac{1}{2}mgr\theta_m^2 = \frac{1}{2}(\bar{I} + mr^2)\omega_m^2$

BUT FOR SHM: $\omega_m = p\theta_m$

$$\frac{1}{2}mgr\theta_m^2 = \frac{1}{2}(\bar{I} + mr^2)(p\theta_m)^2 \qquad p^2 = \frac{mgr}{\bar{I} + mr^2} \quad (1)$$

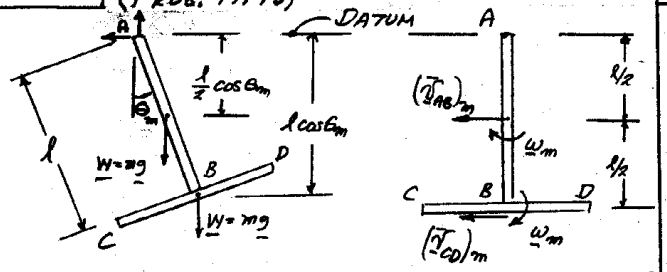
DATA: $W = mg = 8\,lb$, $r = 3\,ft$, $T = 2.2\,s$

$\tau = \frac{2\pi}{p}$; $p = \frac{2\pi}{2.2\,s}$ EQ(1): $\left(\frac{2\pi}{2.2s}\right)^2 = \frac{(8lb)(3ft)}{\bar{I} + \left(\frac{8lb}{32.2ft/s^2}\right)(3ft)^2}$

$\bar{I} + \left(\frac{8}{32.2}\right)(3)^2 = (8)(3)\left(\frac{2.2}{2\pi}\right)^2$: $\bar{I} + 2.24 = 29.4.24$

$\bar{I} = 29.2\ lb\cdot ft\cdot s^2$ ◀

19.68 (PROB. 19.46)



$(\bar{v}_{AB})_m = \frac{l}{2}\omega_m \qquad (\bar{v}_{CD})_m = l\omega_m \qquad \bar{I}_{AB} = \bar{I}_{CD} = \frac{1}{12}ml^2$

POSITION 1: $T_1 = 0$, $V_1 = -mg\frac{l}{2}\cos\theta_m - mgl\cos\theta_m = -\frac{3}{2}mgl\cos\theta_m$

POSITION 2: $V_2 = -mg\frac{l}{2} - mgl = -\frac{3}{2}mgl$

$$T_2 = \frac{1}{2}\bar{I}_{AB}\omega_m^2 + \frac{1}{2}m(\bar{v}_{AB})_m^2 + \frac{1}{2}\bar{I}_{CD}\omega_m^2 + \frac{1}{2}m(\bar{v}_{CD})_m^2$$
$$= \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_m^2 + \frac{1}{2}m\left(\frac{l}{2}\omega_m\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_m^2 + \frac{1}{2}m(l\omega_m)^2$$
$$= \frac{17}{24}ml^2\omega_m^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$
$$0 - \frac{3}{2}mgl\cos\theta_m = \frac{17}{24}ml^2\omega_m^2 - \frac{3}{2}mgl$$
$$\frac{3}{2}mgl(1-\cos\theta_m) = \frac{17}{24}ml^2\omega_m^2$$

FOR SMALL OSCILLATIONS, $(1-\cos\theta_m) = \frac{1}{2}\theta_m^2$

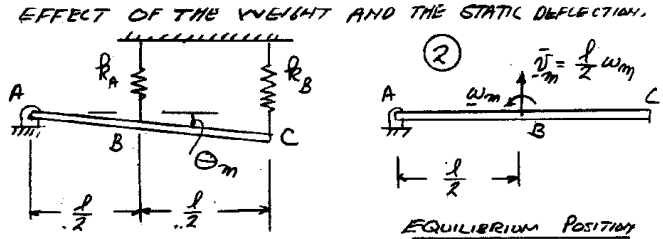
$$\frac{3}{4}mgl\theta_m^2 = \frac{17}{24}ml^2\omega_m^2$$

BUT FOR S.H.M., $\omega_m = p\theta_m$: $\frac{3}{4}mgl\theta_m^2 = \frac{17}{24}ml^2(p\theta_m)^2$

$$p^2 = \frac{18}{17}\frac{g}{l} \qquad f = \frac{p}{2\pi} \qquad f = \frac{1}{2\pi}\sqrt{\frac{18}{17}\frac{g}{l}}$$

19.69

SINCE VIBRATION TAKES PLACE ABOUT THE POSITION OF EQUILIBRIUM, WE SHALL NEGLECT THE EFFECT OF THE WEIGHT AND THE STATIC DEFLECTION.



POSITION OF MAX. DISPLACEMENT

EQUILIBRIUM POSITION

SPRING ELONGATIONS

$$x_B = \frac{l}{2}\theta_m \qquad x_C = l\theta_m$$

POSITION 1: $T_1 = 0$

$$V_1 = \frac{1}{2}k_B x_B^2 + \frac{1}{2}k_C x_C^2 = \frac{1}{2}k_B\left(\frac{l}{2}\theta_m\right)^2 + \frac{1}{2}k_C(l\theta_m)^2$$
$$V_1 = \frac{1}{2}\left(\frac{1}{4}k_B + k_C\right)l^2\theta_m^2$$

POSITION 2: $V_2 = 0$

$$T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_m^2 + \frac{1}{2}m\left(\frac{l}{2}\omega_m\right)^2$$
$$T_2 = \frac{1}{6}ml^2\omega_m^2$$

(CONTINUED)

19.69 CONTINUED

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2}\left(\frac{1}{4}k_B + k_C\right)l^2\theta_m^2 = \frac{1}{6}ml^2\omega_m^2$$

BUT, FOR SHM, $\omega_m = p\theta_m$: $\frac{1}{2}\left(\frac{1}{4}k_B + k_C\right)l^2\theta_m^2 = \frac{1}{6}ml^2(p\theta_m)^2$

$$p^2 = \frac{3}{m}\left(\frac{1}{4}k_B + k_C\right) \qquad (1)$$

NOTE: RESULT IS INDEPENDENT OF LENGTH OF ROD

DATA:

$W = 6$ lb, $k_B = 3$ lb/in. $= 36$ lb/ft, $k_C = 5$ lb/in. $= 60$ lb/ft.

$$p^2 = \frac{3}{(6\text{ lb})/(32.2\text{ ft/s}^2)}\left(\frac{36\text{ lb/ft}}{4} + 60\text{ lb/ft}\right) = 1110.9$$

$p = 33.33$ rad/s $\qquad f = \frac{p}{2\pi} = \frac{33.33}{2\pi}$ $\qquad f = 5.30$ Hz ◀

19.70

SEE SOLUTION OF PROB. 19.69 FOR DERIVATION OF

$$p^2 = \frac{3}{m}\left(\frac{1}{4}k_B + k_C\right) \qquad (1)$$

DATA:

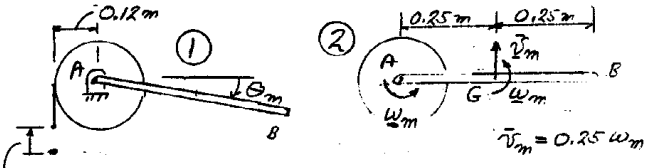
$W = 6$ lb, $k_B = 5$ lb/in. $= 60$ lb/ft, $k_C = 3$ lb/in. $= 36$ lb/ft.

$$p^2 = \frac{3}{(6\text{ lb})/(32.2\text{ ft/s}^2)}\left(\frac{60\text{ lb/ft}}{4} + 36\text{ lb/ft}\right) = 821.1$$

$p = 28.655$ rad/s $\qquad f = \frac{p}{2\pi} = \frac{28.655}{2\pi}$ $\qquad f = 4.56$ Hz ◀

19.71

SINCE VIBRATION TAKES PLACE ABOUT THE POSITION OF EQUILIBRIUM, WE SHALL NEGLECT THE EFFECT OF THE WEIGHT AND THE STATIC DEFLECTION.



$\bar{v}_m = 0.25\omega_m$

$x_m = 0.12\theta_m$ = SPRING ELONGATION

DISK: $m = 8$ kg, $\bar{I}_{DISK} = \frac{1}{2}(8\text{ kg})(0.12\text{ m})^2 = 0.0576$ kg·m²

ROD: $m = 5$ kg, $\bar{I}_{ROD} = \frac{1}{12}(5\text{ kg})(0.5\text{ m})^2 = 0.10417$ kg·m²

POSITION 1: $T_1 = 0$, $V_1 = \frac{1}{2}kx_m^2 = \frac{1}{2}k(0.12\theta_m)^2 = 7.2\times10^{-3}k\theta_m^2$

POSITION 2: $V_2 = 0$, $T_2 = \frac{1}{2}\bar{I}_{DISK}\omega_m^2 + \frac{1}{2}\bar{I}_{ROD}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2$

$$T_2 = \frac{1}{2}(0.0576)\omega_m^2 + \frac{1}{2}(0.10417)\omega_m^2 + \frac{1}{2}(5)(0.25\omega_m)^2 = 0.23714\omega_m^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 7.2\times10^{-3}k\theta_m^2 = 0.23714\omega_m^2$$

BUT, FOR SHM: $\omega_m = p\theta_m$: $7.2\times10^{-3}k\theta_m^2 = 0.23714(p\theta_m)^2$

$$p^2 = 0.030362k \qquad (1)$$

FOR $k = 450$ N/m, $p^2 = 0.030362(450) = 13.663$

$p = 3.696$ rad/s

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{3.696} \qquad \tau = 1.700\text{ s}$$ ◀

19.72

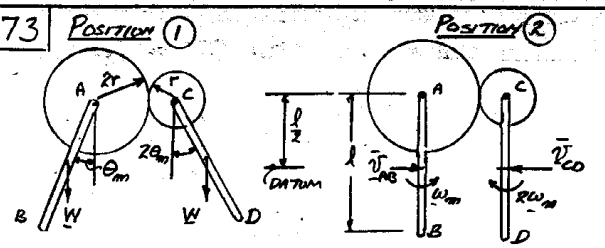
SEE SOLUTION OF PROB. 19.71 FOR DERIVATION OF

$$p^2 = 0.030362k \qquad (1)$$

FOR $\tau = 1.6$ s: $\tau = \frac{2\pi}{p}$; $p = \frac{2\pi}{\tau} = \frac{2\pi}{1.6}$

EQ.(1): $\left(\frac{2\pi}{1.6}\right)^2 = 0.030362k$ $\qquad k = 508$ N/m ◀

19.73



KINEMATICS $\theta_{CD} = 2\theta_{AB} = 2\theta_m$ $\quad \bar{v}_{AB} = \frac{l}{2}\omega_m$ $\quad \bar{v}_{CD} = \frac{l}{2}(2\omega_m)$

$\omega_{CD} = 2\omega_{AB} = 2\omega_m$

POSITION 1: (MAX. DEFLECTION) VELOCITY = 0

$T_1 = 0$ $\quad V_1 = W\frac{l}{2}(1-\cos\theta_m) + W\frac{l}{2}(1-\cos 2\theta_m)$

$\approx W\frac{l}{2}\left[1-(1-\frac{1}{2}\theta_m^2) + 1-(1-\frac{1}{2}(2\theta_m)^2)\right]$

$V_1 \approx \frac{5}{4}Wl\theta_m^2$

POSITION 2: (MAX VELOCITY)

$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}\omega_{AB}^2 + \frac{1}{2}m\bar{v}_{CD}^2 + \frac{1}{2}\bar{I}\omega_{CD}^2$

$= \frac{1}{2}\frac{W}{g}\left(\frac{l}{2}\omega_m\right)^2 + \frac{1}{2}\left(\frac{1}{12}\frac{W}{g}l^2\right)\omega_m^2$

$+ \frac{1}{2}\frac{W}{g}\left[\frac{l}{2}(2\omega_m)\right]^2 + \frac{1}{2}\left(\frac{1}{12}\frac{W}{g}l^2\right)(2\omega_m)^2$

$T_2 = \frac{5}{6}\frac{W}{g}l^2\omega_m^2$ $\quad V_2 = 0$

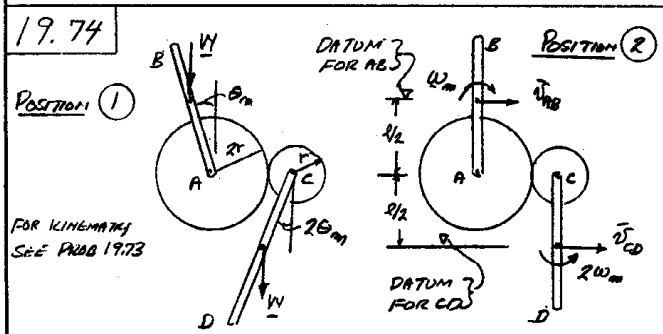
$T_1 + V_1 = T_2 + V_2$ $\quad 0 + \frac{5}{4}Wl\theta_m^2 = \frac{5}{6}\frac{W}{g}l^2\omega_m^2 + 0$

SETTING $\omega_m = \theta_m p$: $\quad \frac{5}{4}Wl\theta_m^2 = \frac{5}{6}\frac{W}{g}l^2\theta_m^2 p^2$

THUS: $p^2 = \frac{3g}{2l}$ $\quad \tau = \frac{2\pi}{p} = 2\pi\sqrt{2l/3g}$ ◀

NOTE: THIS IS ALSO THE PERIOD OF A SINGLE ROD

19.74



FOR KINEMATICS SEE PROB 19.73

POSITION 1 (MAX. DEFLECTION)

$T_1 = 0$ $\quad V_1 = -W\frac{l}{2}(1-\cos\theta_m) + W\frac{l}{2}(1-\cos 2\theta_m)$

$\approx +W\frac{l}{2}\left[-1+(1-\frac{1}{2}\theta_m^2) + 1-(1-\frac{1}{2}(2\theta_m)^2)\right]$

$V_1 \approx \frac{3}{4}Wl\theta_m^2$

POSITION 2 (MAX VELOCITY)

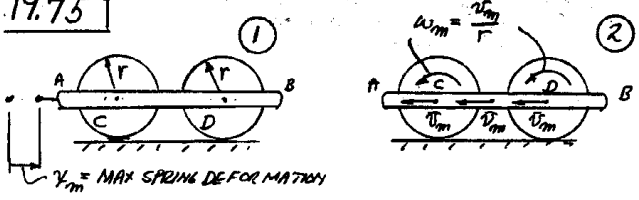
SAME AS IN PROB. 19.73 $\quad T_2 = \frac{5}{6}\frac{W}{g}l^2\omega_m^2$ $\quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$ $\quad 0 + \frac{3}{4}Wl\theta_m^2 = \frac{5}{6}\frac{W}{g}l^2\omega_m^2 + 0$

SETTING $\omega_m = \theta_m p$: $\quad \frac{3}{4}Wl\theta_m^2 = \frac{5}{6}\frac{W}{g}l^2\theta_m^2 p^2 + 0$

$p^2 = \frac{9g}{10l}$ $\quad \tau = \frac{2\pi}{p} = 2\pi\sqrt{10l/9g}$ ◀

19.75



x_m = MAX SPRING DEFORMATION

POSITION 1: $\quad T_1 = 0$, $\quad V_1 = \frac{1}{2}kx_m^2$

POSITION 2: $\quad V_2 = 0$, $\quad T_2 = \frac{1}{2}m_{AB}v_m^2 + 2\left[\frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m_{DISK}\bar{v}_m^2\right]$

$T_2 = \frac{1}{2}m_{AB}v_m^2 + \left(\frac{1}{2}m_{DISK}r^2\right)\left(\frac{v_m}{r}\right)^2 + m_{DISK}v_m^2$

$T_2 = \frac{1}{2}\left(m_{AB} + 3m_{DISK}\right)v_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{2}kx_m^2 = \frac{1}{2}\left(m_{AB}+3m_{DISK}\right)v_m^2$

BUT, FOR SHM, $v_m = px_m$: $\quad \frac{1}{2}kx_m^2 = \frac{1}{2}\left(m_{AB}+3m_{DISK}\right)(px_m)^2$

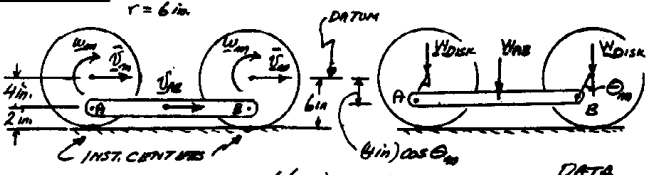
$p^2 = \frac{k}{m_{AB}+3m_{DISK}}$ $\quad$ NOTE: RESULT IS INDEPENDENT OF r

DATA: $k = 5\ kN/m$, $m_{AB} = 9\ kg$, $m_{DISK} = 6\ kg$

$p^2 = \frac{5000\ N/m}{9\ kg + 3(6\ kg)} = 185.185$ $\quad p = 13.608\ rad/s$

$f = \frac{p}{2\pi} = \frac{13.608}{2\pi}$ $\quad f = 2.17\ Hz$ ◀

19.76



KINEMATICS $\quad \bar{v}_m = r\omega_m = \left(\frac{6}{12}ft\right)\omega_m = \frac{1}{2}\omega_m$

$v_{AB} = \left(\frac{2}{12}ft\right)\omega_m = \frac{1}{6}\omega_m$

DATA

$W_{DISK} = 8\ lb$

$W_{AB} = 20\ lb$

POSITION 1: (MAX. DISPLACEMENT)

$T_1 = 0$ $\quad V_1 = -W_{AB}\left(\frac{4}{12}ft\right)\cos\theta_m = -\frac{1}{3}W_{AB}\cos\theta_m$

POSITION 2: (MAX. VELOCITY)

$T_2 = 2\left[\frac{1}{2}m_{DISK}\bar{v}_m^2 + \frac{1}{2}\bar{I}_{DISK}\omega_m^2\right] + \frac{1}{2}m_{AB}\bar{v}_{AB}^2$

$= 2\left[\frac{1}{2}\frac{8}{g}\left(\frac{1}{2}\omega_m\right)^2 + \frac{1}{2}\left(\frac{1}{2}\frac{8}{g}\right)\left(\frac{1}{2}\right)^2\omega_m^2\right] + \frac{1}{2}\frac{20}{g}\left(\frac{1}{6}\omega_m\right)^2$

$T_2 = 3.2778\frac{\omega_m^2}{g}$ $\quad V_2 = -W_{AB}\left(\frac{4}{12}ft\right) = -\frac{1}{3}W_{AB}$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 - \frac{1}{3}W_{AB}\cos\theta = 3.2778\frac{\omega_m^2}{g} - \frac{1}{3}W_{AB}$

$3.2778\frac{\omega_m^2}{g} = \frac{1}{3}W_{AB}(1-\cos\theta)$

BUT: $1-\cos\theta_m \approx \frac{1}{2}\theta_m^2$ AND $W_{AB} = 20\ lb$

$3.2778\frac{\omega_m^2}{g} = \frac{1}{3}(20)\left(\frac{1}{2}\theta_m^2\right)$

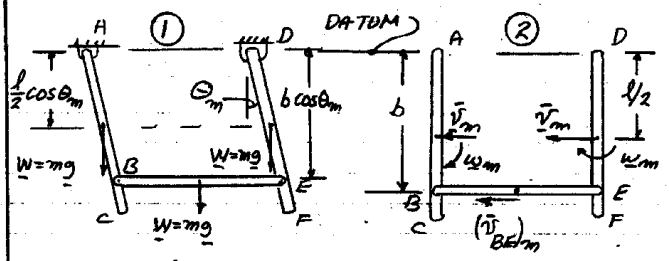
SETTING $\omega_m = \theta_m p$: $\quad 3.2778\frac{\theta_m^2 p^2}{g} = \frac{20}{6}\theta_m^2$

$p^2 = \frac{(20)(32.2)}{(6)(3.2778)} = 32.746$ $\quad p = 5.722\ rad/s$

$f = \frac{p}{2\pi} = \frac{5.722}{2\pi} = 0.9107\ s^{-1}$ $\quad f = 0.911\ Hz$ ◀

19.77

l = LENGTH OF EACH ROD
m = MASS OF EACH ROD



$\bar{v}_m = \frac{l}{2}\omega_m \qquad (\bar{v}_{BE})_m = b\omega_m$

POSITION 1: $T_1 = 0$, $V_1 = -2mg\frac{l}{2}\cos\theta_m - mg\,b\cos\theta_m$

$V_1 = -mg(l+b)\cos\theta_m$

POSITION 2: $V_2 = -2mg\frac{l}{2} - mgb = -mg(l+b)$

$T_2 = 2\left[\frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2\right] + \frac{1}{2}m(\bar{v}_{BE})_m^2$

$= \frac{1}{12}ml^2\omega_m^2 + m\left(\frac{l}{2}\omega_m\right)^2 + \frac{1}{2}m(b\omega_m)^2$

$T_2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\omega_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$:

$0 - mg(l+b)\cos\theta_m = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\omega_m^2 - mg(l+b)$

$mg(l+b)(1-\cos\theta_m) = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\omega_m^2$

FOR SMALL OSCILLATIONS $(1-\cos\theta_m) = \frac{1}{2}\theta_m^2$

$\frac{1}{2}mg(l+b)\theta_m^2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\omega_m^2$

BUT, FOR SHM: $\omega_m = p\theta_m$: $\frac{1}{2}mg(l+b)\theta_m^2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m(p\theta_m)^2$

$$p^2 = \frac{1}{2}g\frac{l+b}{\frac{1}{3}l^2 + \frac{1}{2}b^2} \quad \text{or} \quad p^2 = 3g\frac{l+b}{2l^2+3b^2} \qquad (1)$$

FOR $b = \frac{3}{4}l$, WE HAVE

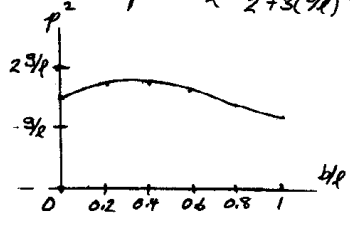
$$p^2 = 3g\frac{l+\frac{3}{4}l}{2l^2+3\left(\frac{3}{4}l\right)^2} = 3g\frac{\frac{7}{4}l}{\frac{57}{16}l^2} = 1.4237\frac{g}{l}$$

$p = 1.1932\sqrt{\frac{g}{l}} \qquad f = \frac{p}{2\pi} = \frac{1.1932}{2\pi}\sqrt{\frac{g}{l}}$

$$f = 0.1899\sqrt{\frac{g}{l}}$$

NOTE: EQ(1) CAN BE WRITTEN AS FOLLOWS:

$$p^2 = 3\frac{g}{l}\frac{1+b/l}{2+3(b/l)^2}$$



FOR $b/l = 0$, WE FIND p^2 OF SINGLE ROD SUPPORTED AT ITS END. WE NOTE THAT p^2 IS MAXIMUM BETWEEN $b/l = 0.2$ AND 0.4

SEE PROB 19.78

19.78

SEE SOLUTION OF PROB 19.77 FOR DERIVATION OF

EQ(1): $$p^2 = 3g\left[\frac{l+b}{2l^2+3b^2}\right]$$

(a) FOR MAXIMUM FREQUENCY WE MUST HAVE

$$\frac{d(p^2)}{db} = 0 \quad \text{OR} \quad \frac{d}{db}\left[\frac{l+b}{2l^2+3b^2}\right] = 0$$

$$\frac{(2l^2+3b^2)(1) - (l+b)(6b)}{(2l^2+3b^2)^2} = 0$$

$$2l^2 + 3b^2 - 6lb - 6b^2 = 0$$

$$-3b^2 - 6lb + 2l^2 = 0$$

$$b^2 + 2lb - \frac{2}{3}l^2 = 0$$

$b = -2.29l$ (IMPOSSIBLE) AND $\qquad b = 0.291l$ ◀

(b) SUBSTITUTE $b = 0.291l$ INTO EQ(1):

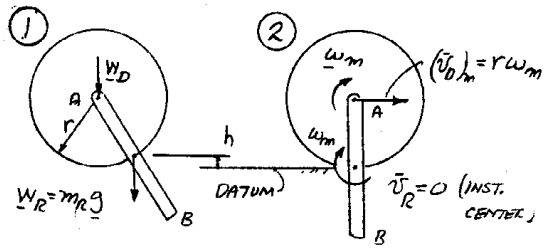
$$p^2 = 3g\frac{l+0.291l}{2l^2+3(0.291l)^2} = 1.7182\frac{g}{l} \qquad p = 1.3108\sqrt{\frac{g}{l}}$$

$$f = \frac{p}{2\pi} = \frac{1.3108}{2\pi}\sqrt{\frac{g}{l}} \qquad f = 0.209\sqrt{\frac{g}{l}}$$ ◀

19.79

D = DISK
R = ROD

$l_{AB} = 2r$



FOR SMALL OSCILLATIONS: $h = r(1-\cos\theta_m) = \frac{1}{2}r\theta_m^2$

POSITION 1: $T_1 = 0$

$V_1 = W_R h = \frac{1}{2}m_R g r\theta_m^2$

POSITION 2: $V_2 = 0$

$T_2 = \frac{1}{2}\bar{I}_D\omega_m^2 + \frac{1}{2}m_D(\bar{v}_D)_m^2 + \frac{1}{2}\bar{I}_R\omega_m^2$

$= \frac{1}{2}\left(\frac{1}{2}m_D r^2\right)\omega_m^2 + \frac{1}{2}m_D r^2\omega_m^2 + \frac{1}{2}\left(\frac{1}{12}m_R(2r)^2\right)\omega_m^2$

$= \frac{1}{2}\left(\frac{3}{2}m_D + \frac{1}{3}m_R\right)r^2\omega_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$:

$0 + \frac{1}{2}m_R g r\theta_m^2 = \frac{1}{2}\left(\frac{3}{2}m_D + \frac{1}{3}m_R\right)r^2\omega_m^2$

BUT, FOR SHM: $\omega_m = p\theta_m$: $\frac{1}{2}m_R g r\theta_m^2 = \frac{1}{2}\left(\frac{3}{2}m_D + \frac{1}{3}m_R\right)r^2(p\theta_m)^2$

$$p^2 = \frac{m_R}{\frac{3}{2}m_D + \frac{1}{3}m_R}\left(\frac{g}{r}\right)$$

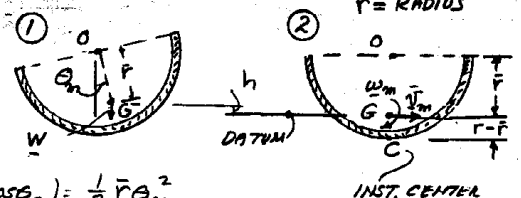
OR $$p^2 = \frac{W_R}{\frac{3}{2}W_D + \frac{1}{3}W_R}\left(\frac{g}{r}\right) \qquad (1)$$

DATA: $W_R = 3$ lb, $W_D = 5$ lb, $r = \frac{4}{12}$ ft

$$p^2 = \frac{3}{\frac{3}{2}(5) + \frac{1}{3}(3)}\left(\frac{32.2}{4/12}\right) = 34.094 \qquad p = 5.839 \text{ rad/s}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{5.839} \qquad \tau = 1.076 \text{ s}$$ ◀

19.80



$\bar{I} = m\bar{k}^2$

$h = \bar{r}(1-\cos\theta_m) \approx \frac{1}{2}\bar{r}\theta_m^2$

$\bar{v}_m = (r-\bar{r})\omega_m$

Position 1: $T_1 = 0$, $V_1 = Wh = \frac{1}{2}mg\bar{r}\theta_m^2$

Position 2: $V_2 = 0$

$T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}m\bar{k}^2\omega_m^2 + \frac{1}{2}m(r-\bar{r})^2\omega_m^2$

Conservation of Energy

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}mg\bar{r}\theta_m^2 = \frac{1}{2}m\left[\bar{k}^2 + (r-\bar{r})^2\right]\omega_m^2$

But, for SHM, $\omega_m = p\theta_m$: $\frac{1}{2}mg\bar{r}\theta_m^2 = \frac{1}{2}m\left[\bar{k}^2 + (r-\bar{r})^2\right](p\theta_m)^2$

$$p^2 = \frac{\bar{r}}{\bar{k}^2 + (r-\bar{r})^2}\,g \qquad (1)$$

For half section of pipe

See Fig 5.88: $\bar{r} = \frac{2r}{\pi}$ $\qquad r - \bar{r} = r\left(1 - \frac{2}{\pi}\right)$

Parallel-axis theorem: $I_O = \bar{I} + m\bar{r}^2$

$I_O = mr^2$; $\quad mr^2 = \bar{I} + m\left(\frac{2r}{\pi}\right)^2$

$\bar{I} = mr^2\left(1 - \frac{4}{\pi^2}\right)$ $\qquad \bar{k}^2 = r^2\left(1 - \frac{4}{\pi^2}\right)$

Eq(1): $p^2 = \dfrac{\frac{2r}{\pi}}{r^2\left(1-\frac{4}{\pi^2}\right) + r^2\left(1-\frac{2}{\pi}\right)^2}\,g$

$$p^2 = \frac{\frac{2r}{\pi}}{r^2\left[1 - \frac{4}{\pi^2} + 1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right]}\,g = \frac{2/\pi}{2 - 4/\pi}\,\frac{g}{r}$$

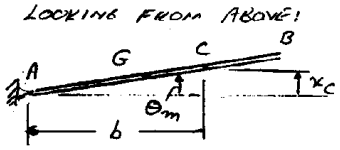
$$p^2 = \frac{2/\pi}{(2\pi-4)/\pi}\,\frac{g}{r} = \frac{2}{2\pi-4}\,\frac{g}{r} = \frac{1}{\pi-2}\,\frac{g}{r}$$

$\tau = \frac{2\pi}{p}$ $\qquad \tau = 2\pi\sqrt{\frac{(\pi-2)r}{g}}$ ◀

19.81

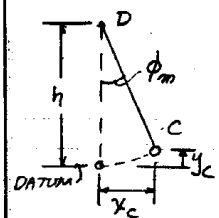
Position 1. (Max deflection)

Looking from above:



Horizontal displacement of C: $x_c = b\theta_m$

Looking from right:



$\phi_m \approx \frac{x_c}{h} = \frac{b}{h}\theta_m$

$y_c = h(1-\cos\phi_m) \approx \frac{1}{2}h\phi_m^2$

$y_c = \frac{1}{2}h\left(\frac{b}{h}\theta_m\right)^2 = \frac{1}{2}\frac{b^2}{h}\theta_m^2$

$\bar{y}_m = y_G = \frac{AG}{AC}y_c = \frac{\frac{1}{2}L}{b}\left(\frac{1}{2}\frac{b^2}{h}\theta_m^2\right)$

$\bar{y}_m = \frac{1}{4}\frac{bL}{h}\theta_m^2$

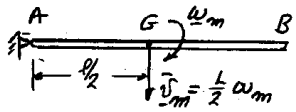
We have $T_1 = 0$ $\quad V_1 = mg\bar{y}_m = \frac{1}{4}\frac{mgbL}{h}\theta_m^2$

(CONTINUED)

19.81 CONTINUED

Position 2 (Max. velocity)

Looking from above.



$T_2 = \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_m^2 + \frac{1}{2}m\left(\frac{L}{2}\omega_m\right)^2$

$T_2 = \frac{1}{6}mL^2\omega_m^2$

$V_2 = 0$

Conservation of Energy

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{4}\frac{mgbL}{h}\theta_m^2 = \frac{1}{6}mL^2\omega_m^2$

But for SHM: $\omega_m = p\theta_m$

$\frac{1}{4}\frac{mgbL}{h}\theta_m^2 = \frac{1}{6}mL^2(p\theta_m)^2$

$p^2 = \frac{3bg}{2hL}$ $\qquad \tau = \frac{2\pi}{p}$ $\qquad \tau = 2\pi\sqrt{\frac{2hL}{3bg}}$ ◀

19.82

See solution of Prob 19.81 for derivation of

$\tau = 2\pi\sqrt{\frac{2hL}{3bg}}$

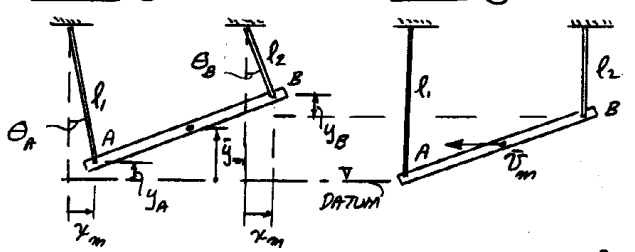
Data: $L = 0.9\text{ m}$, $b = 0.75\text{ m}$, $h = 0.4\text{ m}$

$\tau = 2\pi\sqrt{\frac{2(0.4\text{ m})(0.9\text{ m})}{3(0.75\text{ m})(9.81\text{ m/s}^2)}} = 1.1348\text{ s}$ $\qquad \tau = 1.135\text{ s}$ ◀

19.83

For small displacement x_m, the motion of AB is a translation. ($\omega_m = 0$)

Position ① Position ②



Geometry $\theta_A = \frac{x_m}{l_1}$; $\quad y_A = l_1(1-\cos\theta_m) \approx \frac{1}{2}l_1\theta_m^2 = \frac{1}{2}l_1\left(\frac{x_m}{l_1}\right)^2 = \frac{x_m^2}{2l_1}$

Similarly: $y_B \approx \frac{x_m^2}{2l_2}$

$y_G = \bar{y}_m = \frac{1}{2}(y_A + y_B) \approx \frac{x_m^2}{4}\left(\frac{1}{l_1} + \frac{1}{l_2}\right) = \frac{1}{4}\frac{l_1+l_2}{l_1 l_2}x_m^2$

Position 1: $T_1 = 0$ $\quad V_1 = W\bar{y}_m = \frac{W}{4}\frac{l_1+l_2}{l_1 l_2}x_m^2$

Position 2: $T_2 = \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}\frac{W}{g}\bar{v}_m^2$ $\qquad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{W}{4}\frac{l_1+l_2}{l_1 l_2}x_m^2 = \frac{1}{2}\frac{W}{g}\bar{v}_m^2 + 0$

Setting $\bar{v}_m = x_m p$

$\frac{W}{4}\frac{l_1+l_2}{l_1 l_2}x_m^2 = \frac{1}{2}\frac{W}{g}x_m^2 p^2$

$p^2 = \frac{g(l_1+l_2)}{2l_1 l_2}$ $\qquad \tau = \frac{2\pi}{p}$ $\qquad \tau = 2\pi\sqrt{\frac{2l_1 l_2}{g(l_1+l_2)}}$ ◀

***19.84** **Note:** For a body moving in an ideal fluid there is no friction drag. Thus, this motion is not a damped vibration. However, the kinetic energy acquired by the fluid must be included in the solution. For a sphere, this K.E. is $\frac{1}{4}\rho V v^2$.

(a) Position 1 (Max. deflection):

$T_1 = 0 \qquad V_1 = \frac{1}{2} k x_m^2$

Position 2 (Max. velocity): $V_2 = 0$

$T_2 = T_{sphere} + T_{fluid} = \frac{1}{2} m_s v_m^2 + \frac{1}{4}\rho V v_m^2$

$T_1 + V_1 = T_2 + V_2$: $\frac{1}{2} k x_m^2 = \frac{1}{2}\left(m_s + \frac{1}{2}\rho V\right) v_m^2$

Setting $v_m = x_m p$: $\frac{1}{2} k x_m^2 = \frac{1}{2}\left(m_s + \frac{1}{2}\rho V\right) x_m^2 p^2$

Thus: $p^2 = \dfrac{k}{m_s + \frac{1}{2}\rho V}$ But $k = 3$ lb/in. $= 36$ lb/ft

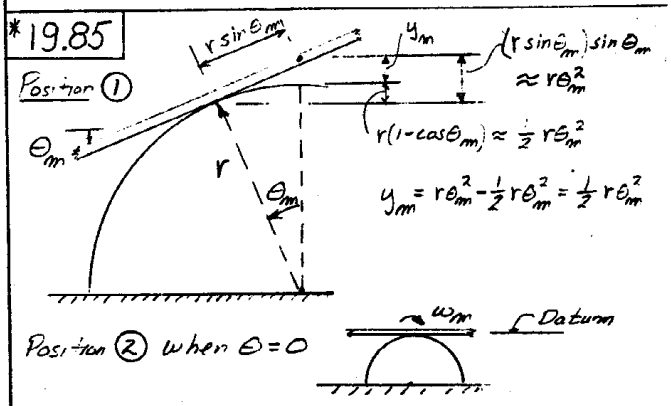
$m_s = (1\text{ lb})/32.2 = 0.03106$

$\frac{1}{2}\rho V = \frac{1}{2}\rho\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\rho r^3 = \frac{2}{3}\pi\left(\frac{62.4}{32.2}\right)\left(\frac{3}{12}\right)^3 = 0.06342$

$p^2 = \dfrac{36}{0.03106 + 0.06342} = 381$, $p = 19.52$, $\tau = \dfrac{2\pi}{p} = 0.322$ s ◀

(b) Accel. changes apparent "weights", but not masses: $\tau = 0.322$ s ◀

***19.85**



Position 1: Max. deflection

$T_1 = 0 \qquad V_1 = W y_m = mg\left(\frac{1}{2} r\theta_m^2\right) = \frac{1}{2} mgr\theta_m^2$

Position 2: Max. velocity

$T_1 = \frac{1}{2} \bar{I}\, \omega_m^2 = \frac{1}{2}\left(\frac{1}{12} m\ell^2\right)\omega_m^2 \qquad V_2 = 0$

$T_1 + V_1 = T_2 + V_2 \qquad 0 + \frac{1}{2} mgr\theta_m^2 = \frac{1}{24} m\ell^2 \omega_m^2$

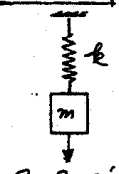
Setting $\omega_m = p\theta_m$ $\qquad \frac{1}{2} mgr\theta_m^2 = \frac{1}{24} m\ell^2 p^2 \theta_m^2$

$$p^2 = \frac{12gr}{\ell^2}$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\ell^2}{12gr}}$$

$$\tau = \frac{\pi\ell}{\sqrt{3gr}}$$ ◀

19.86



$P = P_m \sin\omega t$

EQ(19.33) $x_m = \dfrac{P_m/k}{1-(\omega/p)^2}$

For $m = 4$ kg, $k = 324$ N/m, $P_m = 15$ N

$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{324\text{ N/m}}{4\text{ kg}}} = 9$ rad/s

$P_m/k = 15/324 = 46.296 \times 10^{-3}$ m

(a) $\omega = 6$ rad/s

$x_m = \dfrac{46.296\times10^{-3}\text{ m}}{1-\left(\frac{6\text{ rad/s}}{9\text{ rad/s}}\right)^2} = 83.33\times10^{-3}\text{ m}$ $\qquad x_m = 83.3$ mm ◀

(b) $\omega = 12$ rad/s

$x_m = \dfrac{46.296\times10^{-3}\text{ m}}{1-\left(\frac{12\text{ rad/s}}{9\text{ rad/s}}\right)^2} = -59.52\times10^{-3}\text{ m}$ $\qquad x_m = 59.5$ mm (OUT OF PHASE) ◀

19.87

$P = P_m \sin\omega t$

$p^2 = \dfrac{k}{m}$

Static deflection $= \delta_{st} = \dfrac{P_m}{k}$

Thus, from Eq. (19.33), we write.

$$x_m = \frac{P_m/k}{1-(\omega/p)^2} = \frac{\delta_{st}}{1-(\omega/p)^2}, \qquad \frac{x_m}{\delta_{st}} = \frac{1}{1-(\omega/p)^2} \quad (1)$$

We want $\left|\dfrac{x_m}{\delta_{st}}\right| > 2$ or $\left|\dfrac{\delta_{st}}{x_m}\right| < \frac{1}{2}$ or $-\frac{1}{2} < \dfrac{\delta_{st}}{x_m} < \frac{1}{2}$

Thus we must have $-\frac{1}{2} < 1-(\omega/p)^2 < \frac{1}{2}$

$$-\frac{3}{2} < -(\omega/p)^2 < -\frac{1}{2}$$

$$\frac{1}{2} < (\omega/p)^2 < \frac{3}{2}$$

Since $p^2 = \frac{k}{m}$:

$$\sqrt{\frac{k}{2m}} < \omega < \sqrt{\frac{3k}{2m}}$$ ◀

19.88 See solution of Prob. 19.87 for derivation of

$$\frac{x_m}{\delta_{st}} = \frac{1}{1-(\omega/p)^2} \quad (1)$$

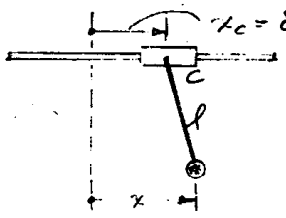
We want $\left|\dfrac{x_m}{\delta_{st}}\right| < 1$. This occurs if

either $1-(\omega/p)^2 > 1$, which means $(\omega/p)^2 < 0$ (impossible)

or $1-(\omega/p)^2 < -1$, which means $(\omega/p)^2 > 2$

Since $p^2 = \frac{k}{m}$: — $\qquad \omega > \sqrt{\dfrac{2k}{m}}$ ◀

19.89



$$f^2 = \frac{g}{l}$$

From Eq. (19.33'):

$$\frac{x_m}{\delta_m} = \frac{1}{1-(\omega/f)^2}$$

We want $\left|\frac{x_m}{\delta_m}\right| < 1$ or $\left|\frac{\delta_m}{x_m}\right| > 1$ or $-1 > \frac{\delta_m}{x_m} > 1$

Thus, we must have $-1 > 1-(\omega/f)^2 > 1$

$$-2 > -(\omega/f)^2 > 0$$

$$0 > (\omega/f)^2 > 2$$

Since $f^2 = \frac{g}{l}$:

$$\omega > \sqrt{2\frac{g}{l}}$$ ◀

$(\omega/f)^2 < 0$ (IMPOSSIBLE)

19.90

From Eq. (19.33'),

$$\frac{x_m}{\delta_m} = \frac{1}{1-(\omega/f)^2}$$

We want $\left|\frac{x_m}{\delta_m}\right| > 2$ or $\left|\frac{\delta_m}{x_m}\right| < \frac{1}{2}$ or $-\frac{1}{2} < \frac{\delta_m}{x_m} < \frac{1}{2}$

Thus, we must have $-\frac{1}{2} < 1-(\omega/f)^2 < \frac{1}{2}$

$$-\frac{3}{2} < -(\omega/f)^2 < -\frac{1}{2}$$

$$\frac{1}{2} < (\omega/f)^2 < \frac{3}{2}$$

Since $f^2 = \frac{g}{l}$

$$\sqrt{\frac{g}{2l}} < \omega < \sqrt{\frac{3g}{2l}}$$ ◀

19.91

(a) SPRING CONSTANT FOR BEAM IS:

$$k = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}} = \frac{(100\ \text{kg})(9.81\ \text{m/s}^2)}{5.5\times10^{-3}\ \text{m}} = 178.36\ \text{kN/m}$$

NATURAL FREQUENCY (RESONANCE)

$$f = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.81\ \text{m/s}^2}{5.5\times10^{-3}\ \text{m}}} = 42.233\ \text{rad/s}$$

$$f = \frac{f}{2\pi} = \frac{42.233}{2\pi} = 6.7216\ \text{Hz} = 403.3\ \text{rpm}$$ ◀ $f = 403$ rpm

(b) $\omega = 720$ rpm $= 75.398$ rad/s

$$P_m = m'a_m = m'r\omega^2 = (0.022\ \text{kg})(0.18\ \text{m})(75.398\ \text{rad/s})^2 = 22.512\ \text{N}$$

$$\frac{P_m}{k} = \frac{22.512\ \text{N}}{178.36\ \text{kN/m}} = 126.22\times10^{-6}\ \text{m}$$

For $\omega = 720$ rpm, with $f = 403$ rpm: $\frac{\omega}{f} = \frac{720}{403.3} = 1.7853$

EQ(19.33): $x_m = \frac{P_m/k}{1-(\omega/f)^2} = \frac{126.22\times10^{-6}\ \text{m}}{1-(1.7853)^2} = -52.71\times10^{-6}\ \text{m}$

$x_m = 52.7\ \mu\text{m}$

(OUT OF PHASE) ◀

19.92

(a) TOTAL SPRING CONSTANT: $k = 45$ kN/m

NATURAL FREQUENCY (RESONANCE)

$$f = \sqrt{\frac{k}{m}} = \sqrt{\frac{45\ \text{kN/m}}{100\ \text{kg}}} = 21.213\ \text{rad/s}$$

$$f = \frac{f}{2\pi} = \frac{21.213}{2\pi} = 3.376\ \text{Hz} = 202.57\ \text{rpm}$$ $f = 203$ rpm ◀

(b) $\omega = 720$ rpm $= 75.398$ rad/s

$$P_m = m'a_m = m'r\omega^2 = (0.022\ \text{kg})(0.18\ \text{m})(75.398\ \text{rad/s})^2 = 22.512\ \text{N}$$

$$\frac{P_m}{k} = \frac{22.512\ \text{N}}{45\ \text{kN/m}} = 500.27\times10^{-6}\ \text{m}$$

For $\omega = 720$ rpm, with $f = 202.57$ rpm: $\frac{\omega}{f} = \frac{720}{202.57} = 3.554$

EQ(19.33): $x_m = \frac{P_m/k}{1-(\omega/f)^2} = \frac{500.27\times10^{-6}\ \text{m}}{1-(3.554)^2} = -43.0\times10^{-6}\ \text{m}$

$x_m = 43\ \mu\text{m}$ ◀

(OUT OF PHASE)

19.93

SINCE AMPLITUDE DECREASES CONTINUOUSLY, WE NOTE THAT $\omega > f$. WE ASSUME THAT VIBRATION IS FORCED BY A MASS m' LOCATED AT A DISTANCE e FROM THE AXIS OF ROTATION

PERIODIC FORCE = $P_m = m'e\omega^2$

EQ(19.33) FOR $\omega > f$:

$$-x_m = \frac{P_m/k}{1-(\omega/f)^2} = \frac{(m'e/k)\omega^2}{1-(\omega/f)^2} \qquad x_m = \frac{(m'e/k)\omega^2}{(\omega/f)^2-1}$$

EXPRESSING BOTH ω AND f IN rpm, WE WRITE

FOR $\omega = 300$ rpm, $x_m = 0.075$ in.

$$\frac{0.075}{12}\ \text{ft} = \frac{(m'e/k)(300)^2(2\pi/60)^2}{\left(\frac{300}{f}\right)^2-1} \qquad (1)$$

FOR $\omega = 400$ rpm, $x_m = 0.040$ in.

$$\frac{0.040}{12}\ \text{ft} = \frac{(m'e/k)(400)^2(2\pi/60)^2}{\left(\frac{400}{f}\right)^2-1} \qquad (2)$$

DIVIDE EQ(1) BY EQ(2):

$$\frac{0.075}{0.040} = \frac{(300)^2}{(400)^2}\cdot\frac{\left(\frac{400}{f}\right)^2-1}{\left(\frac{300}{f}\right)^2-1}$$

$$\left(\frac{300}{f}\right)^2-1 = 0.3\left[\left(\frac{400}{f}\right)^2-1\right]$$

$$\frac{1}{f^2}\left[(300)^2-0.3(400)^2\right] = 1-0.3$$

$$\frac{42\times10^3}{f^2} = 0.7 \qquad f^2 = 60\times10^3\ (\text{rpm})^2$$

$f = 245$ rpm ◀

19.94

FOR ROTATING MACHINERY, SEE PROB 19.93

$$\chi_m = \frac{(m'e/k)\omega^2}{(\omega/p)^2 - 1}$$

WE SHALL EXPRESS BOTH ω AND p IN rpm

FOR $\omega = 300$ rpm, $\chi_m = 0.075$ in.; $\frac{0.075}{12}$ ft $= \dfrac{(m'e/k)(300)^2(2\pi/60)^2}{(\frac{300}{p})^2 - 1}$ (1)

FOR $\omega = 400$ rpm, $\chi_m = 0.040$ in.; $\frac{0.040}{12}$ ft $= \dfrac{(m'e/k)(400)^2(2\pi/60)^2}{(\frac{400}{p})^2 - 1}$ (2)

FOR ω, $\chi_m = 0.125$ in.; $\frac{0.125}{12}$ ft $= \dfrac{(m'e/k)\omega^2(2\pi/60)^2}{(\omega/p)^2 - 1}$ (3)

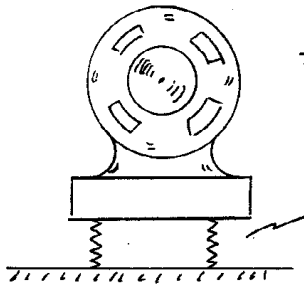
DIVIDE EQ(1) BY EQ(2) AND SOLVE FOR p AS DONE IN PROB 19.93: $p = 245$ rpm

DIVIDE EQ(1) BY EQ(3):

$$\frac{0.075}{0.125} = \frac{(300)^2}{\omega^2} \cdot \frac{(\frac{\omega}{245})^2 - 1}{(\frac{300}{245})^2 - 1}$$

SOLVE FOR ω: $\omega = 274$ rpm ◀

19.95



$m = 18$ kg

FOUR SPRINGS EACH OF CONSTANT 40 kN/m

WE NOTE THAT MOTOR IS CONSTRAINED TO MOVE VERTICALLY.

$k = 4(40 \text{ kN/m}) = 160 \text{ kN/m}$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{160\times10^3 \text{ N/m}}{18 \text{ kg}}} = 94.281 \text{ rad/s} = 900 \text{ rpm}$$

FOR $\omega = 1200$ rpm $= 125.664$ rad/s WE HAVE $\chi_m = 1.5$ mm $= 1.5\times10^{-3}$ m

EQ(19.33): $\chi_m = \dfrac{P_m/k}{1-(\omega/p)^2}$

THUS: $\dfrac{P_m}{k} = \chi_m\left[1-(\omega/p)^2\right]$

$$= (1.5\times10^{-3} \text{ m})\left[1-\left(\frac{125.664}{94.281}\right)^2\right] = -1.164\times10^{-3} \text{ m}$$

(OUT OF PHASE)

$P_m = (160 \text{ kN/m})(1.164\times10^{-3} \text{ m}) = 186.4$ N

(CONTINUED)

19.95 CONTINUED

WE HAVE FOUND: $P_m = 186.4$ N

FOR AN UNBALANCED ROTOR OF MASS $m_R = 4$ kg, ROTATING AT $\omega = 1200$ rpm $= 125.664$ rad/s, WITH THE MASS CENTER AT A DISTANCE $\bar{r}$ FROM THE AXIS OF ROTATION, WE HAVE

$$P_m = m_R \bar{r} \omega^2$$

$$\bar{r} = \frac{P_m}{m_R \omega^2} = \frac{186.4 \text{ N}}{(4 \text{ kg})(125.664 \text{ rad/s})^2} = 2.95\times10^{-3} \text{ m}$$

$\bar{r} = 2.95$ mm ◀

19.96

FROM PROB 19.95:

$p = 94.281$ rad/s $= 900$ rpm

FOR UNBALANCED ROTOR:

$$P_m = m_R \bar{r} \omega^2$$

EQ(19.33) YIELDS:

$$\chi_m = \frac{P_m/k}{1-(\omega/p)^2} = \frac{(m_R\bar{r}/k)\omega^2}{1-(\omega/p)^2} = \left(\frac{m_R\bar{r}}{k}p^2\right)\frac{(\omega/p)^2}{1-(\omega/p)^2} \quad (1)$$

BUT FOR $\omega = 1200$ rpm, $\chi_m = 1.5$ mm, AND $p = 900$ rpm

$$1.5\times10^{-3} \text{ m} = \left(\frac{m_R\bar{r}}{k}p^2\right)\frac{(\frac{1200}{900})^2}{1-(\frac{1200}{900})^2}$$

$$\frac{m_R\bar{r}}{k}p^2 = 1.5\times10^{-3}\left[\frac{1-(\frac{1200}{900})^2}{(\frac{1200}{900})^2}\right] = -0.656\times10^{-3} \text{ m}$$

THE NEGATIVE SIGN INDICATES THAT P_m AND χ_m ARE OUT OF PHASE. SUBSTITUTING INTO (1):

$$\chi_m = (0.656\times10^{-3} \text{ m})\frac{(\omega/900)^2}{1-(\omega/900)^2}$$

(a) FOR $\omega = 450$ rpm:

$$\chi_m = (0.656\times10^{-3} \text{ m})\frac{(450/900)^2}{1-(450/900)^2} = 0.219\times10^{-3} \text{ m}$$

$\chi_m = 219$ μm (IN PHASE) ◀

(b) FOR $\omega = 1600$ rpm

$$\chi_m = (0.656\times10^{-3} \text{ m})\frac{(1600/900)^2}{1-(1600/900)^2} = -0.960\times10^{-3} \text{ m}$$

$\chi_m = 960$ μm (OUT OF PHASE) ◀

(c) FOR $\omega = 900$ rpm

$$\chi_m = (0.656\times10^{-3} \text{ m})\frac{(900/900)^2}{1-(900/900)^2} = \infty$$

$\chi_m = \infty$ (RESONANCE) ◀

19.97 (a) One collar: $(x_m)_1 = 15\text{ mm}$ $\quad f_1^2 = \frac{k}{m}$

(b) Two collars:

$(x_m)_2 = 18\text{ mm}$ $\quad f_2^2 = \frac{k}{2m} = \frac{1}{2} f_1^2$, $\quad \frac{\omega}{f_2} = \sqrt{2}\,\frac{\omega}{f_1}$

(c) Three collars:

$(x_m)_3 =$ unknown, $\quad f_3^2 = \frac{k}{3m} = \frac{1}{3} f_1^2$, $\quad \frac{\omega}{f_3} = \sqrt{3}\,\frac{\omega}{f_1}$

We also note that the amplitude δ_m of the displacement of the base remains constant. Referring to Fig. 19.9, we note that, since $(x_m)_2 > (x_m)_1$ and $\omega/f_2 > \omega/f_1$, we must have $\omega/f_1 < 1$ and $(x_m)_1 > 0$. However, ω/f_2 may be either <1 or >1, with $(x_m)_2$ being correspondingly either >0 or <0.

1. Assuming $(x_m)_2 > 0$:

For one collar:

$$(x_m)_1 = \frac{\delta_m}{1-\left(\frac{\omega}{f_1}\right)^2} \qquad +15\text{ mm} = \frac{\delta_m}{1-\left(\frac{\omega}{f_1}\right)^2} \qquad (1)$$

For two collars:

$$(x_m)_2 = \frac{\delta_m}{1-\left(\frac{\omega}{f_2}\right)^2} \qquad +18\text{ mm} = \frac{\delta_m}{1-2\left(\frac{\omega}{f_1}\right)^2} \qquad (2)$$

Dividing (2) by (1), member by member:

$$1.2 = \frac{1-\left(\frac{\omega}{f_1}\right)^2}{1-2\left(\frac{\omega}{f_1}\right)^2}\text{; we find, } \quad \left(\frac{\omega}{f_1}\right)^2 = \frac{1}{7}$$

Substituting into (1): $\delta_m = (15\text{ mm})\left(1-\frac{1}{7}\right) = \frac{90}{7}\text{ mm}$

For three collars:

$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{f_1}\right)^2} = \frac{\left(\frac{90}{7}\right)\text{mm}}{1-3\left(\frac{1}{7}\right)} = \frac{90}{4}\text{ mm}, \quad (x_m)_3 = 22.5\text{ mm}$$

2. Assuming $(x_m)_2 < 0$:

For two collars, we have: $-18\text{ mm} = \frac{\delta_m}{1-2\left(\frac{\omega}{f_1}\right)^2}$ (3)

Dividing (3) by (1), member by member:

$$-1.2 = \frac{1-\left(\frac{\omega}{f_1}\right)^2}{1-2\left(\frac{\omega}{f_1}\right)^2}$$

$$-1.2 + 2.4\left(\frac{\omega}{f_1}\right)^2 = 1-\left(\frac{\omega}{f_1}\right)^2$$

$$\left(\frac{\omega}{f_1}\right)^2 = \frac{2.2}{3.4} = \frac{1.1}{1.7}$$

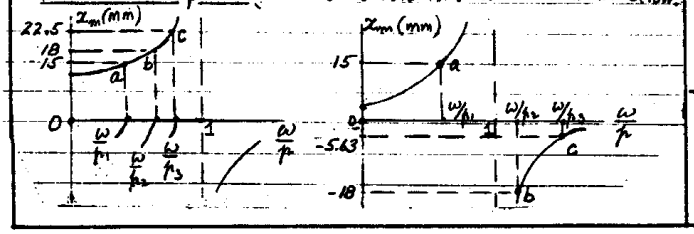
Substituting into (1): $\delta_m = (15\text{ mm})\left(1-\frac{1.1}{1.7}\right) = \frac{9}{1.7}\text{ mm}$

For three collars:

$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{f_1}\right)^2} = \frac{9/1.7}{1-3\left(\frac{1.1}{1.7}\right)} = \frac{9\text{ mm}}{-1.6}, \quad (x_m)_3 = 5.63\text{ mm}$$

(out of phase)

Points corresponding to the 2 solutions are indicated below:



19.98 (a) One collar: $(x_m)_1 = 9\text{ mm}$ $\quad f_1^2 = \frac{k}{m}$

(b) Two collars

$(x_m)_2 = 3\text{ mm}$ $\quad f_2^2 = \frac{k}{2m} = \frac{1}{2} f_1^2$, $\quad \frac{\omega}{f_2} = \sqrt{2}\,\frac{\omega}{f_1}$

(c) Three collars

$(x_m)_3 =$ unknown $\quad f_3^2 = \frac{k}{3m} = \frac{1}{3} f_1^2$, $\quad \frac{\omega}{f_3} = \sqrt{3}\,\frac{\omega}{f_1}$

We also note that the amplitude δ_m of the displacement of the base remains constant. Referring to Fig. 19.9, we note that, since $(x_m)_2 < (x_m)_1$ and $\omega/f_2 > \omega/f_1$, we must have $\omega/f_2 > 1$ and $(x_m)_2 < 0$. However, ω/f_1 may be either <1 or >1, with $(x_m)_1$ being correspondingly either >0 or <0.

1. Assuming $(x_m)_1 > 0$:

For one collar:

$$(x_m)_1 = \frac{\delta_m}{1-\left(\frac{\omega}{f_1}\right)^2} \qquad +9\text{ mm} = \frac{\delta_m}{1-\left(\frac{\omega}{f_1}\right)^2} \qquad (1)$$

For two collars:

$$(x_m)_2 = \frac{\delta_m}{1-\left(\frac{\omega}{f_2}\right)^2} \qquad -3\text{ mm} = \frac{\delta_m}{1-2\left(\frac{\omega}{f_1}\right)^2} \qquad (2)$$

Dividing (1) by (2), member by member:

$$-3 = \frac{1-2\left(\frac{\omega}{f_1}\right)^2}{1-\left(\frac{\omega}{f_1}\right)^2}\text{; we find, } \quad \left(\frac{\omega}{f_1}\right)^2 = \frac{4}{5}$$

From Eq(1) $\delta_m = 1.8\text{ mm}$

For three collars:

$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{f_1}\right)^2} = \frac{1.8\text{ mm}}{1-3\left(\frac{4}{5}\right)} = -1.286\text{ mm}$$

$(x_m)_3 = 1.286\text{ mm}$ (out of phase)

2. Assuming $(x_m)_1 < 0$:

For one collar, we have now

$$-9\text{ mm} = \frac{\delta_m}{1-\left(\frac{\omega}{f_1}\right)^2} \qquad (3)$$

Dividing (3) by (2), member by member:

$$3 = \frac{1-2\left(\frac{\omega}{f_1}\right)^2}{1-\left(\frac{\omega}{f_1}\right)^2}$$

$$3-3\left(\frac{\omega}{f_1}\right)^2 = 1-2\left(\frac{\omega}{f_1}\right)^2 \qquad \left(\frac{\omega}{f_1}\right)^2 = 2$$

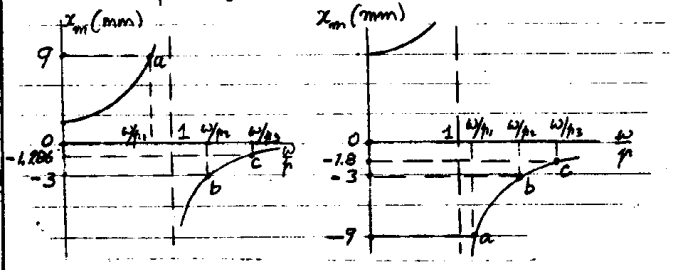
Substituting into (3): $\delta_m = -9\text{ mm}(1-2) = 9\text{ mm}$

For three collars:

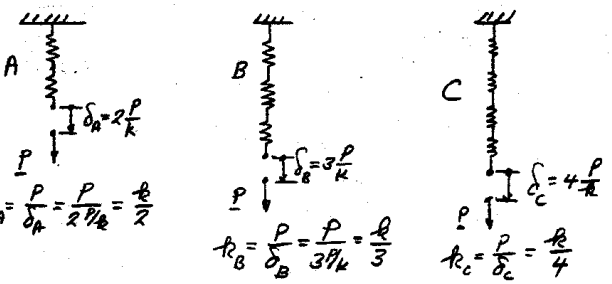
$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{f_1}\right)^2} = \frac{9\text{ mm}}{1-3(2)} = -1.800\text{ mm}$$

$(x_m)_3 = 1.800\text{ mm}$ (out of phase)

Points corresponding to the 2 solutions are indicated below:



19.99 LET k = SPRING CONSTANT OF ONE SPRING



$\delta_A = 2\frac{P}{k}$, $k_A = \frac{P}{\delta_A} = \frac{P}{2P/k} = \frac{k}{2}$

$\delta_B = 3\frac{P}{k}$, $k_B = \frac{P}{\delta_B} = \frac{P}{3P/k} = \frac{k}{3}$

$\delta_C = 4\frac{P}{k}$, $k_C = \frac{P}{\delta_C} = \frac{k}{4}$

(A) TWO SPRINGS: $(\gamma_A)_m = \pm 3$ in., $k_A = \frac{k}{2}$, $f_A^2 = \frac{k}{2m}$, $\left(\frac{\omega}{f_A}\right)^2 = 2\frac{m}{k}\omega^2$

(B) THREE SPRINGS: $(\gamma_B)_m = \pm 1.5$ in. $k_B = \frac{k}{3}$, $f_B^2 = \frac{k}{3m}$, $\left(\frac{\omega}{f_B}\right)^2 = 3\frac{m}{k}\omega^2 = \frac{3}{2}\left(\frac{\omega}{f_A}\right)^2$

(C) FOUR SPRINGS: $(\gamma_C)_m$ = UNKNOWN, $k_C = \frac{k}{4}$, $f_C^2 = \frac{k}{4m}$, $\left(\frac{\omega}{f_C}\right)^2 = 4\frac{m}{k}\omega^2 = 2\left(\frac{\omega}{f_A}\right)^2$

WE NOTE THAT AMPLITUDE δ_m OF THE DISPLACEMENT OF BAR DE REMAINS CONSTANT. REFERRING TO FIG. 19.9, WE NOTE THAT, SINCE $(\gamma_B)_m < (\gamma_A)_m$ AND, $\omega/f_B > \omega/f_A$, WE MUST HAVE $\omega/f_A > 1$ AND, $(\gamma_B)_m < 0$. THUS $(\gamma_B)_m = -1.5$ in. HOWEVER, ω/f_B MAY BE EITHER < 1 OR > 1, WITH $(\gamma_A)_m$ BEING CORRESPONDINGLY EITHER > 0 OR < 0.

1. ASSUMING $(\gamma_A)_m < 0$:

FOR TWO SPRINGS:

$$(\gamma_A)_m = \frac{\delta_m}{1-(\omega/f_A)^2} \qquad -3 \text{ in.} = \frac{\delta_m}{1-(\omega/f_A)^2} \qquad (1)$$

FOR THREE SPRINGS: $(\omega/f_B)^2 = \frac{3}{2}(\omega/f_A)^2$

$$(\gamma_B)_m = \frac{\delta_m}{1-(\omega/f_B)^2} \qquad -1.5 \text{ in.} = \frac{\delta_m}{1-\frac{3}{2}(\omega/f_A)^2} \qquad (2)$$

DIVIDE (1) BY (2):

$$2 = \frac{1-\frac{3}{2}(\omega/f_A)^2}{1-(\omega/f_A)^2}; \text{ WE FIND, } (\omega/f_A)^2 = 2$$

SUBSTITUTE INTO (1): $\delta_m = (-3 \text{ in.})[1-2] = 3$ in.

FOR FOUR SPRINGS: $(\omega/f_C)^2 = 2(\omega/f_A)^2$

$$(\gamma_C)_m = \frac{\delta_m}{1-(\omega/f_C)^2} = \frac{3 \text{ in.}}{1-2(\omega/f_A)^2} = \frac{3}{1-2(2)} = -1 \text{ in.}$$

$(\gamma_C)_m = 1$ in. (OUT OF PHASE) ◀

2. ASSUMING $(\gamma_A)_m > 0$:

FOR TWO SPRINGS:

$$+3 \text{ in} = \frac{\delta_m}{1-(\omega/f_A)^2} \qquad (3)$$

DIVIDING (3) BY (2):

$$-2 = \frac{1-\frac{3}{2}(\omega/f_A)^2}{1-(\omega/f_A)^2}; \text{ WE FIND, } (\omega/f_A)^2 = \frac{6}{7}$$

SUBSTITUTE INTO (3): $\delta_m = (3 \text{ in.})\left[1-\frac{6}{7}\right] = \frac{3}{7}$ in.

FOR FOUR SPRINGS: $(\omega/f_C)^2 = 2(\omega/f_A)^2$

$$(\gamma_C)_m = \frac{\delta_m}{1-(\omega/f_C)^2} = \frac{\delta_m}{1-2(\frac{\omega}{f_A})^2}$$

$$= \frac{(\frac{3}{7} \text{ in.})}{1-2(\frac{6}{7})} = \frac{3/7}{-5/7} = -0.6 \text{ in}$$

$(\gamma_C)_m = 0.6$ in. (OUT OF PHASE) ◀

19.100 SEE SOLUTION OF FIG. 19.99 FOR SPRING CONSTANTS.

(A) TWO SPRINGS: $(\gamma_A)_m = \pm 1.6$ in., $k_A = \frac{k}{2}$, $f_A^2 = \frac{k}{2m}$, $\left(\frac{\omega}{f_A}\right)^2 = 2\frac{m}{k}\omega^2$

(B) THREE SPRINGS: $(\gamma_B)_m = \pm 2.4$ in., $k_B = \frac{k}{3}$, $f_B^2 = \frac{k}{3m}$, $\left(\frac{\omega}{f_B}\right)^2 = 3\frac{m}{k}\omega^2 = \frac{3}{2}\left(\frac{\omega}{f_A}\right)^2$

(C) FOUR SPRINGS: $(\gamma_C)_m$ = UNK., $k_C = \frac{k}{4}$, $f_C^2 = \frac{k}{4m}$, $\left(\frac{\omega}{f_C}\right)^2 = 4\frac{m}{k}\omega^2 = 2\left(\frac{\omega}{f_A}\right)^2$

WE NOTE THAT THE AMPLITUDE δ_m OF THE DISPLACEMENT OF THE BOB REMAINS CONSTANT. REFERRING TO FIG. 19.9, WE NOTE THAT SINCE $\omega/f_B > \omega/f_A$ AND, $(\gamma_B)_m > (\gamma_A)_m$, WE MUST HAVE $\omega/f_A < 1$ AND $(\gamma_A)_m > 0$. THUS $(\gamma_A)_m = +1.6$ in. HOWEVER ω/f_B MAY BE EITHER < 1 OR > 1, WITH $(\gamma_B)_m$ BEING CORRESPONDINGLY EITHER > 0 OR < 0.

1. ASSUMING $(\gamma_B)_m < 0$:

FOR TWO SPRINGS:

$$(\gamma_A)_m = \frac{\delta_m}{1-(\omega/f_A)^2} \qquad 1.6 \text{ in.} = \frac{\delta_m}{1-(\omega/f_A)^2} \qquad (1)$$

FOR THREE SPRINGS $(\omega/f_B)^2 = \frac{3}{2}(\omega/f_A)^2$

$$(\gamma_B)_m = \frac{\delta_m}{1-(\omega/f_B)^2} \qquad -2.4 \text{ in.} = \frac{\delta_m}{1-\frac{3}{2}(\omega/f_A)^2} \qquad (2)$$

DIVIDE (1) BY (2):

$$-\frac{2}{3} = \frac{1-\frac{3}{2}(\omega/f_A)^2}{1-(\omega/f_A)^2}; \text{ WE FIND } (\omega/f_A)^2 = \frac{10}{13}$$

SUBSTITUTE INTO (1): $\delta_m = (1.6 \text{ in.})(1-\frac{10}{13}) = 0.3692$ mm

FOR FOUR SPRINGS: $(\omega/f_C)^2 = 2(\omega/f_A)^2$

$$(\gamma_C)_m = \frac{\delta_m}{1-(\omega/f_C)^2} = \frac{\delta_m}{1-2(\omega/f_A)^2} = \frac{0.3692}{1-2(\frac{10}{13})} = -0.6857 \text{ in.}$$

$(\gamma_C)_m = 0.686$ in. (OUT OF PHASE) ◀

2. ASSUMING $(\gamma_B)_m > 0$

FOR THREE SPRINGS $(\omega/f_B)^2 = \frac{3}{2}(\omega/f_A)^2$

$$(\gamma_B)_m = \frac{\delta_m}{1-(\omega/f_B)^2} \qquad 2.4 \text{ in.} = \frac{\delta_m}{1-\frac{3}{2}(\omega/f_A)^2} \qquad (3)$$

DIVIDING (1) BY (3):

$$\frac{2}{3} = \frac{1-\frac{3}{2}(\omega/f_A)^2}{1-(\omega/f_A)^2}; \text{ WE FIND } (\frac{\omega}{f_A})^2 = \frac{2}{5}$$

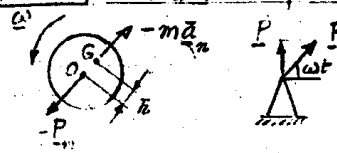
SUBSTITUTE INTO (1): $\delta_m = (1.6 \text{ in.})(1-\frac{2}{5}) = 0.96$ in.

FOR FOUR SPRINGS:

$$(\gamma_C)_m = \frac{\delta_m}{1-(\omega/f_C)^2} = \frac{\delta_m}{1-2(\omega/f_A)^2} = \frac{0.96 \text{ in.}}{1-2(\frac{2}{5})} = \frac{0.96}{0.2} = 4.8 \text{ in.}$$

$(\gamma_C)_m = 4.8$ in. (IN PHASE) ◀

19.101 Force exerted by unbalanced rotating machinery



$P_m = m\bar{a}_n = m\bar{r}\omega^2$

$P = P_m \sin\omega t$

Eq. (19.33): $x_m = \dfrac{P_m/k}{1-(\omega/p)^2} = \left(\dfrac{m\bar{r}}{k}p^2\right)\dfrac{(\omega/p)^2}{1-(\omega/p)^2}$ (1)

When object loses contact with the beam, the maximum acceleration of the beam is $a_m = g$ and, since $a_m = x_m\omega^2$, its maximum deflection is

$$x_m = \pm\frac{g}{\omega^2}$$

For $\omega = \omega_1 = 600$ rpm, we have $x_m > 0$ and $x_m = \dfrac{g}{\omega_1^2}$. Substituting in Eq. (1):

$$\frac{g}{\omega_1^2} = \left(\frac{m\bar{r}}{k}p^2\right)\frac{(\omega_1/p)^2}{1-(\omega_1/p)^2} \qquad (2)$$

For $\omega = \omega_2 = 1200$ rpm, we have $x_m < 0$ and $x_m = -\dfrac{g}{\omega_2^2}$. Substituting in Eq. (1):

$$-\frac{g}{\omega_2^2} = \left(\frac{m\bar{r}}{k}p^2\right)\frac{(\omega_2/p)^2}{1-(\omega_2/p)^2} \qquad (3)$$

Dividing (1) by (2), member by member:

$$-\left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{\omega_1}{p}\right)^2\frac{1-\left(\frac{\omega_2}{p}\right)^2}{1-\left(\frac{\omega_1}{p}\right)^2}$$

$$-\left(\frac{\omega_2}{\omega_1}\right)^2 + \left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{\omega_1}{p}\right)^2 = \left(\frac{\omega_1}{\omega_2}\right)^2 - \left(\frac{\omega_1}{p}\right)^2$$

and, since $\omega_2/\omega_1 = 2$:

$$-4 + 4\left(\frac{\omega_1}{p}\right)^2 = \frac{1}{4} - \left(\frac{\omega_1}{p}\right)^2 \qquad \left(\frac{\omega_1}{p}\right)^2 = \frac{17}{20}$$

$$p = \omega_1\sqrt{\frac{20}{17}} = (600\text{ rpm})\sqrt{\frac{20}{17}} \qquad p = 651\text{ rpm}$$

19.102

Since G describes a circle of radius $r+e$:

$$\bar{a} = (r+e)\omega^2$$

Since the deflection of the shaft is r, the force exerted on the disk is

$$F = kr$$

$F = m\bar{a}$: $\quad kr = m(r+e)\omega^2$

$$\frac{k}{m} = \frac{r+e}{r}\omega^2$$

But $p^2 = \dfrac{k}{m}$. Thus: $\quad p^2 = \dfrac{r+e}{r}\omega^2$

$$rp^2 = r\omega^2 + e\omega^2$$

$$r = \frac{e\omega^2}{p^2-\omega^2}$$

$$r = \frac{e(\omega/p)^2}{1-(\omega/p)^2} \qquad (Q.E.D.)$$

19.103

$$k = \frac{F}{\delta} = \frac{180\text{ N}}{0.3\times10^{-3}\text{ m}} = 600\times10^3\text{ N/m}$$

$$p^2 = \frac{k}{m} = \frac{600\times10^3\text{ N/m}}{25\text{ kg}} = 24\times10^3$$

$$p = 154.92\text{ rad/s} = 1479.4\text{ rpm}$$

(a) RESONANCE OCCURS WHEN

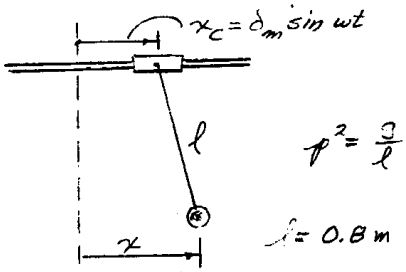
$\omega = p = 1479.4$ rpm $\qquad \omega = 1479$ rpm ◀

(b) FOR $\omega = 1350$ rpm AND $e = 0.2$ mm USE EQUATION DERIVED IN PROB 19.102:

$$r = \frac{e(\omega/p)^2}{1-(\omega/p)^2} = \frac{(0.2\text{ mm})\left(\frac{1350\text{ rpm}}{1479.4\text{ rpm}}\right)^2}{1-\left(\frac{1350\text{ rpm}}{1479.4\text{ rpm}}\right)^2}$$

$r = 0.996$ mm ◀

19.104



$p^2 = \dfrac{g}{\ell}$

$\ell = 0.8$ m

From Eq. (19.33'): $\quad x_m = \dfrac{\delta_m}{1-\left(\frac{\omega}{p}\right)^2}$

with $\delta_m = 10$ mm and $x_m = \pm 30$ mm, we have

$$-\pm 30 = \frac{10}{1-\left(\frac{\omega}{p}\right)^2} \qquad \frac{\omega^2}{p^2} = 1 \pm \frac{1}{3}$$

But, for simple pendulum: $p^2 = \dfrac{g}{\ell}$ Thus

$$\omega^2 = \frac{g}{\ell}\left(1 \pm \frac{1}{3}\right)$$

First solution: $\omega_1^2 = \dfrac{g}{\ell}\left(1+\frac{1}{3}\right) = \frac{4}{3}\frac{g}{\ell} = \frac{4}{3}\cdot\frac{9.81\text{ m/s}^2}{0.8\text{ m}}$

$\omega_1^2 = 16.350 \qquad \omega_1 = 4.0435 \qquad f_1 = \dfrac{\omega_1}{2\pi} = 0.644$ Hz ◀

Second solution: $\omega_2^2 = \dfrac{g}{\ell}\left(1-\frac{1}{3}\right) = \frac{2}{3}\frac{g}{\ell} = \frac{2}{3}\cdot\frac{9.81\text{ m/s}^2}{0.8\text{ m}}$

$\omega_2^2 = 8.175, \qquad \omega_2 = 2.859 \qquad f_2 = \dfrac{\omega_2}{2\pi} = 0.455$ Hz ◀

19.105

EQ(19.33') $x_m = \dfrac{\delta_m}{1-(\omega/p)^2}$

For $l = 600$ mm, $x_m = 24$ mm

$p^2 = \dfrac{g}{l} = \dfrac{9.81\ m/s^2}{0.6\ m} = 16.35$ $\quad p = 4.0435$ rad/s

EQ(19.33'): $\quad 24\ mm = \dfrac{\delta_m}{1-\left(\dfrac{\omega}{4.0435}\right)^2}$ (1)

For $l = 400$ mm, $x_m = 35$ mm

$p^2 = \dfrac{g}{l} = \dfrac{9.81\ m/s^2}{0.4\ m} = 24.525$ $\quad p = 4.9523$ rad/s

EQ(19.33'): $\quad \pm 35\ mm = \dfrac{\delta_m}{1-\left(\dfrac{\omega}{4.9523}\right)^2}$ (2)

WHERE + SIGN INDICATES MOTION IS "IN PHASE" AND − SIGN INDICATES MOTION IS "OUT OF PHASE"

ASSUME MOTION IS "IN PHASE" AND DIVIDE (1) BY (2)

$$+\frac{24}{35} = \frac{1-\left(\dfrac{\omega}{4.9523}\right)^2}{1-\left(\dfrac{\omega}{4.0435}\right)^2}$$

$$24 - 24\left(\frac{\omega}{4.0435}\right)^2 = 35 - 35\left(\frac{\omega}{4.9523}\right)^2$$

$$\omega^2\left[\frac{35}{(4.9523)^2} - \frac{24}{(4.0435)^2}\right] = 11$$

$$\omega^2(1.4271 - 1.4679) = 11$$

$\omega^2 = -270$ IMPOSSIBLE — ASSUMPTION THAT MOTION IS "IN PHASE" IS NOT CORRECT

WE NOW ASSUME THAT MOTION IS "OUT OF PHASE" DIVIDE (1) BY (2)

$$-\frac{24}{35} = \frac{1-\left(\dfrac{\omega}{4.9523}\right)^2}{1-\left(\dfrac{\omega}{4.0435}\right)^2}$$

$$-24 + 24\left(\frac{\omega}{4.0435}\right)^2 = 35 - 35\left(\frac{\omega}{4.9523}\right)^2$$

$$\omega^2\left[\frac{35}{(4.9523)^2} + \frac{24}{(4.0435)^2}\right] = 59$$

$$\omega^2(1.4271 + 1.4679) = 59$$

$$\omega^2 = 20.38$$

$\omega = 4.5144$ rad/s $\quad \omega = 0.718$ Hz ◀

SUBSTITUTE INTO EQ(1):

$$24\ mm = \frac{\delta_m}{1-\left(\dfrac{4.5144}{4.0435}\right)^2}$$

$$\delta_m = 24\ mm\left[1-\left(\frac{4.5144}{4.0435}\right)^2\right] = -5.92\ mm.$$

$\delta_m = 5.92$ mm ◀
(OUT OF PHASE)

19.106

TRAILER HAS TWO SPRINGS. TOTAL SPRING CONSTANT:

$k = 2(100\ lb/in.) = 200\ lb/in.$

$k = 2400\ lb/ft$

$W = 600\ lb$

$$p^2 = \frac{k}{m} = \frac{2400\ lb/ft}{(600\ lb)/(32.2\ ft/s^2)} = 128.8$$

$p = 11.349$ rad/s

(a) SINCE ONE COMPLETE CYCLE OF THE ROAD PROFILE CORRESPONDS TO A LENGTH OF 16 ft, THE VELOCITY v_r AT RESONANCE MUST BE SUCH THAT THE TRAILER TRAVELS 16 ft IN ONE PERIOD $\quad \tau = 2\pi/p$

$$v_r = \frac{16\ ft}{\tau} = (16\ ft)\frac{p}{2\pi} = \frac{(16)(11.349)}{2\pi} = 28.9\ ft/s$$

$v_r = 19.705$ mi/h $\quad v_r = 19.70$ mi/h ◀

(b) NOTING THAT v IS PROPORTIONAL TO ω, WE HAVE FOR $v = 40$ mi/h AND $\delta_m = 1.5$ in.:

$$x_m = \frac{\delta_m}{1-(\omega/p)^2} = \frac{\delta_m}{1-\left(\dfrac{v}{v_r}\right)^2} = \frac{1.5\ in.}{1-\left(\dfrac{40}{19.705}\right)^2} = -0.481\ in$$

$x_m = 0.481$ in. ◀
(OUT OF PHASE)

19.107

SEE PART (a) OF PROB 19.106 FOR COMPUTATION OF VELOCITY v_r AT RESONANCE.

$v_r = 19.705$ mi/h

GIVEN: $\delta_m = 1.5$ in., $x_m = 0.75$ in.

THUS: $\dfrac{x_m}{\delta_m} = 0.5$

SINCE $|x_m/\delta_m| < 1$, WE MUST HAVE $\omega > p$ (Fig 19.9), AND $x_m < 0$. WE ALSO NOTE THAT $v \sim \omega$, THUS

EQ(19.33'): $\quad -\dfrac{x_m}{\delta_m} = \dfrac{1}{1-(\omega/p)^2} = \dfrac{1}{1-\left(v/v_r\right)^2}$

$$-0.5 = \frac{1}{1-\left(v/v_r\right)^2}$$

$$1-\left(v/v_r\right)^2 = -2 \qquad \left(\frac{v}{v_r}\right)^2 = 3$$

$$v = v_r\sqrt{3} = (19.705\ mi/h)\sqrt{3} = 34.13\ mi/h$$

$v = 34.1$ mi/h ◀

FOR ANY SPEED $v < 34.1$ mi/h, THE AMPLITUDE x_m OF THE TRAILER IS LARGER THAN 0.75 in. FOR SPEEDS NEAR ZERO $x_m \approx 1.50$ in.

19.108 Since $c > c_c$, we use Eq. (19.41), where $\lambda_1 < 0$, $\lambda_2 < 0$:

$$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \quad (1)$$

$$v = \frac{dx}{dt} = A\lambda_1 e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t} \quad (2)$$

(a) $t = 0$, $x = x_0$, $v = 0$. Eqs (1) and (2) yield

$$x_0 = A + B \qquad 0 = A\lambda_1 + B\lambda_2$$

Solving for A and B:

$$A = \frac{\lambda_2}{\lambda_2 - \lambda_1} x_0 \qquad B = -\frac{\lambda_1}{\lambda_2 - \lambda_1} x_0$$

Substituting for A and B into (1):

$$x = \frac{x_0}{\lambda_2 - \lambda_1}\left[\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}\right]$$

For $x = 0$ when $t \neq \infty$, we must have

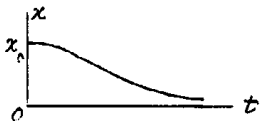
$$\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t} = 0 \qquad \frac{\lambda_2}{\lambda_1} = e^{(\lambda_2 - \lambda_1)t} \quad (3)$$

We recall that $\lambda_1 < 0$, $\lambda_2 < 0$. Choosing λ_1 and λ_2 so that $\lambda_1 < \lambda_2 < 0$, we have

$$0 < \frac{\lambda_2}{\lambda_1} < 1 \quad \text{and} \quad \lambda_2 - \lambda_1 > 0$$

Thus, a positive solution $t > 0$ for Eq. (3) cannot exist, since it would require that e raised to a positive power be less than 1, which is impossible.

The x-t curve for this motion is as shown:



(b) $t = 0$, $x = 0$, $v = v_0$. Eqs. (1) and (2) yield

$$0 = A + B \qquad v_0 = A\lambda_1 + B\lambda_2$$

Solving for A and B:

$$A = -\frac{v_0}{\lambda_2 - \lambda_1} \qquad B = \frac{v_0}{\lambda_2 - \lambda_1}$$

Substituting for A and B into (1):

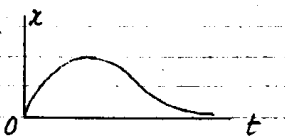
$$x = \frac{v_0}{\lambda_2 - \lambda_1}\left[e^{\lambda_2 t} - e^{\lambda_1 t}\right]$$

For $x = 0$ when $t \neq \infty$, we must have

$$e^{\lambda_2 t} = e^{\lambda_1 t}$$

But, for $c > c_c$; $\lambda_1 \neq \lambda_2$. Thus, no solution can exist for t, and the body will never pass through its position of equilibrium.

The x-t curve for this motion is as shown:



19.109 We substitute the initial conditions $t = 0$, $x = x_0$, $v = v_0$ in Eqs. (1) and (2) of Prob. 19.108.

$$x_0 = A + B \qquad v_0 = A\lambda_1 + B\lambda_2$$

Solving for A and B:

$$A = -\frac{v_0 - \lambda_2 x_0}{\lambda_2 - \lambda_1} \qquad B = \frac{v_0 - \lambda_1 x_0}{\lambda_2 - \lambda_1}$$

Substituting for A and B into (1):

$$x = \frac{1}{\lambda_2 - \lambda_1}\left[(v_0 - \lambda_1 x_0)e^{\lambda_2 t} - (v_0 - \lambda_2 x_0)e^{\lambda_1 t}\right]$$

For $x = 0$ when $t \neq \infty$, we must have

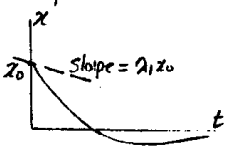
$$(v_0 - \lambda_1 x_0)e^{\lambda_2 t} = (v_0 - \lambda_2 x_0)e^{\lambda_1 t}$$

$$e^{(\lambda_2 - \lambda_1)t} = \frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0}$$

$$t = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0}$$

This defines <u>one solution only</u>. Assuming $\lambda_1 < \lambda_2 < 0$, we note that this solution will exist if the natural log is positive, i.e., if $\frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0} > 1$

We may check that this will occur if $v_0 < \lambda_1 x_0$



19.110 For light damping: $c < c_c$

Eq. (19.45): $x = x_m e^{-\frac{c}{2m}t}\sin(qt + \phi)$

At given max. displacement, $t = t_n$, $x = x_m$:

$\sin(qt_n + \phi) = 1$, $\quad x_n = x_m e^{-\frac{c}{2m}t_n}$

At next max. displacement, $t = t_{n+1}$, $x = x_{n+1}$:

$\sin(qt_{n+1} + \phi) = 1$, $\quad x_{n+1} = x_m e^{-\frac{c}{2m}t_{n+1}}$

But: $qt_{n+1} - qt_n = 2\pi \qquad t_{n+1} - t_n = \frac{2\pi}{q}$

<u>Ratio of successive max. displacements:</u>

$$\frac{x_n}{x_{n+1}} = \frac{x_m e^{-\frac{c}{2m}t_n}}{x_m e^{-\frac{c}{2m}t_{n+1}}} = e^{-\frac{c}{2m}(t_n - t_{n+1})} = e^{+\frac{c}{2m}\frac{2\pi}{q}}$$

Thus: $\ln \frac{x_n}{x_{n+1}} = \frac{c\pi}{mq} \quad (1)$

But, from Eqs. (19.44) and (19.40):

$$q = p\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

Thus:

$$\ln \frac{x_n}{x_{n+1}} = \frac{c\pi}{m}\frac{2m}{c_c}\frac{1}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$$

$$\ln \frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1 - (c/c_c)^2}} \qquad \text{(Q.E.D.)}$$

19.111 Eq. (19.45): $x = x_m e^{-\frac{c}{2m}t} \sin(qt+\phi)$

At max. displacement, $t=t_n$, $x=x_n$:

$\sin(qt_n+\phi)=1 \qquad x_n = x_m e^{-\frac{c}{2m}t_n}$

At max. displacement, $t=t_{n+k}$, $x=x_{n+k}$:

$\sin(qt_{n+k}+\phi)=1 \qquad x_{n+k} = x_m e^{-\frac{c}{2m}t_{n+k}}$

But $qt_{n+k} - qt_n = k(2\pi) \qquad t_{n+k} - t_n = k\frac{2\pi}{q}$

Ratio of max. displacements:

$$\frac{x_n}{x_{n+k}} = \frac{x_m e^{-\frac{c}{2m}t_n}}{x_m e^{-\frac{c}{2m}t_{n+k}}} = e^{-\frac{c}{2m}(t_n - t_{n+k})} = e^{+\frac{c}{2m}\frac{2k\pi}{q}}$$

Thus: $\ln\frac{x_n}{x_{n+k}} = k\frac{c\pi}{mq}$ (2)

But, from Prob. 19.110 Eq. (1)

Log. decrement $= \ln\frac{x_n}{x_{n+1}} = \frac{c\pi}{mq}$

Comparing with Eq. (2):

Log. decrement $= \frac{1}{k}\ln\frac{x_n}{x_{n+k}}$ (Q.E.D.)

19.112 For $c < c_c$, we have

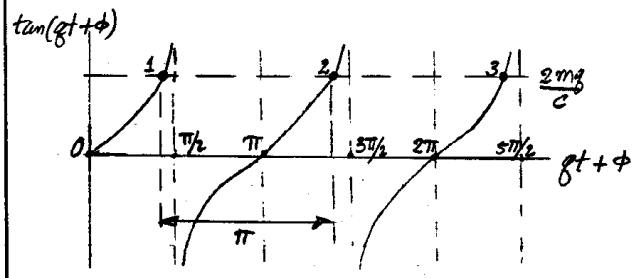
Eq. (19.45): $x = x_m e^{-\frac{c}{2m}t}\sin(qt+\phi)$ (1)

$$\frac{dx}{dt} = x_m e^{-\frac{c}{2m}t}\left[q\cos(qt+\phi) - \frac{c}{2m}\sin(qt+\phi)\right] \quad (2)$$

(a) Maxima (positive or negative) occur when bracket is zero in Eq (2):

$$q\cos(qt+\phi) - \frac{c}{2m}\sin(qt+\phi) = 0$$

or $\tan(qt+\phi) = \frac{2mq}{c}$ (3)



We note from the sketch that the solutions of Eq. (3) occur at intervals $\Delta t = \frac{\pi}{q} = \frac{1}{2}\frac{2\pi}{q} = \frac{1}{2}\tau$ (Q.E.D.)

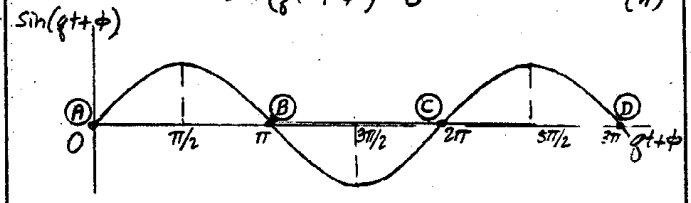
Referring to Eq. (1), we also note that points 1 and 3 correspond to max. positive displacements, and point 2 to a max. negative displacement.

(CONTINUED)

19.112 CONTINUED

(b) Zero displacements occur when [see Eq. (1)]:

$$\sin(qt+\phi) = 0 \quad (4)$$



The solutions of Eq. (4) occur at intervals

$$\Delta t = \frac{\pi}{q} = \frac{1}{2}\frac{2\pi}{q} = \frac{1}{2}\tau \quad \text{(Q.E.D.)}$$

(c) Interval between max. pos. displ. and following zero displacement:

From the first sketch, we observe that the max. positive displ. 1, 3, etc. occur shortly before $\pi/2$, $5\pi/2$, etc. From the second sketch, it appears that the following zero displ. will be B, D, etc., that is, at π, 3π, etc.

It is clear that the angles between 1 and B, between 3 and D, etc., are larger than $\pi/2$ and, thus, that the corresponding time intervals Δt are larger than $\frac{\pi}{2q}$, or $\frac{1}{4}\frac{2\pi}{q}$, or $\frac{1}{4}\tau$ (Q.E.D.)

19.113 (a) Damping factor c/c_c

From Prob 19.110: $\ln\frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$ (1)

From data:

$$\frac{x_n}{x_{n+1}} = \frac{64\text{ mm}}{48\text{ mm}} = \frac{48\text{ mm}}{36\text{ mm}} = \frac{36\text{ mm}}{27\text{ mm}} = \frac{4}{3}$$

$$\ln\frac{x_n}{x_{n+1}} = \ln\frac{4}{3} = 0.28768$$

Substituting into Eq. (1):

$$0.28768 = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$1-\left(\frac{c}{c_c}\right)^2 = \left(\frac{2\pi}{0.28768}\right)^2\left(\frac{c}{c_c}\right)^2$$

$$\left(\frac{c}{c_c}\right)^2 = \frac{1}{477+1} = 2.092\times10^{-3} \qquad \frac{c}{c_c} = 0.04574$$

$$\frac{c}{c_c} = 0.0457 \quad \blacktriangleleft$$

(b) Coefficient of viscous damping c

From answer just obtained: $c = 0.04574\,c_c$

From Eq. (19.40): $c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk}$

Thus: $c = 0.04574\left(2\sqrt{mk}\right) = 0.09148\sqrt{mk}$

Given data: $m = 15$ kg, $k = 1800$ N/m

$$c = 0.09148\sqrt{(15\text{ kg})(1800\text{ N/m})} = 15.03\text{ N-s/m}$$

$$c = 15.03\text{ N-s/m} \quad \blacktriangleleft$$

19.114

FROM PROBS. 19.110 AND 19.111:

$$\frac{1}{n}\ln\left(\frac{x_m}{x_{m+n}}\right)=\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

DATA: $x_0 = 30$ mm, $k = 120$ N/m, $m = 4$ kg
$x_8 = 18$ mm, THUS $n=8$, $k=10$

(a) $$\frac{1}{10}\ln\left(\frac{30\text{ mm}}{18\text{ mm}}\right)=\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$0.051083\sqrt{1-(c/c_c)^2}=2\pi(c/c_c)$$

$$(0.051083)^2\left[1-(c/c_c)^2\right]=4\pi^2(c/c_c)^2$$

$$1-(c/c_c)^2=(123.001)^2(c/c_c)^2$$

$$(c/c_c)^2=66.093\times10^{-6} \qquad c/c_c=8.130\times10^{-3}$$

EQ(19.40): $c_c=2m\sqrt{\frac{k}{m}}=2\sqrt{km}=2\sqrt{(120\text{ N/m})(4\text{ kg})}=43.82$

$c=(c/c_c)c_c=8.13\times10^{-3}(43.82)$ $\qquad c=0.356$ N·s/m

19.115

(a.) CRITICAL DAMPING

$$c=c_c=2mp=2m\sqrt{\frac{k}{m}}=2\sqrt{mk}$$

$$c=2\sqrt{\left(\frac{1200\text{ lb}}{32.2\text{ ft/s}^2}\right)(8{,}000\text{ lb/ft})}=1092.04\text{ lb·s/ft}$$

$c = 1092$ lb·s/ft

(b) MOTION OF A CRITICALLY DAMPED SYSTEM

EQ(19.42): $x=Ae^{-pt}+Bte^{-pt}$ (1)

$$\frac{dx}{dt}=-Ape^{-pt}+B\left[-pte^{-pt}+e^{-pt}\right]$$

$$\frac{dx}{dt}=e^{-pt}\left[-pA+(1-pt)B\right]$$

FOR SYSTEM RELEASED FROM REST:

$\frac{dx}{dt}=0$, WHEN $t=0$: $\quad 0=1\left[-pA+(1-0)B\right]$

$$B=Ap$$

EQ(1): $x=Ae^{-pt}(1+pt)$

FOR $t=0$, $x_0=A$ THUS $x=x_0e^{-pt}(1+pt)$

FOR $x=\frac{1}{2}x_0$: $\quad \frac{1}{2}x_0=x_0e^{-pt}(1+pt)$

$$e^{-pt}(1+pt)=\frac{1}{2} \qquad (2)$$

SOLVE BY TRIAL: $pt=1.6783$

$$t=\frac{1.6783}{p} \qquad (3)$$

FOR GUN: $$p=\sqrt{\frac{k}{m}}=\sqrt{\frac{8{,}000\text{ lb/ft}}{(1200\text{ lb})/(32.2\text{ ft/s}^2)}}$$

$p = 14.651$ rad/s

EQ(3): $t=\frac{1.6783}{14.651}=0.11455$ $\qquad t=0.1146$ s

19.116

SINCE THE COEFFICIENT OF DAMPING HAS NOT CHANGED, FROM PROB 19.115, WE RECALL

$c = 1092.04$ lb·s/ft

(a) THE WEIGHT OF THE BARREL IS NOW $W=1200+300=1500$ lb

$c=c_c=2\sqrt{mk}$ OR $k=\frac{c^2}{4m}$

$$k=\frac{(1092.04\text{ lb·s/ft})^2}{4\left(\frac{1500\text{ lb}}{32.2\text{ ft/s}^2}\right)}=6400\text{ lb/ft}$$

$k = 6400$ lb/ft

ALTERNATE SOLUTION: $c=2\sqrt{mk}$

SINCE c IS UNCHANGED, mk IS ALSO UNCHANGED

$$\left(\frac{1200\text{ lb}}{g}\right)(8{,}000\text{ lb/ft})=\left(\frac{1500\text{ lb}}{g}\right)k$$

$k = 6400$ lb/ft

(b) SEE PART (b) OF PROB 19.115 FOR DERIVATION OF EQ(2)

$$e^{-pt}(1+pt)=\frac{1}{2} \qquad pt=1.6783$$

$$t=\frac{1.6783}{p}$$

FOR MODIFIED GUN:

$$p=\sqrt{\frac{6400\text{ lb/ft}}{\frac{(1500\text{ lb})}{32.2\text{ ft/s}^2}}} \qquad p=11.721\text{ rad/s}$$

$t=\frac{1.6783}{11.721}=0.1432$ $\qquad t=0.1432$ s

19.117

EQ(19.52): $$\frac{x_m}{\delta_m}=\frac{}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

FOR MAXIMUM, WE SET EQUAL TO ZERO THE DERIVATIVE WITH RESPECT TO (ω/p) OF THE SQUARE OF THE DENOMINATOR.

$$2\left[1-\left(\frac{\omega}{p}\right)^2\right]\left[-2\left(\frac{\omega}{p}\right)\right]+8\left(\frac{c}{c_c}\right)^2\left(\frac{\omega}{p}\right)=0$$

REARRANGING WE OBTAIN

$$4\left(\frac{\omega}{p}\right)\left[\left(\frac{\omega}{p}\right)^2-1+2\left(\frac{c}{c_c}\right)^2\right]=0$$

$$\left(\frac{\omega}{p}\right)^2=1-2\left(\frac{c}{c_c}\right)^2 \qquad \frac{\omega}{p}=\sqrt{1-2\left(\frac{c}{c_c}\right)^2}$$

19.118

FROM PROB 19.117, MAXIMUM AMPLITUDE OCCURS WHEN $\frac{\omega}{p}=\sqrt{1-2\left(\frac{c}{c_c}\right)^2}$

(a) FOR SMALL DAMPING, WE NEGLECT THE TERM $2\left(\frac{c}{c_c}\right)$ WHEN COMPARED TO UNITY. THUS $\omega\approx p$

(b) FOR $\omega=p$, THE MAGNIFICATION FACTOR IS OBTAINED FROM EQ(19.52). $\frac{x_m}{\delta_m}=\frac{1}{\sqrt{\left[2\frac{c}{c_c}\right]^2}}$ OR, $\frac{x_m}{\delta_m}\approx\frac{1}{2}\frac{c_c}{c}$

19.119

SPRING CONSTANT:

$$k = \frac{W}{\delta_{st}} = \frac{35\text{ lb}}{0.06\text{ in.}} = 583.3\text{ lb/in.} = 7000\text{ lb/ft}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{7000\text{ lb/ft}}{(35\text{ lb})/(32.2\text{ ft/s}^2)}} = 80.250\text{ rad/s}$$

$\omega = 800\text{ rpm} = 83.776\text{ rad/s}$

UNBALANCED FORCE:

$m' = 1\text{ oz} = \frac{1}{16}\text{ lb}$, $\bar{r} = 5\text{ in.} = \frac{5}{12}\text{ ft}$

$$P_m = m'\bar{r}\omega^2 = \frac{1}{(16)(32.2)}\left(\frac{5}{12}\right)(83.776)^2 = 5.676\text{ lb}$$

(a) NO DAMPING: EQ(19.33)

$$x_m = \frac{P_m/k}{1-\left(\frac{\omega}{p}\right)^2} = \frac{\frac{5.676\text{ lb}}{7000\text{ lb/ft}}}{1-\left(\frac{83.776}{80.250}\right)^2} = -0.00903\text{ ft}$$

$x_m = 0.1084\text{ in.}$ (OUT OF PHASE) ◀

(b) DAMPED: $c/c_c = 0.065$, EQ(19.52)

$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

$$x_m = \frac{\frac{5.676\text{ lb}}{7000\text{ lb/ft}}}{\sqrt{\left[1-\left(\frac{83.776}{80.250}\right)^2\right]^2+\left[2(0.065)\left(\frac{83.776}{80.250}\right)\right]^2}}$$

$x_m = 0.004983\text{ ft}$ $\quad x_m = 0.0598\text{ in.}$ ◀

PHASE ANGLE MAY BE FOUND FROM EQ(19.53). $\phi = \tan^{-1}(-1.511)$

19.120

$k = 4(1000\text{ lb/in.}) = 4000\text{ lb/in.} = 48000\text{ lb/ft}$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{48000\text{ lb/ft}}{(50\text{ lb})/(32.2\text{ ft/s}^2)}} = 175.818\text{ rad/s}$$

UNBALANCED FORCE: $m' = 1\text{ oz} = \frac{1}{16}\text{ lb}$, $\bar{r} = \frac{5}{12}\text{ ft}$

$\omega = 1800\text{ rpm} = 188.495\text{ rad/s}$

$$P_m = m'\bar{r}\omega^2 = \frac{1}{(16)(32.2)}\left(\frac{5}{12}\right)(188.496)^2 = 28.735\text{ lb}$$

(a) NO DAMPING: EQ(19.33)

$$x_m = \frac{P_m/k}{1-\left(\frac{\omega}{p}\right)^2} = \frac{\frac{28.735\text{ lb}}{48000\text{ lb/ft}}}{1-\left(\frac{188.496}{175.818}\right)^2} = -0.00401\text{ ft}$$

$x_m = 0.0481\text{ in.}$ (OUT OF PHASE) ◀

(b) DAMPING: $c/c_c = 0.125$ EQ(19.52)

$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

$$x_m = \frac{\frac{28.735\text{ lb}}{48000\text{ lb/ft}}}{\sqrt{\left[1-\left(\frac{188.496}{175.818}\right)^2\right]^2+\left[2(0.125)\left(\frac{188.496}{175.818}\right)\right]^2}}$$

$x_m = 0.00195\text{ ft}$ $\quad x_m = 0.0234\text{ in.}$ ◀

PHASE ANGLE MAY BE FOUND FROM EQ(19.53). $\phi = \tan^{-1}(-1.794)$

19.121

FROM SOLUTION OF PROB 19.95, WE HAVE

$k = 160\text{ kN/m}$ $\quad m = 18\text{ kg}$

$p = 94.281\text{ rad/s}$ $\quad 900\text{ rpm}$

EQ(19.40): $c_c = 2mp = 2(18)(94.28) = 3394\text{ N·s/m}$

FOR $\omega = 1200\text{ rpm} = 125.664\text{ rad/s}$

$x_m = 1.5\text{ mm} = 1.5\times10^{-3}\text{ m}$ $\quad c = 350\text{ N·s/m}$

EQ(19.52): $$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

$$1.5\times10^{-3}\text{ m} = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{125.664}{94.281}\right)^2\right]^2+\left[2\left(\frac{350}{3394}\right)\left(\frac{125.664}{94.281}\right)\right]^2}}$$

$$1.5\times10^{-3} = \frac{P_m/k}{\sqrt{(0.7765)^2+(0.2749)^2}} = \frac{P_m/k}{0.82376}$$

$\frac{P_m}{k} = 1.2356\times10^{-3}$ $\quad P_m = (1.2356\times10^{-3})(160\times10^3) = 197.7\text{ N}$

FOR ROTOR OF MASS $m' = 4\text{ kg}$ AND $\omega = 125.664\text{ rad/s}$

$$P_m = m'\bar{r}\omega^2 \qquad \bar{r} = \frac{P_m}{m'\omega^2} = \frac{197.7\text{ N}}{(4\text{ kg})(125.664\text{ rad/s})^2}$$

$\bar{r} = 3.13\times10^{-3}\text{ m}$ $\quad \bar{r} = 3.13\text{ mm}$ ◀

19.122

$$k = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}} = \frac{(50\text{ kg})(9.81\text{ m/s}^2)}{6\times10^{-3}\text{ m}} = 81.75\times10^3\text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{81.75\times10^3\text{ N/m}}{50\text{ kg}}} = 40.435\text{ rad/s}$$

UNBALANCED FORCE $m' = 100\text{ g} = 0.1\text{ kg}$, $\bar{r} = 75\text{ mm} = 0.075\text{ m}$

$\omega = 400\text{ rpm} = 41.888\text{ rad/s}$

$$P_m = m'\bar{r}\omega^2 = (0.1\text{ kg})(0.075\text{ m})(41.888\text{ rad/s})^2$$

$P_m = 13.160\text{ N}$

EQ(19.52): $$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

(a) GIVEN $x_m = 0.8\text{ mm} = 0.8\times10^{-3}\text{ m}$

$$0.8\times10^{-3} = \frac{\frac{13.160\text{ N}}{81.75\times10^3\text{ N/m}}}{\sqrt{\left[1-\left(\frac{41.888}{40.435}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{41.888}{40.435}\right)\right]^2}}$$

SOLVE FOR $\frac{c}{c_c} = 0.090475$ $\quad \frac{c}{c_c} = 0.0905$ ◀

(b) EQ(19.40): $c_c = 2mp = 2(50\text{ kg})(40.435\text{ rad/s})$

$c_c = 4043.5\text{ N·s/m}$

FROM PART (a):

$\frac{c}{c_c} = 0.090475$ $\quad c = (0.090475)(4043.5\text{ N·s/m})$

$= 365.8\text{ N·s/m}$

$c = 366\text{ N·s/m}$ ◀

19.123 $k = 2(48\times10^3\ \text{N/m}) = 96\times10^3\ \text{N/m}$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{96\times10^3}{500}} = 13.856\ \text{rad/s}$$

$$f = \frac{p}{2\pi} = \frac{13.856}{2\pi} = 2.205\ \text{Hz}$$

$$c_c = 2mp = 2(500)(13.856) = 13.856\times10^3\ \text{N·s/m}$$

$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}} \qquad (1)$$

We have

$$\frac{\omega}{p} = \frac{2.4\ \text{Hz}}{2.205\ \text{Hz}} = 1.088$$

$c = 1600\ \text{N·s/m}$

$$\frac{c}{c_c} = \frac{1.6\times10^3}{13.856\times10^3} = 0.11547$$

Substituting into (1) with $P_m = 150\ \text{N}$, we have

$$x_m = \frac{150\ \text{N}/96\times10^3\ \text{N/m}}{\sqrt{\left[1-(1.088)^2\right]^2+\left[2(0.11547)(1.088)\right]^2}}$$

$$= \frac{1.5625\times10^{-3}\ \text{m}}{0.3113} = 5.02\times10^{-3}\ \text{m}$$

$$x_m = 5.02\ \text{mm}$$

19.124 Substituting the values found in Prob. 19.123, together with $x_m = 4\times10^{-3}\ \text{m}$ into Eq. (1) of Prob. 19.123, we have:

$$4\times10^{-3} = \frac{150\ \text{N}/96\times10^3\ \text{N/m}}{\sqrt{\left[1-(1.088)^2\right]^2+\left[2\frac{c}{c_c}(1.088)\right]^2}}$$

Solving for $\frac{c}{c_c}$, we find $\frac{c}{c_c} = 0.1584$

Since $c_c = 13.856\times10^3\ \text{N·s/m}$, we have

$$c = (0.1584)(13.856\times10^3\ \text{N·s/m}) = 2.19\times10^3\ \text{N·s/m}$$

$$c = 2.19\ \text{kN·s/m}$$

19.125 (a) NO DAMPING:

$$k = 2(250\ \text{lb/in.}) = 500\ \text{lb/in.} = 6000\ \text{lb/ft}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{6000\ \text{lb/ft}}{(200\ \text{lb})/(32.2\ \text{ft/s}^2)}} = 31.08\ \text{rad/s} = 4.947\ \text{Hz}$$

$$f = 297\ \text{rpm}$$

(b) DAMPED MOTION

$$c = 9\ \text{lb·s/in.} = 108\ \text{lb·s/ft}$$

$$c_c = 2mp = 2\left(\frac{200\ \text{lb}}{32.2\ \text{ft/s}^2}\right)(31.08\ \text{rad/s}) = 386.09\ \text{lb·s/ft}$$

$$\frac{c}{c_c} = \frac{108\ \text{lb·s/ft}}{386.09\ \text{lb·s/ft}} = 0.27973$$

(CONTINUED)

19.125 CONTINUED FOR MAXIMUM AMPLITUDE WE USE THE RESULT OBTAINED IN PROB. 19.117

$$\left(\frac{\omega}{p}\right)^2 = 1-2\left(\frac{c}{c_c}\right)^2 = 1-2(0.27973)^2 = 0.8435$$

$$\frac{\omega}{p} = 0.91842 \qquad \omega = (0.91842)p = (0.91842)(31.08\ \text{rad/s})$$

$$\omega = 28.544\ \text{rad/s} = 4.543\ \text{Hz} \qquad \omega = 273\ \text{rpm}$$

(c) AMPLITUDE

EQ(19.52): $$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

FOR PART (a) WITH $P_m = 125\ \text{lb}$ AND $\omega = p$

$$x_m = \frac{\frac{125\ \text{lb}}{6000\ \text{lb/ft}}}{\sqrt{\left[1-1\right]^2+\left[2(0.27973)(1)\right]^2}} = 0.03724\ \text{ft} \qquad x_m = 0.447\ \text{in.}$$

FOR PART (b) WITH $P_m = 125\ \text{lb}$ AND $\frac{\omega}{p} = 0.91842$

$$x_m = \frac{\frac{125\ \text{lb}}{6000\ \text{lb/ft}}}{\sqrt{\left[1-(0.91842)^2\right]^2+\left[2(0.27973)(0.91842)\right]^2}} = 0.03879\ \text{ft}$$

$$x_m = 0.465\ \text{in.}$$

19.126 (a) NO DAMPING. SAME AS PROB 19.125

$k = 6000\ \text{lb/ft}$, $p = 31.08\ \text{rad/s}$ $f = 297\ \text{rpm}$

(b) DAMPED MOTION. $c = 12\ \text{lb·s/in.} = 144\ \text{lb·s/ft}$

$c_c = 2mp = 386.09\ \text{lb·s/ft}$ (SAME AS IN PROB 19.125)

$$\frac{c}{c_c} = \frac{144\ \text{lb·s/ft}}{386.09\ \text{lb·s/ft}} = 0.37297$$

FOR MAXIMUM AMPLITUDE WE USE THE RESULT OBTAINED IN PROB 19.117.

$$\left(\frac{\omega}{p}\right)^2 = 1-2\left(\frac{c}{c_c}\right)^2 = 1-2(0.37297)^2 = 0.72179$$

$$\frac{\omega}{p} = 0.84958 \qquad \omega = (0.84958)p = (0.84958)(31.08\ \text{rad/s})$$

$$\omega = 26.405\ \text{rad/s} = 4.2025\ \text{Hz} \qquad \omega = 252\ \text{rpm}$$

(c) AMPLITUDE

EQ(19.52): $$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left[2\frac{c}{c_c}\frac{\omega}{p}\right]^2}}$$

FOR PART (a) WITH $P_m = 125\ \text{lb}$ AND $\omega = p$

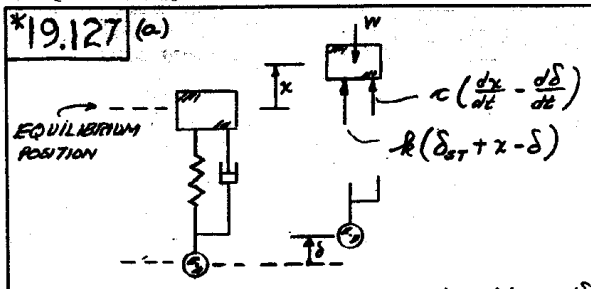
$$x_m = \frac{\frac{125\ \text{lb}}{6000\ \text{lb/ft}}}{\sqrt{\left[1-1\right]^2+\left[2(0.37297)(1)\right]^2}} = 0.02793\ \text{ft} \qquad x_m = 0.335\ \text{in.}$$

FOR PART (b) WITH $P_m = 125\ \text{lb}$ AND $\frac{\omega}{p} = 0.84958$

$$x_m = \frac{\frac{125\ \text{lb}}{6000\ \text{lb/ft}}}{\sqrt{\left[1-(0.84958)^2\right]^2+\left[2(0.84958)(0.37297)\right]^2}}$$

$$x_m = 0.0301\ \text{ft} \qquad x_m = 0.361\ \text{in.}$$

***19.127** (a)



$$+\downarrow \Sigma F = ma: \quad W - k(\delta_{st}+x-\delta) - c\left(\frac{dx}{dt}-\frac{d\delta}{dt}\right) = m\frac{d^2x}{dt^2}$$

RECALLING THAT $W = k\delta_{st}$, WE WRITE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = k\delta + c\frac{d\delta}{dt} \qquad (1)$$

MOTION OF WHEEL IS SINE CURVE. $\delta = \delta_m \sin \omega t$

SINCE SPEED IS V, INTERVAL OF TIME REQUIRED TO TRAVEL DISTANCE L IS $t = L/V$

BUT PERIOD OF ROAD SURFACE IS L, THUS

$$\omega t = 2\pi \qquad \omega\frac{L}{V} = 2\pi \quad \text{OR} \quad \omega = \frac{2\pi V}{L}$$

SUBSTITUTING $\delta = \delta_m \sin \omega t$ INTO (1) WE HAVE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = (k\sin\omega t + c\omega\cos\omega t)\delta_m$$

(b) AMPLITUDE: CONSIDER SEPARATELY THE TWO TERMS OF THE RIGHT-HAND SIDE OF DIFFERENTIAL EQUATION.

FIRST TERM

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = k\delta_m \sin\omega t$$

THIS EQUATION IS SAME AS EQ 19.46, EXCEPT THAT $P_m = k\delta_m$. ASSUME SOLUTION OF FORM

$$x = x_1 \sin(\omega t - \phi)$$

AND PROCEED AS WAS DONE IN EQS. 19.47 THROUGH 19.51, WE FIND

$$x_1 = \frac{k\delta_m}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

SECOND TERM

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = c\omega\delta_m \cos\omega t$$

RECALLING THAT $\cos\omega t = \sin(\omega t + 90°)$, WE HAVE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = c\omega\delta_m \sin(\omega t + 90°)$$

THIS EQUATION IS NOW SIMILAR TO EQ 19.46, WE ASSUME SOLUTION OF FORM

$$x = x_2 \sin(\omega t + 90° - \phi)$$

AND PROCEED AS IN EQS 19.47 THROUGH 19.51, WE FIND

$$x_2 = \frac{c\omega\delta_m}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

ADDING SOLUTIONS FOR THE TWO TERMS, WE HAVE

$$x = x_1 \sin(\omega t - \phi) + x_2 \sin(\omega t + 90° - \phi)$$

$$x = \frac{k\delta_m \sin(\omega t - \phi) + c\omega\delta_m \sin(\omega t + 90° - \phi)}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} \qquad (2)$$

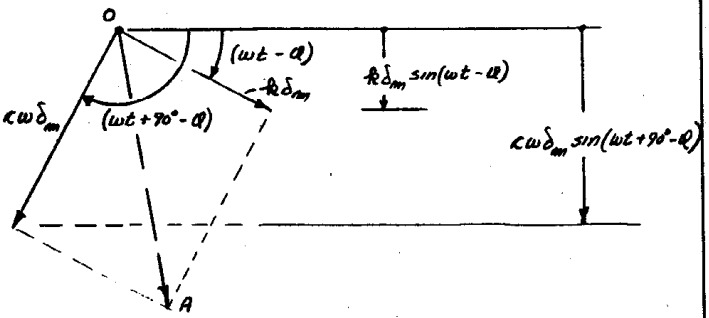
CONTINUED

***19.127 CONTINUED**

WE SHALL SEEK THE MAXIMUM VALUE OF THE NUMERATOR OF EQUATION (2), i.e.

$$k\delta_m \sin(\omega t - \phi) + c\omega\delta_m \sin(\omega t + 90° - \phi)$$

CONSIDERING THESE TWO TERMS AS THE VERTICAL PROJECTIONS OF ROTATING VECTORS OF MAGNITUDE $k\delta_m$ AND $c\omega\delta_m$.



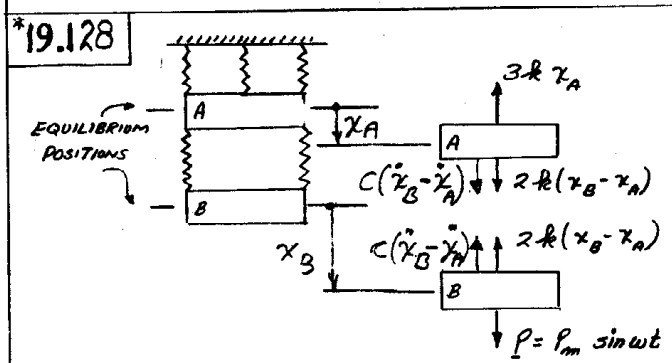
MAXIMUM VALUE OF NUMERATOR = OA

$$OA = \sqrt{(k\delta_m)^2 + (c\omega\delta_m)^2} = \delta_m\sqrt{k^2 + c^2\omega^2}$$

THUS CONSIDERING BOTH NUMERATOR AND DENOMINATOR, WE HAVE

$$x_m = \frac{\sqrt{k^2 + c^2\omega^2}\ \delta_m}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

***19.128**



SINCE THE ORIGINS OF COORDINATE ARE CHOSEN AT THE POSITIONS OF EQUILIBRIUM, WE MAY OMIT THE INITIAL SPRING TENSIONS AND THE EFFECT OF GRAVITY.

FOR LOAD A:

$$+\downarrow \Sigma F = ma_A: \quad -3kx_A + 2k(x_B - x_A) + c(\dot{x}_B - \dot{x}_A) = m\ddot{x}_A \qquad (1)$$

FOR LOAD B:

$$+\downarrow \Sigma F = ma_B: \quad P_m \sin\omega t - 2k(x_B - x_A) - c(\dot{x}_B - \dot{x}_A) = m\ddot{x}_B \qquad (2)$$

REARRANGING EQS. (1) AND (2), WE FIND:

$$m\ddot{x}_A + 5kx_A - 2kx_B - c(\dot{x}_B - \dot{x}_A) = 0$$

$$m\ddot{x}_B - 2kx_A + 2kx_B + c(\dot{x}_B - \dot{x}_A) = P_m \sin\omega t$$

19.129 FOR A MECHANICAL SYSTEM OSCILLATIONS TAKE PLACE IF $c < c_c$. BUT, FROM EQ 19.40,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km}$$

$$\therefore \quad c < 2\sqrt{km} \qquad (1)$$

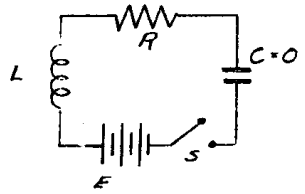
FROM TABLE 19.2: $c \to R$, $m \to L$, $k \to 1/C$

THUS, FOR ELECTRICAL SYSTEM, OSCILLATIONS TAKE PLACE IF

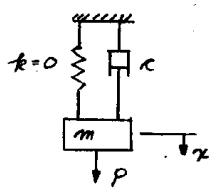
$$R < 2\sqrt{L/C}$$ ◀

19.130

ELECTRICAL SYSTEM

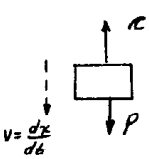


MECHANICAL SYSTEM



CLOSING SWITCH S IS EQUIVALENT TO SUDDENLY APPLYING A CONSTANT FORCE OF MAGNITUDE P TO THE MASS.

(a) FINAL VALUE OF CURRENT CORRESPONDS TO FINAL VELOCITY OF MASS. SINCE CAPATANCE C IS ZERO, THE SPRING CONSTANT IS ALSO ZERO.



$+\downarrow \Sigma F = ma$:

$$P - c\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad (1)$$

FINAL VELOCITY OCCURS WHEN $\frac{d^2x}{dt^2} = 0$,

$$P - c\left.\frac{dx}{dt}\right|_{FINAL} = 0$$

$$P - cv_{FINAL} = 0 \qquad v_{FINAL} = \frac{P}{c}$$

FROM TABLE 19.2: $v \to i$, $P \to E$, $c \to R$.

THUS: $i_{FINAL} = E/R$ ◀

(b) REARRANGING EQ (1), WE HAVE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} = P$$

SUBSTITUTE $\frac{dx}{dt} = Ae^{-\lambda t} + \frac{P}{c}$; $\frac{d^2x}{dt^2} = -A\lambda e^{-\lambda t}$

$$m\left[-A\lambda e^{-\lambda t}\right] + c\left[Ae^{-\lambda t} + \frac{P}{c}\right] = P$$

$$-m\lambda + c = 0 \qquad \lambda = \frac{c}{m}$$

THUS: $\frac{dx}{dt} = Ae^{-\frac{c}{m}t} + \frac{P}{c}$

WHEN $t = 0$, $\frac{dx}{dt} = 0$: $0 = A + \frac{P}{c}$ $\qquad A = -\frac{P}{c}$

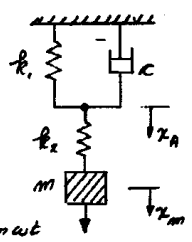
$$v = \frac{dx}{dt} = \frac{P}{c}\left(1 - e^{-\frac{c}{m}t}\right)$$

FROM TABLE 19.2: $v \to i$, $P \to E$, $c \to R$, $m \to L$

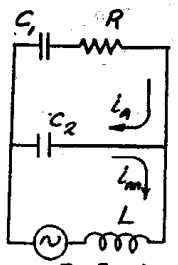
$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

FOR $i = \frac{E}{R}(1 - 1/e)$, WE HAVE $\frac{R}{L}t = 1$ $\qquad t = \frac{L}{R}$ ◀

19.131

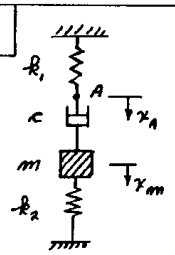


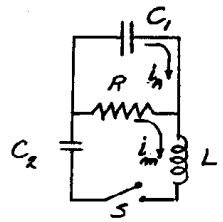
$P = P_m \sin \omega t$



$E = E_m \sin \omega t$

19.132





19.133 SEE FIGURES AT SOLUTION OF PROB. 19.131

(a) DISPLACEMENTS OF MASS AND OF POINT A.

$$m\frac{d^2x_m}{dt^2} + k_2(x_m - x_A) = P_m \sin \omega t$$ ◀

$$c\frac{dx_A}{dt} + k_1 x_A + k_2(x_A - x_m) = 0$$ ◀

(b) ELECTRICAL ANALOGUES.

WE LET: $q_A = \int i_A\,dt$ $\qquad q_m = \int i_m\,dt$

THUS: $i_A = \frac{dq_A}{dt}$ $\qquad i_m = \frac{dq_m}{dt}$

$$L\frac{d^2q_m}{dt^2} + \frac{(q_m - q_A)}{C_2} = E_m \sin \omega t$$ ◀

$$R\frac{dq_A}{dt} + \frac{q_A}{C_1} + \frac{(q_A - q_m)}{C_2} = 0$$ ◀

19.134 SEE FIGURES AT SOLUTION OF PROB 19.132

(a) DISPLACEMENTS OF MASS AND OF POINT A.

$$m\frac{d^2x_m}{dt^2} + k_2 x_m + c\frac{d}{dt}(x_m - x_A) = 0$$ ◀

$$c\frac{d}{dt}(x_A - x_m) + k_1 x_A = 0$$ ◀

(b) ELECTRICAL ANALOGUES.

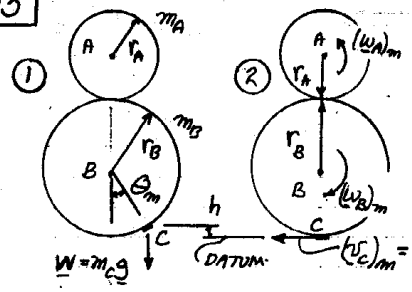
WE LET: $q_A = \int i_A\,dt$ $\qquad q_m = \int i_m\,dt$

THUS: $i_A = \frac{dq_A}{dt}$ $\qquad i_m = \frac{dq_m}{dt}$

$$L\frac{d^2q_m}{dt^2} + \frac{q_m}{C_2} + R\frac{d}{dt}(q_m - q_A) = 0$$ ◀

$$R\frac{d}{dt}(q_A - q_m) + \frac{q_A}{C_1} = 0$$ ◀

19.135



SINCE NO SLIPPING OCCURS:

$r_A(\omega_A)_m = r_B(\omega_B)_m$:

$(\omega_A)_m = \frac{r_B}{r_A}(\omega_B)_m$

FOR SMALL OSCILLATIONS: $h = r_B(1-\cos\theta_m) = \frac{1}{2} r_B \theta_m^2$

POSITION 1: $T_1 = 0$ $\quad V_1 = m_C g h = \frac{1}{2} m_C g r_B \theta_m^2$

POSITION 2: $V_2 = 0$, $\quad T_2 = \frac{1}{2}\bar{I}_A(\omega_A)_m^2 + \frac{1}{2}\bar{I}_B(\omega_B)_m^2 + \frac{1}{2} m_C (v_C)_m^2$

$$T_2 = \frac{1}{2}\left(\frac{1}{2} m_A r_A^2\right)\frac{r_B^2}{r_A^2}(\omega_B)_m^2 + \frac{1}{2}\left(\frac{1}{2} m_B r_B^2\right)(\omega_B)_m^2 + \frac{1}{2} m_C r_B^2 (\omega_B)_m^2$$

$$T_2 = \frac{1}{4}(m_A + m_B + 2m_C) r_B^2 (\omega_B)_m^2$$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{2} m_C g r_B \theta_m^2 = \frac{1}{4}(m_A + m_B + 2m_C) r_B^2 (\omega_B)_m^2$

BUT FOR SHM $(\omega_B)_m = p\theta_m$: $\quad \frac{1}{2} m_C g r_B \theta_m^2 = \frac{1}{4}(m_A + m_B + 2m_C) r_B^2 (p\theta_m)^2$

$$p^2 = \frac{2m_C}{m_A + m_B + 2m_C}\frac{g}{r_B}$$

NOTE: RESULT IS INDEPENDENT OF r_A.

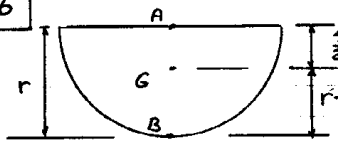
DATA: $m_A = 5$ kg, $m_B = 14$ kg, $m_C = 1.5$ kg

$r_B = 0.200$ m

$$p^2 = \frac{2(1.5)}{5 + 14 + 2(1.5)}\left(\frac{9.81}{0.200}\right) = 6.689 \qquad p = 2.586 \text{ rad/s}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{2.586} \qquad \tau = 2.435$$

19.136

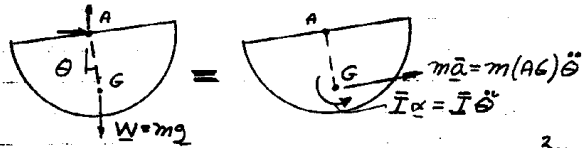


LET m = MASS OF DISK: $\quad I_A = \frac{1}{2} m r^2$

$I_A = \bar{I} + m\left(\frac{4r}{3\pi}\right)^2$; $\quad \frac{1}{2} m r^2 = \bar{I} + m(0.424413\,r)^2$

$$\bar{I} = 0.31987\, m r^2$$

(a) DISK SUSPENDED FROM A:



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad -W(AG)\sin\theta = \bar{I}\ddot{\theta} + m(AG)^2\ddot{\theta}$

$-mg(0.424413\,r)\sin\theta = 0.31987 m r^2 \ddot{\theta} + m(0.424413\,r)^2\ddot{\theta}$

$-\frac{g}{r}(0.424413)\sin\theta = 0.5000\,\ddot{\theta}$

$$\ddot{\theta} + 0.848826\frac{g}{r}\sin\theta = 0$$

(CONTINUED)

19.136 CONTINUED

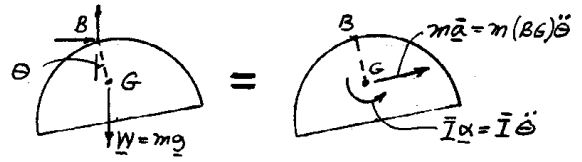
FOR SMALL OSCILLATIONS $\sin\theta \approx \theta$

$\ddot{\theta} + 0.848826\frac{g}{r}\theta = 0$ $\quad p^2 = 0.848826\frac{g}{r}$

$p = \sqrt{0.848826\frac{g}{r}}$ OR $p = \sqrt{\frac{g}{1.1781\,r}}$

$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{1.1781\,r}{g}}$ OR $\tau = 6.82\sqrt{\frac{r}{g}}$ ◀

(b) DISK SUSPENDED FROM B.



$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $\quad -W(BG)\sin\theta = \bar{I}\ddot{\theta} + m(BG)^2\ddot{\theta}$

$-mg(0.575587\,r)\sin\theta = 0.31987 m r^2\ddot{\theta} + m(0.575587\,r)^2\ddot{\theta}$

$-\frac{g}{r}(0.575587)\sin\theta = 0.65117\,\ddot{\theta}$

$$\ddot{\theta} + 0.88393\frac{g}{r}\sin\theta = 0$$

FOR SMALL OSCILLATIONS $\sin\theta \approx \theta$

$\ddot{\theta} + 0.88393\frac{g}{r}\theta = 0$ $\quad p^2 = 0.88393\frac{g}{r}$

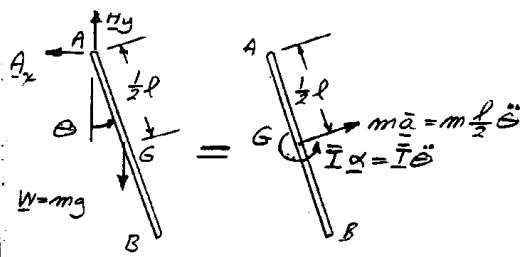
$p = \sqrt{0.88393\frac{g}{r}}$ OR $p = \sqrt{\frac{g}{1.1313\,r}}$

$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{1.1313\,r}{g}}$ OR $\tau = 6.68\sqrt{\frac{r}{g}}$ ◀

19.137

(a) FOR M SUFFICIENT TO PREVENT SLIPPING.

WE HAVE ROTATION ABOUT A.



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad -W\frac{1}{2}l\sin\theta = \bar{I}\ddot{\theta} + m\frac{l}{2}\ddot{\theta}\frac{l}{2}$

$-mg\frac{1}{2}l\sin\theta = \frac{1}{12} m l^2\ddot{\theta} + \frac{1}{4} m l^2\ddot{\theta}$

$$\ddot{\theta} + \frac{3}{2}\frac{g}{l}\sin\theta = 0$$

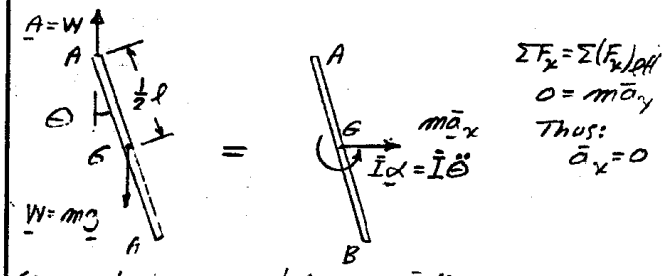
For small oscillations: $\quad \ddot{\theta} + \frac{3}{2}\frac{g}{l}\theta = 0$

$p^2 = \frac{3g}{2l}$ $\qquad p = \sqrt{\frac{3g}{2l}}$

$\tau = \frac{2\pi}{p}$ $\qquad \tau = 2\pi\sqrt{\frac{2l}{3g}}$ ◀

19.137 CONTINUED

(b) FOR $\mu = 0$.
REACTION AT A IS VERTICAL



$\Sigma F_x = \Sigma(F_x)_{eff}$: $0 = m\bar{a}_x$ Thus: $\bar{a}_x = 0$

$\zeta \Sigma M_G = \Sigma(M_G)_{eff}$: $-W\frac{1}{2}\ell\sin\theta = \bar{I}\ddot{\theta}$

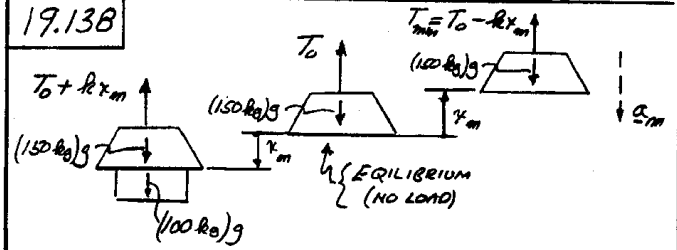
$-mg\frac{1}{2}\ell\sin\theta = \frac{1}{12}m\ell^2\ddot{\theta}$

For $\sin\theta \approx \theta$: $\ddot{\theta} + 6\frac{g}{\ell}\theta = 0$

$p^2 = 6\frac{g}{\ell}$ $\quad p = \sqrt{\frac{6g}{\ell}}$

$\tau = \frac{2\pi}{p}$ $\quad \tau = 2\pi\sqrt{\frac{\ell}{6g}}$ ◀

19.138



(a) FROM FIRST TWO SKETCHES, WE HAVE

$T_0 + kx_m = 250g$ $\quad T_0 = 150g$

THUS: $kx_m = 100g = 981\ N$

BUT: $k = 200\ kN/m$

$(200\ kN/m)x_m = 981\ N$

$x_m = 4.905\times10^{-3}\ m$ $\quad x_m = 4.91\ mm$ ◀

$p^2 = \frac{k}{m} = \frac{200\ kN/m}{150\ kg} = 1333$ $\quad p = 36.51\ rad/s$

$f = \frac{p}{2\pi} = \frac{36.51}{2\pi} = 5.811$ $\quad f = 5.81\ Hz$ ◀

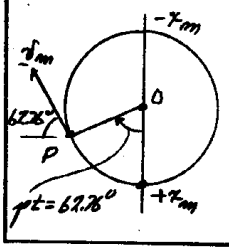
$v_m = x_m p = (4.905\ mm)(36.51\ rad/s)$ $\quad v_m = 179.1\ mm/s$ ◀

(b) FROM LAST SKETCH

$T_{min} = T_0 - kx_m = 150g - 100g = 50g = 50(9.81)$

$T_{min} = 491\ N$ ◀

(c) FOR $t = 0.03\ s$:



$pt = (36.51\ rad/s)(0.03\ s)$
$= 1.0953\ rad$
$= 62.76°$

USING THE AUXILIARY CIRCLE, WE PROJECT v_m ON THE VERTICAL AXIS

$v = v_m \sin pt = (179.1\ mm/s)\sin 62.76°$

$v = 159.2\ mm/s\ \uparrow$ ◀

19.139

$W = 6\ lb$ $\quad k = 450\ lb/in. = 5400\ lb/ft$

$\omega = 1800\ rpm = 188.5\ rad/s$

$x_m = 0.35\ in.$

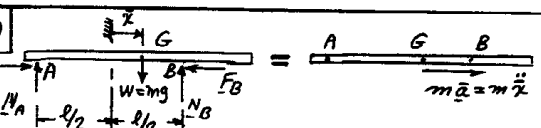
$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{5400\ lb/ft}{(6\ lb)/(32.2\ ft/s^2)}} = 170.235\ rad/s$

EQ(19.38') $\quad x_m = \frac{\delta_m}{1-\left(\frac{\omega}{p}\right)^2}$

$0.35\ in. = \frac{\delta_m}{1-\left(\frac{188.5\ rad/s}{170.235\ rad/s}\right)^2} = \frac{\delta_m}{-0.22603}$

$\delta_m = -0.0791\ in.$ $\quad \delta_m = 0.0791\ in.$ OUT OF PHASE ◀

19.140



$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$: $\ell N_B - (\frac{\ell}{2}+\bar{x})mg = 0$ $\quad N_B = (\frac{1}{2}+\frac{\bar{x}}{\ell})mg$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N_A + (\frac{1}{2}+\frac{\bar{x}}{\ell})mg - mg = 0$ $\quad N_A = (\frac{1}{2}-\frac{\bar{x}}{\ell})mg$

THUS: $F_A = \mu_k N_A = \mu_k(\frac{1}{2}+\frac{\bar{x}}{\ell})mg$

$F_B = \mu_k N_B = \mu_k(\frac{1}{2}-\frac{\bar{x}}{\ell})mg$

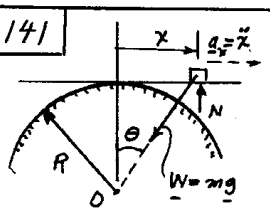
$\rightarrow \Sigma F_x = \Sigma(F_x)_{eff}$: $F_A - F_B = -m\ddot{\bar{x}}$

$-2\mu_k\frac{\bar{x}}{\ell}mg = m\ddot{\bar{x}}$

$\ddot{\bar{x}} + \frac{2\mu_k g}{\ell}\bar{x} = 0$

$p^2 = \frac{2\mu_k g}{\ell}$ $\quad f = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2\mu_k g}{\ell}}$ ◀

19.141



(a) $+\rightarrow \Sigma F = ma_x$

$-mg\sin\theta = m\ddot{x}$

BUT $\sin\theta \approx \tan\theta = \frac{x}{R}$

$\ddot{x} + \frac{g}{R}x = 0$

THUS: $p^2 = \frac{g}{R}$ $\quad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{R}{g}}$

COMPARING WITH EQ. (19.18), WE CONCLUDE THAT THE PERIOD OF OSCILLATION IS THAT OF A SIMPLE PENDULUM OF LENGTH $\ell = R$ (Q.E.D.)

(b) IN SI UNITS:

$\tau = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.37\times10^6\ m}{9.81\ m/s^2}} = 5063\ s$

OR IN U.S. CUSTOMARY UNITS

$\tau = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{(3960\times5280)\ ft}{32.2\ ft/s^2}} = 5063\ s$

$\tau = 84.4\ min$

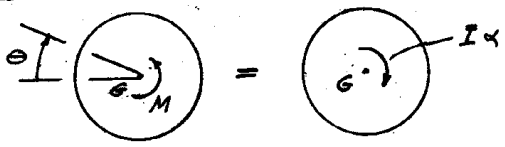
FROM EQ(12.44):

$v_{circ} = \sqrt{\frac{gR^2}{r_0}} = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$

$\tau = \frac{2\pi R}{v_{circ}} = \frac{2\pi R}{\sqrt{gR}} = 2\pi\sqrt{\frac{R}{g}}$

SAME τ AS FOUND IN PART a, Q.E.D.

19.142



SPRING CONSTANT OF WIRE: $K = 0.40$ lb·in./rad

$K = \frac{0.40}{12}$ lb·ft/rad

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad -M = \bar{I}\alpha$

$-K\theta = \bar{I}\ddot{\theta}$

$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$

$p^2 = \frac{K}{\bar{I}} \qquad \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\bar{I}}{K}}$

FOR $\tau = 30$ s AND $K = \frac{0.40}{12}$ lb·ft/rad

$30\text{ s} = 2\pi\sqrt{\frac{\bar{I}}{\frac{0.40}{12}\text{ lb·ft/rad}}} \qquad \bar{I} = 0.760\text{ lb·ft·s}^2$ ◀

WEIGHT: $W = 4$ lb

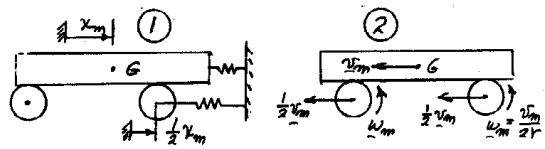
$\bar{I} = m\bar{k}^2$; $\quad 0.760\text{ lb·ft·s}^2 = \frac{4\text{ lb}}{32.2\text{ ft/s}^2}\bar{k}^2$

$\bar{k}^2 = 0.5207 \qquad \bar{k} = 0.7216\text{ ft} \qquad \bar{k} = 8.66\text{ in}$ ◀

19.143

$m_{BEAM} = 15$ kg

$m_{DISK} = 8$ kg



POSITION 1: (MAX. DEFLECTION) $\quad T_1 = 0$

$V_1 = \frac{1}{2}kx_m^2 + \frac{1}{2}k\left(\frac{1}{2}x_m\right)^2 = \frac{1}{2}\frac{5}{4}kx_m^2$

POSITION 2: (MAX. VELOCITY) $\quad V_2 = 0$

$T_2 = \frac{1}{2}(15\text{ kg})v_m^2 + 2\left[\frac{1}{2}(8\text{ kg})\left(\frac{v_m}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}8r^2\right)\left(\frac{v_m}{2r}\right)^2\right] = \frac{1}{2}21v_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{2}\frac{5}{4}kx_m^2 = \frac{1}{2}21v_m^2$

SETTING $v_m = px_m$ AND $k = 300$ N/m

$\frac{1}{2}\frac{5}{4}(300)x_m^2 = \frac{1}{2}(21)(px_m)^2$

$p^2 = 17.857, \qquad p = 4.226$

$\tau = \frac{2\pi}{p} = \frac{2\pi}{4.226} \qquad \tau = 1.487$ s ◀

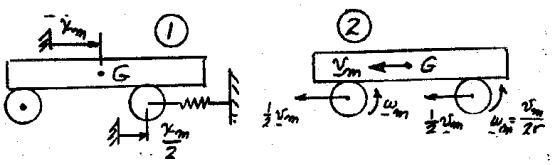
WE NOTE THAT RESULTS ARE INDEPENDENT OF THE RADII OF THE DISKS.

19.144

$m_{BEAM} = 15$ kg

$m_{DISK} = 8$ kg

WE REMOVE THE SPRING ATTACHED TO THE BEAM



POSITION 1: (MAX. DEFLECTION) $\quad T_1 = 0$

$V_1 = \frac{1}{2}k\left(\frac{x_m}{2}\right)^2 = \frac{1}{8}kx_m^2$

POSITION 2: (MAX. VELOCITY) $\quad V_2 = 0$

$T_2 = \frac{1}{2}(15\text{ kg})v_m^2 + 2\left[\frac{1}{2}(8\text{ kg})\left(\frac{v_m}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}8r^2\right)\left(\frac{v_m}{2r}\right)^2\right] = \frac{1}{2}21v_m^2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{8}kx_m^2 = \frac{1}{2}21v_m^2$

SETTING $v_m = px_m$ AND $k = 300$ N/m

$\frac{1}{8}(300)x_m^2 = \frac{1}{2}21(px_m)^2$

$p^2 = 3.571 \qquad p = 1.890 \qquad \tau = \frac{2\pi}{p} = \frac{2\pi}{1.890} = 3.32$ s ◀

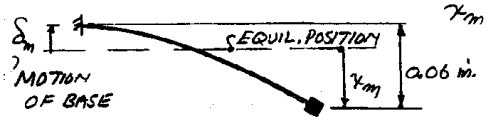
19.145

WE HAVE: $\omega = 600$ rpm, $p = (5\text{ Hz})(60) = 300$ rpm

SINCE $\omega > p$, MOTION IS OUT OF PHASE, $x_m < 0$.

EQ(19.33'): $\quad -x_m = \frac{\delta_m}{1-\left(\frac{\omega}{p}\right)^2} = \frac{\delta_m}{1-\left(\frac{600}{300}\right)^2} = \frac{\delta_m}{1-4} = -\frac{\delta_m}{3}$

$x_m = \frac{1}{3}\delta_m \quad (1)$

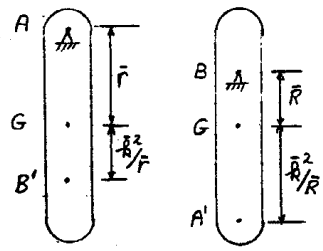


$\delta_m + x_m = 0.06$ in.; $\quad \delta_m + \frac{1}{3}\delta_m = 0.06$ in. $\qquad \delta_m = 0.045$ in. ◀

19.146

FROM PROB. 19.38 OR PROB. 19.40 THE LENGTH OF AN EQUIVALENT SIMPLE PENDULUM IS:

$l_A = \bar{r} + \frac{\bar{k}^2}{\bar{r}}$ AND $l_B = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$.



BUT: $\tau_A = \tau_B$

$2\pi\sqrt{\frac{g}{l_A}} = 2\pi\sqrt{\frac{g}{l_B}}$

THUS: $l_A = l_B$

FOR $l_A = l_B$: $\quad \bar{r} + \frac{\bar{k}^2}{\bar{r}} = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$

$\bar{r}^2\bar{R} + \bar{k}^2\bar{R} = \bar{r}\bar{R}^2 + \bar{k}^2\bar{r}$

$\bar{r}\bar{R}(\bar{r} - \bar{R}) = \bar{k}^2(\bar{r} - \bar{R})$

SINCE $(\bar{r} - \bar{R}) \neq 0$, WE HAVE $\quad \bar{r}\bar{R} = \bar{k}^2$

OR $\quad \bar{r} = \bar{k}^2/\bar{R}$ AND $\bar{R} = \bar{k}^2/\bar{r}$

THUS: $AG = GA'$ AND $BG = GB'$. THAT IS, $A = A'$ AND $B = B'$

NOTING THAT $l_A = l_B = l$

$\tau = 2\pi\sqrt{\frac{l}{g}}$ OR $g = \frac{4\pi^2 l}{\tau^2}$ ◀

PROBLEM 19.C1

AMPLITUDE $= \theta_m$ $\qquad c = \frac{1}{2}\sin\theta_m$

LET $T = 2\pi\sqrt{\frac{L}{g}}\,[B]$ WHERE $B = \left[1 + \left(\frac{1}{2}\right)^2 c^2 + \left(\frac{1\times3}{2\times4}\right)^2 c^4 + \left(\frac{1\times3\times5}{2\times4\times6}\right)^2 c^6 + \cdots\right]$

WE MAY COMPUTE B AS FOLLOWS:

$n = 1$: $\quad B = \left[1 + \left(\frac{2n-1}{2n}c\right)^2\right]$

$n = 2$: $\quad B = \left[1 + \quad + \left(\quad\right)\left(\frac{2n-1}{2n}c\right)^2\right]$

$n = 3$: $\quad B = \left[1 + \quad + \quad + \left(\quad\right)\left(\frac{2n-1}{2n}c\right)^2\right]$

AT EACH STEP THE QUANITY ABOVE THE ⟶ IS THE CHANGE IN B AND IS DENOTED BY "DELTA"

AND THE QUANITY $\left(\frac{2n}{2n-1}c\right)$ IS DENOTED BY $\quad FACTOR = \frac{2n-1}{2n}c$

```
10 '*****************************************************************
20 '*'                    PROBLEM 19.C1                              *
30 '*                     BASIC VERSION                              *
40 '*****************************************************************
50 '
60 '           Definition of Symbols
70 '
80 '         TH = amplitude of oscillations  (degrees)
90 '          C = sin(TH/2)
100 '         B = sum of series in bracket
110 '         N = number of terms included in bracket
120 '     DELTA = change in B as number of terms is increased
130 '
140      PI = 4*ATN(1)
150      K = PI/180
160 '
170      PRINT "--------------------------------"
180      FOR TH = 30 TO 120 STEP 30
190        PRINT USING "    Amplitude = ### degrees";TH:PRINT
200        B = 1
210        DELTA = 1
220        C = (SIN(K*TH/2))
230        PRINT "        N        Bracket    ";:PRINT
240        I = 1
250        FOR N = 1 TO 20 STEP 1
260           FACTOR = ((2*N - 1)/(2*N))*C
270           DELTA = DELTA*FACTOR^2
280           B = B + DELTA
290           IF N < I GOTO 330
300           PRINT USING "       ##       ";N,
310           PRINT USING " #.#####  ";B
320           I = I + I - .0001
330        NEXT
340        PRINT "--------------------------------"
350     NEXT
```

```
C*****************************************************************
C*                    PROBLEM 19.C1                              *
C*                   FORTRAN VERSION                             *
C*****************************************************************
C
C           Definition of Symbols
C
C         TH = amplitude of oscillations   (degrees)
C          C = sin(TH/2)
C          B = sum of series in bracket
C          N = number of terms included in bracket
C      DELTA = change in B as number of terms is increased
C
      INTEGER TH
      REAL K, I
      PI = 4 * ATAN(1.0)
      K = PI / 180.0
C
      WRITE(*,10)
 10   FORMAT('--------------------------------')
      DO 100 TH=30,120,30
         WRITE(*,20) TH
 20      FORMAT('    Amplitude = ',I3,' degrees'/)
         B = 1.0
         DELTA = 1.0
         C = SIN(K*TH/2)
         I = 1.0
         WRITE(*,30)
 30      FORMAT('        N        Bracket',/)
         DO 80 N=1,20,1
            FACTOR = ((2.0*N - 1.0)/(2.0*N))*C
            DELTA = DELTA * FACTOR**2
            B = B + DELTA
            IF (N.LT.I) GOTO 80
               WRITE(*,40) N,B
 40            FORMAT(7X,I2,7X,F7.5)
               I = I + I - 0.0001
 80      CONTINUE
         WRITE(*,10)
 100  CONTINUE
      END
```

SEE NEXT PAGE FOR
REPEAT PRINTING OF PROGRAMS AND FOR ANSWERS

PROBLEM 19.C1 CONTINUED

```
10 '*****************************************************************
20 '*'                      PROBLEM 19.C1                            *
30 '*                       BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         TH = amplitude of oscillations  (degrees)
90 '          C = sin(TH/2)
100 '         B = sum of series in bracket
110 '         N = number of terms included in bracket
120 '     DELTA = change in B as number of terms is increased
130 '
140       PI = 4*ATN(1)
150       K = PI/180
160 '
170     LPRINT "-----------------------------------"
180      FOR TH = 30 TO 120 STEP 30
190       LPRINT USING "    Amplitude = ### degrees";TH:LPRINT
200         B = 1
210         DELTA = 1
220         C = (SIN(K*TH/2))
230       LPRINT "         N         Bracket     ":LPRINT
240         I = 1
250         FOR N = 1 TO 20 STEP 1
260            FACTOR = ((2*N - 1)/(2*N))*C
270            DELTA = DELTA*FACTOR^2
280            B = B + DELTA
290            IF N < I GOTO 330
300           LPRINT USING "        ##        ";N,
310           LPRINT USING " #.######  ";B
320            I = I + I - .0001
330         NEXT
340        LPRINT "-----------------------------------"
350     NEXT
```

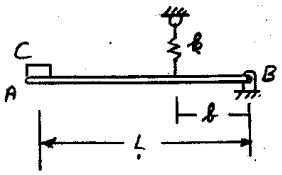
```
-----------------------------------
   Amplitude =  30 degrees

        N         Bracket

        1         1.01675
        2         1.01738
        4         1.01741
        8         1.01741
       16         1.01741
-----------------------------------
   Amplitude =  60 degrees

        N         Bracket

        1         1.06250
        2         1.07129
        4         1.07311
        8         1.07318
       16         1.07318
-----------------------------------
   Amplitude =  90 degrees

        N         Bracket

        1         1.12500
        2         1.16016
        4         1.17704
        8         1.18022
       16         1.18034
-----------------------------------
   Amplitude = 120 degrees

        N         Bracket

        1         1.18750
        2         1.26660
        4         1.33146
        8         1.36460
       16         1.37240
-----------------------------------
```

```
C*****************************************************************
C*                      PROBLEM 19.C1                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C           Definition of Symbols
C
C        TH = amplitude of oscillations   (degrees)
C         C = sin(TH/2)
C         B = sum of series in bracket
C         N = number of terms included in bracket
C     DELTA = change in B as number of terms is increased
C
      INTEGER TH
      REAL K, I
      PI = 4 * ATAN(1.0)
      K = PI / 180.0
C
      WRITE(*,10)
 10   FORMAT('-----------------------------------')
      DO 100 TH=30,120,30
        WRITE(*,20) TH
 20     FORMAT('    Amplitude = ',I3,' degrees'/)
        B = 1.0
        DELTA = 1.0
        C = SIN(K*TH/2)
        I = 1.0
        WRITE(*,30)
 30     FORMAT('         N         Bracket',/)
        DO 80 N=1,20,1
          FACTOR = ((2.0*N - 1.0)/(2.0*N))*C
          DELTA = DELTA * FACTOR**2
          B = B + DELTA
          IF (N.LT.I) GOTO 80
            WRITE(*,40) N,B
 40         FORMAT(7X,I2,7X,F7.5)
            I = I + I - 0.0001
 80     CONTINUE
        WRITE(*,10)
 100  CONTINUE
      END
```

```
-----------------------------------
   Amplitude =  30 degrees

        N         Bracket

        1         1.01675
        2         1.01738
        4         1.01741
        8         1.01741
       16         1.01741
-----------------------------------
   Amplitude =  60 degrees

        N         Bracket

        1         1.06250
        2         1.07129
        4         1.07311
        8         1.07318
       16         1.07318
-----------------------------------
   Amplitude =  90 degrees

        N         Bracket

        1         1.12500
        2         1.16016
        4         1.17704
        8         1.18022
       16         1.18034
-----------------------------------
   Amplitude = 120 degrees

        N         Bracket

        1         1.18750
        2         1.26660
        4         1.33146
        8         1.36460
       16         1.37240
-----------------------------------
```

Problem 19.C2

From Solution of Prob. 19.34



CIRCULAR FREQUENCY: $p = \frac{k}{L}\sqrt{\frac{b}{m_c + m/3}}$ ◁ PERIOD: $\tau = \frac{2\pi}{p}$ ◁

For amplitude of vibration of block C equal to $\delta_0 = y_{max}$, maximum acceleration is $a_{max} = y_{max} p^2$. Since block will leave beam if $a_{max} > g$, allowable value of y_{max} is: $y_{max} = \frac{g}{p^2}$ ◁

```
10 '*****************************************************************
20 '*                      PROBLEM 19.C2                            *
30 '*                      BASIC VERSION                            *
40 '*****************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         L = length of rod AB    (meters)
90 '         B = distance B          (meters)
100 '        M = mass of rod AB      (kg)
110 '        K = spring constant     (N/m)
120 '       MC = mass of block C     (kg)
130 '        P = circular frequency of vibration    (rad/s)
140 '      TAU = period of vibration    (seconds)
150 '     YMAX = maximum allowable displacement of end A  (mm)
160 '
170     LET L = 1.25     'meters
180     LET M = 3        'kg
190     LET K = 900      'N/m
200     LET G = 9.810001       'm/s^2
210     PI = 4*ATN(1)
220 '
230     DATA 0.5,0.75,1,0
240     READ B
250     IF B = 0 GOTO 420
260     PRINT "-----------------------------------------"
270     PRINT USING " Distance B = #.### m ";B:PRINT
280     PRINT " Mass of        Period of        Maximum
290     PRINT " block C        vibration        amplitude
300     PRINT "  (kg)             (s)             (mm)
310     PRINT
320     FOR MC = 0 TO 2 STEP .5
330        P = (B/L)*SQR(K/(MC + M/3))
340        TAU = 2*PI/P
350        YMAX = G/P^2
360        YMAX = YMAX*1000      'express YMAX in millimeters
370        PRINT USING "  #.#          ";MC,
380        PRINT USING "  #.####       ";TAU,
390        PRINT USING "  ###.##       ";YMAX
400     NEXT
410     GOTO 240
420     END
```

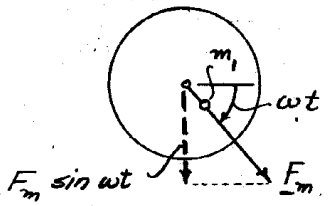
```
-----------------------------------------

Distance B = 0.500 m

Mass of        Period of        Maximum
block C        vibration        amplitude
 (kg)             (s)             (mm)

 0.0            0.5236           68.13
 0.5            0.6413           102.19
 1.0            0.7405           136.25
 1.5            0.8279           170.31
 2.0            0.9069           204.38
-----------------------------------------

Distance B = 0.750 m

Mass of        Period of        Maximum
block C        vibration        amplitude
 (kg)             (s)             (mm)

 0.0            0.3491           30.28
 0.5            0.4275           45.42
 1.0            0.4937           60.56
 1.5            0.5519           75.69
 2.0            0.6046           90.83
-----------------------------------------

Distance B = 1.000 m

Mass of        Period of        Maximum
block C        vibration        amplitude
 (kg)             (s)             (mm)

 0.0            0.2618           17.03
 0.5            0.3206           25.55
 1.0            0.3702           34.06
 1.5            0.4139           42.58
 2.0            0.4534           51.09
```

```
C*****************************************************************
C*                      PROBLEM 19.C2                            *
C*                      FORTRAN VERSION                          *
C*****************************************************************
C
C            Definition of Symbols
C
C              L = length of rod AB    (meters)
C              B = distance B          (meters)
C              M = mass of rod AB      (kg)
C              K = spring constant     (N/m)
C             MC = mass of block C     (kg)
C              P = circular frequency of vibration    (rad/s)
C            TAU = period of vibration    (seconds)
C           YMAX = maximum allowable displacement of end A   (mm)
C
      REAL  B(4), L, M, K, MC
      DATA B/0.5,0.75,1.0,0.0/
      L = 1.25
      M = 3.0
      K = 900.0
      G = 9.81
      I = 1
      PI = 4.0 * ATAN(1.0)
C
 10   CONTINUE
      WRITE(*,*) '-----------------------------------------'
         WRITE(*,20) B(I)
 20      FORMAT(1X,'Distance B = ',F5.3,' m ',/)
         WRITE(*,*) ' Mass of        Period of        Maximum'
         WRITE(*,*) ' block C        vibration        amplitude'
         WRITE(*,*) '  (kg)             (s)             (mm)'
      WRITE (*,*) ' '
         DO 50 J=0,4
            MC = J * 0.5
            P = (B(I))/L)*SQRT(K/(MC + M/3))
            TAU = 2.0*PI/P
            YMAX = (G/P**2)*1000
            WRITE(*,40) MC,TAU,YMAX
 40         FORMAT(2X,F3.1,11X,F6.4,10X,F6.2)
 50      CONTINUE
         I = I + 1
         IF (B(I).GT.0) GOTO 10
      END
```

```
-----------------------------------------

Distance B =  .500 m

 Mass of        Period of        Maximum
 block C        vibration        amplitude
  (kg)             (s)             (mm)

  .0             .5236           68.12
  .5             .6413           102.19
 1.0             .7405           136.25
 1.5             .8279           170.31
 2.0             .9069           204.38
-----------------------------------------

Distance B =  .750 m

 Mass of        Period of        Maximum
 block C        vibration        amplitude
  (kg)             (s)             (mm)

  .0             .3491           30.28
  .5             .4275           45.42
 1.0             .4937           60.56
 1.5             .5519           75.69
 2.0             .6046           90.83
-----------------------------------------

Distance B = 1.000 m

 Mass of        Period of        Maximum
 block C        vibration        amplitude
  (kg)             (s)             (mm)

  .0             .2618           17.03
  .5             .3206           25.55
 1.0             .3702           34.06
 1.5             .4139           42.58
 2.0             .4534           51.09
```

PROBLEM 19.C3

SEE SAMPLE PROB 19.5:



$$F_m = m_1 r \omega^2 = \left(\frac{W_1}{g}\right) r \omega^2 \quad \triangleleft \qquad p = \sqrt{k/m} \quad \triangleleft$$

$$x_{max} = \frac{F_m/k}{1-(\omega/p)^2} \qquad \text{LET: } DENOM = 1-(\omega/p)^2 \quad \triangleleft$$

$$x_{max} = \frac{F_m/k}{DENOM} \quad \triangleleft$$

$$a_{max} = x_{max}\,\omega^2 \quad \triangleleft$$

(NOTE: THE FORCE DUE TO THE UNBALANCE IS DENOTED BY F_m IN THE ABOVE DERIVATION AND BY F IN THE FOLLOWING PROGRAMS)

```
10 '*************************************************************
20 '*                      PROBLEM 19.C3                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '            Definition of Symbols
70 '
80 '         W = weight of motor    (lb)
90 '         K = spring constant    (lb/ft)
100 '        P = circular frequency of free vibration  (rad/s)
110 '
120 '       W1 = magnitude of equivalent unbalance  (lb)
130 '        R = distance of W1 from axis of rotation  (ft)
140 '    OMEGA = speed of motor    (rpm)
150 '        F = magnitude of centrifugal force due to unbalance (lb)
160 '     XMAX = amplitude of vibration  (ft)
170 '     AMAX = maximum acceleration during vibration  (ft/s^2)
180 '
190 '       Data
200     LET  W = 350      'lb
210     LET  G = 32.2     'ft/s^2
220     LET  K = 750      'lb/in., for one spring
230          K = 4*K*12   'express constant of 4 springs in lb/ft
240     LET W1 = 1        'oz
250          W1 = W1/16   'express unbalance in pounds
260     LET  R = 6        'in.
270          R = R/12     'express R in feet
280 '
290     PI = 4*ATN(1)
300     J = 2*PI/60       'factor to change rpm into rad/s
310 '
320     M = W/G           'mass of motor
330     P = SQR(K/M)
340 '
350     PRINT " OMEGA              XMAX              AMAX
360     PRINT "  rpm               inches            ft/s^2
370     PRINT
380 '
390     FOR OMEGA = 300 TO 1200  STEP 50
400         F = (W1/G)*R*(J*OMEGA)^2
410         DENOM = 1 - (J*OMEGA/P)^2
420         XMAX = (F/K)/DENOM
430         AMAX = XMAX*(J*OMEGA)^2
440         XMAX = XMAX*12    'express XMAX in inches for printing
450         PRINT USING " ####           ";OMEGA,
460         PRINT USING " ##.######      ";XMAX,
470         PRINT USING " ####.####      ";AMAX
480     NEXT
```

OMEGA rpm	XMAX inches	AMAX ft/s^2
300	0.000455	0.0374
350	0.000731	0.0818
400	0.001207	0.1765
450	0.002180	0.4034
500	0.005149	1.1764
550	-0.672654	-185.9488
600	-0.006652	-2.1885
650	-0.003757	-1.4507
700	-0.002793	-1.2506
750	-0.002314	-1.1893
800	-0.002029	-1.1866
850	-0.001841	-1.2155
900	-0.001708	-1.2646
950	-0.001610	-1.3281
1000	-0.001535	-1.4028
1050	-0.001476	-1.4868
1100	-0.001428	-1.5788
1150	-0.001389	-1.6781
1200	-0.001356	-1.7841

```
C*************************************************************
C*                      PROBLEM 19.C3                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C           Definition of Symbols
C
C           W = weight of motor    (lb)
C           K = spring constant    (lb/ft)
C           P = circular frequency of free vibration   (rad/s)
C
C          W1 = magnitude of equivalent unbalance   (lb)
C           R = distance of W1 from axis of rotation   (ft)
C       OMEGA = speed of motor     (rpm)
C           F = magnitude of centrifugal force due to unbalance  (lb)
C        XMAX = amplitude of vibration   (ft)
C        AMAX = maximum acceleration during vibration   (ft/s**2)
C
      REAL  W, G, K, W1, R, PI, J, M, P, DENOM, XMAX, AMAX, F
      INTEGER  OMEGA
C
C      Data
      W = 350.0
      G = 32.2
      K = 750.0
      K = 4*K*12
      W1 = 1.0
      W1 = W1/16.0
      R = 6.0
      R = R/12
C
      PI = 4.0*ATAN(1.0)
      J = 2*PI/60
C
      M = W/G
      P = SQRT(K/M)
C
      WRITE(*,*) ' OMEGA              XMAX              AMAX'
      WRITE(*,*) '  rpm               inches            ft/s**2'
      WRITE(*,*) ' '
C
      DO 100 OMEGA=300,1200,50
         F = (W1/G)*R*(J*OMEGA)**2
         DENOM = 1 - (J*OMEGA/P)**2
         XMAX = (F/K)/DENOM
         AMAX = XMAX*(J*OMEGA)**2
         XMAX = XMAX*12
         WRITE(*,50) OMEGA,XMAX,AMAX
 50      FORMAT(2X,I4,12X,F9.6,7X,F9.4)
 100  CONTINUE
      END
```

OMEGA rpm	XMAX inches	AMAX ft/s**2
300	.000455	.0374
350	.000731	.0818
400	.001207	.1765
450	.002180	.4034
500	.005149	1.1764
550	-.672654	-185.9488
600	-.006652	-2.1885
650	-.003757	-1.4507
700	-.002793	-1.2506
750	-.002314	-1.1893
800	-.002029	-1.1866
850	-.001841	-1.2155
900	-.001708	-1.2646
950	-.001610	-1.3281
1000	-.001535	-1.4028
1050	-.001476	-1.4868
1100	-.001428	-1.5788
1150	-.001389	-1.6781
1200	-.001356	-1.7841

```
10 '*************************************************************
20 '*                      PROBLEM 19.C4                        *
30 '*                      BASIC VERSION                        *
40 '*************************************************************
50 '
60 '           Definition of Symbols
70 '
80 '         W = weight of motor  (lb)
90 '         K = spring constant  (lb/ft)
100 '        P = circular frequency of free vibration  (rad/s)
110 '
120 '       W1 = magnitude of equivalent unbalance  (lb)
130 '        R = distance of W1 from axis of rotation  (ft)
140 '    OMEGA = speed of motor  (rpm)
150 '        F = magnitude of centrifugal force due to unbalance  (lb)
160 '     XMAX = amplitude of vibration  (ft)
170 '     AMAX = maximum acceleration during vibration  (ft/s**2)
180 '      PHI = phase difference  (degrees)
190 '        C = coefficient of viscous damping  (lb.s/ft)
200 '       CC = critical damping coefficient  (lb.s/ft)
210 '
220 '      Data
230     LET  W = 350      'lb
240     LET  G = 32.2     'ft/s^2
250     LET  K = 750      'lb/in., for one spring
260          K = 4*K*12   'express constant of 4 springs in lb/ft
270     LET W1 = 1        'oz
280          W1 = W1/16   'express unbalance in pounds
290     LET  R = 6        'in.
300          R = R/12     'express R in feet
310     LET  C = 75       'lb.s/ft
320 '
330     PI = 4*ATN(1)
340     J = 2*PI/60       'factor to change rpm into rad/s
350 '
360     M = W/G           'mass of motor
370     P = SQR(K/M)
380     CC = 2*M*P
390 '
400     PRINT " OMEGA            XMAX           AMAX          PHI
410     PRINT "  rpm            inches         ft/s^2       degrees
420     PRINT
430 '
440     FOR OMEGA = 300 TO 1200  STEP 50
450         F = (W1/G)*R*(J*OMEGA)^2
460         DENOM = (1 - (J*OMEGA/P)^2)
470         PHI = (ATN(2*(C/CC)*(J*OMEGA/P)/DENOM))/(PI/180)
480         XMAX = ((F/K)/DENOM)*COS((PI/180)*PHI)    'see note below
490         AMAX = (XMAX)*(J*OMEGA)^2
500         XMAX = XMAX*12    'express XMAX in inches for printing
510         PRINT USING " ####          ";OMEGA,
520         PRINT USING " ##.######     ";XMAX,
530         PRINT USING "  ##.####      ";AMAX,
540         PRINT USING "  ###.###";PHI
550     NEXT
```

OMEGA rpm	XMAX inches	AMAX ft/s^2	PHI degrees
300	0.000453	0.0372	5.326
350	0.000725	0.0812	7.320
400	0.001187	0.1735	10.513
450	0.002089	0.3867	16.591
500	0.004350	0.9939	32.348
550	-0.008943	-2.4721	-89.238
600	-0.005496	-1.8082	-34.288
650	-0.003540	-1.3669	-19.569
700	-0.002712	-1.2146	-13.786
750	-0.002273	-1.1685	-10.742
800	-0.002005	-1.1724	-8.865
850	-0.001825	-1.2049	-7.587
900	-0.001697	-1.2561	-6.658
950	-0.001602	-1.3210	-5.951
1000	-0.001528	-1.3966	-5.393
1050	-0.001470	-1.4813	-4.940
1100	-0.001423	-1.5738	-4.564
1150	-0.001385	-1.6735	-4.247
1200	-0.001353	-1.7798	-3.975

```
C*************************************************************
C*                      PROBLEM 19.C4                        *
C*                      FORTRAN VERSION                      *
C*************************************************************
C
C          Definition of Symbols
C
C          W = weight of motor  (lb)
C          K = spring constant  (lb/ft)
C          P = circular frequency of free vibration  (rad/s)
C
C         W1 = magnitude of equivalent unbalance  (lb)
C          R = distance of W1 from axis of rotation  (ft)
C      OMEGA = speed of motor  (rpm)
C          F = magnitude of centrifugal force due to unbalance  (lb)
C       XMAX = amplitude of vibration  (ft)
C       AMAX = maximum acceleration during vibration  (ft/s^2)
C        PHI = phase difference  (degrees)
C          C = coefficient of viscous damping  (lb.s/ft)
C         CC = critical damping coefficient  (lb.s/ft)
C
      REAL  W, G, K, W1, R, PI, J, M, P, DENOM, XMAX, AMAX, F, C, CC
      INTEGER  OMEGA
C
C      Data
      W = 350.0
      G = 32.2
      K = 750.0
      K = 4*K*12
      W1 = 1.0
      W1 = W1/16
      R = 6.0
      R = R/12
      C = 75.0
C
      PI = 4.0*ATAN(1.0)
      J = 2*PI/60
C
      M = W/G
      P = SQRT(K/M)
      CC = 2*M*P
C
      WRITE(*,10)
 10   FORMAT(' OMEGA              XMAX              AMAX',
     +     '              PHI')
      WRITE(*,20)
 20   FORMAT('  rpm               inches            ft/s**2',
     +     '          degrees')
      WRITE(*,*) ' '
C
      DO 100 OMEGA=300,1200,50
         F = (W1/G)*R*(J*OMEGA)**2
         DENOM = 1 - (J*OMEGA/P)**2
         PHI = (ATAN(2*(C/CC)*(J*OMEGA/P)/DENOM))/(PI/180)
         XMAX = ((F/K)/DENOM)*COS((PI/180)*PHI)      (See note below)
         AMAX = XMAX*(J*OMEGA)**2
         XMAX = XMAX*12
         WRITE(*,50) OMEGA,XMAX,AMAX,PHI
 50      FORMAT(2X,I4,12X,F9.6,7X,F7.4,7X,F7.3)
 100  CONTINUE
      END
```

OMEGA rpm	XMAX inches	AMAX ft/s**2	PHI degrees
300	.000453	.0372	5.326
350	.000725	.0812	7.320
400	.001187	.1735	10.513
450	.002089	.3867	16.591
500	.004350	.9939	32.348
550	-.008943	-2.4721	-89.238
600	-.005496	-1.8082	-34.288
650	-.003540	-1.3669	-19.569
700	-.002712	-1.2146	-13.786
750	-.002273	-1.1685	-10.742
800	-.002005	-1.1724	-8.865
850	-.001825	-1.2049	-7.587
900	-.001697	-1.2561	-6.658
950	-.001602	-1.3210	-5.951
1000	-.001528	-1.3966	-5.393
1050	-.001470	-1.4813	-4.940
1100	-.001423	-1.5738	-4.564
1150	-.001385	-1.6735	-4.247
1200	-.001353	-1.7798	-3.975

NOTE: THIS EQUATION IS OBTAINED FROM Eq. (19.52) AS FOLLOWS:

$$x_m = \frac{F_m/k}{\sqrt{\left[1-(\omega/p)^2\right]^2+\left[2(c/c_c)(\omega/p)\right]^2}} = \frac{F_m/k}{1-(\omega/p)^2}\cdot\frac{1}{\sqrt{1+\left[\frac{2(c/c_c)(\omega/p)}{1-(\omega/p)^2}\right]^2}}$$

OR, RECALLING FROM Eq. (19.53) THAT $\tan\varphi = \frac{2(c/c_c)(\omega/p)}{1-(\omega/p)^2}$; $x_m = \frac{F_m/k}{1-(\omega/p)^2}\cdot\frac{1}{\sqrt{1+\tan^2\varphi}} = \frac{(F_m/k)\cos\varphi}{1-(\omega/p)^2}$ ◁